Divergence-Free Scalar Electrodynamics

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Abstract

We present results of applying our divergence-free effective action quantum field theory techniques to the theory of scalar electrodynamics, describing the interaction of the electromagnetic photon field with a charged scalar. This gives an example of the applicability of our divergence-free methods to a system with Abelian gauge invariance. Results of loop computations are given, demonstrating gauge invariance of the effective vertices. Whereas an infrared-regulating mass parameter is given to the virtual photon, the masslessness of the effective photon is demonstrated as well.

1 Introduction

The divergence-free effective action approach to quantum field theory[1, 2, 3, 4, 5, 6, 7] is a subtle formulation that evades loop divegences in all quantum field theories while preserving fundamental gauge and coordinate invariances. Here we give another illustration that pertains to the theory of a massless Maxwellian (photon) gauge field and massive matter represented by a charged scalar field.

After presenting the Lagrangian and the associated Feynman rules, we give the results of applying our divergence-free methods to several loop computations, suppressing much of the detailed derivations (given in comprehensive reports elsewhere[8]), and conclude with a brief discussion.

2 The Lagrangian of Scalar Electrodynamics and Graphical Rules

The Lagrangian of scalar electrodynamics is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \nabla_\mu\Phi^*\nabla_\mu\Phi - m^2\Phi^*\Phi \tag{1}$$

Here, $F_{\mu\nu}$ is the field tensor, given in terms of the 4-potential by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2}$$

We also have

$$\nabla_\mu \Phi = \partial_\mu \Phi - ieA_\mu \Phi \qquad \nabla_\mu \Phi^* = \partial_\mu \Phi^* + ieA_\mu \Phi^* \tag{3}$$

Here, e is the (dimensionless) electromagnetic coupling constant. Notice that whereas the charged (complex) scalar Φ, Φ^* has a mass m, the photon field A_μ is massless, which fact is associated with Abelian gauge invariance with respect to the infinitesimal transformations (with real parameter ω):

$$\delta\Phi = ie\omega\Phi \qquad \delta\Phi^* = -ie\omega\Phi^* \qquad \delta A_\mu = \partial_\mu \omega \tag{4}$$

The above Lagrangian can be rewritten in the following form

$$\begin{cases} \frac{1}{2} A_\mu \left(\partial^2 \eta_{\mu\nu} - \partial_\mu \partial_\nu\right) A_\nu - \Phi^* \left(\partial^2 + m^2\right) \Phi \\ +ieA_\mu \left(\Phi^* \partial_\mu \Phi - \Phi \partial_\mu \Phi^*\right) \\ +e^2 A_\mu A_\mu \Phi^* \Phi \end{cases} \tag{5}$$

displaying the bilinear, trilinear, and quartilinear terms.

Now according to the scheme of the effective action, the fields would be split like $A_\mu \to A_\mu + \mathcal{A}_\mu$, $\Phi \to \Phi + \phi$, and $\Phi^* \to \Phi^* + \phi^*$, where $\mathcal{A}, \phi, \phi^*$ are *virtual* fields. Accordingly, the virtual vector $\mathcal{A}_\mu$ will be constrained with the (gauge-invariant) condition $\partial_\mu \mathcal{A}_\mu = 0$. Hence, we derive the following basic graphical rules (in Minkowskian momentum space):

- For every internal or bare propagator of the charged scalar (depicted with an arrow) with momentum p, we write

$$\frac{1}{-p^2 + m^2}$$

- For every internal photon propagator (to be depicted by a wavy line), with momentum q, we write

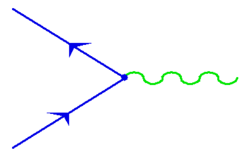
$$\frac{1}{-q^2 + m^2} \left\{ -\eta^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right\}$$

  Notice that we have included a projection operator corresponding to the constraint applied to the virtual vector, and we have included a mass-regulating parameter as well. That the mass-regulating parameter taken here is equal to the mass of the charged scalar should not be of much concern. In general the two masses could be taken different, and the arbitrary mass of the virtual photon would be *the subject of fundamental physical interpretation* in forthcoming works. However, for simplicity in presenting the results of this article, we proceed as indicated above. We stress that the above prescription is gauge invariant.

