

Half proof of Collatz problem

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Abstract

We prove Collatz sequence has not general m-cycle. Already proved result is that there is no less than 68-cycle. We can not prove the possibility Collatz sequence goes to infinity.

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In this paper, we prove partial result of Collatz problem. This problem is bred by Lothar Collatz at 1937. For 80 years, proved best result is 68-cycle case. We prove general m-cycle case.

Collatz problem

$$\begin{cases} n \text{ is even number} & \Rightarrow \text{divides } 2 \\ n \text{ is odd number} & \Rightarrow \text{times } 3 \text{ plus } 1 \end{cases}$$

repeat this process, this sequence reaches to 1

Next result is known already.

Theorem 1.1. *(68-cycle)*

Except trivial case, some number does not go to same number in the case less than 68 times increase from odd number less than 68 times decrease from even number.

example:

$$43 \rightarrow 130 \rightarrow 65 \rightarrow 196 \rightarrow 49 \rightarrow 148 \rightarrow 37 \rightarrow 112 \rightarrow 7 \rightarrow 22 \rightarrow 11 \\ \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 13 \rightarrow$$

This sequence has possibility to be the part of 8 or more cycle.

Theorem 1.2. *(Half proof of Collatz problem)*

Except trivial case, In the Collatz sequence, some number does not go to same number

Corollary 1.1. *Collatz sequence is gradually increase and goes to infinity or gradually decrease and goes to 1.*

proof. We prove the theorem. We assume next formula.

$$\left(\frac{1}{2}\right)^n \left(\frac{3}{2}\right)^m N + \alpha = M \\ \alpha = \left(\frac{1}{2}\right)^{n_1} + \left(\frac{1}{2}\right)^{n_2} \left(\frac{3}{2}\right) + \left(\frac{1}{2}\right)^{n_3} \left(\frac{3}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^{n_{m-1}} \left(\frac{3}{2}\right)^{m-1} \\ n_1 \leq n_2 \leq n_3 \leq \dots \leq n_{m-1}$$

M is the number the $(m - 1)$ th odd number from N .

example.

$N = 43, M = 13$ case.

$$M(= 13) = \left(\frac{1}{2}\right)^6 \left(\frac{3}{2}\right)^7 N(= 43) + \alpha \\ \alpha = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \left(\frac{3}{2}\right) + \left(\frac{1}{2}\right)^2 \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^5 \left(\frac{3}{2}\right)^3 + \left(\frac{1}{2}\right)^6 \left(\frac{3}{2}\right)^4 \\ + \left(\frac{1}{2}\right)^7 \left(\frac{3}{2}\right)^5 + \left(\frac{1}{2}\right)^7 \left(\frac{3}{2}\right)^6$$

In this formula, $\frac{1}{2}$ appear at the time $\frac{3N+1}{2}$ occur. Thereafter, we multiply $\frac{3}{2}, \frac{1}{2}$ according to calculation. This formula easily can be checked by computer.

We assume that M and N is equal and lead contradiction. For easy to understand, we take $M = 13, N = 43$ and assume $M = N$. Ofcourse, $43 \neq 13$.

$$N = \left(\frac{1}{2}\right)^6 \left(\frac{3}{2}\right)^7 N + \alpha$$

We multiply 2^{13} .

$$2^{13}N = 3^7N + 2^{11} + 2^{10}3 + 2^93^2 + 2^53^3 + 2^33^4 + 23^5 + 3^6$$

We calculate α 's value. $\alpha = 1.520386 \dots$. $N = M$,

$$N\left(1 - \left(\frac{1}{2}\right)^n 3^m\right) = \alpha$$

$$N(2^n - 3^m) = 2^{n-l} + 2^{n-l'}3 + \dots + 3^{m-1}$$

We assume $m \geq 2$. The case $N = 1, n = 2, m = 1, l = 2$ exists.

$$2^{13}N - 3^7N = 2^{11} + 2^{10}3 + 2^93^2 + 2^53^3 + 2^33^4 + 23^5 + 3^6$$

This formula is transformed as

$$2^{13}(N - 1) - 3^7(N - 1) = -2^{13} + 2^{11} + 2^{10}3 + 2^93^2 + 2^53^3 + 2^33^4 + 23^5 + 3^6 + 3^7$$

Right side is divided 2 and 3. So this is 0. Left side is 0. We get $N = 1$. This is the contradiction. m -cycle can not exist. \square

Finally we calculate 1-cycle.

$$N = \left(\frac{1}{2}\right)^n \frac{3}{2}N + \left(\frac{1}{2}\right)^{n+1}$$

multiply 2^{n+1} .

$$2^{n+1}N = 3N + 1 \Rightarrow ((2^{n+1}) - 3)N = 1$$

$$(2^{n+1} - 3)N = 1, n = 1, N = 1$$

1 is trivial sequence $1 \rightarrow 4 \rightarrow 1 \rightarrow 4$. This result is easily checked.

$$1 = \left(\frac{1}{2}\right) \frac{3}{2} \times 1 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$$