A theory of the Comprehensive Endosemasiopasigraphic Algebraico-Predicate Organon and its conformal catlogographic interpretations:

A general analytical solution of trial decision problems for first-order predicate calculus

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«Results by A. Church based on papers by K. Gödel show that the quest for a general solution of the decision problem must be regarded as hopeless.»
Hilbert and Ackermann [1950, p. 124]

Abstract

In contrast to Church, who proved in 1936, based on papers by Gödel, that a *dual decision problem* for the conventional axiomatic first-order predicate calculus is unsolvable, I have solved a *trial decision problem algebraically* (and hence analytically, not tabularily) for a properly designed *axiomatic first-order algebraico-predicate calculus*, called briefly the *trial logic* (*TL*), and have successfully applied the pertinent *algebraic decision procedures* to all conceivable logical relations of academic or practical interest, including the 19 categorical syllogisms. The structure of the TL is a *synthesis* of the structure of a *conventional axiomatic first-order predicate calculus* (briefly *CAPC*) and of the structure of an *abstract integral domain*. Accordingly, the TL contains as its autonomous parts the so-called *Predicate-Free Relational Trial Logic* (*PFRTL*), which is parallel to a *conventional axiomatic sentential calculus* (CASC*)*, and the so-called *Binder-Free Predicate Trial Logic* (*BFPTL*), which is parallel to the predicate-free part of a *pure* CAPC. This treatise, presenting some of my findings, is alternatively called "*the Theory of Trial Logic*" ("*the TTL*") or "*the Trial Logic Theory*" ("*the TLT*"). The treatise *reopens* the entire topic of symbolic logic that is called "*decision problem*" and that Church actually *closed by the fact of synecdochically calling* the specific *dual* decision problem, the insolvability of which he had proved, by the *generic* name "decision problem", without the qualifier "*dual*". Any additional axiom that is incompatible with the *algebraic decision method* of the trial logic and that is therefore detrimental for that method is regarded as one belonging to either to another logistic system or to mathematics

Table of contents

Preface

1. The background of the treatise

1) Based on two papers by Kurt Gödel [1930, 1931], Alonzo Church [1936a, 1936b] proved that the *decision problem* for a *conventional axiomatic first-order predicate calculus* is unsolvable (see also Rosser [1939]). Regarding his unsolvable decision problem, Church [1936b, p. 41, footnote 6] says:

> «By the Entscheidungproblem of a system of symbolic logic is here understood the problem to find an effective method by which, given any expression Q in the notation of the system, it can be determined whether or not Q is provable in the system. »

At the same time, Hilbert and Ackermann [1950, p. 124] comment on the papers of Church thus:

> « Results by A. Church based on papers by K. Gödel show that the quest for a general solution of the decision problem must be regarded as hopeless. We cannot report on these researches in detail within the limits of this book. We shall only remark that a general method of decision would consist of a certain recursive procedure for the individual formulas which would finally yield for each formula the value truth or the value falsehood. Church's work proves, however, the non-existence of such a recursive procedure; at least, the necessary restrictions would not fall under the general type of recursion set up by Church, who has given to the somewhat vague intuitive concept of recursion a certain precise formalization.»

Thus, the decision problem, which was dealt with by Church, should have been explicitly called a *dual* (*two-valued*, *two-fold*) *decision problem* in the sense that, if existed, its solution for a given relation would have discriminated between the pertinent *positive value* of the relation as its *provability* or *truth* (*validity*) and the respective *negative value* as its *improvability* or *untruth* (*falsehood, invalidity*). However, modern *formal logic* is *dual* (*two-valued*) and therefore it has not dealt with any decision problems other than dual ones. Consequently, the generic name "decision problem" was unfortunately used in the literature on logic, particularly by Church himself and by the commentators on his works, *synecdochically* instead of the more correct specific name “dual decision problem” – just as the generic name “formal logic” is as a rule used *synecdochically* instead of the more correct specific name “dual formal logic”. Since the dual character of Church’s decision problem was blurred, therefore by the fact of proving its insolvability Church actually eliminated the entire subject category called “*decision problem*” from the *subject taxonomy* (*partition*) *of symbolic logic*. The logicians of the generation, succeeded that of Church and his contemporaries, have in fact abandoned the very concept of decision problem – just as long ago the physicists abandoned their concept of *ether* and just as long ago the mathematicians abandoned their concept of *infinitesimals* as being supposedly infinitely small but nonzero real numbers. In the modern mathematics the latter notion is replaced by the so-called *ε&δ-language* (*epsilon-and-delta-language*). Thus, the theorem of Church, which was of course a distinguished achievement of symbolic logic, paradoxically became at the same time detrimental to symbolic logic from the standpoint of prospective trends of its further development. Particularly, it was discouraging logicians to attempt formulating and solving a *trial* (*three-fold*) *decision problem* of some kind so as to contradict neither to the results of Gödel nor to the results of Church. I employ the first sentence of the above quotation of Hilbert and Ackermann [1950, p. 124], given above in the item 1, as an epigraph to my treatise in order to emphasize the fact that the generic name “decision problem” without either additional qualifier “dual” or “trial” is a *misnomer* that results in confusion, while the fact that the trial decision problem has turned out to be solvable does not contradict the results of Church and agrees with the results of Gödel.

2) After Whitehead and Russell [1910; 1962, p. 6ff], relations of any *conventional axiomatic logical calculus* (briefly *CALC*), a *sentential* one (briefly *CASC*) or a *first-order predicate* one (briefly *CAFOPC* or synecdochically *CAPC*), is supposed to be *propositional* or *dualistic truth-functional* in the sense that every relation of any CALC that is not paradoxical can be either *true* or *untrue* (*false*), the understanding being that the *negation* of a true relation is an untrue (false) relation and vice versa. In general, the *validity* or *invalidity* of a relation of *dual formal logic* can be qualified as a *truth-functional* one, and likewise the *truth* or *untruth* (*falsehood*) of a relation of dual formal logic can be qualified as a *validity-functional* one, in the sense that a relation of dual formal logic is said to be *valid* if and only if it

is *true* and *invalid* if and only if it is *untrue* (*false*). Consequently, the *negation* of a valid (true) relation is an invalid (untrue, false) relation and vice versa.

3) Based on the results of Church, the skepticism of Hilbert and Ackermann regarding possibility to solve the dual decision problem for first-order predicate calculus has been shared by some other authoritative logicians, who have not, however, explicitly mentioned that the problem in question is dual – just as Hilbert and Ackermann and Church have not explicated this fact. Here follows one of the most categorical statements, if not the most categorical one, regarding such a decision problem by Suppes [1957, pp. 69–70]:

> «In chapter 2 we saw that there was a mechanical method (by use of truth tables) for testing the truth-functional validity or invalidity of an argument. Such a mechanical method is often called a *decision procedure*. In one sense the existence of a decision procedure for truth-functional arguments trivializes the subject. Fortunately or unfortunately, no such trivialization of the logic of quantification is possible. It was rigorously proved in 1936 by the contemporary American logician Alonzo Church that there is no decision procedure, that is, no mechanical test, for the validity of arbitrary formulas in first-order predicate logic.[*] Since all of mathematics may be formalized within first-order predicate logic,[†] the existence of such a decision procedure would have startling consequences: a machine could be built to answer any mathematical problem or to decide on the validity or invalidity of any mathematical argument. But Church's theorem ruins at a stroke all such daydreams of students of logic and mathematics. Not only there is no known decision procedure: his theorem establishes that there never be any.
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> [*]First-order predicate logic is the logic of sentential connectives and quantifiers for individual variables, that is, the logic of the formulas defined in Chapter 3. "First-order" refers to the fact that no quantification of predicates is permitted.
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> [†]The standard developments of axiomatic set theory has one of their aims to establish this fact in substantive details.»

Unlike Hilbert and Ackermann, who associate solution of the decision problem for a given relation with the possibility to decide whether the relation is true or false, Suppes associates solution of the decision problem for a given relation with the possibility to decide whether the relation is valid or invalid. However, in accordance with the above item 2 the values truth and falsehood of a propositional (dualistic truth-functional) relation are tantamount to its values validity and invalidity respectively. Therefore, Suppes speaks about the same dual decision problem as Hilbert and Ackermann.

4) Should the dual decision problem be solvable, Suppes misinterprets implications of its solution in mathematics for the following reasons. First, a system of *class*, or particularly *set*, *theory* is a *semantic theory* that cannot be equivalent to any system of first-order predicate calculus. Particularly, a class theory should necessarily contain a *class-builder* such as '$\{\boldsymbol{x}|\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n\rangle\}$', which puts a class-valued *term* $\{\boldsymbol{x}|\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n\rangle\}$ into a correspondence to the pertinent *relation* $\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n\rangle$ ('$\boldsymbol{P}$', '$\boldsymbol{x}$', '$\boldsymbol{x}_1$', '$\boldsymbol{x}_2$', …, '$\boldsymbol{x}_n$' are atomic placeholders having the appropriate ranges). Such a term cannot be bound by the same quantifiers as those binding $\boldsymbol{x}$, and it is not introduced by formation rules of any first-order predicate calculus. Particularly, any axiomatic system of set theory has a certain axiom, which makes that system *self-consistent* (*non-paradoxical*) and which necessarily involves, explicitly or implicitly, a certain *set-builder*. This axiom was originally called "*Axiom of Ausonssonderung*" by Zermello [1908], i.e. "*Axiom of sifting*", and it is most often called in English "*Axiom of specification*" (e.g., in Halmos [1960, p. 6]) or "*Axiom of separation*". Also, if a class (or set) theory involves nonempty individuals then the latter can be introduced only by *verbal* axioms (cf., e.g., Fraenkel *et al* [1973, pp. 24–25]) and hence *informally.* Lastly, a system of axiomatic set theory (e.g.) cannot be regarded as a system of first-order predicate calculus at all. To be specific, here follow two different but equivalent formulations of Axiom of specification.[2]

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[2]The same axiom is informally (in the intuitive manner of Halmos' formulation) stated in Bernays [1958, p. 11] under the name *Axiom of Subsets.* Supposedly the same axiom is semi-formally (in the semi-formal manner of Suppes' formulation) stated in Fraenkel *et al* [1973, p. 31] under the name *Axiom of*

«**Axiom of specification**. *To every set A and to every condition S*(*x*) *there corresponds a set B whose elements are exactly those elements x of A for which S*(*x*) *holds*.» Halmos [1960, p. 6].

«... the *axiom schema of specification:*

$$(\exists B)(\forall x)\big(x \in B \leftrightarrow x \in A \,\&\, \varphi(x)\big).$$

It is understood in the axiom schema of specification that the variable '*B*' is not free in $\varphi(x)$.» Suppes [1960, p. 21].

It is evident that the informal verbal universal quantifiers, which occur in Axiom of specification by Halmos can conventionally be written symbolically as '$(\forall A)(\forall S)(\exists B)(\forall x)$', while the quantifier '$(\forall S)$' should be understood as: «for every predicate *S* that is defined in terms of $\in$ and perhaps of some sentential connectives present in the given set theory». Suppes' operand of his axiom schema of specification should be bound by the quantifiers '$(\forall A)(\forall\varphi)$' that are similar to '$(\forall A)(\forall S)$', subject to the like reservation regarding '$(\forall\varphi)$'. In this case, besides '*x*', either condition '$S(x)$' or '$\varphi(x)$' may involve any number *n* of additional atomic terms such as 'x_1', 'x_2', …, 'x_n', which should be bound by the respective universal quantifiers '$(\forall x_1)$', '$(\forall x_2)$', …, '$(\forall x_n)$'.

5) In contrast to the results of Church, I have in my treatise *algebraically* and hence *analytically* (*not tabularily*) solved a sequence of interrelated *trial* (*three-valued*, *three-fold*) *decision problems* for a certain properly designed *algebraico-predicate calculus of first order* and have successfully applied the pertinent *algebraic*

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comprehension. However, in the latter formulation of the axiom, the condition analogous to the condition '$x \in A$' in Suppes' axiom schema is missing. Therefore, Axiom of comprehension of Fraenkel *et al* is contradictory (paradoxical). In the set-theoretic system by Bourbaki [1960], the axiom separation schema is stated under the logographic name S8 and verbal name "La schéma de sélection et réunion", i.e. "The schema of selection and reunion" [*ibid*. Chap. II, §1, n°6]. There occurs in Bourbaki's schema the syntactic variable ***R***, whose range is [the set of] the so-called *relations* of the theory, i.e. the well-formed sentence-valued formulas of the theory.

decision procedures to all conceivable logical relations of interest, including the 19 categorical syllogisms. The above calculus is qualitatively described along with all its trial *algebraic decision methods* (*ADM's*) in Preface of the treatise, in general outline, and in its Introduction, in depth. The fact that all pertinent trial decision problems have turned out to be solvable does not contradict the results of Church and agrees with the results of Gödel. But in order to develop that calculus and to treat of its successive constituent parts, and also in order to solve the above decision problems, I have developed the entire system of new notions, to which the conventional dualistic terminology that is *from the very beginning* based on using semantic terms such as "proposition", "truth", and "falsehood" is inapplicable, except for the case of restricted dualistic interpretation of the *final* results. Accordingly, I have developed the appropriate new system of nomenclature, i.e. of *pasigraphic* (*euautographic* and *logographic*) *notation* and *phonographic* (*wordy*, *verbal*) *terminology*. Voltaire said, «If you wish to converse with me, define your terms». Therefore, some most conspicuous peculiarities of the algebraico-predicate calculus of first order in question and of the solutions of the associated trial decision problems are explicated below along with some indispensable elements of the pertinent nomenclature.

2. The trial (three-valued, three-fold) decision problems that are solved in the treatise

2.1. "Organon" and "Psychologistics"

1) Some elements of the new comprehensive terminology are used (but not mentioned) from the very beginning in *the title of the treatise*, which should be understood as follows.

i) The entire calculus addressed in the treatise is denoted logographically by '$\mathcal{A}_1$' and is called (denoted phonographically) the *Combined Algebraico-Predicate Organon* (*CAPO*) or *Combined Advanced Algebraico-Logical Organon* (*CAALO*), and also the *Psychologistic Trial Formal Logic* (*PLTFL*). The principal *semantically uninterpreted* (*genuinely self-referential, chess-like*) calculus of $\mathcal{A}_1$ is denoted by 'A_1' and is called the *Comprehensive Euautographic Algebraico-Predicate Organon* (*CEAPO*) or *Comprehensive Euautographic Advanced Algebraico-Logical Organon* (*CEAALO*), whereas *the calculus of placeholders of euautographic relations of* A_1 is denoted by '$\mathbf{A}_1$' and is called the *Comprehensive Panlogographic Algebraico-Predicate Organon* (*CPLAPO*) or *Comprehensive Panlogographic Advanced Algebraico-Logical Organon* (*CPLAALO*). I use the term "Organon" in analogy with Aristotelian «*Organon*» and also in analogy with Bacon's «*Novum Organum*», but I attach it with the specific sense of the *description* (*descriptive name*) "*master logical calculus having an inseparable associated trial* (*three-valued, three-fold*) *algebraic decision method*", the understanding being that "*algebraic*" implies "*analytical*" ("*not tabular*"). The qualifier "*comprehensive*" to "organon" means «*having an infinite number of branches that share the same trialistic algebraic decision method*»; "*euautographic*" means «*graphic* (*written*) *and genuinely self-referential*»; and "*panlogographic*" means «*logographic over* (*assuming, taking on, interpretable by*) *euautographic values*». Accordingly, A_1 or $\mathbf{A}_1$ is a tree-like algebraico-predicate calculus of first order, comprising an infinite number of branches that have the same *trialistic advanced algebraic decision method (TAADM*) in common, which is denoted by 'D_1' or '$\mathbf{D}_1$' and which is called the *Euautographic* or *Panlohographic AADM* – briefly *EAADM* or *PLAADM*, respectively. The above proper names of D_1 and $\mathbf{D}_1$ are variants of the proper "ALO"-names of A_1 and $\mathbf{A}_1$ without the qualifier "Comprehensive" and with "ADM" in place of "ALO".

ii) D_1 is *the conjunction of current* (at any given moment) *rules of inference* (*transformation*) *and decision of* A_1, *primary* (*postulated) ones*, i.e. *subject* (*intrinsic*) *axioms* and *meta-axioms*, and *secondary* (*inferred*) *ones*, i.e. *subject* (*intrinsic*) *theorems* and *meta-theorems*. Therefore, D_1 belongs to the *inclusive metalanguage* (*IML*) of A_1, i.e. to the treatise in question, and not to A_1, which is prescinded from the IML The rules comprised in D_1 are expressed in terms of two *categoremata* (special terms) $\mathsf{0}$ and $\mathsf{1}$ (in this font) and some *syncategoremata* (*kernel-ssigns* and *punctuation marks*), *belonging to both* A_1 *and* $\boldsymbol{\mathsf{A}}_1$, and also in terms of some *categoremata* (*terms* and *relations*) *of* $\boldsymbol{\mathsf{A}}_1$, being at the same time *panlogographic placeholders* (*PLPH's*), *whose ranges are certain classes of euautographic categoremata* (correspondingly, *terms* or *relations*) *of* A_1, Therefore, D_1 and $\boldsymbol{\mathsf{D}}_1$ are in fact *two hypostases* (*aspects*) *of the same TAADM*, so that the above remarks regarding D_1 apply to $\boldsymbol{\mathsf{D}}_1$ as well.

iii) The qualifier "advanced" to either generic name "Algebraico-Logical Organon" ("ALO") or "algebraic decision method" ("ADM") is used by way of emphatic comparison with either one of the qualifiers "*rich basic*" and "*basic*" (or "*depleted basic*"), which will be used in the sequel for distinguishing two certain parts of A_1 or $\boldsymbol{\mathsf{A}}_1$ and of the respective two parts of D_1 or $\boldsymbol{\mathsf{D}}_1$.

iv) The union and superposition of A_1 and $\boldsymbol{\mathsf{A}}_1$ is denoted by 'A_1' and called the *Comprehensive Biune Euautographic and Panlogographic Algebraico-Predicate Organon* (*CBUE&PLAPO*) or concisely the *Comprehensive Endosemasiopasigraphic Algebraico-Predicate Organon* (*CBEnSPGAPO*) – the name that occurs (is used but not mentioned) in the title of the treatise. The occurrence of the generic name "*Algebraico-Predicate Organon*" ("*APO*") in either of the above two synonymous terms can be used interchangeably (synonymously) with an occurrence of the generic name "*Advanced Algebraico-Logical Organon* ("*AALO*"), so that A_1 is alternatively (synonymously) called the *Comprehensive Biune Euautographic and Panlogographic Advanced Algebraico-Logical Organon* (*CBUE&PLAALO*) or the *Comprehensive Endosemasiopasigraphic Advanced Algebraico-Logical Organon* (*CEnSPGAALO*). The occurrence of the qualifier "*Biune*" ("*BU*") in the former term means: «*being the union and at the same time a superposition of the two pertinent APO's*, or *ALO's*». The adjective "*pasigraphic*", being a combining form of the complex *monomial* qualifier "*endosemasiopasigraphic*" (abbreviated as "*EnSPGR*"), means «*either* *euautographic or panlogographic* (in general, *logographic*)». Etymologically, I have derived the adjective "pasigraphic" from the Greek adjective "πᾶς" \pás\ meaning *all* or *every* so that, lexically, it means «*commonly intelligible, i.e. capable of being shared by all people independent of the languages they use*», – like «logographic» and «pictographic» («iconographic»). Consequently, the qualifier "endosemasio-pasigraphic" to A_1 means that all relations and all terms of A_1 are *pasigraphic,* i.e. either those of A_1 or those of $\boldsymbol{\mathsf{A}}_1$, while the complex prepositive prefix "*endosemasio*" (in contrast to "*exosemasio*") emphasizes the fact that *any pasigraph of* A_1 *neither has nor assumes* (*takes on*) *any signification* (*import value*) *beyond* A_1, i.e. that A_1 is *semantically close.* Etymology of all unconventional terms that I use is explained in the treatise. The *ADM of* A_1 is logographically denoted by 'D_1' and is alternatively called the *Biune Euautographic and Panlogographic Advanced Algebraic Decision Method* (*BUE&PLAADM*) or the *Endosemasiopasigraphic Advanced Algebraic Decision Method* (*EnSPGAADM*). These proper names of D_1 are variants of the respective proper "ALO"-names of A_1 without the qualifier "Comprehensive" and with "ADM" in place of "ALO" (cf. a like relation between the proper "ALO"-name of A_1 or $\boldsymbol{\mathsf{A}}_1$ and the proper name of D_1 or $\boldsymbol{\mathsf{D}}_1$ respectively). It is understood that D_1 is the union and superposition of D_1 and $\boldsymbol{\mathsf{D}}_1$. In accordance with the alternative name "the *Psychologistic Trial Formal Logic*" of $\mathcal{A}_1$, the treatise can, alternatively but less informatively, be called *the Theory of Psychologistic Trial Formal Logic* (*the TPLTFL*) or *the Psychologistic Trial Formal Logic Theory* (*the PLTFLT*). In subsequent refences to the treatise, the abbreviations "TPLTFL" and "PLTFLT" will be abbreviated further as "*TTL*" (for "*Theory of Trial Logic*") and "*TLT*" (for "*Trial Logic Theory*") respectively.

v) My treatise, i.e. my TTL, has the following important aspect. The occurrence of the noun "*Principia*" in the title "*Principia Mathematica*" of the known 3-volume monograph by A. N. Whitehead and B. Russell [1910–13] means *dual-logic principles of.* Accordingly, *I regard my treatise* as *Principia Nova Mathematica*, where the occurrence of the substantive "*Principia Nova*", i.e. "*new principles of*", means *trial-logic principles of.* In this case, the trial-logic principles are the higher logical principles, which allow answering epistemological questions and solving logical problems (the decision problem is among them), being beyond the scope of

Principia Mathematica and also beyond the scope of any other *dual* logical theory that has stemmed from or been inspired by the above monograph.

vi) In order to solve the *trial decision problem* for any *relation of interest of* A_1, i.e. either of A_1 or of $\mathbf{A}_1$, which is qualified as a *slave relation* (*SR*), I *algebraically prove* (*deduce*) for it the pertinent *master*, or *decision*, *theorem* (*MT* or *DT*). In accordance with the [syntactic] form of the master theorem, I unambiguously classify the slave relation as a [syntactically] *valid* one or as an *antivalid* one, or else as a *vav-neutral* (*vav-indeterminate*, *neither valid nor antivalid*) one. Since the calculus A_1 is semantically close (endosemasiopasigraphic), therefore a *panlogographic relation* (*PLR*) *of* $\mathbf{A}_1$ is valid if and only if every *euautographic relation* (*ER*) *of* A_1 *in its range* is valid, and similarly with "antivalid" in place of "valid". For the same reason, the range of a vav-neutral PLR may, in the general case, comprise ER's of all the three classes: valid, antivalid, and vav-neutral. The slave relation of an MT (DT) is also *the slave relation of the proof of the MT*, which is alternatively called an *algebraic decision procedure* (*ADT*) *for the slave relation*. An SR, MT (DT), and ADT are called *euatographic* (*E*) *ones*, i.e. briefly an *ESR*, *EMT* (*EDT*), and *EADT* respectively, if they belong to A_1, and *panlogographic* (*PL*) *ones*, i.e. briefly a *PLSR*, *PLMT* (*PLDT*), and *PLADT* respectively, if they belong to $\mathbf{A}_1$. The notion of a proposition is not applicable to the *dramatis personae* of an APD – either in the Aristotelian sense of "proposition" as a *truth-functional declarative sentence* or in the Frege-Church sense of "proposition" as *the [Platonic] sense a truth-functional declarative sentence.*

2) The banner "*Psychologistics*", under which this treatise (TTL) is included, is an abbreviation of the description "*Psychological foundations of logic and logical foundations of psychology*" ("*PFL & LFP*"). It is understood that Psychologistics is a *biune* field of study and discourse, so that the *psychology*, called the *psychologistic psychology* (*PLP*), and the *logic*, called the *psychologistic logic* (*PLL*) or *psycho-logic*, which are *complementary conceptual hypostases* (*ways of existence*, *aspects*) of Psychologistics, can be distinguished and contrasted, but they cannot be separated from each other, – like *matter* and *form* of a thing.

i) By "*psychologistic* psychology" ("*PLP*"), I mean traditional *introspective psychology* (as opposed to various trends of modern *extrospective psychology*), or more precisely *cognitive* and *conative* aspects (as opposed to *affective* ones) *of* *introspection* (*introspective psychology*) *of my own*, along with the doctrine of *physicalistic monism* (relegated to *philosophical psychology*), according to which *my mind is my cerebral cortex and vice versa.*

ii) For convenience in description and study, the *psychologistic logic* (*PLL*) can in turn be divided into two parts, one of which is called the *principal*, or *first*, *PLL* (*PPLL*), and the other one is called the *auxiliary*, or *applied*, or *second*, *PLL* (*APLL*).

a) The PPLL is a certain *trial* (*three-valued*) *logic* (*TL*), so that it is more specifically called the *trial PLL* (*TPLL*) or *psychologistic TL* (*PLTL*). In accordance with Aristotelian *principle* (*doctrine*) *of opposition and unity of matter and form of a being*, which is called *hylomorphism*, – from the Greek nouns: “ύλη” \íli\ (pl. “ύλαι” \íle\), meaning *a matter*, and “μορφή” \morfí\ (dual “μορφά” \morfá\, pl. “μορφαί” \morfé\), meaning *a form*, – the PLTL (TPLL, PPLL) has two *complementary conceptual hypostases* (*ways of existence*, *aspects*), namely, the *psychologistic trial formal logic* (*PLTFL*) and the *psychologistic trial material logic* (*PLTML*) *adjoint of the PLTFL*, the understanding being that the two can be distinguished and contrasted, but they cannot be separated from each other (cf. two aspects PFL and LFP of Psychologistics).

a_1) The PLTFL is denoted by ‘$\mathcal{A}_1$’ and therefore it is alternatively called the *Combined Algebraico-Predicate Organon* (*CAPO*) or the *Combined Advanced Algebraico-Logical Organon* (*CAALO*). Therefore, the expressions “*A theory of the Combined Algebraico-Predicate Organon*” and “*A theory of the Combined Algebraico-Logical Organon*” could be used as two other alternative titles of the treatise. $\mathcal{A}_1$ can be thought of as *the sequence of the four interrelated logistic systems* $\mathbf{A}_1$, A_1, I_1, *and* A_1 *in that order*, the first two of which are the organons that have been described earlier. The four systems are interrelated as follows.

$\mathbf{A}_1$ is the calculus of *panlogographic relations* (*PLR’s*), which are *panlogographic placeholders* (*PLPH’s*) of *euautographic* (*genuinely autographic*, *semantically uninterpreted*) *relations* (*ER’s*) of the calculus A_1, so that a PLR is the *panlogographic interpretans* (*anti-interpretand*, pl. “*interpretantia*”) of the ER’s that are condensed (comprised) in its range, while the ER’s are *euautographic interpretands* of the PLR.

I_1 is the so-called *conservative conformal catlogographic* (*CCFCL*) *interpretation of* A_1, which is the set of *CCFCL interpretations* of ER’s of A_1 of three kinds: (a) some selective *valid ESR's*, (b) some selective *vav-neutral ESR's*, (c) the *EMT's* (*EDT*'s) of the selective vav-neutral ESR's. The totality of rules of I_1, denoted by 'I_1', comprises replacements of the occurrences of *atomic euautographic ordinary terms (AEOT's*), as u to z, u_1 to z_1, u_2 to z_2, etc, and $\emptyset$, and of *atomic euautographic relations* (*AER's*), as p to s, p_1 to s_1, p_2 to s_2, etc, throughout the above *euautographic interptetantia* (*interpreted euautographic relations*) with occurrences of the respective *atomic conformal catlogographic terms* (*ACFCLT's*) u to z, u_1 to z_1, u_2 to z_2, etc, and $\varnothing$ and *atomic conformal catlogographic relations* (*ACFCLR's*) p to s, p_1 to s_1, p_2 to s_2, etc, without any quotation marks. The function of I_1 is in principle analogous to that of D_1 or $\mathbf{D}_1$ with the only difference that it is trivial. In the result of the above *conformal catlogographic replacements*, a valid ESR is transduced into the respective so-called *formally tautologous* (*f-tautologous*, *universally f-true*) *conservative catlogographic relation* (*CCLR*), a vav-neutral ESR is transduced into the respective so-called *f-ttatt-neutral* (*f-ttatt-indeterminate, neither f-tautologous nor f-antitautologous*) *CCLR*, and the EMT (EDT) of a vav-neutral ESR is transduced into the *CCFCLMT* (*CCFCLDT*) of the respective *slave* f-ttatt-neutral CCLR.

A_1 is the so-called *progressive conformal catlogographic* (*PCFCL*) *interpretation of* A_1, i.e. the *PCFCL interpretations* of some selective *vav-neutral ESR's of* A_1 with the help of the respective PCFCL interpretations of the their EMT's (EDT's). Unlike I_1, A_1 is another organon, which is alternatively called the *Comprehensive Catlogographic Algebraico-Predicate Organon* (*CCLAPO*) or *Comprehensive Catlogographic Advanced Algebraico-Logical Organon* (*CCLAALO*). Accordingly, A_1 has a certain TAADM, which is denoted by 'D_1' and is called the *catlogographic AADM* (*CLAADM*). All rules of D_1 are CCFCL interpretands of the rules of D_1, so that formally

$$D_1 = I_1(\mathsf{D}_1). \tag{2.1}$$

That is to say, A_1 has no transformation (inference) and no decision rules other than those comprised in D_1. At the same time, A_1 has no formation rules of its own either: some selective *ttatt-neutral output CLR's of* I_1 can be used as *input CLR's of* A_1. Strictly some of the input CLR's can be *postulated*, *permanently* or *ad hoc*, *to be f-veracious* (*accidentally f-true*) by replacing their CCFCLMT's with *the pertinent progressive CFCL master postulate* (*PCFCLMP*). In this case, the CCFCLMT's of some other input CLR can be developed with the help D_1 further with the purpose to deduce the pertinent *progressive CFCLMT's* (*PCFCLMT*) and to decide thus whether the given slave CLR is *f-veracious* (*accidentally f-true*) or *f-antiveracious* (*accidentally f-antitrue*) or else *f-vravr-neutral* (*f-vravr-indeterminate, neither f-veracious nor f-antiveracious*); it is understood that an f-vravr-neutral (f-vravr-indeterminate) CLR is an *f-tat-neutral* (*f-tat-indeterminate*, *neither f-true nor f-antitrue*) and vice versa. Thus, I_1 plays two interrelated roles: first, it is the *most immediate interpretational supplement to* A_1 and, second, it is the *interpretational interface between* A_1 *and* A_1.

a_2) The PLTML is the union of *two* sets of English *declarative sentences* (*DS's*). One of the two sets contains m-true and m-ttatt-neutral DS's that are explicitly used as examples illustrating material interpretations of certain f-true and f-ttatt-neutral CLR's. The other set of DS's of PLTML comprises *assertive* and hence *materially true* (*m-true*), i.e. *m-tautologous* (*universally m-true*) *and m-veracious* (*accidentally m-true*, *fact-conformable*) *DS's of the IML* (*inclusive metalanguage*) *of* $\mathcal{A}_1$, i.e. DS's of the treatise, which *are used but not mentioned*, and which are *latent* (*implicit*) *physical* (*substitutional*) *sentential interpretands of certain formally-true* (*f-true*), i.e. *f-tautologous* (*universally t-true*) *and f-veracious* (*accidentally f-true*) *CLR's of* $\mathcal{A}_1$. To be more specific, the PLTFL, $\mathcal{A}_1$, involves a system of *euautographic* (*genuinely autographic*, *semantically uninterpreted*) kernel-signs (operators), including logical connectives, relational logical contractors (pseudo-quantifiers and pseudo-qualifiers), and substantival algebraic contractors (pseudo-multipliers), whose use is determined by the rules of formation, transformation (inference), and decision of A_1. At the same time, there are in the *exclusive metalanguage* (*XML*) *of* $\mathcal{A}_1$ some standard *phonographic* (*wordy*) operators (conjunctions and adverbs), which are associated with certain *euautographic* operators (kernel-signs) in the sense that they are supposed to apply to the appropriate declarative sentential clauses as their *operata* in accordance with the same rules, according to which their counterpart euautographic operators apply to the appropriate *euautographic* or *logographic operata of* $\mathcal{A}_1$. The correspondence between calogographic, and hence phonographic, occurrences and euautographic occurrences of the same operators will be made explicit in due course later on. Meanwhile, I shall remark that I *associate*:

"not"¸ "it is not the case that", or "it is not the true that" *with* '¬',

"or" or "ior" ("inclusive or"), i.e. "vel" in Latin, *with* '∨', “and” or “&” *with* ‘$\wedge$’,

“if … then –” or “… only if –” *with* ‘$\Rightarrow$’,

“if” *with* ‘$\Leftarrow$’,

‘if and only if” or “iff” *with* ‘$\Leftrightarrow$’,

“neither … nor –” *with* ‘$\not\vee$’ or ‘$\overline{\vee}$’,

“not both … and –” *with* ‘$\not\wedge$’ or ‘$\overline{\wedge}$’,

“but not” *with* ‘$\overline{\Rightarrow}$’;

“not … but –” *with* ‘$\overline{\Leftarrow}$’,

“either … or – but not both” or “xor” (“exclusive or”), i.e. “auf” in Latin, *with* ‘$\overline{\Leftrightarrow}$’,

“for some $*$:” or “for at least one $*$:” or “there exists at least one $*$ such that” *with* ‘$\bigvee_*$’,

“for all $*$:” or “for every $*$:” *with* ‘$\bigwedge_*$’,

“for some but not all $*$:” or “for strictly some $*$:” *with* ‘$\breve{\bigvee}_*$’,

“for at most one $*$:” or “there exists at most one $*$ such that” *with* ‘$\frown\!\!\!\!\bigvee_*^1$’,

“for exactly one $*$:” or “there exists exactly one $*$ such that” *with* ‘$\bigvee_*^1$’,

“the product of … over $*$” *with* ‘$\hat{\cdot}_*$...’

in all occurrences of the above-mentioned wordy operators. It is understood that alike ellipses that occur in a group of synonymous operators should be replaced alike by the appropriate concrete operata. In view of the analogy that exists between the binary disjunction operator ‘$\vee$’ and the existential quantifier ‘$(\exists *)$’ and in view of the like analogy that exists between the binary conjunction operator ‘$\wedge$’ and the universal quantifier ‘$(\forall *)$’, which are explicated in the treatise, I employ the *binder* (*contractor*) *signs* ‘$\bigvee_*$’ and ‘$\bigwedge_*$’ instead of ‘$(\exists *)$’ and ‘$(\forall *)$’ respectively.

b) Every *metaterm* (metalinguistic *term*) and particularly every *taxonym* (*name of a taxon*, i.e. *of a taxonomic class*) of the APLL is a *description*, or more explicitly *description of the species, through a genus and the difference*, or *differences*, – briefly *DcTrG&D*, *DcSTrG&D*, *DcTrG&Ds*, or *DcSTrG&Ds* in that order, in Latin *descriptio*, or *descriptio species, per genus et differentiam*; or *differentias*, respectively. A *definition* whose definiens is a DcTrG&D or DcTrG&Ds is a traditional *definition through the genus and difference* (*differentia*), or *differences* (*differentiae*), – briefly a *DfTrG&D* or *DfTrG&Ds*, in Latin *definitio per genus et differentiam*, or *differentias*, which was introduced by Aristotle [350 BCE, *Posterior Analytics*] (referred to as [APstAM]) and which is often called a *real*, or *explicative*, *definition*. Therefore, the APLL is alternatively called the *onomastic PLL* (*OPLL*) and also the *psychologistic onomatology* (*PLO*). The APLL (OPLL, PLO) comprises *three* self-subsistent *egocentric systems of psychologistic terminology,* i.e. systems, whose elements have *definite significations with respect to me* and, by transcendental extrapolation, *analogous significations with respect to you*. Two of the three systems are *egocentric terminological esperantos*, one which is called the *first psychologistic onomastics* and also "*onymology*" or "*nymology*", because any one of its elements is a *monomial description* of Greek origin, having either allomorph "*onym*" or "*nym*" *as its root* (*generic name*). The constituent graphonyms "*graphonym*" and "*phononym*" of *onymological* (*nymological*) *terms* are abbreviated respectively as "*graph*" and as *"phon"*¸ which are used as the pertinent *effective roots*. Another egocentric terminological esperanto is called the *second psychologistic onomastics* and also *onology*, because any one of its elements is a *monomial description* of Greek origin, having the morpheme "*on*" *as its root* (*generic name*). The third system of psychologistic terminology, called the *third psychologistic onomastics,* is an inhomogeneous system of *univocal* (*single-valued*, *monosemantic*) monomial and polynomial descriptions, involving chaste English or Anglicized Latin words, and hence it is not a terminological esperanto.

c) In accordance with the pertinent terms that have been introfuced at the begining of this item ii and at its sub-items a and b, the treatise can alternatively be called "*The psychologistic logics*" or more specifically "*The psychologistic trial and psychologistic onomastic logics*" in reference to both branches of psychologistic logics (PLL) or briefly "*The psychologistic trial logic*", thus putting the APLL backward.