- For the bare trilinear vertex, we write

$$-e(2p_\mu + r_\mu)$$

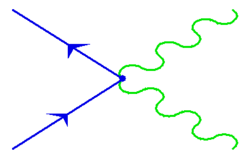
  Here, p is the momentum of the incoming charged scalar, and r is the momentum of the incoming photon, according to the following depiction:



- For the quartic vertex, we write

$$-2e^2\eta_{\mu\nu}$$

corresponding to the following depiction



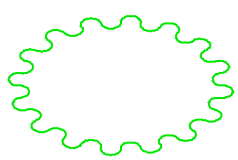
- We must associate a factor of i for each propagator, a factor of i for each vertex, and an overall factor of $-i$ for each graph.
- We must supply the appropriate combinatoric factors for each graph.
- Most importantly, we must supply the appropriate *regularizing parameters* and the corresponding *pole-removing operators*, together with the gamma functions factors, and Feynman parameter combinations, all according to our divergence-free methods.[1]

In the following section, we shall display associated graphics and computational results suppressing all details.

3 Vacuum contributions

3.1 One-Loop Contributions

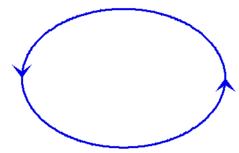
For the one-loop vacuum contributions we have two graphs. The 1st one corresponds to the virtual photon, depicted below:



This gives the result:

$$\frac{3m^4}{128\pi^2}\left\{-3+2\ln(m^2)\right\} \tag{6}$$

The 2nd contribution corresponds to the virtual charged scalar, with the depiction:



And the result:

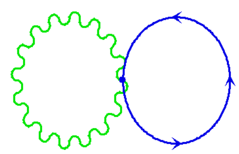
$$\frac{m^4}{64\pi^2}\left\{-3+2\ln(m^2)\right\} \tag{7}$$

Hence the total one-loop vacuum contribution is given by

$$\frac{m^4}{128\pi^2}\left\{-3+2\ln(m^2)\right\} \tag{8}$$

3.2 Two-Loop Contributions

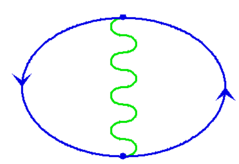
For 2-loop vacuum contributions we have two graphs. The 1st one



gives exactly

$$\frac{3e^2m^4}{256\pi^4}\left\{-1+\ln(m^2)\right\}^2 \tag{9}$$

The 2nd graph



gives (approx.)

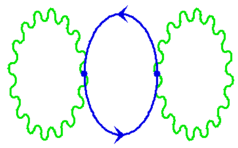
$$-\frac{e^2m^4}{430080\pi^4}\left\{1767-8780\ln(m^2)+8400\ln^2(m^2)\right\} \tag{10}$$

Hence the total 2-loop contribution is given by

$$-\frac{e^2m^4}{430080\pi^4}\left\{-3273+1300\ln(m^2)+3360\ln^2(m^2)\right\} \tag{11}$$

3.3 Three-Loop Contributions

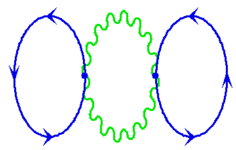
For 3-loop vacuum contributions we have seven graphs. The 1st graph



gives exactly

$$-\frac{9e^4m^4}{4096\pi^6}\left\{-1+\ln(m^2)\right\}^2\left\{1+\ln(m^2)\right\} \tag{12}$$

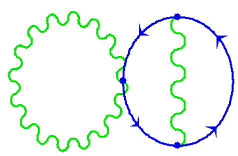
The 2nd graph



gives

$$-\frac{3e^4m^4}{4096\pi^6}\left\{-1+\ln(m^2)\right\}^2\left\{1+\ln(m^2)\right\} \tag{13}$$

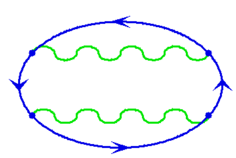
The 3rd graph



gives (approx.)

$$\frac{3e^4m^4}{5734400\pi^6}\left\{10627-17047\ln(m^2)+6420\ln^2(m^2)\right\} \tag{14}$$

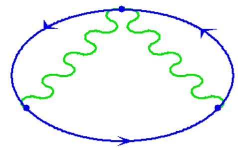
The 4th graph



gives (approx.)

