P- ^{3}H AND P- ^{3}He ELASTIC DIFFERENTIAL SCATTERING

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ABSTRACT

One pion exchange mass dependent Yukawa Potential Energy is used to estimate neutron-nucleus total elastic scattering cross-section. When this total cross-section is extrapolated to zero-energy for neutron- Triton case it agrees pretty well with the known experimental value. Estimations for 5.54 MeV P-^{3}H andP-^{3}He differential scattering cross-section are carried out. It is shown that for 15 degree center of mass angle of scattering the P-^{3}He differential scattering is about 300times that of P-^{3}H ,and this needs experimental confirmation to fix the mass dependence of the Yukawa Potential Energy.

Key Words:Neutron Total Scattering,Mass dependent Yukawa Potential energy, Proton differential scattering,Triton,Helium-3.
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1. NEUTRON-NUCLEUS SCATTERING.

In recent years, there has been a rapid advance in solving the four nucleon (4N) scattering problem with realistic Hamiltonians [1-4].The theoretical description of A=4 systems constitutes a challenging problem from the-point of nuclear few-body theory. The new computational techniques opened the possibility for accurate calculations of 4N observables using realistic models for NN and 3N forces. Moreover experiments also have reached a level of precision such that models become accessible to experimental investigation.
The mass-dependent Yukawa Potential Energy function is one which can be put to test because it accounts for many gross-properties of any nucleus provided a proper choice of the interacting masses is made. It is given by,

$$V(r) = -\frac{g^2}{r}\frac{\hbar c \quad m_n M e^{-ar}}{M_0^2} \text{, where ,} \tag{1.1}$$

m_n is the rest mass of the neutron and M is the mass of scattering nucleus. For one pion exchange the parameter a in the Yukawa factor e^{-ar} is given by,

$$a = a_1 = \frac{cm_{\pi\pm}}{\hbar} = 6.930125 \times 10^{12}\frac{1}{cm}, \tag{1.2}$$

$$a = a_2 = \frac{cm_{\pi 0}}{\hbar} = 6.839960 \times 10^{12}\frac{1}{cm}. \tag{1.3}$$

The other parameter M_0^2 is specific to each nucleus. This parameter for various nuclei which are relevant here is obtained in Section-2. The constant g^2 is same for all nuclei and is given by,[5-8],

$$g^2 = \frac{e^2}{0.2254} = \frac{1}{137}\left(\frac{1}{0.2254}\right) = 0.032384. \tag{1.4}$$

Here,$sin^2\vartheta_W = 0.2254$, is the Weinberg mixing parameter of the standard model [9,10].For the Yukawa potential energy given by Eq.(1.1),the scattering amplitude for the elastic scattering of a neutron by a Triton can readily be obtained. In the first Born approximation it is given by,

$$f(\vartheta) = \frac{2\mu}{\hbar^2}\frac{g^2\hbar c m_n M}{M_0^2}\frac{1}{[a^2+2k^2(1-cos\vartheta)]} \quad . \tag{1.5}$$

Here, the wave number, $k^2 = \frac{2\mu E}{\hbar^2}$,E being the CM energy of the incident neutron. Moreover

$$M(Triton) = 5.008271 \times 10^{-24} gm, \quad \text{and} \tag{1.6}$$

$$M_0^2 = 1.103243 \times 10^{-48} gm^2 \ . \tag{1.7}$$

This value in Eq.(1.7) will be justified in the next Section. The reduced mass of the neutron is given by,

$$\mu = \frac{m_n M}{m_n + M} \,. \tag{1.8}$$

The differential scattering cross-section is obtained by squaring the amplitude given by Eq.(1.5).The total elastic scattering cross-section is obtained by integrating the differential scattering cross-section. This is given by,

$$\sigma_T(E) = \left[\frac{2\mu}{\hbar^2}\frac{g^2\hbar c m_n M}{M_0^2}\right]^2 \frac{4\pi}{[a^4+4a^2k^2]} \quad . \tag{1.9}$$

The above expression gives the neutron-nucleus total elastic scattering cross-section [11].It can be extrapolated to zero energy to give,

$$\sigma_T(0) = 4\pi r_c^2 \ , \tag{1.10}$$

Where,

$$r_c = \frac{2\mu}{\hbar^2}\frac{g^2\hbar c m_n M}{M_0^2}\frac{1}{a^2} \quad . \tag{1.11}$$

From the above expressions we can readily compute the total elastic scattering of a neutron by a triton nucleus at zero energy. The total cross section at zero energy is computed for the two values of a given by Eqs. (1.2 &1.3) and the average value is given by,

$$\sigma_T(0) = \frac{[1.6839+1.7731]}{2} b = 1.7285 \, barn \quad . \tag{1.12}$$

