New Einstein gravity field equation and New Einstein-Maxwell Equation

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ABSTRACT

In the general relativity theory, we discover New Einstein's gravity field equation. We treats New Einstein-Maxwell Equation. In this time, we can think Kaluza-Klein theory. We need 5 Dimension-New Kaluza-Klein theory that can express New Einstein-Maxwell Equation. Reissner-Nodstrom solution, Cosmology, etc in present Einstein gravity field equation are the same in new Einstein gravity field equation

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1.Introduction

We discover new Einstein gravity field equation.

New gravity field equation is

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = \frac{C_1 \pi G}{c^4} T_{\mu\upsilon} + \frac{C_2 \pi G}{c^4} T^{\lambda}{}_{\mu} g_{\lambda\upsilon}$$

$$T^{\lambda}{}_{\mu} = g^{\lambda\alpha} T_{\mu\alpha},\ C_1, C_2 \text{ is constant}$$

(1)

Eq(1) is

$$(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R)_{;\mu} = \frac{C_1 \pi G}{c^4} T_{\mu\upsilon;\mu} + \frac{C_2 \pi G}{c^4} T^{\lambda}{}_{\mu;\mu} g_{\lambda\upsilon} = 0$$

$$g^{\mu\upsilon}(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R)$$

$$= R - 2R = \frac{C_1 \pi G}{c^4} T^{\lambda}{}_{\lambda} + \frac{C_2 \pi G}{c^4} T^{\lambda\upsilon} g_{\lambda\upsilon} \ , g^{\mu\upsilon} T^{\lambda}{}_{\mu} = T^{\lambda\upsilon}$$

$$= \frac{C_1 \pi G}{c^4} T^{\lambda}{}_{\lambda} + \frac{C_2 \pi G}{c^4} T^{\lambda}{}_{\lambda} = \frac{(C_1 + C_2)}{c^4} \pi G T^{\lambda}{}_{\lambda}$$

$$R = -\frac{(C_1 + C_2) \pi G}{c^4} T^{\lambda}{}_{\lambda} \tag{2}$$

Hence,

$$R_{\mu\upsilon} = \frac{C_1 \pi G}{c^4} T_{\mu\upsilon} - \frac{1}{2} \frac{(C_1 + C_2) \pi G}{c^4} g_{\mu\upsilon} T^{\lambda}{}_{\lambda} + \frac{C_2 \pi G}{c^4} T^{\lambda}{}_{\mu} g_{\lambda\upsilon} \tag{3}$$

2.Newton limitation and Weak gravity field approximation

In this theory, Newton limitation is

$$g_{\mu\upsilon} \approx \eta_{\mu\upsilon},\ |T_{ij}| << T_{00}$$

$$R_{ij} - \frac{1}{2} g_{ij} R \approx 0 \rightarrow R_{ij} \approx \frac{1}{2} \delta_{ij} R$$

$$R \approx -R_{00} + \sum_{i=1}^{3} R_{ii} = -R_{00} + \frac{3}{2} R$$

$$R \approx 2R_{00} \tag{4}$$

Hence, Newton limitation of Eq(1)

$$R_{0000} \approx 0, R_{i0j0} \approx \frac{1}{2} \frac{\partial^2 g_{00}}{\partial x^i \partial x^j}$$

$$R_{00} - \frac{1}{2} g_{00} R$$

$$\approx R_{00} + \frac{1}{2}R \approx 2R_{00} \approx \nabla^2 g_{00} \approx \frac{C_1 \pi G}{c^4} T_{00} + \frac{C_2 \pi G}{c^4} T^{\lambda}{}_{0} g_{\lambda 0}$$

$$= \frac{C_1 \pi G}{c^4} T_{00} + \frac{C_2 \pi G}{c^4} T_{00} = -\frac{8\pi G}{c^4} T_{00}, T^{\lambda}{}_{0} \approx g^{\lambda 0} T_{00}, g^{\lambda 0} g_{\lambda 0} = g^{00} g_{00} = 1$$

Hence, $C_1 + C_2 = -8$ (5)

Weak gravity field approximation is

$$g_{\mu\upsilon} = \eta_{\mu\upsilon} + h_{\mu\upsilon}$$

$$R_{\mu\upsilon} = \frac{C_1 \pi G}{c^4} T_{\mu\upsilon} - \frac{1}{2} \frac{(C_1 + C_2)\pi G}{c^4} g_{\mu\upsilon} T^{\lambda}{}_{\lambda} + \frac{C_2 \pi G}{c^4} T^{\lambda}{}_{\mu} g_{\lambda\upsilon}$$

