INTRODUCTION:

In this paper, I will be looking functional equation satisfied by Riemann zeta function actually a non-cooperative game of perfect information between its constituent terms(here different mathematical functional symbols) in which the best strategy adopted by each player to locate zeros on mathematical field leads to discovering the most stable arrangement of physical location non-trivial zeros of Riemann zeta function, which in turn leads to TRUTHFULNESS OF RIEMANN HYPOTHESIS..

As visualized by David Hilbert- Advanced mathematics is actually a game between different mathematical symbols, where different symbols follow certain defined rules. I am going to extend his view stating that entire(number theory in particular) itself is actually a game, where different players play a non-cooperative game to reach at the most stable equilibrium stage.

The mathematical theory of games was invented by John von Neumann and Oskar Morgenstern (1944).Game theory is the study of the ways in which *strategic interactions* among *agents* produce *outcomes* with respect to the *preferences* (or *utilities*) of those agents, where the outcomes in question might have been intended by none of the agents..All situations in which at least one agent can only act to maximize his utility through anticipating (either consciously, or just implicitly in his behavior) the responses to his actions by one or more other agents is called a *game*. Agents involved in games are referred to as *players*. If all agents have optimal actions regardless of what the others do, as in purely parametric situations or conditions of monopoly or perfect competition we can model this without appeal to game theory; otherwise, we need it.

Each player in a game faces a choice among two or more possible *strategies*. A strategy is a predetermined 'programme of play' that tells her what actions to take in response to *every possible strategy other players might use*. The significance of the italicized phrase here will become clear when we take up some sample games below.

The simplest games (from the perspective of logical structure) are those in which agents have *perfect information*, meaning that at every point where each agent's strategytellshertotakeanaction, she knowseverythingthathashappenedin the game up to that point. This is so because in such games (as long as the games arefinite,thatis,terminate after a known numberof actions)playersandanalysts canuseastraightforwardprocedureforpredictingoutcomes.A playerinsucha gamechoosesherfirst actionbyconsideringeachseriesofresponsesand counter-responses that will result from each action open to her. She then asks herselfwhichofthe availablefinaloutcomesbringsherthehighestutility,and chooses the action that starts the chain leading to this outcome. This process is called*backwardinduction*(becausethereasoningworks backwardsfrom eventual outcomes to present choice problems).

Nashequilibrium(NE)stagesarethesetofstrategiessothatnoplayercan maximize payoff by unilateral deviations. John Forbes Nash devised a theorem for itwhichareknownasNashequilibrium stages. SNE(strongNashequilibrium stagesaretherefinementsofNashequilibriumsuchthatno playerisgoingto benefit by unilateral or bilateral defimitions.

I will prominently use the tools of game theory to find out different Nash equilibrium stages in this functional game played between mathematical symbols and discovering the Strong Nash Equilibrium (SNE) stage to find out the most equilibrium state and thus the preferred adopted set of strategy between different players.

Mathematics, here numbers are basically language to represent some physical aspect in nature. Numbers are materialistic representations of some physical aspects of nature, These physical aspects are inter-connected with each other in nature ,that is well known by BELL's theorem of interconnectedness of space in nature . Numbers are basically materialistic manifestation of this very aspect of nature on mathematical domain. This physical abstract of interconnectedness makes numbers plying GAME OF PERFECT INFORMATION, where each number exactly knows its relative location relative to other and each number exactly knows everything about the history of game and other numbers(players). In fact, players (mathematical symbols) don't exist independently. Each player exists because, other exists. They all are connected with each other. In the sense that if anyone ceases to exist,all other would do so.,Or better say, If one changes, others would change automatically.

I mean all the players (here numbers) say e.g. -5,-4,-3,-2,-1,0 ,1,2,3,4,5,6 all exist wrt each other. i.e. 5 doesn't exist unless 4,3,2,1 etc. exists. And -5 doesn't exist unless 5 exists. So, each number well knows its relative location wrt each other knowing the history of entire number system, that's how numbers have originated in Nature.