Similarly the average for the parameter r_c is given by

$$r_c = \frac{[3.66059+3.75633]}{2} \times 10^{-13} cm = 3.70846\ \times 10^{-13} cm \quad . \quad (1.13)$$

The experimental values are given by [see Ref 1],

$$\sigma_T(0) = 1.70 \pm 0.03\ \ b \text{ , and} \qquad (1.14)$$

$$r_c = 3.59 \pm 0.02 \times\ \ 10^{-13}\ \text{ cm.} \qquad (1.15)$$

This elaborate introduction indicates that the mass dependent YPE is indeed the right choice to account for the gross experimental facts of any nucleus. In Sec .2,we indicate the procedure for obtaining the parameter M_0^2 for some nuclei .In Secs. 3 &4 proton scattering by Triton and Helium-3 are presented.Sec.5 contains our conclusions.

2 EVALUATION OF M_0^2

The parameter M_0^2 will be evaluated for some nuclei as in Refs.[5-8].The Triton nucleus contains two neutrons and a proton. The core for this nucleus is proton and neutron combination. The core mass $m_c = (m_n + m_p)$.The Yukawa Potential energy for this nucleus with a neutron outside the core is,

$$V(r) = -\frac{g^2 \hbar c m_c m_n}{M_0^2\, r} e^{-ar} \ . \qquad (2.1)$$

when the Yukawa factor e^{-ar} is set equal to one, the resulting central potential allows a closed solution of the Schrödinger equation. The total spin of this nucleus in the ground state is $J = \left(\frac{1}{2}\right)^+$ and hence the orbital angular momentum of the neutron must be zero. As $l = 0, the\ principal\ quantum\ number\ n\ \ must$ be 1 in the ground state of this nucleus. These quantum numbers are similar to those of Hydrogen atom. The energy spectrum for this nucleus is given by,

$$E_{nl} = -\frac{\mu}{2\hbar^2}\left[\frac{g^2\hbar c m_c m_n}{M_0^2}\right]^2 \frac{1}{n^2} = -\frac{8.4817\ MeV}{n^2} \ , \tag{2.2}$$

Where the principal quantum number n=1, 2, 3 etc. The binding energy of this nucleus is 8.4817 MeV.The reduced mass μ is given by,

$$\mu = \frac{m_n m_c}{m_n + m_c} = 1.116357 \times 10^{-24} gm \ . \tag{2.3}$$

From Eq. (2.2) it is now a simple matter to obtain the parameter M_0^2.It is given by,

$$M_0^2 = 1.103243 \times 10^{-48} gm^2 \ . \tag{2.4}$$

The ground-state wave function and other excited states can as well be obtained and put to test.The central potential part is the dominant one over the shielding Yukawa factor.

The nucleus ${}_2^3He$ contains two protons and a neutron. The binding energy of this nucleus is 7.178 MeV and $J = \left(\frac{1}{2}\right)^+$.So, $l = 0,1,2$ ...for this nucleus. There is a proton outside a core of proton and a neutron. The core mass is also same as for the Triton core mass.The potential energy in this case is given by,

$$V(r) = -\frac{g^2\hbar c m_p m_c}{M_0^2 r} e^{-ar} + \frac{e^2\hbar c}{r} \ . \tag{2.5}$$

Again we set the shieldingYukawa factor equal to unity and solve the Schrödinger equation. This gives,

$$E_{nl} = -\frac{\mu}{2\hbar^2}\frac{1}{n^2}\left(\frac{g^2\hbar c m_c m_p}{M_0^2} - \frac{e^2\hbar c}{1}\right)^2 = -\frac{7.1780 MeV}{n^2}. \tag{2.6}$$

The reduced mass in this case is given by,

$$\mu = \frac{m_c m_p}{m_c + m_p} = 1.115332 \times\times 10^{-24} gm \ . \tag{2.7}$$

The principal quantum number n=1, 2, 3…..From this we can readily evaluate the parameter M_0^2 for the Helium-3 nucleus. It is given by,

$$M_0^2 = 1.142021 \times 10^{-48} gm^2 \ . \tag{2.8}$$

A similar procedure is followed for the nucleus $_2^4He$ also to evaluate this parameter [5-8].

$$M_0^2 (Helium - 4) = 0.935301 \times 10^{-48} gm^2. \tag{2.9}$$

It should be noted that helium- 3 and helium-4 have different values for this parameter. The nucleus with only two protons is not observed and this forced us to believe that the parameter M_0^2 for a diproton or Helium-2 nucleus is very nearly equal to m_p^2 and this led us to relate g^2 to the fine structure constant. To account for the small binding energy of the Deuteron the Weinberg mixing parameter is brought into play in Eq. (1.4). The near equality of the parameter M_0^2 to m_p^2 has an important consequence. The attractive force between two protons is quite small compared to the attractive force between a proton and an anti-proton. This causes the proton-antiproton to come so nearer as to result in their annihilation, That is how probably matter dominates over anti-matter.