2.2. Incoherent (binder-free and predicate-free) restrictions of $\mathcal{A}_1$

3) The qualifier "*Advanced"* ("*A*"), occurring in the proper name "the *Combined Advanced* Algebraico-*Logical Organon*" ("the *CAALO*)" of $\mathcal{A}_1$, is relevant to the fact that $\mathcal{A}_1$ includes as its autonomous but inseparable part an organon, which is denoted by '$\mathcal{A}_1^0$' is qualified *Rich Basic*, whereas the latter organon includes as its autonomous and separable part an organon, which is denoted by '$\mathcal{A}_0$' and is qualified

Basic or *Depleted Basic*. To be specific, $\mathcal{A}_1^0$ is called the *Combied Rich Basic Algebraico-Logical Organon* (*CbRBALO*) and also the *Combined Binder-Free,* or *Contractor-Free, Algebraico-Predicate Organon* (*CbBFAPO* or *CbCFAPO*), whereas $\mathcal{A}_0$ is called the *Combined Basic,* or *Combined Depleted Basic,* or *Combined Predicate-Free, Algebraico-Logical Organon* (*CbBALO* or *CbDBALO* or *CbPFALO*). Accordingly, with "PLTFL" being as before an abbreviation for "Psychologistic Trial Formal Logic", $\mathcal{A}_1^0$ and $\mathcal{A}_0$ can alternatively be called the *Rich Basic PLTFL* (*RBPLTFL*) or *Binder-Free Predicate Trial Logic* (*BFPTL* and the *Basic*, or *Depleted Basic, PLTFL* (*BPLTFL* or *DBPLTFL*), or *Predicate-Free Relational Trial Logic* (*PFRTL*), respectively, while $\mathcal{A}_1$ can, more precisely, be called the *Advanced PLPFL* (*APLPFL*), and not just the PLPFL. For the sake of brevity, both $\mathcal{A}_1^0$ and $\mathcal{A}_0$ are set up as constituent parts of $\mathcal{A}_1$, but every categorem (formula, term or relation) of $\mathcal{A}_1$ is unambiguously recognizable either as one of $\mathcal{A}_1^0$ or as one of $\mathcal{A}_0$, or else as none of the two kinds.

4) Like $\mathcal{A}_1$, the organon $\mathcal{A}_1^0$ is *the sequence of four interrelated logistic systems* $\mathbf{A}_1^0$, A_1^0, I_1^0, and A_1^0 (in this order), being *autonomous but inseparable constituent parts* of $\mathbf{A}_1$, A_1, I_1, and A_1 respectively, whereas the organon $\mathcal{A}_0$ is in turn *the sequence of four interrelated logistic systems* $\mathbf{A}_0$, A_0, I_0, and A_0 (in this order), being *autonomous and separable constituent parts* of $\mathbf{A}_1^0$, A_1^0, I_1^0, and A_1^0 and hence those of $\mathbf{A}_1$, A_1, I_1, and A_1, respectively. Using as before the abbreviations "APO" for "Algebraico-Predicate Organon" and "ALO" for "Algebraico-Logical Organon", A_1^0 is called the *Comprehensive Euautographic Binder-Free,* or *Contractor-Free, APO* (*CEBFAPO* or *CECFAPO*) and also the *Comprehensive Euautographic Rich Basic Algebraico-Logical Organon* (*CERBALO*), whereas $\mathbf{A}_1^0$ is called the *Comprehensive Panlogographic Binder-Free,* or *Contractor-Free, Algebraico-Predicate Organon* (*CPLBFAPO* or *CPLCFAPO*) and also the *Comprehensive Panlogographic Rich Basic Algebraico-Logical Organon* (*CPLRBALO*); the latter two names are variants of the former two with "Panlogographic" ("PL") in place of "Euautographic" ("E"). A_0 is called the *Euautographic Predicate-Free,* or *Euautographic Basic,* or *Euautographic Depleted Basic* (in contrast to *Euautographic Rich Basic*), *ALO* (briefly *EPFALO* or *EBALO* or *EDBALO*), whereas $\mathbf{A}_0$ is called the *Panlogographic*

Predicate-Free, or *Panlogographic Basic*, or *Panlogographic Depleted Basic* (in contrast to *Panlogographic Rich Basic*), *ALO* (briefly *PLPFALO* or *PLBALO* or *PLDBALO*); the latter two names are again variants of the former two with "Panlogographic" ("PL") in place of "Euautographic" ("E").

5) The *union* and superposition of two interrelated organons A_1^0 and $\mathbf{A}_1^0$, or A_0 and $\mathbf{A}_0$, is denoted by 'A_1^0', or by 'A_0', respectively, so that A_1^0 is an *autonomous but inseparable constituent part* of A_1, whereas A_0 is an *autonomous and separable constituent part* of A_1^0 and hence that of A_1. Consequently, in analogy with the corresponding names of A_1, A_1^0 is called the *Comprehensive Biune Euautographic and Panlogographic Binder-Free*, or *Contractor-Free*, *APO* (*CBUE&PLBFAPO* or *CBUE&PLCFAPO*) and also the *Comprehensive Biune Euautographic and Panlogographic Rich Basic ALO* (*CBUE&PLRBALO*), whereas A_0 is called the *Biune Euautographic and Panlogographic Predicate-Free ALO* (*BUE&PLPFALO*), and also the *Biune Euautographic and Panlogographic Basic*, or *Deleted Basic* (in contrast to *Rich Basic*), *ALO* (*BUE&PLBALO*). The occurrence of the qualifier "*Biune Euautographic and Panlogographic*" ("*BUE&PL*") in any of the above names can be used interchangeably (synonymously) with an occurrence of the qualifier "*Endosemasiopasigraphic*" ("*EnSPG*"). The *autonomy* of A_1^0 relative to the host organon A_1 or the autonomy of A_0 relative to either host organon A_1^0 or A_1 means that an autonomous organon is *physically* or *as if physically* fitted into the host organon (like a Russian matreshka into a larger one), and is not just prescinded from the IML (inclusive metalanguage) of A_1. A like remark applies, *mutatis mutandis*, to the autonomy of any logistic system $\mathbf{A}_1^0$, A_1^0, I_1^0, A_1^0, $\mathbf{A}_0$, A_0, I_0, or A_0 relative to its only or its either host logistic system.

6) The ADM's of

$$\mathbf{A}_1^0,\ \mathsf{A}_1^0,\ \mathsf{I}_1^0,\ \mathit{A}_1^0,\ \mathbf{A}_0,\ \mathsf{A}_0,\ \mathsf{I}_0,\ \mathit{A}_0,\ \mathsf{A}_1^0,\ \mathsf{A}_0 \tag{2.2}$$

are denoted by

$$\text{'}\mathbf{D}_1^0\text{', '}\mathsf{D}_1^0\text{', '}\mathit{I}_1^0\text{', '}\mathit{D}_1^0\text{', '}\mathbf{D}_0\text{', '}\mathsf{D}_0\text{', '}\mathit{I}_0\text{', '}\mathit{D}_0\text{', '}\mathsf{D}_1^0\text{', '}\mathsf{D}_0\text{'} \tag{2.3}$$

respectively. In this case,

$$I_1^0 = I_1,\ D_1^0 = I_1\left(\mathsf{D}_1^0\right),\ D_0 = I_0\left(\mathsf{D}_0\right). \tag{2.4}$$

Also, D_1^0 is the union and superposition of D_1^0 and $\mathbf{D}_1^0$, whereas D_0 is the union and superposition of D_0 and $\mathbf{D}_0$. I recall that the proper names of D_1, $\mathbf{D}_1$, and D_1 are variants of the proper "ALO"-names of A_1, $\mathbf{A}_1$ and A_1 without the qualifier "Comprehensive" and with "ADM" in place of "ALO". The proper names of

$$\mathbf{D}_1^0,\ \mathsf{D}_1^0,\ I_1^0,\ D_1^0,\ \mathbf{D}_0,\ \mathsf{D}_0,\ I_0,\ D_0,\ \mathsf{D}_1^0,\ \mathsf{D}_0 \tag{2.5}$$

are formed of the proper "ALO"-names of the respective organons (2.2), *mutatis mutandis*, in a like way. Consequently, the three RBALO's A_1^0, $\mathbf{A}_1^0$, and A_1^0, or the three BALO's A_0, $\mathbf{A}_0$, and A_0, and their verbal names, full and abbreviated, are interrelated in the same way as the three AALO's A_1, $\mathbf{A}_1$, and A_1 and as their verbal names, respectively. Likewise, the three RBADM's D_1^0, $\mathbf{D}_1^0$, and D_1^0, or the three BADM's D_0, $\mathbf{D}_0$, and D_0, and their verbal names, full and abbreviated, are interrelated in the same way as the three AADM's D_1, $\mathbf{D}_1$, and D_1 and as their verbal names, full and abbreviated, respectively. Also, the three organons A_1^0, A_0, and A_1, or $\mathbf{A}_1^0$, $\mathbf{A}_0$, and $\mathbf{A}_1$, or A_1^0, A_0, and A_1, and their verbal names, full and abbreviated, are interrelated similarly. Lastly, the four logistic systems $\mathbf{A}_1^0$, A_1^0, I_1^0 (i.e I_1), and A_1^0, or $\mathbf{A}_0$, A_0, I_0, and A_0, are interrelated in the same way as $\mathbf{A}_1$, A_1, I_1, and A_1.

7) Just as D_0 (D_0 or $\mathbf{D}_0$), D_1^0 (D_1^0 or $\mathbf{D}_1^0$) concerns exclusively with occurrences of the *logical* (*ordinary*) *connectives* and of the *algebraic* (*special*) *sign of equality* $\hat{=}$ in *euautographic* and *panlogographic relations* (*ER's* and *PLR's) of* A_1, and not with occurrences of any *binders* (*contractors*), if present. That is to say, D_1^0 and D_0 are *functionally* the same set of rules, but the domains of application of D_1^0 and D_0 differ in *euautographic* and *panlogographic operata* (singular "*operatum*") of the above *kernel-signs* (*KS's*), i.e. in *euautographic* and *panlogographic formulas* (*EF's* and *PLF's*) *of* A_1 that are *united* (*acted upon*) *by the KS's* to produce the respective *euautographic* and *panlogographic relations* (*ER's* and *PLR's*) *of* A_1 as the *operands* (*scopes*) *of the KS's*. In general outline, the difference between D_1^0 and D_0 is that the domain of application of D_1^0 is the class of ER's and PLR's of the organon A_1^0, which is, like A_1, *branched* (*comprehensive*, *inhomogeneous)*, while the domain

of application of D_0 is the class of ER’s and PLR’s of the organon A_0, which is, unlike A_1, *unbranched* (*homogeneous*).

8) The occurrence of the adjective “*Algebraic*” in every one of the phonographic (wordy, verbal) names of organons and of their associated ADM’s, as suggested above, means «*involving the laws of algebra*», while the occurrence of the suffixed connective vowel “o”, followed by the hyphen, in any one of the above names of organons means «*and*». The algebraic part of any given one of the organons

$$\mathsf{A}_1,\ \mathbf{A}_1,\ \mathrm{A}_1,\ A_1,\ \mathsf{A}_1^0,\ \mathbf{A}_1^0,\ \mathrm{A}_1^0,\ A_1^0,\ \mathsf{A}_0,\ \mathbf{A}_0,\ \mathrm{A}_0,\ A_0 \tag{2.6}$$

is called the *integronic domain of that organon*. Owing to its integronic domain, *all phases and all branches* of any given one of the *AALO’s* A_1, $\mathbf{A}_1$, A_1, and A_1, *have the same built-in AADM in common,* which is denoted by ‘D_1’, , ‘$\mathbf{D}_1$’, ‘D_1’, or ‘D_1’ respectively. The remaining organons on the list (2.6) are *not AALO’s* and hence they are *neither phases nor branches of the respective AALO’s* A_1, $\mathbf{A}_1$, A_1, and A_1, Accordingly, a *restriction* of an AALO is called (i) a *coherent one*, if it is the restriction of the AALO either to some one of its phases or to some one or some more of its branches, and (ii) an *incoherent one*, if it is the restriction of the restriction of the AALO either to *its Rich Basic constituent part* or to *its Basic constituent part*. At the same time, owing to their simplicity, A_0, $\mathbf{A}_0$, A_0, and A_0 can be regarded as introductions into A_1, $\mathbf{A}_1$, A_1, and A_1 respectively, so that, in reference to this role, the former can be called *the first zero quasi-phases of the respective latter*, whereas A_1^0, $\mathbf{A}_1^0$, A_1^0, and A_1^0, having the like simple ADM’s can be called *the second zero quasi-phases of the respective* A_1, $\mathbf{A}_1$, A_1, and A_1.

9) The post positive occurrence of the adjective “*Predicate*” in any pertinent verbal name of any one of the organons A_1, $\mathbf{A}_1$, A_1, A_1, A_1^0, $\mathbf{A}_1^0$, A_1^0, and A_1^0, should be understood as an abbreviation of the adjective equivalent “concerning *both in predicate-containing and in predicate-free relations*”. Accordingly, the occurrence of the adjective equivalent “*Predicate-Free*” in the pertinent verbal name of A_0, $\mathbf{A}_0$, A_0 , or A_0 means «*concerning in predicate-free relations*». In this case, A_0 is in a sense parallel to a *conventional axiomatic sentential calculus* (*CASC*), particularly to the *Russell logistic system*, denoted by ‘$\mathrm{P_R}$’, and to the equivalent *Russell-Bernays logistic*

system, denoted by 'P_{RB}' (see, e.g., Hilbert and Ackermann [1950, §10, pp. 27–30], Church [1956, §25, pp. 136–138; §29, p. 157], or Bourbaki [1960, chap. I, §3, S1–S4]). To be recalled, P_R is based on the five axioms, which were published in Russell [1908] and which were afterwards used in Whitehead and Russell [1910; 1962, pp. 96, 97]) as the items **∗1·2–∗1·6.** Bernays [1926] discovered the non-independence of Russell's axiom **∗1·5**, so that P_{RB} is based on the remaining four Russell axioms. At the same time, $\mathbf{A}_0$ is parallel to a CASC that is set up in terms of the pertinent *axiom schemata*, – such a CASC e.g. as the schematic version of P_{RB} given in Bourbaki [1960, chap I, §3, S1–S4]. In spite of the fact that both A_0 and $\mathbf{A}_0$ are parallel to a CASC, in forming verbal names of A_0 and $\mathbf{A}_0$, I utilize one of the compound qualifiers "*Predicate-Free Algebraico-Logical*" and "*Basic Algebraico-Logical*" instead of either of the conventional qualifiers "*sentential*" and "*propositional*" (cf. Hilbert and Ackermann [1950, pp. 27, 165, 166], Church [1956, pp. 27, 28, 69, 119], Suppes [1957, p. 3], Lyndon [1966, pp. 20, 35]), because the latter two are *incompatible* with any of the qualifiers "*euautographic*", "*panlogographic*", and "*endosemasiopasigraphic*".

10) Owing to its special simplicity, A_0 can serve as an introduction into A_1 (cf. the above item 8). Still, for saving room and labor, I started in the treatise directly from A_1 and developed A_0 as the simplest one of an infinite number of other restrictions of A_1. In this case, the statement that A_0 is an autonomous constituent part of A_1, i.e. that A_0 and $\mathbf{A}_0$ are autonomous constituent parts of A_1 and $\mathbf{A}_0$ respectively, means that, firstly, A_0 can be set up and executed independently of A_1 and that, secondly, in addition to the attributes of A_0 such as its *euautographic atomic basis* (*EAB*) and its rules of formation, transformation, decision, and interpretation, which are at the same time some *basic* (*elementary*) attributes of A_1, the latter has some other, *advanced*, attributes of the same classes. Therefore, the statements that are relevant to advanced attributes of A_1 are irrelevant to A_0 and conversely some statements related exclusively to A_0 are inapplicable to A_1 as a whole. For instance, there are no *ordinary terms* (*OT's*) in A_0, neither *euautographic* ones (*EOT's*) nor *panlogographic* ones (*PLOT's*), so that all terms of A_0 are *special* ones (*SpT's*), called also *integrons* (*I's*), – either *euautographic* ones (*ESpT's* or *EI's*) or *panlogographic*

ones (*PLSpT's* or *PLI's*). At the same time, there are some statements that are relevant to both A_0 and A_1. In this case, if A_0 and A_1 have a certain feature in common then A_1^0 has the same feature. Therefore, in order not to make redundant statements, especially in preliminary discussions as this one, I shall employ two devices. First, I shall state properties of A_1 and then I shall supplement a relatively complete passage relevant to properties of A_1 with a statement or statements of the changes, if any, which should be introduced in that passage in case of A_0 or A_1^0 in place of A_1. Second, when possible and advisable to emphasize some properties that A_0 and A_1, and hence A_1^0, have in common, I treat of A_0 and A_1 simultaneously. In order to do this conveniently, I use the symbol 'A_n' as a placeholder for either of the two symbols 'A_0' and 'A_1'. From a somewhat different viewpoint, 'A_n' is just an abbreviation of the phrase 'A_0 or A_1'. Like remarks and a like definition apply, *mutatis mutandis*, with any one of the letters 'A', '$\mathbf{A}$', 'I', 'A', 'D', '$\mathbf{D}$', 'D', 'I', and 'D' (e.g) in place of 'A'. Also, the above definition of the meaning of the subscript '$_n$' on any one of these and some other appropriate letters is formalized and generalized as follows.

11) Every statement that contains some of the logographic symbols 'A_n', '$\mathbf{A}_n$', 'A_n', 'I_n', 'A_n', 'D_n', '$\mathbf{D}_n$', 'D_n', 'I_n', and 'D_n' or some other logographic similar symbols, which may be introduced after this manner in the sequel, and each of which consists of a capital base letter of an appropriate distinctive type and of the Roman (upright) subscript '$_n$' and perhaps of some other labels, should be understood as a schema of the two statements, one of which corresponds to '$_0$' and the other one to '$_1$' in place of '$_n$'. Under the above definition, A_n, $\mathbf{A}_n$, and A_n, are called an *EALO*, a *PLALO*, and an *EnSPSGALO* or *BUE&PLALO*, whereas D_n, $\mathbf{D}_n$, and D_n are called an *EADM*, *PLADM*, and *EnSPSGADM* or *BUE&PLADM*, respectively. As before, I use the abbreviations: "E" for "euautographic", "PL" for "panlogographic", "EnSPSG" for "Endosemasiopasigraphic", "BUE&PL" for "Biune Euautographic and Panlogographic", "ALO" for "algebraico-logical organon", and "ADM" for "algebraico-logical method".

12) Once the organons A_1 and $\mathbf{A}_1$, i.e. the single whole organon A_1, are set up and learned, they can be executed without mentioning their theory – just as a native

language is used in everyday communication without mentioning its grammar. Particularly, all inference and decision procedures of A_1, i.e. all executions of its AADM, D_1, turn out to be almost as simple as computational procedures of primary school arithmetic with integers. Especially simple are executions of D_0, and D_1^0, while D_1, contains some additional, more sophisticated rules for handling the binders (contractors) of A_1. However, all executions of D_1 are after all as straightforward and intelligible as executions of D_0, and D_1^0. The most difficult problems concerning the organons A_1 and $\mathbf{A}_1$ are setting them up and explicating various *epistemological aspects* of them, including significant (semantic) interpretations of A_1, – the problems, which lie far beyond the scope of the primary school arithmetic. In order to solve these problems and to instruct the reader how to execute D_1 and $\mathbf{D}_1$, I have set up A_1 and $\mathbf{A}_1$ and the rules of interpretation of A_1 within their IML (inclusive metalanguage) that is identical with *the theory of* $\mathcal{A}_1$, i.e. with the treatise. The IML is a complicated self-consistent linguistic construction which, in addition to the *pasigraphic* (*euautographic* and *logographic*) *nomenclature* of A_1 and $\mathbf{A}_1$ and of the other relevant object logistic systems, contains extensive and extremely ramified unconventional self-consistent syntaxic phonographic (wordy, verbal) terminology concerning both the object logistic systems and the IML itself.

2.3. The organon A_1 and its trial algebraic decision method (ADM)

13) A_1 is a *tree-like*, *phased and branched*, *euautographic* (*uninterpreted and immediately uninterpretable semantically*) *algebraico-predicate calculus of first order*, whose structure remotely reminds both the structure of a *conventional axiomatic first-order predicate calculus* (briefly *CAFOPC* or synecdochically *CAPC*, pl. "*CAFOPC'i*" or "*CAPC'i*" respectively), especially the structure of the calculus F^1 of Church [1956, chaps. III and IV], and the structure of an abstract *integral domain* (as framed, e.g., in Birkhoff & Mac Lane [1965, pp. 1, 2] or Mac Lane & Birkhoff [1967, pp. 132–134]). The algebraic part of A_1 is called the *integronic domain of* A_1. Owing to its integronic domain, *all phases and all branches of* A_1 *have the same built-in ADM in common*, which is denoted logographically by 'D_1' and which is called (denoted phonographically) the *Advanced ADM* (*AADM*) *of* A_1 or the *Euautographic AADM* (*EAADM*). In order to set up A_1 along with its D_1 as a single while *algebraico-logical* organon, the set of admissible *primary* (*undefined*) *atomic* (*functionally*

indivisible) *euautographs* (*graphic chips*) *of* A_1, which is denoted logographically by ‘B_1’ and called (denoted phonographically) the *euautographic atomic basis* (*EAB*) *of* A_1, is assumed (postulated) to consist of two parts: the *ordinary* (*non-special*), or *logical*, *EAB* (*OEAB* or *LEAB*), denoted by ‘B_{1O}’, and the *special* (*unordinary*), or *algebraic, EAB* (*SpEAB* or *AlEAB*), denoted by ‘B_{1Sp}’. In order to set up A_1 as a *branching tree-like* organon, B_{1O} is composed of two parts: the *mandatory*, or *obligatory*, *ordinary basis*, denoted by ‘B_{1OM}’, and the *selective ordinary basis*, denoted by ‘B_{1OS}’. The union of B_{1OM} and B_{1Sp} is called (denoted phonographically) *the mandatory*, or *obligatory, basis of* A_1 and it is denoted [logographically] by ‘B_{1M}’. An element of B_1 is called a *basic*, or *primary atomic, euautograph* (*BscE* or *PAE*), “*primary*” meaning *postulated* or, concurrently, *undefined*. Consequently, an element of B_{1O} is called a *primary atomic ordinary*, or *logical*, *euautograph* (*PAOE* or *PALE*) and an element of B_{1Sp} is called a *primary atomic special*, or *algebraic*, *euautograph* (*PASpE* or *PAAlE*). The qualifiers “ordinary” and “logical”, or “special” and “algebraic”, can be used interchangeably (synonymously) when they apply either to the respective part of B_1 or to a PAE and generally to *any euautographic term*s. When, however, they apply to *combined euautographic relations* (*CbER’s*), they remain synonyms in some cases and cease to be synonyms in some other cases.

14) In accordance with the previous item, B_1 is postulated to comprise the following PAE’s:

I) The ordinary (logical) basis, B_{1O}

a) The mandatory (obligatory) ordinary (logical) basis, B_{1OM}

i) The square and round brackets: [ ] ( )

ii) The comma: ,

iii) The *primary universal logical connective* (*connective-sign*): $\veebar$

iv) The *primary logical sign of contraction* (*binding*): ∃

v) *Atomic pseudo-variable ordinary terms* (*APVOT’s*), called also (unredundantly) *pseudo-variable ordinary terms* (*PVOT’s*):

$$u, v, w, x, y, z, u_1, v_1, w_1, x_1, y_1, z_1, u_2, v_2, w_2, x_2, y_2, z_2, \ldots \qquad (2.7)$$

b) The selective (optional) ordinary (logical) basis, B_{1OS}

vi) *Atomic pseudo-variable ordinary relations* (*APVOR’s*), called also (unredundantly) *atomic euautographic relations* (*AER’s*) or *atomic pseudo-variable relations* (*APVR’s*):

$$p, q, r, s, p_1, q_1, r_1, s_1, p_2, q_2, r_2, s_2, \ldots, \tag{2.8}$$

vii) *Atomic pseudo-variable ordinary predicate-signs* (*APVOPS's*), *singular* ones:

$$f^1, g^1, h^1, f_1^1, g_1^1, h_1^1, f_2^1, g_2^1, h_2^1, \ldots, \tag{2.9^1}$$

binary ones:

$$f^2, g^2, h^2, f_1^2, g_1^2, h_1^2, f_2^2, g_2^2, h_2^2, \ldots, \tag{2.9^2}$$

and so on.

viii) Any one and only one of the three *primary binary atomic pseudo-constant ordinary predicate signs* (briefly, *PBAPCOPS's* or *primary BAPCOPS's* or *primary binary APCOPS's*): =, called the *ordinary equality sign*; $\subseteq$, called the *rightward mass-inclusion predicate-sign*; $\in$, called the *rightward class-membership predicate-sign*.

ix) In the presence of $\subseteq$ or $\in$, two *atomic pseudo-constant ordinary terms* (*APCOT's, PCOT's*) $\emptyset$ and $\emptyset'$, the first of which is the *systemic* (*permanent*) one, called the *euautographic ordinary zero*, or *pseudo-empty*, *term* (*EOZT* or *EOPET*), while the second one, called the *subsidiary* (*temporary*) *EOZT* or *EOPET*, is used exclusively for proving that $\emptyset = \emptyset'$, i.e. that $\emptyset$ is unique, and is disregarded after doing this duty.

II) The special (algebraic) basis, $\mathrm{B}_{1\mathrm{Sp}}$

x) The *special* (*algebraic*) *kernel-signs*: $\hat{-}$, the *singular sign of additive inversion*; $\hat{+}$, the *binary sign of addition*; $\hat{\cdot}$, the *binary sign of multiplication* and at the same time the *base transcendental sign of multiplication*, called also (in this hypostasis) the *base sign of multiplicative contraction* (*binding*); $\hat{=}$, the *binary sign of equality*.

xi) The *transformative special* (*algebraic*) *singulary kernel-sign*: V, which is called *the validity-sign* (or, when regarded as an abbreviation of $V(\;)$, *the validity-operator*) *of termizing* (*substantivating*, *substantivizing*) *a relation*, because its function is *converting a relation into a computable special term* (*substantive*), which is called the *primary*, or *initial*, *validity-integron* (*PVI* or *IVI*) *of that relation*.

xii) The two *primary atomic special* (*algebraic*) *terms*: 0, called the *zero integron* or the *special* (*algebraic*) *zero-term* (*SpZT*), and 1, called the *unity* *integron* or the *special* (*algebraic*) *unity-term* (*SpUT*). Collectively, the two terms are called the *primary atomic euautographic integrons* (*PAEI's*) or the *idempotent digital integrons* (*IDI's*). In order to connote certain *dual properties* of 0 and 1 in the EAADM D_1, 0 is called the *validity-integron validity* or alternatively the *antivalidity-integron antivalidity,* while 1 is called the *validity-integron antivalidity* or alternatively the *antivalidity-integron validity*. Accordingly, 0 and 1 are collectively called the *digital validity-integrons* (*DVI's*) or alternatively the *digital antivalidity-integrons* (*DAVI's*).

Any one of the infinite (open) ordered lists of congeneric or conspecific PAE's (2.7)–(2.9^2) and so on is called the *alphabet of the PAE's*, while their order on the list is called the *alphabetic order of the PAE's*.

15) A *single PAE* or a finite *juxtaposition* (*linear sequence*) *of PAE's without blanks* is called a *primary atomic euautographic assemblage* (*PAEA*) or a *primary combined euautographic assemblage* (*PCbEA*) respectively, and also indiscriminately a *primary euautographic assemblage* (*PEA*), *of* A_1. A PEA is called a *primary euautographic ordinary*, or *ordinary euautographic, assemblage* (*PEOA* or *POEA*) if it comprises PAOE's and a *primary euautographic special*, or *special euautographic, assemblage* (*PESpA* or *PSpEA*) if it contains at least one PASpE and some or no PAOE's. In accordance with certain *meta-axioms*, having the form of *interrelated recursive semantic definitions* and called the *primary formation rules of* A_1, a PEA of A_1, which is admitted as a *primary genuine euautographic expression* of A_1 and which is called a *primary euautographic formula* (*PEF*), or *primary euautographic categorem* (*PEC,* pl. "*PEC'ta*"), *of* A_1, is one of the following *four* kinds (classes): a *primary euautographic ordinary term* (*PEOT*), a *primary euautographic special term* (*PESpT*) called also a *primary euautographic integron* (*PEI*), a *primary euautographic ordinary relation* (*PEOR*), and a *primary euautographic special relation* (*PESpR*). A PEOT or a PESpT (PEI) is indiscriminately called a *primary euautographic term* (*PET*), whereas a PEOR or a PESpR is indiscriminately called *primary euautographic relation* (*PER*). Accordingly, a PEF is either a PET or a PER. Also, a PEF is called a *primary euautographic ordinary formula* (*PEOF*) or *primary euautographic ordinary categorem* (*PEOC*) if it is either a PEOT or a PEOR and a *primary euautographic special formula* (*PESpF*) or *primary euautographic special*

categorem (*PESpC*) if it is either a PESpT (PEI) or a PESpR. Besides the PFR's, there are certain meta-axioms, having the form of *asymmetric synonymic definitions* (*ASD's*) and called the *secondary formation rules* (*SFR's*) of A_1, which define *secondary euautographic formulas* (*SEF's*), or *secondary euautographic categoremata* (*SEC'ta*) of the following *three* kinds (classes): *secondary euautographic special terms* (*SESpT's*), i.e. *secondary euautographic integrons* (*SEI's*), *secondary euautographic ordinary relations* (*SEOR's*), and *secondary euautographic special relations* (*SESpR's*), either in terms of the *respective PEF's* or in terms of some other *SEF's of the respective kind*, which have been defined earlier. A PEI (PESpT) or an SEI (SESpT) is indiscriminately called an *EI* (*ESpT*), a PEOR or an SEOR is indiscriminately called an *EOR*, a PESpR or an SESpR is indiscriminately called an *ESpR*, and an EOR or an ESpR is indiscriminately called an *ER*. At the same time, a PEOT is a *primary atomic EOT* (*PAEOT*) and also an *atomic EOT* (*AEOT*), and vice versa, and it is briefly and unambiguously called an *EOT*, because there are no EOT's that could be qualified either *combined* or *secondary*. A PEF (PEC) or an SEF (SEC) is indiscriminately called a *euautographic formula* (*EF*), and also a *formulary*, or *categorematic*, *euautograph* (*FE* or *CtgE*).

16) The qualifiers "*special*" ("*unordinary*") and "*ordinary*" ("*non-special*") to an endosemasiopasigraph, i.e. to a euautograph or a panlogograph, in general or to one of a specific class as an endosemasiopasigraphic formula, term, relation, or sign are antonymous technical metaterms (metalinguistic terms) of the treatise, which have the following meanings:

a) "Special" ("unordinary") means «specially designed for setting up D_1 (the AADM of A_1) or serving as a tool of D_1, or being a by-side product of D_1, and having therefore no analogues in any CALC and in its metalanguage».

b) "Ordinary" ("non-special") means «having none of the above features», i.e. «not specially designed for setting up D_1, not serving as a tool of D_1, and not being a by-side product of D_1, but being exclusively an object of the pertinent *ADP* (*algebraic decision procedure*) and having therefore an analogue or an interpretand in some CALC or in its metalanguage».

i) When the qualifier "ordinary" applies to a euautograph of A_1, it does not necessarily mean that the euautogaph can be interpreted directly by a certain logograph of a CALC. A euautograph of A_1 can be qualified as an ordinary one also if it is used for defining some other, secondary euatographs that have direct interpretands in a CALC. For instance, the primary universal logical connective $\not\vee$, which will be called the *former*, or *primary, antidisjunction sign*; and which is dual of *Sheffer's stroke* that I denote as $\not\wedge$, has no *direct* interpretand in any CALC. Nevertheless, it is qualified as an ordinary one, because I shall use it as the definiens for defining *twelve secondary elemental euautographic logical connectives* of the following cumulative list:

$$\not\vee,\ \neg,\ \vee,\ \wedge,\ \Rightarrow,\ \Leftarrow,\ \Leftrightarrow,\ \not\wedge,\ \overline{\vee},\ \overline{\wedge},\ \overline{\Rightarrow},\ \overline{\Leftarrow},\ \overline{\Leftrightarrow}. \qquad (2.10)$$

In the exclusion of $\neg$, which is the only *singulary* logical connective, the rest of logical connectives on the list (2.10) are *binary* ones. The secondary connectives will be distinguished by the following proper names (not quoted for the sake of brevity): $\neg$, the *negation*, or *denial*, *sign*; $\vee$, the *inclusive disjunction sign*; $\wedge$, the *conjunction sign*; $\Rightarrow$, the *rightward implication sign*; $\Leftarrow$, the *leftward implication sign*; $\Leftrightarrow$, the *biimplication*, or *equivalence*, *sign*; $\not\wedge$, the *former anticonjunction sign*; $\overline{\vee}$, the *latter antidisjunction sign*; $\overline{\wedge}$, the *latter anticonjunction sign*; $\overline{\Rightarrow}$, the *rightward antiimplication sign*; $\overline{\Leftarrow}$, the *leftward antiimplication sign*; $\overline{\Leftrightarrow}$, the *anti-biimplication*, or *antiequivalence*, or *exclusive disjunction, sign*. The occurrence of the word "sign" in any of the above metaterms should be understood as an abbreviation of the compound noun "kernel-sign" as opposed to the name "punctuation sign" or "punctuation mark". Also, any of the above metaterms has been abbreviated by omission of the prepositive qualifier "*formal*" (as opposed to "*material*") that should immediately follow the definite article occurring in the metaterm. The first seven binary logical connectives on the list (2.10) are called *positive* ones, whereas the remaining five are called *negative* ones. The former are *atomic*, whereas the latter are *molecular*, because the *overbar of an adjustable length*, $\overline{}$, can be regarded as an *overscript synonym of the adscript negation sign* $\neg$.

ii) I have mentioned in the point a_2 of the item 2ii that in view of the analogy that exists between the binary disjunction operator '$\vee$' and the existential quantifier '$(\exists *)$ and in view of the like analogy that exists between the binary conjunction operator '$\wedge$' and the universal quantifier '$(\forall *)$', I employ the *binder* (*contractor*) *signs* '$\bigvee_*$' and '$\bigwedge_*$' instead of '$(\exists *)$' and '$(\forall *)$' respectively. The binder sign '$\bigwedge_*$'

along with the three other secondary binder signs ‘$\breve{\vee}_{*}$’, ‘$\overset{\frown}{\vee}{}^{1}_{*}$’, and ‘$\vee^{1}_{*}$’ are defined in terms of ‘$\vee_{*}$’. In this case, I qualify all five binder signs:

$$\vee_{*},\ \wedge_{*},\ \breve{\vee}_{*},\ \overset{\frown}{\vee}{}^{1}_{*},\ \vee^{1}_{*} \tag{2.11}$$

ordinary, although some of the latter three binders can have no direct interpretands in any CALC’i.

17) A euautograph, which is not an FE (CtgE) and which can be united with some other euautographs, formulary or not, to produce an FE (CtgE) is called a *syncatecorematic euautograph* (*SCtgE*) or a *euautographic syncategorem* (*ESC*, pl. “*ESC’ta*”). There are two kinds of ESC’ta in A_1, namely the *main*, or *principal*, ones, called also *euautographic kernel-signs* (*EKS’s*), i.e. *kernel-signs of euautographic operators*, and the *auxiliary* ones, called also *euautographic punctuation marks* (*EPM’s*). A *euautographic operator* (*EO*) is an EKS along with the pertinent EPM’s. Still, an EKS is often equivocally called an EO, while the pertinent EPM’s are obviously understood. Like an ER, an EKS is a *primary* one (*PEKS*) or a *secondary* one (*SEKS*), an *atomic* one (*AEKS*) or a *combined* one (CbEKS), an *ordinary* one (*EOKS*) or a *special* one (*ESpKS*), and a *logical* one (*ELKS*) or an *algebraic* one (*EAlKS*). An SEKS is always defined by a certain *asymmetric synonymic definition* (*ASD*) as its by-side *contextual* (*implicit*) *effectual definiendum.* An ELKS of A_1 is one of the following three kinds: a *euautographic logical*, or *ordinary*, *connective* (*ELCn* or *EOCn*), a *euautographic logical*, or *ordinary*, *predicate-sign* (*ELPS* or *EOPS*), or a *euautographic logical binder* (*ELB*) called also *euautographic logical contractor* (*ELCt*). An EAlKS of A_1 is any one of the primary atomic EAlKS’s given in the items 14x and 14xi or the secondary binary atomic EAlKS of subtraction $\hat{-}$, defined as an abbreviation of $\hat{+}\,\hat{-}$, or else a *euautographic pseudo-multiplier* (as $\hat{\cdot}_{x}$), called also a *euautographic algebraic binder* (*EAlB*) or *euautographic algebraic contractor* (*EAlCt*). By contrast, an EPM is, like an EOT, necessarily a *primary*, *ordinary*, and *logical* one simultaneously. In addition, a *comma*, being the only *EPM of separation* of A_1, is obviously an *atomic* one, whereas various *pairs of brackets*, namely, the pair [ ], serving as an *EPM of aggregation*, and the pairs ( ), ⟨ ⟩, and | ⟩, all serving as *EPM’s of description* (⟨ ⟩ and | ⟩ are used only in $\mathbf{A}_1$), are *molecular* ones. The EF’s, which are united by a EKS to produce a new EF, are called the *operata* (singular “*operatum*”) *of the EKS* and also the *operata of the new EF*, while the latter

EF is called the *operand*, or *scope*, of the EKS. An EKS occurring in a given EF is called the *principal EKS* (*PEKS*) of the EF if it is either the only EKS of the EF or if it is the one of two or more EKS's of the EF, which is executed in the last place, so that the EF is the *operand*, or *scope, of its PEKS* in either case. The operata of the PEKS of an EF are called the *principal operata* of the EF. An ESpR, whose PEKS is the *special equality sign* $\hat{=}$, is called a *euautographic algebraic relation* (*EAlR*) or a *euautographic algebraic*, or *special, equality* (*EAlE* or *ESpE*). A *valid* (to be defined) EAlE (ESpE) is called a *euautographic algebraic*, or *special, identity* (*EAlI* or *ESpI*) and vice versa.

