New perspectives on the relationship of space and time within General Relativity and Quantum Mechanics

Nikola Perkovic[1]

[1]University of Novi Sad, Faculty of Natural Sciences, Institute of Physics, Serbia.

Correspondence: percestyler@gmail.com

Abstract: This paper innovates the way we comprehend time and its relationship with space by proposing a new concept within General relativity and, to some extent, Quantum Mechanics in order to explain some of the natural aspects that still puzzle scientists that study the very nature of the Universe. Gravitation, the arrow of time, cosmic inflation and many more subjects that are crucial to modern day physics shall be debated within the most accurate theory, General relativity, but also with some concepts developed in terms of Quantum Mechanics.

I Introduction

1. Manifold

The model we will use for spacetime is a Lorentzian manifold, as we mentioned before that is the manifold type used in General Relativity. We have a pair (M, g) where (M) is a connected (4)-dimensional Hausdorf (C^{∞}) manifold and (g) is a Lorentzian metric with a signature $(+2)$ on (M).

(M, g) and (M', g') will be taken as equivalent if they are isometric, meaning that there should be a diffeomorphism $(\theta: M \to M')$ which would take the metric (g) into the metric (g'), that is:

$$(1)\ \theta_* \, g = g'$$

The metric (g) enables the non-zero vectors at a point $(p \in M)$ to be divided into three classes: a non-zero vector $(X \in T_p)$ that is timelike, spacelike or null if $(g(X, X))$ is negative, positive or zero, respectively.

The order of differentiability (r) of the metric should be sufficient for the field equations to be defined. They can be defined in a distributional sense if the metric coordinate components (g_{ab}) and (g^{ab}) are continuous and have locally square integrable generalized first derivatives with respect to local coordinates. A set of functions $(f_{;a})$ on (R^n) is said to be a generalized derivative of a function (f) on (R^n) if for any (C^{∞}) function (Ψ) on (R^n) with compact support:

$$(2)\ \int f_{;a}\, \Psi \, d^n x = \int f \left(\frac{\partial \Psi}{\partial x^a}\right) d^n x$$

However, this condition is too weak since it does not guarantee neither the existence nor the uniqueness of geodesics. We will now assume that the metric is at least (C^2). The (C^r) pair (M', g') is a (C^r)-extension of (M, g) if there is an isometric (C^r) imbedding $(\mu: M \to M')$.

We require that the model (M, g) is (C^r)-inextensible, meaning that there is no (C^r) extension (M', g') of (M, g) where $(\mu(M))$ does not equal (M').

A pair (M, g) is (C^r) locally inextensible if there is no open set $(U \subset M)$ with non-compact closure in (M), such that the pair $(U, {}^{g}/_{n})$ has an extension (U', g') in which closure of the image of (U) is compact.

2. Matter fields

We denote the matter fields as:

$$(3)\ \Psi(i)^{a\ldots b}{}_{c\ldots d}$$

where the sub-script (i) numbers the fields considered.

The following two postulates on the nature of the equations obeyed by the $(\Psi(i)^{a\ldots b}{}_{c\ldots d})$ are common to both Special and General Relativity.

2.1. The first postulate: Local causality

The equations governing the matter fields must be such that if (U) is a convex normal neighborhood and (p) and (q) are points in (U), then a signal can be sent in (U) between (p) and (q) if and only if (p) and (q) can be joined by a (C^1) curve lying entirely in (U), whose tangent vector is everywhere non-zero and is either timelike or null; hence called "non-spacelike".

Whether the signal is sent from (p) to (q), or vice versa, will depend on the direction of time in (U).

This postulate is what sets apart the metric (g) from other fields on (M) and gives it its distinctive geometrical character.

If $(\{x^a\})$ are normal coordinates in (U) about (p), then we can conclude that the points that can be reached from (p) by non-spacelike curves in (U) are those whose coordinates satisfy:

$$(4)\ (x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 \leq 0$$

The boundary of these points is formed by the image of the null cone of (p) under the exponential map, that is the set of all null geodesics through (p). Therefore by observing which points can communicate with (p), we can determine the null cone (N_p) in (T_p). Once the (N_p) is known, the metric at (p) may be determined up to a conformal factor.

