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The Electromagnetic Wave in a Spherical Capacitor and the Nature of Earth Magnetism

Abstract

A solution of the Maxwell equations for the electromagnetic wave in a spherical capacitor which is included in an alternating current circuit or in an constant current circuit is proposed. A hypothesis of the Earth magnetism nature is presented on the basis of this solution.

Contents

1. Introduction

The electromagnetic wave in a capacitor in an alternating current or constant current circuit is investigated in [1, 2]. In this paper, a spherical capacitor in a sinusoidal current circuit or an constant current circuit is considered. The capacitor electrodes are two spheres having the same center and radii $R_2 > R_1$. A hypothesis of the Earth magnetism nature is proposed on the basis of this solution. A model of the ball lightning was substantiated previously in a similar manner [3].

2. Solution of the Maxwell Equations in the Spherical Coordinate System

Let us first consider a spherical capacitor in a sinusoidal current circuit. Fig. 1 shows the spherical coordinate system (ρ, θ, φ). Expressions for the rotor and the divergence of vector **E** in these coordinates are given in Table 1 [4]. The following notation is used:

E - electrical intensities,
H - magnetic intensities,
μ - absolute magnetic permeability,
ε - absolute dielectric constant.

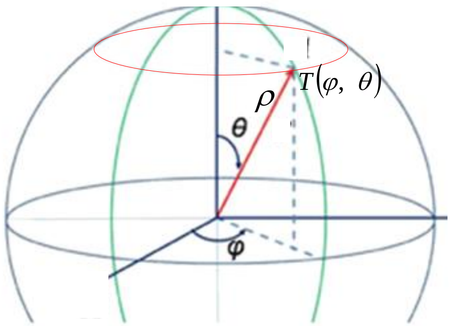

Fig. 1.

Table 1.

1	**2**	**3**
1	$\mathrm{rot}_{\rho}(E)$	$\frac{E_{\varphi}}{\rho \mathrm{tg}(\theta)}+\frac{\partial E_{\varphi}}{\rho \partial \theta}-\frac{\partial E_{\theta}}{\rho \sin(\theta) \partial \varphi}$
2	$\mathrm{rot}_{\theta}(E)$	$\frac{\partial E_{\rho}}{\rho \sin(\theta) \partial \varphi}-\frac{E_{\varphi}}{\rho}-\frac{\partial E_{\varphi}}{\partial \rho}$
3	$\mathrm{rot}_{\varphi}(E)$	$\frac{E_{\theta}}{\rho}+\frac{\partial E_{\theta}}{\partial \rho}-\frac{\partial E_{\rho}}{\rho \partial \varphi}$
4	$\mathrm{div}(E)$	$\frac{E_{\rho}}{\rho}+\frac{\partial E_{\rho}}{\partial \rho}+\frac{E_{\theta}}{\rho \mathrm{tg}(\theta)}+\frac{\partial E_{\theta}}{\rho \partial \theta}+\frac{\partial E_{\varphi}}{\rho \sin(\theta) \partial \varphi}$

With no charge on and no current between the spherical capacitor electrodes, the Maxwell equations in the spherical coordinate system take the form presented in Table 2.

Table 2.

1	**2**
1.	$\mathrm{rot}_{\rho} H-\frac{\varepsilon}{c} \frac{\partial E_{\rho}}{\partial t}=0$

2.	$\mathrm{rot}_\theta H - \frac{\varepsilon}{c}\frac{\partial E_\theta}{\partial t} = 0$
3.	$\mathrm{rot}_\varphi H - \frac{\varepsilon}{c}\frac{\partial E_\varphi}{\partial t} = 0$
4.	$\mathrm{rot}_\rho E + \frac{\mu}{c}\frac{\partial H_\rho}{\partial t} = 0$
5.	$\mathrm{rot}_\theta E + \frac{\mu}{c}\frac{\partial H_\theta}{\partial t} = 0$
6.	$\mathrm{rot}_\varphi E + \frac{\mu}{c}\frac{\partial H_\varphi}{\partial t} = 0$
7.	$\mathrm{div}(E) = 0$
8.	$\mathrm{div}(H) = 0$

Below the solution will be sought for in form of functions E, H, which presented in Table. 3, where the functions of the form $E_{\varphi\rho}(\rho)$ to be calculated. It is important to note that

- these functions are independent of the argument φ;
- if $E(\theta) = \sin(\theta)$, then

$$\frac{E}{\mathrm{tg}(\theta)} + \frac{\partial E}{\partial \theta} = 2\cos(\theta). \tag{11}$$

Table 3.

1	**2**
	$E_\rho = E_{\rho\rho}(\rho)\cos(\theta)\sin(\omega t)$
	$E_\theta = E_{\theta\rho}(\rho)\sin(\theta)\sin(\omega t)$
	$E_\varphi = E_{\varphi\rho}(\rho)\sin(\theta)\sin(\omega t)$
	$H_\rho = H_{\rho\rho}(\rho)\cos(\theta)\cos(\omega t)$
	$H_\theta = H_{\theta\rho}(\rho)\sin(\theta)\cos(\omega t)$
	$H_\varphi = H_{\varphi\rho}(\rho)\sin(\theta)\cos(\omega t)$

We substitute the functions E, H from the Table 3 in Table 1 and take into account (11). Then we obtain Table 4.

Table 4.