18) A_1 has an infinite number of *coherent restrictions*, some of which are developed to be *its branches,* and it also has *two incoherent restrictions, not being its branches*. Most conspicuous restrictions of A_1 *of academic or practical interest* (to be explained), especially most conspicuous branches of A_1 and its both incoherent restrictions, are distinguished by the appropriate logographic constants, obtained by modifications 'A_1', and by the appropriate phonographic (wordy, verbal) names in the form of a *description through the genus,* denoted by the appropriate one of the abbreviated generic names "EAPO", "APO", and "ALO", *and through the pertinent differentia* (*difference*), denoted by the appropriate qualifier.

i) A *coherent restriction of* A_1 is *equivocally* denoted by 'a_1' and is *commonly* called an *EAPO*. The EAB of any given a_1 and its *selected part*, i.e. *the complement of* B_{1M} *in the EAB,* are denoted by 'b_1' and by 'b_{1OS}' respectively. b_{1OS} is selected out of B_{1OS} so as to necessarily include either at least one of the infinite alphabets: (2.9[1]), (2.9[2]), etc or at least exactly one of the three BAPCOPS's =, $\subseteq$, or $\in$, indicated in the item 14viii. In the latter case, b_{1OS} may also include $\emptyset$ and $\emptyset'$, in accordance with the item 14ix. In all other respects, the choice of b_{1OS} out of B_{1OS} is unrestricted.

ii) A_1 has three *comprehensive branches,* which are denoted by '$A_{1=}$', '$A_{1\subseteq}$', and '$A_{1\in}$' and which are called *the major branches of* A_1. The EAB of $A_{1=}$, $A_{1\subseteq}$, or $A_{1\in}$ and its *selective part*, i.e. *the complement of* B_{1M} *in the EAB,* are denoted by '$B_{1=}$' and '$B_{1OS=}$', '$B_{1\subseteq}$' and '$B_{1OS\subseteq}$', or '$B_{1\in}$' and '$B_{1OS\in}$' respectively. $B_{1OS=}$, $B_{1OS\subseteq}$, or $B_{1OS\in}$ contains the respective BAPCOPS =, $\subseteq$, or $\in$ as the *primary one* and it also contains all other PAOE's of B_{1OS} except for $\emptyset$ and $\emptyset'$ in the case of $B_{1OS=}$ as indicated in the item 14ix.

iii) Each one of the comprehensive branches $A_{1=}$, $A_{1\subseteq}$, and $A_{1\in}$ has an infinite number of *restricted branches*, which are equivocally denoted by '$a_{1=}$', '$a_{1\subseteq}$', and '$a_{1\in}$'. The EAB of any given $a_{1=}$, $a_{1\subseteq}$, or $a_{1\in}$ and its *selective part*, i.e. *the complement of* B_{1M} *in the EAB*, are equivocally denoted by '$b_{1=}$' and '$b_{1OS=}$', '$b_{1\subseteq}$' and '$b_{1OS\subseteq}$', and '$b_{1\in}$' and '$b_{1OS\in}$' respectively. Besides the respective predicate-sign $=$, $\subseteq$, or $\in$ and also besides $\emptyset$ and $\emptyset'$ associated with $\subseteq$ or $\in$, $b_{1OS=}$, $b_{1OS\subseteq}$, or $b_{1OS\in}$ contains either *strictly some*, i.e. some but not all, or *none* of the infinite sets of PAOE's of b_{1OS}, in agreement with the above point i.

iv) In agreement with the item 4, one of the two *incoherent restrictions* of A_1 is denoted by 'A_1^0' and is called the *Comprehensive Euautographic Binder-Free*, or *Contractor-Free, Algebraico-Predicate Organon* (*CEBFAPO* or *CECFAPO*) and also the *Comprehensive Euautographic Rich Basic Algebraico-Logical Organon* (*CEFBALO*), and the other one is denoted by 'A_0' and is called the *Euautographic Predicate-Free*, or *Euautographic* [*Depleted*] *Basic*, *Algebraico-Logical Organon* (*EPFALO* or *EDBALO* or *EBALO*). The meanings of the verbal names of A_1^0 and A_0 are predetermined by following facts of the EAB's of A_1^0 and A_0. The EAB of A_1^0, denoted by 'B_1^0', includes B_{1OS} and B_{1Sp} as its constituent parts, while the *mandatory ordinary* constituent part of B_1^0, denoted by 'B_{1OM}^0', contains all PAOE's of B_{1OM} *in the exclusion of* $\exists$. The EAB of A_0, denoted by 'B_0', includes B_{1Sp} and in addition it contains the PAOE's indicated in the points i, iii, and vi. Owing to its EAB, A_1^0 has three comprehensive branches $A_{1=}^0$, $A_{1\subseteq}^0$, and $A_{1\in}^0$, which are incoherent restrictions of the respective branches $A_{1=}$, $A_{1\subseteq}$, and $A_{1\in}$ of A_1 and which have D_1^0, i.e. the ERBADM of A_1, as their common ADM. By contrast, owing to its EAB, A_0 is an *unbranched* (*indivisible*, *single whole*) EALO (euautographic algebraico-logical organon), which is, in accordance with the items 9 and 13, a kind of *synthesis* of a CASC (as P_R or P_{RB}) and of the abstract integral domain. The ADM of A_0, which is denoted by 'D_0' and is called the Euautographic Basic, or Euautographic Depleted Basic, ADM (EBADM or EDBADM) of A_1,, is functionally the same as D_1^0 (cf. the item 5). In fact, A_0 is a *schema of* A_1^0 and therefore D_0 is a s*chema of* D_1^0. Consequently, A_0 is a *coherent restriction* of A_1^0. Since D_0 and D_1^0 differ from D_1, therefore A_0 and A_1^0 are *not AALO's* and hence they are *neither phases nor branches of* A_1, as has been stated in the item 8. It has also been stated there that, owing to its simplicity, A_0 can be regarded as an introduction into A_1 so that, in reference to this role, A_0 has been called *the first zero quasi-phase of* A_1, whereas A_1^0 having the like simple ADM has been called *the second zero quasi-phase of* A_1.

19) In stating any FR's of A_1, either primary or secondary, the division of B_{1O} into B_{1OM} and B_{1OS} and hence the condition of mutual incompatibility of the BAPCOPS's =, $\subseteq$, and $\in$ as primary ones and the condition of association of APCOT's $\emptyset$ and $\emptyset'$ with $\subseteq$, or with $\in$, which have been stated above in the items 14viii and 14ix, are *ignored* (*not utilized*). For instance, in accordance with the pertinent PFR, the PEA's $=(x,y)$, $\subseteq(x,y)$, and $\in(x,y)$ are PEOR of A_1, like the PEA's $f^2(x,y)$, $g^2(x,y)$, and $h^2(x,y)$. In accordance with the pertinent SFR, the former relations, given in the *Clairaut-Euler form*, can be presented in the *bilinear* (*homogeneous, algebraic*) form as $[x=y]$, $[x\subseteq y]$, and $[x\in y]$, while the latter relations can be represented similarly as $[xf^2y]$, $[xg^2y]$, and $[xh^2y]$. Hence, in this respect, any one of the three BAPCOPS's is indistinguishable from any binary APVOPS and is therefore indistinguishable from any one of the two other BAPCOPS's. That is to say, *with respect to its FR's*, A_1 *is a single whole EAPO* that is based on B_1 as its *effectual mandatory EAB*. This property of entirety of A_1 is preserved through the whole *initial phase of the setup of* A_1, which is called the *Primordial*, or, by way of its allegoric association with a tree, *Root, EAPO* (*PEAPO* or *REAPO*), and which is denoted logographically by 'A_{1P}' or 'A_{1R}'. A_{1P} is based on, and includes, *all PFR's of* A_1 and also *certain SFR's of* A_1, which are *sufficient* for setting A_{1P} up conveniently. Accordingly, all these FR's can be called the *primordial FR's* (*PmFR's*) *of* A_1.

20) The calculus A_{1P} is set up as a single whole EAPO that has an *associated built-in* and hence *inseparable EAADM*, which is denoted by 'D_1' and whose *region of applicability* (*domain of definition*) is *the class of ER's of* A_1, *including* A_{1P}. D_1 is by definition a *current conjunction in progress of intrinsic* (*subject*) *and extrinsic* (*metalinguistic*) *rules of inference* (*transformation*) *and decision of* A_{1P}, which consists of two sequential parts, the first of them being properly denoted by 'D_1^a' and called the *a̲xiomatic*, or *primary, EAADM*, and also *EAADM in intension*, *of* A_1 and

the other one being commonly (equivocally) denoted by 'D_1^t' and called the *current theorematic extension of*, or *theorematic supplement to*, D_1^a in each given place in the process of executing of A_{1P} after D_1^a is laid down. Therefore, by way of emphatic comparison with the last one of above three names of D_1^a, D_1 can alternatively be called *the EAADM in extension of* A_1, while D_1^a can alternatively be called *the kernel of* D_1. *The result of application of* D_1 *to any given ER of* A_1, and particularly of *to any given ER of* A_{1P}, *is determined exclusively by* D_1^a.

21) D_1^a is a relatively compact *totality* (actually, *conjunction*) *of typical* (*unspecific, branch-independent, branch-indifferent*) *axiomatic* (*primary*) *rules of inference and decision, whose domain of definition* (*region of applicability*) *is the class of all ER's of* A_1, i.e. of all ER's that are determined by the PmFR's and by all subsequent SER's of A_1. Once D_1^a is laid down, the current D_1 is identified with it Therefore, until D_1 is augmented by the *abbreviative rules* of inference and decision comprised in D_1^t, 'D_1' and 'D_1^a' can be used interchangeably. D_1^a has the following general features.

i) In the exclusion of the *concrete intrinsic* (*subject*) *euautographic logico-algebraic axiom* $\neg[0 \hat{=} 1]$, all other axioms comprised in D_1^a are either *panlogographic schemata of an infinite number of intrinsic euautographic axioms* or semi-formal *meta-axioms* (*metalinguistic*, or *extrinsic*, *axioms*). All the axioms relate *euautographic* and *panlogographic integrons* (*EI's* and *PLI's*) *to one another* or *to euautographic* and *panlogographic relations* (*ER's* and *PLR's*). D_1^a *does not include any atypical* (*specific, branch-dependent, branch-determining*) *intrinsic* (*subject*) *euautographic* or *panlogographic axiom* that may later be imposed, either directly or obliquely, on a certain concrete PAOE (PAE of B_{1OS}), and particularly on any primary BAPCOPS or on either APCOT with the purpose to distinguish that PAOE from all other PAOE's of the same nomenclature, i.e. of the same genus or of the same species.

ii) Every intrinsic (subject) euautographic or panlogographic axiom, typical or atypical, involves no words and it is by definition a *valid ER* or a *valid PLR* respectively but not necessarily vice versa. By contrast, every meta-axiom necessarily involves some words of the IML, but it is also regarded as (postulated to be) a *valid relation of the IML*.

iii) A meta-axiom is called a *rule of inference* if it determines a *single act*, by which a *valid* ER or PLR can be *immediately inferred* as *conclusion* from appropriate *valid* ER's or PLR's as *premises* (*premisses*). A meta-axiom is called a *rule of decision* if it determines a *single act*, by which a *valid* ER or PLR can be *immediately inferred* as *conclusion* from appropriate *valid* ER's or PLR's as *premises* (*premisses*). A finite sequence of valid ER's or PLR's is called a *proof* or *argument* if each ER or PLR of is immediately inferred from preceding valid ER's or PLR's in the sequence by means of one of the rules of inference. A proof is called an *algebraic proof* if each of its sequential valid relations is an *algebraic identity*. A proof is called a proof, more precisely, *master*-proof, *of* the last valid relation in the sequence¸ while that relation is called an *intrinsic* (*subject*) *master-theorem*

iv) Since D_1^a *does not include any atypical* (*branch-determining*) *axiom*, therefore all PAOE's, being congeneric or conspecific from the standpoint of PmFR's, and being hence indistinguishable in this sense, remain so also from the standpoint of D_1^a. At the same time, D_1^a *does include the meta-axioms* being *rules of using*, i.e. rules *how to use, atypical axioms* in subsequent applications of D_1^a to *atypical ER's*, once such axioms are laid down. These two facts guarantee that D_1^a will remain universal and unaltered in any subsequent branch of A_1, which necessarily involves, in addition to D_1^a, some atypical subject axioms imposed on some selected PAOE's.

v) Any one of the of the *lexigraphs* (*atomic logographs*) '**P**' to '**S**', '$\mathbf{P}_1$' to '$\mathbf{S}_1$', '$\mathbf{P}_2$' to '$\mathbf{S}_2$', etc is an *atomic panlogograph* (*APL*), i.e. an *atomic panlogographic placeholder* (*APLPH*), which is alternatively called an *analytical atomic panlogographic relation* (*AnAPLR*) *of* A_1, because its range is the class of all ER's of A_1, unless stated otherwise. Hence, when '**P**', e.g., is used *xenonymously*, **P** *is a certain* (*concrete but not concretized*) *ER of* A_1. In this case, the identity (valid equality)

$$V(\mathbf{P}) \hat{=} V(\mathbf{P}) \tag{2.12}$$

is an intrinsic euautographic axiom that is more specifically called a *euautographic master-axiom for* (or *with respect to*) **P**; **P** is called the *euautographic slave-relation* (*ESR*), or *ER-slave*, *of that axiom*; and *V*(**P**) is called the *primary*, or *initial*, *validity-*

integron (*PVI* or *IVI*) *of* **P**. It is postulated that for any ER **P** of A_1, V(**P**) satisfies the *idempotent law*:

$$V(\mathbf{P}) \hat{\cdot} V(\mathbf{P}) \hat{=} V(\mathbf{P}), \tag{2.13}$$

which is another euautographic master-axiom for **P**. The identities (2.12) and (2.13) are some *rules of inference of* (*belonging to, comprised in*) $\mathsf{D}_1^{\mathrm{a}}$. When '**P**' is mentally used *autonymously*, i.e. as a *tychautograph* (*accidental autograph*), either for mentioning itself or for mentioning its any *homolographic* (*photographic*, *congruent* or *proportional)* token, the identities (2.12) and (2.13) are regarded as a *panlogographic master-axioms for* '**P**', which belong to the *axiomatic*, or *primary*, *part* $\mathbf{D}_1^{\mathrm{a}}$ of the *PLAADM* $\mathbf{D}_1$. The qualifier "*panlogographic*" means «*of*, i.e. *belonging to,* $\mathbf{A}_1$», whereas $\mathbf{A}_1$ is the *background calculus of logographic placeholders of the appropriate foreground euautographic formulas of* A_1. In reference to both mental hypostases of the identities (2.12) and (2.13), either one of them is said to be a *panlogographic master-axiom schema of* (*belonging to*) $\mathbf{A}_1$, *of an infinite number of euautographic master-axioms* A_1.

vi) $\mathsf{D}_1^{\mathrm{a}}$ includes some intrinsic euautographic axioms, by means of which V(**P**) can be reduced to an algebraic form with respect to the PVI's of the principal relation-operata of **P**; the latter PVI's can be reduced likewise; and so on with the following proviso. Prior performing a reduction of the above kind of each next algebraic form in turn, it is reduced by performing all its performable algebraic operations by means of the appropriate rules of inference that are also comprised in $\mathsf{D}_1^{\mathrm{a}}$. The identity (2.13) is one of such rules, by means of which it also follows, e.g., that

$$V(\mathbf{P}) \hat{\cdot} V(\neg\mathbf{P}) \hat{=} V(\mathbf{P}) \hat{\cdot} [1 \hat{-} V(\mathbf{P})] \hat{=} 0\,, \tag{2.14}$$

where $\neg$ is the *EOKS* (*ELKS, logical connective*) *of negation*. Thus, given an ER **P**, the sequence of the above reductions of V(**P**) can be represented in the *staccato style* as a certain *algebraic proof from the axiom* (2.12) of the form:

$$V(\mathbf{P}) \hat{=} V(\mathbf{P}), V(\mathbf{P}) \hat{=} \mathbf{i}_1|\mathbf{P}\rangle, V(\mathbf{P}) \hat{=} \mathbf{i}_2|\mathbf{P}\rangle, ..., V(\mathbf{P}) \hat{=} \mathbf{i}_{n-1}|\mathbf{P}\rangle, V(\mathbf{P}) \hat{=} \mathbf{i}_n|\mathbf{P}\rangle\,, \tag{2.15}$$

or in the *legato style* as:

$$V(\mathbf{P}) \hat{=} \mathbf{i}_1|\mathbf{P}\rangle \hat{=} \mathbf{i}_2|\mathbf{P}\rangle \hat{=} ... \hat{=} \mathbf{i}_{n-1}|\mathbf{P}\rangle \hat{=} \mathbf{i}_n|\mathbf{P}\rangle\,, \tag{2.15a}$$

where $\mathbf{i}_1|\mathbf{P}\rangle$ to $\mathbf{i}_{n\text{-}1}|\mathbf{P}\rangle$ are successive *reducible*, or *intermediate, secondary euautographic validity-integrons* (*RSEVI*) *of* **P**, while $\mathbf{i}_n|\mathbf{P}\rangle$ is the *irreducible*, or *ultimate*, *euautographic validity-integron* (*UEVI*) *of* **P**, called also the *euautographic validity-identifier*, or *euautographic validity-index*, (briefly *EVID* in both cases), *of* **P**. Each one of the *n* euautographic validity-integrons satisfies the respective varia`nt of the idempotent law (2.13):

$$\mathbf{i}_k\left|\mathbf{P}\right\rangle \hat{\cdot}\, \mathbf{i}_k\left|\mathbf{P}\right\rangle \hat{=} \mathbf{i}_k\left|\mathbf{P}\right\rangle \text{ for each } k \in \omega_{1,n}; \tag{2.16}$$

$\omega_{1,n}$ is the set of Arabic numerals from 1 to *n*, The statement that $\mathbf{i}_n\left|\mathbf{P}\right\rangle$ is irreducible means that the final identity in the proof (2.15) has exactly one of the following three forms:

$$V(\mathbf{P}) \hat{=} \mathbf{i}_n\left|\mathbf{P}\right\rangle \hat{=} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \mathbf{i}\left|\mathbf{P}\right\rangle & \text{(c)} \end{cases}, \tag{2.17}$$

where $\mathbf{i}\left|\mathbf{P}\right\rangle$ is a certain UEVI, other than either 0 or 1, which satisfies the idempotent law:

$$\mathbf{i}\left|\mathbf{P}\right\rangle \hat{\cdot}\, \mathbf{i}\left|\mathbf{P}\right\rangle \hat{=} \mathbf{i}\left|\mathbf{P}\right\rangle, \tag{2.18}$$

– just as 0 or 1 does. The digit 0 is called the *validity-integron* (*VI*) *validity*, the digit 1 is called the *VI antivalidity*, and the UEVI $\mathbf{i}\left|\mathbf{P}\right\rangle$ is called *a euautographic VI* (*EVI*) *neutrality* (and also *an EVI indeterminacy*), or, alternatively, with "*VID*" in place of "*VI*" in each case.

vii) It is understood that the three equalities (a), (b), and (c) of the *metalinguistic scheme* (*pattern*) (2.17), i.e. a scheme that belongs to the *exclusive metalanguage* (*XML*) of both A_1 and $\mathbf{A}_1$, is *not an identity*, so that it cannot be used assertively as a valid tychautographic relation. These equalities are used here xenonymously as three mutually independent *ad hoc conditions on* **P**, i.e. on *accidental euautographic denotata of* '**P**'. However, the pertinent one of the three conditions (a), (b), and (c) of (2.17), which a given ER **P** satisfies, turns *ipso facto* into an *identity*, which is denoted by '$\mathsf{T}_{1+}(\mathbf{P})$', '$\mathsf{T}_{1-}(\mathbf{P})$', or '$\mathsf{T}_{1\sim}(\mathbf{P})$' respectively or indiscriminately by '$\mathsf{T}_1(\mathbf{P})$' and is called the *euautographic master-theorem* (*EMT*), or *euautographic decision-theorem* (*EDT*), *for* **P**, or more generally an *MT* (*master-theorem*), or *DT* (*decision-theorem*), *of* A_1. Accordingly, the metalinguistic three-fold scheme (2.17) is called the *EMT*, or *EDT*, *scheme*, or *pattern*, *for* **P**.

viii) Thus, given an ER **P**, the algebraic proof (2.15) or (2.15a) of the pertinent one of the three euautographic identities (2.17, a–c) as the EMT (EDT) for **P** is called a *euautographic algebraic decision procedure* (*EADP*) *for* **P** or less explicitly an *EADP of* A_1, while **P**, i.e. the *ER proceeded*, is called the *euautographic slave-relation* (*ESR*), or *euautographic relation-slave* (*ER-slave*), *of both the EADP and the EMT* (*EDT*). The qualifier "*algebraic*" in the above full verbal name of an EADP of A_1 implies that the EADP is *analytical* (*computational), and not tabular*. Each *rule of inference* (*transformation*) that is used in EADP's is alternatively and more specifically called a *rule of EADP's* or an *EADP rule* (*EADPR*). An EADP is called a *rich basic* one (*RBEADP*) if it is performed by means of D_1^0, a *basic* one (*BEADP*) if it is performed by means of D_0, and an *advanced* one (*AEADP*) if it involves applications of at least one rule of D_1 not belonging either to D_1^0 or to D_0. A RBEADP for **P** of A_1^0 is denoted by '$\mathsf{D}_1^0(\mathbf{P})$', whereas the pertinent EDT $\mathsf{T}_{1+}(\mathbf{P})$, $\mathsf{T}_{1-}(\mathbf{P})$, or $\mathsf{T}_{1\sim}(\mathbf{P})$ is, when desired, denoted more specifically by '$\mathsf{T}_{1+}^0(\mathbf{P})$', '$\mathsf{T}_{1-}^0(\mathbf{P})$', or '$\mathsf{T}_{1\sim}^0(\mathbf{P})$' respectively or indiscriminately by '$\mathsf{T}_1^0(\mathbf{P})$' instead of '$\mathsf{T}_1(\mathbf{P})$'. Analogously, an BEADP for **P** of A_0 is denoted by '$\mathsf{D}_0(\mathbf{P})$', whereas the pertinent EDT $\mathsf{T}_{1+}(\mathbf{P})$, $\mathsf{T}_{1-}(\mathbf{P})$, or $\mathsf{T}_{1\sim}(\mathbf{P})$ is, when desired, denoted more specifically by '$\mathsf{T}_{0+}(\mathbf{P})$', '$\mathsf{T}_{0-}(\mathbf{P})$', or '$\mathsf{T}_{0\sim}(\mathbf{P})$' respectively or indiscriminately by '$\mathsf{T}_0(\mathbf{P})$' instead of '$\mathsf{T}_1(\mathbf{P})$'. An ER **P** of A_1 is called a *decided ER* (*DdER*) and it is more specifically denoted by '$\mathbf{P}_*$', if it either is an intrinsic (subject) axiom of A_1, denoted by '$\mathbf{P}^a$' or '$\mathbf{P}_+^a$' or if it has an EDT (EMT) $\mathsf{T}_1(\mathbf{P})$.

ix) An ER of A_1 may have several EADP's, which differ in orders of using the appropriate rules of D_1 in the different EADP's. The different EADP's for a given ER result in the same EDT. However, one of the EADP's may turn out to be shorter and simpler than another one. Therefore, in spite of the fact that any EADP is mechanical, choice of the optimal EADP for a given ER-slave is a kind of art that is acquired by experience – just as in the case of arithmetical calculations with natural numbers.

x) In accordance with the distinctive form of an EDT (EMT), *its ESR* (*ER-slave*) *is decided* to be an ER of exactly one of the three *decision classes* as stated in the following *decision rule* [*for ER's*] *of* A_1:

A DdER **P** is said to be:

a) *valid*, and it is specifically denoted by **'$\mathbf{P}_{+}$'**, if it either is $\mathbf{P}^{a}$ or if its EDT has the form (2.17a);

b) *antivalid*, and it is specifically denoted by **'$\mathbf{P}_{-}$'**, if its EDT has the form (2.17b);

c) *vav-neutral* (or *vav-indeterminate*), i.e. *neutral* (or *indeterminate*) *with respect to the validity-values validity and antivalidity* or, in other words, to be *neither valid nor antivalid*, and it is specifically denoted by **'$\mathbf{P}_{\_}$'**, if its EDT has the form (2.17c) subject to (2.18).

Therefore, a DdER is, more specifically but redundantly, called a *vavn-decided ER* (*vavn-DdER*), i.e. *decided with respect to the validity-values validity, antivalidity, and vav-neutrality*. Accordingly, in reference to an ER of A_1, the noun "*decision*", kindred of the adjective "*decided*", is understood as *decision with respect to the above three validity-values*. Particularly, the abbreviations "EADP", "EDT", and "DT", introduced above, can, more precisely but redundantly, be replaced with the abbreviations "*vavn-EADP*", "*vavn-EDT*", and "*vavn-DT*" respectively. The division of the DdER's of A_1 into three classes: the valid ER's, the antivalid ER's, and the vav-neutral (vav-indeterminate) ER's is called the *primary*, or *basic*, *decisional trichotomy* (*trisection*, *trifurcation*) *of the DdER's*.

xi) A DdER of A_1 is said to be: *invalid* if it is antivalid or vav-neutral, *non-antivalid* if it is valid or vav-neutral, and *vav-unneutral* or *vav-determinate* if it is valid or antivalid. In accordance with these definitions, the DdER's of A_1 are divided into *two complementary classes in three ways*, namely: (a) the *valid* ER's and the *invalid* ER's, (b) the *antivalid* ER's and the *non-antivalid* ER's, (c) the *vav-neutral*, or *vav-indeterminate*, ER's and the *vav-unneutral*, or *vav-determinate*, ER's. These three divisions the DdER's are called the *secondary*, or *subsidiary*, *decisional dichotomies* (*bisections*, *bifurcations*) *of the DdER's*.

xii) In all *metaterms* (*taxonyms*, *taxonomic names*) that are relevant to the pertinent DdER's of A_1, the words "*neutral*", "*unneutral*", "*neutrality*", and "*unneutrality*" are used interchangeably with the words "*indeterminate*", "*determinate*", "*indeterminacy*", and "*determinacy*" respectively. In this case, although I use the qualifier "indeterminate" as a synonym of qualifier "neutral', *there is no indeterminacy* (*uncertainty*) *in relegating an DdER of* A_1 *to the class of vav-neutral* (*vav-indeterminate*) *ER's if it is so*. A vav-neutral ER of A_1 is *not an* *improvable relation of the Gödelian type*, because it *is proved to be vav-neutral* – just as a valid ER, other than a euautographic axiom of A_1, is proved to be valid and just as an antivalid ER is proved to be antivalid. That is to say, application of the pertinent EADPR of $\mathsf{D}_1^{\mathrm{a}}$ to the EDT of **P** in place of **P** yields:

$$
\begin{aligned}
&[V(\mathbf{P}) \hat{=} 0] && \text{if and only if } [V(V(\mathbf{P}) \hat{=} 0) \hat{=} 0], && \text{(a)} \\
&[V(\mathbf{P}) \hat{=} 1] && \text{if and only if } [V(V(\mathbf{P}) \hat{=} 1) \hat{=} 0], && \text{(b)} \qquad (2.19) \\
&[V(\mathbf{P}) \hat{=} \mathbf{i}|\mathbf{P}\rangle] && \text{if and only if } [V(V(\mathbf{P}) \hat{=} \mathbf{i}|\mathbf{P}\rangle) \hat{=} 0], && \text{(c)}
\end{aligned}
$$

i.e.

$$
\begin{aligned}
&\mathsf{T}_{1+}(\mathbf{P}) \text{ if and only if } \mathsf{T}_{1+}(\mathsf{T}_{1+}(\mathbf{P})), \text{(a)} \\
&\mathsf{T}_{1-}(\mathbf{P}) \text{ if and only if } \mathsf{T}_{1+}(\mathsf{T}_{1-}(\mathbf{P})), \text{(b)} \qquad (2.19\text{a}) \\
&\mathsf{T}_{1\sim}(\mathbf{P}) \text{ if and only if } \mathsf{T}_{1+}(\mathsf{T}_{1\sim}(\mathbf{P})). \text{(c)}
\end{aligned}
$$

xiii) It is proved (inferred) by the pertinent EADPR of $\mathsf{D}_1^{\mathrm{a}}$ that

$$V(\neg\mathbf{P}) \hat{=} 1 \hat{-} V(\mathbf{P}), \qquad (2.20)$$

which is the EDT for $\neg$**P**, $\neg$ being the logical connective of negation (cf. (2.14)). Therefore, any one of the three identity schemes (2.17,a–c) subject to (2.18) holds if and only if the respective one of the following three identity schemes holds:

$$V(\neg\mathbf{P}) \hat{=} \begin{cases} 1 & \text{(a)} \\ 0 & \text{(b)} \\ \mathbf{i}|\neg\mathbf{P}\rangle & \text{(c)} \end{cases}, \qquad (2.21)$$

where, in accordance with (2.18),

$$\mathbf{i}|\neg\mathbf{P}\rangle \hat{=} 1 \hat{-} \mathbf{i}|\mathbf{P}\rangle, \qquad (2.22)$$

In this case, it follows from (2.13) and (2.18) by (2.20) and (2.22) that

$$V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{P}) \hat{=} V(\neg\mathbf{P}), \qquad (2.23)$$

$$\mathbf{i}|\neg\mathbf{P}\rangle \hat{\cdot} \mathbf{i}|\neg\mathbf{P}\rangle \hat{=} \mathbf{i}|\neg\mathbf{P}\rangle. \qquad (2.24)$$

Comparison of (2.17) and (2.21) shows that *the negation of a valid*, or *antivalid*, *ER* **P** *is an antivalid*, or correspondingly *valid*, *ER* $\neg$**P**, *and vice versa*, whereas *the negation of a vav-neutral* (*vav-indeterminate*) *ER* **P** is *another vav-neutral* (*vav-indeterminate*) *ER* $\neg$**P**.

xiv) A *valid*, or *antivalid*, *DdER of* A_1 is called a *euautographic slave-theorem*, or *slave-antitheorem*, *of* A_1 respectively. The qualifier (first appositive noun) “slave” in either of the above two metaterms is used as an antonym of either one of the qualifiers “master” and “decision”, which are used in the synonymous meta terms “euautographic master-theorem” (“EMT”) and “euautographic decision-theorem” (“EDP”). A master-theorem or a slave-theorem is indiscriminately called a *theorem*.

xv) I select ER’s of A_1 to subject them to EADP’s in accordance with the following informal criterion that they have academic or practical interest. I say that an ER of A_1 is one of *academic or practical interest* (*API*) if it is a *comprehensible*, i.e. not unreasonably long and complex, ER of at least one of the following kinds:

a) an illustration of certain aspects of A_1

b) an illustration of the effectiveness of D_1;

c) a master relation of a certain subject of logic;

d) a general formal solution of one of the logical paradoxes;

e) formal groundwork upon which a system of reasoning is erected in the treatise or can be erected in logic or mathematics in the sequel;

f) an instructive example of mental experience.

EOR’s of A_1 have analogues or interpretands among relations of *conventional axiomatic logical calculi* (*CALC’i*), *sentential* ones (*CASC’i*) or *predicate* ones (*CAPC’i*), while ESpLR’s are either *tools* or *by-side products* of D_1 that have no analogues in any CALC. Therefore, an EOR can have either practical interest or academic interest or both, whereas some ESpR’s can have *academic interest* only.

xvi) Although it has not happened so far, should it happens in the sequel that a certain ER of A_1 is subjected to all conceivable would-be EADP’s, all of which fail because the relation is too complicated and too long, so that any one of the EADP’s cannot be completed or comprehended, or because some unknown rules of inference are missing, the relation will be called a *vavn-undecided*, or simply *undecided, ER.* In addition to the vavn-decided ER’s and some supposedly vavn-undecided ER’s, A_1 has an infinite number of ER’s, which are determined by the formation rules of A_1, but which are not subjected to any EADP’s or even are not written down. These relations will be called *vavn-suspended* ones. Accordingly, the vavn-decided and, if detected, vavn-undecided ER’s are collectively called the *vavn-unsuspended ER’s*. Vavn-undecided ER’s (if detected some) and vavn-suspended ER’s will collectively be called *vavn-nondecided*, or simply *nondecided*, *ER’s of* A_1.

22) Immediately upon laying it down, D_1^a can be used and is used to prove that certain intrinsic ER's of A_{1P} are *intrinsic* (*subject*) *master or slave theorems of* A_{1P} so that they are *typical* (*specific*, *branch-independent*, *branch-indifferent*) *intrinsic* (*subject*) *euautographic master or slave theorems of* A_1. At the same time, certain informal intuitive substitutions, conjunctions, and disjunctions can be applied to some meta-axioms of D_1^a to deduce some other *valid meta-relations*, i.e. *meta-theorems*, of A_{1P}. In some cases or universally, some of the above euautographic theorems or meta-theorems or both can be used instead of certain sequences of interrelated axioms of D_1^a, thus abbreviating proof of other master-theorems, in analogy with usage of the secondary (deduced or composite) arithmetic operations on an integral domain in algebra. Once such an auxiliary theorem is proved with the help of D_1^a, being the initial D_1, it is supplemented to D_1^a so that if the theorem is equivocally denoted by 'ΔD_1^t' then the new EAADM is denoted by 'D_1^a *and* ΔD_1^t', i.e. by 'D_1 *and* ΔD_1^t'. Then 'D_1' is mentally freed of its previous denotatum, D_1^a, and is mentally redefined to denote the latter conjunction. Thus, in any given moment of executing A_{1P} after laying down D_1^a, D_1 can, as was stated in the item 20, be represented as 'D_1^a *and* D_1^t' if 'D_1^t' equivocally stands for the conjunction of *all* pertinent typical theorems, which been established by that moment and which can be used in subsequent EADP's of D_1. It is understood that the current denotatum that 'D_1' has in any given place is an *abstraction*, so that no changes in its denotatum are made explicit. If in the proof of a euautographic theorem, I use a euautographic theorem, a master one or a slave one, that has been established earlier then this fact signifies that the latter theorem is included in D_1. In accordance with this criterion, I regard as included into D_1^t the typical (branch-independent, branch-indifferent) *slave-theorem*, which I call the *General Law of Nonexistence of Russell's Paradox* (*GLNERP*), and which is proved with the help of the version of D_1, preceding it. The GLNERP is valid for any ER, whose PEKS (principal euautographic kernel-sign) is *any predicate-sign of weight* 2 *or higher*.

23) The part of the IML, i.e. of the treatise, where D_1^a is laid down, is relatively compact. At the same time, any typical theorem comprised or to be comprised in D_{1P} is readily distinguishable from any atypical theorem, which is

proved with the help of D_{1P}, but is irrelevant to D_{1P}. Therefore, the part of the IML dealing with D_1^t and hence the part of A_{1P} associated with D_1^t is *piecewise continuous* (*not compact*), i.e. typical theorems or groups of typical theorems are scattered throughout the part of the treatise following the statement of D_1^a. However, I have, as far as possible, attempted to concentrate *all most typical* theorems of A_{1P}, in a certain compact portion of the treatise following the statement of D_1^a and preceding the setups of the major branches of A_1. In any case, in every place after setting up D_1^a, D_1 is, like D_1^a, a *branch-independent EAADM* in the sense that it does not include any atypical euautographic axiom and hence it does not include any atypical euautographic theorem either. At the same time, D_1 is, like D_1^a, *does include* the rules how to apply atypical euautographic axioms and hence atypical euautographic theorems in EADP's. Therefore, the EDT, being the result of application of D_1 to an atypical ER of a given branch of A_1, depends on the atypical euautographic axioms of the branch. Consequently, if a certain EDT of A_1 is independent of any atypical euautographic axiom of A_1 then the ER-slave of the EDT belongs to A_{1P} and not to any advanced phase of A_1.

24) Once A_{1P} or at least the most essential part of it is set up and the current EAADM D_1 is laid down, the division of B_{1O} into B_{1OM} and B_{1OS} and hence the stipulations stated in the items 14viii and 14ix are recovered. Then the three major branches of A_1 and their coherent restriction can be set up in agreement with the rules that were stated in the item 18. Thus, owing to A_{1P}, A_1 has two hypostases (aspects, ways of existence).

a) A_1 *is a single whole EAPO, not only with respect to its FR's, but also with respect to its EAADM,* D_1.

b) A_1 is a *bunch* (*bundle*) of an infinite number of EAPO's, called *coherent restrictions*, which are intermixed for conveniently treating them simultaneously as a single whole. Some of the coherent restrictions are branches of A_1. All coherent restrictions and hence all branches of A_1 have as an inseparable part of each of them *one and the same built-in EAADM,* D_1.

