Gedankenexperiment for start of generation of gravitons, in initial cosmology, using V. Balck's Tunneling in cosmology variant of the Wheeler De Witt equation.

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Abstract. We use a quantum tunnelling equation as given by V. Balck in the proceedings "Relativity and Gravitation, 100 years after Einstein in Prague" to delineate a relationship between spatial dimensions, and energy. Afterwards, we then scale the energy as related to formation of primordial black holes, in the onset of the big bang, and how they decay, to evaluate graviton production, and by extension graviton 'particle' induced entropy. The first section affirms that in principle we may have far fewer dimensions than was deemed necessary in string theory. In doing so, we compare the results with earlier work done where we used an argument by Haggard and Rovelli as far as the introduction of quantum effects, in the early universe. Note that Haggard and Rovelli delineated an outer radius as to the range of quantum effects, which extends past the Schwartzshield radius This is defined as 7/3 times the mass of the initial cosmological system. We also have a range of perturbative effects as delineated by Turok's article which gives a range of values of $\tilde{k}_0 \cdot \eta$ for which second order perturbative terms in cosmological evolution may play a role, where we have second order perturbation terms for which $\varepsilon < \tilde{k}_0 \cdot \eta < 1/\varepsilon$. Right afterwards, there are no perturbative behavior and no perturbation if $\tilde{k}_0 \cdot |\eta| \sim 1/\varepsilon$ .These two comparisons, i.e. graviton production and the introduction of quantum effects are contrasted with each other. Finally, we bring up would be entropy issues, if we use infinite quantum statistics, and conflate the number of gravitons, with entropy. The result is an enormous figure, as of say a million primordial black holes contributing, within the purported quantum radius r, as brought up in this document, up to 10^60 , for non dimensionalized entropy. This figure of 10^60 for relic graviton production entropy should be contrasted with present entropy of at least 10^100 or more in the present era.

Key words: Wheeler De Witt Equation, Schwartzshield Radius, Quantum Effects, Infinite quantum statistics, Entropy, Primordial Black holes.

1. Introduction. Arguing for far less dimensions than given in String theory.

We will begin with the observation that V. Balck, in [1], that there is, after changing a Wheeler De Witt equation to read, similarly to the WKB approximation equation, in [1], with the first part given in [1] as

$$\begin{aligned} &\left[-\partial_r^2 - r^{-2}\partial_\phi^2 + .25\cdot r^2\cdot\left(1-\varepsilon\cdot r^2 + 2\varepsilon\cdot\gamma\cdot r^2\cdot\phi^2\right)\right]\Psi = E\Psi \\ &\&\Psi = \sin(\alpha\cdot r\cdot\phi) \end{aligned} \tag{1}$$

Here, if the abbreviation H.O.T. for higher order terms is used, and $|\varepsilon| << 1, |\phi| << 1$, we then have

$$r^2 \sim 4\cdot E + H.O.T. \tag{2}$$

Eq. (2) should be compared with the Virial theorem results, if P.E. ~ k* r^2 of the P.E. being half of the total energy, i.e. Eq. (2) should be compared with [2], and is given more substance in [3], so that

$$2\langle T\rangle = n\langle V_{\mathrm{TOT}}\rangle, if \quad V_{TOT} \sim r^n \tag{3}$$

What the Eq.(2) is arguing is that the initial energy of the universe is directly proportional to a S.H.O. plus some higher order terms, which is astounding, since, Eq. (2) can be reconciled with Eq.(3) only if n ~ 2.

We then need to refer to the approximations as given for energy, and black holes via [4] . According to [4], for times as of about the Big Bang, in Planckian time length, the mass of initial black holes was of the order of , if the mass of the sun is about, say 1.989 × 10^33 g, so the initial B.H.s. were about **10^-38** times smaller in mass than the sun today, with

$$M_{initial.B.H.mass} \sim 10^{-5} grams \tag{4}$$

Then by [4] we have that the initial black holes would have a lifetime of, say, if 1 year = 31536000 seconds

$$\begin{aligned} &\tau\left(M_{initial.B.H.mass}\right) \sim 10^{64}\cdot\left(M_{initial.B.H.mass}/M_{sun}\right)^3 years \\ &\sim 10^{-50} years \sim 3.1536\times 10^{-42}\sec \end{aligned} \tag{5}$$

This would lead to, a decay of a micro sized black hole roughly 8-10 times the length of Planck time, i.e. so small as to indicate, if inflation holds, a regime of space time well before the end of inflation, i.e. inflation allegedly ended approximately 10^-32 seconds after the big bang, so the radii, and the timing of the decay of the micro black holes was well before the end of inflation, i.e. [5] has it graphed out, as well does [6] .

i.e. the decay of the black holes, commences in a region that would be consistent with a multiple of a Planck radii, or of **~ 10^-50 to 10^-55 or so for a scale factor**, as given in [7]

This result, i.e. especially the comparison between Eq. (2) and Eq. (3) argues in favor of a finite dimensional universe. Possibly one with four dimensions and with far fewer dimensions than is alleged by String theory. This may be an artifact of the choices made in Eq. (1) above, but if Eq. (1) receives some experimental confirmation the implications of such are profound. Having said this, we will next discuss the issues brought up as to quantum behavior and its range.