1	**2**	**3**
1	$\mathrm{rot}_\rho(E)$	$\frac{2E_{\varphi\rho}}{\rho}\cos(\theta)\ \sin(\omega t)$
2	$\mathrm{rot}_\theta(E)$	$-\left(\frac{E_\varphi}{\rho}+\frac{\partial E_\varphi}{\partial\rho}\right)\sin(\theta)\ \sin(\omega t)$
3	$\mathrm{rot}_\varphi(E)$	$\left(\frac{E_\theta}{\rho}+\frac{\partial E_\theta}{\partial\rho}\right)\sin(\theta)\ \sin(\omega t)$
4	$\mathrm{div}(E)$	$\left(\left(\frac{E_\rho}{\rho}+\frac{\partial E_\rho}{\partial\rho}\right)+\frac{2E_{\theta\rho}}{\rho}\right)\cos(\theta)\ \sin(\omega t)$

Expressions for the rotor and divergence function H differ from those shown in the Table. 4 only in that instead of factors $\sin(\omega t)$ are factors $\cos(\omega t)$.

Substituting the expression for the curl and divergence in Maxwell's equations (see Table 2), differentiating with respect to time and reducing common factors, we obtain a new form of Maxwell's equations - see Table. 5.

Table 5.

1	**2**
1	$\frac{2E_{\varphi\rho}}{\rho}-\frac{\omega\mu}{c}H_{\rho\rho}=0$
2	$-\left(\frac{E_{\varphi\rho}}{\rho}+\frac{\partial E_{\varphi\rho}}{\partial\rho}\right)-\frac{\omega\mu}{c}H_{\theta\rho}=0$
3	$\left(\frac{E_{\theta\rho}}{\rho}+\frac{\partial E_{\theta\rho}}{\partial\rho}\right)-\frac{\omega\mu}{c}H_{\varphi\rho}=0$
4	$\left(\left(\frac{E_{\rho\rho}}{\rho}+\frac{\partial E_{\rho\rho}}{\partial\rho}\right)+\frac{2E_{\theta\rho}}{\rho}\right)=0$
5	$\frac{2H_{\varphi\rho}}{\rho}-\frac{\omega\varepsilon}{c}E_{\rho\rho}=0$
6	$-\left(\frac{H_{\varphi\rho}}{\rho}+\frac{\partial H_{\varphi\rho}}{\partial\rho}\right)-\frac{\omega\varepsilon}{c}E_{\theta\rho}=0$

7	$\left(\frac{H_{\theta\rho}}{\rho}+\frac{\partial H_{\theta\rho}}{\partial\rho}\right)-\frac{\omega\varepsilon}{c}E_{\varphi\rho}=0$
8	$\left(\left(\frac{H_{\rho\rho}}{\rho}+\frac{\partial H_{\rho\rho}}{\partial\rho}\right)+\frac{2H_{\theta\rho}}{\rho}\right)=0$

3. The solution of Maxwell's equations for the vacuum

First, we consider the equations for a vacuum where in the GHS system we have: $\varepsilon=\mu=1$. At the same table. 5 takes the following form:

Table 5a.

1	**2**
1	$\frac{2E_{\varphi\rho}}{\rho}-qH_{\rho\rho}=0$
2	$-\left(\frac{E_{\varphi\rho}}{\rho}+\frac{\partial E_{\varphi\rho}}{\partial\rho}\right)-qH_{\theta\rho}=0$
3	$\left(\frac{E_{\theta\rho}}{\rho}+\frac{\partial E_{\theta\rho}}{\partial\rho}\right)-qH_{\varphi\rho}=0$
4	$\left(\left(\frac{E_{\rho\rho}}{\rho}+\frac{\partial E_{\rho\rho}}{\partial\rho}\right)+\frac{2E_{\theta\rho}}{\rho}\right)=0$
5	$\frac{2H_{\varphi\rho}}{\rho}-qE_{\rho\rho}=0$
6	$-\left(\frac{H_{\varphi\rho}}{\rho}+\frac{\partial H_{\varphi\rho}}{\partial\rho}\right)-qE_{\theta\rho}=0$
7	$\left(\frac{H_{\theta\rho}}{\rho}+\frac{\partial H_{\theta\rho}}{\partial\rho}\right)-qE_{\varphi\rho}=0$
8	$\left(\left(\frac{H_{\rho\rho}}{\rho}+\frac{\partial H_{\rho\rho}}{\partial\rho}\right)+\frac{2H_{\theta\rho}}{\rho}\right)=0$

where

$$q=\frac{\omega}{c}. \tag{12}$$

Then Maxwell's equations are completely symmetrical with respect to the intensities E and H. Find the sum pairs of (1-4) and (5-8). Then we get:

$$\frac{2W_{\varphi\rho}}{\rho} - qW_{\rho\rho} = 0, \tag{13}$$

$$\left(\frac{W_{\varphi\rho}}{\rho} + \frac{\partial W_{\varphi\rho}}{\partial \rho}\right) + qW_{\theta\rho} = 0, \tag{14}$$

$$\left(\frac{W_{\theta\rho}}{\rho} + \frac{\partial W_{\theta\rho}}{\partial \rho}\right) - qW_{\varphi\rho} = 0, \tag{15}$$

$$\left(\left(\frac{W_{\rho\rho}}{\rho} + \frac{\partial W_{\rho\rho}}{\partial \rho}\right) + \frac{2W_{\theta\rho}}{\rho}\right) = 0, \tag{16}$$

where

$$W = E + H. \tag{17}$$

The system of 4 equations (13-16) defines 3 unknown functions - the system is overdetermined. We show that there is a solution that satisfies all equations