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4} S_{\mu\upsilon}$$

$$S_{\mu\upsilon} = -\frac{1}{8}[C_1 T_{\mu\upsilon} - \frac{1}{2}(C_1 + C_2) g_{\mu\upsilon} T^{\lambda}{}_{\lambda} + C_2 T^{\lambda}{}_{\mu} g_{\lambda\upsilon}]$$

$$h_{\mu\upsilon}(t, \vec{x}) = \frac{4G}{c^2} \int d^4 x' \frac{S_{\mu\upsilon}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|}$$

$$h_{00}(\vec{x}) \approx -\frac{1}{8} \frac{4G}{rc^2} \int d^3 x' [C_1 T_{00} - \frac{1}{2}(C_1 + C_2) T_{00} + C_2 T^{\lambda}{}_{0} g_{\lambda 0}], T^{\lambda}{}_{0} \approx g^{\lambda 0} T_{00}$$

$$= -\frac{G}{2rc^2} \int d^3 x' [C_1 T_{00} - \frac{1}{2}(C_1 + C_2) T_{00} + C_2 T_{00}]$$

$$= -\frac{G}{2rc^2} [\frac{1}{2}(C_1 + C_2) M] \quad , C_1 + C_2 = -8$$

$$= \frac{2GM}{rc^2}$$

$$h_{ij}(\vec{x}) = -\frac{1}{8} \frac{4G}{rc^2} \int d^3 x' [C_1 T_{ij} + \frac{1}{2}\delta_{ij}(C_1 + C_2) T_{00} + C_2 T^{\lambda}{}_{i} g_{\lambda j}], T^{\lambda}{}_{i} \approx g^{\lambda j} T_{ij}$$

$$\approx -\frac{1}{8} \frac{4G}{rc^2} \int d^3 x' [\frac{1}{2}\delta_{ij} \cdot -8 T_{00}] = \frac{2GM}{rc^2} \delta_{ij}, C_1 + C_2 = -8$$

$$c^2 d\tau^2 = -g_{\mu\upsilon} dx^{\mu} dx^{\upsilon} \approx (1 - \frac{2GM}{rc^2}) c^2 dt^2 - (1 + \frac{2GM}{rc^2}) \delta_{ij} dx^i dx^j \qquad (6)$$

In Eq(3), if $T_{\mu\upsilon} = 0$,

$$R_{\mu\upsilon} = 0 \qquad (7)$$

The solution of Eq(7) is Schwarzschild solution.

$$c^2 d\tau^2 = -g_{\mu\upsilon} dx^\mu dx^\upsilon = (1 - \frac{2GM}{rc^2}) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \qquad (8)$$

3. New Einstein-Maxwell Equation

New gravity field equation is

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = \frac{C_1 \pi G}{c^4} T_{\mu\upsilon} + \frac{C_2 \pi G}{c^4} T^\lambda{}_\mu g_{\lambda\upsilon}$$

$$T^\lambda{}_\mu = g^{\lambda\alpha} T_{\mu\alpha}, \; C_1, C_2 \text{ is constant} \qquad (9)$$

Energy-Momentum Tensor of Electro-magnetic field in General Relativity is

$$T^{\mu\upsilon} = \frac{1}{4\pi} (F^\mu{}_\rho F_\upsilon{}^\rho - \frac{1}{4} g^{\mu\upsilon} F_{\rho\sigma} F^{\rho\sigma})$$

$$T_{\mu\upsilon} = \frac{1}{4\pi} (F_{\mu\rho} F_\upsilon{}^\rho - \frac{1}{4} g_{\mu\upsilon} F_{\rho\sigma} F^{\rho\sigma})$$

$$T^\lambda{}_\mu = g^{\lambda\alpha} T_{\mu\alpha} = \frac{1}{4\pi} (g^{\lambda\alpha} F_{\mu\rho} F_\alpha{}^\rho - \frac{1}{4} g^{\lambda\alpha} g_{\mu\alpha} F_{\rho\sigma} F^{\rho\sigma}) \qquad (10)$$

Hence, New Einstein-Maxwell Equation is

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R$$

$$= \frac{C_1 G}{4c^4} (F_{\mu\rho} F_\upsilon{}^\rho - \frac{1}{4} g_{\mu\upsilon} F_{\rho\sigma} F^{\rho\sigma}) + \frac{C_2 G}{4c^4} (g^{\lambda\alpha} F_{\mu\rho} F_\alpha{}^\rho - \frac{1}{4} g^{\lambda\alpha} g_{\mu\alpha} F_{\rho\sigma} F^{\rho\sigma})$$

$$C_1 + C_2 = -8 \qquad (11)$$

$$F^{\mu\upsilon}{}_{;\upsilon} = 0$$

$$\frac{\partial F_{\mu\upsilon}}{\partial x^\rho} + \frac{\partial F_{\upsilon\rho}}{\partial x^\mu} + \frac{\partial F_{\rho\mu}}{\partial x^\upsilon} = 0 \qquad (12)$$

4. Conclusion

Therefore, we discover new Einstein-Maxwell Equation. In this time, we can think Kaluza-Klein theory.

The metric tensor of 5 Dimension-Kaluza-Klein theory is

$$\overline{g}_{MN} = \begin{pmatrix} g_{\mu\upsilon} + \kappa\phi A_\mu A_\upsilon & -\kappa\phi A_\mu \\ -\kappa\phi A_\upsilon & \phi \end{pmatrix}, \; \overline{g}^{MN} = \begin{pmatrix} g^{\mu\upsilon} & \kappa A^\mu \\ \kappa A^\upsilon & \frac{1}{\phi}(1 + \kappa^2 \phi A_\rho A^\rho) \end{pmatrix} \qquad (13)$$

In this time, $(M, N) = (0,1,2,3,4)$

5 Dimension-Kaluza-Klein theory can express the present Einstein-Maxwell Equation. But we need 5 Dimension-New Kaluza-Klein theory that can express the New Einstein-Maxwell Equation.

Reissner-Nodstrom solution, Cosmology, etc in present Einstein gravity field equation are the same in new Einstein gravity field equation

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