2.2. The second postulate: Local conservation of energy and momentum

The equations governing the matter fields are such that there exists a symmetric tensor (T^{ab}), known as the energy momentum tensor, which depends on the fields, their covariant derivatives and the metric; all of which has properties:

1) (T^{ab}) vanishes on an open set (U) if and only if all the matter fields vanish on (U).
2) (T^{ab}) obeys the equation:

$$(5)\ T^{ab}{}_{;b} = 0$$

3. Lagrangian formulation

Let (L) be a Lagrangian which is a scalar function of the fields $(\Psi(i)^{a\ldots b}{}_{c\ldots d})$, their first covariant derivatives and the metric. We obtain the equations of the fields by requiring that the action:

$$(6)\ \int_D L\, dv$$

be stationary under variations of the fields in the interior of a compact four-dimensional region (D). By variation of the fields $(\Psi(i)^{a\ldots b}{}_{c\ldots d})$ we mean a one-parameter family of fields $(\Psi(i)(u,r))$ where $(u \in (-\varepsilon,\varepsilon))$ and $(r \in M)$, such that:

1) (7) $\Psi(i)(0,r) = \Psi(i)(r)$
2) (8) $\Psi(i)(u,r) = \Psi(i)(r)$ when $(r \in M - D)$

We denote $(\partial\Psi(i)(u,r)/\partial u|_{u=0})$ by $(\Delta\Psi(i))$.

Then:

$$(9)\ \left.\frac{\partial I}{\partial u}\right|_{u=0} = \sum_{(i)} \int_D \left(\frac{\partial L}{\partial \Psi(i)^{a\ldots b}{}_{c\ldots d}} \Delta\Psi(i)^{a\ldots b}{}_{c\ldots d} + \frac{\partial L}{\partial \Psi(i)^{a\ldots b}{}_{c\ldots d;e}} \Delta(\Psi(i)^{a\ldots b}{}_{c\ldots d;e})\right) dv$$

where $(\Psi(i)^{a\ldots b}{}_{c\ldots d;e})$ are the components of the covariant derivatives of $(\Psi(i))$ but $(\Delta\left(\Psi(i)^{a\ldots b}{}_{c\ldots d;e}\right) = \Delta(\Delta\Psi(i)^{a\ldots b}{}_{c\ldots d;e}))$, hence the second term can be expressed as:

$$(10)\ \sum_{(i)} \int_D \left[\left(\frac{\partial L}{\partial \Psi(i)^{a\ldots b}{}_{c\ldots d;e}} \Delta\Psi(i)^{a\ldots b}{}_{c\ldots d}\right)_{;e} - \left(\frac{\partial L}{\partial \Psi(i)^{a\ldots b}{}_{c\ldots d;e}}\right)_{;e} \Delta\Psi(i)^{a\ldots b}{}_{c\ldots d}\right] dv$$

The first term in this expression can be written as:

$$(11)\ \int_D Q^a{}_{;a}\, dv = \int_{\partial D} Q^a\, d\sigma_a$$

Where (Q) is a vector whose components are:

$$(12)\ Q^e = \sum_{(i)} \frac{\partial L}{\partial \Psi(i)^{a\ldots b}{}_{c\ldots d;e}} \Delta\Psi(i)^{a\ldots b}{}_{c\ldots d}$$

This integral is zero as condition two states that $(\Delta\Psi(i))$ vanishes at the boundary (∂D). Hence in order that $(\partial I/\partial u|_{\partial u=0})$ should vanish for all variations on all volumes (D), it is necessary and sufficient that the Euler-Lagrange equations:

$$(13)\ \frac{\partial L}{\partial \Psi(i)^{a\ldots b}{}_{c\ldots d}} - \left(\frac{\partial L}{\partial \Psi(i)^{a\ldots b}{}_{c\ldots d;e}}\right)_{;e} = 0$$

hold for all (i).

We obtain the energy momentum tensor from the Lagrangian by considering the change in the action induced by a change in the metric.

