Simple theory of thermal failure in electric circuits with a negative temperature coefficient of resistance

I. INTRODUCTION

Using a simple mathematical model, the paper clearly illustrates elevated risks of thermal failure in the electric systems with a negative temperature coefficient of resistance (in particular, for the electronics elements made of carbon and semiconductors). If the temperature coefficient of resistance is positive, the thermal equilibrium exists at any temperature below the melting point. But if the temperature coefficient of resistance is negative, there are three potential cases depending on cooling: (1) for a relatively low cooling rate, a thermal equilibrium is not feasible and the temperature goes up unlimitedly; (2) for a relatively high cooling rate, there are two thermal equilibrium states, stable and unstable; (3) in the borderline case, there is just one unstable thermal equilibrium.

As known, the heat produced in an undercooled electric circuit elevates a risk of thermal failure (for instance, Central Processing Units can generate a notable heat and crash if overheated). Such a risk is higher in the electric circuits with negative temperature coefficients of resistance [1-4], in particular for the elements made of semiconductors (silicon, germanium, etc.). The goal is to illustrate the relevant thermal effects using a simple engineering theory.

II. SIMPLE MATHEMATICAL MODEL

Consider an electrical circuit consisting of a constant voltage source and resistor (FIG. 1) with the electrical resistance

$$R = R_0 (1 + \alpha t) \qquad (1)$$

where R_0 is the resistance at Celsius scale temperature t, α is the temperature coefficient of resistance which is positive for most conductors (the electrical resistance increases with temperature). However for graphite, amorphous carbon, and semiconductors (in particular, germanium), the temperature coefficient of resistance is negative (the electrical resistance decreases with temperature). Let the output thermal power transferred from the resistor into ambient air be described by equation

$$P_{out} = A\,t \tag{2}$$

where A is a positive constant referred to as the surface heat conductance, or coefficient of heat transfer [5].

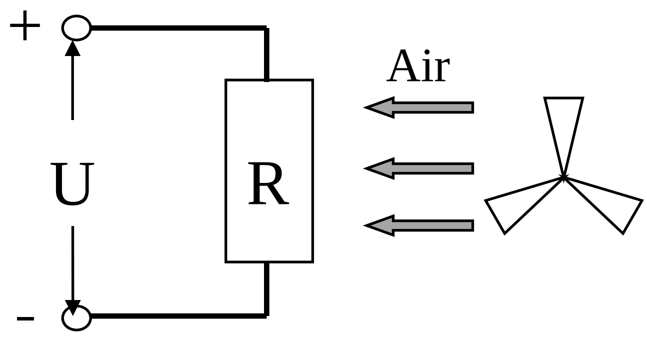

FIG. 1. Simplified model of air- cooled electrical circuit

Eq. (2) is valid if the forced or free thermal convection is a dominant mechanism of heat transfer and the Celsius scale temperature of ambient air equals zero. The latter condition is not principal because the results obtained can be also applied if the ambient temperature t_{amb} is different from zero: in this case, (a) the value t represents the difference between the circuit temperature and ambient temperature, (b) R_0 is approximated by the circuit resistance at the ambient temperature t_{amb}. The heat input is given by

$$P_{in} = \frac{U^2}{R} \tag{3}$$

where U is the constant voltage applied to the resistor. Using Eqs (1) - (3), the condition of thermal equilibrium $(P_{in} = P_{out})$ can be written as

$$\frac{U^2}{R_0(1+\alpha\,t)} = A\,t \tag{4}$$

that can be transformed into quadratic equation

$$\alpha\,t^2 + t - \frac{U^2}{R_0 A} = 0$$

with two roots

$$t_{1,2} = \frac{-1 \pm \sqrt{1+D}}{2\alpha} \qquad (5)$$

representing the thermal equilibrium points. Here, the dimensionless parameter

$$D = \frac{4\,\alpha\,U^2}{R_0\,A}. \qquad (6)$$

It should be noted that such thermal equilibriums may be unstable or even in contradiction to the real physical properties.

Consider two main cases:

(1) the temperature coefficient of resistance is positive $(\alpha > 0)$,

(2) the temperature coefficient of resistance is negative $(\alpha < 0)$.

III. THERMAL EQUILIBRIUM IF $\alpha > 0$

Here, the lower root of Eq. (5) is physically unreal because in this case the electrical resistance given by Eq. (1) gets negative. The higher root of Eq. (5) can be obtained by plotting the left and right parts of Eq. (4) in the 2-D coordinate system for temperature (X) and thermal power (Y). Here, the left part of Eq. (4), describing the input power, is represented by the hyperbola HG. The straight line OC through the coordinate origin stands for the output power defined by the right part of Eq. (4). The intersection point T of the two lines represents the mathematical solution. To the left of point T, the hyperbola HG lies over the straight line OC, so, the heat input exceeds the heat output and therefore the temperature of the resistor should increase to the equilibrium point T. To the right of point T, the hyperbola HG lies below the straight line OC, so, the heat output prevails upon the heat input so that the temperature of the resistor should reduce to the equilibrium point T. Hence, this thermal equilibrium is stable unless the temperature T exceeds the melting point of the resistance material.

