Analyzing the 13S1, 13S2 and 13S3 Normal Modes

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Abstract: The use of specialized filtering techniques allows the identification of three eigen-resonances of the earth near 4800 μHz, which are sensitive to the inner core. Careful programming prevents the formation of interfering intermodulation products and lowers the noise level. The accurately measured frequencies differ by about 1.1% from the PREM predictions.

Introduction

After earthquakes, the Earth vibrates like a bell and a set of different natural frequencies is recorded by various instruments. According to the theory, the inner core of the earth influences the modes ${}_{13}S_1$, ${}_{13}S_2$ and ${}_{13}S_3$ [[1]]. The analysis of those oscillations may serve as a probe to gain more knowledge about its properties like shape, rotation and the type of processes occurring at the boundary between the solid inner core and the liquid outer core.

The Preparation of the data records

In a first step, the raw data (the most common sampling time is one second) of all available SG stations between 2004-12-26 and 2004-12-31 were made machine readable. One week after the earthquake, the amplitudes of most rapid natural oscillations of the earth disappear in the background noise. The influence of atmosphere pressure variation on the gravity data was omitted because the air mass above the instruments changes much slower than the oscillation time of the analyzed normal modes (about 206 seconds).

In order not to worsen the low SNR of the SG records, the very intense tides must be effectively reduced by a comb filter in the *first* processing step. The huge dynamic range of the data (more than 100 dB) plus a safety margin of about 40 dB inside the filter programs must not override the math co-processor of the computer. Otherwise there is a risk that the large dynamic range leads to an inaccurate representation of weak signals with too few significant figures, equivalent to a low measurement resolution. This form of non-linearity may produce unwanted intermodulation products which can not be distinguished from actual spectral lines. The application of a comb filter is very simple: all samples are shifted by 103 seconds and subtracted from the original record. The ratio of this time difference to the period(s) of the oscillations produces the desired effect: The very intense amplitudes of the slow tides ($T \approx$ several hours) are largely compensated, while the amplitude of the much faster ${}_{13}S_x$ oscillations is doubled. A low-pass filter (f < 8 mHz), followed by a second "*shift and subtract*" procedure generates the final data string. A multiple application does not bring further improvement. After the comb filter has reduced the dynamic range of the data, the bandwidth around the region of interest is limited to 1200 μHz and the mean frequency is shifted to the intermediate frequency 1000 μHz, using the mixing method[[2]]. This low value allows to extend the sampling rate of the records to 120 seconds without any loss of information. In order not to further degrade the noisy data and to avoid any distortion of the spectra, nearly all filtering is done with the time consuming convolution with *windowed Sinc* functions. FIR filters are seldom used and IIR filters are strictly forbidden because of their nonlinear phase response and ringing artefacts.