3 PROTON-TRITON SCATTERING

The differential scattering of a proton by ^{3}H and ^{3}He nucleus is an important topic to evaluate the scattering variables of 4N systems [see Ref. 2].For the scattering of a proton by the Triton nucleus the complete Yukawa potential energy along with the Coulomb potential energy is given by,

$$V(r) = -\frac{g^2\hbar c}{M_0^2}\frac{m_p M}{r}e^{-ar} + \frac{e^2\hbar c}{r} \ , \tag{3.1}$$

Where, M is the mass of the Triton nucleus,[Eq. (1.6)], and e^2 is the fine structure constant. The in-coming proton together with the triton is equivalent to a ${}_2^4He$ nucleus. This requires that the parameter M_0^2 in Eq. (3.1) be identified with that given by, Eq. (2.9).On the other-hand the neutron-triton scattering discussed in Sec. 1 indicates that the parameter M_0^2 in (3.1) may nearly be equal to that given by, Eq. (2.7). The differential elastic scattering is evaluated for both the values of $M_0^2 and\ a$ and tabulated below.

In the first-Born approximation the Scattering amplitude is given by,

$$f(\vartheta) = f_1(\vartheta) - f_2(\vartheta) \text{ , where,} \tag{3.2}$$

$$f_1(\vartheta) = \frac{2\mu}{\hbar^2}\frac{g^2\hbar c m_p M}{M_0^2}\frac{1}{[2k^2(1-cos\vartheta)+a^2]} \text{ , and} \tag{3.3}$$

$$f_2(\vartheta) = \frac{2\mu}{\hbar^2}\frac{e^2\hbar c}{[2k^2(1-cos\vartheta)]} \ . \tag{3.4}$$

The reduced mass $\mu = \frac{m_p M}{m_p+M} = 1.253861 \times 10^{-24} gm$ and the wave number $k^2 = \frac{2\mu E}{\hbar^2}$, E being the CM energy of the incident proton which in the present case is 5.54 MeV.The elastic differential scattering cross section is given by,

$$\sigma(\vartheta) = f_1^2 + f_2^2 - 2f_1 f_2 \ . \tag{3.5}$$

In the table given below the differential scattering cross is evaluated for the 5.54 MeV proton by ^{3}H nucleus.

Angle in degrees	$\frac{\sigma_1(\vartheta)+\sigma_2(\vartheta)}{2}$ $in\ mb$ $M_0^2 = 1.103243 \times 10^{-48} gm^2$	$\frac{\sigma_1(\vartheta)+\sigma_2(\vartheta)}{2}$ $in\ \ mb$ $M_0^2 = 0.935301 \times 10^{-48} gm^2$
15	(0.708+ 0.304)/2= 0.506	(1.378+2.295)/2=1.8365
30	(50.842+57.174)/2=54.008	(84.244+89.681)/2=86.96
45	(61.995+ 65.135)/2=63.57	(91.003+95.488)/2=93.25
60	(53.653+55.894)/2 =54.77	(77.224+80.395)/2=78.81
75	(43.297+44.836)/2=44.067	(61.798+63.965)/2=62.88
90	(34.624+35.692)/2=35.158	(49.205+50.706)/2=49.96
105	(28.155+28.923)/2=28.539	(39.911+40.99)/2=40.45
120	(23.575+24.158)/2=23.867	(33.367+34.185)/2=33.78
135	(20.46+21.449)/2=20.955	(28.93+29.585)/2=28.26
150	(18.463+18.861)/2=18.662	(26.09+26.65)/2=26.37
165	(17.351+17.713)/2=17.532	(24.512+25.031)/2=24.77
180	(16.994+17.345)/2=17.170	(24.005+24.498)/2=24.25

TABLE-1

In the above TABLE-1 the differential elastic scattering $\sigma(\vartheta)$ is evaluated for both the values of a , Eq. (1.2 & 1.3) and the columns are given for the possible values of the parameter M_0^2.The angle ϑ is the CM angle of scattering for unpolarized protons. For 4N systems the measured differential scattering cross-section at $\vartheta = 90\ degrees$ is reported [see Ref.2] to be 50 mill barns. In Column 3, the same value appears.