4. Field equations

To determine what the field equations should be we shall determine the Newtonian limit. Since the Newtonian gravitational field equation does not include time, correspondence with the Newtonian theory should be made with a metric that is static, meaning a metric that admits a timelike Killing vector field (K) which is orthogonal to a family of spacelike surfaces, which can be regarded as surfaces of constant time and may be labeled by the parameter (t).

We define the unit timelike vector (V) as ($f^{-1}K$) where ($f^2 = -K^a K_a$). Then ($V^a{}_{;b} = -\dot{V}_a V_b$) where ($\dot{V}^a = V^a{}_{;b}V^b = f^{-1}f_{;b}g^{ab}$) represents the departure from geodesity of the integral curves of (V), which are also the curves of (K). Note that ($\dot{V}^a V_a = 0$).

These integral curves define the static frame of reference. We can derive an equation for the Newtonian gravitational potential by considering the divergence of ($\dot{V}^a$):

$$(14)\ \dot{V}^a{}_{;a} = \left(V^a{}_{;b}V^b\right)_{;a} = V^a{}_{;b;a}V^b + V^a{}_{;b}V^b{}_{;a} = R_{ab}V^aV^b + \left(V^a{}_{;b}\right)_{;b}V^b + \left(V_b\dot{V}^b\right)^2$$
$$= R_{ab}V^aV^b$$

But:

$$(15)\ \dot{V}^a{}_{;a} = \left(f^{-1}f_{;b}g^{ab}\right)_{;a} = -f^{-2}f_{;a}f_{;b}g^{ab} + f^{-1}f_{;ba}g^{ab}$$

and:

$$(16)\ f_{;ab}V^aV^b = -f_{;a}V^a{}_{;b}V^b = -f^{-1}f_{;a}f_{;b}g^{ab}$$

So we find:

$$(17)\ f_{;ab}(g^{ab} + V^aV^b) = f\, R_{ab}V^aV^b$$

We therefore obtain agreement with the Newtonian theory in the limit of a weak field, when ($f \cong 1$), if the term on the right is equal to ($4\pi G$) times the matter density plus terms which are small in the weak field limit. This will be the case if there is a relation:

$$(18)\ R_{ab} = K_{ab}$$

where (K_{ab}) is a tensorial function of the energy momentum tensor and the metric, which is such that ($(4\pi G)^{-1}\, K_{ab}V^aV^b$) is equal to the matter density plus terms which are small in the Newtonian limit since (R_{ab}) satisfies the contracted Bianchie identities ($R_a{}^b{}_{;b} = \frac{1}{2}R_{;a}$), implies that:

$$(19)\ K_a{}^b{}_{;b} = \frac{1}{2} K_{;b}$$

which shows that the apparently natural equation ($K_{ab} = 4\pi G T_{ab}$) cannot be correct due to the equation above (19) and the conservation equations ($T_a{}^b{}_{;b} = 0$) would imply ($T_{;a} = 0$).

The only first order identities satisfied by the energy-momentum tensor are the conservation equations. The only tensorial function (K_{ab}) of the energy-momentum tensor and the metric, which obeys the identities ($K_a{}^b{}_{;b} = \frac{1}{2} K_{;b}$) for all energy-momentum tensors is the:

$$(20)\ K_{ab} = k\left(T_{ab} - \frac{1}{2} T\, g_{ab}\right) + \lambda g_{ab}$$

where (k) and (λ) are constants. The values of these constants can be determined from the Newtonian limit, for example a perfect fluid with energy density (μ) and pressure (p) whose flow lines are the integral curves of the Killing vector, hence:

$$(21)\ f_{;ab}(g^{ab} + V^a V^b) = f\left(\frac{1}{2} k(\mu + 3p) - \lambda\right)$$

In the Newtonian limit the pressure is usually very small compared to energy density. We would then obtain approximate agreement with Newtonian theory if ($k = 8\pi G$) and if (λ) is very small. We shall use the units of mass where ($G = 1$), in these units a mass of ($10^{28} g$) corresponds to a length of ($1 cm$).