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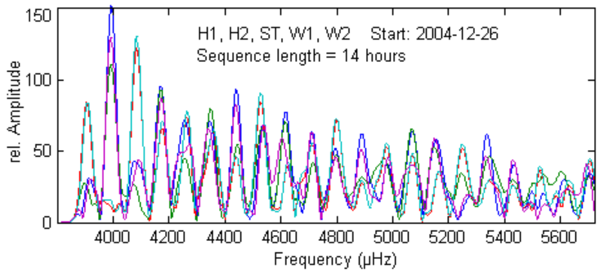
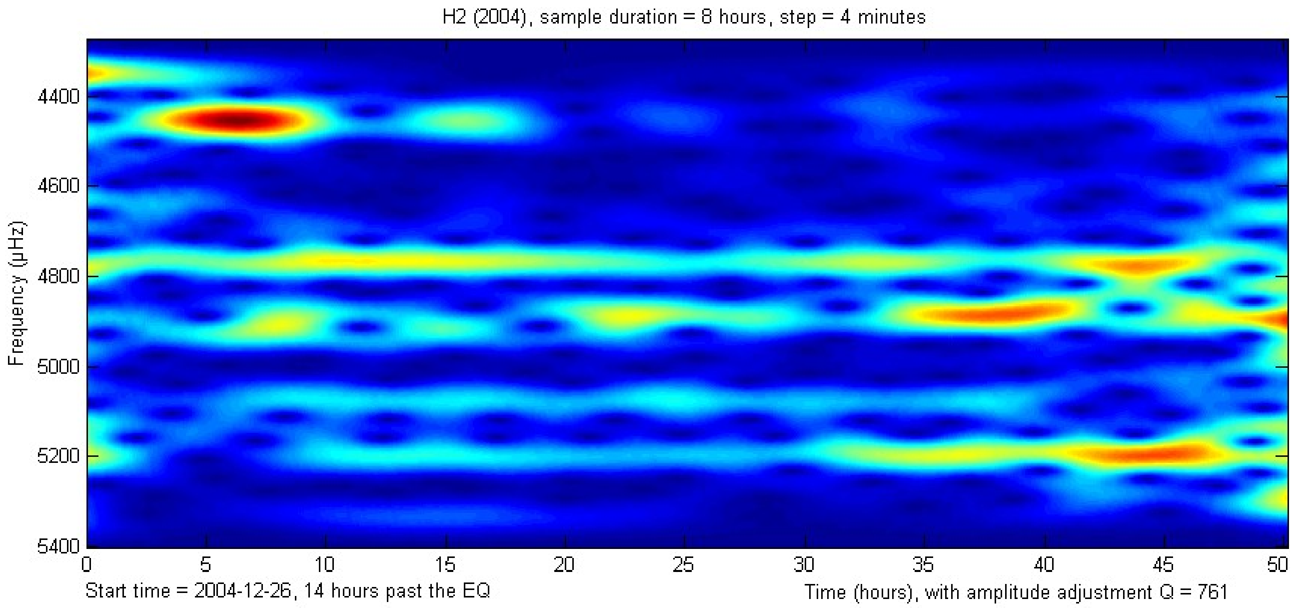
The Spectrogram

After a very strong earthquake, several hundred normal modes are excited which decay at different rates. Some, such as ${}_0S_0$, can be measured even months later, others disappear in the noise after a few hours. The picture shows the spectrum, taken immediately after the earthquake, of the region where - according to theoretical calculations - the natural resonances ${}_{13}S_1$, ${}_{13}S_2$ and ${}_{13}S_3$ are assumed.



All we see is a regular forest of equidistant spectral lines with a mutual distance of about 89 µHz, covering weaker signals. According to the published frequency tables of normal modes[3], these intense lines are generated by the modes ${}_0S_{32}$ to ${}_0S_{52}$, which are of no further interest here.

The PREM model provides an important indication of how the weak ${}_{13}S$ signals still can be found. The ${}_0S_{xx}$ vibrations are strongly attenuated ($Q \approx 140$) and soon disappear in the noise, while the ${}_{13}S$ vibrations with their much higher quality factor ($Q \approx 900$) should be longer measurable. Skipping the first 14 hours after the earthquake, the spectrogram below shows a very different picture.



The all covering picket fence is gone and all natural resonances with low attenuation remain. The spectrogram is composed of 752 individual spectra, each one generated from a sequence of 240 samples (Hamming windowed and then zero-padded to the total length 8192) and whose start times advance in steps of 240 s. The exponential decay of the amplitude has been compensated by the factor $\exp\frac{\omega t}{2Q}$. The value of $Q \approx 760$ gives the most uniform coloring of the peaks. Even more important is the elimination of the exponential modulation, resulting in a substantial reduction of the FWHM and a better selectivity. The relative short sequence length causes a poor frequency resolution, because a spectrogram has the primary task to show the temporal resolution. (Below, the frequencies are measured with more precision.)

The spectral line near 4447 µHz seems to be amplitude-modulated, the maxima follow at intervals of nine hours. The first three peaks have the amplitudes 0.502, 0.247 and 0.115. The resulting value $Q = 620$ is lower than the PREM prediction and the frequency differs markedly from 4497 µHz. The same applies for all other lines in the spectrogram.