25) In agreement with the rules of the item 18, the setup of any branch a_1 of A_1 should begin from restricting B_{1OS} to specify b_{1OS}. Then the EF's of A_{1P} should

mentally (imaginarily) be sifted to free them of all PAOE’s of B_{1OS} that are not included in b_{1OS}. However, the coherent restriction of A_{1P} thus obtained, to be equivocally denoted by ‘a_{1P}’ and be commonly called *a PEAPO* or *REAPO* in accordance with the item 19, is just a restriction of the region of applicability of D_1 and *not a branch of* A_1. In order to turn a_{1P} into a branch of A_1, i.e. into a_1, at least one specific (atypical) subject axiom should be laid down for at least one PAOE of b_{1OS}. Otherwise, reducing A_{1P} to a_{1P} is aimless. Therefore, in the framework of specifying b_{1OS}, a certain one of the three BAPCOPS’s =, $\subseteq$, and $\in$ should be selected as the *primary* one; and if the selected predicate-sign is either $\subseteq$, or $\in$ then $\emptyset$ and $\emptyset'$ should also be included into b_{1OS}, in accordance with the item 14ix. Any PEAPO a_{1P}, whose b_{1OS} contains =, $\subseteq$, or $\in$, is equivocally denoted by ‘$a_{1P=}$’, ‘$a_{1P\subseteq}$’, or ‘$a_{1P\in}$’, while its EAB is equivocally denoted by ‘$b_{1P=}$’, ‘$b_{1P\subseteq}$’, or ‘$b_{1P\in}$’, respectively. The most general way to specify b_{1OS} is to retain all PAOE’s given in the points vi and vii of the item 14 and to make a selection indicated in the points viii and ix of the item 14, in agreement with the item 18. The corresponding $b_{1P=}$, $b_{1P\subseteq}$, and $b_{1P\in}$ are denoted by ‘$B_{1P=}$’, ‘$B_{1P\subseteq}$’, and ‘$B_{1P\in}$’, while the corresponding $a_{1P=}$, $a_{1P\subseteq}$, and $a_{1P\in}$ are denoted by ‘$A_{1P=}$’, ‘$A_{1P\subseteq}$’, and ‘$A_{1P\in}$’, respectively.

26) The selected primary predicate-sign = of $A_{1P=}$ is called the *equality predicate-sign for nonempty individuals*, whereas the only *secondary BAPCOPS of* $A_{1P=}$ is $\neg$=, which is abbreviated as $\not\equiv$ and is called the *anti-equality predicate-sign for nonempty individuals*. The selected primary predicate-sign $\subseteq$ of $A_{1P\subseteq}$ is called the *rightward mass-inclusion predicate-sign*, whereas *the pertinent secondary equality sign for masses, equivocally* depicted as =, *is postulated by contextually* (*implicitly*) *defining it in terms of* $\subseteq$. The selected primary predicate-sign $\in$ of $A_{1P\in}$ is called the *rightward class-inclusion predicate-sign*, whereas *the pertinent secondary rightward class-inclusion predicate-sign, equivocally* depicted as $\subseteq$, and *the pertinent secondary predicate-sign of equality for classes, equivocally* depicted as =, *are postulated by contextually defining* $\subseteq$ *in terms* of $\in$, and = *in terms of* $\subseteq$. In addition, in either of the above two cases, the pertinent secondary BAPCOPS $\subset$, called the *rightward strict mass-inclusion predicate-sign*, when it is utilized in $A_{1P\subseteq}$, and the *rightward strict class-inclusion predicate-sign*, when it is utilized in $A_{1P\in}$, is postulated by contextually defining it in terms of the respective predicate-sign $\subseteq$ and of *its negation*

$\neg\subseteq$, which is abbreviated as $\overline{\subseteq}$ and is called the *rightward mass-anti-inclusion* in the former case or the *rightward class-anti-inclusion, predicate-sign*. The predicate-sign $\subset$ is contextually expressible in terms of the respective predicate-signs $\subseteq$ and =, so that it can alternatively be defined in this way. Accordingly, the secondary binary *molecular* pseudo-constant predicate-sign $\neg\subset$ is abbreviated as $\overline{\subset}$ and is called the *rightward strict mass-anti-inclusion predicate-sign*, when utilized in $\mathsf{A}_{1\mathsf{P}\subseteq}$, and the *rightward strict class-anti-inclusion predicate-sign*, when utilized in $\mathsf{A}_{1\mathsf{P}\in}$. The *leftward* (*mirror-symmetrical*) *version* of a primary or secondary binary atomic or molecular pseudo-constant predicate-sign of $\mathsf{A}_{1\mathsf{P}\in}$ or $\mathsf{A}_{1\mathsf{P}\subseteq}$, is a *secondary* predicate-sign of the same calculus.

27) In order to state a *binary asymmetric synonymic definition* (*BASD*) conveniently and formally, I make use of either one of the horizontal arrows $\rightarrow$ and $\leftarrow$, which belong to the IML and which are called the *universal rightward synonymic definition sign* and the *universal leftward one* respectively. At the head of an arrow I write the *material definiens* – the graphonym, which is already known either from a previous definition or from another source. At the base of the arrow I write the *material definiendum* – the new graphonym, which is being introduced by the definition and which is designed to be used instead of or interchangeably with the definiens. Accordingly, the arrow $\rightarrow$ is rendered into ordinary language thus: "*is to stand as a synonym for*" or straightforwardly "*is the synonymous definiendum of*", and $\leftarrow$ thus: "*can be used instead of or interchangeably with*" or straightforwardly "*is the synonymous definiens of*". The [material] definiendum and [material] definiens of a BASD are indiscriminately called the *terms* of the definition. A BASD, which is made with the help of $\rightarrow$ or $\leftarrow$, is said to be a *formal BASD* or briefly an *FBASD*. Neither the definiendum nor the definiens of an FBASD should involve any *function symbols*, particularly any outermost (enclosing) quotation marks, that are not their constituent parts and that are therefore used but not mentioned with the following proviso. If it is necessary to indicate the integrity of the definiendum or of the definiens then that term of the definition can be enclosed in *square brackets as metalinguistic punctuation marks*, which do not, by definition, belong to the enclosed term and which are therefore used but not mentioned. If an arrow stands between a definiendum schema and a definiens schema then the arrow is supposed to aplly

simultaneously to the schemata and to every pair of interrelated instances (denotata, interpretands) of the schemata, unless stated otherwise.

In order to state formally that two old or two new graphonyms are or are to be used interchangeably (synonymously), I write the graphonyms, without any quotation marks that are not their constituent parts, in either order on both sides of the two-sided arrow ↔ belonging to the IML. Such a relation is called a *formal binary symmetric synonymity*, or *concurrency*, *relation* (FBSSR), whereas ↔ is accordingly called a *synonymity*, or *concurrency*, *sign*. The two graphonyms standing on both sides of ↔ are called the *terms* of the FBSSR. If an FBSSR is a *corollary* from the pertinent FBASD stated previously then ↔ is read as "*is concurrent to*" or, alternatively, **"**— ↔ …**"** is read as **"**— *and … are concurrent***"** or as **"**— *and … are synonyms***"**, where alike ellipses should be replaced alike and then the bold-faced double quotation marks should be replaced with the light-faced ones. If an FBSSR is stated in no connection with any previous FBASD then the FBSSR is said to be a *formal binary symmetric synonymic definition* (*FBSSD*), whereas ↔ is called the *symmetric*, or *two-sided*, *synonymic definition sign*. In this case ↔ is read as "*is to be concurrent to*" or, alternatively, **"**— ↔ …**"** is read as **"**— *and … are to be concurrent***"** or as **"**— *and … are to be synonyms***"**, where alike ellipses should, as before, be replaced alike, while the bold-faced double quotation marks are placeholders for the light-faced ones. Just as in the case of → or ←, if ↔ stands between schemata then the arrow is supposed to aplly simultaneously to the schemata and to every pair of interrelated instances (denotata, interpretands) of the schemata, unless stated otherwise.

28) Let '**I**'' and '**P**'' be analytical atomic panlogogographic placrholders of euautographic integron-definienda and euautographic relation-definienda of certain classes, whereas '**I**' and '**P**' are analytical atomic panlogogographic placrholders of the respective euautographic integron-definientia and of the respective euautographic relation-definientia. Hence, **I**'→**I**, or **P**'→**P**, is supposed to be an ASD, in which the placeholders '**I**'' and **I**', or '**P**'' and '**P**', are used *xenonymously*, so that → applies to all pertinent pairs of EI's **I**' and **I**, or to all pertinent pairs of ER's **P**' and **P**, and not to the pair of placeholders '**I**'' and '**I**', or '**P**'' and '**P**'. Subject to the above notation, $\mathsf{D}_1^{\mathrm{a}}$ includes the following two *meta-axiomatic rules of inference*:

i) If $[\mathbf{I}' \rightarrow \mathbf{I}]$ then $\vdash[\mathbf{I}' \hat{=} \mathbf{I}]$, i.e. an *asserted* ASD $[\mathbf{I}' \rightarrow \mathbf{I}]$ implies the euautographic *identity* (*valid equality* [*relation*]) $[\mathbf{I}' \hat{=} \mathbf{I}]$.

ii) If $[\mathbf{P}' \rightarrow \mathbf{P}]$ then $\vdash[\mathbf{P}' \Leftrightarrow \mathbf{P}]$ and equivalently $\vdash[V(\mathbf{P}') \hat{=} V(\mathbf{P})]$, i.e. an *asserted* ASD $[\mathbf{P}' \rightarrow \mathbf{P}]$ implies the euautographic *valid equivalence* [*relation*] $[\mathbf{P}' \Leftrightarrow \mathbf{P}]$ and also the equivalent euautographic *identity* $[V(\mathbf{P}') \hat{=} V(\mathbf{P})]$.

28) Every definition has its scope. Therefore, the general rule is that within the scope of an FBASD, which is stated by means of either sign $\rightarrow$ or $\leftarrow$, or within the scope of an FBSSD, which is stated by means of the $\leftrightarrow$, *isotokens of the terms* (*members*) *of the respective definition* can be related by:

a) the sign $\hat{=}$ if the isotokens belong to *the class of special terms* (*integrons*) *of a logistic system*, on which that sign is defined;

b) the sign = if the isotokens belong to *the class of ordinary terms of a logistic system*, on which that sign is defined;

c) the sign $\Leftrightarrow$ if the isotokens belong to *the class of relations of a logistic system*, on which that sign is defined.

The scope of a defininition does not include the definition itself. Accordingly, in a definition itself, which is stated by means of a certain one of the signs $\rightarrow$, $\leftarrow$, and $\leftrightarrow$, that sign can be replaced with the respective one of the signs $\overset{\rightarrow}{\hat{=}}$, $\overset{\leftarrow}{\hat{=}}$, and $\overset{\leftrightarrow}{\hat{=}}$ in the case a, with the respective one of the signs $\overset{\rightarrow}{=}$, $\overset{\leftarrow}{=}$, and $\overset{\leftrightarrow}{=}$ in the case b, and with the respective one of the signs $\overset{\rightarrow}{\Leftrightarrow}$, $\overset{\leftarrow}{\Leftrightarrow}$, and $\overset{\leftrightarrow}{\Leftrightarrow}$ in the case c. The signs $\overset{\rightarrow}{\hat{=}}$, $\overset{\leftarrow}{\hat{=}}$, and $\overset{\leftrightarrow}{\hat{=}}$ are called the *special rightward, leftward, and two-sided signs of equality by definition*; the signs $\overset{\rightarrow}{=}$, $\overset{\leftarrow}{=}$, and $\overset{\leftrightarrow}{=}$ are called the *ordinary rightward, leftward, and two-sided signs of equality by definition*; the signs $\overset{\rightarrow}{\Leftrightarrow}$, $\overset{\leftarrow}{\Leftrightarrow}$, and $\overset{\leftrightarrow}{\Leftrightarrow}$ are called the *[ordinary] rightward, leftward, and two-sided signs of equivalence by definition.*]

29) Upon introducing the secondary predicate-signs =, $\subset$, $\overline{\subseteq}$, $\equiv$, and $\overline{\subset}$ in $A_{1P\subseteq}$ or the secondary predicate-sign $\subseteq$ and the pertinent homographs of the above five secondary predicate-signs of $A_{1P\subseteq}$ in $A_{1P\in}$, D_1 can be applied to certain EOR's of $A_{1P\subseteq}$ or $A_{1P\in}$, of academic or practical interest, each of which involves a certain one of *its* secondary predicate-signs. In the result of the respective EADP's, some of the proceeded EOR's of $A_{1P\in}$ or $A_{1P\subseteq}$ are decided to be *valid*, i.e. they are *euautographic*

slave-theorems of $A_{1P\in}$ or $A_{1P\subseteq}$ respectively. In this case, an EOR of $A_{1P=}$ being a homograph of a slave-theorem of $A_{1P\subseteq}$ is not necessarily a slave-theorem of $A_{1P=}$ and likewise an EOR of $A_{1P\subseteq}$ being a homograph of a slave-theorem of $A_{1P\in}$ is not necessarily a slave-theorem of $A_{1P\subseteq}$. This fact evidences that any such slave-theorem is an *atypical* (*specific*) *slave-theorem* of its branch and therefore it is not included into D_1. Also, this fact evidences that the valid ER's of $A_{1P\subseteq}$, or $A_{1P\in}$, which are intrinsic equivalents of the extrinsic (metalinguistic) ASD's that contextually introduce the secondary binary predicate-sign of $A_{1P\subseteq}$, or $A_{1P\in}$, and from which the above slave-theorems follow, are, in agreement with the previous item, *atypical* (*specific, branch-determining*) *axioms,* which are not included into D_1.

30) The homographs of the sign =, which are employed in $A_{1P=}$, $A_{1P\subseteq}$, and $A_{1P\in}$, or the homographs of either one of the signs $\subseteq$ and $\subset$, which are employed $A_{1P\subseteq}$ and $A_{1P\in}$, are different signs that have different interpretations. Therefore, for avoidance of confusion, they might, and probably should, have been depicted differently. For instance, instead of the sign = of $A_{1P\subseteq}$, I might have employed, say, =', whereas instead of the signs $\subseteq$, = , and $\subset$ of $A_{1P\in}$, I might have employed, say, $\dot{\subseteq}$, $\doteq$, and $\dot{\subset}$ respectively. However, according to the item 14viii, the three signs =, $\subseteq$, and $\in$ of B_{1OS} are incompatible. Therefore, once the three full-scale branches $A_{1=}$, $A_{1\subseteq}$, and $A_{1\in}$ are developed from their *primordial phases* $A_{1P=}$, $A_{1P\subseteq}$, and $A_{1P\in}$, they do not intermix, i.e. they are incompatible as well. At the same time, if the primary predicate-sign = of $A_{1=}$ is equivocally used as a secondary predicate-sign of both $A_{1\subseteq}$ and $A_{1\in}$ and if the primary predicate-sign $\subseteq$ of $A_{1\subseteq}$ is equivocally used as a secondary predicate-sign of $A_{1\in}$ then *all axioms of* $A_{1=}$ formally turn out to be *theorems of both* $A_{1\subseteq}$ *and* $A_{1\in}$ and *all axioms of* $A_{1\subseteq}$ *other than the above-mentioned axioms of* $A_{1P\subseteq}$ formally turn out to be *theorems of* $A_{1P\in}$. At the same time, the axioms of $A_{1=}$, and every theorem following from them, or the axioms of $A_{1\subseteq}$, and every theorem following from them, should be interpreted differently from the theorems of $A_{1\in}$, being their homographs. The three comprehensive branches of A_1, which are developed from $A_{1P=}$, $A_{1P\subseteq}$, and $A_{1P\in}$, and are denoted by '$A_{1=}$', '$A_{1\subseteq}$' and '$A_{1\in}$' respectively, are described below in some detail.

31) The first member of the branch triple, denoted by '$A_{1=}$', is called the *Egalitarian EAPO* (*EgEAPO*), because it involves the *atomic ordinary* (*logical*)

predicate-sign of equality for nonempty individuals, =, as the pertinent distinguished optional element of B_{1OS}. This sign should not be confused with the *atomic special* (*algebraic*) *predicate-sign of equality for integrons*, $\hat{=}$. In the next and final phase of $A_{1=}$ following $A_{1P=}$, the *conventional laws of reflexivity, symmetry, and transitivity of* =, along with an additional axiom that is called the *incidence law for anti-equalities*, are laid down as *specific* (*atypical*) *subject* (*intrinsic*) *axioms of* $A_{1=}$. The branch $A_{1=}$ does not involve the PCOT's $\emptyset$ and $\emptyset'$, while the PVOT's of $A_{1=}$ are alternatively called *nonempty pseudo-individuals*, because they are not interrelated by either one of the predicate-signs $\subseteq$ and $\in$, which are not available in $A_{1=}$. Accordingly, $A_{1=}$ is alternatively called the *Pseudo-Nonempty-Individual EAPO* (*PNEIEAPO*), i.e. the *EAPO of Pseudo Nonempty Individuals*. The *restriction* (*quasi-branch*) of $A_{1=}$, which results by disregarding the alphabet of APVOR's, is denoted by '$A_{1=}^{p}$' and is called the *Pure Functional EgEAPO* or the *Pure Functional PNEIEAPO*. When desired, $A_{1=}^{p}$ can be restricted further by disregarding some or all other *ballast* PAOE's of B_{1OS}, which occur in it.

32) The second member of the branch triple, denoted by '$A_{1\subseteq}$', is called the *Pseudo-Mass EAPO* (*PMsEAPO*), meaning the *EAPO of Pseudo-Masses*, because it involves the *atomic ordinary inclusion* (*part-to-whole*) *predicate-sign* $\subseteq$ *for masses* as the pertinent distinguished optional element of basis b_{1OS} and because therefore it can therefore serve as the underlying calculus of a full-scale [*one-individual*] *theory of masses* as opposed to a *one-individual theory of classes* (see the next item). It is understood that $A_{1\subseteq}$ also involves both PCOT's: $\emptyset$, which is in this case called the *euautographic empty pseudo-mass* (*EEPMs*) or the *euautographic empty pseudo-individual* (*EEPIl*), and $\emptyset'$, which is called the *subsidiary EEPMs* or the *subsidiary EEPIl*. At the same time, the EOT's, i.e. PCOT's and PVOT's, of $A_{1\subseteq}$ are alternatively called *pseudo-masses*, because they are interrelated by the predicate-sign $\subseteq$ and are not interrelated by the predicate-sign $\in$, which is not available in $A_{1\subseteq}$. The setup of $A_{1\subseteq}$ is conveniently divided into four successive phases (stages), which are called the *Primordial*, *Ground*, *Defficient*, and *Sufficient PMsEAPO's*, and which are denoted by '$A_{1P\subseteq}$', '$A_{1G\subseteq}$', '$A_{1D\subseteq}$', and '$A_{1S\subseteq}$' respectively. Each phase includes all previous ones, so that $A_{1S\subseteq}$ is $A_{1\subseteq}$.

a) The phase $A_{1P\subseteq}$ is the coherent restriction of A_{1P}, which has been defined in the item 25, whereas $A_{1G\subseteq}$ is the phase, which has been described in the item 29. That is to say, it is $A_{1G\subseteq}$, where all secondary predicate-signs of $A_{1\subseteq}$ are introduced and where all possible slave-theorems are established for those signs exclusively from their *axiomatic definitions*.

b) In $A_{1D\subseteq}$, the *conventional laws of reflexivity and transitivity of* $\subseteq$ and an additional *incidence law for anti-inclusions of pseudo-masses* are laid down as *subject axioms of* $A_{1\subseteq}$, and all most fundamental *subject theorems*, including those for the predicate-signs = and $\subset$, which was introduced in $A_{1P\subseteq}$, are proved from them with the help of D_1. Among the theorems proved, are the conventional laws of reflexivity, symmetry, and transitivity of =, and also the pertinent incidence law for anti-equalities of pseudo-masses – the laws that are homographs of the respective *axiomatic laws of* $A_{1=}$.

c) In $A_{1S\subseteq}$, the axioms of pseudo-emptiness of $\emptyset$ and $\emptyset'$, namely $\emptyset \subseteq x$ and $\emptyset' \subseteq x$, are stated and the most fundamental slave-theorems, including the identity $\emptyset' = \emptyset$, are proved from them.

The *coherent restriction of* $A_{1\subseteq}$, which results by disregarding the alphabet of APVOR's and which is a branch of A_1 but not a branch of $A_{1\subseteq}$, is denoted by '$A^{p}_{1\subseteq}$' and is called the *Pure Functional PMsEAPO*. When desired, $A^{p}_{1=}$ can be restricted further by disregarding some or all other *ballast* PAOE's of B_{1OS}, which occur in it.

A certain part of this IML (this treatise), with the help of which and within which $A_{1\subseteq}$ is developed (set up and executed), is called the *Euautographic Pseudo-Mass Theory* (*EPMsT*) or alternatively and more precisely the *Pseudo-Unrestricted*, or *Pseudo-Unconfined*, *EPMsT* – in contrast to that called the *Pseudo-Restricted EPMsT*, in which a version of $A_{1\subseteq}$, denoted by '$\bar{A}_{1\subseteq}$' and called the *Pseudo-Restricted PMsEAPO*¸ is developed (see the item 52 below).

33) The third member of the branch triple is denoted by '$A_{1\in}$' and is called the *Pseudo-Class EAPO* (*PCsEAPO*), meaning the *EAPO of Pseudo-Classes,* because it involves the *ordinary atomic ordinary class-membership predicate-sign* $\in$ as the pertinent distinguished optional element of its basis b_{1OS} and because, therefore, it can serve as the underlying calculus of a full-scale *one-individual theory of classes*. $A_{1\in}$ also involves both PCOT's: $\emptyset$, which is in this case alternatively called the

euautographic empty pseudo-class (*EEPCs*) or *euautographic empty pseudo-individual* (*EEPIl*), and $\emptyset'$, which is alternatively called the *subsidiary EEPCs* or *subsidiary EEPIl*. At the same time, the EOT's, i.e. PCOT's and PVOT's, of $\mathsf{A}_{1\in}$ are alternatively called *pseudo-classes* or *pseudo-elements*, because they are interrelated by the predicate-sign $\in$ and also by the predicate-signs $\subseteq$ and =, which are defined in terms of $\in$. Like the setup of $\mathsf{A}_{1\subseteq}$, the setup of $\mathsf{A}_{1\in}$ is conveniently divided into successive phases (stages), but these are five in number, which are called the *Primordial*, *Ground*, *Defficient*, *Sufficient*, and *Aristotelian* (or *Syllogistic) PCsEAPO's*, and which are denoted by '$\mathsf{A}_{1P\in}$', '$\mathsf{A}_{1G\in}$', '$\mathsf{A}_{1D\in}$', '$\mathsf{A}_{1S\in}$', and '$\mathsf{A}_{1A\in}$' respectively. As before, each phase includes all previous ones. However, while the first four phases of $\mathsf{A}_{1\in}$ are analogous to those of $\mathsf{A}_{1\subseteq}$, $\mathsf{A}_{1A\in}$ is a peculiar phase, which essentially differs from the previous one. Therefore, I identify $\mathsf{A}_{1\in}$ with $\mathsf{A}_{1S\in}$, while $\mathsf{A}_{1A\in}$, which is alternatively denoted by 'A_{1A}' and which is briefly called the *Aristotelian*, or *Syllogistic*, *EAPO* (*AEAPO* or *SEAPO*), will be considered separately after $\mathsf{A}_{1\in}$ ($\mathsf{A}_{1S\in}$) as its *extra phase*.

a) Just as in the case of $\mathsf{A}_{1\subseteq}$, the phase $\mathsf{A}_{1P\in}$ is the coherent restriction of A_{1P}, which has been defined in the item 25, whereas $\mathsf{A}_{1G\in}$ is the phase, which has been described in the item 29. That is to say, it is $\mathsf{A}_{1G\in}$, where all secondary predicate-signs of $\mathsf{A}_{1\in}$ are introduced and where all possible slave-theorems are established for those signs exclusively from their *axiomatic definitions*.

b) In $\mathsf{A}_{1D\in}$, two *subject axioms,* namely the *asymmetry law*: $\neg[[x \in y] \wedge [y \in x]]$ and the *incidence law with respect to the pseudo-class term*: $\bigvee_u [x \in u]$ ($\bigvee_u$ is a synonym of $(\exists u)$), are stated and most fundamental slave-theorems are proved from them with the help of D_1. Among the slave-theorems are homographs of all laws of the predicate-signs =, $\subseteq$, and $\subset$, which have previously been laid down as axioms or proved as slave-theorems of $\mathsf{A}_{1\subseteq}$ or of both $\mathsf{A}_{1=}$ and $\mathsf{A}_{1\subseteq}$. In this case, the homograph of the *incidence law for anti-inclusions of pseudo-masses* is called the *incidence law for anti-inclusions of pseudo-classes.*

c) In $\mathsf{A}_{1S\in}$, the axioms of pseudo-emptiness of $\emptyset$ and $\emptyset'$, namely $\neg[x \in \emptyset]$ and $\neg[x \in \emptyset']$, are stated and the most fundamental slave-theorems, including the identity

$\emptyset' = \emptyset$ and the universal pseudo-class-inclusion law $\emptyset \subseteq x$ (cf. its homograph in the item 32c, are proved from them.

The *coherent restriction* of $\mathsf{A}_{1\in}$, which results by disregarding the alphabet of APVOR's and which is a branch of $\mathsf{A}_{1\in}$, is denoted by '$\mathsf{A}^{\mathrm{p}}_{1\in}$' and is called the *Pure Functional PCsEAPO*. When desired, $\mathsf{A}^{\mathrm{p}}_{1\in}$ can be restricted further by disregarding some or all other *ballast* PAOE's of $\mathsf{B}_{1\mathsf{OS}}$, which occur in it.

A certain part of this IML (this treatise), with the help of which and within which $\mathsf{A}_{1\in}$ is developed (is set up and executed), is called the *Euautographic Pseudo-Class Theory* (*EPCsT*) or alternatively and more precisely the *Pseudo-Unrestricted*, or *Pseudo-Unconfined*, *EPCsT* – in contrast to that called the *Pseudo-Restricted EPCsT*, in which a version of $\mathsf{A}_{1\in}$, denoted by '$\bar{\mathsf{A}}_{1\in}$' and called the *Pseudo-Restricted PCsEAPO*¸ is developed (see the items 61–63 below).

34) $\mathsf{A}_{1\mathsf{A}}$, i.e. $\mathsf{A}_{1\mathsf{A}\in}$, results by supplementing $\mathsf{A}_{1\in}$ with *axiomatic definitions* of various sets of 19 *euautographic ordinary relations* (*EOR's*) in each set – the EOR's, which have the same structure as 19 *categorical syllogisms* of Aristotelian logic (see, e.g. Hilbert and Ackermann [1950, Chapter II], Łukasiewicz [1951], or Lamontagne and Woo [2008]), which are collectively called the *euautographic syllogistic implications* (*ESI's*). Separate ESI's are distinguished by the same catchwords as those identifying separate categorical syllogisms, e.g. "Barbara", "Bamalip", etc, but these are set in the Roman Arial Narrow Type, and are furnished with additional *alphanumeric subscripts*. Also, for a certain reason, which is, in general outline, relevant to a certain unconventional classification of the ESI's and of the categorical syllogisms, being their so-called *conformal catlogographic* (*CFCL*) *interpretands,* I have replaced the conventional catchword "Darapti" with "Barapti". Together with its subscripts, each modified catchword, is the *euautographic predicate* of the pertinent ESI, and it is therefore called a *ternary euautographic syllogistic predicate* (*TESP*). Any ESI comprises three binary EOR's that are called *euautographic syllogistic judgments* (*ESJ's*). There are *four* types of ESJ's in each set of 19 ESI's, which are distinguished from one another by their binary euautographic predicates that are called *binary euautographic syllogistic predicates* (*BESP's*). The latter are denoted by the letters 'A', 'O', 'E', and 'I' furnished with the appropriate subscripts – the letters, which are associated with the conventional *catch letters* 'A', 'O', 'E', and 'I', or 'a',

‘o’, ‘e’, and ‘i’, serving as logical predicates of the *separate judgments* (the *premises* and *conclusion*) *of a categorical syllogism* of Aristotelian logic. There are two kinds of BESP’s: *binary pseudo-variable syllogistic predicates* (*BPVSP’s*) and *binary pseudo-constant syllogistic predicates* (*BPCSP*’s). A BPVSP is defined in terms of an *atomic pseudo-variable predicate-sign* (as f^2, g^2, or h^2) and is not interpretable by any syllogistic judgment of Aristotelian logic. A BPCSP is defined in terms of $\in$ or in terms of some one of the predicate-signs $\subseteq$ and $\subset$, which are in turn defined in terms of $\in$. Some such BPCSP’s are interpretable by the syllogistic judgments of Aristotelian logic. The purpose of A_{1A} is to apply D_1 to all defined ESI’s and to calculate their UEVI’s, hich are tantamount to their validity-values. In this way, I have proved that 15 categorical syllogisms, other than Bamalip, Barapti (former Darapti), Felapton, and Fesapo, are *tautologous*, i.e. *universally true*, because they are the so-called *conservative* CFCL (*C*CFCL) interpretands of the respective *valid* ESI’s. The latter four categorical syllogisms are *neutral*, or *indeterminate*, *with respect to the tautologousness-values tautologousness and antitautologousness* (*contradictoriness*) – briefly *ttatt-neutral* or *ttatt-indeterminate,* i.e. they are *neither tautologous nor antitautologous* (*nor contradictory*), because they are theCCFCL) interpretands of the respective *vav-neutral* (*vav-indeterminate*) ESI’s. These four are turned into *veracious*, i.e. *accidentally true*, *syllogisms* owing to certain *additional veracious catlogographic axioms*. This result is in agreement with the finding of Hilbert and Ackermann [1950, pp. 48–54, 53ff] that all categorical syllogisms in the exclusion of the above four are deducible from Boolean algebra. Incidentally, in view of the additional veracious catlogographic axioms, the four peculiar syllogisms cannot, strictly speaking be qualified *categorical*, i.e. *unconditional.*

35) According to the above items 31–33, all laws (valid relations) for the predicate-sign =, which are postulated (taken for granted) and laid down as *axioms of* $\mathsf{A}_{1=}$, turn out to be *slave-theorems of* $\mathsf{A}_{1\subseteq}$, whereas all laws for the predicate-sign $\subseteq$, which are laid down as *axioms of* $\mathsf{A}_{1\subseteq}$, turn out to be *slave-theorems of* $\mathsf{A}_{1\in}$. Consequently, all laws for the predicate-sign =, which are laid down as axioms of $\mathsf{A}_{1=}$, are theorems of $\mathsf{A}_{1\in}$ as well. That is to say, $\mathsf{A}_{1\subseteq}$ *formally* includes $\mathsf{A}_{1=}$, whereas $\mathsf{A}_{1\in}$ *formally* includes both $\mathsf{A}_{1\subseteq}$ and $\mathsf{A}_{1=}$. Since $\mathsf{A}_{1=}$ and $\mathsf{A}_{1\subseteq}$ *as if converge into* $\mathsf{A}_{1\in}$, it seems therefore that the earlier splitting of A_1 into the triple of $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, and $\mathsf{A}_{1\in}$ can be regarded as a *virtual* one or be *disregarded* at all and that hence $\mathsf{A}_{1\in}$ can be

identified with the entire A_1 that includes also A_{1A}, described in the previous item as an extra phase of $A_{1\in}$. From this viewpoint, $A_{1\in}$ is the *main branch of* A_1 that may therefore be alternatively called the *Trunk*, or *Stem*, *EAPO*, i.e. the *trunk*, or *stem*, *of* A_1, whereas $A_{1=}$ and $A_{1\subseteq}$ may figuratively be called *boughs*, or *limbs*, *of* A_1. These considerations are however true only as long as A_1 remains semantically uninterpreted. The PVOT's (APVOT's) of A_1, given on the list (2.7), are interpretable in three different ways, when they are employed in $A_{1=}$, $A_{1\subseteq}$, or $A_{1\in}$.

36) *Tokens* of a given *prototypal artificial* (*man-made*) *sensible entity* (e.g. visible, audible, or tangible) are called *isotokens* if they are entities of the same genesis so that they have the same sensorial properties, and the tokens are called *paratokens* if they are entities of another genesis so that they have either the same or other sensorial properties. For instance, *phonic tokens* of a graphonym or *graphic tokens* of a phononym are paratokens of the respective prototypal onym. A euautograph or a logograph is indiscriminately called a *pasigraph*. A pasigraph has an indefinite number of isotokens and no phonic paratokens. A pasigraph is called (a) a *homolograph* if it has only *congruent* or *proportional tokens* (*isotokens*), called *homolographic*, or *photographic*, ones; (b) an *analograph* if, besides homolographic isotokens, it has *recognizably same but stylized* isotokens, qualified *iconographic* or *pictographic*. An isotoken of a pasigraph is called a *strictly analographic token* (*isotoken*) if it is *not homolographic* (*not photographic*). A pasigraph, i.e. a euautograph or logograph, particularly a panlogograph or a catlogograph, is called a *conformal*, or *analo-homolographic, token* (*isotoken*) *of a given prototypal pasigraph* if it is a *strictly analographic token of the latter that has only homolographic* (*photographic*) *tokens* (*isotokens*) *of its own*.

37) The only values that a euautograph can have or assume within A_1 are its *autonymous values* such as the *class of its homolographic* (*photographic*) *isotokens*, a *concrete member* of the above class as the euautograph itself, or a *common* (*general*, *certain*) *member* of that class, which is just another *hypostasis* (*way of existence*, *aspect*) of that same class. By the complete absence of any semantic properties, a euautograph is analogous to a chessman or to a position of chessmen on the chessboard or, more precisely, it is analogous to a figure of either of the above objects in a textbook on chess (as Chernev [1958]). To be more specific, within A_1, a euautograph, – *atomic* or *combined* and at the same time, *categorematic* (*formulary*,

self-subsistent) or *syncatergorematic* (*not formulary*), – is *functional but insignificant*, i.e. it is a *graphic chip* (*fish*) or a *pattern group* (*combination*) *of such chips*, which has a *certain syntactic function* or *functions* in itself or with respect to other pasigraphs (euautographs or panlogographs or both), especially those of its immediate surrounding (when applicable), but which has no psychical (mental) significations (imports, values) except autonymous ones. Therefore, a euautograph of any kind (class) *is incapable either of having or of assuming* (*taking on*) *any denotatum* (*denotation value*, pl. "*denotata*"). Particularly, within A_1, a *euautographic relation* (*ER*) *is incapable either of having or assuming psychically* (*mentally*) *any truth-value*, or of being *physically replaced* with any significant graphic relation such as a *propositional* (*dualistic truth-valued*) *functional form* or a *propositional declarative sentence* of any *written native language* (*WNL*). That is to say, a euautograph can be *neither a variable nor a constant* and *neither a qualifier nor a quantifier*. Consequently, for the purpose of description or reference, a euautograph is, when applicable, called a *pseudo-variable* or a *pseudo-constant*, in accordance with its function or in accordance with its subsequent *syntactico-semantic interpretation outside of* A_1 by a variable or by a constant respectively, or in accordance with both reasons. Particularly, an EKS (euautographic kernel-sign) of A_1, whose function is to *bind* every occurrence of an APVOT in the scope of the EKS to the first occurrence of that APVOT in the EKS itself, is impartially called a *euautographic binder* (*EB*) or *euautographic contractor*, whereas the APVOT that it binds is called a *bound*, or *dummy*, *APVOT*. An EB, which is united with an ER (euautographic relation) to produce another ER, and whose logical status is therefore similar to that of a *logical quantifier*, is called a *euautographic logical*, or *ordinary*, *pseudo-qualifier* (*EPQl*, pl. "*EPQl's*") if it is utilized in $A_{1\subseteq}$ and a *euautographic logical*, or *ordinary*, *pseudo-quantifier* (*EPQn*, pl. "*EPQn's*") if it is utilized in $A_{1\in}$. An EB, which is united with an EI (euautographic integron), i.e. with ESpT (euautographic special term), to produce another EI (ESpT), and whose logical status is therefore similar to that of an *algebraic multiplier* (*multiplication operator*) over occurrences of a dummy variable, is called a *euautographic algebraic*, or *special*, *binder* (*EAlB* or *ESpB*) or a *euautographic algebraic*, or *special*, *contractor* and also a *euautographic pseudo-multiplier* (*EPM*).