Hence, mathematical field of numbers, being the manifestation of physical aspect of nature inherits the physical, metaphysical contents of Nature, which governs mathematical axioms, logic, and intuition and so on. This was righly interpreted by Leibniz—"Mathematics has more of metaphysical content than it's generally recognized".

Here, in particular, I visualize the functional equation satisfied by Riemann zeta function as game between different constituent terms which are connected through multiplication sign on both side of equality sign. Moreover, numbers play a game of perfect information, and hence it must have at least one SNE (Strong Nash equilibrium) stage corresponding to the arrangement solution of the functional equation .

It's further verified by kuhn's theorem that every extensive finite form game of perfect information must have at least 1 solution . As this has exactly 1 SNE stage corresponding to the location of non-trivial zeros on the critical line in 0<R(s) <1.

No arrangement of players can lead to gain individual payoff by unilateral or bilateral deviations from this SNE stage.

So, what I would be doing is- finding the locations of trivial & non-trivial zeros by looking the arithmetic structure of Riemann zeta function and by applying the two basic arithmetic of numeric '0' to find out different set of possibilities of taking zero value by different constituent terms.

In a nutshell, I will NOT go into finding the zeros of this functiuon,rather I will be visualizing the arithmetic structure of FUNCTIONAL EQUATION ,in which different constituent terms are connected through multiplicative sign and using game theory find the SNE stage to locate zeros. So, it has hardly anything to do with anything else than game theory and slight arithmetic of numeric 0.

Readers should also try to visualize and realize INTUITIVELY how numbers are well- informed about each other, their relative location and their history that makes it a game of PERFECT INFORMATON.

The Riemann zeta function ((s) is a function of a complex variable s = o + it (here, s, o and t are traditional notations associated to the study of the Ç-function). The following infinite series converges for all complex numbers s with real part greater than 1, and defines ((s) in this case:

$$\zeta(s) = \sum^{\infty} n^{-s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots \qquad \sigma = \Re(s) > 1.$$

The Riemann zeta function is defined as the analytic continuation of the function defined for o > 1 by the sum of the preceding series.

The Riemann zeta function satisfies the functional equation

$$\zeta(s) = 2^s \pi^{s-1}\ \sin\left(\frac{\pi s}{2}\right)\ \Gamma(1-s)\ \zeta(1-s).$$

where T(s) is the gamma function which is an equality of meromorphic functions valid on the whole complex plane. This equation relates values of the Riemann zeta function at the points s and 1 – s. The gamma function has a simple pole at every non-positive integer, therefore, the functional equation implies that ((s) has a simple zero at each even negative integer s = – 2n — these are the trivial zeros of ((s).

$$\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = (1 - 2^{1-s})\zeta(s).$$

Incidentally, this relation is interesting also because it actually exhibits ((s) as a Dirichlet series (of the y-function) which is convergent (albeit non-absolutely) in the larger half- plane o > 0 (not just o > 1), up to an elementary factor.

Riemann also found a symmetric version of the functional equation, given by first defining

$$\xi(s) = \frac{1}{2}\pi^{-s/2}s(s-1)\Gamma\left(\frac{s}{2}\right)\zeta(s).$$

STATEMENT of Riemann Hypothesis: All non-trivial zeros of Riemann zeta function in the critical space 0<R(s)<1 lies on R(s)=1/2 .

Preliminaries :

Number syctem is actually a syctem ,where each number can be compared to a player . All the (numbers)players actually play a game of perfect information. Numbers are the languages to represent come physical aspectc of nature . These physical aspests are interlinked with each other in nature (locality-at-a distance). This makes numbers behaving in an interconnectedness manner and that's why there exists locality-at- dictance on complex field leading to the interconnectedness of trivial & non-trivial zeros of Riemann zeta function. That’s why numerical values taken by Riemann zeta function on different point on complex field are inter-linked with each other and are not independent but relatively defined. This makes the entire complex field an interconnected system and appear mysterious.