The peculiar modulation at 4900 µHz is probably caused by the interference of two closely spaced spectral lines, resulting in temporary extinction.

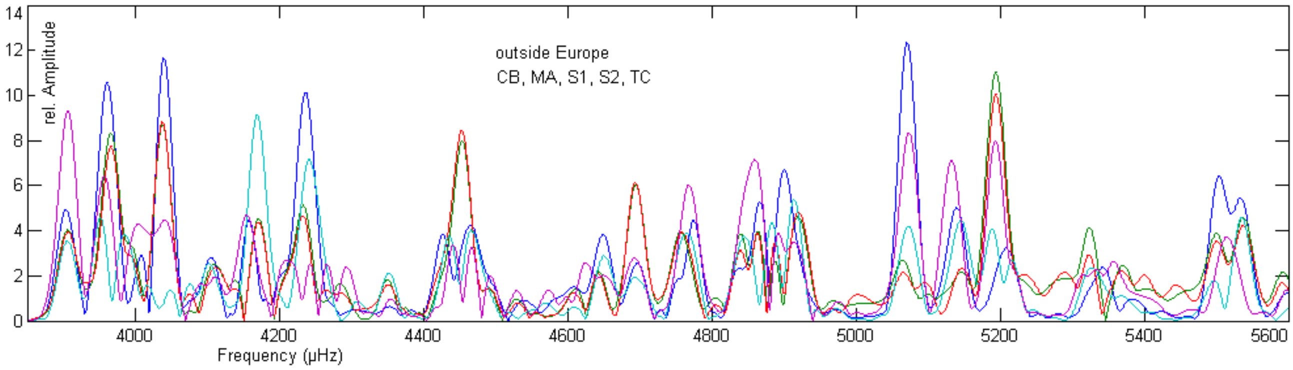
The table lists the average values taken from the best spectrograms and compares with PREM[3].

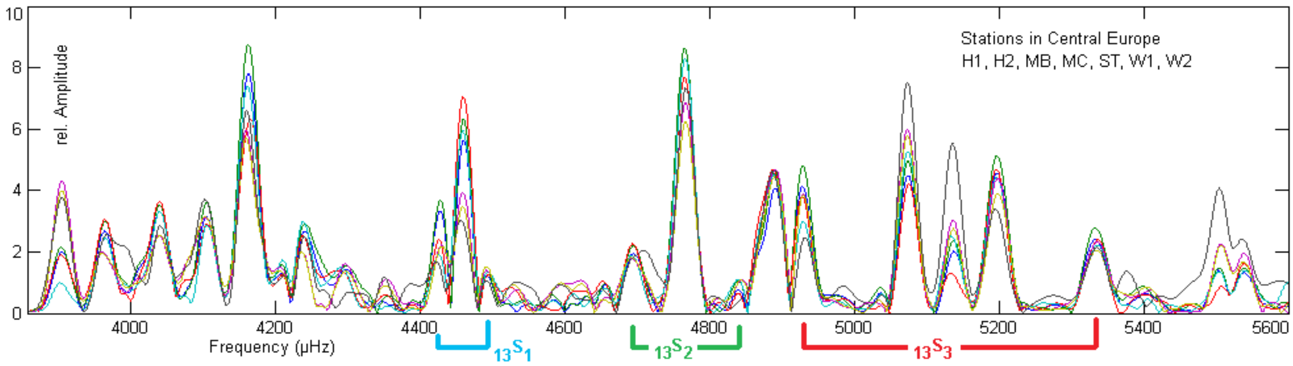
	Spectrogram		PREM		
Mode	Frequency (µHz)	measured Q	Frequency (µHz)	predicted Q	$\Delta f = f_{exp} - f_{PREM}$ (µHz)
13S1	4454	620	4497	735.1	-43
13S2 (m= 0)	4765	760	4846	878.5	-81
??	≈ 4916	≈ 760			
13S3 (m= -1)	5075	700			
13S2 (m= 0)	5142	??	5195	908.6	-53
13S3 (m= +1)	5197	850			
13S2 (m= +2)	5335	≈ 700			