Direct substitution can be seen that the equations (14, 15) has the following solution:

$$W_{\theta\rho} = A \cdot \frac{-i}{\rho} \exp\left(iq(\rho - R) + \beta\right), \tag{18}$$

$$W_{\varphi\rho} = -A \cdot \frac{1}{\rho} \exp\left(iq(\rho - R) + \beta\right), \tag{19}$$

where $A,\ R,\ \omega,\ \beta,\ c$ - constants. We find from equations (13, 18):

$$W_{\rho\rho} = \frac{2W_{\varphi\rho}}{\rho}\frac{c}{\omega} = -\frac{2A}{q\rho^2} \exp\left(iq(\rho - R) + \beta\right), \tag{20}$$

$$\frac{\partial W_{\rho\rho}}{\partial \rho} = A\left(\frac{2i}{q\rho^3} - \frac{2}{\rho^2}\right) \exp\left(iq(\rho - R) + \beta\right). \tag{21}$$

Substituting equations (19-21) to (16), we see that equation (16) turns into the identical relation 0=0. Therefore, three functional relations (18-20) comply with four equations (13-16), which was to be proved.

The decision does not change if instead of (17) will be used condition

$$W = (E + H)\frac{2}{(1 + i)}. \tag{22}$$

Next, we will look for a solution in which

$$E = iH. \tag{23}$$

From (76, 77), we find:

$$W = (1+i)H\frac{2}{(1+i)} = 2H \tag{24}$$

or

$$H = W/2\,. \tag{25}$$

From (77, 79), we find:

$$E = Wi/2\,. \tag{26}$$

From (18-20, 79, 80), we find:

$$H_{\theta\rho} = \frac{-Ai}{2\rho}\exp\left(iq(\rho - R) + \beta\right), \tag{27}$$

$$H_{\varphi\rho} = \frac{-A}{2\rho}\exp\left(iq(\rho - R) + \beta\right), \tag{28}$$

$$H_{\rho\rho} = \frac{-A}{q\rho^2}\exp\left(iq(\rho - R) + \beta\right), \tag{29}$$

$$E_{\theta\rho} = \frac{A}{2\rho}\exp\left(iq(\rho - R) + \beta\right), \tag{30}$$

$$E_{\varphi\rho} = \frac{-Ai}{2\rho}\exp\left(iq(\rho - R) + \beta\right), \tag{31}$$

$$E_{\rho\rho} = \frac{-Ai}{q\rho^2}\exp\left(iq(\rho - R) + \beta\right). \tag{32}$$

The solution obtained is a complex value. It is known that the real part of a complex solution is also a solution. It follows that one can take the real parts of functional relations (27-32) as a solution instead of these functional relations:

$$H_{\theta\rho} = \frac{A}{2\rho}\sin\left(q(\rho - R) + \beta\right), \tag{33}$$

$$H_{\varphi\rho} = \frac{-A}{2\rho}\cos\left(q(\rho - R) + \beta\right), \tag{34}$$

$$H_{\rho\rho} = \frac{-A}{q\rho^2}\cos\left(q(\rho - R) + \beta\right), \tag{35}$$

$$E_{\theta\rho} = \frac{A}{2\rho}\cos\left(q(\rho - R) + \beta\right), \tag{36}$$

$$E_{\varphi\rho} = \frac{A}{2\rho}\sin\left(q(\rho - R) + \beta\right), \tag{37}$$

$$E_{\rho\rho} = \frac{A}{q\rho^2}\sin(q(\rho - R) + \beta). \tag{38}$$

To check this solution, one can substitute these functions into equations in Table 3 to make sure that these equations become equalities.

Thus, the solution of Maxwell's equations for the spherical vacuum capacitor has the form of equations (33-38).

To find all these functions, it suffices to know the values of constants $A,\ R,\ \omega,\ \beta,\ c$. This solution means that **<u>an electromagnetic wave does exist in the spherical capacitor in a sinusoidal current circuit.</u>**

The solution of Maxwell's equations for the case when the dielectric is not a vacuum is given in Application 1 and for the case when the dielectric has some electrical conductivity – в Application 2.

4. Electric and magnetic intensities

Let us consider a point T with coordinates $\varphi,\ \theta$ on a sphere of radius ρ .Vectors E_φ and E_θ, going from this point are in plane P, tangent to this sphere at point $T(\varphi,\ \theta)$ - see Fig. 2.

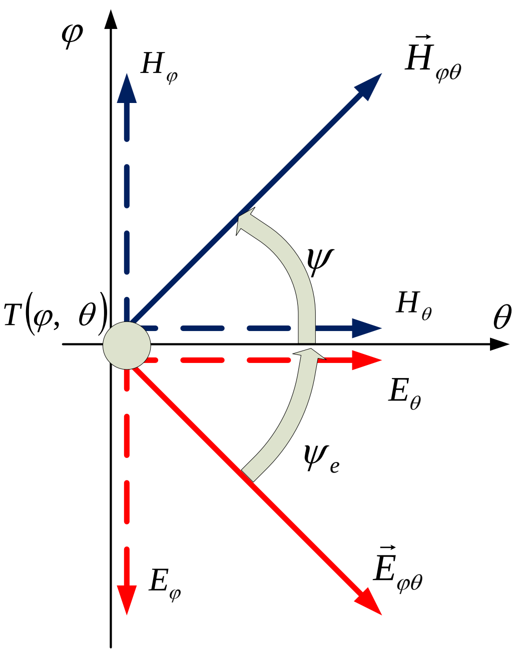

Fig. 2.