4 PROTON- ^{3}He SCATTERING

The complete Yukawa Potential Energy along with the Coulomb part for the P- ^{3}He scattering is given by,

$$V(r) = -\frac{g^2\hbar c}{M_0^2}\frac{m_p M_{He}}{r}e^{-ar} + \frac{2e^2\hbar c}{r} \quad \text{, where} \tag{4.1}$$

$$M_{He} = 5.00823 \times 10^{-24}\ gm\ . \tag{4.2}$$

The scattering amplitude is given by,

$$f(\vartheta) = f_1(\vartheta) - f_2(\vartheta)\ \text{, where} \tag{4.3}$$

$$f_1(\vartheta) = \frac{2\mu}{\hbar^2}\frac{g^2\hbar c m_p M_{He}}{M_0^2}\frac{1}{[2k^2(1-cos\vartheta)+a^2]}\ \text{, and} \tag{4.4}$$

$$f_2(\vartheta) = \frac{2\mu}{\hbar^2}\frac{2e^2\hbar c}{[2k^2(1-cos\vartheta)]}\ . \tag{4.5}$$

The reduced mass $\mu = \frac{m_p M_{He}}{m_p+M_{He}} = 1.253859 \times 10^{-24}$gm. The wave number $k^2 = \frac{2\mu \text{E}}{\hbar^2}$, E being the CM energy of the proton The cross-section $\sigma(\vartheta)$ is obtained by squaring Eq. (4.3) [see{3.5}].

TABLE-2

Angle in degrees	$\frac{\sigma_1(\vartheta)+\sigma_2(\vartheta)}{2}\ in\ mb$ $M_0^2 = 1.142021 \times 10^{-48} gm^2$	$\frac{\sigma_1(\vartheta)+\sigma_2(\vartheta)}{2}\ in\ mb$ $M_0^2 = 0.935301 \times 10^{-48} gm^2$
15	(176.560+169.208)/2=172.88	(114.62+114.96)/2=114.8
30	(15.154+17.071)/2=16.113	(37.33+40.98)/2=39.154
45	(37.878+40.256)/2=39.067	(66.19+70.03)/2=68.109
60	(38.754+40.597)/2=39.676	(63.45+66.33)/2=64.887
75	(33.436+34.743)/2=34.090	(53.39+55.40)/2=54.393
90	(27.652+28.574)/2=27.113	(43.61+45.02)/2=44.313
105	(22.919+23.589)/2=23.254	(35.89+36.92)/2=36.403
120	(19.416+19.927)/2=19.672	(30.28+31.05)/2=30.665
135	(16.913+17.384)/2=17.179	(26.40+27.02)/2=26.711
150	(15.389+15.743)/2=15.566	(23.89+24.42)/2=24.156

165	(14.487+14.813)/2=14.650	(22.49+22.97)/2=22.728
180	(14.206+14.516)/2=14.361	(22.03+22.51)/2=22.27

The Table contains the differential cross sections as in the case of Triton. The CM energy of the proton is 5.54 MeV as in the case of the Triton. The differential cross sections are estimated for two different values of the parameter M_0^2 given by, Eqs. (2.8 & 2.9). The two tables are similar in all respects but the results are different. For 15 degree the differential scattering cross-section for the Proton-Triton case is very small compared to the Proton-Helium-3 scattering. Both are 4N systems!