2.4. The conservative conformal catlogographic interpretation of A_1

38) In accordance with the item 11, I use the following system of interrelated notations subject to any one of the three substituends '$_0$', '$^0_1$', and '$_1$' for '$_n$'.

i) I_n is the class of *euautographic integrons* (*EI's*) of A_n.

ii) R_n is the class of *euautographic relations* (*ER's*) of A_n, so that $\mathsf{R}_n \to \mathsf{R}_n^{\mathrm{O}} \cup \mathsf{R}_n^{\mathrm{Sp}}$ subject to the following two definitions.

a) $\mathsf{R}_n^{\mathrm{O}}$ is the class of *ordinary ER's* (*OER's* or *EOR's*) of A_n.

b) $\mathsf{R}_n^{\mathrm{Sp}}$ is the class of *special ER's* (*SpER's* and *ESpR's*) of A_n.

iii) $\hat{\mathsf{R}}_n$ is the class of *decided euautographic relations* (*DdESR's*) of A_n, so that $\hat{\mathsf{R}}_n \to \mathsf{R}_{n*} \cup \mathsf{R}_{n\oplus}^{*}$ subject to the following two definitions a and b.

a) R_{n*} is the class of *decided euautographic slave relations* (*DdESR's*) of A_n, so that R_{n+}¸R_{n-}¸ and $\mathsf{R}_{n\sim}$ are the classes of *valid*, *antivalid*, and *vav-neutral* (*vav-indeterminate*) *DdESR's* of A_n respectively. In this case, $\mathsf{R}_{n*} \to \mathsf{R}_{n*}^{\mathrm{O}} \cup \mathsf{R}_{n*}^{\mathrm{Sp}}$, where

a$_1$) $\mathsf{R}_{n*}^{\mathrm{O}}$ is the class of *decided euautographic [slave] ordinary relations* (*DdESOR's* or *DdEOR's*) of A_n subject to $\mathsf{R}_{n*}^{\mathrm{O}} \to \left[\mathsf{R}_{n+}^{\mathrm{O}} \cup \mathsf{R}_{n-}^{\mathrm{O}}\right] \cup \mathsf{R}_{n\sim}^{\mathrm{O}}$, the understanding being that $\mathsf{R}_{n+}^{\mathrm{O}}$, $\mathsf{R}_{n-}^{\mathrm{O}}$, and $\mathsf{R}_{n\sim}^{\mathrm{O}}$ are the classes of *valid*, *antivalid*, and *vav-neutral* (*vav-indeterminate*) *DdEOR's* of A_n; a DdEOR is necessarily a DdESOR and vice versa,

a$_2$) $\mathsf{R}_{n*}^{\mathrm{Sp}}$ is the class of *decided euautographic slave special relations* (*DdESSpR's*) of A_n subject to $\mathsf{R}_{n*}^{\mathrm{Sp}} \to \left[\mathsf{R}_{n+}^{\mathrm{Sp}} \cup \mathsf{R}_{n-}^{\mathrm{Sp}}\right] \cup \mathsf{R}_{n\sim}^{\mathrm{Sp}}$, the understanding being that $\mathsf{R}_{n+}^{\mathrm{Sp}}$, $\mathsf{R}_{n-}^{\mathrm{Sp}}$, and $\mathsf{R}_{n\sim}^{\mathrm{O}}$ are the classes of *valid*, *antivalid*, and *vav-neutral* (*vav-indeterminate*) *DdESSpR's* of A_n respectively.

b) $\mathsf{R}_{n\oplus}^{*}$ is the class of EMT's (EDT's) of DdESR's in the class R_{n*}, so that $\mathsf{R}_{n\oplus}^{+}$, $\mathsf{R}_{n\oplus}^{-}$, and $\mathsf{R}_{n\oplus}^{\sim}$ are the classes of EMT's (EDT's) of DdESR's in the classes R_{n+}¸R_{n-}¸ and $\mathsf{R}_{n\sim}$ respectively. In this case, $\mathsf{R}_{n\oplus}^{*} \to \mathsf{R}_{n\oplus}^{\mathrm{O}*} \cup \mathsf{R}_{n\oplus}^{\mathrm{Sp}*}$, where

b$_1$) $\mathsf{R}_{n\oplus}^{\mathrm{O}*}$ is the class of EMT's (EDT's) of DdEOR's in the class $\mathsf{R}_{n*}^{\mathrm{O}}$, so that $\mathsf{R}_{n\oplus}^{\mathrm{O}*} \to \left[\mathsf{R}_{n\oplus}^{\mathrm{O}+} \cup \mathsf{R}_{n\oplus}^{\mathrm{O}-}\right] \cup \mathsf{R}_{n\oplus}^{\mathrm{O}\sim}$, the understanding being that $\mathsf{R}_{n\oplus}^{\mathrm{O}+}$,

$\mathsf{R}_{n\oplus}^{O-}$, and $\mathsf{R}_{n\oplus}^{O\sim}$ are the classes of EMT's (EDT's) of DdEOR's in the classes R_{n+}^{O}, R_{n-}^{O}, and $\mathsf{R}_{n\sim}^{O}$ respectively,

b2) $\mathsf{R}_{n\oplus}^{Sp*}$ is the class of EMT's (EDT's) of DdESSpR's in the class R_{n*}^{Sp}, so that $\mathsf{R}_{n\oplus}^{Sp*} \to [\mathsf{R}_{n\oplus}^{Sp+} \cup \mathsf{R}_{n\oplus}^{Sp-}] \cup \mathsf{R}_{n\oplus}^{Sp\sim}$, the understanding being that $\mathsf{R}_{n\oplus}^{Sp+}$, $\mathsf{R}_{n\oplus}^{Sp-}$, and $\mathsf{R}_{n\oplus}^{Sp\sim}$ are the classes of EMT's (EDT's) of DdESSp's in the classes R_{n+}^{Sp}, R_{n-}^{Sp}, and $\mathsf{R}_{n\sim}^{Sp}$ respectively.

iv) The class $\dot{\mathsf{R}}_n$ defined as $\dot{\mathsf{R}}_n \to [[\mathsf{R}_{n+}^{O} \cup \mathsf{R}_{n\sim}^{O}] \cup \mathsf{R}_{n\oplus}^{O\sim}] \leftrightarrow [\mathsf{R}_{n+}^{O} \cup [\mathsf{R}_{n\sim}^{O} \cup \mathsf{R}_{n\oplus}^{O\sim}]]$ is called *the* [*class of*] *output ER's* (*OptER's*) *of* A_n.

v) The above items i–iv apply with '**A**', '**I**', '**R**', and "*panlogographic*" ("*PL*") in place of 'A', 'I', 'R', and "*euautographic*" ("*E*") respectively.

vi) In accordance with *the above item iii*, $\hat{R}_n$ is *the class of CCFCL inteprretands of the DdESR's in the class* $\hat{\mathsf{R}}_n$, so that $\hat{R}_n \to R_{n*} \cup R_{n\oplus}^{*}$ subject to the following two definitions a and b.

a) R_{n*} is *the class of CCFCL interpretands of the DdESR's in the class* R_{n*}, so that R_{n+}, R_{n-}, and $R_{n\sim}$ are *the classes of CCFCL interpretands of the valid, antivalid,* and *vav-neutral* (*vav-indeterminate*) *DdESR's in the classes* $\mathsf{R}_{n+}, \mathsf{R}_{n-}$, and $\mathsf{R}_{n\sim}$ respectively. In this case, $R_{n*} \to R_{n*}^{O} \cup R_{n*}^{Sp}$, where

a1) R_{n*}^{O} is *the class of CCFCL interpretands of DdEOR's* in the class R_{n*}^{O}, subject to $R_{n*}^{O} \to [R_{n+}^{O} \cup R_{n-}^{O}] \cup R_{n\sim}^{O}$, the understanding being that R_{n+}^{O}, R_{n-}^{O}, and $R_{n\sim}^{O}$ are the classes of the *CCFCL interpretands of valid, antivalid,* and *vav-neutral* (*vav-indeterminate*) *DdEOR's in the classes* R_{n+}^{O}, R_{n-}^{O}, and $\mathsf{R}_{n\sim}^{O}$ respectively.

a2) R_{n*}^{Sp} is the class of the *CCFCL interpretands of the DdESSpR's in the class* R_{n*}^{Sp}, subject to $R_{n*}^{Sp} \to [R_{n+}^{Sp} \cup R_{n-}^{Sp}] \cup R_{n\sim}^{Sp}$, the understanding being that R_{n+}^{Sp}, R_{n-}^{Sp}, and $R_{n\sim}^{Sp}$ are *the classes of CCFCL interpretands of the valid, antivalid,* and *vav-neutral* (*vav-indeterminate*) *DdESSpR's in the classes* R_{n+}^{Sp}, R_{n-}^{Sp}, and $\mathsf{R}_{n\sim}^{O}$ respectively.

b) $R^{*}_{n\oplus}$ is *the class of CCFCL interpretands of the EMT's* (*EDT's*) *in the class* $\mathrm{R}^{*}_{n\oplus}$, so that $R^{+}_{n\oplus}$, $R^{-}_{n\oplus}$, and $R^{\sim}_{n\oplus}$ are *the classes of CCFCL interpretands of the EMT's* (*EDT's*) *in the classes* $\mathrm{R}^{+}_{n\oplus}$, $\mathrm{R}^{-}_{n\oplus}$, and $\mathrm{R}^{\sim}_{n\oplus}$ respectively. In this case, $R^{*}_{n\oplus} \to R^{O*}_{n\oplus} \cup R^{Sp*}_{n\oplus}$, where

b1) $R^{O*}_{n\oplus}$ is *the class of CCFCL interpretands of the EMT's* (*EDT's*) *in the class* $\mathrm{R}^{*}_{n\oplus}$, so that $R^{O*}_{n\oplus} \to \left[R^{O+}_{n\oplus} \cup R^{O-}_{n\oplus}\right] \cup R^{O\sim}_{n\oplus}$, the understanding being that $R^{O+}_{n\oplus}$, $R^{O-}_{n\oplus}$, and $R^{O\sim}_{n\oplus}$ are *the classes of CCFCL interpretands of the EMT's* (*EDT's*) *in the classes* $\mathrm{R}^{O+}_{n\oplus}$, $\mathrm{R}^{O-}_{n\oplus}$, and $\mathrm{R}^{O\sim}_{n\oplus}$ respectively,

b2) $R^{Sp*}_{n\oplus}$ is *the class of CCFCL interpretands of the EMT's* (*EDT's*) *in the class* $\mathrm{R}^{Sp*}_{n\oplus}$, so that $R^{Sp*}_{n\oplus} \to \left[R^{Sp+}_{n\oplus} \cup R^{Sp-}_{n\oplus}\right] \cup R^{Sp\sim}_{n\oplus}$, the understanding being that $R^{Sp+}_{n\oplus}$, $R^{Sp-}_{n\oplus}$, and $R^{Sp\sim}_{n\oplus}$ are *the classes of CCFCL interpretands of the EMT's* (*EDT's*) *in the classes* $\mathrm{R}^{Sp+}_{n\oplus}$, $\mathrm{R}^{Sp-}_{n\oplus}$, and $\mathrm{R}^{Sp\sim}_{n\oplus}$ respectively.

vii) The class $\dot{R}_n$ defined as $\dot{R}_n \to \left[\left[R^{O}_{n+} \cup R^{O}_{n\sim}\right] \cup R^{O\sim}_{n\oplus}\right] \leftrightarrow \left[R^{O}_{n+} \cup \left[R^{O}_{n\sim} \cup R^{O\sim}_{n\oplus}\right]\right]$ is formally related to the class $\dot{\mathrm{R}}_n$, defined in the above item iv, as $\dot{R}_n = I_n\left(\dot{\mathrm{R}}_n\right)$ (cf. (2.1) and (2.4)) and is called *the CCFCL interpretand of* $\dot{\mathrm{R}}_n$ and also *the* [*class of*] *CCFCL interpretands of the OptER's of* A_n or briefly and loosely *the CCFCL interpretand of* A_n.

The class $\dot{\mathrm{R}}_1$, e.g., defined in the above item iv, comprises the valid and vav-neutral DdEOR's of the classes R^{O}_{1+} and $\mathrm{R}^{O}_{1\sim}$ respectively and it also comprises the EMT's (EDT's) of the class $\mathrm{R}^{O\sim}_{1\oplus}$, i.e. the EMT's (EDT's) for the vav-neutral DdEOR's of the class $\mathrm{R}^{O}_{1\sim}$. Hence, $\dot{\mathrm{R}}_1$ is the *least inclusive class of the ER's of* A_1, whose *conservative conformal catlogographic* (*CCFCL*) *interpretands* are sufficient for recovering such interpretands of *all* DdEOR's of A_1 and of *their* EDT's (EDT's), the understanding being that *any EMT* (*EDT*) *is a valid ESpR*, but not necessarily vice versa. Therefore, the class $\dot{\mathrm{R}}_1$ is called *the* [*class of*] *output ER's* (*OptER's*) *of* A_1, whereas the class $\dot{R}_1$, defined in the above item vii, is called *the* [*class of*] *CCFCL interpretands of the OptER's of* A_1. The above remarks with either one of the indices '$_0$' and '$^0_1$' in place of '$_1$' remain true.

The expression "*its conformal catlogographic interpretations*" that occurs in the title of the treatise refers to the *sequence of two interrelated systems of conformal catlogographic* (*CFCL*) *interpretations of* (*acts of interpreting*) *the OptER's of* A_1 (*but not of* $\mathbf{A}_1$), which I regard as the *principal interpretations of* A_1. In accordance the item 2ii.a$_1$, the first CFCL interpretation of A_1 in the sequence is denoted by 'I_1' and is called the *conservative CFCL* (*CCFCL*) *interpretation of* A_1, whereas the second one is denoted by 'A_1' and is called the *progressive CFCL* (*PCFCL*) *interpretation of* A_1. In this case, I_1 is the class of ordered pairs of an OptER in the class $\dot{\mathsf{R}}_1$ and of its CCFCL interpretand in the class $\dot{R}_1$. By the pertinent definition of the item 36, the qualifier "*conformal*" (*CF*") to "interpretation" and to any other relevant substantive (as "interpretand") can be used interchangeably (synonymously) with the qualifier "*analo-homolographic*". Each one of the two CFCL interpretations has two *hypostases* (*aspects, way of existence*), which are analogous respectively to the *grammar of an object WNL* and to *its semantics*. The CCFCL interpretation of A_1 is described below in this subsection, and the PCFCL interpretation of A_1 will be described in the next subsection.

39) The first hypostasis (aspect) of the CCFCL interpretation of A_1 is denoted by 'I_1' and is called the *CCFCL interpretation of* A_1 *in intension* or the *rules of the CCFCL interpretation of* A_1 *in extension* and also the *CCFCL Interpretational Decision Method* (*CCFCLIDM*) *of* A_1, whereas the second hypostasis of the CCFCL interpretation of $\mathsf{A}_1$is denoted by 'I_1' and is called the *CCFCL interpretation of* A_1 *in extension*.

a) Thus, I_1 is the *cumulative rule of* I_1, which comprises (i) the *cumulative rule* I_1' *of syntactic substitutions of conformal catlexigraphs* (*conformal atomic catlogographs*) *for the occurrences of EOT's* (*AEOT's*) *and AER's* (*APVR's, APVOR's*) *throughout any given OptER of* A_1 (*IptER of* I_1) and (ii) the *cumulative rule* I_1'' *of conservative semantic interpretation of the above catlexigraphs*, so that $I_1 = I_1'' \circ I_1'$. These constituent parts of I_1 along with the pertinent definitions and comments are made explicit in the next three items.

b) An OptER of A_1 is alternatively called an *input euautographic relation* (*IptER*) *of* I_1. The act of interpreting of an OptER of A_1 (IptER of I_1) in accordance with I_1 is called the *CCFCL interpretation* of that OptER of A_1 (IptER of I_1). The result of that act is called the *CCFCL interpretand of the OptER of* A_1 (*IptER of* I_1) and also less explicitly an *output conservative catlogographic relation* (*OptCCLR*) *of* I_1 or simply a *CCLR*, whereas the OptER of A_1 (IptER of I_1) being the euautographic (insignificant) template of its CCFCL interpretand is called the *conformal*, or *template*, *euautographic* (*CFE*) *interpretans* (pl. "*interpretantia*") *of the CCFCL interpretand*. Thus, I_1 the class of acts of CCFCL interpretation of the OptER's of A_1 or alternatively the class of ordered pairs of an OptER's of A_1 and its CCFCL interpretand. In accordance with the pertinent conventional terminology of the theory of sets of ordered pairs, a separate ordered pair belonging to I_1, i.e. the ordered pair of an OptER and its CCFCL, is called *the cut of* I_1 *at the OptER* or, less explicitly, a *cut of* I_1 (cf. Bourbaki [1960, chap. II, §3, 1]). Consequently, associated with both I_1 and I_1 is the *pertinent decisional trichotomy of CCLR's*, which will also be explicated below.

c) I_1 is a *concretum* in the sense that it is actually stated in full, whereas I_1 is an *abstractum* in the sense that only some instances of it are realized (written down, made explicit).

40) *The cumulative rule* I_1' *of syntactic substitutions of conformal catlexigraphs for atomic euautographic formulas* (*categoremata*).

i) In order to obtain the *CCFCL* interpretand of a given OptER of A_1 (IptER of I_1), the pertinent ones of the following substitutions should be performed throughout the OptER at first place:

$$u \mapsto \mathsf{u},\ v \mapsto \mathsf{v},\ w \mapsto \mathsf{w},\ x \mapsto \mathsf{x},\ y \mapsto \mathsf{y},\ z \mapsto \mathsf{z}, \tag{2.25}$$

$$\emptyset \mapsto \varnothing,\ \emptyset' \mapsto \varnothing', \tag{2.26}$$

$$p \mapsto \mathsf{p},\ q \mapsto \mathsf{q},\ r \mapsto \mathsf{r},\ s \mapsto \mathsf{s}, \tag{2.27}$$

while all other euautographs that may occur in the OptER, including ESC'ta (euautographic syncategoremata), i.e. primary and secondary EKS's (euautographic kernel-signs) and atomic punctuation marks, and also including digital integrons 0, 1, 2, etc, *remain unaltered*. The *barred arrow* is the metalinguistic *sign of substitution*, which is directed from a *substituend*, i.e. from a *replacing catlexigraph*, to the respective *substituens* (pl. "*substituentia*"), i.e. to the *replaced atomic euautograph*.

The substitutions (2.25) and (2.27) should be understood both as ones for the index-free APVOT's and APVOR's and for the base letters of the indexed APVOT's and APVOR's. For instance, $u \mapsto \mathsf{u}$ means, not only the substitution of the catlexigraph *u* for the APVOT u, but it also implies the substitutions: $u_1 \mapsto \mathsf{u}_1$, $u_2 \mapsto \mathsf{u}_2$, etc. The conjunction of rules (2.25)–(2.27) is denoted by 'I_1'' and is called the *cumulative advanced syntactic rule of CCFCL interpretation of* A_1 and also that of A_1^0. The rule of substitutions (2.27) alone is denoted by 'I_0'' and is called the *syntactic rule of CCFCL interpretation of* A_0 and also the *basic syntactic rule of CCFCL interpretation of both* A_1 *and* A_1^0. I_0' or I_1' is indiscriminately denoted by 'I_n'', so that is said to be the *syntactic rule of CCFCL interpretation of* A_n.

a) Any one of the following *lexigraphs* (*atomic logographs*), i.e. the interior of any one of the following so-called *homolographic*, or *photographic*, *autonymous quotations* (*HAQ's*), which is conventionally mentioned by using its HAQ (see the subsection 2.4 for greater detail):

$$\text{`}u\text{' to `}z\text{', `}u_1\text{' to `}z_1\text{', `}u_2\text{' to `}z_2\text{', etc,} \tag{2.28}$$

is a *logographic variable* that is called a *variable*, or redundantly *atomic variable*, *catlogographic ordinary term* (briefly *VCLOT* or *AVCLOT*).

b) Either one of the *lexigraphs* '∅' and '∅′', i.e. again either one of the logographs therein depicted between light-faced single quotation marks, is a *logographic constant* that is called a *constant*, or redundantly *atomic constant*, *catlogographic ordinary term* (briefly *CCLOT* or *ACCLOT*) and also, more explicitly, a *catlogographic ordinary zero*, or *empty, term* (*CLOZT* or *CLOET*). The CCLOT '∅' is the *systemic* CLOZT, whereas '∅′' is the *subsidiary* CLOZT that is introduced exclusively as the CFCL interpretand of $\mathtt{0}'$, so that ∅′=∅, in accordance with the item 14ix.

c) Any one of the lexigraphs:

$$\text{`}p\text{' to `}s\text{', `}p_1\text{' to `}s_1\text{', `}p_2\text{' to `}s_2\text{', etc} \tag{2.29}$$

is a *logographic variable*, to be called an *atomic*, or redundantly *atomic variable*, *catlogographic relation* (briefly *ACLR* or *AVCLR*).

d) A VCLOT (AVCLOT) or a CCLOT (ACCLOT) is indiscriminately called an *catlogographic*, or redundantly *atomic catlogographic*, *ordinary term* (briefly *CLOT* or *ACLOT*). An ACLOT (CLOT) or an ACLR (*AVCLR*) is indiscriminately called an *ordinary catlexigraph* or *atomic ordinary catlogograph* and also an *atomic catlogographic ordinary formula* (*ACLOF*) or *atomic catlogographic ordinary categorem*.

ii) Each one of the letters u to z and p to s is a homolograph of the Light-Faced Roman Arial Narrow Type, while each of the letters *u* to *z* and *p* to *s* is a homolograph of the *Light-Faced Italic Times New Roman Type*. At the same time, any one of the former ten letters and the respective one of the latter ten letters are *analographic* (*analogous graphic*) isotokens of each other. Like remarks apply, *mutatis mutandis*, to the pasigraphs $\emptyset$ and $\emptyset'$ on the one hand and $\varnothing$ and $\varnothing'$ on the other hand. Therefore, the substitutions (2.25)–(2.27), and any similar substitutions, are called *analo-homolographic* ones in the sense that in this case a *homolograph of one type is replaced with an analographic homolograph of another type*.

41) *The rules of conservative semantic interpretation of the CLOT's.*

i) Depending on a branch $A_{1=}$, $A_{1\subseteq}$, or $A_{1\in}$, in which the PVOT being the CFE interpretans of a given AVCLOT is employed, the range of the AVCLOT is the respective one of the following three *universals*:

a) a class comprising *nonempty individuals* if its CFE interpretans is employed in $A_{1=}$;

b) some mass comprising both *nonempty masses* and *the empty mass*, i.e. *the empty individual*, if its CFE interpretans is employed in $A_{1\subseteq}$;

c) a class comprising *elements*, i.e. both *nonempty classes* and *the empty class*, i.e. *the empty individual* again, if its CFE interpretans is employed in $A_{1\in}$.

In this case, the range of a VCLOT is said to be *designated by* the VCLOT or to be the *designatum* (*designation value*¸ pl. "*designata*") *of* the VCLOT. Consequently, a VCLOT can assume as its *accidental* (*circumstantial*) *denotatum* (*denotation value*¸ pl. "*denotata*") any *instance* of its range, namely: a *nonempty individual* in the case a), some *nonempty mass* or *the empty mass*, i.e. the *empty individual*, in the case b), and a *nonempty class* or *the empty class*, i.e. the *empty individual*, in the case c). Therefore, a VCLOT is called a *nonempty-individual-valued* one in the case a), a *mass-valued* one in the case b), and a *class-valued* one in the case c). Therefore, a VCLOT is called a *nonempty-individual-valued* one in the case a), a *mass-valued* one in the case b), and a *class-valued* one in the case c). In each one of the above three cases a)–c), separate accidental denotata of a VCLOT have the following properties.

a′) A nonempty individual has neither elements nor parts and it cannot be predicated of any other substance.

b′) Mass has no elements, but it has the empty part (empty submass) and it also has nonempty parts (nonempty submasses) if it is nonempty itself. Mass can be predicated of some other masses.

c′) A nonempty class has both *elements* (*members*) and *parts* (*nonempty subclasses* and *the empty subclass*), and it can be *predicated* of some other classes, nonempty ones and the empty one. The empty class has no elements (no members), but it is a part (subclass) of itself and it can therefore be predicated of another class by stating that the latter is the empty one.

Hence, the *universals of three kinds*, indicated in the above points a)–c) are *incomparable* (*not intersecting*). Accordingly, just as the predicate-sign $\subseteq$, a PCOV, which is employed in $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, and $\mathsf{A}_{1\in}$, is *a three-fold homograph*, i.e. *a euautograph that has three different hypostases*. In this connection, it is noteworthy that in the English translations of Aristotle [350 BCE, *Categories*] by Edghill (referred to as [ACE]) and by Owen (referred to as [ACO]) and in studies of that treatise (e.g., in Studtmann [2008]), the terms "*primary substances*" and "*secondary substances*" are used for denoting the *entities* (*beings* – τᾶ ὄντα \tá ónta\, singular "τό ὀντότης" \tó ontótis\ *s. f.*), which are respectively called "*nonempty individuals*" and "*classes*" in the presently common terminology. At the same time, in Aristotelianism, immediate classes of nonempty individuals are called *species*, whereas a *superclass* (*whole*) *of a species* is called a *genus* (see, e.g., [ACE, Part 5] or [ACO, Chapter V]). Since A_1 allows distinguishing formally (axiomatically) between classes (including sets) and masses, therefore, I divide the Aristotelian subcategory of *secondary substances* into two distinct narrower subcategories: *classes* and *masses*. Any substance of a given domain of classes, i.e. a *nonempty class* (particularly a *nonempty set*), the *empty class* (being at the same time the *empty set*), i.e. the empty *individual*, or a *nonempty individual* is indiscriminately called an *element of the domain*. Thus, a nonempty individual is a primary substance and vice versa, whereas a class or some mass, nonempty or empty, is a secondary substance. A nonempty individual can be an element, i.e. a member, of a class, but it cannot be a *subclass*, i.e. a part, of a class.

ii) The entity (object) $\varnothing$ or $\varnothing'$, denoted by the respective ACCLOT '$\varnothing$' or '$\varnothing'$', is called:

a) the *empty mass* if the PCOT $\emptyset$ or $\emptyset'$, being the CFE interpretans of '$\varnothing$' or '$\varnothing'$' respectively, is employed in $\mathsf{A}_{1\subseteq}$ or $\mathsf{A}_{1\in}$;

b) the *empty class* if the PCOT $\emptyset$ or $\emptyset'$, being the CFE interpretans of '$\varnothing$' or '$\varnothing'$' respectively, is employed in $\mathsf{A}_{1\in}$.

The empty mass of a *domain* (*theory*) *of masses* or the empty class of a *domain* (*theory*) *of classes* is indiscriminately called the *indivisible empty substance* (*entity, object*) or briefly the *empty individual*. Still, the two domains are incompatible and hence their empty individuals are incompatible as well. Thus, just as the predicate-sign $\subseteq$, the PCOT $\emptyset$ or $\emptyset'$, which is employed in $\mathsf{A}_{1\subseteq}$ and $\mathsf{A}_{1\in}$, is *a two-fold homograph*, i.e. *a euautograph that has two different hypostases*.

iii) While the rules of substitutions (2.25)–(2.27) constituting I_1' are independent of a branch of A_1, the rules of semantic interpretations of VCLOT's and CCLOT's, which are stated in the points i and ii and which are comprised in I_1'', do depend on the branch. That is to say, I_1'' that is associated with $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, or $\mathsf{A}_{1\in}$ should more specifically be denoted by '$I_{1=}''$', '$I_{1\subseteq}''$', or '$I_{1\in}''$', the understanding being that $I_{1=}''$, $I_{1\subseteq}''$, or $I_{1\in}''$ should include the pertinent rule of semantic interpretation of the ACLR's, denoted by 'I_0''' (to be specified in the item 44), provided that the primary atomic basis of $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, or $\mathsf{A}_{1\in}$ involves the list (2.8). Consequently, I_1 that is associated with $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, or $\mathsf{A}_{1\in}$ should more specifically be denoted by '$I_{1=}$', '$I_{1\subseteq}$', or '$I_{1\in}$', defined as $I_{1=} \to I_{1=}'' \circ I_1'$, $I_{1\subseteq} \to I_{1\subseteq}'' \circ I_1'$, or $I_{1\in} \to I_{1\in}'' \circ I_1'$, and be called the *cumulative rule of CCFCL interpretation* [*of OptER's*] *of* $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, or $\mathsf{A}_{1\in}$, and also the *CCFCL interpretation* [*of OptER's*] *of* $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, or $\mathsf{A}_{1\in}$ *in intension*; and similarly with '$\mathsf{A}_{1=}^0$', '$\mathsf{A}_{1\subseteq}^0$', and '$\mathsf{A}_{1\in}^0$' in place of '$\mathsf{A}_{1=}$', '$\mathsf{A}_{1\subseteq}$', and '$\mathsf{A}_{1\in}$' respectively.

42) An OptER of A_1 that does not involve any AER of the list (2.8) is called an *output pure predicate euautographic relation* (*OptPPER*) *of* A_1 or an *input PPER* (*IptPPER*) *of* I_1, whereas its *CCFCL interpretand* is called an *output conservative catlogographic pure predicate relation* (*OptCCLPPR*) *of* I_1 or simply a *PPCCLR*. An OptPPER is a relation among EOT's, which is established by uniting the latter by means of some *euautographc syncategoremata* (*euautographic kernel-signs* and

punctuation marks) *of* A_1. In making the CCFCL interpretand of an OptPPER of A_1 (IptPPER of I_1), the rule I_1 does not affect any euautographs occurring in the latter other than EOT's. Therefore, the CCFCL interpretand of an OptPPER of A_1 is a *biune* PPCCLR, one *component* (*aspect*) of which is the relation among the pertinent *CLOT's* (*ACLOT's*), while the other one is the analogous relation among the *universals designated by the CLOT's*. The PPCCLR preserves the *validity-value of its CFE* (*conformal catlogographic*) *interpretans* and it is therefore a *vavn-decided PPCCLR*. In this case however, in order to express the fact that, in contrast to the latter, the former is *significant* (*biune*), I say that a PPCCLR is *tautologous* (*tautological*, *tautologously true*, *universally true*) if and only if its CFE interpretans and hence the PPCCLR itself is *valid* and that a PPCCLR is *neutral* (*indeterminate*) *with respect to the tautologousness-values tautologousness* (*universal truth*) *and antitautologousness* (*universal antitruth, universal falsehood, contradictoriness*), i.e. *neither tautologous nor antitatologous* (*nor contradictory*) – briefly *ttatt-neutral* (*ttatt-indeterminate*), if and only if its CFE interpretans and hence the PPCCLR itself is *vav-neutral* (*vav-indeterminate*). By definition, among the above PPCCLR's there are no antivalid ones. However, the negation of a valid and hence tautologous PPCCLR is by definition an *antivalid* and hence *antitautologous* (*antitautological*, *universally antitrue*, *universally false*, *contradictory*) *PPCCLR*, whereas the negation of a vav-neutral (vav-indeterminate) and hence ttatt-neutral (ttatt-indeterminate) PPCCLR is *another ttatt-neutral* (*ttatt-indeterminate*) *PPCCLR*. For instance, the following PPCCLR's are *tautologous* ones, i.e. *tautologies*:

a) $\bigvee_u [u = v]$ and $\bigvee_v [u = v]$, being the CCFCL interpretand of the [homographic] theorems $\bigvee_u [u = v]$ and $\bigvee_v [u = v]$ of $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, or $\mathsf{A}_{1\in}$;

b) $\bigvee_u [u \subseteq v]$, $\bigvee_v [u \subseteq v]$, $\bigvee_u \neg[u \subset v]$, and $\bigvee_v \neg[u \subset v]$, being the CCFCL interpretands of the [homographic] theorems $\bigvee_u [u \subseteq v]$, $\bigvee_v [u \subseteq v]$, $\bigvee_u \neg[u \subset v]$, and $\bigvee_v \neg[u \subset v]$ of $\mathsf{A}_{1\subseteq}$ or $\mathsf{A}_{1\in}$;

c) '$\bigvee_u \neg[u = v]$', being the CCFCL interpretand of the pertinent axiom $\bigvee_u \neg[u = v]$ of $\mathsf{A}_{1=}$ and of the homographic theorems of $\mathsf{A}_{1\subseteq}$, and $\mathsf{A}_{1\in}$;

d)'$\bigvee_u \neg[u \subseteq v]$', being the CCFCL interpretand of the pertinent axiom $\bigvee_u \neg[u \subseteq v]$ of $\mathsf{A}_{1\subseteq}$ and of the homographic theorem of $\mathsf{A}_{1\in}$.

All the above-mentioned theorems of $A_{1=}$, $A_{1\subseteq}$, or $A_{1\in}$ are proved by the pertinent EADP's of A_1, At the same time, the tautologous character of the PPCCLR's occurring in the above points a and b is evident because in these three cases the operata of '$\vee_u$' can particularized as '$[v=v]$', '$[v\subseteq v]$', and '$\neg[v\subset v]$', whereas the operata of '$\vee_v$' can particularized as '$[u=u]$', '$[u\subseteq u]$', and '$\neg[u\subset u]$', respectively. Since $\wedge_* \to \neg\vee_*\neg$, i.e. $(\forall *)\to\neg(\exists *)\neg$ in the conventional notation, therefore the negation of any tautologous PPCCLR, being an *antitautology* (*contradiction*) and the negation of any *kyrologous* (*valid*) *ER*, being an *antikyrology* (*antivalid ER*), can equivalently be rewritten with $\wedge_*\neg$ in place of $\neg\vee_*$ while all occurrence of $\neg\neg$ can be omitted.

43) Thus, in addition to or instead of its *inherent validity-value validity*, *antivalidity*, or *vav-neutrality* (*vav-indeterminacy*), a PPCCLR assumes exactly one respective *tautologousness-value*s: *tautologousness* (*universal truth*, *tautologous truth*) or *antitautologousness* (*universal antitruth*, *contradictoriness*) or *ttatt-neutrality* (*neither tautologousness nor antitatologousness*), which is *inclusive of* and is, hence, *compatible with its validity-value*. The above division of the PPCCLR's into three classes: the tautologous ones, the antitautologous ones, and the ttatt-neutral (ttatt-indeterminate) ones is called the *specific primary* (or *specific basic*) *decisional trichotomy* (*trisection*, *trifurcation*) *of the PPCCLR's* (cf. the item 8). A PPCCLR is said to be: (a) *atautologous* if it is antitautologous or ttatt-neutral, (b) *non-antitautologous* if it is tautologous or ttatt-neutral, (c) *ttatt-unnutral* if it is tautologous or antitautologous. Consequently, there are three *specific secondary* (or *specific subsidiary*) *decisional dichotomies* (*bisections*, *bifurcations*) *of the PPCCLR*'s into: (a′) the *tautologous* ones and *atautologous* ones, (b′) the *antitautologous* ones and *non-antitautologous* ones, (c′) the *ttatt-neutral* ones and the *ttatt-unneutral* ones (cf. the item 10).

44) *The rules of conservative semantic interpretations of ACLR's and taxonomies of the CCLR's.* Since every AER of the list (2.8) is *vav-neutral* (*vav-indeterminate*), therefore any ACLR of the list (2.29) that serves as the CCFCL interpretand of the respective AER is also vav-neutral (vav-indeterminate), so that it is postulated to be *ttatt-neutral* (*ttatt-indeterminate*), unless stated otherwise. Consequently, the trichotomy and three dichotomies of CCLPPR's, which have been

established in the previous item, apply to *all CCLR's*. In this connection, it is noteworthy that isotokens of any AER of the list (2.8) that occur either alone or as constituent parts of some IptER's in the different branches $I_{1=}$, $I_{1\subseteq}$, and $I_{1\in}$, are homographs that are replaceable with *vav-neutral* IptEPPER's of the same branch unconditionally in the former case or under certain conditions of *well-formedness* in the latter case. Accordingly, isotokens of any ACLR of the list (2.29) that occur either alone or as constituent parts of some OptCCLR's in the different branches $I_{1=}$, $I_{1\subseteq}$, and $I_{1\in}$, are homographs that are interpretable *physically* by replacing them with *ttatt-neutral OptPPCCLR's* of the same branch unconditionally in the former case or under the pertinent conditions of well-formedness in the latter case.

45) In accordance with the items 29–35, $A_{1=}$, $A_{1\subseteq}$, and $A_{1\in}$ are three different calculi, which have the same AADM, D_1, and which are therefore conveniently treated simultaneously as a single whole calculus, A_1, in terms of common (but homographic) nomenclature. However, such a treatment becomes impossible at the stage of the CCFCL interpretations of $A_{1=}$, $A_{1\subseteq}$, and $A_{1\in}$, because the CCFCL interpretands of the three calculi are *incompatible* and hence *incomparable*. I regard $A_{1=}$, $A_{1\subseteq}$, and $A_{1\in}$ along with their CCFCL interpretations as underlying logical calculi of the *prospective relevant full-scale semantic theories*. Namely, $A_{1=}$ is the underlying logical calculus of a prospective *theory of nonempty individuals*; $A_{1\subseteq}$ is the underlying logical calculus of a prospective *theory of masses*, which should unavoidably be a *one-individual theory*, because it may involve the *empty mass*, i.e. the *empty individual*, as its only individual; $A_{1\in}$ is the underlying logical calculus of a prospective *theory of classes*, which should unavoidably also be a *one-individual theory*, because it may involve the *empty class*, i.e. the *empty individual*, as its only individual. In this case, a *many-individual class theory* cannot in principle be derived either from $A_{1\subseteq}$ or from $A_{1\in}$ because, in accordance with the primary formation rules of A_1, common to $A_{1\subseteq}$ and $A_{1\in}$, any of its EOT's can stand in ER's to the right of either predicate-sign $\subseteq$ or $\in$ and not only to the left of it. Therefore, the identity of two nonempty individuals cannot be stated with the help of the sign =, which is defined in terms of the sign $\subseteq$, either as a primary (axiomatic) one or as a secondary one that is defined in terms of $\in$. Incidentally, all existing *many-individual class theories* and particularly *many-individual set theories* are *verbal* (*phonographic*) ones

and not *logographic* (see, e.g., Fraenkel *et al* [1973, pp. 24–25]). This fact can be explained as follows. Since $\emptyset$ and $\emptyset'$ can stand in ER's of $\mathsf{A}_{1\in}$ to the right of the predicate-sign $\in$, therefore the property of emptiness of $\emptyset$ and $\emptyset'$ and hence the property of their individuality (indivisibility) can be expressed by negating the relations $[\mathbf{x} \in \emptyset]$ and $[\mathbf{y} \in \emptyset']$ for all possible EOT's $\mathbf{x}$ and $\mathbf{y}$ of $\mathsf{A}_{1\in}$, i.e. by asserting '$\neg[\mathbf{x} \in \emptyset]$' and '$\neg[\mathbf{y} \in \emptyset']$' (e.g.) as panlogographic axiom schemata of $\mathsf{A}_{1\in}$, which imply that $\emptyset = \emptyset'$ and that $[\emptyset \subseteq \mathbf{x}]$. By contrast, a name of a nonempty individual is prohibited to stand to the right of the predicate-sign $\in$, while there is no euautographic predicate in $\mathsf{A}_{1\in}$ to be a parasynonym of the verbal predicate "*is not a class*". Therefore, to introduce nonempty individuals into a formal (logographic) *axiomatic class theory* (*ACT*), there is no way other than a verbal one.