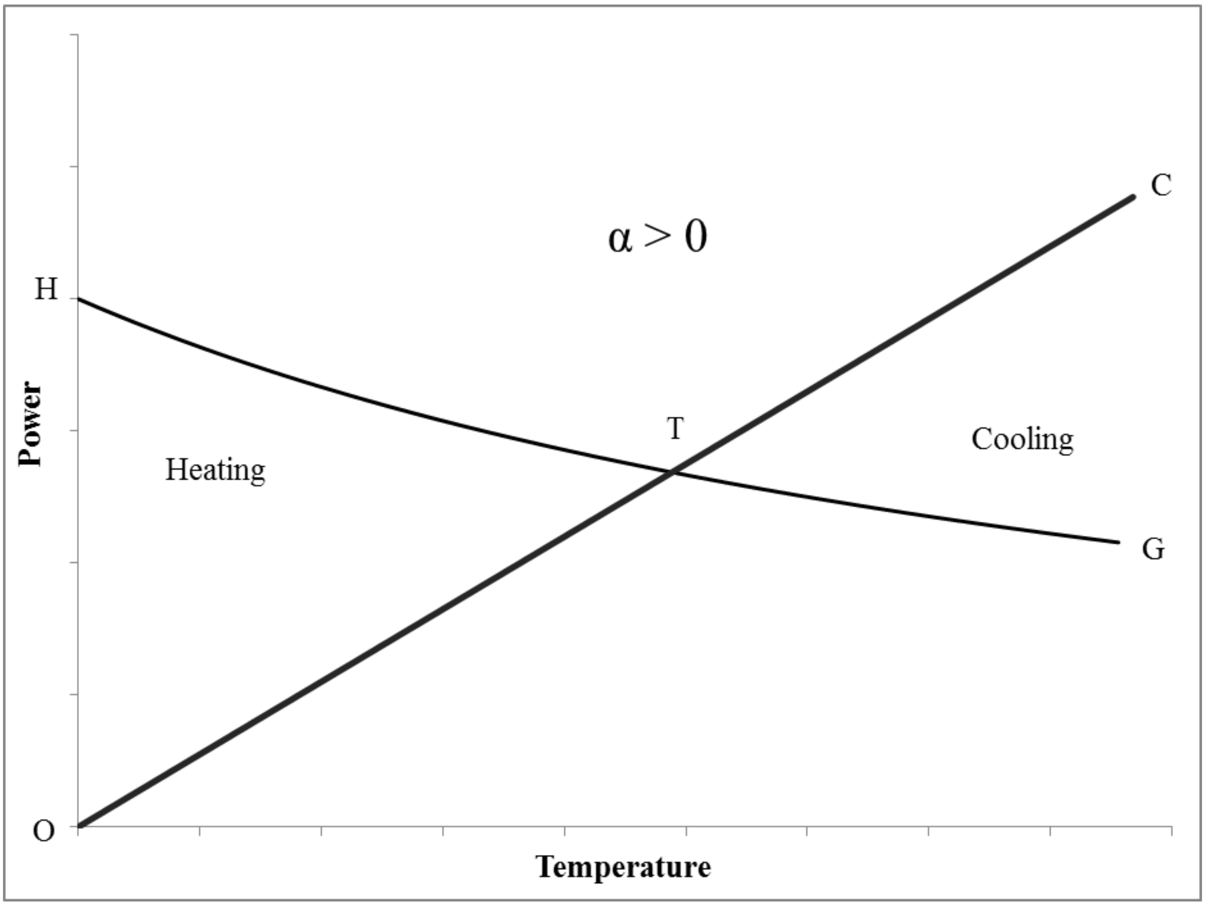

FIG. 2. Graphical solution of Eq. (4) in case if the temperature coefficient of resistance is positive.

IV. THERMAL EQUILIBRIUM IF $\alpha < 0$

Here, there are three straight lines OC_1, OC_2, and OC_3, representing respectively high, moderate (critical), and low cooling regimes. They illustrate the three important mathematical conditions in Eq. (5): $|D| < 1, |D| = 1, \text{and} |D| > 1$ (here, the parameter D given by Eq. (6) is negative because it is proportional to the temperature coefficient of resistance). For the low cooling regime, the heat input exceeds the heat output at any temperature: the straight line OC_3 lies below the hyperbola HG. For the moderate (critical) cooling, the hyperbola HG is over the straight line OC_2 in the whole range, except for point T_3. The temperature to the right of point T_3 can infinitely increase since the heat output prevails upon the heat input. Therefore, the thermal equilibrium at point T_3 is unstable.