2. Analyzing what is necessary for the quantum bounce, if higher dimensions, are not essential.

We start with what Turok [8] wrote up as to the initial starting point of analysis, as to where he described the cosmological evolution to describe a perfect bounce," in which the universe passes smoothly through the initial singularity". A perfect bounce is a way to describe an interference free, simple matter-energy transition from a prior universe to the present universe. In what we analyze four our purposes, we have that the 2nd order perturbative term of $h^{T(n)}$ for cosmological perturbations obey, here with a 2nd order contribution we can set as

$$\psi^{(2)}(\eta, x) \sim \frac{A^2}{12} \cdot \left[\exp\left(-\frac{2}{\sqrt{3}} i\tilde{k}_0 \eta \right) \right] \cdot (1 + 2\cos(2k_0 x)) + \ldots \tag{6}$$

Which is a 2nd order perturbative term for the equation for the evolution of h, if $J^n(\eta, x)$ is nonlinear

$$\frac{\partial^2 h^{T(n)}}{\partial \eta^2} + \frac{2}{\eta} \cdot \frac{\partial h^{T(n)}}{\partial \eta} - \frac{\partial^2 h^{T(n)}}{\partial x^2} = -J^n(\eta, x) \tag{7}$$

Then setting a conformal time as approaching early universe conditions requires that

$$\eta \xrightarrow[a \to a_{INITIAL} \sim 10^{-55}]{} -10^{\xi}; \xi \approx very \quad big \neq \infty \tag{8}$$

Our supposition is, then that we have the following for well-behaved GW (gravitational waves) and early cosmological perturbations being viable, in the face of cosmological evolution with modifying the formalism of Turok [8] to obtain

$$\tilde{k}_0 |\eta| \sim \tilde{k}_0 \times 10^{\xi} < 1/\varepsilon \Leftrightarrow \tilde{k}_0 < 10^{-\xi} / \varepsilon \tag{9}$$

In practical terms near the initial expansion point it would mean that near the beginning of cosmological expansion we would have an initial energy density of the order of

$$\rho(initial - energy - density) \sim \hbar \cdot 10^{-\xi} / l_P^3 \varepsilon \tag{10}$$

If so then , if we assume that gravitons, of initial mass about 10^-62 grams, i.e. and that we have Planck mass of about 10^-5 grams, if gravitons were the only 'information' passed into a new universe, making use of the following expression for the initiation of quantum effects, i.e. by Haggard and Rovelli [9] of

$$r \sim \frac{7}{3} m \tag{11}$$

We should reflect upon what Eq. (11) is saying. It is stating that quantum effects, in the early universe are proportional to mass, and below, we are bringing up what the particulars of the quantum effect inducing mass should be.

Then, we would have, the initiation of quantum effects as of about [9]

$$r_{entropy-gravitons.contribution} \sim \frac{7}{3} \times S\left(entropy - count\right) \times 10^{-57} \times l_P \tag{12}$$

Then by making use of Eq. (10) we could, by dimensional analysis, start the comparison by setting values from Eq. (9) and Eq. (12) to obtain

$$10^{-\xi} / \varepsilon \sim \frac{7}{3} \times S\left(entropy - count\right) \times 10^{-57} \tag{13}$$

So that to first order, a graviton count, for a radii of about the order of l_P (Planck length, approximately 10^-33 centimeters) would be if we take the entropy as dimensionally scaled by the expression given in Eq. (14) .

$$S\left(entropy - count\right) \sim 10^{57} \times \frac{3}{7} \times 10^{-\xi} / \varepsilon \qquad (14)$$

Depending upon what comes up out of Eq. (10) above as well as $\varepsilon < \tilde{k}_0 \cdot \eta < 1/\varepsilon$, Eq. (14) with its connections to density of energy, and then subsequently to Eq. (12). This will then lead to a condition for which Eq. (6) vanishes, which is the next chapter to consider.

3. Considerations of what could lead to Eq.(6), i.e. 2nd order perturbation to cosmological evolution, vanishing

The simple short course as to the radius achieving its starting point to being quantum mechanical in its effects, from the big bang initiating from a quantum bounce is to have the following threshold for quantum effects to be in action, to the vanishing of Eq. (6). Here the quantum effects start with a value of

$$r(quantum - effects) \sim \left(10^{-\xi} / \varepsilon\right) \times l_P \qquad (15)$$

Note that the term, *l* with subcript p is for the Planck Length. Eq. (15) is indicating that the quantum effects start at the beginning of cosmological expansion.

