GENERAL RELATIVITY AND MORPHED GRAVITATIONAL INTERACTION

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Einstein's tensor equation for a homogeneous, isotropic constant curvature universe is simplified to a scalar equation. It is a mass dominated universe. This equation is related to the Hubble's constant and to a nucleus through the uncertainty principle and morphed gravitational interaction. The sizes of some nuclei are obtained and all nuclei have radii of the order of a femtometer.

KEY WORDS; Robertson-Walker metric, constant curvature, Hubble's constant, uncertainty principle, Morphed gravitational potential energy, nuclear size

1.INTRODUCTION

One of the great problems of this century is to unify general relativity and quantum mechanics .General relativity has a negative dimensional coupling constant (Newton's constant) and hence is not renormalizable in the usual sense [1]. Before going into this problem let us recall the procedure that one uses to find the escape speed from a planet with Newtonian gravity. If there is only a planet and a rocket the total energy of the rocket on the surface of the planet is given by

$$E = \frac{1}{2}mv^2 - G\frac{mM'}{R} \ , \qquad (1.1)$$

Where G is the universal constant of gravitation, and R is the radius of the planet. The rocket of mass m will escape the gravitational pull of the planet if the above energy is zero on the planet. The energy will also be zero at an infinite distance from the center of the planet. Whenever E=0, on the planet as in Eq.(1.1) ,the corresponding speed is known as the escape speed. This will be a reminder for what we find below and it will stand in good stead.

Let us assume an ansatz for the metric that solves Einstein's equations, for example, the Robertson-Walker metric. We assume that the metric tensor is radially symmetric, with all angular dependence omitted, and that the time dependence of the metric is represented by a single function of(t) , which sets the scale of the universe and acts as an effective "radius" of the universe. We assume that,

$$ds^2 = dx^\mu g_{\mu\nu} dx^\nu = dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \qquad (1.2)$$

Where k is a constant .The tensor equation of the general relativity reduces to a homogeneous, isotropic, constant curvature universe with a scalar differential equation given by,

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \left(\frac{8\pi G}{3}\right)\rho, \qquad (1.3)$$

Where ρ is the mass density of the universe. Here $\dot{R}$ measures the time rate of change in the size of the universe. The universe looks the same in every direction. It is a mass dominated universe .That means it is a universe where mass rather than other forms of energy constitutes the dominant force. It is this that allowed us to replace Einstein's general energy momentum tensor with a scalar quantity that measures mass.

When k=1,it is a closed universe, when k=0, it is a flat universe, and when k= -1 , the universe is open. In EQ.(1.3) let k be zero and assume that there is only a planet of mass M'.The ratio $\dot{R}/R$ is equal to V/R on the surface of the planet. Here V is Hubble's speed and R is the radius of the planet. This speed is identical to the expression that one obtains from Eq.(1.1)whenever $E = 0$.That means V is the escape speed from the planet which is the only one exerting gravitational force that is due to only mass energy M' of the planet. In other words,

$$\left(\frac{V}{R}\right)^2 = \frac{8\pi G}{3} \frac{M'}{(4/3)\pi R^3} . \tag{1.4}$$

The above expression means a fired rocket of mass m will have a total energy E equal to zero:

$$\frac{1}{2} mV^2 - \frac{GM'm}{R} = E = 0 . \tag{1.5}$$

This means the Hubble's speed is the escape speed from the planet. Equation (1.5) is no different from what we have shown for the escape speed from Newtonian gravity.

2 HUBBLES CONSTANT AND THE NUCLEAR CONTEXT

Let us consider the case of a flat universe where k=0.The ratio $\frac{\dot{R}}{R}$ is equal to the Hubble constant which is $\frac{V}{d}$ in the astronomical case. For a flat nucleus, d should be the radius or distance from the center of the nucleus to the outermost particle of mass m. From the uncertainty principle it just follows that,

$$\frac{V}{d} = \frac{mV}{mr} = \frac{mVr}{mr^2} \approx \frac{\hbar}{mr^2} . \tag{2.1}$$

For a nucleus the Hubble's parameter is given by $\frac{\hbar}{mr^2}$. Here m is the mass of the outermost particle. For a flat nucleus, [k=0, Euclid geometry],

$$\left(\frac{\hbar}{mr^2}\right)^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}\mathrm{G}\rho \ , \tag{2.2}$$

Where the constant G is the universal constant of gravitation. Let the remaining mass of the nucleus be M.The nucleus is supposed to be a core of mass M and this core is exerting a gravitational force on the nuclear piece having a mass m.The density of the core is given by ,

$$\rho = \frac{M}{4/3}\frac{1}{\pi}\frac{1}{r^3} \ . \tag{2.3}$$

From Eq.(2.2) we readily note that,

$$\frac{1}{m^2}\left(\frac{\hbar}{r^2}\right)^2 = 2GM\frac{1}{r^3} \ . \tag{2.4}$$

