Counter-Examples to the Kruskal-Szekeres Extension: In Reply to Jason J. Sharples

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FOREWORD

Jason J. Sharples of the University of New South Wales wrote an undated article titled 'On Crothers' counter-examples to the Kruskal-Szekeres extension', in which he asserted, "*The claims relating to the counter-examples made by the author in [1] about the invalidity of the Kruskal-Szekeres extension and the Schwarzschild black hole are completely erroneous.*" However, Sharples failed to understand the arguments I adduced and consequently committed serious errors in both mathematics and physics. In his endeavour to prove me 'erroneous', Sharples introduced what he calls "*an inverted radial coordinate*". Contrary to Sharples' allegation, there is no 'inverted radial coordinate' involved. Sharples simply failed to comprehend the geometry of the problem. Sharples' mathematical proof that I am 'erroneous' violates the rules of pure mathematics. The Kruskal-Szkeres extension and the Schwarzschild black hole it facilitates are fallacious because they violate the rules of pure mathematics.

Jason J. Sharples is an Associate Professor of applied mathematics at the University of New South Wales, Australian Defence Force Academy, in Canberra, Australia. In response to a paper I wrote, titled 'The Kruskal-Szekeres "Extension": Counter-Examples' [1], Sharples penned an undated article titled 'On Crothers' counter-examples to the Kruskal-Szekeres extension' [2]. Sharples' article seems not to have been formally published but Sharples has made it freely available on the World Wide Web, and it is even cited by my critics [3] as 'proof' of errors I have allegedly committed.

Sharples takes exception to my demonstration that the Kruskal-Szekeres 'extension' to produce a black hole is fallacious. In particular he focuses on two counter-examples I presented in my paper [1].

The starting point is the so-called 'Schwarzschild solution' for a 'point-mass',

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

$$0 \leq r \tag{1}$$

where $d\Omega^2 = \left(d\theta^2 + sin^2\theta d\varphi^2\right)$ and $r = \sqrt{x^2 + y^2 + z^2}$. Eqs.(1) are not in fact Schwarzschild's solution [4], but Hilbert's solution [5]. Schwarzschild's actual solution does not permit the black hole. In the prologue to his article Sharples [2] asserts, concerning Eqs.(1) above, that

> "*the stationary, spherically symmetric solution to the Einstein equations corresponding to a point mass, has a removable coordinate singularity at the Schwarzschild radius and a curvature singularity at the location of the point mass*".

Prima facie the line-element of Eqs.(1) appears to be undefined at $r = 2m$ (the 'Schwarzschild radius') and at $r = 0$ (the 'physical singularity' or 'curvature singularity' "*at the location of the point mass*" [2]). Such a superficial finding is based on mere inspection and the false assumptions, first introduced by of D. Hilbert, r is the radius and that $0 \leq r$. When confronted with the line-element of Eqs.(1) there is no reason to suppose that r is the radius, or to suppose $0 \leq r$. Yet according to the cosmologists, the 'Schwarzschild radius' is the radius of the event horizon of a black hole, at which the escape speed is the speed of light, although nothing can leave or escape from the event horizon, including light [6, 7]*. It is not difficult to prove that the 'Schwarzschild radius' is neither the radius nor even a distance in Eqs.(1) [6, 8–10, 12, 13]. Nonetheless, Sharples, as is the wont of cosmology, incorrectly treats r in Eqs.(1) as the radius. Furthermore, in accordance with standard cosmologist practice, Sharples places a 'point-mass' at $r = 0$ in Eqs.(1). Assuming, merely for the sake of argument, that there is a 'point-mass' present in Eqs.(1), it is not located at $r = 0$, but at $r = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2} = 2m$ [6, 8–10, 12, 13], at which the actual radius for Eq.(1) is precisely zero†. Cosmologists lend

*Contrary to the theory of black holes, nothing can have and not have an escape speed simultaneously at the same place.