The assignments in the left column are an anticipation of the more accurate spectra, which are shown below. Should this assignment be true, it is noticeable that the predicted frequencies are clearly too high by about 1.1%. The main argument for this assignment is that the attenuation factors of ${}_{13}S_1$, ${}_{13}S_2$ and ${}_{13}S_3$ are substantially higher than the Q-values of other normal modes in this frequency range, and the measured values roughly correspond to the theoretical values[3][1].

Spectra of European and non-European stations

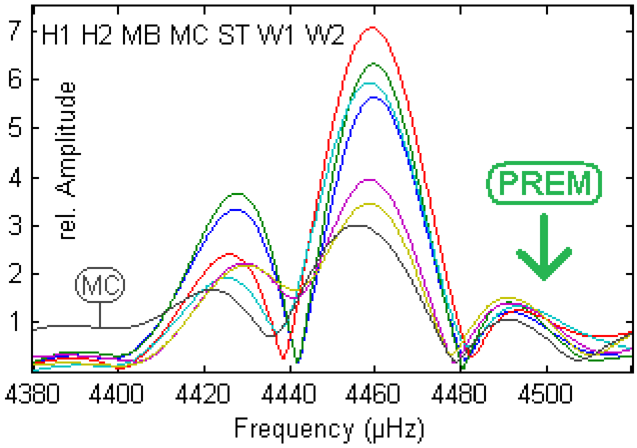
The poor frequency resolution of the spectrogram can be increased by extending the sequence length to 23 hours, resulting in the following twelve spectra.





The very good matching spectra of Central European stations are the basis for the assignment in the left column of the table. Striking is the peculiar and asymmetric waveform around 4900 µHz. The superimposed spectra of the non-European stations show that this area is clearly affected by a very pronounced interference of two different signal sources, leading to irregular results of the FFT and the spectrogram. Perhaps this is cross-coupling or resonance between different modes, which changes the frequencies of the singlets.

$_{13}S_1$ en detail

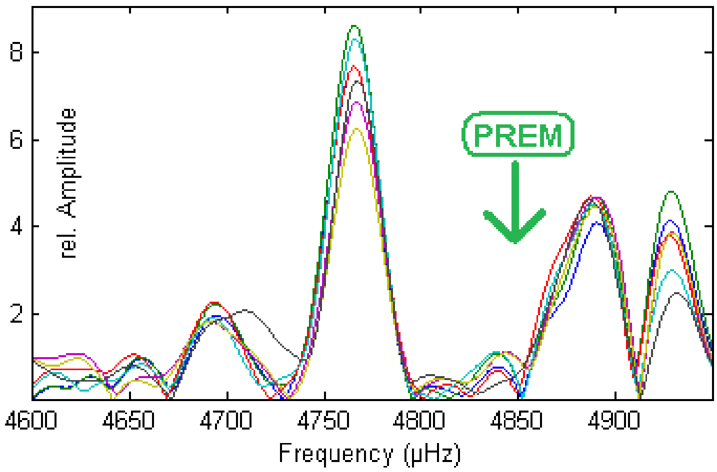


This eigenresonance splits into three components. Their average frequencies are calculated with the jackknife method.

$f\,(_{13}S_1,\ m = -1) = 4426.87 \pm 1.07\ \mu Hz$
$f\,(_{13}S_1,\ m = 0) = 4458.81 \pm 0.49\ \mu Hz$
$f\,(_{13}S_1,\ m = +1) = 4492.21 \pm 0.43\ \mu Hz$

In the spectrogram, the three frequencies can not be separated because their distance (32.67 µHz) from the central carrier is lower than the resolution of the spectrogram. Therefore, the interaction of the two sidebands acts like an amplitude modulation with the rhythm of 8.5 hours. Notably, the proposed frequency by PREM is higher by 0.9% [1][3][4][5][6]. This deviation exceeds by far the error bounds of the measurement and the FWHM of the spectral lines.