$$\frac{e^4m^4}{20643840\pi^6}\left\{-63747+89597\ln(m^2)+19788\ln^2(m^2)-45360\ln^3(m^2)\right\} \tag{15}$$

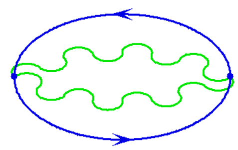
The 5th graph



gives (approx.)

$$\frac{e^4 m^4}{13762560\pi^6}\left\{12381 - 13842\ln(m^2) - 13868\ln^2(m^2) + 15120\ln^3(m^2)\right\} \tag{16}$$

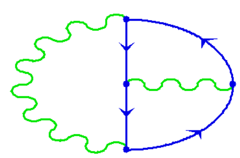
The 6th graph



gives (approx.)

$$\frac{e^4 m^4}{41287680\pi^6}\left\{-21003 + 5693\ln(m^2) + 60900\ln^2(m^2) - 45360\ln^3(m^2)\right\} \tag{17}$$

The 7th graph



gives (approx.)

$$-\frac{e^4 m^4}{5898240\pi^6}\left\{15613 - 44338\ln(m^2) + 25536\ln^2(m^2)\right\} \tag{18}$$

Hence the total 3-loop vacuum contribution is given (approx.) by

$$\frac{e^4 m^4}{206438400\pi^6}\left\{-560309 + 1032359\ln(m^2) + 698760\ln^2(m^2) - 1058400\ln^3(m^2)\right\} \tag{19}$$

3.4 Fixing the Vacuum

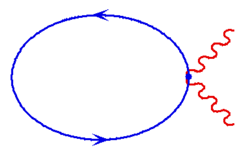
According to the scheme adopted in earlier papers, consistency requires that we must determine the value of $ln(m^2)$ by setting the total vacuum contribution equal to zero, and inverting the perturbative series. The value of $ln(m^2)$ begins with $\frac{3}{2}$ and receives serial contributions in the coupling constant (here e^2). This scheme can be applied easily to the above results (compare with the procedures in other articles[6, 7]).

However, we should note that when more than one mass parameter is present in the vacuum contributions, we must find consistency conditions to relate masses to each other. Basically there must be one central $ln(m^2)$ to be determined perturbatively. On the other hand, physical interpretations of infrared-regulating mass parameters, such as associated with the virtual photon (the virtual gluons and the virtual graviton as well) would be important at the fundamental level. These ideas will be treated much more deeply in other papers.[8]

4 Bilinear Photon Contributions

4.1 One-Loop Contributions

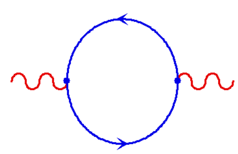
For 1-loop bilinear photon contributions we have two graphs. For the 1st,



we obtain

$$\frac{e^2 m^2}{16\pi^2}\left\{-1+\ln(m^2)\right\}\eta_{\mu\nu} \tag{20}$$

For the 2nd,



we obtain

$$\frac{e^2}{960m^2\pi^2}\left\{\begin{array}{l} r_\mu r_\nu\left\{r^2-10m^2\ln(m^2)\right\} \\ -\eta_{\mu\nu}\left\{r^4-10m^2r^2\ln(m^2)+60m^4\left(-1+ln(m^2)\right)\right\}\end{array}\right\} \tag{21}$$

Here, we have computed the contribution to 4th order in the (external) momentum r of the effective photon.

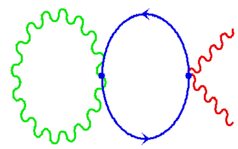
Adding the above two contributions, we obtain

$$\frac{e^2}{960\pi^2}\left\{\frac{r^2}{m^2}-10\ln(m^2)\right\}\left\{r_\mu r_\nu-r^2\eta_{\mu\nu}\right\} \tag{22}$$

This result demonstrates gauge invariance as well as masslessness of the effective photon, at the 1-loop level.