†There is no mass present in the metric of Eqs.(1) because it is the solution to a set of equations that contains no matter by mathematical construc-

the quantity r in Eqs.(1) many different meaningless, or otherwise platitudinous, names. It has been variously and vaguely called "*the areal radius*" [14–18]*, "*the Schwarzschild r- coordinate*" [19], "*the distance*" [20], "*the radius*" [21–23], "*the radius of a 2-sphere*" [24], "*the radial coordinate*" [16,19,21,25–27], "*the reduced circumference*" [28]†, "*the radial space coordinate*" [30]. To this list must be added Sharples' "*inverted radial coordinate*" [2]. That r in Eqs.(1) goes by so many vagarious names attests to confusion.

The cosmologists maintain that Earth, Sun, and stars each have a 'Schwarzschild radius', but it is buried within them:

> "*For ordinary stars, the Schwarzschild radius lies deep in the stellar interior.*" [26]

> "*For example, a Schwarzschild black hole of mass equal to that of the Earth, $M_E = 6 \times 10^{27}g$, has $r_s = 2GM_E/c^2 \approx$ 1 cm. ...A black hole of one solar mass has a Schwarzschild radius of only 3km.*" [15]

> "*The Schwarzschild radius for the Earth is about 1.0 cm and that of the Sun is 3.0 km.*" [31]

> "*'ordinary' stars and planets contain matter ($T_{\mu\nu} \neq 0$) within a certain radius $r > 2M$, so that for them the validity of the Schwarzschild solution stops there*" [21]

The two counter-examples I presented in [1] are,

$$ds^2 = \left(1 - \frac{2m}{2m - r}\right)dt^2 - \left(1 - \frac{2m}{2m - r}\right)^{-1} dr^2 - (r - 2m)^2\, d\Omega^2 \tag{2}$$

and

$$ds^2 = \left(1 - \frac{2m}{4m - r}\right)dt^2 - \left(1 - \frac{2m}{4m - r}\right)^{-1} dr^2 - (r - 4m)^2\, d\Omega^2 \tag{3}$$

In §2 of his article, Sharples [2] remarks on Eqs.(2) and (3) above,

> "*Crothers [1] calculates that the line-element (2) has a coordinate singularity at $r = 0$ and a point singularity at $r = 2m$. Similarly for the line-element (3), a coordinate singularity is found at $r = 2m$ and a point singularity at $r = 4m$.*"

This is inaccurate. Here is what I wrote [1] concerning Eq.(2) above (i.e. Eq.(3) in [1]):

> "*I now apply to Eq. (3) the very same methods that the astrophysical scientists apply to Eq. (1) and so assume that $0 \leq r < \infty$ on Eq. (3), and that 'the origin' $r = 0$ marks the point at the centre of spherical symmetry of the manifold. By inspection there are two 'singularities'; at $r = 2m$ and at $r = 0$, just as in the case of Eq. (1).*" [1]

Here is what I wrote [1] concerning Eq.(3) above (i.e. Eq.(5) in [1]):

> "*Once again, applying the very same methods of the astrophysical scientists, assume that $0 \leq r < \infty$ and that $r = 0$ is the 'origin'. Then by inspection there are singularities at $r = 4m$ and at $r = 2m$.*"

In other words, I apply to Eq.(2) and Eq.(3) the very same assumptions that the cosmologists apply when confronted with the line-element of Eqs.(1) above:

1. r is the radius.
2. $0 \leq r$.

The result for Eqs.(2) and (3) is that the 'infinitely dense' point-mass 'singularity' is encountered before the event horizon, making a mockery of the theory of black holes. Sharples [2] dislikes me doing this because it leads to obvious nonsense. However, the very same assumptions on Eqs.(1) produce nonsense, but there it is not obvious, which was the point of my paper [1].

