Meaning of Twin Paradox and Special Relativity Theory

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Abstract

About the twin paradox of special relativity theory, there are some resolutions. But these might not be the best fit resolution considering the core concept of special relativity theory. Here we will approach the concept of special relativity theory thinking the resolution of twin paradox.

1. Introduction

Typical scenario of twin paradox solution is;

1) Time and space for each of twin is integrated respectively from starting through returning to meeting again,
2) The paradox is recognized resolved by the fact time and space is equal for both of twin when they meet again.

This may admit paradox situation during their travel. If so, this means paradox is not resolved completely.

Here we reconsider this paradox and reasonable resolution.

2. View from s system

We consider about following two systems

s system(2dimensions(ct, x)) and s' system(2dimensions(ct', x')). [1]

Here both are moving relatively with velocity v.

This situation can be shown as Minkowsky graph. (Fig.1)

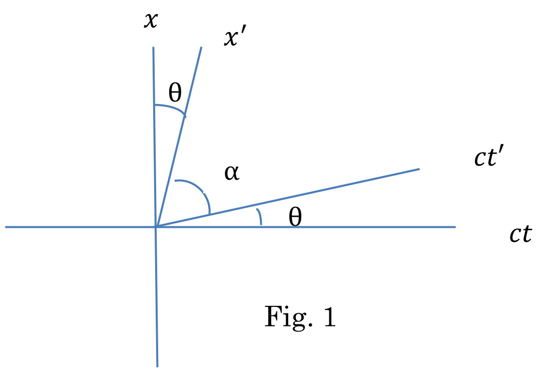


Fig. 1

On Fig.2, point A is a spot time in s' system. Its simultaneous line is $\overline{PQ}$.

Also point B is a spot time in s' system. Its simultaneous line is $\overline{RS}$.

Time between A and B is elapse time in s' system but not in s system. Then regarding to the elapse time, relevant elapse time for fix point of s system is time between T and

U, for example. On this situation, elapse time $\overline{AB}$ for s' system is recognized as elapse time $\overline{TU}$ for s system.

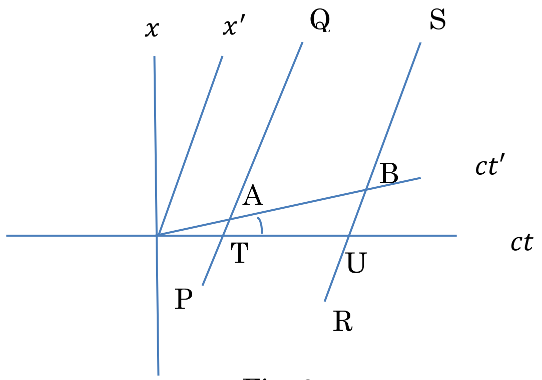


Fig. 2

Lorentz equation is

$$ct' = \frac{ct - \frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (1)$$

$$x' = \frac{-vt + x}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (2)$$

Here we set

ct' value of point A: ct'_A,

ct' value of point B: ct'_B,

ct value of point T: ct_T,

ct value of point U: ct_U,

elapse time $\overline{AB} = t'$

elapse time $\overline{TU} = t$

Here ct' value at T point is equal to ct'_A.

ct' value at U point is equal to ct'_B.

Then from (1),

$$ct'_A = \frac{ct_T - \frac{v}{c}\times 0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (3)$$

$$ct'_B = \frac{ct_U - \frac{v}{c}\times 0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (4)$$

$$t'_A - t'_B = \frac{t_T - t_U}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (5)$$

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (6)$$

(6) has been called time delay of moving object.

This is

$\overline{AB}$ is real existence of time-space distance t' for s' system and s system.

It is moving for s system. Fixed point to s system views it $\overline{TU}$ t.

Then s system views slower time of moving object than real existence.

3. View from s' system

Same as above, we look points of s system from s' system.

On Minkowsky graph. (Fig.3), point A is a spot time in s system. Its simultaneous line is $\overline{PQ}$.

Also point B is a spot time in s system. Its simultaneous line is $\overline{RS}$.

Time between A and B is elapse time in s system but not for s' system. Then regarding to the elapse time, relevant elapse time for fix point of s' system is time between T and U, for example. On this situation, elapse time $\overline{AB}$ for s system is recognized as elapse time $\overline{TU}$ for s' system.

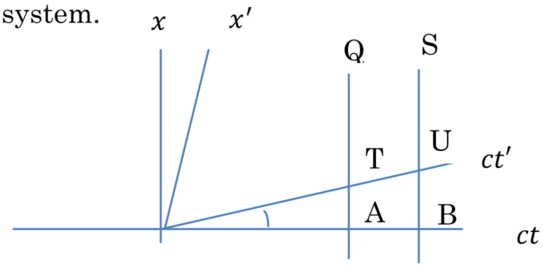


Fig. 3

Lorentz inverse transformation equation is

$$ct = \frac{ct' + \frac{v}{c}x'}{\sqrt{1-\frac{v^2}{c^2}}} \quad (7)$$

$$x = \frac{vt' + x'}{\sqrt{1-\frac{v^2}{c^2}}} \quad (8)$$

Here we set

ct' value of point T: ct'_T,

ct' value of point U: ct'_U,

ct value of point A: ct_A,

ct value of point B: ct_B,

elapse time $\overline{AB} = t$

elapse time $\overline{TU} = t'$

Here ct value at T point is equal to ct_A.

ct value at U point is equal to ct_B.

