The Gödel Universe and the Einstein-Podolsky-Rosen Paradox

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Abstract

Notion of causality in Gödel universe, is compared with what is implied by the Einstein-Podolski-Rosen (EPR) type experiments of quantum mechanics. Red shift of light from distant galaxies is explained by employing Segal's compact time coordinate - in the Gödel universe - which indeed was considered by Gödel in his seminal paper. Various possibilities for the rotation of the universe are discussed. It is note worthy that in recent years, the scope of research on Gödel univese, has expanded to include topics in string theory, supersymmetry and embedding of black holes.

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1 Introduction

In this paper, certain issues relating to Gödel universe [1], namely - (1) causality, (2) cosmological red shift and (3) universe's rotation - are addressed. Causality in Gödel universe is related to what is implied by Einstein-Podolsky-Rosen (EPR) [2] type experiments of quantum mechanics [3, 4] [1]. Cosmological red shift is explainable by Segal's [5] method of compatifying the time coordinate or by combining expansion and rotation - as in the Gödel-Obukhov universe [6]. Various possible approaches to issue of rotation are discussed.

Renewed interest in Gödel universe [1], was triggered by the papers [7, 8, 9], which showed that it was a solution of the string theory. There followed a number of papers in string theory and supersymmetry with the Gödel universe as the base space. Their bibliography is extensive and is omitted here. Some of them examined the causality issue due to presence of Closed Time-like Curves (CTCs). At the same time, it was found that various black holes could be embedded in the Gödel universe (for instance, see [10, 11] and references therein).

Earlier set of papers, concentrated on geodesics, isometries and Closed Time-like Curves (CTCs) of the Gödel universe. Camci [12] gives an excellent bibliography of these. Grave et.al [13] investigate visualization in Gödel universe. Various quantum fields [14] in Gödel universe have also been investigated.

2 Comparasion of Causality in EPR experiments and Gödel universe

Experiments in quantum mechanics - with the recent ones being by Hensen et. al. [3], Salart et. al. [4]; show that Bell's inequalities [15] are indeed violated. These experiments are the physical realization of the Gedanken experiments visualized in the landmark paper by Einstein-Podolsky and Rosen [2]. Wiseman [16] discusses implication of these experiments with concept of local realism. The usual view of causality is based upon light cones [17], - i.e., a point x is causally related to another point y if either x lies in the

[1] Coincidentally, Gödel was a friend of Einstein, Podolsky and Rosen, at the Institute of Advanced Studies, Princeton

future or the past light cone of y. If separation between x and y is spacelike, then there is no causal connectivity. However, as shown below - in Gödel universe, points which are locally seperated by a space-like distance can still have causal connectivity using **multiple intersecting light cones**. This therefore shows that, causality in Gödel universe and in EPR type experiments of Quantum Mechanics can be related - i.e., the results of EPR type experiments are not paradoxial in Gödel universe - as far as notion of causality is concerned. In the next sub-section, concept of "multiple intersecting light cones" is highlighted in conformal compactification of Minkowski universe. This is followed by a sub-section on extending this idea to Gödel universe.

2.1 Causality as equivalence relationship in conformal compactification of Minkowski Universe

As a prelude consider causal relationship between points in space-time with the topology $S^3 \times S^1$ - namely, the conformal compactification of the Minkowski universe with topology $R^3 \times R^1$. This compactified spacetime will be referred to as $M_{3,1}$. Let V_x^+ and V_x^- be the future and past light cones, respectively - at x. In what follows, the light like boundary ∂V_x^+ of V_x^+ is considered to be included in V_x^+ - i.e.,

$$V_x^+ \rightarrow \partial V_x^+ \cup V_x^+ \tag{1}$$

Consider two points x and y, which are having timelike or lightlike separation. Herein a spacetime point x is considered synonymous with "the event at x". Let xCy, denote the causal relationship "x causes y". In terms of light cones, this implies -

$$y \in V_x^+ \tag{2}$$

It can be shown that postulating transivity for C, also makes it reflexive and symmetric - and thus it becomes an "equivalence relationship". Transitivity means -