If we also look at the foundation of mathematics (here complex-valued functions and the physical mechanism of analytic continuation) we will be able to see that there are certain invariance of physicalities while performing the analytic continuation e.g. Collinearity of Zeros trivial and non-trivial both and Infiniteness of Number of Zeros(Bpth Trivial and hence Nn-Trivial Zeros).If we become more imaginative, this invariance of physical characteristics is sufficient to state that Riemann Hypotheis is TRUE because Non-trivial zeros will also be infinite like Trivial counterparts and also collinear . Collienarity means they will have to lie on the same line and hence R(z)=1/2 ! Otherwise if it lies anywhere on other ,it wont be collinear and violate the invariance of physical characteristic.

But here we look at Game theoretic aspects .

Given the physical aspects of numbers and its complex field, I visualize numbers and their mathematical functions playing a perfectly informed game and exercising move to reach the most equilibrium stage (SNE) .

A) Game Theory

1. The Normal Form Representation

Basic Notations:

n: number of players.
s : a (pure) strategy of player i.
S = {s , ..., s }: the strategy space (or strategy set) of player i. Here, player i has m strategies in her strategy space.

s = (s , ..., s): the strategy profile of the n players; the “outcome” of the game. s =
(s , ..., s , s , ..., s): the strategy profile of the other n – 1 players.

s = (s , s) when that is convenient.

u (s , s) = the payoff to player i ac a function of the ctrategy profile played by the n playerc in the game. Payoff s should be thought of as utilities of the outcomes, though we will occasionally

S: the set of all possible strategy profiles.

Lemmas and theorem:

Nash Equilibrium

Best Response: For player i, a strategy o is a best response to the strategy profile o if u (o , o) “ u (s , o) for all s < S .

Note that o is a specific strategy profile that could be played by the other players in the game.

Since o may not be the only best response to o , we will call BR (o) the set of best responses for player i to o , and note that o < BR (o).

We can also consider the set BR(&) of best responses of player i to her belief & about the strategies being played by the other players.

Note that a (strictly) dominated strategy is never a best response.

Nash Equilibrium : A strategy profile s = (s , ...s) is a Nash Equilibrium if each player's strategy is a best response to the strategy profile played by the other players in the game.

I.e., s is a Nash equilibrium if s < BR (s) for all players i,

or equivalently, if u (s , s) “ u (s , s) for all s < S and for all players i.

Definitions :

A game of perfect information is a game of complete information in which all information sets in the game tree are singletons. I.e., whenever a player is called upon to take an action, she knows exactly where she is in the tree, or equivalently, she knows the exact history of the game.

Kuhn's Theorem: (1953) : Every finite extensive form game with perfect information has at least one solution by backward induction.

In games of perfect information, solutions by backward induction correspond to SNE (Strong Nash equilibrium)

Existence: Since every subgame of a finite game (of complete information) has a corresponding finite normal form, and every finite normal form game has at least one NE (in mixed strategies), every finite game (of complete information) must have at least one SNE.

Thus, SNE is a refinement of NE that has two desirable properties. First, SNE are

NE that do not involve incredible threats or promises. Second, every finite game (of complete information) has at least one SNE.

In context of functional equation game played by Riemann zeta functions in the ebthere are two players A & B where A corresponds to sin()=0 and B corresponds to sin() NOT=0.

A solution concept in game theory :

In game theory, Nash equilibrium states/ refinement of Nash equilibrium stages correspond to the solution. Backward Induction in agame of perfect information always lead to at least 1 nash equilibrium stage Number system as an originally found quantum group:, which corresponds to the solution,here the physical location of non-trivial zeros of Riemann zeta function.

Numbersystemisaphysically FOUND commutativeGROUPasin quantum world.

a) For every a, b there exists a+b on the number line(group)
b) There exists an identity element I su ch that a+I=I+a=a

This will also be valid. if operator + is changed to *.
Any way ^,* are derived out of +
So, with any one operator, the number system is DEFINITELY QUANTUM GROUP FNATURE.

On numbr line for everyelement

1) a, there exists 1/a,

2) a there exists –a.