4.2 Two-Loop Contributions

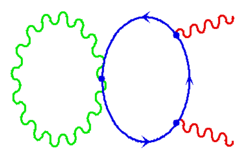
For 2-loop bilinear photon contributions we have eight graphs. Here, we compute to 2nd order in the effective photon momentum. For the 1st graph,



we obtain

$$-\frac{3e^4m^2}{256\pi^4}\left\{-1+\ln(m^2)\right\}\ln(m^2)\eta_{\mu\nu} \tag{23}$$

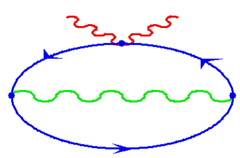
For the 2nd graph,



we obtain

$$\frac{e^4}{512\pi^4}\left\{-1+\ln(m^2)\right\}\left\{r_\mu r_\nu-\eta_{\mu\nu}\left(r^2-6m^2\right)\right\} \tag{24}$$

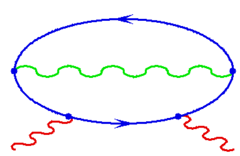
For the 3rd graph,



we obtain (approx.)

$$\frac{e^4m^2}{30720\pi^4}\left\{-439+114\ln(m^2)+360\ln^2(m^2)\right\}\eta_{\mu\nu} \tag{25}$$

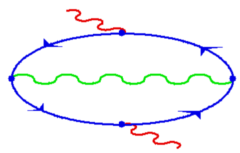
For the 4th graph,



we obtain (approx.)

$$\begin{aligned}&\tfrac{e^4}{1290240\pi^4}\left\{3013-2308\ln(m^2)\right\}r_\mu r_\nu\\&+\tfrac{e^4}{2580480\pi^4}\left\{(-3425+3632\ln(m^2))r^2-48m^2\left(-1049+453\ln(m^2)+630\ln^2(m^2)\right)\right\}\end{aligned} \tag{26}$$

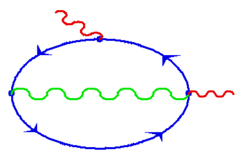
For the 5th graph,



we obtain (approx.)

$$\frac{e^4}{7741440\pi^4}\left\{21613-17532\ln(m^2)\right\}r_\mu r_\nu + \frac{e^4}{7741440\pi^4}\left\{(-37579+32868\ln(m^2))r^2-108m^2\left(-309-394\ln(m^2)+840\ln^2(m^2)\right)\right\} \tag{27}$$

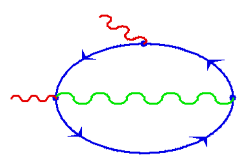
For the 6th graph,



we obtain (approx.)

$$\frac{e^4}{645120\pi^4}\left\{169-97\ln(m^2)\right\}r_\mu r_\nu + \frac{e^4}{2580480\pi^4}\left\{(3557-3164\ln(m^2))r^2+42m^2\left(-439+114\ln(m^2)+360\ln^2(m^2)\right)\right\} \tag{28}$$

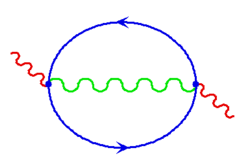
For the 7th graph,



we obtain (approx.)

$$\frac{e^4}{645120\pi^4}\left\{169-97\ln(m^2)\right\}r_\mu r_\nu + \frac{e^4}{2580480\pi^4}\left\{(3557-3164\ln(m^2))r^2+42m^2\left(-439+114\ln(m^2)+360\ln^2(m^2)\right)\right\} \tag{29}$$

For the 8th graph,



we obtain (approx.)

$$\begin{aligned}&\tfrac{e^4}{161280\pi^4}\left\{-57+32\ln(m^2)\right\}r_\mu r_\nu\\&+\tfrac{e^4}{645120\pi^4}\left\{(2307-1886\ln(m^2))r^2+21m^2\left(-259-66\ln(m^2)+360\ln^2(m^2)\right)\right\}\end{aligned} \tag{30}$$

At this point, we note that adding the first two, 2-loop contributions that are obtained exactly, we obtain the gauge-invariant result:

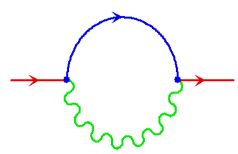
$$\frac{e^4}{512\pi^4}\left\{-1+\ln(m^2)\right\}\left(r_\mu r_\nu-r^2\eta_{\mu\nu}\right) \tag{31}$$

However, the remaining six, 2-loop contributions that have a structure involving overlapping momenta, and that can only be evaluated approximately (as far as we know), cannot be combined into a gauge-invariant result. We shall have more to say about this situation later on and in other articles.[8]

5 Scalar Bilinear Contributions

5.1 One-Loop Contributions

For the 1-loop contributions to the bilinears of the charged scalar matter we have two graphs. For the 1st graph,

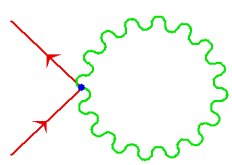


we have the result:

$$\frac{3e^2}{16\pi^2}\left\{1+\ln(m^2)\right\}r^2 \tag{32}$$

Here r is the (external) momentum of the effective scalar field.