$$xCy \;\wedge\; yCz \;\Rightarrow\; xCz \tag{3}$$

which in terms of light cones is -

$$y \in V_x^+ \;\wedge\; z \in V_y^+ \;\Rightarrow\; z \in V_z^+ \tag{4}$$

For simplicity, let the radii of the spatial factor S^3 and the temporal factor S^1 be equal. This implies equivalence of advanced and retarded waves - i.e., the advanced waves are nothing but retarded waves, returning to their origin after circling the universe in a single time cycle. In contrast, in Minkowski universe, advanced waves are interpreted as retarded waves under time reversal - which gives it Time symmetry [18]. In $M_{3,1}$ time reversal need not be evoked to explain advanced waves - which gives it time asymmetry - as far as electromagnetism is concerned. In terms of light cones it means that future and past light cones are identical -

$$V_x^+ \equiv V_x^- \; and \, V_y^+ \equiv V_y^- \tag{5}$$

Thus xCx and C becomes a reflexive relationship. Now, equation (2) implies -

$$x \in V_y^- \tag{6}$$

which in conjunction with equation (5) implies -

$$x \in V_y^+ \tag{7}$$

Therefore yCx - i.e., C becomes symmetric. Thus C is - reflexive, symmetric and transitive and therefore an equivalence relationship between points which are seperated by a timelike or lightlike separation.

Next, let A be a spacelike hypersurface in $M_{3,1}$. Let x and y are two points on A - i.e., have spacelike separation. Let z be a point lying in intersection of future light cones at x and y -

$$z \in V_x^+ \cap V_y^+ \tag{8}$$

Clearly, $y \in V_z^-$, and since $V_z^+ \equiv V_z^-$, therefore, $y \in V_z^+$, which means zCy. Since, xCz, we have from transitivity - xCy. By a similar argument yCx.

Hence, C is symmetric for points separated by a spacelike interval. Reflexivity xCx follows from equality of past and future light cones. Therefore, C is an equivalence relatioship for points lying on a spacelike hypersurface A. Thus, there exists a causal relationship between x and y, even though they have spacelike separation. Clearly, this depends upon the existence of the non-empty intersection of V_x^+ and V_y^+. That this is indeed the case, can be easily seen from the fact, that there exists a time $t_0 = \pi r/(2c)$, (where, r is the radius of the spatial S^3 factor, and c is the velocity of light), when the interior of the wave fronts is the complete S^3 universe. The points of intersection of V_x^+ and V_y^+ range from in the time interval $[t_1, t_2]$, with -

$$t_1 = d/(2c) \tag{9}$$

and,

$$t_2 = (2\pi r - d/2)/c \tag{10}$$

where, d is the distance between spatial projections of x and y and c is the velocity of light.

2.2 Causality in Gödel Universe

Here in, Gödel spacetime will be denoted as $G_{3,1}$ - where the subscripts refer to the 3 dimensional space and the 1 dimensional time. Let x and y be two point in $G_{3,1}$ having space-like separation. Sahdev et. al. [19] have showed that any two points in the Gödel universe can be connected by a time-like world line - not necessarily a geodesic. Let γ_{xy} be a such a curve connecting x with y. There exists a series of points $p_1, p_2, p_3, ...p_n$, with their light cones satisfying the following condition -

$$p_1 \in V_x^+, \quad p_2 \in V_{p_1}^+, \quad p_3 \in V_{p_2}^+, \dots p_n \in V_{p_{n-1}}^+, \quad y \in V_{p_n}^+ \tag{11}$$

Thus there follows the following series of causal relationships -

$$xCp_1, \quad p_1Cp_2, \quad p_2Cp_3, \dots p_{n-1}Cp_n, \quad p_nCy \tag{12}$$

From transitivity of C, it follows that xCy. From [19], there exists another time like world line γ_{yx}, connecting y with x. By a similar reasoning , as above, we have yCx - and hence C becomes symmetric. Reflexivity xCx follows trivially. Thus C is also an equivalence relation in $G_{3,1}$. Note that -

$$\gamma_{xy} \cup \gamma_{yx} \tag{13}$$

is a Closed Time-like Curve (CTC).

Where as in $M_{3,1}$, C is an equivalence relation due to S^1 topology of time, in $G_{3,1}$, it is an equivalence relation due to prevalence of Closed Time-like Curves (CTCs) connecting all the points of the whole space-time. This is essentially due to tilting and opening of the light cones [20] - as one travels away from a choosen origin (say x or y).