The above Equation can be written in the following way,

$$\frac{1}{2m}\frac{\hbar^2}{r^2} = \frac{GMm}{r} \ . \tag{2.5}$$

For a particle of mass m, and at a distance r from the center of the nuclear core of mass M, the LHS of Eq.(2.5) is the kinetic energy. In other words, If E is the total energy of the outer particle of mass m with a gravitational potential energy, $-GMm/r$,

$$E = \frac{1}{2m}\frac{\hbar^2}{r^2} - \frac{GMm}{r} = 0 \ . \tag{2.6}$$

The above relation is obtained from the mass dominated Universe containing only a nucleus of total mass M + m.The nucleus is not a bound system. The radius of the nucleus can be obtained from the above relation. The total energy E is zero and positive and hence the outer particle escapes and can escape to infinite distance from the center of the core. In other words, for a nucleus that exerts only the gravitational interaction on the other piece of mass m the total energy is zero if we identify the first term of Eq.(1.3) with Hubble's speed divided by its radius. The uncertainty principle is also used to arrive at Eq.(2.6).The radius of the nucleus can be computed from the above relation.

$$r = \frac{\hbar^2}{2m^2 GM} \,. \tag{2.7}$$

When we use the above equation to find r for deuteron for which the core is neutron (M) and the outer particle is a proton,(m), we note that,

$$r = 1.77921 \times 10^{24}\text{cm}. \tag{2.8}$$

The above value is so big one cannot think of the applicability of the uncertainty relation to such a big nucleus. One may dismiss this number by saying that gravity plays no role whatsoever in the case of nuclear domain. But we have a different suggestion.

3.The Morphed Gravitational Interaction

Recently we have shown [2] that the strong interaction is closely related to the Newtonian potential energy. The nuclear potential energy consists of two parts, (1) central part and (2) the Yukawa factor which restricts the interaction to short range. The nuclear potential energy is given by,

$$V_N = -G\frac{M_P^2}{M_0^2}\,\frac{m_1 m_2}{r}\; e^{-\mu r[1-f(N)]}\,, \tag{3.1}$$

Where M_P is a limiting mass in the sense that whenever, either $m_1 \geq M_P$ or $m_2 \geq M_P$, $M_P^2 = M_0^2$. (3.2)

On the other hand f(N) is a function of $M_P^2/m_1 m_2$ such that whenever Eq.(3,2) holds ,

$$f(N) = 1. \tag{3.3}$$

and whenever both,

$$m_1 < M_P and m_2 < M_p \ , \tag{3.4}$$

Eq.(3.1) holds and whenever (3.4) is true f(N)= 0.That means that if a nucleus can be split into interacting masses satisfying Eq. (3.4), then the nuclear potential energy is given by,

$$V_N = -\frac{GM_P^2}{M_0^2}\frac{m_1 m_2}{r}e^{-\mu r} \ . \tag{3.5}$$

The limiting mass M_P is shown to be given by,[2,3,4,5],

$$M_P^2 = \frac{g^2 \hbar c}{G} = 15.34122 \times 10^{-12} gm^2 \ . \tag{3.6}$$

To account for the binding energy of deuteron, helium and many other nuclei we were led to the following value forg^2,

$$g^2 = \frac{e^2}{0.2254} = \frac{1}{137}\left[\frac{1}{0.2254}\right] = 0.032384 \ . \tag{3.7}$$

The limiting mass M_P involves universal constants and it does not change with the speed of the interacting masses. The parameter M_0 also does not change with the speed of the interacting pieces. The Nuclear potential energy is given by,

$$V_N = -\frac{GM_P^2}{M_0^2}\frac{m_1 m_2}{r}e^{-\mu r} = -\frac{g^2 \hbar c}{M_0^2}\frac{m_1 m_2}{r}e^{-\mu r} \ . \tag{3.8}$$

The nucleus is supposed to be divided in to $m_1\ and m_2$ which satisfy Eq.(3.4). The factor other than the Yukawa factor $e^{-\mu r}$, is what we called the morphed gravitational interaction.For the present we set the Yukawa factor equal to one. In Eq.(3.8) M_0^2 is specific to each pair of interacting masses $m_1 and m_2$. The factor $g^2\hbar c/M_0^2$ is something like a variable gravitational constant. It is this that should be used in Eq.(2.7) in place of G.