Since Eq.(2) and Eq.(3) herein both satisfy all the conditions set by Einstein for a solution to his so-called 'field equations of gravitation in the absence of matter' [29], $R_{\mu\nu} = 0$, they must be equivalent to Eqs.(1). Sharples, in the usual fashion of cosmology, failed to understand this point. Consequently, the assumptions that r is the radius and that $0 \leq r$ apply to Eqs.(1), (2) and (3), are false. In Eqs.(1) the range on r is $2m \leq r$ and the metric of Eqs.(1) is undefined (i.e. singular) only at $r = 2m$. Sharples rewrites Eq.(2) and Eq.(3) above, together, in the following form,

$$ds^2 = \left(1 - \frac{2m}{\rho}\right)dt^2 - \left(1 - \frac{2m}{\rho}\right)^{-1} d\rho^2 - \rho^2 d\Omega^2 \tag{4}$$

where $\rho = M_0 - r$, $-\infty < \rho \leq M_0$, and M_0 takes the value of $2m$ or $4m$ to recover the line-elements Eq.(2) and Eq.(3) respectively. Note that Eq.(4) above is Eq.(5) in Sharples' article [2]. The line-element Eq.(4) has precisely the same form as the line-element in Eqs.(1). Consequently, ρ cannot take on negative values because r in Eq.(1) cannot take on negative values. Sharples therefore simply truncates ρ to $0 \leq \rho \leq M_0$ with the following incorrect argument:

> "*It is important to note that since $0 \leq r < \infty$, it follows that $-\infty < \rho \leq M_0$. However, to avoid degeneracy of the metric we require that $\rho \geq 0$.*

tion [6, 8–10, 12, 13]. Consequently Eqs.(1) have no physical meaning.

*Because $A = 4\pi r^2$.

†Because $C = 2\pi r$.

Thus $0 \leq \rho \leq M_0$ *and the line-element (5) is seen to represent a subset of the Schwarzschild spacetime.*" [2]

Hence, according to Sharples, Eq.(4) with $0 \leq \rho \leq M_0$ is a subset of Eqs.(1) with its $0 \leq r$. This conclusion is false because in Eq.(2) $r \leq 0$ and in Eq.(3) $r \leq 2m$ [6, 8–10, 12, 13, 32, 33]. Thus, there is no truncation; $2m \leq \rho$ in both cases. This is amplified by the equation,

$$\rho = M_0 - r \tag{5}$$

where $M_0 = 2m$ or $M_0 = 4m$. Consider the case $M_0 = 2m$. Then $\rho = 2m - r$. The assumptions that r is the radius and $0 \leq r$ are incorrect, not just on Eq.(2) and Eq.(3), but also on Eqs.(1). In Eq.(2) $r \leq 0$ so that in Eq.(4) $2m \leq \rho$. In Eq.(3) $r \leq 2m$ so that in Eq.(4) $2m \leq \rho$. It immediately follows from this that the Kruskal-Szekeres 'extension' is fallacious because it violates the rules of pure mathematics [1, 6, 8–10, 12, 13, 32, 33]. Similarly, the Painlevé-Gullstrand extension [32] and all other alleged 'coordinate extensions' used to generate black holes without a 'coordinate singularity' at the black hole 'event horizon', are false [6, 8–10, 12, 13, 33]. Consequently, there are no 'curvature' singularities anywhere in Eqs.(1), (2) and (3).

The correct statement of Eqs.(1) is [6, 8–10, 12, 13, 32–35],

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

$$2m \leq r \tag{5}$$

For a solution to his static, vacuum field, Einstein [29, 36] set the following prescription:

1. It must be static.
2. It must be spherically symmetric.
3. It must satisfy $R_{\mu\nu} = 0$.
4. It must be asymptotically flat.

The following infinite equivalence class satisfies Einstein's prescription:

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2 d\Omega^2$$

$$R_c = (|r - r_0|^n + \alpha^n)^{\frac{1}{n}} \quad r, r_0 \in \Re, \quad n \in \Re^+ \tag{6}$$

where α is a non-negative real constant, and r_0 and n are arbitrary constants. Eqs.(5) constitute an element of this infinite equivalence class. Eq.(2) and Eq.(3) are also elements of the infinite equivalence class. For instance, setting $r_0 = \alpha = 2m$, $n = 1$, $r_0 \leq r$ yields Eqs.(5). Since every element of Eqs.(6) is equivalent, if any element of the infinite equivalence class cannot be 'extended', then no element of the class can be extended. It is immediately clear that Eqs.(6) cannot be extended, as the element $r_0 = 0, n = 2$ amplifies. In this particular case the mathematical theory of black holes requires that the square of a real number must take on values less than zero, which is a violation of the rules of pure mathematics, and therefore false. Since $r_0 = \alpha, n = 1, r_0 \leq r$ (i.e. Eqs.(5)) is an element of the equivalence class, it cannot be extended to Eqs.(1). In the same fashion Sharples' arguments violate the rules of pure mathematics and so they are false.

Sharples ignored the physical arguments I adduced in [1], precluding the black hole. By way of summary thereof,

1. There are forces in General Relativity but gravity is not one of them, because it is 'spacetime curvature'. According to the theory of black holes the finite mass of a black hole is concentrated at its 'physical singularity', where volume is zero, density is infinite, and 'spacetime curvature' is infinite (infinite spacetime curvature $\Rightarrow$ infinite gravity). But no finite mass has zero volume, infinite density, and infinite gravity, anywhere.

2. There is no matter in the 'field equations' $R_{\mu\nu} = 0$ by mathematical construction, and consequently no matter in the solution thereto, because matter cannot be both present and absent by means of the very same mathematical constraint (namely $T_{\mu\nu} = 0$). In the 'field equations' $R_{\mu\nu} = \lambda g_{\mu\nu}$, where λ is the so-called 'cosmological constant', there is no matter present because $T_{\mu\nu} = 0$. The solution to this latter set of equations is de Sitter's empty universe, which has no physical meaning because it is empty, precisely because $T_{\mu\nu} = 0$. Consequently, there is no matter in the universe modelled by $R_{\mu\nu} = 0$, where $T_{\mu\nu} = 0$. Moreover, according to Einstein, matter is everything except his gravitational field, and matter and spacetime are not independent. According to his theory, if there is no matter there is no spacetime and hence there is no universe. This means that the components of his 'field equations' must vanish identically; to yield $0 = 0$, not $R_{\mu\nu} = 0$ or $R_{\mu\nu} = \lambda g_{\mu\nu}$.

3. A material source is inserted *post hoc* into Hilbert's solution by improper insinuation of the Newtonian expression for escape speed; an implicit two-body relation into what is alleged to be a solution for a one-body problem. This is evident when Hilbert's metric is written explicitly with the Newtonian gravitational constant G and the speed of light in vacuo c. Then $2m = 2GM/c^2$ in Eqs.(1) herein. The 'Schwarzschild radius' r_s then reads, $r_s = 2GM/c^2$. Solving this for c yields the Newtonian expression for escape speed; an implicit two-body relation (one body 'escapes' from another body).

Finally, it is worth noting that the so-called Cosmic Microwave Background (CMB) is inextricably intertwined with

Big Bang cosmology. Without the CMB, Big Bang cosmology and all its elements, thus including black holes, disappear. The reasons why the CMB does not exist are simply stated [37]:

1. Kirchhoff's Law of Thermal Emission is false.
2. Due to 1. Planck's equation for thermal spectra is not universal.

Without Kirchhoff's Law of Thermal Emission and universality of Planck's equation, the spectroscopic assignment of a mean temperature to the Universe violates the laws of thermal emission. When Penzias and Wilson assigned a temperature to their residual signal and the theoreticians assigned this signal to the Cosmos, they violated the laws of thermal emission. An interesting fact follows: Nuclear Magnetic Resonance (NMR) and Magnetic Resonance Imaging (MRI) would not exist if Kirchhoff's Law of Thermal Emission were true, and Planck's equation universal, because both NMR and MRI utilise spin-lattice relaxation, which would be impossible if Kirchhoff's Law of Thermal Emission were true. NMR and MRI are thermal processes.

> "*Kirchhoff's Law remains without theoretical or experimental confirmation and is directly refuted by the very existence of clinical MRI.*"
>
> Robitaille [38]

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