# Decay of a Black Hole

P. R. Silva – Retired associate professor – Departamento de Física – ICEx – Universidade Federal de Minas Gerais (UFMG) – email: prsilvafis@gmail.com

Abstract- Non-linear Klein-Gordon-Yukawa equations, jointly with time-energy uncertainty relations, are used as a means to estimate the extinction time of a black hole. In a first step we produce the expression originally obtained by Hawking. In a second step we found a massive boson mediating the black hole decay. This boson can also be associated to the size of a vanishingly interacting string. Finally a modification is done in the treatment performed in the first step, going in the direction of Page's results for the black hole decay time.

## 1 – Introduction

Classically: "black holes have no hair", as put forward by John Archibald Wheeler [1], as a means to state that these objects can be completely characterized by only three externally observable classical parameters: mass, electric charge and angular momentum. Indeed according the proper Wheeler [1], this phrase was coined before by Jacob Bekenstein. However, applying quantum mechanics to the study of the problem, Stephen Hawking [2] found that black holes can radiate energy in the same way as done by a black body. This emission of energy by a black hole received the name of Hawking Radiation (HR).

In this work we aim to describe HR in alternative ways usually done in the literature [3]. Section 2 deals with the black hole radiation as a particle decay. There we use a non-linear Klein-Gordon-Yukawa (KGY) equation where two masses scales are first considered: the black hole mass and a particle mass tied to the range of the Yukawa field. Combined with the time-energy uncertainty relation, we find an expression for the black hole decay time. Besides this, relating these two mass to the Planck mass, we recover the Hawking [2] original expression for the black hole lifetime. In section 3, we treat the black hole decay by using another KGY equation, where the decaying occurs mediated by a massive boson. When compared with the result of section 2, we obtain a relation linking the boson and the black hole masses to the Planck mass. In section 4 the mass of the boson is again obtained through of reasonings tide to the length of a string with vanishingly small values of the string coupling. In section 6, the black hole decay time calculations performed in the previous sections are reformulated in order to compare with the Page [4] estimates. Section 7 deals with some concluding remarks.

## 2- Black hole extinction and particle decay

Let us consider the non-linear KGY equation ( $\hbar = c = 1$)

$$\partial^2\phi/\partial r^2 - \partial^2\phi/\partial t^2 = m^2\,\phi - M^2\,\phi^3. \qquad (1)$$

In (1) $\phi$ is a scalar field, M the black hole mass, and m is a very special test particle. Give the Planck mass $M_{Pl}$, these three masses are linked through the relation

$$M\,m = M_{Pl}^2. \qquad (2)$$

We observe from (2) that the reduced Compton length of the particle of mass m is equal to half of the Schwarzschild radius of the black hole.

Putting relation (1) equal to zero and solving for $\phi^2$, we find

$$\phi^2 = m^2/M^2. \qquad (3)$$

Next we consider the time-energy uncertainty relation

$$M\, t_1 = 1. \qquad (4)$$

Thus the width $\Gamma_1$ will be given by

$$\Gamma_1 = (1/t_1)\, \phi^2 = m^2/M. \qquad (5)$$

Therefore, black hole decay time reads

$$\tau_1 = 1/\Gamma_1 = M/m^2 = M^3/M_{Pl}^{\,4}. \qquad (6)$$

In the last equality of (6), we have used relation (2).

We notice that (6) agrees with the expression first obtained by Hawking [H], which stated that: "The black hole would therefore have a finite life of the order of $10^{71}\, (M/M_{sun})^3$ s."

## 3 – A Massive Boson at Work

Let us consider another non-linear KGY equation, writing

$$\partial^2\Psi/\partial r^2 - \partial^2\Psi/\partial t^2 = m^2\, \Psi - M_B^{\,2}\, \Psi^2. \qquad (7)$$

In equation (7), m is the mass of the test particle as before, and $M_B$ is the mass of the boson which mediates the reaction. Making both sides of equation (7) equal to zero, we find that

$$\Psi^2 = (m / M_B)^4. \qquad (8)$$

Next we write a time-energy uncertainty relation for the particle of mass m. We have

$$t_2\, m = 1. \quad (\hbar = c = 1) \qquad (9)$$

Thus, the width of this reaction reads

$$\Gamma_2 = (1 / t_2)\, \Psi^2 = m^5 / M_B{}^4. \qquad (10)$$

We observe that here we are looking at the decay of the particle of mass m. Its lifetime will be given by