Then from (7),

$$ct_A = \frac{ct'_T + \frac{v}{c}\times 0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{9}$$

$$ct_B = \frac{ct'_U + \frac{v}{c}\times 0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{10}$$

$$t_A - t_B = \frac{t'_T - t'_U}{\sqrt{1-\frac{v^2}{c^2}}} \tag{11}$$

$$t = \frac{t'}{\sqrt{1-\frac{v^2}{c^2}}} \tag{12}$$

(12) has been called time delay of moving object.

This is

$\overline{AB}$ is real existence of time-space distance t for s system and s' system.

It is moving for s' system. Fixed point to s' system views it $\overline{TU}$ t'.

Then s' system views slower time of moving object than real existence.

4. Time-space distance

On Fig.2 for example, $\overline{AB}$ is time-(space) distance and real existence for s' system. Also $\overline{AB}$ is time-space distance and real existence for s system. For s system, its time element is

$$\frac{\frac{v}{c}(x'_B - x'_A)}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{13}$$

space element is

$$\frac{x'_B - x'_A}{\sqrt{1-\frac{v^2}{c^2}}} \tag{14}$$

based on Lorentz inverse equation.

But actually s system feel time elapse as own time elapse view. It is its time elapse at same position of s system.

One example of it is $\overline{TU}$. It is a projection of $\overline{AB}$ as time elapse for s system. Here real existence of time-space distance is only one or common for every inertia system. On the other hand, every inertia systems have own projection as time elapse.

5. Re-description of the Twin Paradox

Based on above consideration, twin paradox could be re-described as following.

Time elapse t for one of twin is in s system. Time elapse t' for another twin is in s' system.

From s system, time elapse for another twin can be seen as $\sqrt{1-\frac{v^2}{c^2}}t'$ based on the time delay of moving object.

From s' system, time elapse for another twin can be seen as $\sqrt{1-\frac{v^2}{c^2}}t$ based on the time delay of moving object.

On such situation, the twin could be viewed different age each other. This could be contradiction.

Here paradox is regarding to 'one sees' or projection. But because there is only one unique real time-space distance even how one sees it, there is no paradox about the real existence. When they meet each other, two systems become same system. Then at that timing both see same elapse time projection of real time-space distance. Then contradiction regarding to projection could disappear.

6. Another approach

Whole above story is:

-Lorentz transformation is derived on the following definition. [1]

Definition: Time moves toward time direction also toward space direction with speed c. (a)

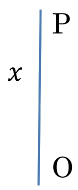
-Time delay of moving object is derived on the Lorentz transformation and Minkowsky graph which draws the relation of the Lorentz transformation.

-Time delay depends on each view to see an object. But real time-space existence is unique. Then twin paradox is resolved on such recognition.

From here, time delay is explained directly on the definition (a) for intuitive understanding

7. Explanation of time delay

There are two point O, P. O is at position 0 and P is at position x.



In the case that point O is moving with velocity v toward to upper, time stamp of staying system including point P is behind $v\frac{x}{c}$ from moving system including point O.
This means time stamp of moving system is ahead $v\frac{x}{c}$ from staying system.
This has been derived from the definition (a) and x-axis for moving system is defined. [1] (Fig.4)

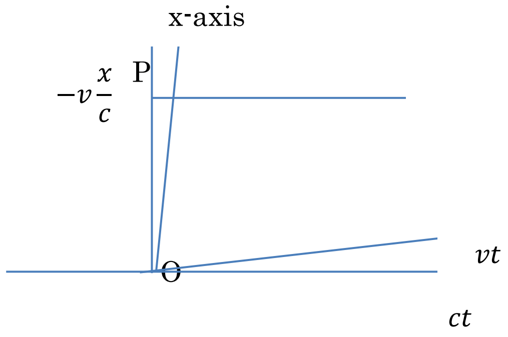


Fig. 4

When time ct passed for point O, point O is at position vt for staying system.
Time of point O of this position is ahead $v\frac{x}{c} = v\frac{vt}{c}$ for staying system.
Staying system views passed time of moving system at own position zero which is same position when time started.
Then elapse time should be behind $v\frac{vt}{c}$

$$ct - v\frac{vt}{c} = ct(1 - \frac{v^2}{c^2}) \tag{15}$$

Here oblique frame of reference indication and scaling should be applied because moving points are on oblique system to staying system. [1] Then (15) is

$$t\left(1 - \frac{v^2}{c^2}\right)\frac{\cos\theta\sqrt{\sin\alpha}}{\sin\alpha} = t\left(1 - \frac{v^2}{c^2}\right)\frac{\cos\theta}{\sqrt{\sin\alpha}} = t\left(1 - \frac{v^2}{c^2}\right)\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = t\sqrt{1 - \frac{v^2}{c^2}}$$

This is delay of moving object.

8. Conclusion

Lorentz transformation is derived on the definition (a). [1]
Time delay of moving object is derived on the Lorentz transformation.
Also time delay of moving object is derived based on the definition directly.
On both ways, twin paradox is resolved.

Reference

[1] Tsuneaki Takahashi, viXra:1611.0077,(http://vixra.org/abs/1611.0077)
" Deductive Derivation of Lorentz Transformation for the Special Relativity Theory"