These vectors are perpendicular to each other. Hence, at each point $(\varphi,\ \theta)$ the sum vector

$$\vec{H}_{\varphi\theta} = \vec{H}_{\varphi} + \vec{H}_{\theta} \tag{39}$$

is in plane P and has an angle of ψ to a parallel line. As it follows from (33, 34) and the Table. 3, the module of this vector and the angle ψ defined by the following formulas:

$$\left|\vec{H}_{\varphi\theta}\right| = \frac{A}{2\rho} \tag{40}$$

$$\cos(\psi) = \frac{H_{\theta\rho}}{\left|\vec{H}_{\varphi\theta}\right|} = \sin\left(\frac{\omega}{c}(\rho - R) + \beta\right)$$

or

$$\psi = \frac{\pi}{2} - \frac{\omega}{c}(\rho - R) - \beta\,. \tag{41}$$

Similarly, the same relationships exist for the vectors $\vec{E}_{\varphi}$ and $\vec{E}_{\theta}$. At each point $(\varphi,\ \theta)$ the total vector

$$\vec{E}_{\varphi\theta} = \vec{E}_{\varphi} + \vec{E}_{\theta} \tag{42}$$

lies in the plane P and is directed at an angle ψ_e to a line parallel. It follows from (36, 37) and Table 3, the module of this vector and the angle ψ_e defined by the following formulas:

$$\left|\vec{E}_{\varphi\theta}\right| = \frac{A}{2\rho} \tag{43}$$

$$\cos(\psi_e) = \frac{E_{\theta\rho}}{\left|\vec{E}_{\varphi\theta}\right|} = \cos\left(\frac{\omega}{c}(\rho - R) + \beta\right)$$

or

$$\psi_e = \frac{\omega}{c}(\rho - R) - \beta \tag{44}$$

or

$$\psi_e = \frac{\pi}{2} - \psi\,. \tag{45}$$

The angle between $\vec{H}_{\varphi\theta}$ и $\vec{E}_{\varphi\theta}$ in the plane P is straight.

Therefore, in a spherical capacitor we can consider only one vector of the electrical field intensities $\vec{E}_{\varphi\theta}$ and only one vector of the magnetic field intensities $\vec{H}_{\varphi\theta}$. As these vectors lie on the sphere, they will be called <u>spherical vectors</u>.

In Fig. 3 shows the vectors $\vec{H}_{\varphi\theta}$ and $\vec{E}_{\varphi\theta}$ lying in the plane P, and vectors $\vec{H}_{\rho}$ and $\vec{E}_{\rho}$ lying on a radius.

Note that there are many solutions distinguished by value β. This fact reflects the arbitrary rule in the choice of mathematical coordinate axes.

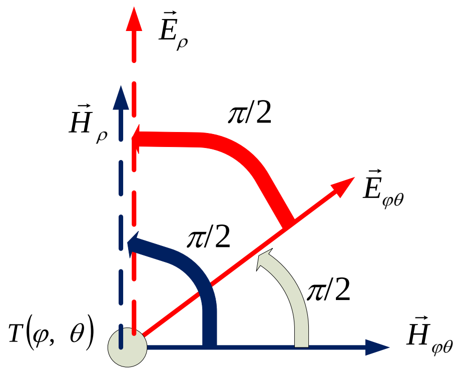

Fig. 3.

Angle ψ (30) is constant for all vectors $\vec{H}_{\varphi\theta}$ for a given radius ρ. This means that the directions of all vectors $\vec{H}_{\varphi\theta}$ constitute the same angle ψ with all parallels on a sphere with a radius of ρ. This implies in turn that there are the <u>magnetic equatorial plane</u> inclined to the mathematical equatorial plane at angle ψ, <u>magnetic axis</u>, <u>magnetic poles</u>, and <u>magnetic meridians</u>, along which vectors $\vec{H}_{\varphi\theta}$ are directed – see Fig. 4, where thin lines mark the mathematical meridional grid, thick lines mark the magnetic meridional grid, the mathematical axis *mm*, and magnetic axis *aa* and electric axis *bb* are shown. It is important to note that the magnetic axis *aa*, electric axis *bb* and all vectors $\vec{E}_{\varphi\theta}$ и $\vec{H}_{\varphi\theta}$ are perpendicular.

When $\frac{\omega}{c} \approx 0$ and $\beta = 0$ the magnetic axis coincides with the mathematical axis.

Spherical vectors depend on $\sin(\theta)$. Radial vectors depend on $\cos(\theta)$ – see Table 3. Therefore, there are the radial intensities only in locations where the spherical intensity is zero.

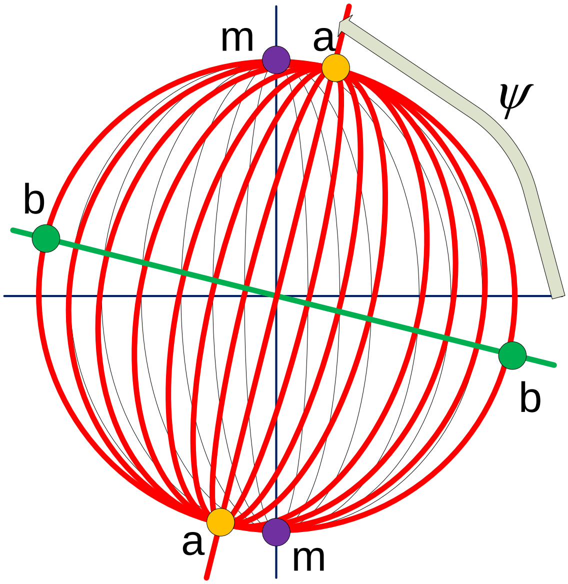

Fig. 4.

5. An Electromagnetic Wave in a Charged Spherical Capacitor

A solution of the Maxwell equations for a parallel-plate capacitor being charged [2] stems from a solution of these equations for a parallel-plate capacitor in a sinusoidal current circuit [1]. In this paper the method described in [1] will be used in solving the Maxwell equations for a spherical capacitor being charged.