In case of the high cooling regime, there are two thermal equilibriums: at the intersection points T_1 and T_2 of the straight line OC_1 and the hyperbola HG. Between the points T_1 and T_2, the hyperbola HG lies below the straight line OC_1; here, the heat output prevails and the temperature should go down. To the left of point T_1 and to the right of point T_2, the hyperbola HG is over the straight line OC_1; here, the heat input exceeds the heat output, so, the temperature should go up.

Hence, the only stable thermal equilibrium is that at point T_1. But a catastrophic thermal failure is inevitable to the right of point T_2 even if this temperature is below the melting point of the material. Such an unfavorable transient process is not described in the monograph [6] but can be considered like a thermal catastrophe.

V. CONCLUSIONS

In contrast to the electric circuits with a positive temperature coefficient of resistance, the circuits with a negative temperature coefficient of resistance (common for electronics) are prone to a higher risk of thermal catastrophe. Basing on a simple straightforward model, the following theoretical results were obtained:

(1) If the temperature coefficient of resistance is positive, a stable equilibrium can be achieved below the melting temperature.

(2) If the temperature coefficient of resistance is negative but the air cooling is high, the thermal equilibrium may formally exist at two temperatures, only one of such equilibrium states (at the lower temperature) being stable. The equilibrium at the higher temperature is unstable with a notable risk of catastrophic thermal failure.

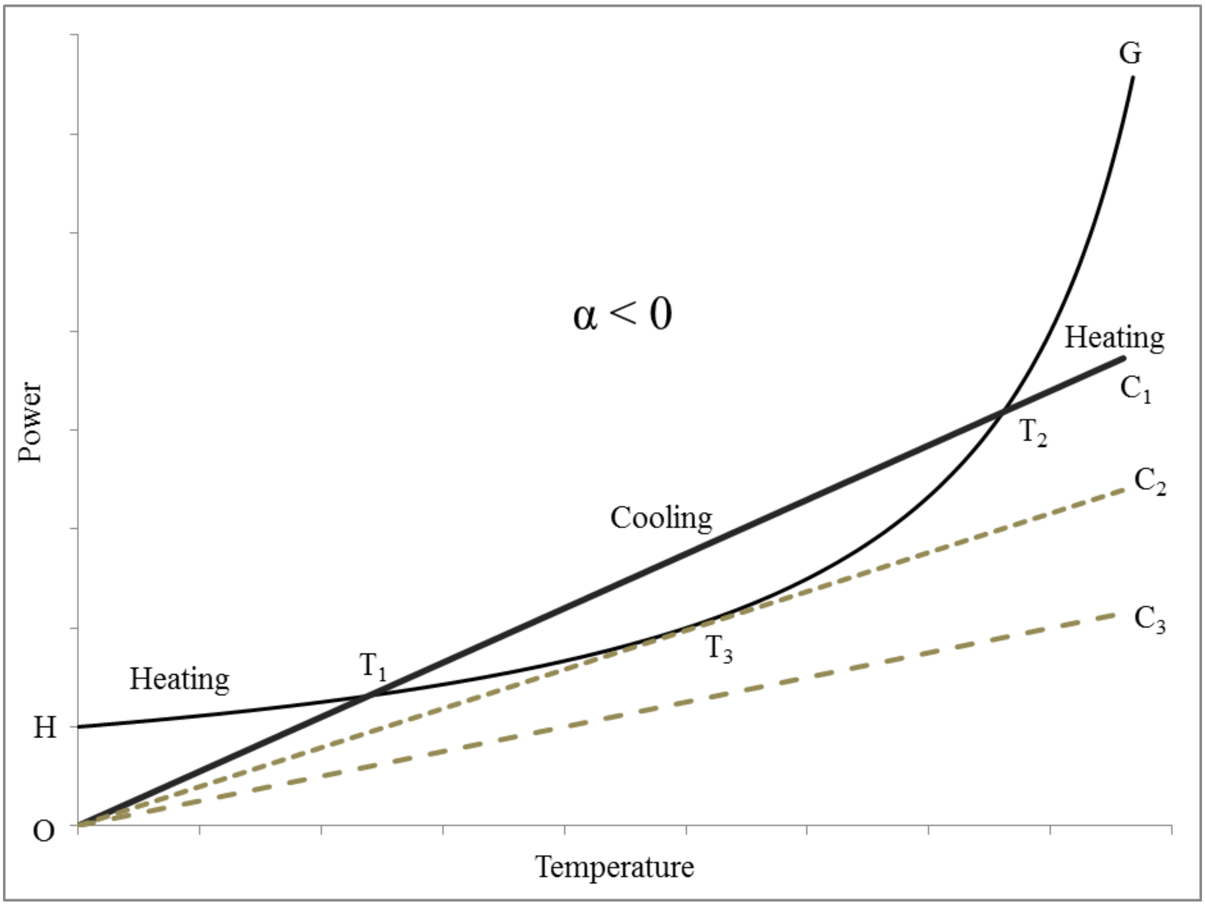

FIG. 3. Graphical solution of Eq. (4) in case if the temperature coefficient of resistance is negative.

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