3 Cosmological Red Shift in Gödel Type Universes

3.1 Segal's model with Compact Time Dimension

In his seminal paper Gödel [1] does consider a closed time (property 5), which means an S^1 topology of time. On the plane perpendicular to the axis of rotation, light rays return to their origin. Thus, a natural time period T for the compactified time coordinate is the time taken by the light rays to return. This time period is [20]-

$$T = \frac{2\pi}{\omega}[\sqrt{2} - 1] \tag{14}$$

where ω is the angular velocity of the universe. A Lorentzian manifold is said to possess Zollfrei metric, if all its null geodesics are closed [21]. If one removes the flat z coordinate the resultant $(2 + 1)$ $-D$ space time becomes a new example of Zollfrei metric [22]. As per author's knowledge, this is the only known example of Zollfrei metric in which the spatial component is having the topology R^2. Other known examples of Zollfrei metric are - $S^n \times S^1$ and $P^n \times S^1$ - in which the topology of spatial factors is closed.

Segal [5] developed cosmology on conformal compactification of Minkowski universe. He was motivated to develop the cosmological redshift for the closed and static universe, with S^3 topology - which was favored by Einstein. He replaced the R^1 temporal factor of Einstein's cylinder with S^1. He showeed that red shift z in $M_{3,1}$ is given by -

$$z = \tan^2 \frac{\tau}{2} \tag{15}$$

where τ is the periodic time parameter. He showed that this relationship was good agreement for red shift of distant quasars.

3.2 The Gödel-Obukhov Universe

Consider the Gödel-Obukhov universe [6] with the line element -

$$ds^2 = dt^2 - 2e^x R(t)dydt - (R(t))^2(dx^2 - \frac{e^{2x}}{2}dy^2 - dz^2) \tag{16}$$

Here $R(t)$ is the scale factor, which gives expansion and the cosmological red shift. As in the FRW universe, the Gödel-Obukhov universe has a singularity at -

$$R(t = 0) = 0 \tag{17}$$

This can be avoided by choosing a scale factor of the form -

$$(R(t))^2 = A \sin \frac{2\pi t}{T} + B \tag{18}$$

which gives an oscillating universe with periodicity T, and without the singularity at $t = 0$. A and B are dimensionless constants, and can be regarded as amplitude and phase of the oscillation of the universe.

4 Universe's rotation, density and Caismir energy

The evidence for rotation of universe in context of polarization of radiation propagating over large (cosmological scale) distances, has been examined by

Birch [23] and by Nodland and Ralston [24]. Jain, Modgil and Ralston [25] have done statistical analysis on Type Ia Supernova data [26] and found that anisotropy in cosmological red shift, predicted in Gödel-Obukhov universe [6], could not be ruled out.

As an interesting example of paradigm shift [27] consider the Gödel-Brahe universe [28] in which the angular velocity of the universe is 7.27×10^{-5} rad/sec. Gödel's equation relating the angular velocity of the univese ω and the density of the universe ρ is

$$\omega = 2(G\rho)^{1/2} \tag{19}$$

where, G is the Gravitational constant. Pluging in the angular velocity in this equation gives universe's density $\rho = 6.3 \times 10^{-3}$ gm/cm^3. This is in marked contrast to the conventional estimate of universe's density, which is $\rho \cong 10^{-29}$ gm/cm^3. On the other hand, when zero point energy of quantum fields is taken into consideration, the density of vacuum becomes 10^{120}. The picture becomes further complicated when dark matter and dark energy is taken into account. The solution proposed in this paper is to use Casimir energy of the compactified Gödel universe as the energy density - with appropriate infrared and ultraviolet cutoffs.

5 Conclusions

5.1 Occham's Razor

Occham's razor [29] can be stated as - "*Among competing hypotheses, the one with the fewest assumptions should be selected*". The ideas present in this paper relate diverse standing physical questions, namely - (1) causality implied by results of EPR experiments, (2) the cosmological red shift, (3) the rotation of universe, (4) Vacuum energy, (5) embedding of local curvature sources (e.g., Schwarszchild and Kerr metric) in back ground of the universe - with a single idea - namely the rotating, acausal Gödel universe.

5.2 A short remark on collapse of Wave function

EPR paradox [2] not only raises the issue of causality, but also the intertwined question, of quantum measurement. Penrose [30] points out that a theory of quantum gravity should also explain EPR results and have a satisfactory method for collapse of wave function. The central theme of this paper needs to be suplemented with such a theory.

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