Let us consider the field intensities in the form of functions presented in Table 6. These functions differ from functions of Table 3 only by the type of time dependence: in Table 3, E and H functions depend on time as $\sin(\omega t)$, $\cos(\omega t)$, respectively, while in Table 6, E and H functions depend on time as $(1-\exp(\omega t))$, $(\exp(\omega t)-1)$, respectively.

Although the indicated substitution, the solution of Maxwell's equations remain unchanged.

Bias Current

$$J_\rho = \frac{d}{dt} E_\rho = -\omega E_{\rho\rho}(\rho)\cos(\theta)\exp(\omega t) \tag{46}$$

Table 6.

1	2
	$E_\rho = E_{\rho\rho}(\rho)\cos(\theta)(1-\exp(\omega t))$
	$E_\theta = E_{\theta\rho}(\rho)\sin(\theta)(1-\exp(\omega t))$
	$E_\varphi = E_{\varphi\rho}(\rho)\sin(\theta)(1-\exp(\omega t))$
	$H_\rho = H_{\rho\rho}(\rho)\cos(\theta)(\exp(\omega t)-1)$
	$H_\theta = H_{\theta\rho}(\rho)\sin(\theta)(\exp(\omega t)-1)$
	$H_\varphi = H_{\varphi\rho}(\rho)\sin(\theta)(\exp(\omega t)-1)$

Fig. 6 presents intensities components and their time derivatives as well as the bias current as a function of time for $\omega = -300$: H_ρ is shown with a solid line, with a dashed line, and J_ρ with dotted line. It is evident that with $t \Rightarrow \infty$ the amplitudes of all intensities components tend to a constant together, while the current amplitude approaches zero. This corresponds to the capacitor charging via a fixed resistor.

When the capacitor becomes fully charged, the current stops to flow. However, the stationary flow of the electromagnetic energy is maintained according to [2].

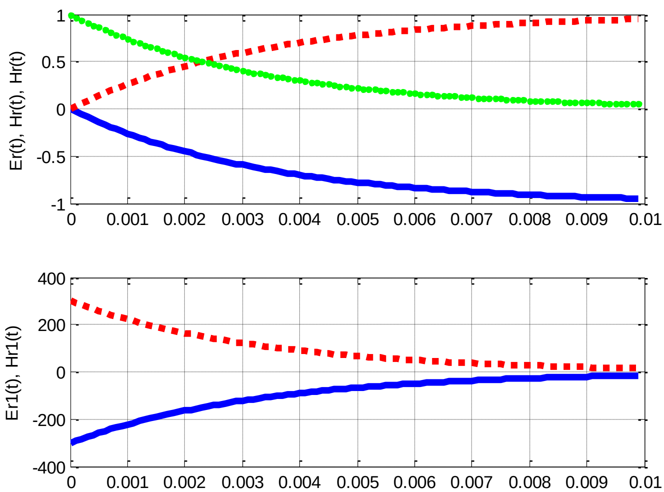

Fig.6. (SSMB6.1)

Thus, the solution of the Maxwell equations for a capacitor being charged and the solution for a capacitor in a sinusoidal current circuit differ only in that the former includes exponential time functions while the latter contains sinusoidal time functions.

The electromagnetic wave structure remains the same - see Section 3. It is evident from Section 3 that there is an electromagnetic wave in a spherical capacitor with only spherical vectors $\vec{E}_{\varphi\theta}$, $\vec{H}_{\varphi\theta}$ and radial vectors $\vec{E}_{\rho}$, $\vec{H}_{\rho}$.

Thus, we can say that the spherical capacitor is a device equivalent to the magnet and, simultaneously, electrets, which are perpendicular.

6. The Magnetic and the Electrical Field of the Earth

It is known that the Earth electrical field can be considered as a field "between spherical capacitor electrodes" [5]. These electrodes are the Earth surface having a negative charge and the ionosphere having a positive charge. The charge of these electrodes is maintained by continuous atmospheric thunderstorm activities.

It is also known that there is the Earth magnetic field. However, in this case no generally accepted explanation of the source of this field is available. "The problem of the origin and retaining of the field has not been solved as yet." [7].

It flows from the above mentioned that **the Earth electrical field is responsible for the Earth magnetic field.**

Let us consider this problem in more details.

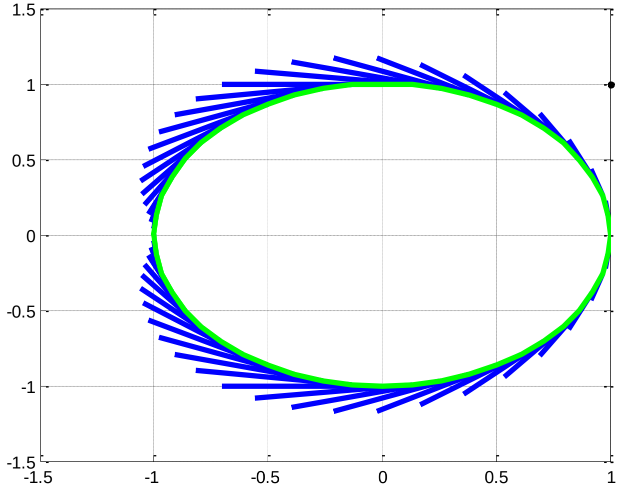

FIG. 8. (Sfera.88)

