Conjecture on an infinity of Poulet numbers which are also triangular numbers

Abstract. I was studying the sequences of primes obtained applying concatenation to some well known classes of numbers, when I discovered that the second Poulet number, 561 (also the first Carmichael number, also a very interesting number – I wrote a paper dedicated to some of its properties), is also a triangular number. Continuing to look, I found, up to the triangular number T(817), if we note T(n) = n*(n + 1)/2 = 1 + 2 +...+ n, fifteen Poulet numbers. In this paper I state the conjecture that there exist an infinity of Poulet numbers which are also triangular numbers.

Conjecture:

There exist an infinity of Poulet numbers which are also triangular numbers.

The triangular numbers T(n):
(Sequence A000217 in OEIS)

: 0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300, 325, 351, 378, 406, 435, 465, 496, 528, 561, 595, 630, 666, 703, 741, 780, 820, 861, 903 (...)

The Poulet numbers:
(Sequence A001567 in OEIS)

: 341, 561, 645, 1105, 1387, 1729, 1905, 2047, 2465, 2701, 2821, 3277, 4033, 4369, 4371, 4681, 5461, 6601, 7957, 8321, 8481, 8911, 10261, 10585, 11305, 12801, 13741, 13747, 13981, 14491, 15709 (...)

The sequence of Poulet numbers which are also triangular numbers:

: 561 (T(33)), 2701 (T(73)), 4371 (T(93)), 8911 (T(133)), 33153 (T(257)), 41041 (T(286)), 49141 (T(313)), 93961 (T(433)), 104653 (T(457)), 115921 (T(481)), 157641 (T(561)), 226801 (T(673)), 289941 (T(761)), 314821 (T(793)), 334153 (T(817)...

Observations:

Note the interesting fact that T(561) = 157641 is a Poulet number! Question: are there other Poulet numbers p such that T(p) is also a Poulet number?

Note the numbers obtained when these Poulet, also triangular numbers, are concatenated to the right with 1: there are primes among them (27011, 43711, 410411, 939611, 3341531) or semiprimes x*y having the property that x and y have the same last digit (1 or 9) and sometimes the property that x + y - 1 is prime (5611 = 31*181 and 3 + 181 - 1 = 211, prime; 89111 = 11*8101 and 11 + 8101 - 1 = 8111, prime; 331531 = 19*17449 and 19 + 17449 - 1 = 17467, prime; 491411 = 59*8329 and 59 + 8329 - 1 = 8387, prime; 1046531 = 139*7529; 1159211 = 359*3229; 2899411 = 1601*1811).

Note that many of these Poulet, also triangular numbers, admit a deconcatenation in two primes: 561 (5 and 61), 2701 (2 and 701), 4371 (43 and 71), 8911 (89 and 11), 33153 (331 and 53), 49141 (491 and 41), 157641 (157 and 641), 226801 (2 and 26801), 314821 (3 and 14821).