46) The *CFCL pre-interpretation of the atomic euautographic ordinary formulas of* A_1. The PVOT's on the list (2.7), the AER's (APVR's) on the list (2.8), and the PCOT's (APCOT's) $\emptyset$ and $\emptyset'$ of the item 14ix are collectively called the *atomic euautographic ordinary formulas* (*AEOF's*), or *atomic euautographic ordinary categoremata, of* A_1. In view of the above CCFCL interpretation of the AEOF's, the latter formulas can *a posteriori* be defined *synonymously* by the following set of *asymmetric synonymic definitions* (*ASD's*):

$$u \to `u\text{' to } z \to `z\text{'},\ u_1 \to `u_1\text{' to } z_1 \to `z_1\text{'},\ u_2 \to `u_2\text{' to } z_2 \to `z_2\text{', etc;} \quad (2.30)$$

$$\emptyset \to `\varnothing\text{'},\ \emptyset' \to `\varnothing'\text{'}, \quad (2.31)$$

$$p \to `p\text{' to } s \to `s\text{'},\ p_1 \to `p_1\text{' to } s_1 \to `s_1\text{'},\ s_2 \to `s_2\text{' to } s_2 \to `s_2\text{', etc;} \quad (2.32)$$

where *the definientia are the HAQ's* (*homoloautographic quotations*), *and not their interiors*, *being their denotata*. The above ASD's are called the *CFCL pre-interpretations of the AEOF's*, being their synonymous definienda, while the definientia of the ASD's are alternatively called the *CFCL pre-interpretands of the definienda*. That is to say, just as the definienda, their *synonymous definientia*, i.e. the respective HAQ's, are used *autonymously*. In the general case, however, I use an HAQ as a common name, i.e. as a *xeno*graph (graphic *xeno*nym), whose range is the *homolographic* (*photographic*) *isotoken-class*, i.e. the *class of homolographic* (*photographic*, *proportional* or particularly *congruent*) *isotokens*, *of its interior*. Accordingly, without any added words, an HAQ is used in a natural *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I mentally experience and mention the range of the HAQ as *my as if extramental* (*exopsychical*) *object that represents the whole homolographic isotoken-class.* This object is called a *common* (*general*, *certain*, *particular but not particularized*) *member* of that class. Particularly, the HAQ's on the lists (2.28) and (2.29) are used in the above projective mental mode. The ranges of the catlexigraphs occurring in the definientia in (2.30)–(2.32) depend on a branch of A_1, to which their CFE interpretantia (definienda) belong. However, the AEOF's as defined by (2.30)–(2.32) *are irrelevant to the ranges of the interiors of their definientia*. Definitions (2.30)–(2.32) are not used either in the setup of A_1 or in the EADP's (euautographic algebraic decision procedures) for ER's of A_1, but they make explicit all peculiarities of A_1 as a *euautographic* calculus, particularly those outlined in the item 37. Particularly, owing to definitions (2.30)–(2.32), all euautographs, atomic and combined, are always used autonymously. Accordingly, all euautographs are in a sense *constants*. I classify the atomic euautographs on the lists (2.7) and (2.8) as *pseudo*-variables and $\emptyset$ and $\emptyset'$ as *pseudo*-constants in accordance with classification of their CCFCL interpretands as variables and as constants respectively.

47) The digits 0 and 1, as introduced in the item 14xii, are PAEI's, i.e. *primary atomic euautographic special terms* (*PAESpT*), and hence they are *primary atomic euautographic formulas* (*PAEF's*, like the AEOF's. 0 and 1 can be defined in analogy with (2.30)–(2.32) with the following proviso. When the digits 0 and 1 in this (current) type occur as subscripts, they are used autonymously like 0 and 1. Assuming therefore that '*0*' and '*1*' in this type are logographic numerals that are especially designed to be primarily used xenonymously for denoting the respective natural numbers, 0 and 1 can be defined as:

$$0 \to \text{'}\mathit{0}\text{'},\ 1 \to \text{'}\mathit{1}\text{'}. \tag{2.33}$$

2.5. The progressive conformal catlogographic interpretation of A_1

48) Let '$\mathbf{P}_{+}$' be a *panlogographic placeholder* (*PLPH*) whose range is the class $\mathsf{R}^{\mathrm{O}}_{1+}$ of *valid* IptEOR's of I_1 (OptEOR's of A_1) and let '$\mathbf{P}_{\sim}$' be a PLPH whose range is the class $\mathsf{R}^{\mathrm{O}}_{1\sim}$ of *vav-neutral* IptEOR's of I_1 (OptEOR's of A_1), whereas '$\mathsf{T}_{1\sim}(\mathbf{P}_{\sim})$' is a PLPH whose range is the class $\mathsf{R}^{\mathrm{O}\sim}_{1\oplus}$ of *EMT's* (*EDT's*) *for all* $\mathbf{P}_{\sim} \in \mathsf{R}^{\mathrm{O}}_{1\sim}$, i.e. *for all vav-neutral IptEOR's of* I_1, the understanding being that such an EMT is an *IptESpR of* I_1 (*OptESpR of* A_1) and vice versa. Let '$\boldsymbol{P}_{+}$', '$\boldsymbol{P}_{\sim}$', and '$\mathsf{T}_{1\sim}(\boldsymbol{P}_{\sim})$' be

pancatlogographic placeholders (*PCLPH's*) for the respective *OptCCLR's of* I_1 in the classes R^{O}_{1+}, $R^{\mathrm{O}}_{1\sim}$, and $R^{\mathrm{O}\sim}_{1\oplus}$ respectively, so that

$$\boldsymbol{P}_{+} = I_1(\mathbf{P}_{+}), \boldsymbol{P}_{\sim} = I_1(\mathbf{P}_{\sim}), \mathsf{T}_{1\sim}(\boldsymbol{P}_{\sim}) = I_1(\mathsf{T}_{1\sim}(\mathbf{P}_{\sim})) = \mathsf{T}_{1\sim}(I_1(\mathbf{P}_{\sim})) \tag{2.34}$$

subject to $\mathsf{T}_{1\sim}(\mathbf{P}_{\sim}) \to [V(\mathbf{P}_{\sim}) \hat{=} \mathbf{i}|\mathbf{P}_{\sim}\rangle]$ and $\mathsf{T}_{1\sim}(\boldsymbol{P}_{\sim}) \to [V(\boldsymbol{P}_{\sim}) \hat{=} \mathbf{i}|\boldsymbol{P}_{\sim}\rangle]$.

Let D_1 be the CCFCL interpretand of D_1, defined by (2.1). The set of rules that is denoted by 'D_1' is called the *catlogographic advanced algebraic decision method* (*CLAADM*). Since $\boldsymbol{P}_{+}$ is by definition a tautologous and hence valid CCLR, therefore it cannot be modified either syntactically or semantically. By contrast, $\boldsymbol{P}_{\sim}$ is a ttatt-neutral CCLR and it can therefore either remain *unattended* (*suspended*) or be treated in one of the following two alternative ways.

i) $\boldsymbol{P}_{\sim}$ can *be postulated to be veracious* (*atautologously*, or *accidentally*, *true*). As a result, $\boldsymbol{P}_{\sim}$ turns into a *veracious catlogographic slave postulate,* $\boldsymbol{P}^{\mathrm{p}}_{\sim+}$, which satisfies the *progressive*, or *transformative, conformal catlogographic* (*PCFCL* or *TFCFCL*) *master postulate* $V(\boldsymbol{P}^{\mathrm{p}}_{\sim+}) \hat{=} 0$ instead of the *CCFCL MT* (*DT*) $\mathsf{T}_{1\sim}(\boldsymbol{P}^{\mathrm{p}}_{\sim+})$, i.e $V(\boldsymbol{P}^{\mathrm{p}}_{\sim+}) \hat{=} \mathbf{i}|\boldsymbol{P}^{\mathrm{p}}_{\sim+}\rangle$. A postulate $\boldsymbol{P}^{\mathrm{p}}_{\sim+}$ is, more specifically, called a *veracious catlogographic slave axiom,* $\boldsymbol{P}^{\mathrm{a}}_{\sim+}$, if it is a *permanent* slave postulate and a *veracious catlogographic slave hypothesis*, $\boldsymbol{P}^{\mathrm{h}}_{\sim+}$, if it is an *ad hoc* slave postulate.

ii) If $\mathsf{T}_{1\sim}(\boldsymbol{P}_{\sim})$, being the CCFCL MT (DT) for $\boldsymbol{P}_{\sim}$, contains as its constituent parts tokens (isotokens, occurrences) of some catlogographic slave postulates that have been laid down earlier then $\mathsf{T}_{1\sim}(\boldsymbol{P}_{\sim})$ can be developed further with the help of D_1 into a *catlogographic algebraic decision procedure* (*CLADP*) $D_1(\boldsymbol{P}_{\sim})$ *for* $\boldsymbol{P}_{\sim}$ as its *catlogographic slave-relation* (*CLSR*), or *catlogographic relation-slave* (*CLR-slave*). The CLADP $D_1(\boldsymbol{P}_{\sim})$ is similar to an EADP $\mathsf{D}_1(\mathbf{P})$, so that it terminates in a certain *progressive catlogographic master*, or *decision, theorem* (*PCLMT* or *PCLDT*) $T_1(\boldsymbol{P}_{\sim})$ *for* $\boldsymbol{P}_{\sim}$ of exactly one of the following three forms:

$$V(\boldsymbol{P}_{\sim}) \hat{=} \boldsymbol{i}_m|\boldsymbol{P}_{\sim}\rangle \hat{=} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \boldsymbol{i}|\boldsymbol{P}_{\sim}\rangle & \text{(c)} \end{cases},$$

which are similar to those given by the euautographic decision theorem (EDT) scheme (2.17) and which are therefore denoted by '$T_{1+}(\boldsymbol{P}_{\sim})$', '$T_{1-}(\boldsymbol{P}_{\sim})$', or '$T_{1\sim}(\boldsymbol{P}_{\sim})$'

respectively, in analogy with '$\mathsf{T}_{1+}(\mathbf{P})$', '$\mathsf{T}_{1-}(\mathbf{P})$', and '$\mathsf{T}_{1\sim}(\mathbf{P})$' introduced in the item 21vii. It is understood that the CLADP $D_1(\boldsymbol{P}_\sim)$ results in the CLMT (CLDT) of the respective one of three possible forms a–c is a development of the CCFCL MT (DT) $V(\boldsymbol{P}_\sim)\hat{=}\mathbf{i}|\boldsymbol{P}_\sim\rangle$ subject to $\boldsymbol{i}_n|\boldsymbol{P}_\sim\rangle\vec{\hat{=}}\mathbf{i}|\boldsymbol{P}_\sim\rangle\vec{\hat{=}}\mathbf{i}_n|\boldsymbol{P}_\sim\rangle$ and $m>n$.

49) In order to indicate that $T_{1*}(\boldsymbol{P}_\sim)$, i.e. $T_{1+}(\boldsymbol{P}_\sim)$, $T_{1-}(\boldsymbol{P}_\sim)$, or $T_{1\sim}(\boldsymbol{P}_\sim)$, is the pertinent catlogographic development of $\mathsf{T}_{1\sim}(\boldsymbol{P}_\sim)$, all occurrences of the validity-operator V throughout $T_{1*}(\boldsymbol{P}_\sim)$ can be (but is not recommended to be) replaced with occurrences of the *CFCL veracity-operator V*, which has exactly the same properties, and then the pertinent CLADP $D_1(\boldsymbol{P}_\sim)$ is performed with V in place of V. Formally, the analo-homolographic substitution

$$V \mapsto V \tag{2.35}$$

can in this case be included as an additional rule in I_1' and hence in I_1, which has been used in the item 48.

50) In analogy with the decisional terminology introduced in the item 21x, a *ttatt-neutral CCLR* $\boldsymbol{P}_\sim$ is said to be (a) *veracious* (*atautologously*, or *accidentally*, *true*), and it is denoted by '$\boldsymbol{P}_{\sim+}$', if it either is $\boldsymbol{P}^{\mathrm{p}}_{\sim+}$ or if its PCLDT is $T_{1+}(\boldsymbol{P}_\sim)$; (b) *antiveracious* (*accidentally antitrue*), and it is denoted by '$\boldsymbol{P}_{\sim-}$', if its PCLDT is $T_{1-}(\boldsymbol{P}_\sim)$; (c) *vravr-neutral* (or *vravr-indeterminate*), i.e. *neutral* (or *indeterminate*) *with respect to the veracity-values veracity and antiveracity* or, in other words, *neither veracious nor antiveracious*, and it is denoted by '$\boldsymbol{P}_{\sim\sim}$', if its PCLDT is $T_{1\sim}(\boldsymbol{P}_\sim)$. A ttatt-neutral CCLR $\boldsymbol{P}_{\sim+}$, which is proved to be veracious, is called a *veracious catlogographic slavetheorem*.

51) By definition, "veracious" and "antiveracious" mean *accidentally true* and *accidentally antitrue* (*accidentally false*) – in contrast to *universally true* (*tautologously true*, *tautologous*) and *universally antitrue* (*tautologously anitrue*, *tautologously false*, *antitautologous*, *contradictory*), respectively. Accordingly, "veracity" and "antiveracity" mean *accidental truth* and *accidental antitruth* (*accidentally falsehood*) – in contrast to *universal truth* (*tautologous truth*, *tautologousness*) and *universally antitruth* (*tautologous anitruth*, *tautologous falsehood*, *antitautologousness*, *contradictoriness*), respectively. It is understood that

the negation of a veracious relation is an antiveracious relation and vice versa, whereas the negation of a vravr-relation is another a vravr-relation.

52) Once a *ttatt-neutral CCLR*, i.e. *ttatt-neutral OptCCLR's of* I_1, $\boldsymbol{P}_{\sim}$ is provided with any one of the above three veracity-values, it turns into its own *homograph* that is called a *progressive CLR* (*PCLR*), while the name "*catlogographic relation*" ("*CLR*") *without either prepositive qualifier* "*conservative*" ("*C*") *or* "*progressive*" ("*P*") *equivocally applies to both homographs*. Thus, a PCLR is syntactically indistinguishable from the *CCLR* being its *predecessor*. A PCLR $\boldsymbol{P}_{\sim}$ is more specifically denoted by **'** $\boldsymbol{P}_{\sim *}$ **'** and is alternatively called a *vravrn-decided ttatt-neutral CLR.* A totality (set) of compatible (mutually consistent) catlogographic postulates of A_1 along with all PCLMT's (PCLDT's) that can be proved from those postulates with the help of D_1 and also along with the *catlogographic slave relations* (*CLSR's*) of the PCLMT's is called a *progressive CFCL* (*PCFCL*) *interpretation of* A_1 [*in extension*]. The class of PCLR's, being the result of a PCFCL interpretation of A_1, is called a *PCFCL interpretand of* A_1. The division of the PCLR's into three classes as indicated in the item 50, namely veracious, antiveracious, and vravr-neutral (vravr-indeterminate), is called the *primary*, or *basic*, *decisional trichotomy* (*trisection*, *trifurcation*) *of the PCLR's.* At the same time, a PCLR is said to be: *unveracious* if is antiveracious or vravr-neutral, *non-antiveracious* if it is veracious or vravr-neutral, and *vravr-unnutral* or *vravr-determinate* if it is veracious or antiveracious. Consequently, there are three *secondary*, or *subsidiary*, *decisional dichotomies* (*bisections*, *bifurcations*) *of the PCLR*'s:

a) the *veracious* ones and the *unveracious* ones,

b) the *antiveracious* ones and the *non-antiveracious* ones,

c) the *vravr-neutral* (*vravr-indeterminate*) ones and the *vravr-unneutral* (*vravr-determinate*) ones.

53) All PCFCL interpretations of A_1 have the class of ttatt-neutral OptCCLR's of I_1 as their common source of *vravrn-decided CCLR's* and they also have the CLAADM D_1 as their common ADM. Therefore, the PCFCL interpretations of A_1 form a *single whole interpreted logistic system*, i.e. a *formalized language*, which is denoted by 'A_1' and which is called the *Comprehensive Catlogographic Algebraico-Predicate Organon* (*CCLAPO*) or the *Comprehensive Catlogographic Advanced Algebraico-Logical Organon* (*CCLAALO*). In this case, in accordance with the items

48–52, $\mathcal{A}_1$ has no formation, no transformation (inference), and no decision rules other than those comprised in $\mathcal{D}_1$. Particularly, by the item 48, some selective ttatt-neutral OptCCLR's of I_1 are used as *input CCLR's* (*IptCCLR's*) *of* $\mathcal{A}_1$, whereas the CLAADM $\mathcal{D}_1$ of $\mathcal{A}_1$ is, by (2.1), the CCFCL interpretand of the EAADM D_1 of A_1. Therefore, $\mathcal{D}_1$ can alternatively be called the *PCFCL interpretation of* A_1 *in intension*, the understanding being that this is unique. At the same time, there is an indefinite number of *PCFCL interpretations of* A_1 *in extension*, each of which is accomplished within $\mathcal{A}_1$. In this case, I_1 plays two interrelated roles: first, it is the *most immediate interpretational supplement to* A_1 and, second, it is the *interpretational interface between* A_1 *and* $\mathcal{A}_1$.

54) *The progressive conformal catlogographic interpretations of* A_1^0 *and* A_0. It goes without saying that $\mathcal{A}_1$ contains as its *autonomous* (*self-subsistent*) parts two organons, which are denoted by '$\mathcal{A}_1^0$' and '$\mathcal{A}_0$' and which stand to A_1^0 and A_0 via I_1^0 and I_0 respectively in the same interpretational relations as that, in which $\mathcal{A}_1$ stands to A_1 via I_1. Accordingly, $\mathcal{A}_1^0$, or $\mathcal{A}_0$, is called the *PCFCL interpretation of* A_1^0, or A_0, respectively. Alternatively, $\mathcal{A}_1^0$ is called the *Comprehensive Catlogographic Binder-Free*, or *Contractor-Free, Algebraico-Predicate Organon* (*CCLBFAPO* or *CCLCFAPO*) and also the *Comprehensive Catlogographic Rich Basic Algebraico-Logical Organon* (*CCLRBALO*), whereas $\mathcal{A}_0$ is called the *Catlogographic Predicate-Free*, or *Catlogographic* [*Depleted*] *Basic*, *Algebraico-Logical Organon* (*CLPFALO* or *CLDBALO* or *CLBALO*). $\mathcal{A}_1^0$ and $\mathcal{A}_0$ have ADM's, which are denoted by '$\mathcal{D}_1^0$' and '$\mathcal{D}_0$' and which are, in accordance with (2.1), related to D_1^0 and D_0 by (2.4). $\mathcal{D}_1^0$ is called the *Catlogographic Rich BADM* (*CLRBADM*) *of* $\mathcal{A}_1$ and $\mathcal{D}_0$ is called the *Catlogographic BADM* (*CLBADM*) *of* $\mathcal{A}_1$, in accordance with the corresponding names of D_1^0 and D_0 suggested in the item 6 of this section. Just as in the case of $\mathcal{A}_1$, a totality (set) of compatible (mutually consistent) catlogographic postulates of $\mathcal{A}_1^0$ or $\mathcal{A}_0$ along with all PCLMT's (PCLDT's) that can be proved from those postulates with the help of $\mathcal{D}_1^0$ or $\mathcal{D}_0$ and also along with the *catlogographic slave relations* (*CLSR's*) of the PCLMT's is called a *progressive CFCL* (*PCFCL*) *interpretation of* $\mathcal{A}_1^0$ or $\mathcal{A}_0$ [*in extension*] respectively. Hence, there is an indefinite number of PCFCL

interpretations of A_1^0 or A_0, each of which is accomplished within A_1^0 or A_0 respectively (cf. the item 53).

55) *Psychical interpretations of an ACLR by formal veracity-values.* All AER's on the list (2.8) are *vav-neutral* and *insignificant*, whereas all ACLR's on the list (2.29) are *ttatt-neutral*, i.e. *vav-neutral* and *significant*. Therefore, for instance, the semi-verbal statements:

i) "p is vav-neutral", "p is neither valid nor antivalid", "$\mathsf{p}\vee\mathsf{q}$ is vav-neutral", "$\mathsf{p}\vee\neg\mathsf{p}$ is valid", and "$\mathsf{p}\wedge\neg\mathsf{p}$ is antivalid"

are valid, whereas the statements

ii) "p is ttatt-neutral", "p is neither tautologous nor antitautologous", "$p\vee q$ is ttatt-neutral", "$p\vee\neg p$ is tautologous", and "$p\wedge\neg p$ is antitautologous"

are both valid and tautologous, so that all those statements are legitimate, so that they can be asserted (used assertively). By contrast, the statements:

iii) "Let p be valid", "Let p be antivalid", and "If p is valid then $\mathsf{p}\vee\mathsf{q}$ is valid",

and also the statements:

iv) "Let p be tautologous", "Let p be antitautologous", and "If p is tautologous then $p\vee q$ is tautologous"

are illegitimate (inapplicable, incorrect). At the same time, the variants of the statements on the list i) with 'p' and 'q' in place of 'p' and 'q' remain legitimate, whereas the like the variants of the statements of the list iii) and also the variants of the statements on the list iv) with 'p' and 'q' in place of 'p' and 'q' remain illegitimate. Also, I may not, for instance, render the relation $\neg\mathsf{p}$ in words either as "It is not the case that p" or as "It is false that p", because p is insignificant. For the same reason, any one of the statements:

v) "Let p be veracious", "Let p be antiveracious", "Let p be vravr-neutral", and "If p is veracious then $\mathsf{p}\vee\mathsf{q}$ is veracious",

and of their variants with "true" in place of "veracious" is illegitimate – just as illegitimate are the phrases "Let ♪ be true" and "Let ♫ be false". On the other hand, the variants of all statements on the list v) with 'p' and 'q' in place of 'p' and 'q', i.e.

vi) "Let p be veracious", "Let p be antiveracious", "Let p be vravr-neutral", and "If p is veracious then $p\vee q$ is veracious",

and also the statements:

vii) “Let *p* be true”, “Let *p* be antitrue”, “Let *p* be tat-neutral”, and “If *p* is true then $p \vee q$ is true”

are legitimate, because ‘*p*’ and ‘*q*’ are significant. However, since either ‘*p*’ or ‘*q*’ is not and cannot be assumed to be tautologous, i.e. universally true, therefore all occurrences of the word “true” in the statements on the list vii) should be understood as occurrences of the qualifier “accidentally true” and hence as occurrences of the qualifier “veracious”, in agreement with the list vi). Therefore, for avoidance of confusion, statements such as those on the list vii) should be avoided.

56) In accordance with the above item, any ACLR of the list (2.29), e.g. ‘*p*’, can be interpreted *psychically* (*mentally*) within A_0 and hence within A_1 by assigning to it exactly one of the three *formal* (*f*-) *veracity-values*: *f-veracity, f-antiveracity*, and *f-vravr-neutrality* (*f-vravr-indeterminacy*) in accordance with the following rules. *p* is said to be *formally* (*f*-)

a) *veracious* if it is a *catlogographic postulate* (*CLP*) *of* A_0 or a *catlogographic slave-theorem* (*CLST*) that is proved to be so from some other CLP’s of A_0;

b) *antiveracious* if it is a *CLST* that is proved to be so from some CLP’s of A_0;

c) *vravr-neutral* if it is irrelevant to any CLP’s of A_0.

A totality (set) of compatible (mutually consistent) catlogographic postulates of A_0 along with all CLMT’s (CLDT’s) that can be proved from those postulates with the help of D_0 and also along with the *catlogographic slave-relations* (*CLSR’s*) of the CLMT’s is called a *progressive CFCL* (*PCFCL*) *interpretation of* A_0 [*in extension*] and also a *basic* PCFCL interpretation of A_1 [in extension]. The class of PCLR’s, being the result of a PCFCL interpretation of A_0, is called a *PCFCL interpretand of* A_0 and also a *basic* PCFCL interpretand of A_1.

57) In spite of the fact that I classify A_1 and its autonomous parts A_1^0’ and A_0 as organons and provide them with long pretentious proper names, these are not full-scale effective logistic calculi, but rather they are weak (practically ineffective) semantic supplements to A_1, A_1^0, and A_0, which have with respect to the latter primarily illustrative academic interest. Particularly, A_1 illustrates that the class of *ttatt-neutral OptCCLR’s of* l_1 is the source of *mathematical catlogographic postulates* (*veracious catlogographic axioms* and *veracious catlogographic hypotheses*) and of *mathematical catlogographic theorems*. In addition, A_1 illustrates the difference

between a *tautologous* (*universally true*) relation and a *veracious* (*atautologously*, or *accidentally*, *true*) relation, which is necessarily a *ttatt-neutral* one. Hence, A_1 also illustrate the difference between a veracious relation and a true relation, which is either a tautologous one or a veracious ttatt-neutral one. The resources of A_1 provide the most general underlying concepts to allow distinguishing with complete rigor between masses and classes but they do not allow distinguishing between irregular (proper) classes and sets (regular, or small, classes). In order to develop a full-scale class, set, or mass theory, A_1 should be augmented by an *additional formation rule*, according to which to any given condition-relation $\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\dots,\boldsymbol{x}_n\rangle$ ('$\boldsymbol{P}$' is an atomic placeholder for a ttatt-neutral CLR, while each one of the logographs '$\boldsymbol{x}$', '$\boldsymbol{x}_1$', '$\boldsymbol{x}_2$', …, '$\boldsymbol{x}_n$' is an atomic placeholder for any *ACFCLT* mentioned in the item 2 of this section), there corresponds a class (particularly, set) or mass, which is denoted by '$\{\boldsymbol{x}|\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\dots,\boldsymbol{x}_n\rangle\}$'. This formation rule is in fact a *contextual definition* of the *operator of abstraction* { | }, which is called a *general builder of an ordinary term* and which allows prescinding a class, set, or mass $\{\boldsymbol{x}|\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\dots,\boldsymbol{x}_n\rangle\}$ (depending on the given theory) from the given condition-relation $\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\dots,\boldsymbol{x}_n\rangle$. All other operators that are used in a class or mass theory, – such operators, e.g., as the *binary operators* ∪, ∩, and **-** *of union*, *intersection*, and *difference of classes* or the *operator of aggregation* { , ,…, } *of elements* (*classes* or *sets*), called also a *concrete set-builder*, – can contextually be defined in terms of the operator { | }. For instance, in the case of classes or particularly sets,

$$\boldsymbol{x}_1 \cup \boldsymbol{x}_2 \to \{\boldsymbol{x}|\boldsymbol{x} \in \boldsymbol{x}_1 \text{ or } \boldsymbol{x} \in \boldsymbol{x}_2\},\ \boldsymbol{x}_1 \cap \boldsymbol{x}_2 \to \{\boldsymbol{x}|\boldsymbol{x} \in \boldsymbol{x}_1 \text{ and } \boldsymbol{x} \in \boldsymbol{x}_2\},$$
$$\boldsymbol{x}_1 - \boldsymbol{x}_2 \to \{\boldsymbol{x}|\boldsymbol{x} \in \boldsymbol{x}_1 \text{ and} \neg[\boldsymbol{x} \in \boldsymbol{x}_2]\},$$

whereas a *singleton,* i.e. a *one-member set,* and an *unordered pair*, i.e. an *unordered two-member set*, are conventionally defined as

$$\{\boldsymbol{x}_1\} \to \{\boldsymbol{x}|\boldsymbol{x} = \boldsymbol{x}_1\} \text{ and } \{\boldsymbol{x}_1, \boldsymbol{x}_2\} \to \{\boldsymbol{x}|\boldsymbol{x} = \boldsymbol{x}_1 \text{ or } \boldsymbol{x} = \boldsymbol{x}_2\}$$

respectively and then recursively

$$\{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_{n-1}, \boldsymbol{x}_n\} \to \{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_{n-1}\} \cup \{\boldsymbol{x}_n\} \text{ for each } n \geq 3.$$

Also, sets (but not irregular classes and not masses) should be allowed to be *domains of definitions* of various order relations and thus to become *ordered*. It is understood that all the above operators and all order relations should be subjected to or be introduced by the appropriate *semantic axioms along with the appropriate definitions*.

Therefore, a full-scale class, set, or mass theory cannot have any decision method after the manner of D_1 and D_1. Thus, my solution of the trial decision problem do not fulfill all enthusiastic expectations regarding would-be decisional proofs of mathematical theorems, which were unjustifiably associated with a hypothetical solution of the dual decision problem before the latter was proved to be unsolvable (see, e.g., the quotation of Suppes [1957, pp. 69–70] in the item 3 of section 1). However, the trial algebraic decision method is a powerful and simple tool for various three-fold classifications of all logical relations of practical or academic interest and of some mathematical relations and it is also an indispensable source of logical, mathematical, and linguistic wisdom.

58) A CLR, concervative or progressive, is said to be

a) *true* if it is either *tautologous* or *veracious*;

b) *antitrue* or *false* if it is either *antitautologous* (*contradictory*) or *antiveracious;*

c) *neutral* (*indeterminate*) *with respect to the truth*-values *truth and antitruth*, i.e. *neither true nor antitrue* – briefly *tat-neutral* (*tat-indeterminate*), if it is vravr-neutral (vravr-indeterminate).

The above division of the CLR's into three classes: the true CLR's, the antitrue (false) CLR's, and the tat-neutral (tat-indeterminate) CLR's is called the *general primary* (or *general basic*) *decisional trichotomy* (*trisection*, *trifurcation*) *of the CLR's*. A CLR is said to be: (a′) *untrue* if is antitrue (false) or tat-neutral, (b′) *non-antitrue* or *non-false* if it is true or tat-neutral, (c′) *tat-unnutral* (*tat-determinate*) if it is true or antitrue (false). Consequently, there are three *general secondary* (or *general subsidiary*) *decisional dichotomies* (*bisections, bifurcations*) *of the CLR*'s:

a″) the *true* ones and the *untrue* ones,

b″) the *antitrue* (*false*) ones and the *non-anttrue* (*false*) ones,

c″) the *tat-neutral* ones and the *tat-unneutral* (*tat-determiate*) ones.

59) The most essential decisional terminology can be recapitulated as follows.

i) The *trichotomal* qualifiers "valid", antivalid", and "vav-neutral" ("vav-indeterminate"), and also their *antonyms* (*dichotomal complements*) "invalid", "non-antivalid", and "vav-unneutral" ("vav-determinate") are *syntactic* characteristics of *graphic relations*, whereas the following qualifiers are *semantic* characteristics of graphic relations:

a) the *trichotomal* qualifiers “tautologous” (“tautological”), “antitautologous” (“antitautological”, “contradictory”), and “ttatt-neutral” (“ttatt-indeterminate”), and also their antonyms (dichotomal complements) “atautologous” (“atautological”), “non-antitautologous” (“non-antitautological”, “uncontradictory”), and “ttatt-unneutral” (“ttatt-determinate”);

b) the *trichotomal* qualifiers “veracious”, “antiveracious”, and “vravr-neutral” (“vravr-indetrminate”), and also their antonyms (dichotomal complements) “unveracious”, “non-antiveracious”, “vravr-unneutral” (“vravr-determinate”);

c) the *trichotomal* qualifiers “true”, “antitrue” (“false”), and “tat-neutral” (“tat-indeterminate”) and also their antonyms (dichotomal complements) “untrue”, “non-antitrue” (“non-false”), and “tat-unneutral” (“tat-determinate”).

ii) A like remark applies to the respective *decisional values* (*classes*) of graphic relations. Namely, the *trichotomal validity-values* validity, antivalidity, and vav-neutrality (vav-indeterminacy), and also their *dichotomal complements* invalidity, non-antivalidity, and vav-unneutrality (vav-determinacy) are *syntactic* attributes of graphic relations, whereas the following decision-values are *semantic* attributes of graphic relations:

a′) the *trichotomal tautologousness-values* tautologousness, antitautologousness (contradictoriness), and ttatt-neutrality (ttatt-indeterminacy), and also their *dichotomal complements* atautologousness, non-antitautologousness (uncontradictoriness), and ttatt-unneutrality (ttatt-determinacy);

b′) the *trichotomal veracity-values* veracity, antiveracity, and vravr-neutrality (vravr-indetrminacy), and also their *dichotomal complements* unveracity, non-antiveracity, and vravr-unneutrality (vravr-determinacy);

c′) the *trichotomal truth-values* truth, antitruth (falsehood), and tat-neutrality (tat-indeterminacy), and also their *dichotomal complements* untrueth, non-antitruth (non-falsehood), and tat-unneutrality (tat-determinacy).

All decision values that are *assigned to* (*acquired by*) *a CLR* in the result of the pertinent *ADP’s* (*algebraic decision procedures*) are qualified *formal*. It is understood that exactly one of the formal decision values of a CLR is necessarily syntactic, while at least one of them is necessarily semantic.

60) All *euautographic kernel-signs* (*EKS's*), i.e. *logical connectives* and *binder-signs* (*contractor-signs*), remain unaltered under both the conservative and progressive CFCL interpretations of A_1, except *V*, which is, under the latter interpretation, replaced with *V* in accordance with the additional rule (2.35), for avoidance of confusion. However, an isotoken of an EKS that occurs in a CFR applies to both its *syntactic operata* (sing. "*operatum*"), i.e. *operated catlogographic formulas* (*categoremata, terms* or *relations*) and to their *semantic significands*, i.e. to the mental entities that the operata signify. Therefore, in contrast to its euautographic prototype that is insignificant and that has *no phonic* (*spoken*, *verbal*) *paratokens*, its isotoken in question is significant and has phonic patatokens and it is therefore called *catlogographic kernel-sign* (*CLKS*). Accordingly, the CLKS's (but not their euautographic prototypes) are replaceable with their *wordy counterparts* (*verbal paratokens*), being their *connotative interpretands* and at the same time *denotative interpretantia*, namely:

‘$\neg$’ with "not" or "It is not the case that" or "It is not the true that",

‘$\vee$’ with "or" or "ior" ("inclusive or") or Latin "vel",

‘$\wedge$’with "and" or "&";

‘$\Rightarrow$’ with "if … then –" or "only if",

‘$\Leftarrow$’ with "if",

‘$\Leftrightarrow$’ with ‘if and only if" or "iff",

‘$\not\vee$’ or ‘$\overline{\vee}$’ with "neither … nor –",

‘$\not\wedge$’ or ‘$\overline{\wedge}$’ with "not both … and –",

‘$\overline{\Rightarrow}$’ with "but not", ‘$\overline{\Leftarrow}$’ with "not … but –",

‘$\overline{\Leftrightarrow}$’ with "either … or – but not both" or "xor" ("exclusive or") or Latin "auf",

‘$\bigvee_*$’ with "for some $*$:" or "for at least one $*$:" or "there exists at least one $*$ such that",

‘$\bigwedge_*$’ with "for all $*$:" or "for every $*$:",

‘$\breve{\bigvee}_*$’ with "for some but not all $*$:" or "for strictly some $*$:",

‘$\frown{\bigvee}_*^1$’ with "for at most one $*$:" or "there exists at most one $*$ such that",

‘$\bigvee_{*}^{1}$’ with “for exactly one ∗:” or “there exists exactly one ∗ such that”,

‘$\hat{\cdot}_{*}$...’ with “the product of … over ∗”.

It is understood that alike ellipses that occur in a group of synonymous operators should be replaced alike by the appropriate concrete operata. In view of the analogy that exists between the binary disjunction operator ‘∨’ and the existential quantifier ‘(∃∗)…’ and in view of the like analogy that exists between the binary conjunction operator ‘∧’ and the universal quantifier ‘(∀∗)…’, which are explicated in the treatise, I employ the kernel signs ‘$\bigvee_{*}$’ and ‘$\bigwedge_{*}$’ instead of ‘(∃∗)’ and ‘(∀∗)’ respectively.