If Eq.(6) is zero due to $x = r(quantum - effects)$ and we want Eq.(6) to vanish, it leads to the following for the vanishing of the 2nd order perturbative effect, with λ the critical value of wavelength for which Eq.(6) vanishes, i.e. hence , borrowing from the spin offs of [8]

$$\begin{aligned}
&\cos(k_0 \cdot r(quantum - effects)) = -1/2 \\
&\Leftrightarrow k_0 \cdot r(quantum - effects) = \frac{2\pi}{3} \\
&\Leftrightarrow k_0 = \left(\frac{2\pi}{3} \times \frac{\varepsilon}{l_P} \times 10^{\xi}\right) \sim \frac{2\pi}{\lambda} \\
&\Leftrightarrow \lambda \sim \frac{3 \cdot l_P}{\varepsilon} \times 10^{-\xi}
\end{aligned} \qquad (16)$$

It means that there is the following interval may be our best Quantum Mechanical perturbative indicator in terms of Eq. (6) , that is making use of [9] .

$$\frac{l_P}{\varepsilon} \times 10^{-\xi} < x < \frac{3 \cdot l_P}{\varepsilon} \times 10^{-\xi} \tag{17}$$

4. Comparing the variance in position given in Eq.(17) with modified HUP

Note this very small value of spatial variable x comes from a scale factor, if we use a very large red shift [10,11] $z \sim 10^{55} \Leftrightarrow a_{scale-factor} \sim 10^{-55}$, i.e. 55 orders of magnitude smaller than what would normally consider, but here note that the scale factor is not zero, so we do not have a space – time singularity. [10, 11] . The scale factor is 1 in the present era, so this tiny scale factor as given by $z \sim 10^{55} \Leftrightarrow a_{scale-factor} \sim 10^{-55}$, is at the onset of cosmological expansion.

Then

$$\begin{aligned} &\frac{l_P}{\varepsilon} \times 10^{-\xi} < \left(x = \Delta l \right) < \frac{3 \cdot l_P}{\varepsilon} \times 10^{-\xi} \\ &\Delta l \cdot \Delta p \geq \frac{\hbar}{2} \end{aligned} \tag{18}$$

We will next discuss the implications of this point in the next section, of a nonzero smallest scale factor

We will be using the approximation given by Unruh [12] , of a generalization we will write as

$$\begin{aligned} (\Delta l)_{ij} &= \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \\ (\Delta p)_{ij} &= \Delta T_{ij} \cdot \delta t \cdot \Delta A \end{aligned} \tag{19}$$

If we use the following, from the Roberson-Walker metric [10, 11] .

$$\begin{aligned} g_{tt} &= 1 \\ g_{rr} &= \frac{-a^2(t)}{1 - k \cdot r^2} \\ g_{\theta\theta} &= -a^2(t) \cdot r^2 \\ g_{\phi\phi} &= -a^2(t) \cdot \sin^2\theta \cdot d\phi^2 \end{aligned} \tag{20}$$

Following Unruh [12] , write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110}, r \equiv l_P \sim 10^{-35} meters \tag{21}$$

Then, the surviving version of Eq. (12) and Eq. (13) is, then, if [10,11] $\Delta T_{tt} \sim \Delta\rho$

$$V^{(4)} = \delta t \cdot \Delta A \cdot r$$
$$\delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} \geq \frac{\hbar}{2} \tag{22}$$
$$\Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}$$

5. Eq. (22) may, with refinements of r=x , in the four dimensional Volume delineate the new HUP, in our problem

If from Giovannini [13] we can write

$$\delta g_{tt} \sim a^2(t) \cdot \phi << 1 \tag{23}$$

Refining the inputs from Eq. (23) means more study as to the possibility of a nonzero minimum scale factor, as well as the nature of an inflaton like scalar field of ϕ as specified by Giovannini [13]. Then we will assert that if **r=x then** if we use $\frac{l_P}{\varepsilon} \times 10^{-\xi} < (x = r) < \frac{3 \cdot l_P}{\varepsilon} \times 10^{-\xi}$ and then the volume $V^{(4)} = \delta t \cdot \Delta A \cdot r$, as used in [10,11,12]

$$(\delta t \cdot \Delta A) \times \frac{l_P}{\varepsilon} \times 10^{-\xi} < V^{(4)} < (\delta t \cdot \Delta A) \times \frac{3 \cdot l_P}{\varepsilon} \times 10^{-\xi} \tag{24}$$