Thus covering each and every point on the number line.

Every 'a' can generate another number 'b' on it through operators +,-,*,/ ,^

4)Entire negative part of number line is mapped onto the domain 0-1 e.g. 2^-3=1/8,2^-2=1/4,2^-1=1/2

5) and every positive side on number line can be mapped onto the domain after 1

e.g. ,2^1=2 ,2^2=4,2^3=8 etc.

6) 0 is mapped onto 1 to maintain the symmetry of GROUP i.e. 2^0=1

What I am interested here is to so why NUMBER SYSTEM is an INTERCONNECRED GROUP.

So, I wd like to say that THIS HAS ALL THE PROPERTY OF A QUANTUM SYMMETRY GROUP FOUND IN NATURE and being a qwuantum group IT"S INTERCONCTED and perfectly informed group.

Refe. Eugen Merzbacher -Quantum mechanics–Chapter 17, Groups and Symmetrical invariancy in ORIGINALLY FOUND GROUP IN nature.

PROOF:

Functional equation satisfied by Players Ç (s) & Ç(1-s) in the entire complex domain 'C' is

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\, \zeta(1-s).$$

Æs one and only one term on each side of = sign can and must be zero as 0³0 is not equal to 0 & 0 ³non-zero number= 0

- A ={ C- s ,sin(wsl2)=0,s‡0} as s=0 is the location for pole

- B = { C-A,s‡0}

- Player A has two options to exercise .It can exercise only one of the two.

1. Ç(s) =0 for R(s)>1l2, Ç(s)‡0 for R(s)<1l2
2. Ç(s)=0 for R(s)<1l2, Ç(s)‡0 for R(s)>1l2

Sim ilarly,

Player B has also two options .It can also exercise one of the two.

1. Ç(s)=0 for R(s)> 1l2 and simultaneously for R(s)<1l2

2. Ç(s)=0 for s= ½+it

But,

Ç(s)‡ 0 for R(s)>1

Now, we look at the different permutations of strategies adopted in this game and find their payoff matrix.

Payoff matrix of this game

	Player A exercises 1 option	Player A exercises 2nd option
Player B exercises 1 option	0,0	0,0

Player B exercises 2nd option	0,0	1,1

Sij =[Xi,Yj]

ui (Xi,Yj)= 0 for 1“i,j“2,i=j ‡2

=1 for i=j =2

We see that

BRi (Yj) = [Xi] for 1“i“2,1“j<2 BRi

(Xj) = [Yi] fori=j=2

This has 4 Nash equilibrium states by looking at the set of BR

But As Number line is a complete group i.e. For every X {X: R,-X:R} And {X:

R, 1/X: R}

Applying Kuhn’s theorem states there must be at least 1 SNE in this game of perfect information.

So, let us now find SNE?

As u2(X2, Y2) = max {u2(Xi, Yj)}

Also,

U1(X2, Y2)=max{ u1(Xi,Yj)}

for 1“i,j“2

Thus S(X2, Y2) is the SNE in this game by Backward induction, THIS EXISTENCE OF solution(SNE stage)IS SUPPORTED BY KUHN"S THEORM . and hence would be preferred by all the players to maximize the payoff.

Although this game has 4 Nash equilibrium states as none can benefit as none can benefit by unilateral deviations, but the 4th set of strategy would be the (SNE)STRONG NASH ERQUILIBRIUM state as this is immune to both unilateral deviations and coalition deviations. And hence both the players must exercise the 2nd options to maximize the payoff.

This asserts the truthfulness of the Riemann hypothesis that trivial zeros lie on the points s=2k,K<0 and trivial zeros lie on the R(s)=1/2 .Thus,

It implies that

Ç(s) =0 for R(s)=1l2+it for 0<R(s)<1

and also,Ç(s) ‡ 0 for R(s)>1l2

QED

Reference:

1. *GAMETHEORY* http://plato.stanford.edu/entries/game-theory/#Games

2. *Extensive form finite game of perfect information*

3. *Riemann hypothesis by Enrico Bombieri*