2.6. Pseudo-Restricted EAPO’s versus Pseudo-Unrestricted ones

61) When PVOT’s of the list (2.7) are employed in $\mathsf{A}_{1\in}$, the AVCLOT’s of the list (2.28), being their CCFCL interpretands, are *counterparts*, i.e. either *tokens* or *tantamount variants*, of the respective conventional *unrestricted* atomic variables, which are employed in CALC’i, e.g. in Whitehead and Russell [1910; 1962, pp. 4, 5]. Accordingly, the PVOT’s are qualified *pseudo-unrestricted* or *pseudo-unconfined*, no matter in which branch, $\mathsf{A}_{1\subseteq}$ or $\mathsf{A}_{1\in}$, they are employed. In this connection, I explore the possibility of introducing into both $\mathsf{A}_{1\subseteq}$ and $\mathsf{A}_{1\in}$ a certain *primary atomic pseudo-constant extraordinary term* (*PAPCXOT*), U, which will be called the *atomic pseudo-constant universal term* (*APCUT*). The EAPO’s resulted by introducing U, along with the pertinent subject axioms, into $\mathsf{A}_{1\subseteq}$ and $\mathsf{A}_{1\in}$ is denoted logographically by ‘$\bar{\mathsf{A}}_{1\subseteq}$’ and ‘$\bar{\mathsf{A}}_{1\in}$’ and is called verbally (phonographically) the *Pseudo-Restricted*, or *Pseudo-Confined*, *PMsEAPO* and the *Pseudo-Restricted*, or *Pseudo-Confined*, *PCsEAPO* respectively.

62) U does not belong to either of the two EAPO’s $\mathsf{A}_{1\subseteq}$ and $\mathsf{A}_{1\in}$. Therefore, U is introduced into each of the two EAPO’a by *separate formation rules*, according to which the new binary euautographic extraordinary relations are formed by placing a token of U on either side of ⊆, or ∈, and hence on either side of any other pedicate-sign that is defined in terms of ⊆ or ∈ respectively, while the other side of a predicate-sign is occupied by **x** or by another token of U; ‘**x**’ is a PLPH for any EOT including any PVOT of the list (2.7) and also including either PCOT $\emptyset$ or $\emptyset'$ of the item 14ix. In order to express the property of universality of U, the new binary ER’s $U\subseteq U$, $\mathbf{x}\subseteq U$, and $\neg[U\subseteq\mathbf{x}]$ are taken for granted as specific (atypical) subject axioms of $\bar{\mathsf{A}}_{1\subseteq}$,

whereas the new binary ER's $\neg[U \in U]$, $\mathbf{x} \in U$, and $\neg[U \in \mathbf{x}]$ are taken for granted as specific (atypical) subject axioms of $\bar{A}_{1\in}$. Accordingly, U is alternatively called the *universal pseudo-mass* if it is employed in $\bar{A}_{1\subseteq}$ and the *universal pseudo-class* if it is employed in $\bar{A}_{1\in}$. Some other new ER's of academic or practical interest that involve U are proved to be valid or antivalid either with the help of D_1 or with the help of the pertinent new rules of inference and decision. Along with these rules, D_1 is denoted by '$\overline{D_1}$'. Just as the EF's of $A_{1\subseteq}$ or $A_{1\in}$, the EF's of $\bar{A}_{1\subseteq}$ or $\bar{A}_{1\in}$ are divided into *ordinary* ones and *special* ones with the proviso that an EF is qualified either an *extraordinary* one or an *extraspecial* one if it is a variant respectively of an ordinary EF or of a special EF of $A_{1\subseteq}$ or $A_{1\in}$, in which at least one occurrence of $\mathbf{x}$ is replaced with an occurrence of U. A certain part of this IML (this treatise), with the help of which and within which $\bar{A}_{1\subseteq}$, or $\bar{A}_{1\in}$, is developed (set up and executed) is called the *Pseudo-Restricted EPMsT*, or the *Pseudo-Restricted EPCsT*, respectively (cf. the items 32 and 33). The CCFCL interpretand of U is denoted by '*U*' and be called the *universal mass* if U is the *universal pseudo-mass* and the *universal class* if U is the *universal pseudo-class*. Since $\bar{A}_{1\subseteq}$, or $\bar{A}_{1\in}$ have the same ADM, $\overline{D_1}$¸ therefore the can be regarded as branches of a single *comprehensive EAPO* (*CEAPO*), which is called the *Pseudo-Unrestricted CEAPO*.

63) The organons $A_{1\subseteq}$ and $A_{1\in}$ are branches of A_1 and therefore they are treated systematically. By contrast, the organons $\bar{A}_{1\subseteq}$ and $\bar{A}_{1\in}$ are minor digressions from $A_{1\subseteq}$ and $A_{1\in}$, which are neither branches nor phases of A_1, The former include the latter in such a way that all axioms and all theorems of $A_{1\subseteq}$ and $A_{1\in}$ retain in $\bar{A}_{1\subseteq}$ and $\bar{A}_{1\in}$ and do not interfere with any additional axioms and theorems involving U. Therefore, $\bar{A}_{1\subseteq}$ and $\bar{A}_{1\in}$ are treated fragmentarily in order to emphasize only the aspects, by which they differ from $A_{1\subseteq}$ and $A_{1\in}$. $\bar{A}_{1\subseteq}$ and $\bar{A}_{1\in}$ demonstrate that $A_{1\subseteq}$ and $A_{1\in}$ are self-consistent and that the fact that the latter two are treated as pseudo-unrestricted organons does not lead to any loss of generality, while essentially simplifying their structure.

2.7. Interpretations of CLR's by declarative sentences

2.7.1. Rational declarative sentences versus paralogous ones

64) In the context of a *natural domain* such as a modern field of study and discourse or a concrete modern scientific treatise, a *declarative sentence* (*DS*) and *its negation* are said to be:

a) *paralogous* (*bare, mere, unsubstantial*, *absurd*, *paradoxical*) *DS's* and also, more generally, *paralogies* if their *designata* are the *empty class* and if hence the notions of *truth* and *antitruth* (*falsity*) *are not applicable to them*;

b) *rational ones* (*RDS's*) if the *designatum* of at least one of them is a *nonempty class* so that the notions of *truth* and *antitruth* (*falsity*) *are applicable to them*.

That is to say, the negation of a paralogous DS is another paralogous DS, whereas the negation of an RDS is another RDS. Etymologically, the adjective "paralogous" and the noun "paralogy" are derived from the Greek adjective "παράλογος" \paráloγos\ meaning *unreasonable* or *absurd*. Particularly, a DS is paralogous if it contains, as one of its congruent parts, a *substantive,* i.e. a *noun* or *noun equivalent,* which either belongs to a *supernatural domain,* – such a domain, e.g., as mythology, theology, or a concrete heroic or religious legend, or such as an antiquated geographic, political, or geopolitical reality or an antiquated scientific theory, – or which is just a *contradictio in adjecto*, or else, which has both peculiar properties. In the context of a natural domain, the *designatum* (*range*) of such a substantive is the *empty class*, so that it has no denotatum and is hence a paralogy. Here follow some examples of paralogous DS's:

> "A centaur is a mammal", "A centaur is not a mammal", "A centaur is a mammal or a centaur is not a mammal", "The present king of Israel is as wise as the king Solomon", "The present king of Israel is not as wise as the king Solomon", "Abraham Lincoln was the 16^{th} president of the USA in the years 1913–21", "Abraham Lincoln was not the 16^{th} president of the USA in the years 1913–21", "Abraham Lincoln was or was not 16^{th} president of the USA in the years 1913–21", "The capital of the USSR in AD2000 was in Europe", "The capital of the USSR in AD2000 was not in Europe", etc.

In the domain of *biology* and generally in the *natural universe*, the designatum of the count noun "centaur" is *empty* – in contrast to the domain of Greek mythology, where it is not empty. Therefore, the *limited* (*articled*) *common name* "a centaur" is a *contradictio in adjecto* that cannot be used in the *projective* (*polarized*) *mental mode* for denoting a common (general) member of its designatum because the latter has no members. Hence, “a centaur” has no denotatum and is therefore a paralogy. The nounal constructions: “the present king of Israel”, “the 16^{th} president of the USA in the years 1913–21”, and “the capital of the USSR in AD2000” are also *contradictiones in adjecto* and are therefore paralogies. Every sentence involving an occurrence of any one of the above paralogies, in which it is used but not mentioned, is paralogous as well, so that its designatum is empty. In contrast to the meaningless expression “the 16^{th} president of the USA in the years 1913–21”, any one of the expressions (e.g.): “the 16^{th} president of the USA”, “the 16^{th} president of the USA in the years 1861–65”, “the president of the USA in the years 1913–21”, and “the 28^{th} president of the USA in the years 1913–21” is meaningful. Therefore, any given one of the DS’s: “Abraham Lincoln was the 16^{th} president of the USA”, “Abraham Lincoln was the 16^{th} president of the USA in the years 1861–65”, “Woodrow Wilson was the president of the USA in the years 1913–21”, and “Woodrow Wilson was the 28^{th} president of the USA in the years 1913–21” is veracious, the negation of that DS is antiveracious, and the disjunction of the two DS’s is a tautologous DS. In connection with the expression “the capital of the USSR in AD2000” and the DS’s involving it, it will be recalled that on December 25, 1991, the USSR was officially dissolved and then consigned to oblivion by an agreement among the heads of its member republics to form Commonwealth of Independent States (CIS). Before that date, the name “the USSR” and hence the name “the capital of the USSR” were meaningful (had denotata) for approximately 70 years. In that time, the DS “The capital of the USSR is in Europe” was veracious (accidentally true), while the DS “The capital of the USSR is not in Europe” was antiveracious (accidentally antitrue, accidentally false). Nowadays, when the pertinent geopolitical state of affairs has changed, both names “the USSR” and hence the name “the capital of the USSR” has become bare and so are the all DS’s, in which those names are supposed to be used but not mentioned. In general, a syntactically congruous graphonym that contains another paralogous graphonym as its constituent part is also paralogous one. Particularly, a complex-coordinate (complex) or complex-subordinate (compound) DS is a paralogy if at least one of its clauses is a paralogy. Therefore, from the standpoint of semantic analysis, the DS: “A centaur is a mammal or a centaur is not a mammal”, e.g., is a paralogy, although it is, syntactically, a *valid relation*.

2.7.2. Interpretation of ACLR's by ttatt-neutral rational simple declarative affirmative sentences (RSDAS's)

65) Beyond the scope of both I_1 and A_1, every ACLR on the list (2.29) can be regarded as a catlogographic placeholder, whose range is a certain class of *ttatt-neutral* (*ttatt-indeterminate*), i.e. *neither tautologous nor contradictory*, *rational simple declarative affirmative sentences* (*RSDAS*) of written English or of another WNL, whose sense is relevant to a certain *natural domain.* In this case, a *sentential variant* of a *predicate-free CCLR* (*PFCCLR*) is a *rational declarative sentence* (*RDS*) that is obtained from the PFCCLR by replacements of occurrences (tokens) of ACLR's of the list (2.29) with occurrences of English RSDAS's in such a way that two occurrences of the same ACLR remain occurrences of the same RSDAS, and two occurrences of different ACLR's remain occurrences of different RSDAS's. A tautologous, antitautologous, ttatt-neutral (ttatt-indeterminate), atautologous, non-antitautologous, or ttatt-unneutral (ttatt-indeterminate) PFCCLR is more specifically called an *f*-one, whereas its sentential variant is more specifically called an *m*-one, where '*f*' and '*m*' are abbreviations for "*formally*" and "*materially*" respectively. Consequently, an ACLR is replaceable with (is interpretable by) any sentence of its range. In accordance with Simpson [1968, p. 547], the sense of a sentence is called a *sententia* (pl. "*sententiae*", adj. "*sentantious*"). Conversely, a sententia is the sense of a certain sentence or, perhaps, the sense that several different sentences either of the same or of different native languages (NL's) have in common. A sententia is qualified by the same modifiers as those qualifying the English sentence expressing the sententia. Particularly, a *rational simple declarative affirmative or negative* (*RSDA* or *RSDN*) *sententia* (briefly *RSDASa* or *RSDNSa*) is the sense of a *RSDA* or *RSDN sentence* (briefly *RSDAS* or *RSDNS*)respectively and vice versa.

66) If in a given spatio-temporal situation or universally the sense of a certain *rational simple declarative sentence* (*RSDS*) *conforms to* (*matches*) a certain psychophysical (physopsychical, physicopsychical) complex object of mine, which I know from another source, a linguistic one (e.g. from some other RSDS's stated earlier) or a nonlinguistic (e.g. by acquaintance from my sensorial experience), and which is called a *state of affairs* and also a *fact*, *case*, *relation*, *event*, *phenomenon*, *situation*, *circumstance*, etc, then that sentence is said to be *veracious*, or more precisely *materially veracious* (*m-veracious*), with respect to me in the given situation

or universally respectively – in contrast to a PCLR, which is said to be *formally veracious* (*f-veracious*). The absence of a certain state of affairs is another state of affairs. For instance, if the *assertive* sentence "It is raining" denotes the respective state of affairs here and now then the *assertive* sentence "It is not raining" may, there and now, denote another state of affairs, which is the absence of the former one. If in a given spatio-temporal situation or universally *the sense of the negation of an RSDS conforms to a certain fact* (*state of affairs*), while *the sense of the sentence itself contradicts that fact and does not conform to any other fact*, then that sentence is said to be *materially antiveracious* (*m-antiveracious*) in the given situation or universally respectively with respect to me. Hence, *the negation of an m-veracious sentence is an m-antiveracious sentence and vice versa*. If the sense of an RSDS *neither conforms to nor contradicts any known relevant fact* then that sentence is said to be *neither m-veracious nor m-antiveracious* and also to be *m-vravr-neutral* (*m-vravr-indeterminate*), but again in the given situation or universally with respect to me. Hence, *the negation of an m-vravr-neutral sentence is another m-vravr-neutral sentence*. For instance, if I do not know what are the weather conditions in Broadway of New York at this moment then either sentence "It is raining in Broadway" or "It is not raining in Broadway" is here and now m-vravr-neutral with respect to me. In the above definitions, the words "true" and "antitrue" can be used instead of "veracious" and "antiveracious" provided that the former are understood as abbreviations of the expressions "accidentally m-true" and "accidentally m-antitrue" respectively. An RSDS is said to be: (a) *m-unveracious* if it is m-antiveracious or m-vravr-neutral (m-vravr-indeterminate); (b) *m-non-antiveracious* if it is m-veracious or m-vravr-neutral (m-vravr-indeterminate); (c) *m-vravr-unnutral* or *m-vrav-determinate* if it is m-veracious or m-antiveracious. It follows from the above-said that if the sense of an RSDS conforms to a certain fact or if, on the contrary, it contradicts that fact and does not conform to any fact then the sentence itself does so.

67) The state of affairs (fact) denoted by an RDS is the *matter* of the sentence in contrast to its *form*. This fact has the following two implications.

a) The material property of an RSDS and of its sense to be m-veracious, m-antiveracious, or m-vravr-neutral (m-vravr-indeterminate) and also that to be m-unveracious, m-non-antiveracious, or m-vravr-unneutral (m-vravr-determinate) are *semantic matter-of-fact properties*, i.e. semantic properties that are concerned with facts and not imaginative or fanciful ones. Accordingly, the kindred substantives (noun equivalents) of the above adjectival qualifiers, namely “m-veracity”, “m-antiveracity”, “m-vravr-neutrality” (“m-vravr-indeterminacy”), “m-unveracity”, “m-non-antiveracity”, and “m-vravr-unneutrality” (“m-vravr-determinacy”) carry the abbreviation “m” for the prepositive adjectival qualifier “material” – as opposed to “formal” abbreviated as “f”.

b) The act of interpreting of an ACLR by a ttatt-neutral RSDAS (e.g.) belongs to *material logic* and is therefore beyond the scope of *formal logic*. The organon A_1, i.e. the union and superposition of A_1 and $\boldsymbol{\mathsf{A}}_1$, is *endo*semasiopasigraphic, i.e. pasigraphic (not wordy) and *semantically close*. The organon A_1 is pure syntactic and therefore it can also be regarded as endosemasiopasigraphic. By contrast, the union of A_1 and I_1 is an *exo*semasiopasigraphic, i.e. pasigraphic and *semantically open*, logistic system. If the ACLR’s occurring in this system are allowed to be replaced with RSDAS’s then this system turns into one that should be qualified as *exo*semasio*xeno*graphic.

68) When I use an m-veracious sentence for mentioning the state of affairs, to which it conforms, and thus turn the latter into the *intended import value* of the sentence, I say that the state of affairs *is denoted* by the sentence or that it is the *denotatum* (*denotation value*, pl. “*denotata*”), or *meaning*, of the sentence. By contrast, an m-unveracious, i.e. m-antiveracious or m-vravr-neutral, sentence denotes nothing, i.e. it has no denotatum, but rather it just expresses its own sense. An m-veracious sentence is alternatively called a *meaningful sentence*, whereas an m-unveracious sentence is alternatively called a *meaningless sentence*. In order to indicate that a sentence is m-veracious and that hence it denotes the pertinent state of affairs (fact), the sentence is put in a certain conventional or properly defined unconventional format, which is called an *assertive format*. Such an m-veracious sentence is said to be *asserted* or *assertive*. An m-veracious sentence is said to be *unasserted* or *unassertive* if it is *put* (*presented*, *exhibited*, *demonstrated*, *ostended*, *written* or *uttered*) *in an unassertive* (*not assertive*) *format within an assertive context*. An m-unveracious, i.e. m-antiveracious or m-vravr-neutral, sentence cannot be asserted (be put in an assertive format). That is to say, an m-unveracious sentence is an unassertive sentence, i.e. a sentence that is put in an unassertive format within an

assertive context. Hence, if a ttatt-neutral sentence is asserted then it is supposed to be an m-veracious one.

69) Ttatt-neutral RSDS's are classified further as follows.

i) An m-veracious sentence is said to be:

a) an *enduringly*, or *permanently*, *m-veracious sentence* and also a *proper m-veracious*, or *m-veracious proper, sentence* if it confirms to a certain *enduring* (*lasting*, *permanent*) *unique fact* of nature or human society, e.g. an astronomic, historical, geographic, or geopolitical one,

b) a *transitorily*, or *temporarily*, *m-veracious sentence* and also a *common m-veracious*, or *m-veracious common*, *sentence* if the fact, to which it conforms in the given circumstances (spatio-temporal situation) with respect to me, is one of many similar *transitory* (*temporary*) states of affairs occurring occasionally here or there and now or then.

ii) An m-antiveracious sentence is said to be:

a) an *enduringly*, or *permanently*, *m-antiveracious sentence* and also a *proper m-antiveracious*, or *m-antiveracious proper, sentence* if its negation is a proper m-veracious sentence;

b) a *transitorily*, or *temporarily*, *m-antiveracious sentence* and also a *common m-antiveracious*, or *m-antiveracious common*, *sentence* if its negation is a common m-veracious sentence in the given circumstances with respect to me.

iii) An m-veracious or m-antiveracious proper sentence is indiscriminately called a *proper sentence*.

iv) An m-vravr-neutral sentence is alternatively called an *m-vravr common sentence* (in contrast to *m-tautologous common sentences* to be defined before long), because in accordance with the points i.b and ii.b, whenever there is a fact (state of affairs), which an m-vravr-neutral (common) sentence either confirms to or contradicts, that sentence becomes a *common m-veracious* or *common m-antiveracious sentence* respectively.

70) Here follow some examples of ttatt-neutral RSDS's of the different kinds indicated in the previous item.

a) Proper m-veracious sentences.

"Sir Walter Scott is the author of *Waverley*", "Abraham Lincoln was the 16th president of the USA in the years 1861–65", "Woodrow Wilson was the 28th president of the USA in the years 1913–21", "London is in Europe", "Chicago is North of New York", "Moscow is the capital of Russia".

b) Proper m-antiveracious sentences.

"Sir Walter Scott is not the author of *Waverley*", "Abraham Lincoln was the 28th president of the USA in the years 1913–21", "Abraham Lincoln was not the 16th president of the USA in the years 1861–65", "Abraham Lincoln was the 28th president of the USA in the years 1913–21", "London is in Asia", "Chicago is South of New York", "Moscow is not the capital of Russia".

c) M-vravr-neutral common sentences.

"It is raining", "It is not raining", "The night is light", "The night is dark", "This water is cold", "This water is hot", "This meal is testy", "This meal is not testy", "I am hungry", "I am full up".

Any sentence of the point a) is an m-veracious (accidentally m-true) proper sentence with respect to me because it *conforms to* (*denotes* when asserted) the pertinent historical, geographical, or or present or present geopolitical fact. Any sentence of the point is an m-antiveracious (accidentally m-antitrue) proper sentence with respect to me because it contradicts the historical, geographical, or present geopolitical fact, which a certain m-veracious proper sentence of the point a) conforms to. Any possible state of affairs of the range of any sentence of the point c), which that sentence can conform to (denote when asserted), has the quality of *thisness* (*haecceity*), i.e. of *being here and now,* and hence it is local and transient. Incidentally, the first sentence of the point a) and the first sentence of the point b) are relevant to the historical fact that Walter Scott published his twenty-nine *Waverley Novels* anonymously, and that he kept his authorship of *Waverley* secret. Therefore, either one of the two sentences was an m-vravr-neutral common sentence for any person who did not know the identity of the mysterious author of *Waverley*, particularly for the English King George IV. Once the identity of the author of *Waverley* had been revealed, the first sentence became m-veracious and the second one m-antiveracious. Likewise, any other sentence of the point a) and its negation or its contrary of the point b) are m-vravr-neutral ones with respect to every person who does not know the facts, which the sentences of the point a) confirm to. The above-mentioned historical fact is discussed by Whitehead and Russell [1910; 1962, p. 67] and by Church [1956, pp. 5–6] for exemplifying their treatments of the denotata and senses of proper names.

2.7.3. Tautologous and antitautologous declarative sentences

71) By the pertinent *euautographic algebraic decision procedures* (*EADP's*), it has been proved that the *predicate-free* ER's (*PFER's*) $p\vee\neg p$, which is called the *law of excluded middle* (*tertium non datur* in Latin), and $[p\Rightarrow q]\vee[q\Rightarrow r]$ are *valid*; the binary kernel-signs (logical connectives) $\vee$ and $\Rightarrow$ are ones of inclusive disjunction and of implication respectively. By definition, the ER $[p\Rightarrow q]\vee[q\Rightarrow r]$ is equivalent to $[\neg p\vee q]\vee[\neg q\vee r]$. By the pertinent EADP's, it has been proved that the kenrnel-sign $\vee$ satisfies the *commutative* and *associative laws*:

$$[p_1\vee p_2]\Leftrightarrow[p_2\vee p_1] \text{ and } [p_1\vee[p_2\vee p_3]]\Leftrightarrow[[p_1\vee p_2]\vee p_3],$$

where $\Leftrightarrow$ is the kernel-sign of equivalence. Owing to these laws, $[[\neg p\vee q]\vee[\neg q\vee r]]$ can be written in various equivalent forms, differing from one another by the orders of $\neg p$, q, $\neg q$, and r and by the arrangements of pairs of square brackets, e.g. as $[[q\vee\neg q]\vee[\neg p\vee r]]$, $[[[q\vee\neg q]\vee\neg p]\vee r]$, or $[[\neg p\vee r]\vee[q\vee\neg q]]$. There is an indefinite number of *valid* PFER's, each of which is reducible (equivalent) either to a certain *valid affirmative* (or *positive*) *n-fold* (or undistributively *repeated*) *binary disjunctive PFER* or to the *negation of such a PFER*, which is called a *valid negative n-fold binary disjunctive PFER*. In this case, the qualifier "*n*-fold" is descriptive of the total number *n* of occurrences (tokens) of the *binary disjunctive operator* $[\ \vee\ ]$ in a PFER. In turn, the latter PFER is reducible (equivalent) to the respective *valid affirmative* (or *positive*) *n-fold binary conjunctive PFER*, whereas the former PFER is reducible (equivalent) to the respective *valid negative n-fold binary conjunctive PFER*. In this case, the qualifier "*n*-fold" is descriptive of the total number *n* of occurrences (tokens) of the *binary conjunctive operator* $[\ \wedge\ ]$ in a PFER. Under the substitutions (interpretations) (2.27), a valid PFER of a certain one of the above names turns into the *f-tautologous* (*universally f-true*) *predicate-free CLR* (*PFCLR*) of the like variant name with "f-tautologous" (or "universally f-true") in place of "valid" and with "PFCLR" in place of "PFER". In this case, the abbreviation "PFCLR" stands for both "*PFCCLR*" and "*PFPCLR*", in accordance with the item 52. Consequently, a DS, having a certain one of the above-mentioned f-tautologous CLR's as its form (schema) and involving the wordy interpretands of the pertinent EKS's indicated in the item 60 is distinguished by the respective variant name with "*m*" for "*materially*" in place of "*f*" for "*formally*" and with "DS" in place of "PFCLR". Particularly, under the above interpretations, the valid PFER's $p\vee\neg p$ and $[p\Rightarrow q]\vee[q\Rightarrow r]$ turn into the f-tautologous (universally f-true) PFCLR's '$p\vee\neg p$', i.e. "*p* or not *p*', being an f-tautologous affirmative 1-fold binary disjunctive PFCLR, and '$[p\Rightarrow q]\vee[q\Rightarrow r]$', i.e. "[if *p* then *q*] or [if *q* then *r*]" or[*p* only if *q*] or [*q* only if *r*]", which are schemata (forms) of m-tautologous (universally m-true) complex-coordinate declarative sentences. The PFCLR "$[p\Rightarrow q]\vee[q\Rightarrow r]$' is equivalent to $[[\neg p\vee q]\vee[\neg q\vee r]]$', being an f-tautologous affirmative 3-fold binary disjunctive PFCLR, while the latter can be written in various equivalent forms such as '$[[q\vee\neg q]\vee[\neg p\vee r]]$', '$[[[q\vee\neg q]\vee\neg p]\vee r]$', or '$[\neg p\vee r]\vee[q\vee\neg q]$'. In reference to its *principal binary disjunctive operator* [$\vee$], a universally m-true DS of any of the above forms is alternatively called an *m-tautologous* (or *m-tautological*) *binary disjunctive DS*. The sentences "It is raining or it is not raining" and "Abraham Lincoln was the 16th president of the USA or Abraham Lincoln was not the 16th president of the USA" of the form '$p\vee\neg p$', which can be abbreviated as the respective *contracted sentences* "It is or is not raining" and as "Abraham Lincoln was or was not the 16th president of the USA", and the sentence "Abraham Lincoln *was* the 16th president of the USA only if it *is* raining, or it *is* raining only if Brutus loved not Caesar less but Rome more" of the form '$[p\Rightarrow q]\vee[q\Rightarrow r]$' and its variant with "*was not*" in place of "*was*" or that with "*is not*" in place of "*is*" or that with both are some examples of such DS's. In these examples, "raining" should be understood as *raining here* or *there* (e.g. in Broadway of New York) *and now*.

72) For the sake of being specific, let an m-tautologous *n*-fold binary disjunctive DS be given. No matter which one or more of the alternative states of affairs that are mentioned in that sentence are realized here and now, the DS is universally m-true by virtue solely of its *valid syntactic form* that has been imported in it via the pertinent f-tautologous PFCLR (as '$p\vee\neg p$' or '$[p\Rightarrow q]\vee[q\Rightarrow r]$') from the conformal euautographic interpretans of the latter (as $p\vee\neg p$ or $[p\Rightarrow q]\vee[q\Rightarrow r]$ respectively). Consequently, like the f-tautologous PFCLR, being the immediate catlogographic interpretans of the m-tautologous DS, and also like the valid PFER, being its immediate euautographic interpretans of the f-tautologous PFCLR, the m-tautologous DS in question can always be used *assertively*. However, when used

assertively, that sentence denotes a certain abstract object (state of affairs), which differs from any one of the *n* separate states of affairs conformable to its *n*+1 disjunctive clauses, – an object that can be defined as follows. The *designatum* (*range*) of the given m-tautologous *n*-fold binary disjunctive sentence is *the union of the designata of its n disjuncts* (*clauses*). Consequently, when I assert the sentence, I use it, along with its designatum, in *the projective* (*polarized*, *extensional*, *connotative*) *mental mode* (cf. the item 46), in which I *mentally experience* the designatum as *my as if extramental* (*exopsychical*) *object* that I call a *common* (*general*, *certain*, *particular but not particularized*) *element* (*member*) *of the designatum* and also a *common denotatum of the sentence*. The common element of the designatum *represents the whole designatum*, thus being just *another hypostasis* (*way of existence*, *aspect*) *of the latter*. In this case, I also say that both *the sentence and its* [*original*, *unpolarized*] *designatum are used for mentioning* the common denotatum of the sentence, i.e. the common element of the designatum of the sentence, or that, less explicitly, *they are used but not mentioned*, whereas the designatum is said to be *connoted by*, or to be *the connotatum* (*connotation value*, pl. "*connotata*") *of*, *the sentence*. Thus, an m-tautologous *n*-fold binary disjunctive sentence is a *common sentence* or more specifically an *m-tautologous common sentence* in contrast to a *vravr-neutral common sentence* (cf. the item 69iv).

73) The negation of a valid PFER is an antivalid PFER and vice versa. Therefore, the negation of an f-tautologous PFCLR is an f-antitautologous (universally f-antitrue, universally f-antitrue, f-contradictory) PFCLR and vice versa. The last statement remains true with "m" in place of "f" and "DS" in place of "PFCLR". Since $\neg[p_1 \vee p_2] \rightarrow [p_2 \wedge p_1]$, therefore '$\neg[p \vee \neg p]$' is equivalent to '$p \wedge \neg p$', whereas

'$\neg[[p \Rightarrow q] \vee [q \Rightarrow r]]$', '$\neg[[\neg p \vee q] \vee [\neg q \vee r]]$', '$\neg[[q \vee \neg q] \vee [\neg p \vee r]]$',

'$\neg[[[q \vee \neg q] \vee \neg p] \vee r]$', and '$\neg[[\neg p \vee r] \vee [q \vee \neg q]]$',

e.g., are equivalent to

'$[p \Rightarrow q] \wedge [q \Rightarrow r]$', '$[\neg p \vee q] \wedge [\neg q \vee r]$', '$\neg[q \vee \neg q] \wedge [\neg p \vee r]$',

'$[[q \vee \neg q] \vee \neg p] \wedge r$', and '$[[\neg p \vee r] \wedge [q \vee \neg q]]$',

respectively, and to one another. Any one of the above PFCLR's having the conjunction $\wedge$ as its *principal kernel-sign* is called an *f-antitautologous binary conjunctive PFCLR*. Making use the distributive law for $\wedge$ over $\vee$:

$$[p_1 \wedge [p_2 \vee p_3]] \Leftrightarrow [[p_1 \wedge p_2] \vee [p_1 \wedge p_3]]$$

and the one for ∨ over ∧:

$$[p_1 \vee [p_2 \wedge p_3]] \Leftrightarrow [[p_1 \vee p_2] \wedge [p_1 \vee p_3]]$$

in that order, any one of the above f-antitautologous binary conjunctive PFCLR's can be represented as equivalent f-antitautologous PFCLR's of various forms. Still, the main implications of the above-said are the following two. First, *the negation of an f-tautologous binary disjunctive PFCLR is an f-antitautologous binary conjunctive PFCLR and vice versa*. Second, up to the ACLR's used, any f-antitautologous binary *conjunctive* PFCLR other than '*p*∧¬*p*' and other than the variants of '*p*∧¬*p*' with any ACLR of the list (10) in place of '*p*' is reducible either to the f-antitautologous binary conjunctive PFCLR '[*p*∧¬*p*]∧***R***' or to its pertinent variant, ***R*** being a certain PFCLR. The above two implications apply, *mutatis mutandis*, with "m" in place of "f" and "DS" in place of "PFCLR". At the same time, no mutually exclusive states of affairs, as that of raining and that of not raining, can happen in the same place at the same time. Therefore, the designatum of an m-antitautologous binary conjunctive DS, being the intersection of the designata of its conjuncts, i.e. of the disjuncts of the respective m-tautologous binary disjunctive sentence, is the *empty class*. But, a sentence whose designatum is empty cannot be used in the projective (polarized, extensional, connotative) mental mode for denoting (mentioning) a common member of the designatum simply because the designatum has no members. That is to say, an m-antitautologous (m-contradictory) DS can *never be used assertively*, so that it can be classified as an *unassertive universally antitrue and hence meaningless sentence*, the understanding being that it is neither a proper sentence nor a common one. *In contrast to a paralogous DS, which is meaningless along with its negation*, the negation of an m-antitautologous DS is the respective *meaningful* m-tautologous DS. All m-antitautologous DS's can be disregarded for being impracticable. The notion of m-antitautologous DS's is however necessary for completeness of the following *specific* taxonomy of DS's and also for completeness of the *general* taxonomy of DS's that is established in the next item.

i) A DS that is *neither m-tautologous nor m-antitautologous* (*nor m-contradictory*) is said to be *neutral* (*indeterminate*) *with respect to the tautologousness-values m-tautologousness* (*universal m-truth*) *and m-*

antitautologousness (*universal m-antitruth*, *universal m-falsehood*, *m-contradictoriness*) – briefly an *m-ttatt-neutral* (*m-ttatt-indeterminate*) *sentence*.

ii) An m-tautologous sentence is alternatively called an *m-tautologous common sentence* – in contrast to *m-vravr-neutral common sentence* (cf. the items 69iv and 72).

iii) It is understood that *an m-veracious*, *m-antiveracious*, or *m-vravr-neutral sentence* is *an m-ttatt-neutral* (*m-ttatt-indeterminate*) *sentence* and vice versa. Hence, if an m-ttatt-neutral sentence is asserted then it is supposed to be an m-veracious one.

iv) A DS is said to be: (a) *m-atautologous* if is m-antitautologous or m-ttatt-neutral, (b) *m-non-antitautologous* if it is m-tautologous or m-ttatt-neutral, (c) *m-ttatt-unnutral* (*m-ttatt-determinate*) if it is m-tautologous or m-antitautologous.

v) The division of DS’s into three classes: m-tautologous, m-antitautologous (m-contradictory), and m-ttatt-neutral (m-ttatt-indeterminate) is called the *specific primary* (or *specific basic*) *trichotomy* (*trisection*, *trifurcation*) *of the DS’s*. The three divisions of the class of DS’s into two complementary classes each, namely: (a′) *m-tautologous* and *m-atautologous*, (b′) *m-antitautologous* and *m-non-antitautologous*, (c′) *ttatt-neutral* (*ttatt-inteterminate*) and *ttatt-unneutral* (*ttatt-determinate*) are called the *specific secondary* (or *specific subsidiary*) *dichotomies* (*bisections*, *bifurcations*) *of the DS’s*.