For the 2nd graph,



we have the result:

$$-\frac{3e^2}{16\pi^2}\left\{-1+\ln(m^2)\right\}m^2 \tag{33}$$

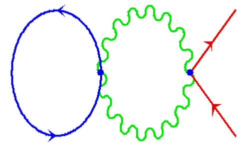
Adding the above two results, we obtain for the total 1-loop contribution to the scalar bilinear:

$$\frac{3e^2}{16\pi^2}\left\{1+\ln(m^2)\right\}r^2-\frac{3e^2}{16\pi^2}\left\{-1+\ln(m^2)\right\}m^2 \tag{34}$$

Needless to tell the alert reader that the first is a correction to the kinetic term, and the second is a corection to the mass term, and that like all computed contributions in our scheme are divergence-free, and that $\ln(m^2)$ can be replaced by its value, obtained by fixing the vacuum, to this order being $\frac{3}{2}$.

5.2 Two-Loop Contributions

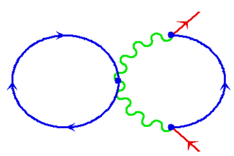
For the 2-loop contributions to the bilinears of the charged scalar matter we compute 10 graphs. For the 1st graph,



we obtain

$$\frac{3e^4m^2}{128\pi^4}\left\{-1+\ln(m^2)\right\}\ln(m^2) \tag{35}$$

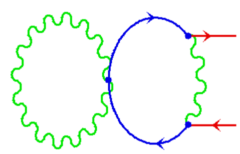
For the 2nd graph,



we obtain

$$-\frac{3e^4}{256\pi^4}\left\{-1+\ln(m^2)\right\}r^2 \tag{36}$$

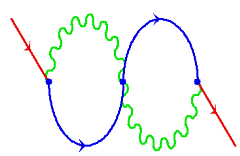
For the 3rd graph,



we obtain

$$-\frac{9e^4}{512\pi^4}\left\{-1+\ln(m^2)\right\}r^2 \tag{37}$$

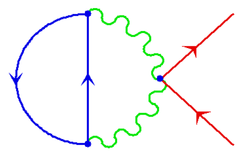
For the 4th graph,



we obtain

$$-\frac{9e^4}{512\pi^4}\left\{-1+\ln(m^2)\right\}^2 r^2 \tag{38}$$

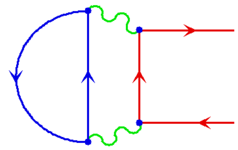
For the 5th graph,



we obtain (approx.)

$$-\frac{e^4 m^2}{7680\pi^4}\left\{73+38\ln(m^2)+240\ln^2(m^2)\right\} \tag{39}$$

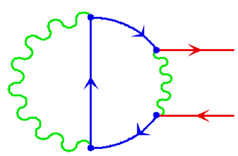
For the 6th graph,



we obtain (approx.)

$$-\frac{e^4}{7680\pi^4}\left\{33-29\ln(m^2)+30\ln^2(m^2)\right\}r^2 \tag{40}$$

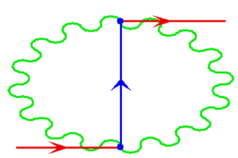
For the 7th graph,



we obtain (approx.)

$$\frac{3e^4}{5120\pi^4}\left\{65+82\ln(m^2)+60\ln^2(m^2)\right\}r^2 \tag{41}$$

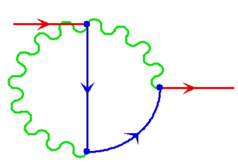
For the 8th graph,



we obtain (approx.)

$$\begin{aligned}&\tfrac{e^4}{122880\pi^4}\left\{403-396\ln(m^2)\right\}r^2\\ &+\tfrac{e^4}{61440\pi^4}\left\{-859-146\ln(m^2)+1080\ln^2(m^2)\right\}m^2\end{aligned} \tag{42}$$

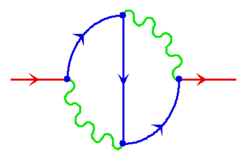
For the 9th graph,



we obtain (approx.)

$$-\frac{e^4}{20480\pi^4}\left\{387+466\ln(m^2)+360\ln^2(m^2)\right\}r^2 \tag{43}$$

For the 10th graph,



we obtain (approx.)