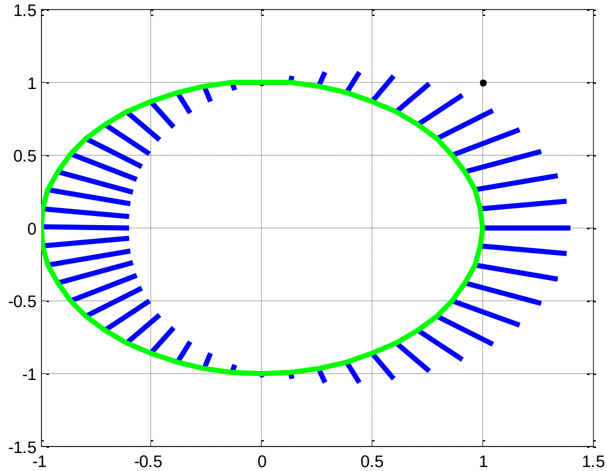

FIG. 9. (Sfera.88)

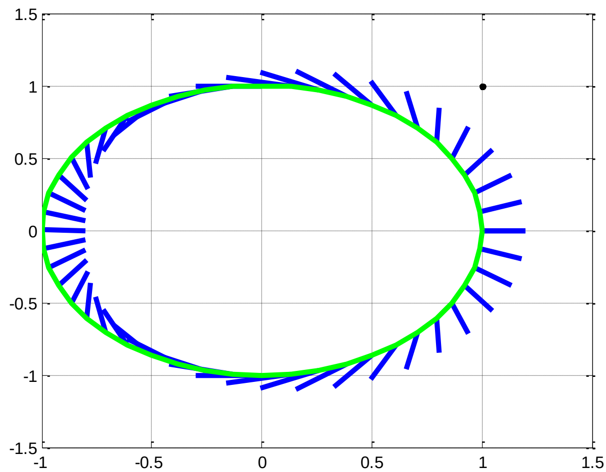

FIG. 10. (Sfera.88)

The vector field $\vec{H}_{\varphi\theta}$ in a diametral plane passing through the magnetic axis is shown in Fig. 8. Here, $\left|\vec{H}_{\varphi\theta}\right| = 0.7;\ \rho = 1.$ The vector field $\vec{H}_{\rho}$ in a diametral plane passing through the magnetic axis is shown in Fig. 9. Here, $\left|\vec{H}_{\rho}\right| = 0.4;\ \rho = 1.$ The vector field $\vec{H} = \vec{H}_{\varphi\theta} + \vec{H}_{\rho}$ in a diametral plane passing through the magnetic axis is shown in Fig. 10. Here, $\left|\vec{H}_{\varphi\theta}\right| = 0.3;\ \left|\vec{H}_{\rho}\right| = 0.2;\ \rho = 1.$

Application 1. Solution of Maxwell's equations for the medium

The solution of equations for the vacuum was considered above, where in the GHS system, $\varepsilon = \mu = 1$. At this time, we take a look at the more general case, where $\varepsilon \neq \mu$.

We consider again Table 5. We shall call

$$E = gE', \tag{60}$$

$$g = \sqrt{\mu/\varepsilon}. \tag{61}$$

Then Table 5 becomes Table 7. We perform simple transforms in Table 7 and get Table 8. In Table 5a:

- In lines 1, 2, 3, 4 the equations are divided by g,
- At the same time, in lines 1, 2, 3 before variable H appears coefficient

$$q = \frac{\omega\mu}{c} \Big/ g = \frac{\omega}{c}\sqrt{\mu\varepsilon}, \tag{62a}$$

- In lines 5, 6, 7 the coefficient before variable E' is replaced with for

$$q = \frac{\omega\varepsilon}{c} g = \frac{\omega}{c}\sqrt{\mu\varepsilon}. \tag{62в}$$

Therefore, in this case the solution also has the form (33-38). The only difference is in the value of coefficient q: compare (12) and (62).

Next, intensities E are defined by (60). Thus, in this case equations (33-38) become:

$$H_{\theta\rho} = \frac{A}{2\rho}\sin(q(\rho - R) + \beta), \tag{63}$$

$$H_{\varphi\rho} = \frac{-A}{2\rho}\cos(q(\rho - R) + \beta), \tag{64}$$

$$H_{\rho\rho} = \frac{-A}{q\rho^2}\cos(q(\rho - R) + \beta), \tag{65}$$

$$E_{\theta\rho} = \frac{Ag}{2\rho}\cos(q(\rho - R) + \beta), \tag{66}$$

$$E_{\varphi\rho} = \frac{Ag}{2\rho}\sin(q(\rho - R) + \beta), \tag{67}$$

$$E_{\rho\rho} = \frac{Ag}{q\rho^2}\sin(q(\rho - R) + \beta). \tag{68}$$

Table 7.

1	**2**
1	$\frac{2E'_{\varphi\rho}}{\rho}g - \frac{\omega\mu}{c}H_{\rho\rho} = 0$
2	$-\left(\frac{E'_{\varphi\rho}}{\rho} + \frac{\partial E'_{\varphi\rho}}{\partial\rho}\right)g - \frac{\omega\mu}{c}H_{\theta\rho} = 0$
3	$\left(\frac{E'_{\theta\rho}}{\rho} + \frac{\partial E'_{\theta\rho}}{\partial\rho}\right)g - \frac{\omega\mu}{c}H_{\varphi\rho} = 0$
4	$\left(\left(\frac{E'_{\rho\rho}}{\rho} + \frac{\partial E'_{\rho\rho}}{\partial\rho}\right) + \frac{2E'_{\theta\rho}}{\rho}\right)g = 0$
5	$\frac{2H_{\varphi\rho}}{\rho} - \frac{\omega\varepsilon}{c}E'_{\rho\rho}g = 0$
6	$-\left(\frac{H_{\varphi\rho}}{\rho} + \frac{\partial H_{\varphi\rho}}{\partial\rho}\right) - \frac{\omega\varepsilon}{c}E'_{\theta\rho}g = 0$
7	$\left(\frac{H_{\theta\rho}}{\rho} + \frac{\partial H_{\theta\rho}}{\partial\rho}\right) - \frac{\omega\varepsilon}{c}E'_{\varphi\rho}g = 0$
8	$\left(\left(\frac{H_{\rho\rho}}{\rho} + \frac{\partial H_{\rho\rho}}{\partial\rho}\right) + \frac{2H_{\theta\rho}}{\rho}\right) = 0$