We then integrate the previous equation over a compact region (F) at the three surface where ($t = constant$), and transform the left hand into an integral of the gradient of (f) over the bounding two-surface (∂F):

$$(22)\ \int_F f\big(4\pi(\mu + 3p)\big) d\sigma = \int_F f_{;ab}(g^{ab} + V^a V^b) d\sigma = \int_{\partial F} f_{;a}(g^{ab} + V^a V^b) d\tau_b$$

where ($d\sigma$) is the volume element of the three-surface where ($t = const$), in the induced metric, and ($d\tau_b$) is the surface element of the two-surface (∂F) in the three surface.

Here we realize two important differences from the Newtonian case:

1) A factor (f) appears in the integral on the right-hand side.
2) The pressure contributes to the total mass.

Hence we form the equations:

$$(23)\ R_{ab} = 8\pi\left(T_{ab} - \frac{1}{2} T g_{ab}\right) + \lambda g_{ab}$$

known as “Einstein field equations” and can also be written in the form:

$$(24)\ \left(R_{ab}-\frac{1}{2}Rg_{ab}\right)+\lambda g_{ab}=8\pi T_{ab}$$

Since both sides are symmetric, these form a set of ten coupled non-linear partially differential equations in the metric and its first and second derivatives. However, the covariant divergence of each side vanishes identically:

$$(25)\ \left(R^{ab}-\frac{1}{2}Rg^{ab}+\lambda g^{ab}\right)_{;b}=0\ \ and\ T^{ab}{}_{;b}=0$$

hold independent of the field equations, which means that the field equations really provide only six independent differential equations of the metric, which is the correct number of equations to determine the spacetime, since four of the ten components of the metric can be given arbitrary values by use of the four degrees of freedom to make coordinate transformations. Einstein equations can also be determined by requiring that the action:

$$(26)\ I=\int_D\ (A(R-2\lambda)+L)dv$$

be stationary under variations of (g_{ab}), where (L) is the matter Lagrangian and (A) is a suitable constant for:

$$(27)\ \Delta\big((R-\lambda)dv\big)=\left((R-2\lambda)\frac{1}{2}g^{ab}\Delta g^{ab}+R_{ab}\Delta g^{ab}+g^{ab}\Delta R_{ab}\right)dv$$

which can be written as:

$$(28)g^{ab}\Delta R_{ab}dv=g^{ab}\big((\Delta\Gamma^{c}{}_{ab})_{;c}-(\Delta\Gamma^{c}{}_{ab})_{;b}\big)dv=(\Delta\Gamma^{c}{}_{ab}g^{ab}-\Delta\Gamma^{d}{}_{ad}g^{ac})_{;c}dv$$

Thus it can be transformed into an integral over the boundary (∂D), which vanishes as ($\Delta\Gamma^{a}{}_{bc}$) vanishes the boundary:

$$(29)\ \left.\frac{\partial I}{\partial u}\right|_{u=o}=\int_D\ \left\{A\left(\left(\frac{1}{2}r-\lambda\right)g^{ab}-R^{ab}\right)+\frac{1}{2}T^{ab}\right\}\Delta g_{ab}dv$$

hence if ($\partial I/\partial u$) vanishes for all (Δg_{ab}), we obtain the Einstein equations on the setting ($A=(16\pi)^{-1}$).

5. Fieldless equations

Equations without any fields are necessary to effectively explain the earliest stages of the Universe, from the beginning of the Big Bang, an event known as “cosmic inflation”. The period of cosmic inflation was a small segment of a second. Only time and space are necessary for the fieldsless equations which is the reason for the name “fieldless”. Having in mind that, within General Relativity, space-time consists of three spatial dimensions and another, temporal dimension, fieldless equations can also be called “dimensional equations”.

The point of the fieldless equations is to explain the fundament nature of the Universe:

1. Why does it expand at an accelerated rate?
2. What is the reason for its homogenous and isometric nature?
3. What is (c) known as the speed of light and why is it the most crucial and important constant?
4. What are the material properties of time?
5. What is the relationship between space and time?

We have to abandon the idea of “arrow of time” and instead define a new term called “temporal motion” which means that time is expanding without a specific direction, in all directions positive to the observer; temporal motion also causes spatial expansion which will be shown later and that it accelerates the spatial expansion as well.

