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Gravitation
and
Relativistic Centrifugal Force

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Centrifugal Force usually occurs when a mass is moving not in a straight line. Its value depend on mass m, mass velocity V and the local radius of curvature R .
The direction of the centrifugal force is always in the direction of the radius R .
And the velocity V perpendicular to R
The Newtonian Centrifugal Force F is given by

1] $$F = \frac{mV^2}{R} = \frac{mV^2}{2} \cdot \frac{2}{R} = E_K \cdot \frac{2}{R}$$

Where E_K is the kinetic energy of the moving mass $(V\_|\_R)$.
To find the relativistic centrifugal force let use the relativistic kinetic energy
According to Special Relativity

2] $$E^2 = \left[E_k + E_0\right]^2 = p^2c^2 + E_0^2$$

Where

3] $$E_0 = m_0c^2$$

and

4] $$\beta = \frac{V}{c}$$

And c is the speed of light.
And the relativistic momentum is

5] $$p = mV = \frac{m_0c}{\sqrt{1-\beta^2}}\beta$$

From eq-2 and eq. 5

6] $$E^2 = \left[E_k + E_0\right]^2 = p^2c^2 + E_0^2 = \frac{(m_0c)^2\beta^2}{1-\beta^2}c^2 + E_0^2 = \frac{E_0^2\beta^2}{1-\beta^2} + E_0^2$$

Expanding the last equation we get

7] $$E_k{}^2 + 2E_kE_0 + E_0^2 = \frac{E_0^2\beta^2}{1-\beta^2} + E_0^2$$

That reduces to

8] $$E_k{}^2 + 2E_0E_k - E_0^2\frac{\beta^2}{1-\beta^2} = 0$$

The last equation is a quadratic equation

9] $$ax^2 + bx + c = 0$$

So, the relativistic kinetic energy is

10] $$E_k = -E_0 + \frac{E_0}{\sqrt{1-\beta^2}} = \frac{-E_0\left(\sqrt{1-\beta^2} - 1\right)}{\sqrt{1-\beta^2}}$$

Using Taylor expansion

11] $$\sqrt{1-\beta^2} \approx 1 - \frac{\beta^2}{2} - \frac{\beta^4}{8} - \frac{3\beta^6}{48}$$

We find that

12] $$E_k = \frac{-E_0\left[1 - \frac{\beta^2}{2} - \frac{\beta^4}{8} - \frac{3\beta^6}{48}\right] + m_0c^2}{\sqrt{1-\beta^2}} \approx \frac{E_0\left[\frac{\beta^2}{2} + \frac{\beta^4}{8} + \frac{3\beta^6}{48}\right]}{\sqrt{1-\beta^2}}$$

And from the first term and eq. 3 we find that the energy for low velocity is approximately the Newtonian energy

13] $$E_k = \frac{m_0c^2\frac{\beta^2}{2}}{\sqrt{1-\beta^2}} \approx \frac{mc^2\beta^2}{2} = \frac{mV^2}{2}$$

So the final expression for the relativistic centrifugal force is derived from eq.1 and eq. 10 to be

14] $$F_{centrifugal} = \frac{-2E_0}{R}\left(\frac{\sqrt{1-\beta^2} - 1}{\sqrt{1-\beta^2}}\right)$$

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Conclusions

1] Now, if we see a mass that move not in a straight line we conclude that the mass induce a centrifugal force. According to eq 14 the centrifugal force is zero only when the velocity β is zero (R is finite and less or equal to the radius of the Universe). The unsolved question is: what thus it means zero velocity. Velocity is usually measured relative to an observer. From equation 14 we need an absolute frame of reference not a relative one.

2] According to Newtonian gravitation the force between a planet and the sun is:

$$F = G\frac{mM}{R^2} = G\frac{m_0 c^2 M}{c^2 R^2\sqrt{1-\beta^2}} = G\frac{E_0 M}{c^2 R^2\sqrt{1-\beta^2}}$$

if the gravitational force is balanced by the centrifugal force then,

$$F = \frac{-2E_0}{R}\left(\frac{\sqrt{1-\beta^2}-1}{\sqrt{1-\beta^2}}\right) = G\frac{E_0 M}{c^2 R^2\sqrt{1-\beta^2}}$$

So

$$G\frac{M}{R} = -2\left(\sqrt{1-\beta^2}-1\right)c^2$$

Planets velocity β is derived from the planet movement against background stars
The gravitational constant G is well known
From the last equation we can find the ratio of $\frac{M}{R}$ but not M or R separately
The distance to the Moon is measured with a Laser.

3] According to Coulomb's law the force between two charges is:

$$F = \kappa\frac{qQ}{R^2}$$

If the Coulomb's force is balanced by the centrifugal force then,

$$F = \frac{-2E_0}{R}\left(\frac{\sqrt{1-\beta^2}-1}{\sqrt{1-\beta^2}}\right) = \kappa\frac{qQ}{R^2}$$

and

$$-2E_0\left(\frac{\sqrt{1-\beta^2}-1}{\sqrt{1-\beta^2}}\right) = \kappa\frac{qQ}{R}$$

If we arrange the equation such that all constant are on one side , The radius of rotation depend strongly on velocity β. This velocity is usually relativistic for electrons in atoms

$$R = \kappa\frac{qQ}{-2E_0}\left(\frac{\sqrt{1-\beta^2}}{\sqrt{1-\beta^2}-1}\right)$$

4] In [3] and [4] the mass does not move in a straight line because of gravitational or Coulombs force. The centrifugal force plus the gravitational or Coulombs force sum to zero. So the total force is zero but the mass path is curved

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with a radius R. According to General Relativity the curved path is the "straight line" of space time near black holes stars etc.

5]   On Earth we feel weight

$$F = m_0 g = \frac{-2m_0 c^2}{R}\left(\frac{\sqrt{1-\beta^2}-1}{\sqrt{1-\beta^2}}\right)$$

The acceleration of gravity is $g = 9.81\left[m/\sec^2\right]$, and since velocity is not relativistic

$$g = \frac{-2c^2}{R}\left(\frac{\sqrt{1-\beta^2}-1}{\sqrt{1-\beta^2}}\right) \approx \frac{V^2}{R}$$

Since $g = 9.81\left[m/\sec^2\right]$ and $R = 6378[km]$ the centrifugal force need a velocity of $V = 7.9[km/\sec]$ to cancel weight, but on the equator our velocity is only 465m/sec.

because of that we feel g

$\left(2\pi R/24[hour] = 2\pi \cdot 6378[km]/24[hour] = 465[m/\sec]\right)$

If we move with a velocity of $V = 7.9[km/\sec]$ we feel no weight but the curvature of space is still R.

This example explains why g and why on ground we feel as if we are in an elevator that accelerates upward.

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