An Alternative Formulation of Special Relativity

A. Blato

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This article presents an alternative formulation of special relativity which can be applied in any inertial reference frame. In addition, a new universal force is proposed.

Introduction

The intrinsic mass $(\, m \,)$ and the frequency factor $(\, f \,)$ of a massive particle are given by:

$$m \doteq m_o$$

$$f \doteq \left(1 - \frac{\mathbf{v} \cdot \mathbf{v}}{c^2}\right)^{-1/2}$$

where $(\, m_o \,)$ is the rest mass of the massive particle, $(\, \mathbf{v} \,)$ is the velocity of the massive particle and $(\, c \,)$ is the speed of light in vacuum.

The intrinsic mass $(\, m \,)$ and the frequency factor $(\, f \,)$ of a non-massive particle are given by:

$$m \doteq \frac{h\, \kappa}{c^2}$$

$$f \doteq \frac{\nu}{\kappa}$$

where $(\, h \,)$ is the Planck constant, $(\, \nu \,)$ is the frequency of the non-massive particle, $(\, \kappa \,)$ is a positive universal constant with dimension of frequency and $(\, c \,)$ is the speed of light in vacuum.

In this article, a massive particle is a particle with non-zero rest mass and a non-massive particle is a particle with zero rest mass.

The Alternative Kinematics

The special position $(\,\bar{\mathbf{r}}\,)$, the special velocity $(\,\bar{\mathbf{v}}\,)$ and the special acceleration $(\,\bar{\mathbf{a}}\,)$ of a (massive or non-massive) particle are given by:

$$\bar{\mathbf{r}} \doteq \int f\,\mathbf{v}\,dt$$

$$\bar{\mathbf{v}} \doteq \frac{d\bar{\mathbf{r}}}{dt} = f\,\mathbf{v}$$

$$\bar{\mathbf{a}} \doteq \frac{d\bar{\mathbf{v}}}{dt} = f\,\frac{d\mathbf{v}}{dt} + \frac{df}{dt}\,\mathbf{v}$$

where $(\,f\,)$ and $(\,\mathbf{v}\,)$ are the frequency factor and the velocity of the particle.

The Alternative Dynamics

If we consider a (massive or non-massive) particle with intrinsic mass $(\,m\,)$ then the linear momentum $(\,\mathbf{P}\,)$ of the particle, the angular momentum $(\,\mathbf{L}\,)$ of the particle, the net force $(\,\mathbf{F}\,)$ acting on the particle, the work $(\,\mathrm{W}\,)$ done by the net force acting on the particle, and the kinetic energy $(\,\mathrm{K}\,)$ of the particle are given by:

$$\mathbf{P} \doteq m\,\bar{\mathbf{v}} = m\,f\,\mathbf{v}$$

$$\mathbf{L} \doteq \mathbf{P} \mathbin{\dot{\times}} \mathbf{r} = m\,\bar{\mathbf{v}} \mathbin{\dot{\times}} \mathbf{r} = m\,f\,\mathbf{v} \mathbin{\dot{\times}} \mathbf{r}$$

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} = m\,\bar{\mathbf{a}} = m\left[f\,\frac{d\mathbf{v}}{dt} + \frac{df}{dt}\,\mathbf{v}\right]$$

$$\mathrm{W} \doteq \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 \frac{d\mathbf{P}}{dt}\cdot d\mathbf{r} = \Delta\,\mathrm{K}$$

$$\mathrm{K} \doteq m\,f\,c^2$$

where $(\,f,\,\mathbf{r},\,\mathbf{v},\,\bar{\mathbf{v}},\,\bar{\mathbf{a}}\,)$ are the frequency factor, the position, the velocity, the special velocity and the special acceleration of the particle and $(\,c\,)$ is the speed of light in vacuum. The kinetic energy $(\,\mathrm{K}_o\,)$ of a massive particle at rest is $(\,m_o\,c^2\,)$ § On the other hand, $(\,\mathbf{a} \mathbin{\dot{\times}} \mathbf{b} = \mathbf{b} \times \mathbf{a}\,)$ or $(\,\mathbf{a} \mathbin{\dot{\times}} \mathbf{b} = \mathbf{b} \wedge \mathbf{a}\,)$

The Kinetic Force

The kinetic force $\mathbf{K}^a_{ij}$ exerted on a particle i with intrinsic mass m_i by another particle j with intrinsic mass m_j is given by:

$$\mathbf{K}^a_{ij} \;=\; -\left[\;\frac{m_i\,m_j}{\mathbb{M}}\,(\,\bar{\mathbf{a}}_i - \bar{\mathbf{a}}_j\,)\;\right]$$

where $\bar{\mathbf{a}}_i$ is the special acceleration of particle i, $\bar{\mathbf{a}}_j$ is the special acceleration of particle j and $\mathbb{M}$ ($= \sum_z m_z$) is the sum of the intrinsic masses of all the particles of the Universe.