Application 2. Solution of Maxwell's equations for conductive dielectric

In Application 1 was considered the solution of equations for the dielectric, which was $\varepsilon \neq \mu$. Next, assume that the dielectric has a certain electrical conductivity. In this case, the equation of the form

$$\mathrm{rot}H - \frac{\varepsilon}{c}\frac{\partial E}{\partial t} = 0 \tag{71}$$

is replaced by the equation of the form

$$\mathrm{rot}H - \frac{\varepsilon}{c}\frac{\partial E}{\partial t} - \sigma E = 0 \tag{72}$$

Instead Table 3 in this case we use the Table 9, where ϕ - the phase angle between the magnetic and electric field intensities – see Fig. 11.

At the same time the system of Maxwell's equations can be replaced by two independent systems of equations: in the first system is used the term $\sin(\phi)\sin(\omega t)$ from the Table 9, and in the second system - the term $\sigma\cos(\phi)\cos(\omega t)$ from the Table 9. After receiving the decision of the system the general solution is defined as the sum of the solutions found (by the linearity of systems). The solution of the first system is given in Appendix 1.

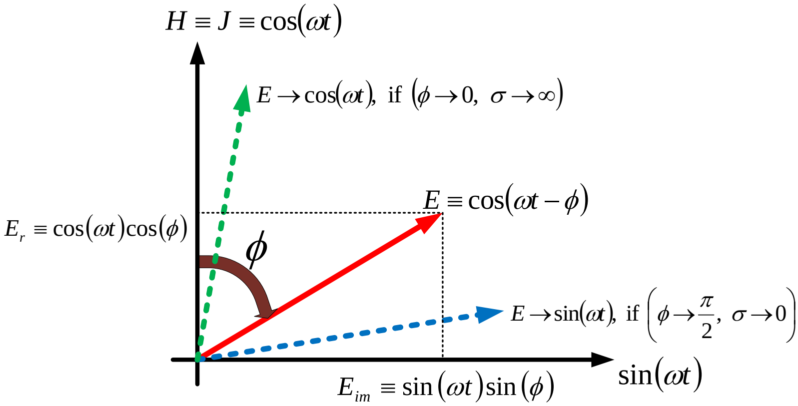

Fig. 11.

Table 9.

1	2
	$E_\rho = E_{\rho\rho}(\rho)\cos(\theta)(\sin(\phi)\sin(\omega t) + \sigma\cos(\phi)\cos(\omega t))$
	$E_\theta = E_{\theta\rho}(\rho)\sin(\theta)(\sin(\phi)\sin(\omega t) + \sigma\cos(\phi)\cos(\omega t))$
	$E_\varphi = E_{\varphi\rho}(\rho)\sin(\theta)(\sin(\phi)\sin(\omega t) + \sigma\cos(\phi)\cos(\omega t))$
	$H_\rho = H_{\rho\rho}(\rho)\cos(\theta)\cos(\omega t)$
	$H_\theta = H_{\theta\rho}(\rho)\sin(\theta)\cos(\omega t)$
	$H_\varphi = H_{\varphi\rho}(\rho)\sin(\theta)\cos(\omega t)$

Table. 5 for the second system takes the form of Table 10 (modified formulas (5-7)). Next will also argue, as in Application 1. Let

$$E = gE'. \tag{73}$$

when

$$g = \sqrt{\frac{\mu}{\varepsilon \cdot \sigma \cdot \cos(\phi)}}. \tag{74}$$

Then the Table 10 takes the form of the Table 11 (similar transformations are presented in Table 7), and again we obtain Table 5a:

- In lines 1, 2, 3, 4 the equations are divided by g,
- At the same time, in lines 1, 2, 3 before variable H appears coefficient

$$q = \frac{\omega\mu}{c} \Big/ g = \frac{\omega}{c}\sqrt{\mu\varepsilon\sigma \cdot \cos(\phi)}, \tag{75}$$

- In lines 5, 6, 7 the coefficient before variable E' is replaced with for

$$q = \sigma \cdot \cos(\phi) \cdot g = \frac{\omega}{c}\sqrt{\mu\varepsilon\sigma \cdot \cos(\phi)}. \tag{76}$$

Therefore, in this case the solution also has the form (33-38). The only difference is in the value of coefficient q: compare (12) and (75). Next, intensities E are defined by (73). Therefore, in this case the solution also has the form (63-68). The only difference is in the value of coefficient g - compare (61) and (74).