This Eq. (24) will be put into $\delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}$, if $\Delta T_{tt} \sim \Delta\rho$, it means that $\delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}$ that this is defined for all x as to where and when $\frac{l_P}{\varepsilon} \times 10^{-\xi} < (x = r) < \frac{3 \cdot l_P}{\varepsilon} \times 10^{-\xi}$ holds, with the lower value for x signifying the spatial range of x for which quantum mechanics is valid, with three times that value connected as to when the perturbative methods break down. Thereby influencing the range of values for $V^{(4)} = \delta t \cdot \Delta A \cdot r$ in $\delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}$. Furthermore we have, if there is an eventual weak field approximation according to Katti [14] gravitational spin off according to $g_{ij} = \eta_{ij} + h_{ij}$, with a gravitational wave signal according to, if $V^{(3)} = \Delta A \cdot r$ [10, 11]

$$h_{kl}(x^i) = -\frac{4G}{c^2} \cdot \int_{V^{(3)}} \frac{T^*_{kl}(x'^i)}{R} \cdot d^3x' \equiv -\frac{4G}{c^2} \cdot \int_{V^{(3)}} \frac{T^*_{kl}(x'^i)}{\left[\eta_{ij} \cdot (x^i - x'^i) \cdot (x^j - x'^j)\right]^{1/2}} \cdot d^3x' \tag{25}$$

If the contribution from Pre Planckian to Planckian is due to the stress energy tensor as given in $\Delta T_{tt} \sim \Delta\rho$ form [10, 11] , it means that the relevant relic GW signal will be of the form, with D^{ij} a small quadrupole tensor

$$h_{00}(=h_{44}, Katti) \approx 2G \cdot \left(\frac{m}{r} + \frac{3}{2r^5} \cdot D^{ij} \cdot x_i \cdot x_j \right) \tag{26}$$

The **m** here is the mass of a graviton, times the relic entropy, with entropy given by Eq. (14) with an estimated magnitude of about 10^20 to 10^36. This equation 26, plus its consequences will be examined later on, while we assume, r is the radial distance variable.

Of further interest to the author and potentially others may be the generalization of initial conditions given in reference [15] which may recast the fluctuations and Eq. (26) in a different form later on. Further care must be taken to keep whatever initial conditions and our choice of inputs into Eq. (26) as being in fidelity with [16] experimental considerations of relativity and cosmology. While also reviewing [17]. In addition, it is important to note that fine tuning of Eq. (26) has to take into consideration inputs from [18] as to the epoch making discovery of gravitational waves, by LIGO, for experimental veracity, and that also, the input from Eq.(26) , if suitably dealt with would be vita for the purpose of determination of if scalar-tensor gravity, or General Relativity is the definitive theory of gravity. Dr. Corda's work in [19] will be vital in terms of determination of the significance of both Eq. (25) and Eq.(26) and a through understanding of Eq. (26) and Eq. (25) may enable fuller comprehension of [11] to foundational cosmology and particle astrophysics.

6. Conclusion. What about the Entropy issue, and production of Gravitational Waves and Gravitons?

Through judicious use of [1] and [4] we can, after our analysis point directly to the real life implications of our analysis. We have, as through the dimensional analysis of Eq. (3) and Eq. (4), given an argument that we are forsaking the use of higher dimensions. In doing so, we can go straight to [4] and use directly what is in page 46 of [4] that in figure 3.4 of [4] there is a statement that for a nonrotating black hole that approximately .1% of the mass M of a black hole , if n=0 (no higher dimensions) as we have argued in the beginning will be transferred to gravitons. We specified that there was, indeed for the extremely small black hole, say 10^-5 grams, i.e. for a fast decaying black hole of 10^-5 grams which would disappear before the end of inflation, 10^-5 grams comes to , if a graviton is approximately 10^-62 grams in rest mass, about 10^57 / 1000 ~ 10 ^ 54 gravitons, for black holes which decay within the regime of quantum radii effects , of the universe as given by Haggard and Rovelli [9]. In saying this, each black hole, even if primordial, will if 1/1000 of its mass degenerates as to before the end of inflation, that due to what we have stated, an astounding figure emerges.

1 decaying primordial black hole ~ 10^54 gravitons (27)

Assuming that what Ng postulated as to infinite quantum statistics, this says that even if we abide by the regime of the quantum radii effects, as in [9], that if there are one million black holes initially produced, that the total entropy, initially within the quantum effects radii as in [9] becomes

S (entropy total) ~ 10^6 times 10^ 54 ~ 10^60 (at or before Electro weak) (28)

We argue that this comparatively enormous figure will have cosmological implications which we should explore thoroughly. It is much smaller than the entropy of 10^36 cited as due to quantum fluctuations, which argues that the entropy production needs further study and analysis to reconcile Eq. (28) with Eq. (12)

Acknowledgements

This work is supported in part by National Nature Science Foundation of China grant No. 11375279

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