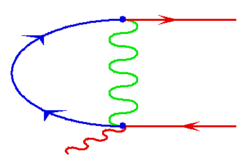
$$\frac{e^4}{61440\pi^4}\left\{-33+226\ln(m^2)\right\}r^2 \tag{44}$$

Adding the above ten results, we obtain for the total 2-loop contribution to the scalar bilinear (approx.):

$$\begin{aligned}&\tfrac{e^4}{20480\pi^4}\left\{-481-630\ln(m^2)+200\ln^2(m^2)\right\}m^2\\&-\tfrac{e^4}{122880\pi^4}\left\{-3607+4292\ln(m^2)+480\ln^2(m^2)\right\}r^2\end{aligned} \tag{45}$$

6 Trilinear Photon-Scalar Contributions

Here we only give the 1-loop contributions. There are 3 implicated graphs. For the 1st graph,

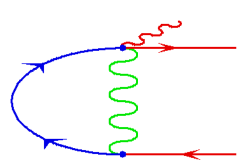


we obtain

$$\frac{3e^3}{16\pi^2}\left\{1+\ln(m^2)\right\}(r_\mu+s_\mu) \tag{46}$$

Here and in the followings, r is the momentum of the incoming charged scalar, and s is the momentum of the incoming photon. We shall give the result to first order in the external momenta, just enough to check the corrections to the fundamental Lagrangian vertices.

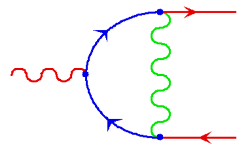
For the 2nd graph,



we obtain

$$\frac{3e^3}{16\pi^2}\left\{1+\ln(m^2)\right\}(r_\mu) \tag{47}$$

For the 3rd graph,



we obtain zero.

The sum of the above contributions gives the total 1-loop contribution to the trilinear vertex:

$$\frac{3e^3}{16\pi^2}\left\{1+\ln(m^2)\right\}(2r_\mu + s_\mu) \tag{48}$$

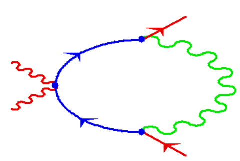
Now, comparing with the 1-loop contribution to the scalar bilinear, namely,

$$\frac{3e^3}{16\pi^2}\left\{1+\ln(m^2)\right\}r^2 \tag{49}$$

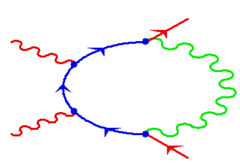
and recalling our momentum-space graphic rules for bare bilinears and trilinears, we realize that the above result is gauge invariant. What this means is that the one-loop corrections to the coordinate-space terms $\partial_\mu\Phi^*\partial_\mu\Phi$ and $ieA_\mu\left(\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right)$ are just the same. The following section will supplement this by computing the correction to $A_\mu A_\mu \Phi^*\Phi$.

7 Quartic Photon-Scalar Vertex Contributions

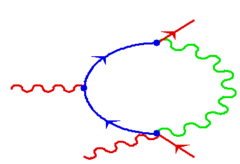
Here we only give the 1-loop contributions to the quartic vertex with two external photons and two external scalars. There are 5 implicated graphs. We shall be interested in contributions without external momenta (no derivatives in the effective vertex), so we can check the corrections to the pertinent bare vertex and the associated gauge invariance. For the 1st graph,



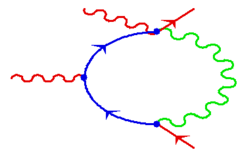
we obtain zero (to the 0th order in external momenta). Also the 2nd graph,



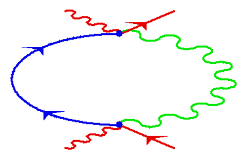
the 3rd graph,



and the 4th graph,



all give zeros. However, the 5th graph,



gives the result:

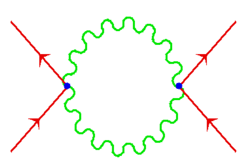
$$\frac{3e^4}{16\pi^2}\left\{1+\ln(m^2)\right\}\eta_{\mu\nu} \tag{50}$$

Comparing with the results for 1-loop bilinears, and the results for trilinears in the preceding section, the above result complements the scene by showing that the one-loop corrections to the coordinate-space terms $\partial_\mu\Phi^*\partial_\mu\Phi$, $ieA_\mu\left(\Phi^*\partial_\mu\Phi-\Phi\partial_\mu\Phi^*\right)$, and $A_\mu A_\mu\Phi^*\Phi$, or rather the components of $\nabla_\mu\Phi^*\nabla_\mu\Phi$, are all the same, verifying gauge invariance.

8 Quartilinear Scalar Vertex

Here we give 1-loop contributions to an effective vertex with four external charged scalars (two of either charge). Notice that this vertex has no bare counterpart in the basic Lagrangian. However, our divergence-free scheme can produce finite results already, and would not need bare counterterms. We have four pertinent graphical contributions. We shall give the results to 2nd order in external momenta (or derivatives in the effective vertex).

For the 1st graph,

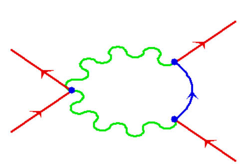


we obtain

$$\frac{5e^4}{64m^2\pi^2}(r+s)^2-\frac{3e^4}{16\pi^2}\ln(m^2) \tag{51}$$

Here and in what follows, r and s are momenta of external Φ and Φ^*, respectively.

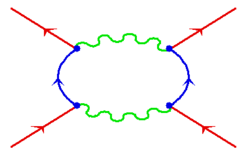
For the 2nd graph,



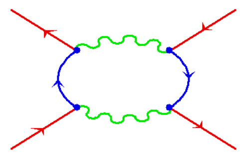
we obtain

$$-\frac{3e^4}{16m^2\pi^2}(r\cdot s) \tag{52}$$

For the 3rd graph,



and for the 4th graph,



we obtain zeros (to the 2nd order demanded in external momenta).

Hence the total 1-loop contribution to the effective quartic scalar vertex is

$$\frac{e^4}{64m^2\pi^2}\left\{5r^2+5s^2-2r\cdot s\right\}-\frac{3e^4}{16\pi^2}\ln(m^2) \tag{53}$$

Notice that (putting $\ln(m^2)\rightarrow\frac{3}{2}$) the last term gives a coupling constant for $(\Phi^*\Phi)^2$ equal to $(9e^4/32\pi^2)$. The first term gives derivative couplings that should be easy to decipher.

9 Discussion

We have presented results of computing loop conributions in the theory of scalar electrodynamics. Our computations are based on the framework of divergence-free quantum field theory and the associated effective action development. It should be clear from our results that our effective action divergence-free approach would preserve gauge invariances, and offers greater simplicity than conventional regularization and renormalization schemes.

However, in the light of our work, we realize the existence of an important issue that requires deeper consideration and perhaps newer strategies. In dealing with higher-loop contributions we notice the existence of certain integrals that (as far as we know) can only be computed approximatively. These are basically the *multiloop integrals with overlapping momenta*. Our inability to compute these integrals exactly seems to stand in the way of achieving perfectly gauge-invariant results.

But the strength of the gauge principle should lead us to devise some strategy for obtaining our needed exact results without going through the ordeal of approximating integrals. Such a strategy would be indispensable when we come to deal with much more complicated and important systems such as non-Abelian gauge theories and Einstein-like gravidynamic theories. Actually, we have been able to formulate certain powerful strategies and make real progress in applying our divergence-free program to the aforementioned important theories.[8]

On the other hand, the (gauge-invariant) inclusion of an infrared-regulating mass parameter in conjunction with virtual gauge fields (photon, gluon, or graviton) seems to us an extremely important step that cannot be avoided. In actual fact, the conventional approach to these theories that tolerates the existence of infrared divergences does seem to be totally inconsistent. Whereas our use, so far, of the infrared-regulating mass is only formal, we seem to have been led to a novel paradigm that endows this approach with a deeper physical content. This would be a subject worth amplification in forthcoming reports.[8]

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