4.APPLICATIONS

We will first apply the variable gravitational constant to estimate the sizes of some nuclei with Eq.(2.7).For the nucleus Deuteron there is a neutron of mass M exerting the morphed gravitational force on a proton of mass m.From Eq.(2.7),we readily note that,

$$r_d = \frac{\hbar^2}{2m_P^2}\frac{1}{(g^2\hbar c/M_0^2)m_n} = 1.08007 \times 10^{-13} cm. \qquad (4.1)$$

In Eq.(2.7), G is replaced by the expression $g^2\hbar c/M_0^2$.The value gives an order of magnitude because it is based on Hubble's law and uncertainty relation .For deuteron $M_0^2 = 0.931826 \times 10^{-48}$ gm^2.[3].This value of r should be compared to that given by Eq.(2.8).But this suggests that a variable G should be used to quantize gravitational interaction. To drive this point we determine the ground state energy for this nucleus using the morphed gravity and uncertainty principle. The total energy is given by,

$$E = \frac{\hbar^2}{2m_p r^2} - \frac{g^2\hbar c}{M_0^2}\frac{m_n m_p}{r}\ . \qquad (4.2)$$

The above energy will be minimum whenever,

$$r_{min} = \frac{\hbar^2 M_0^2}{g^2 \hbar c m_p^2 m_n} \quad . \tag{4.3}$$

The corresponding minimum energy of the Deuteron nucleus is given by,

$$E = -\frac{1}{2}\left(\frac{g^2 \hbar c m_p m_n}{M_0^2}\right)^2 \frac{m_p}{\hbar^2} = -4.44714\ MeV\ . \tag{4.4}$$

The above energy is twice the binding energy of deuteron nucleus. But if we use reduced mass for the mass of the proton at the end of the above expression we obtain an exact value for the binding energy of this nucleus. Eqs.(4.2,4.3,and4.4) have nothing to do with relativity except to highlight the morphed gravitational interaction in the context of nuclear interactions.

We can find the approximate sizes for many nuclei from Eq.(2.7).In the case of oxygen-17 ,nucleus the mass M=$\left(8m_p + 8m_n\right)$ and thereis a neutron outside this core, and $M_0^2 = 0.88696 \times 10^{-48} gm^2$for this nucleus. From Eq.(2.7) using the variable G, we find that,

$$r_o = 0.064121 \times 10^{-13}\text{cm.} \tag{4.5}$$

This size is also very reasonable.

Aluminium-27 nucleus has a core of mass M=$\left(13m_p + 13m_n\right)$. There is a neutron outside this core. For this Nucleus,[3,4,5], $M_0^2 = 1.094011 \times 10^{-48} gm^2$. From Eq.(2.7),we note that,

$$r_{Al} = 0.048671 \times 10^{-13} cm\ . \tag{4.6}$$

All these are consistent with the nuclear sizes .These are approximate and are all obtained from general relativity, uncertainty principle, and morphed gravitational constant. The point is that quantization of gravitation needs a variable G.

5.CONCLUSIONS.

In this note the Newtonian gravitational potential energy is applied to a rocket fired from a planet. Its total energy is shown to be zero .It escapes to infinity assuming that there is only this planet and no other forces are there upon it. A mass dominated universe obeys general relativity and for a rocket fired from a planet also has zero total energy on the planet,Eq.(1.5). If we know the escape speed and the mass of the planet, we can determine the radius of the planet. A similar analysis is carried out for a nucleus of radius r. When its radius is found using the universal constant of gravitation G ,the radius turns to be very big.This suggested that in place of the universal constant of gravitation G we should use a variable constant of gravitation when the interacting masses satisfy Eq.(3.4).This also shows that the strong interaction is due to Morphed gravitational interaction. Finally quantum gravity is due to variable G and also due to a limiting mass,[see Eq.(3.4)].The strong interaction is given by Eq.(3.1).Whenever the nucleus is envisaged as made up of two masses $m_1 and m_2$, their potential energy is given by Eq.(3.1), if both these masses are less than the limiting mass M_P .The short range Yukawa factor may involve different bosons in addition to the $\pi - mesons$.These are all intermediate bosons corresponding to the Morphed gravitational interaction. There is also the Coulomb interaction among the two envisaged parts $m_1 and m_2$ of the nucleus As and when the condition stipulated by Eq.(3.4) is satisfied the potential energy reduces to the usual Newtonian potential energy.The short-range Yukawa factor also disappears. The Morphed potential energy contains electroweak parameters through the limiting mass parameter showing the close relation of all the basic interactions,[6,7].

References

[1] Michio Kaku ,!993,Quantum Field Theory,(Oxford Uni.Press)Pp643

[2] Cvavb.Chandra Raju,2013,J.Mod.Phys.**4**,459

[3] Cvavb.Chandra Raju,2013, J.Mod.Phys.**4 ,**1180

[4] Cvavb.Chandra Raju ,2013,J.Nucl.Part.Phys. **3**,(2) ,25

[5] Cvavb.Chandra Raju, 2014,Mod.Phys.Lett. A, **29,** 145002

[6] Cvavb. Chandra raju 2015,J.Mod.Phys.**6,22**

[7] Cvavb. Chandra raju 2015,j.mod phys. (to be published)