We form an equation by defining that there are two imaginary angles (α_x, α_y) formed by the intersection of an imaginary line that will represent an imaginary axis of ($a(t)$) and another imaginary line (d) that will represent a diameter of space. The two angles are always:

$$(30)\ \alpha_x = \alpha_y = 90^{\circ}$$

These two imaginary angles will represent the difference between the present and the ongoing “temporal motion” that is the future. We will represent this temporal motion with an ($\twoheadrightarrow$) symbol, in honor of the previous idea of the “arrow of time”.

The ($a(t)$) is the scale factor that depends on time and it will describe the expansion of time, that influences the spatial expansion, hence the expansion of the whole Universe. We declare that ($a(t)$) is heading to a *realistic maximum* ($rmax$). It is not certain if this maximum exists so we also define that ($rmax = \infty$) if there is no maximum or ($rmax \neq \infty$) if there is a maximum. If there is a maximum than the Universe will not expand infinitely from a temporal perspective, meaning indefinitely, but will instead have an end in the distant future.

Now we define a set of equations to explain how the relationship between time and space functions due to temporal motion:

$$(31)\ a(t)\begin{cases} t(x) = \log_{\lim\limits_{x\to\infty}\left(\frac{\twoheadrightarrow}{x}+\twoheadrightarrow\right)^{x}}(x)\,; a \to 0 \\ t(y) = \log_{\lim\limits_{y\to\infty}\left(\frac{\twoheadrightarrow}{y}+\twoheadrightarrow\right)^{y}}(y)\,; a \to 0 \\ t(z) = z+\twoheadrightarrow; \qquad\qquad a \to 0 \end{cases}$$

This is the reason why our spacetime is nearly flat. If the (z) spatial dimension were to expand as the (x) and (y) do, our spacetime would be shaped like a saddle.

Or in other words, we imagine the spacetime as:

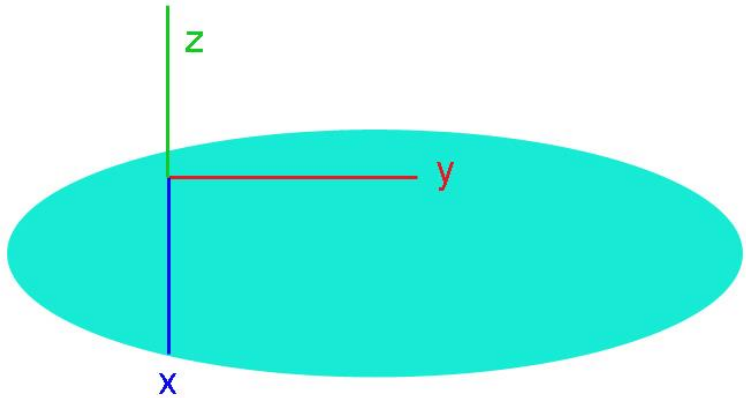

Figure 1: the flat spacetime

Meaning that the (x) and (y) are expanding while spacetime is “moving on”, or “expanding on”, the (z) dimension. On an image it would look like this:

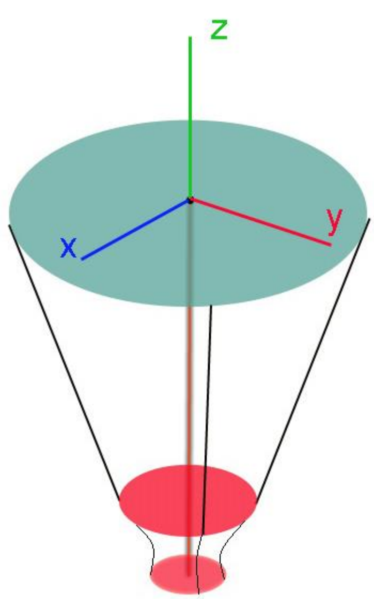

Figure 2: spacetime from the Big Bang to the present

Time expands in all directions as represented by ($a(t)$), therefore making an observer feel like time is "passing". Time also forces the spatial dimensions to expand hence forming the spacetime continuum and by expanding in all directions we could say that time inflates space and it has done so from the soul beginning of the Big Bang due to temporal motion.