The kinetic force $\mathbf{K}^u_i$ exerted on a particle i with intrinsic mass m_i by the Universe is given by:

$$\mathbf{K}^u_i \;=\; -\,m_i\,\frac{\sum_z m_z\,\bar{\mathbf{a}}_z}{\sum_z m_z}$$

where m_z and $\bar{\mathbf{a}}_z$ are the intrinsic mass and the special acceleration of the *z*-th particle of the Universe.

From the above equations it follows that the net kinetic force $\mathbf{K}_i$ ($= \sum_j \mathbf{K}^a_{ij}$ $+\ \mathbf{K}^u_i$) acting on a particle i with intrinsic mass m_i is given by:

$$\mathbf{K}_i \;=\; -\,m_i\,\bar{\mathbf{a}}_i$$

where $\bar{\mathbf{a}}_i$ is the special acceleration of particle i.

Now, substituting ($\mathbf{F}_i = m_i\,\bar{\mathbf{a}}_i$) and rearranging, we obtain:

$$\mathbf{T}_i \;\doteq\; \mathbf{K}_i + \mathbf{F}_i \;=\; 0$$

Therefore, the total force $\mathbf{T}_i$ acting on a particle i is always zero.

Bibliography

A. Einstein, Relativity: The Special and General Theory.

E. Mach, The Science of Mechanics.

W. Pauli, Theory of Relativity.

Appendix I

System of Equations I

$$
\begin{tikzcd}
 & \boxed{[1]} \arrow[d, "dt"] & \\
\boxed{[4]} \arrow[d, "dt"] & \boxed{[2]} \arrow[l, "\dot{\times}\,\mathbf{r}"'] \arrow[d, "dt"] & \\
\boxed{[5]} & \boxed{[3]} \arrow[l, "\dot{\times}\,\mathbf{r}"'] \arrow[r, "\int d\mathbf{r}"] & \boxed{[6]}
\end{tikzcd}
$$

$$[\,1\,] \qquad \frac{1}{\mu}\left[\,\int \mathbf{P}\,dt \;-\; \int\!\!\int \mathbf{F}\,dt\,dt\,\right] = 0$$

$$[\,2\,] \qquad \frac{1}{\mu}\left[\,\mathbf{P} \;-\; \int \mathbf{F}\,dt\,\right] = 0$$

$$[\,3\,] \qquad \frac{1}{\mu}\left[\,\frac{d\mathbf{P}}{dt} \;-\; \mathbf{F}\,\right] = 0$$

$$[\,4\,] \qquad \frac{1}{\mu}\left[\,\mathbf{P} \;-\; \int \mathbf{F}\,dt\,\right] \dot{\times}\, \mathbf{r} \;=\; 0$$

$$[\,5\,] \qquad \frac{1}{\mu}\left[\,\frac{d\mathbf{P}}{dt} \;-\; \mathbf{F}\,\right] \dot{\times}\, \mathbf{r} \;=\; 0$$

$$[\,6\,] \qquad \frac{1}{\mu}\left[\,\int \frac{d\mathbf{P}}{dt}\cdot d\mathbf{r} \;-\; \int \mathbf{F}\cdot d\mathbf{r}\,\right] = 0$$

$[\,\mu\,]$ is an arbitrary constant with dimension of mass $(\,\mathrm{M}\,)$

Appendix II

System of Equations II

$$
\begin{tikzcd}
 & \boxed{[1]} \arrow[d, "dt"] & \\
\boxed{[4]} \arrow[d, "dt"] & \boxed{[2]} \arrow[l, "\dot{\times}\ \mathbf{r}"'] \arrow[d, "dt"] & \\
\boxed{[5]} & \boxed{[3]} \arrow[l, "\dot{\times}\ \mathbf{r}"'] \arrow[r, "\int d\mathbf{r}"] & \boxed{[6]}
\end{tikzcd}
$$

$$[1] \qquad \frac{1}{\mu}\left[m\,\bar{\mathbf{r}} - \iint \mathbf{F}\, dt\, dt \right] = 0$$

$$[2] \qquad \frac{1}{\mu}\left[m\,\bar{\mathbf{v}} - \int \mathbf{F}\, dt \right] = 0$$

$$[3] \qquad \frac{1}{\mu}\left[m\,\bar{\mathbf{a}} - \mathbf{F} \right] = 0$$

$$[4] \qquad \frac{1}{\mu}\left[m\,\bar{\mathbf{v}} - \int \mathbf{F}\, dt \right] \dot{\times}\ \mathbf{r} = 0$$

$$[5] \qquad \frac{1}{\mu}\left[m\,\bar{\mathbf{a}} - \mathbf{F} \right] \dot{\times}\ \mathbf{r} = 0$$

$$[6] \qquad \frac{1}{\mu}\left[m\, f\, c^2 - \int \mathbf{F} \cdot d\mathbf{r} \right] = 0$$

$[\,\mu\,]$ is an arbitrary constant with dimension of mass $(\,\mathrm{M}\,)$