By combining this solution of the second system with the solution the first system, we finally obtain:

$$E_{\rho\rho} = \frac{Ag}{q\rho^2}\sin(q(\rho - R) + \beta). \tag{77}$$

$$H_{\theta\rho} = \frac{A}{2\rho}\left(\sin(q_1(\rho - R) + \beta_1) + \sin(q_2(\rho - R) + \beta_2)\right), \tag{78}$$

$$H_{\varphi\rho} = \frac{-A}{2\rho}\left(\cos(q_1(\rho - R) + \beta_1) + \cos(q_2(\rho - R) + \beta_2)\right), \tag{79}$$

$$H_{\rho\rho} = \frac{-A}{\rho^2}\left(\frac{1}{q_1}\cos(q_1(\rho - R) + \beta_1) + \frac{1}{q_2}\cos(q_2(\rho - R) + \beta_2)\right), \tag{80}$$

$$E_{\theta\rho} = \frac{A}{2\rho}\left(g_1\cos(q_1(\rho - R) + \beta_1) + g_2\cos(q_2(\rho - R) + \beta_2)\right), \tag{81}$$

$$E_{\varphi\rho} = \frac{A}{2\rho}\left(g_1\sin(q_1(\rho - R) + \beta_1) + g_2\sin(q_2(\rho - R) + \beta_2)\right), \tag{82}$$

$$E_{\rho\rho} = \frac{A}{\rho^2}\left(w_1\sin(q_1(\rho - R) + \beta_1) + w_2\sin(q_2(\rho - R) + \beta_2)\right), \tag{83}$$

where

$$q_1 = \frac{\omega}{c}\sqrt{\mu\varepsilon}\,, \tag{84}$$

$$q_2 = \frac{\omega}{c}\sqrt{\mu\varepsilon\sigma\cdot\cos(\phi)}\,. \tag{85}$$

$$g_1 = \sqrt{\frac{\mu}{\varepsilon}}\,, \tag{86}$$

$$g_2 = \sqrt{\frac{\mu}{\varepsilon\cdot\sigma\cdot\cos(\phi)}}\,, \tag{87}$$

$$w_1 = \frac{g_1}{q_1} = \sqrt{\frac{\mu}{\varepsilon}}\Bigg/\left(\frac{\omega}{c}\sqrt{\mu\varepsilon}\right) = \frac{c}{\omega\varepsilon}\,, \tag{88}$$

$$w_2 = \frac{g_2}{q_2} = \sqrt{\frac{\mu}{\varepsilon\cdot\sigma\cdot\cos(\phi)}}\Bigg/\left(\frac{\omega}{c}\sqrt{\mu\varepsilon\sigma\cdot\cos(\phi)}\right) = \frac{c}{\omega\varepsilon\sigma\cdot\cos(\phi)}\,. \tag{89}$$

Table 10.

1	**2**
1	$\frac{2E_{\varphi\rho}}{\rho} - \frac{\omega\mu}{c}H_{\rho\rho} = 0$
2	$-\left(\frac{E_{\varphi\rho}}{\rho} + \frac{\partial E_{\varphi\rho}}{\partial\rho}\right) - \frac{\omega\mu}{c}H_{\theta\rho} = 0$
3	$\left(\frac{E_{\theta\rho}}{\rho} + \frac{\partial E_{\theta\rho}}{\partial\rho}\right) - \frac{\omega\mu}{c}H_{\varphi\rho} = 0$
4	$\left(\left(\frac{E_{\rho\rho}}{\rho} + \frac{\partial E_{\rho\rho}}{\partial\rho}\right) + \frac{2E_{\theta\rho}}{\rho}\right) = 0$
5	$\frac{2H_{\varphi\rho}}{\rho} - \sigma\cos(\phi)E_{\rho\rho} = 0$
6	$-\left(\frac{H_{\varphi\rho}}{\rho} + \frac{\partial H_{\varphi\rho}}{\partial\rho}\right) - \sigma\cos(\phi)E_{\theta\rho} = 0$
7	$\left(\frac{H_{\theta\rho}}{\rho} + \frac{\partial H_{\theta\rho}}{\partial\rho}\right) - \sigma\cos(\phi)E_{\varphi\rho} = 0$
8	$\left(\left(\frac{H_{\rho\rho}}{\rho} + \frac{\partial H_{\rho\rho}}{\partial\rho}\right) + \frac{2H_{\theta\rho}}{\rho}\right) = 0$

Table 11.

1	**2**
1	$\frac{2E_{\varphi\rho}}{\rho}g-\frac{\omega\mu}{c}H_{\rho\rho}=0$
2	$-\left(\frac{E_{\varphi\rho}}{\rho}+\frac{\partial E_{\varphi\rho}}{\partial\rho}\right)g-\frac{\omega\mu}{c}H_{\theta\rho}=0$
3	$\left(\frac{E_{\theta\rho}}{\rho}+\frac{\partial E_{\theta\rho}}{\partial\rho}\right)g-\frac{\omega\mu}{c}H_{\varphi\rho}=0$
4	$\left(\left(\frac{E_{\rho\rho}}{\rho}+\frac{\partial E_{\rho\rho}}{\partial\rho}\right)+\frac{2E_{\theta\rho}}{\rho}\right)g=0$
5	$\frac{2H_{\varphi\rho}}{\rho}-\sigma\cos(\phi)E_{\rho\rho}g=0$
6	$-\left(\frac{H_{\varphi\rho}}{\rho}+\frac{\partial H_{\varphi\rho}}{\partial\rho}\right)-\sigma\cos(\phi)E_{\theta\rho}g=0$
7	$\left(\frac{H_{\theta\rho}}{\rho}+\frac{\partial H_{\theta\rho}}{\partial\rho}\right)-\sigma\cos(\phi)E_{\varphi\rho}g=0$
8	$\left(\left(\frac{H_{\rho\rho}}{\rho}+\frac{\partial H_{\rho\rho}}{\partial\rho}\right)+\frac{2H_{\theta\rho}}{\rho}\right)=0$

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