6. Temporal motion

Unlike spatial motion, temporal motion requires no direction. Instead of a trajectory it needs expansion and it needs a velocity. Time expands in all direction and it influences spatial expansion, hence it "inflates space", which forms the spacetime continuum.

We define that on the quantum level:

$$(32)\ \delta \twoheadrightarrow = \delta \int d\text{Д}\, L(a(t), \dot{a}(t))$$

where the (dД) is the accelerating quantum and ($\dot{a}(t)$) is the velocity. We also define that:

$$(33)\ \dot{a}(t) = c$$

Where (c) is "the speed of light". This is the reason for time dilatation caused by velocity and why (c) is the speed necessary to achieve maximal time dilatation. What (c) actually is, is the speed of temporal motion. Any velocity will cause time dilatation to some extent as every mass of a celestial body will cause gravitational time dilatation to some extent.

The accelerating quantum (dД) constantly tries to accelerate temporal motion but it cannot since temporal motion cannot exceed (c), which leads the accelerating quantum to influence the expansion of space instead, manifesting as "dark energy" on the large scale and accelerating spatial expansion. Spatial expansion of the Universe is well beyond the speed (c) since dark energy has been massing to the point that it has an overwhelming presence in the Universe.

Therefore:

$$(34)\ \delta \twoheadrightarrow = \delta \int d\text{Д}\, L(a(t), c)$$

In order to prove that time inflates space and has inflated it since the beginning of the Big Bang, causing "cosmic inflation", we go back to the early Universe in the period known as the "radiation dominated era".

As ($t \to 0$) and ($a \to 0$) the only term is the radiation one, meaning that the Universe was dominated by radiation, around ($z \gtrsim 3200$).

For the early, radiation dominated era we can approximate a solution:

$$(35)\ a(t) \simeq \left(2H_0\sqrt{\Omega_{r;0}}\,t\right)^{1/2}$$

The early, radiation dominated Universe expanded as ($a \propto \sqrt{t}$).

We use the initial data to write the fieldless equations for the initial inflation of the Universe:

$$(36)\ a(t)\begin{cases} t(x) = \log_{\lim\limits_{a\to 0}\left(\frac{\twoheadrightarrow}{\alpha_x}+\twoheadrightarrow\right)^{\frac{\twoheadrightarrow}{\alpha_x}}}(x)\,;\, x \to \infty \\ t(y) = \log_{\lim\limits_{a\to 0}\left(\frac{\twoheadrightarrow}{\alpha_y}+\twoheadrightarrow\right)^{\frac{\twoheadrightarrow}{\alpha_y}}}(y)\,;\, y \to \infty \\ t(z) = z+\twoheadrightarrow; \qquad\qquad z \to \infty \end{cases}$$

Which means that time inflates space and has always inflated it, causing what we call "cosmic inflation". With time the accelerating quantum influenced space so that the Universe is now dominated by the (λ) factor, or in other words "dark energy" is dominant in the present Universe.

II Conclusion

In the conclusion of the paper we observe the equation (34) where ($d Д$) is the accelerating quantum that is responsible for the existence of "dark energy".

This means that dark energy can be explained as "temporal kinetic energy" since it is caused by temporal motion.

$$(37)\ \langle \widehat{Д} \rangle = \left\langle \Psi \left| \sum_{\twoheadrightarrow=1}^{HU(t)} \frac{-\hbar^2}{2m_\lambda}\nabla^2_{\twoheadrightarrow} \right| \Psi \right\rangle = -\frac{\hbar^2}{2m_\lambda} \sum_{\twoheadrightarrow=1}^{HU(t)} \langle \Psi | \nabla^2_{\twoheadrightarrow} | \Psi \rangle$$

where (H) is the Hamiltonian of the system, ($\nabla^2_{\twoheadrightarrow}$) is the Laplacian, ($HU(t)$) is time-evolution and (m_λ) is the mass which manifests as dark matter.

When the Universe was young the (λ) factor was significantly lesser than it is now, therefore the rate of acceleration of the expansion was much lesser than it is at the present period of time.

III References

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