$$\tau_2 = 1 / \Gamma_2 = M_B{}^4 / m^5. \qquad (11)$$

4 – Comparing the two descriptions of black hole decay

By considering that $\Gamma_1$ (eq. (5)) and $\Gamma_2$ (eq. (10)) are two different ways of describing the black hole extinction and making the requirement of the equality these two quantities, we get

$$M_B{}^4 = M\, m^3. \qquad (12)$$

Using (2) into (12) and after some simplification, we obtain

$$M\, M_B{}^2 = M_{Pl}{}^3. \qquad (13)$$

From (13) we find that if the boson which intermediates the black hole decay have the mass equal to the electron mass, the mass of the black hole will be approximately 6.2 x $10^6$ $M_{sun}$. However, if the boson mass is equal to the proton mass, the mass of the black hole will be approximately equal to 1.8 times the mass of the sun. In the case of a micro black hole of mass equal to the Planck mass, the three masses collapse in a single value, and the black hole lifetime is only determined by the time-energy uncertainty relation. While the boson mediating the weak interaction is a virtual boson, may be in the present situation it have a real existence and escapes from the black hole in a form of a particle beam.

## 5 – Black holes and strings

In this section we obtain the equation (13) which defines the boson mass, in an alternative way we have done in the last section. Starting from equation (12) and multiplying both sides by the Planck mass, we get

$$M\,(m\,M_{Pl}) = M_{Pl}^3. \qquad (14)$$

Now we identify

$$M_{Pl}\,m = M_B^2. \qquad (15)$$

We notice that this identification recovers equation (13). But, what is a possible interpretation of eq. (15)? In the Flory's model of a polymer [5] in 4-dimensions, if we take $1/m$ as the length of the polymer chain, $1/M_{Pl}$ as the "monomer" size, $1/M_B^2$ corresponds to the radius of gyration of the polymer chain (please see also [6]).

As was pointed out by Kalyana Rama [7]: "It is known that at vanishingly small values of the string coupling g, the length $R_0$ of the string scales as $R_0^2 = a^3 M$, where $a = \sqrt{\alpha'}$ is the string scale, and M is the string mass [8,9]." This relation, given by Rama [7], can be compared with relation (13) of the present work.

## 6 – Toward Page results

The time of evaporation of a Schwarzschild black hole of mass M, was calculated by Page [4] in 1976. For a mass much larger than $10^{17}$ grams, he deduced that electron emission can be neglected, and the black hole evaporates via massless electron and muon neutrinos, photons and gravitons. The time of extinction obtained in this case by Page is given by

$$\tau_{Page} = 8.66 \times 10^{-27} (M/grams)^3 \ s. \qquad (16)$$

Going in the direction of Page results, firs we modify the uncertainty relation

$$M\, t_1 = 2\pi\, \hbar/2 = \pi. \qquad (\hbar = c = 1) \qquad (17)$$

Due to the non-abelian character of the quantum gravity (please see reference [10]), we change the field $\phi$ of equation (1) for

$$\varphi = \pi\, \phi. \qquad (18)$$

Besides this, let us consider the black hole decay as governed by the electromagnetic interaction of strength $\alpha$. We write

$$\Gamma_{new} = (1/t_1)\, \alpha^2\, \varphi^2 = (M/\pi)\, \alpha^2\, \pi^2\, (m/M)^2. \qquad (19)$$

The new black hole extinction time reads

$$\tau_{new} = [1/(\pi\, \alpha^2)]\, (M/m^2) = [1/(\pi\, \alpha^2)]\, (M^3/M_{Pl}^4). \qquad (20)$$

We can also write

$$\tau_{new} \approx 6 \times 10^3\, (M^3/M_{Pl}^4) \approx 6 \times 10^{74}\, (M/M_{sun})^3\ \mathrm{s}. \qquad (21)$$

We observe that (21) gives a time of extinction approximately $6 \times 10^3$ greater than that first obtained by Hawking [2+].

7 – Concluding remarks

Relation (15) of this paper, namely: $M_{PL}\, m = M_B^2$, can be interpreted within the scheme of a Flory's model of a polymer [5,6], where $1/m$ is the length of the chain, $1/M_{Pl}$ is the monomer's size and $1/M_B$ is the radius of gyration of the polymer. Using this same scheme in order to interpret relation (2), namely: $M\, m = M_{Pl}^2$, we verify that we are visiting the trans-planckian region of the problem [1], once we are always taking M greater than $M_{PL}$ in this work.

Finally recovering the ħ and c, we can express relation (6) in the form introduced by Damour [3], where $t_{Pl}$ is the Planck time.

$$\tau = t_{Pl}\, (M/M_{Pl})^3. \qquad (22)$$

References


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