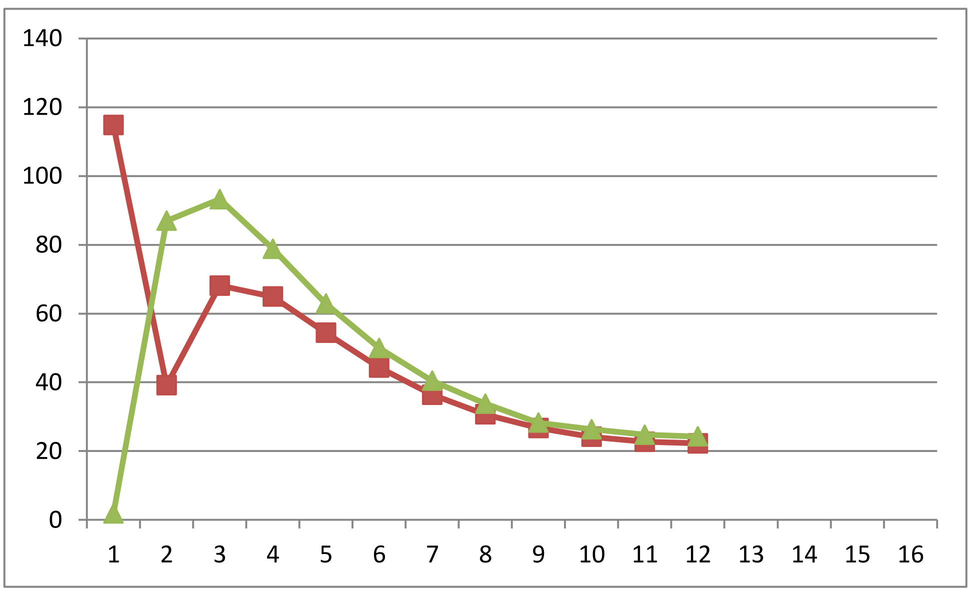

Fig. 1 (Red line Helium -3and green line with triangles is for Triton both for $M_0^2 = 0.935301 \times 10^{-48} gm^2$ .X-axis angle,1=15^0,2=30^0 etc; and Y-axis cross section in milli-barn as in the above tables)

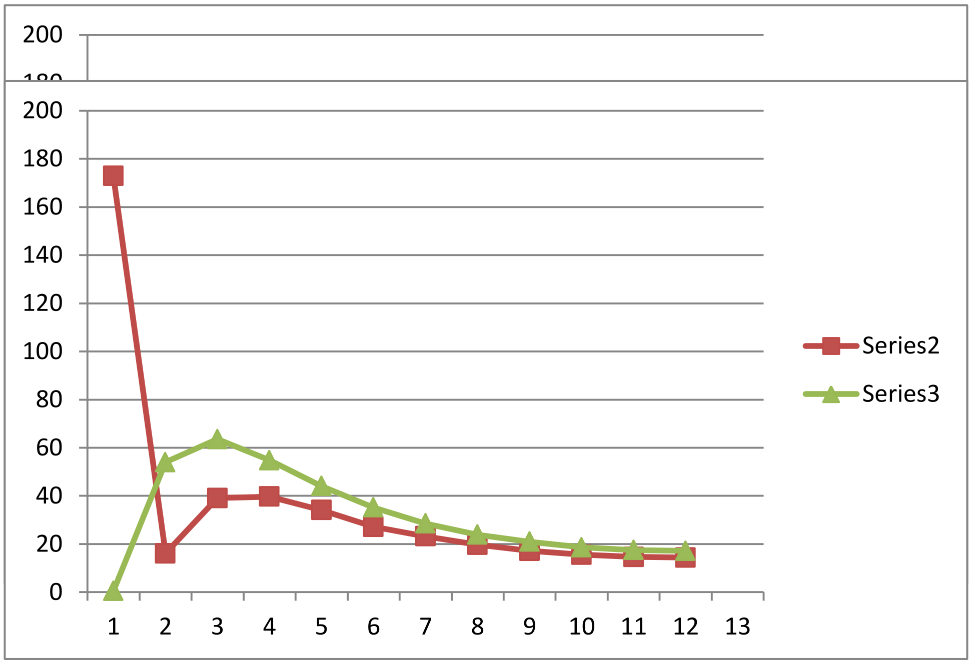

Fig.2 (Red line for Helium-3,with cross section on Y-axis and angle on X-axis with $M_0^2 = 1.142021 \times 10^{-48} gm^2$ and the other series with triangles is for Triton with $M_0^2 = 1.103243 \times 10^{-48} gm^2$).

5.CONCLUSIONS

The tabulated cross sections are displayed in the above graphs .From these graphs it is evident that the differential scattering cross-section is quite different for the Helium-3 and Triton nucleus .It is quite large for the Helium-3 nucleus at 15degree CM angle when compared to the Triton-proton differential cross- section.Proton-Helium-3 and Proton-Triton are both 4N systems !It should be noted that while explaining the total cross-section of a neutron by a Triton nucleus at zero energy no new numbers or cut-off parameters are used.Whatever numbers that are necessary are all there already for a different context! From Ref.(2) it should be noted that the differential cross-section of a 5.54 MeV. Proton by a N=3 nucleus is about 50 milli-barn when the angle is 90^0.This is an observed value.This value is also obtained here theoretically. There are many laboratories which can undertake to verify the conspicuous difference in the differential cross-section pointed here. When this prediction is confirmed the mass dependent Yukawa Potential Energy for any nuclear system stands established. This also shows the close connection between gravitational interaction and strong interaction. Above all it will be the beginning of quantum gravity.

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