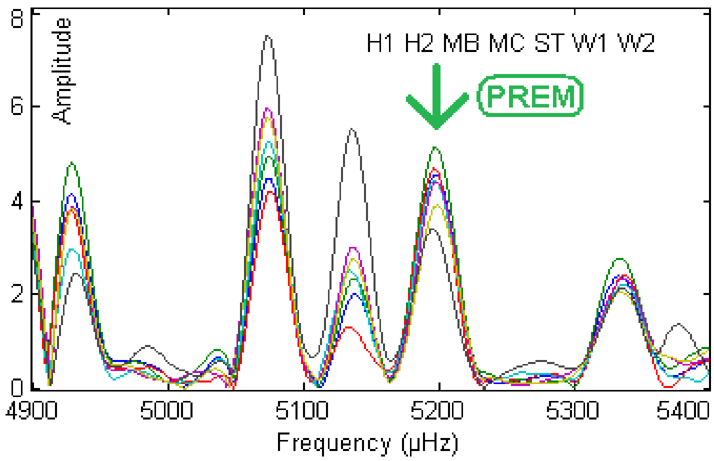
$_{13}S_2$ en detail



This normal mode should split into five lines, but only four can be identified. The two peaks near 4693 µHz and 4841 µHz are symmetrical to the central frequency and most likely correspond to m = ± 1. Probably the line near 4655 µHz corresponds to m = -2. Then the unidentifiable counterpart might cause the peculiar asymmetry of the peak at 4890 µHz.

$f\,(_{13}S_2,\ m = -2) = 4654.80 \pm 0.99\ \mu Hz$
$f\,(_{13}S_2,\ m = -1) = 4693.33 \pm 0.45\ \mu Hz$
$f\,(_{13}S_2,\ m = 0) = 4764.28 \pm 1.29\ \mu Hz$
$f\,(_{13}S_2,\ m = +1) = 4840.92 \pm 0.76\ \mu Hz$

$_{13}S_3$ en detail



This normal mode should split into seven lines, but only five of them can be identified. All four sideband frequencies are symmetrical to the central frequency.

$f\,(_{13}S_3,\ m = -2) = 4929.41 \pm 0.49\ \mu Hz$
$f\,(_{13}S_3,\ m = -1) = 5074.15 \pm 0.27\ \mu Hz$
$f\,(_{13}S_3,\ m = 0) = 5136.22 \pm 0.60\ \mu Hz$
$f\,(_{13}S_3,\ m = +1) = 5197.28 \pm 0.43\ \mu Hz$
$f\,(_{13}S_3,\ m = +2) = 5335.34 \pm 0.59\ \mu Hz$

Summary

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[7]. Almost all the calculations were performed using Matlab. The programs used therein may be requested from the author.

[1] J. Irving, A. Deuss, J. Woodhouse, Normal mode coupling due to hemispherical anisotropic structure in Earth's inner core, Geophys. J. Int. (2009) 178, 962–975
[2] H. Weidner, A New method for High-resolution Frequency Measurements, http://viXra.org/abs/1506.0005
[3] A. Dziewonski, D. Anderson, Preliminary reference Earth model, Physics of the Earth and Planetary Interiors, 25 (1981) 297–356
[4] J. Durek, B. Romanowicz, Inner core anisotropy inferred by direct inversion of normal mode spectra, Geophys. J. Int. (1999) 139, 599–633
[5] J. Irving, A. Deuss, Stratified anisotropic structure at the top of Earth's inner core: A normal mode study, Physics of the Earth and Planetary Interiors 186 (2011) 59–69
[6] A. Mäkinen, A. Deuss, Normal mode splitting function measurements of anelasticity and attenuation in the Earth's inner core, Geophys. J. Int. (2013) 194, 401–416
[7] The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html