74) In this treatise, the notion of m-tautologies as statements that are universally m-true by virtue solely of the *abstract validity* (and hence by virtue of the *f-tautogousness*) *of their syntactic forms* applies, not only to a syntactically valid affirmative *n*-fold binary disjunctive or conjunctive sentence with a finite number *n*+1 of coordinated clauses as its *disjuncts* or *conjuncts*, but it also applies to *syntactically valid quantified* (*bound, contracted*) *statements* of the forms

$$‘\bigvee_u \boldsymbol{P}\langle u\rangle’,\ ‘\bigwedge_v \boldsymbol{Q}\langle v\rangle’,\ ‘\breve{\bigvee}_w \boldsymbol{R}\langle w\rangle’,\ ‘\overset{\frown}{\bigvee}{}^1_x \boldsymbol{S}\langle x\rangle’,\ ‘\bigvee^1_y \boldsymbol{P}_1\langle y\rangle’ \qquad (2.36)$$

(cf. the item 60), whereas $‘\bigvee_u \boldsymbol{P}\langle u\rangle’$ and $‘\bigwedge_v \boldsymbol{Q}\langle v\rangle’$ are equivalent to $‘\neg\bigwedge_u \neg\boldsymbol{P}\langle u\rangle’$ and $‘\neg\bigvee_v \neg\boldsymbol{Q}\langle v\rangle’$ respectively (cf. the item 42). In the general case, each one of the *metalinguistic* relations (2.36) is a schema, whose range is a class of CFCL interpretands of the respective *valid and vav-neutral output ESR’s* (*OptESR’s*) *of* A_1, which are condensed into the range of the respective one of the schemata

$$‘\bigvee_u \mathbf{P}\langle u\rangle’,\ ‘\bigwedge_v \mathbf{Q}\langle v\rangle’,\ ‘\breve{\bigvee}_w \mathbf{R}\langle w\rangle’,\ ‘\overset{\frown}{\bigvee}{}^1_x \mathbf{S}\langle x\rangle’,\ ‘\bigvee^1_y \mathbf{P}_1\langle y\rangle’. \qquad (2.37)$$

The CLR, being the CFLR interpretand of a certain *pseudo-quantified* (*bound, contracted*) *OptESR*, is obtained by the pertinent ones of the substitutions (2.25)–(2.27) subject to the items 14 (points v, vi, and ix), 39, and 40. It is understood that, besides the dummy (bound) PVOT u, v, w, x, or y, every occurrence of which in the respective operata $\mathbf{P}\langle u\rangle$, $\mathbf{Q}\langle v\rangle$, $\mathbf{R}\langle w\rangle$, $\mathbf{S}\langle x\rangle$, or $\mathbf{P_1}\langle y\rangle$ is bound to its first occurrence in the pertinent *pseudo-quantifier* (*binder, contractor*), the operata may involve occurrences (tokens) of some other PVOT's of the list (2.7) (free, bound, or both), occurrences of $\emptyset$, and occurrences of some ACLR's of the list (2.8), – the occurrences, which are not indicated in the schemata (2.37). Consequently, it is supposed that the respective operata $\boldsymbol{P}\langle\text{'}u\text{'}\rangle$, $\boldsymbol{Q}\langle\text{'}v\text{'}\rangle$, $\boldsymbol{R}\langle\text{'}w\text{'}\rangle$, $\boldsymbol{S}\langle\text{'}x\text{'}\rangle$, or $\boldsymbol{P_1}\langle\text{'}y\text{'}\rangle$ involves occurrences (tokens) of the CFCL interpretands of the latent ACLOF's, which are determined by the substitutions (2.25)–(2.27). This is not the appropriate place for discussing any ESR's comprised in the ranges of the placeholders (2.37) or any CLR's comprised in the ranges of the placeholders (2.36) in detail. I shall only remark that, for instance, a CLR $\bigvee_u \boldsymbol{P}\langle u\rangle$ can be regarded as a *disjunction of an infinite number of disjuncts*, whereas $\bigwedge_v \boldsymbol{Q}\langle v\rangle$ can be regarded as a *conjunction of an infinite number of conjuncts*. Also,given a natural domain, the tokens of VCLOT's of the list (2.28), occurring in a given CLR of the range of a certain placeholder of the list (2.36), can be assigned with the appropriate classes as their designata (ranges), such as some taxa of a *biological taxonomy of bionts* or such as some sets of mathematical objects, e.g. the sets of various numbers, the underlying set of vectors of a linear space, the underlying set of points of an affine space, etc. At the same time, the tokens of CLKS's, occurring in the CLR can be provided with the wordy interpretands that have been given in the item 60. In the result, the CLR turns into a DS of *rich* written English or of another *rich* WNL, i.e. of a WNL that is augmented (enriched) by all necessary nomenclature (logographic notation and wordy terminology). In this case, the DS is said to be *m*-tautologous (*materially* tautologous) if the CLR is *f*-tautologous (*formally* tautologous) and *m*-ttatt-neutral (*m*-ttatt-indeterminate) if the CLR is *f*-ttatt-neutral (*f*-ttatt-indeterminate). Some examples of f-tautologous CLR's have been given in the item 42. The negation of an *m*-tautologous DS said to be *m*-antitautologous DS, whereas an *m*-ttatt-neutral DS is more specifically said to be an m-veracious, m-antiveracious, or m-vravr-neutral if the ttatt-neutral CLR, being its CFCL interpretans (form, schema), is f-veracious, f-antiveracious, or f-vravr-neutral respectively. Thus, the specific taxonomy of DS's that has been explicated in the previous item includes both *quantifier-free DS's* and *quantifier-involving DS's.* Also, all specific taxonomies of DS's that have been discussed in the items 65–73 and above in this item are generalized (unified) as follows.

i) A DS is said to be:

a) *m-true* if it is either *m-tautologous* (*universally m-true*) or *m-veracious* (*accidentally m-true*);

b) *m-antitrue* or *m-false* if it is either *m-antitautologous* (*universally m-antitrue, universally m-false, m-contradictory*) or *m-antiveracious* (*accidentally m-antitrue, accidentally m-false*)*;*

c) *neutral* (*indeterminate*) *with respect to the truth-values m-truth and m-antitruth*, i.e. *neither m-true nor m-antitrue,* – briefly *m-tat-neutral* (*m-tat-indeterminate*), if it is *m-vravr-neutral* (*m-vravr-indeterminate*).

In this case*, the negation of an m-true sentence is an m-antitrue* (*m-false*) *sentence and vice versa*, whereas *the negation of an m-tat-neutral* (*m-tat-indeterminate*) *sentence is another m-tat-neutral* (*m-tat-indeterminate*) *sentence*.

ii) An m-tautologous sentence or a common m-veracious is indiscriminately called a *common m-true sentence* (cf. the items 69iv and 73).

iii) A DS is said to be: (a) *m-untrue* if it is m-antitrue or m-tat-neutral (m-tat-indeterminate); (b) *m-non-antitrue* or *m-non-false* if it is m-true or m-tat-neutral (m-tat-indeterminate); (c) *m-tat-unnutral* or *m-tat-determinate* if it is m-true or m-antitrue (m-false).

iv) The division of the class of DS's into three classes: *m-true*, *m-antitrue* (*m-false*), and *m-tat-neutral* (*m-tat-indeterminate*) is called the *general primary* (or *general basic*) *trichotomy* (*trisection*, *trifurcation*) *of the declarative sentences.* The three divisions of the class of DS's into two complementary classes each, namely: (a′) *m-true* and *m-untrue*, (b′) *m-antitrue* and *m-non-antitrue* (*m-non-false*), (c′) *m-tat-neutral* (*m-tat-inteterminate*) and *m-tat-unneutral* (*m-tat-determinate*) are called the *general secondary* (or *general subsidiary*) *dichotomies* (*bisections*, *bifurcations*) *of the DS's*.

2.8. The trial psychologistic logic versus conventional dual logic

75) The way, by which I have developed $\mathcal{A}_1$, can be generalized as the following definition.

***Definition of the term** "**formal logic**"*. A) *Formal logic* (*FL*) is a *study of the **form of reasoning** in abstraction from its matter*, along with or without a *study of the **form** of relationship between the former form and its matter*.

B) The matter of reasoning comprises *complex objects* of a logician (thinker, interpreter, sapient subject), which are called *states of affairs* and also *facts*, *events*, etc, and which *are not objects of FL*. However, in accordance with the above definition, a study of relationship between form and matter of reasoning is a part of FL. Therefore, within the *trial FL* (*TFL*), that study has been done as *a study of the relationship of conformity of valid and vav-neutral* (*vav-indeterminate*, *neither valid nor antivalid*) *euautographic* (*genuinely autographic*, *pure syntactic*) *relations* (*ER's*) *of the organon* A_1 *to the **formal matters*** (***f-matters***) *of those ER's in the form of the respective conformal catlogographic* (*CFCL*) *f-tautologous* (*f-tautological, universally f-true*) *and f-veracious* (*accidentally f-true*) *f-ttatt-neutral* (*f-ttatt-indeteminate, neither f-tautologous nor f-antitautologous*) *relations* (briefly *CFCLR's*), being *the respective CFCL interpretands of the ER's*. In other words, the relationship of conformity under study is *the relationship of conformity of the validity-values* (*validity-classes*) *validity and vav-neutrality* (*vav-indeterminacy*), i.e. *neutrality* (*indeterminacy*) *with respect to the validity-values validity and antivalidity*, *of ER's of* A_1 *to the **f-matters** of the CFCL interpretands of the ER's in the form of the f-tautologousness-value* (*f-tautologousness-class*) *f-tautologousness and in the form of the f-veracity-value f-veracity of the respective CFCL interpretands.*

C) In the case of the TFL, the negation of a valid relation is an antivalid relation and vice versa, whereas the negation of a vav-neutral relation is another vav-neutral relation; the negation of an f-tautologous relation is an f-antitautologous relation and vice versa, whereas the negation of an f-ttatt-neutral relation is another f-ttatt-neutral relation; the negation of an f-veracious relation is an f-antiveracious relation and vice versa, whereas the negation of an f-vravr-neutral (f-vravr-indeterminate, neither an f-veracious nor an f-antiveracious) relation is another an f-vravr-neutral relation. Also, relations of the TFL satisfy the following secondary taxonomies.

i) A relation of the TFL is said to be:

a) *f-true* if it is either *f-tautologous* (*universally f-true*) or *f-veracious* (*accidentally f-true*);

b) *f-antitrue* (*f-false*) if it is either *f-antitautologous* (*universally f-antitrue*, *f-contradictory*) or *f-antiveracious* (*universally f-antitrue*);

c) *neutral* (*indeterminate*) *with respect to the f-truth-values f-truth and f-antitruth*, i.e. *neither f-true nor f-antitrue,* – briefly *f-tat-neutral* (*f-tat-indeterminate*), if it is *f-vravr-neutral* (*f-vravr-indeterminate*).

In this case, the negation of an f-true relation is an f-antitrue relation and vice versa, whereas the negation of an f-tat-neutral relation is another an f-tat-neutral relation. The qualifiers "tat-neutral" ("tat-indeterminate") and "vravr-neutral" ("vravr-indeterminate") are synonyms.

ii) A relation of the TFL is said to be:

a) *invalid* if it is antivalid or vav-neutral, *non-antivalid* if it is valid or vav-neutral, and *vav-unneutral* (or *vav-determinate*) if it is valid or antivalid;

b) *f-atautilogous* if it is f-antitautologous or f-ttatt-neutral, *f-non-antitautologous* if it is f-tautologous or f-ttatt-neutral, and *f-ttatt-unneutral* (or *f-ttatt-determinate*) if it is f-tautologous or f-antitautologous;

c) *f-unveracious* if it is f-veracious or f-vravr-neutral, *f-non-antiveracious* if it is f-veracious or f-vravr-neutral, and *f-vravr-unneutral* (or *f-vravr-determinate*) if it is f-veracious or f-antiveracious;

d) *f-untrue* if it is f-true or f-tat-neutral, *f-non-antitrue* if it is f-true or f-tat-neutral, and *f-tat-unneutral* (or *f-tat-determinate*) if it is f-true or f-antitrue.

D) In the case of *dual FL* (*DFL*), vav-neutral, f-ttatt-neutral, f-vravr-neutral, and f-tat-neutral relations are disregarded. Since f-veracious (accidentally f-true) and f-antiveracious (accidentally f-antitrue, accidentally f-false) relations are f-ttatt-relations, therefore the former relations are disregarded along with the latter. Consequently, the qualifiers to relations of DFL of each one of the following four lists i–iv are synonyms:

i) "f-valid" and "f-non-antivalid";

ii) "f-antivalid" and "f-invalid";

iii) "f-tautologous" ("universally f-true"), "f-true", "f-non-antitautologous" ("f-uncontradictory"); and "f-non-antitrue";

iv) "f-antitautologous" ("f-contradictory", "universally f-antitrue", "universally f-false"), "f-antitrue" ("f-false"), "f-atautologous".

Also, the *validity-value validity* or *invalidity* of a relation of DFL can be qualified as an *f-truth-functional* one and likewise the *f-truth-value f-truth* or *f-untruth* of a relation of DFL can be qualified as a *validity-functional* one in the sense that a relation of DFL is said to be *valid* if and only if it is *f-true* and *invalid* if and only if it is *f-untrue* (*f-false*). That is to say, the qualifiers to relations of DFL of either one of the following two pairs are also synonyms:

a) "valid" and "f-true",

b) "antivalid" and "f-antitrue".

Therefore, the six qualifiers of the lists i and iii or those of the lists ii and iv are synonyms.

E) From the previous item, it follows that the relationship of the TFL, which models *the relationship between the form of reasoning, abstracted from its matter, and that same matter itself*, i.e. *between the TFL and the pertinent TML, becomes an identity, and hence it disappears, in DFL*. Hence, the definition of *formal logic* (*FL*), which was made in the item A of this definition, applies only to *the TFL*, i.e. with "*Trial formal logic* (*TFL*)" in place of "*Formal logic* (*FL*)". In the case of *DFL*, that item should be restated as follows.

F) *Dual formal logic* (*DFL*) is a *study of the form of reasoning in abstraction from its matter*. The relationship between the DFL and the *material logic* (*ML*), being *its matter*, is beyond the scope of the DFL.

G) In accordance with the items A and F, TFL or DFL is a *single trial* or *dual formal logical system* (*TFLS* or *DFLS*) or a *totality of TFLS's* or *DFLS's*, respectively. In this case, a *formal logical system* (*FLS*) is either a *totality of separate axiomatic rules of inference of one type* or an *axiomatic logical calculus*, *basic* (*canonic, plain, non-modal*) or *modal*, the understanding being that some of the former axiomatic rules of inference can be theorems of a certain axiomatic logical calculus. Any *FLS*, *dual* or *trial*, is or is supposed to be developed with the help and within its *inclusive metalanguage* (*IML*), which is called *the theory of the FLS* or generally a *logical theory*. A logical theory is said to be a *dual one* (*DLT*), if it determines a DFLS, and a *trial one* (*TLT*), if it determines a TFLS. Several logical theories may determine recognizably the same FLS. Besides the FLS that is prescinded from its theory, that theory may determine the pertinent *material logical system* (*MLS*) (e.g., a certain system of English declarative sentences) and also determine the relationship between the FLS and the MLS.

H) The noun "logic" alone, without qualifiers, is an equivocal (multisemantic) generic term. By default, this noun most generally means (denotes, is used for mentioning) a *certain totality of logical theories*. At the same time, "logic" can be used synecdochically for mentioning, e.g. either FL, i.e. a single FLS or a totality of FLS's, or a single logical theory.

76) The term "tautology" has arisen in the conventional (dual) truth-functional FL after Wittgenstein [1921], who applied that term to any quantifier-free or quantified statement, being *universally true* by virtue solely of the *abstract truth-functional validity of its syntactic form.* Such use of the term "tautology" has been adopted by all modern logicians and mathematicians. At the same time, Wittgenstein suggested as a thesis the doctrine that *all logic* and *all mathematics* is tautological. This thesis has commonly been regarded as one that is difficult to defend and therefore it has never been adopted by logical and mathematical society (cf. Quine [1951, p. 55]). The item 75 allows reaching complete clarity regarding Wittgenstein's thesis.

77) In accordance with the item D of that definition, the only *truth-values truth* and *antitruth* (*falsehood*) that exist in DFL are *formal truth-values* (*f-truth-values*) *universal formal truth* (*universal f-truth*) and *universal formal antitruth* (*universal f-antitruth*), i.e. *f-tautologousness-values f-tautologousness* and *f-antitautologousness* (*f-contradictoriness*), respectively. Hence, if «*all logic*» is understood either as *all DFL* or as *all dual logic* (*DL*), i.e. as *all DFL together with the pertinent dual material logic* (*DML*), *adjoint of the DFL,* in the hypostasis of a certain rich WNL (e.g. rich written English), *whose materially tautologous* (*m-tautologous, universally m-true*) *DS's interpret f-tautologous relations of DFL*, – which *was obviously meant* by Wittgenstein, – then the part of Wittgenstein's thesis concerning «*all logic*» is *accidentally* (*not universally, not tautologously*) *true*.

78) By contrast, in TFL, besides the above *universal f-truth-values,* there are the *f-veracity-values f-veracity* and *f-antiveracity,* i.e. the *accidental f-truth-values accidental f-truth* and *accidental f-antitruth,* and there is also the *f-veracity-value f-vravr-neutrality* (*f-vravr-indeterminacy*), i.e. *neutrality* (*indeterminacy*) *with respect to the f-veracity-values f-veracity and f-antiveracity*. In this case, an *f-veracious*, *f-*

aniveracious, or *f-vravr-neutral* (*f-vravr-indeterminate*) *relation* of TFL is an *f-ttatt-neutral* (*f-ttatt-indeterminate*) *relation*, i.e. *neutral* (*indeterminate*) *with respect to the f-tautologousness-values f-tautologousness and f-antitautologousness*. An f-veracious relation of TFL is *interpretable* (*replaceable*) by any appropriate *m-veracious* (*accidentally m-true, fact-conformable*) *DS*, which belongs to a *certain rich WNL* (e.g. *rich written English*), being the *pertinent trial material logic* (*TML*) *adjoint of the TFL*. Here follow some examples of m-veracious sentences.

a) Each DS of the item 70a is is a *proper m-veracious DS*, because it conforms to the respective *permanent* historical, geographical, or geopolitical fact:

b) Each one of the following DS's is also a *proper m-veracious DS*, because it conforms to the respective *permanent* mathematical fact:

"To each natural number there is a strictly larger natural number", "The sum of angles of a triangle equals π", "Two infinite straight parallel lines in a 3-dimensional affine real Euclidean space do not intersect", "2>1", "$3^2+4^2=5^2$".

c) "It is raining" is a *common m-veracious DS* and "It is not raining" is a *common m-antiveracious DS there and then, where and when it is raining*, while on the contrary, "It is raining" is a *common m-antiveracious DS* and "It is not raining" is a *common m-veracious DS there and then, where and when it is not raining.*

d) I have proved that, in the exclusion of Bamalip, Barapti (renamed Darapti), Felapton, and Fesapo, the remaining 15 of 19 categorical syllogisms are *f-tautologous*, i.e. *universally f-true*. The former four categorical syllogisms are *f-ttatt-neutral* (*f-ttatt-indeterminate*) ones, which turn into *f-veracious*, i.e. *accidentally f-true*, ones owing to certain *additional f-veracious catlogographic axioms* (cf. the item 34).

The above examples have the following general implications.

i) If «*all logic*» is understood either as *all TFL* or as *all trial logic* (*TL*), i.e. *all TFL together with the pertinent trial material logic* (*TML*) *of a certain rich WNL* (e.g. *rich written English*), *whose m-veracious* (*fact-conformable*) *DS's interpret f-veracious relations of TFL*, – which *was not obviously meant* by Wittgenstein, – then the part of Wittgenstein's thesis concerning «*all logic*» is *accidentally* (*not universally, not tautologonusly*) *m-antitrue* (*m-false*).

ii) The class of ttatt-neutral DS's is an inexhaustible source of *mathematical postulates*, both *permanent* ones called *axioms* and *ad hoc* ones called *hypotheses*, and also of *mathematical theorems*, which are therefore m-veracious (accidentally m-true) and not m-tautological (not universally m-true). Hence, the part of Wittgenstein's thesis concerning *supposed tautologousness of all mathematics* is also *accidentally m-antitrue* (*m-false*).

It is noteworthy that all unquantified f-tautologous relations of the TFL and a few quantified ones (as the 15 tautologous categorical syllogisms) are effective mainly as *rules of inference*, whereas all quantified f-tautologous relations of the TFL (as those demonstrated in the item 42) are substantiated relations in themselves like most f-veracious ones.

79) After the manner of the established term "tautology" and its derivatives, I introduce the following *monomial synonyms* of the taxonyms of the decision classes of ER's of A_1. A *valid, antivalid, vav-neutral* (*vav-indeterminate*), *invalid, non-antivalid*, or *vav-unnutral* (*vav-determinate*) *ER* is alternatively (synonymously) called a *kyrology, antikyrology*, *kak-udeterology* (*kak-anorismenology*), *akyrology, anantikyrology*, or *kak-anudeterology* (*kak-orismenology*) respectively. Consequently, a kyrology is either a euautographic axiom or a euautographic theorem, whereas an antikyrology is either a euautographic antiaxiom or a euautographic antitheorem. It goes without saying that "*kak*" is an abbreviation for "*kyrology-antikyrology*", so that "kak-udeterology" ("kak-anorismenology") means *neither a kyrology nor an antikyrology*. In accordance with Pring [1982] (see also Dict A1.1), the prefix "*kyro*"- is derived from the Greek noun "κῦρος" \kíros, kýros\ meaning *validity*, the prefix "*udetero*"- is derived from the Greek adjective "ουδέτερος" \uðéteros, uthéteros\ meaning *neutral* or (gram.) *neuter* and from the homonymous pronoun meaning *neither*, and the prefix "*orismeno*"- is derived from the Greek adjective "ωρισμένος" \orisménos\ meaning *determinate, determined*, or *certain*.

2.9. The Frege-Church theory of the meaning of dualistic truth-functional proper declarative sentences

80) Unlike the *trialistic* (*three-fold*) interpretation of the ACLR's on the list (2.29) that is indicated in the item 65, homographs of the ACLR's, which are employed in various CALC'i, are conventionally interpreted *dualistically* by assuming that each of them is a variable, whose range is the set of two *formal truth-values,* namely *truth* and *falsehood,* i.e. *antitruth* in the terminology of this treatise. Consequently, when used but not mentioned, such a variable, e.g. '*p*', is said to be *either true or false* (*antitrue*), so that it can be called a *double-valued truth-functional*

variable (*DVTFV*). In this case, it is tacitly assumed that '*p*' (e.g.) is a placeholder for some declarative sentences (formal or informal graphic relations), so that if *p* is said to be true, or false, then every sentence that is supposed to be substituted for '*p*' should be true, or false, respectively. Also, it goes without saying that *p* can be either universally true or universally false, i.e. be either a *tautology* or a *contradiction* (*antitautology*). In English-based Aristotelianism, a dualistic truth-functional sentence is called a *proposition*. In modern logic, the common name "*a proposition*" is used as a synonym of the descriptive name "*the sense of a dualistic truth-functional sentence*", while a *dualistic truth-functional sentence* is alternatively (synonymously) called a *propositional sentence*. Accordingly, Whitehead and Russell [1925; 1962, p. 5] call their DVTFV's "*propositional letters*", whereas Church [1956, p. 27] calls his DVTFV's "*propositional variables"*.

81) In order to define his propositional variables and justify their use, Church adapted for English *the German-based theory of the meaning of proper names and proper sentences* «*Ueber Sinn und Bedeutung*» by Frege [1892]. I call the English version of the Fregean theory as presented in Church [1956, pp. 3–9, 25–28], "*the Frege-Church theory of the meaning of dualistic truth-functional proper declarative sentences*" or briefly "*the Frege-Church theory*" and also "*the FCT*". The FCT is based on the following radical doctrine, which is cited without two pertinent footnotes (*ibid.*, p. 25):

> «Therefore, with Frege, we postulate[66] two abstract objects called *truth-values*, one of them being *truth* and the other one *falsehood*. And we declare all true sentences to denote the truth-value truth, and all false sentences to denote the truth-value falsehood. In alternative phraseology, we shall also speak of a sentence as *having* the truth-value truth (if it is true) or having the truth-value falsehood (if it is false)[67].

The main postulate of FCT that a truth-functional sentence *denotes* one of the truth-values, truth or falsehood, is obscure. As I have already mentioned, a true sentence denotes a certain complex object that is most generally called a state of affairs, whereas a false (antitrue) sentence denotes nothing, but rather it just expresses its sense, i.e. a *false proposition*. Also, both Frege and Church were thoroughgoing Platonic realists. Church employs the term "*proposition*" as a parasynonym (translation) of the German term "*Gedanke*" of Frege. Accordingly, in order to explain the Platonic ontological status of a proposition, as he understands it, Church [1956, p. 26] cites the saying of Frege [1892] about the Platonic ontological status of *ein Gedanke*:

> «nicht das subjective Thun des Denkens, sondern dessen objectiven Inhalt, der fähig ist, gemeinsames Eigenthum von Vielen zu sein»,

which means (in my own interpretation):

> «not the subjective activity of thought, but the objective content capable of being property of many».

I do not adopt the FCT in this treatise. Instead, I follow closely the theory of the meaning of xenographs of my own – the theory that particularly applies to propositional (dualistic truth-functional) sentences.

2.10. Postscript on A_1: the interrelations of A_1 and $\mathbf{A}_1$

82) A *pasigraph* (*pasigraphonym*) *of* A_1, i.e. a *euautograph* (*euautographonym*) *of* A_1 or a *panlogograph* (*panlogographonym*) *of* $\mathbf{A}_1$, is called an *endosemasiopasigraph* (*endosemasiopasigraphonym*) in the sense that it *has certain syntactic functions* with respect to some other *pasigraphs of* A_1, each of which is also called *endosemasiopasigraph* (*EnSPG*, pl. "*EnSPG's*"), and that at the same time it *neither has nor assumes* (*takes on*) *any significations* (*imports*, *values*) *beyond* A_1. Accordingly, A_1 is a *semantically closed* logistic system of EnSPG's (euautographs and panlogographs), and therefore it is qualified *endosemasiopasigraphic* (equivocally abbreviated as "*EnSPG*"). In contrast to A_1, a logistic system of logographs, which have or assume significations beyond it, is qualified *exosemasiologographic* (*ExSPG*). The EnSPG properties of A_1 are explicated below.

i) Within A_1 and hence within A_1, a euautograph is always used autonymously, so that the only values that an EF can have or assume are, as was indicated in the item 37, its *autonymous values* such as a *concrete member* of the *class of its homolographic* (*photographic*) *isotokens*, as the euautograph itself, or a *common* (*general*, *certain*) *member* of that class, which is just another *hypostasis* (*way of existence*, *aspect*) of that same class.

ii) Like a euautograph, a panlogograph of $\mathbf{A}_1$, and hence of A_1, has only *homolographic*, i.e. *photographic* (*congruous or proportional*), *isotokens*; it does not have either *analographic*, i.e. *stylized* (*not photographic*) *iconographic* (*pictographic*) *isotokens*, or *phonic* (*oral*, *spoken*) *paratokens*. The range of a panlogograph of $\mathbf{A}_1$ is a *certain class of euautographs* of A_1 so that it is one of *my psychical* (*mental*, *imaginary*, *ideal*, *abstract*) *entities*. A panlogograph is called a *panlogographic formula* (*PLF*), *panlogographic ordinary term* (*PLOT*), a *panlogographic special term* (*PLSpT*) or *panlogographic integron* (*PLI*), or a *panlogographic relation* (*PLR*) if its range is a class of EF's, a class of EOT's, a class of ESpT's, i.e. of EI's, or a class of ER's respectively.

A panlogograph (PL) is called an *atomic panlogograph* (*APL*) or a *panlexigraph* if it is functionally indivisible and a *combined panlogograph* (*CbPL*) if it is not atomic, i.e. if it is a combination of two or more *atomic EnSPG's*, at least one of which is a panlexigraph.

A panlogograph can contain euautographs as its constituent parts. Particularly, the PKS (principal kernel-sign) of a *combined PLR* (*CbPLR*) or of a *combined PLI* (*CbPLI*) can be either an EKS (euautographic kernel-sign) or a *PLKS* (*panlogographic kernel-sign*), whereas the latter can, in turn, contain some euautographs.

A panlogograph of $\mathbf{A}_1$ condenses a large number (usually an infinite or indefinite number) of euautographs of A_1 in its range. Therefore, I can *mentally* (*psychically*) use a panlogograph, or, more precisely, a homolographic (photographic) isotoken of a certain *prototypal panlogograph*, either (a) *xenonymously*, i.e. in a *xenonymous mental mode,* as a *genuine* (*active*, *assertive*) *panlogograph*, called a *eupanlogograph*, for mentioning all euautographs of its range simultaneously or (b) I can use the same or another isotoken of the prototypal panlogograph *autonymously*, i.e. in an *autonymous mental mode*, as a *tychautograph* (*accidental*, or *circumstantial*, *autograph*) for mentioning either any member of its homolographic token-class or for mentioning itself. The two mental modes of using a panlogograph are explicated below.

a) When I *prescind* a panlogograph from its context (graphic surrounding) and hence from any possible *added words*, which may effectively alter the panlogograph and thus alter its range, and when I use the panlogograph *xenonymously*, i.e. as a *eupanlogograph*, I can use the panlogograph, like any xenograph (see, e.g. the item 44), along with its range in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode,* in which I *mentally experience* the range as *my as if extramental* (*exopsychical*) *object*, which I call a *common* (*general*, *certain*, *particular but not particularized*) *euautographic denotatum of the panlogograph* and also a *common element* (*member*) *of its range*; the common element *represents the whole range*, thus being just *another hypostasis* (*way of existence*, *aspect*) *of the latter*. In this case, I also say that both *the panlogograph and its* [*original*, *unpolarized*] *range are used for mentioning a common element of the range* or that, less explicitly, *they are used but not mentioned,* whereas the range is said to be *connoted by,* or to be *the connotatum* (*connotation value,* pl. "*connotata*") *of, the panlogograph*. Alternatively, I can, when necessary or desired, *physically replace* the panlogograph with any concrete euautograph of its range without any special rules of substitution. The act or process of psychically (mentally) using a panlogograph of $\mathbf{A}_1$ for mentioning a *common* (*certain*) euautograph of A_1 of its range is said to be a *psychical* (*mental, imaginary*) *euautographic interpretation of that panlogograph*. The act of presenting (printing or writing) a *concrete* (*particular*) euautograph of the range of a panlogograph of $\mathbf{A}_1$ is called a *physical* (*real, material*) *euautographic interpretation of that panlogograph*. Accordingly, a common euautograph of A_1 of the range of a panlogograph of $\mathbf{A}_1$ is said to be a *psychical* (*mental, imaginary*) *euautographic interpretand* (*interpretation value*) *of that panlogograph*, whereas a concrete euautograph of A_1 of that range is said to be a *physical* (*real, material, substitutional*) *euautographic interpretand*, or a *concrete* (*particular*) *euautographic instance, of that panlogograph*. Conversely, the panlogograph is said to be a *physical panlogographic interpretans* (*anti-interpretand*) both of its psychical euautographic interpretand and of each one its concrete euautographic interpretands (instances). It will be recalled that a psychical interpretand of a panlogograph of $\mathbf{A}_1$ is just *another mental hypostasis* (*form of existence*) *of its range* and is therefore tantamount to the latter. Thus, A_1 can be called the *psychophysical*, or *physopsychical*, *euautographic interpretand of* $\mathbf{A}_1$ and, conversely, $\mathbf{A}_1$ can be called the *physical panlogographic interpretans* (pl. "*interpretantia*") *of* A_1.

b) When I use a panlogograph *autonymously*, i.e. as a tychautograph, the properties that the panlogograph has *ad hoc* with respect to me are analogous to the permanent (intrinsic) properties of a euautograph as outlined in the item 37. Namely, a tychautograph is, like a euautograph, a functional but insignificant graphic chip that has a certain syntactic function or functions in itself or with respect to other pasigraphs (euautographs or panlogographs or both), especially those of its immediate surrounding (when applicable), but which has no psychical (mental) significations (imports, values) except autonymous ones. In this case, like an autonymous value of a euautograph, an autonymous value of a panlogograph is either its homolographic token-class or one of its homolographic isotokens, a concrete one or a common (general, certain) one, being another hypostasis of the isotoken-class.

83) In accordance with the previous item, $\mathbf{A}_1$ can be regarded as a *formalized logographic* and hence essentially *graphic* (*written*) *metalanguage*, which is designed for stating and processing infinite numbers of conspecific or congeneric euautographs

of A_1 simultaneously and which also allows, when necessary or desired, interpreting (illustrating) a panlogograph of $\mathbf{A}_1$ by concrete euautographs of A_1. Particularly, $\mathbf{A}_1$ allows setting up A_1 in the general form, while being itself a by-side product of the setup of A_1. Consequently, the setup of $\mathbf{A}_1$ is, to a great extent, a by-side product of the setup of A_1. At the same time, the setup of $\mathbf{A}_1$ has some remarkable digressions from the setup of A_1. Namely, while any FR of A_1 either is or can be restated so as to be simultaneously an FR of $\mathbf{A}_1$, *some FR's of* $\mathbf{A}_1$ do not introduce any new EF's of A_1 and are either *generalization rules* or *sortation rules of EF's of* A_1. A mapping from ER's of A_1 to PLF's of $\mathbf{A}_1$ is a *surjection* (*onto-mapping*, *onto-function*), not being an *epimorphism* (for morphisms of algebraic systems, see, e.g., Mac Lane & Birkhoff [1967, pp. 60–62]). In this case, $\mathbf{A}_1$ can be regarded as an *extension and generalization of* A_1 and, conversely, A_1 is a *restriction and specification of* $\mathbf{A}_1$. Also, A_1 is the *psychophysical*, or *physopsychical*, *euautographic interpretand of* $\mathbf{A}_1$ and, conversely, $\mathbf{A}_1$ is the *physical panlogographic interpretans of* A_1.

84) Alternatively, $\mathbf{A}_1$ can be regarded as an *axiomatic quasi-algebraic system*, whose objects are EF's, or, more precisely, ER's, of A_1, whereas A_1 is the pertinent *biune axiomatic quasi-algebraic system*, which concerns both with the EF's of A_1 and with the *panlogographic formulas* (*PLF's*) *of* $\mathbf{A}_1$ and which also concerns with interrelations of formulas of the two classes. Letting aside the fact that there are in algebra no algebraic systems analogous to $\mathbf{A}_1$, the character of the *EnSPG's* (*endosemasiopasigraphs*), i.e. *euautographs* and *panlogographs*, that are employed in $\mathbf{A}_1$ and A_1 essentially differ from the character of *logographs,* i.e. *variables and constants*, that are employed in a *conventional axiomatic algebraic system* (*CAAS*), and therefore the mental modes, in which I use the above pasigraphs, differ from the mental modes of using the logographs of a CAAS. To be specific, a CAAS is a logographic nomenclatural system that is used for representing and mentioning certain abstract (imaginary) objects (as points, vectors, or numbers) and their interrelations – entities, which cannot be exhibited on a material surface (as that of a sheet of paper or that of the screen of a computer monitor). The logographic symbols themselves are used but not mentioned – just as xenographic symbols of an *alphabetic* or *syllabic WNL* (*AbWNL* or *SbWNL)* are used but not mentioned in everyday life. Accordingly, *logographic variables* and *logographic constants* that are used in a CAAS are *purely representing and not place-holding ones*. By contrast, formulas of both mentally

superimposed systems of A_1, viz. EF's of A_1 and PLF's of $\mathbf{A}_1$, occur on the same material surface. In this case, an EF of A_1 is always used autonymously, because it is a euautograph, whereas a PLF of $\mathbf{A}_1$ can be used either xenonymously a eupanlogograph or autonymously as a tychautograph, because it is a panlogograph and is therefore significant.

85) In order to indicate syntactically that I use a panlogograph autonymously for mentioning *itself*, I use instead of the panlogograph its *proper name*, which is formed by enclosing the panlogograph in *slant light-faced single quotation marks*, *′ ′*, and which is called a *kyrioautographic*, or *proper autographic*, *quotation* (*KAQ*). That is to say, a KAQ denotes *its interior* prescinded from all its xenonymous values and from all its autonymous value except the interior itself, while the pair of *KAQ marks*, being *its exterior*, indicates (denotes) the above mental attitude of me towards the interior of the KAQ, and it also indicates the analogous mental attitude, which any interpreter of the KAQ should take towards its interior. By contrast, in order to indicate syntactically that I use a panlogograph autonomously for mentioning a *common* (*general*, *certain*) *member of its homolographic isotoken-class*, which is just another hypostasis (way of existence, aspect) of that class, I use instead of the panlogograph it its *common name*, which is formed by enclosing the panlogograph in *curly* or *upright straight light-faced single quotation marks*, ‘ ’ or ' ', and which is called a *homoloautographic*, or *photoautographic*, *quotation* (*HAQ*). The pair of *HAQ marks*, being the *exterior* of the HAQ, indicates (denotes) the above mental attitude of me towards the interior of the HAQ, and it also indicates the analogous mental attitude, which any interpreter of the HAQ should take towards its interior. It is understood that a KAQ, or an HAQ, has the denotatum of the above kind if the quotation is prescinded from any added words that may alter its meaning. If a panlogograph is not enclosed in any *special quotation marks* to indicate explicitly that it is used autonymously then I can, in accordance with the previous item, use it either xenonymously or autonymously.

86) Most often, however, I use an *unquoted* panlogograph that is prescinded from its symbolic surrounding, especially from the added words (if any), in both opposite mental modes, *xenonymous and autonymous, as if simultaneously but actually equivocally and intermittently by repeatedly switching, involuntary but consciously, from one mental attitude towards the panlogograph to the other* – just as

I perceive any one of Escher's *Convex and Concave* pictures, e.g. "*Cube with Magic Ribbons*" (see, for instance, Ernst [1985, p. 85f]). The class of conceptional or sensational mental phenomena of perceiving *graphonyms as having two opposite alternating hypostases* will be properly called by the *count name* "*alternation of opposites*" (without any article) – or by the limited (articled) and capitalized version of that name "*the Alternation of Opposites*" as the *intended proper class-name of the phenomena*, whereas a concrete instance (member, phenomenon) of this class will be commonly called "*an alternation of opposites*". I regard an alternation of opposites as a concrete manifestation of *the general dialectic principle of unity of opposites* due to Hegel. The class of involuntary but conscious alternations between the autonymous and xenonymous perceptions of a xenograph in general and of a logograph in particular, being a subclass of the Alternation of Opposites, is properly called by the count name "*tychautograph and euxenograph alternation*" or briefly "*tychauto-euxenograph alternation*" ("*TAEXA*"). The two count names can be limited and capitalized as "*the Tychautograph and Euxenograph Alternation*" and "*the Tychauto-Euxenograph Alternation*" to become thus the pertinent intended proper class-names. The TAEXA is a wide class of mental phenomena of perceptions of xenographs including representing and place-holding logographs and particularly including panlogographs, i.e. panlogographic placeholders (PLPH's). A concrete instance (member, phenomenon) of this class will be commonly called "*a tychauto-euxenograph alternation*" ("*a TAEXA*"), the understanding being that this name can, when necessary, be attributed with an appropriate postpositive qualifier such as "*of a xenograph*", "*of a logograph*", or "*of a panlogograph*". Likewise, the set of Escher's *Convex and Concave* pictures, being another subclass of Alternation of Opposites, will be called the count name "*Escher convex and concave alternation*" ("*ECCA*") or by the limited and capitalized intended proper class-name "*the Escher Convex and Concave Alternation*". Either of the latter two names can also be used as an *allegorical name* of the entire class of alternations of opposites, although the mental processes underlying ECCA's, being pure *sensational* (*sensorial*) *alternations of opposites*, differ from the mental processes underlying TAEXA's, being *conceptional ones*.

87) The TAEXA of a panlogograph is a *spontaneous* (*involuntary*) *but conscious mental process* of treating the panlogograph simultaneously as a large

number (usually an indefinite or infinite number) of euautographs condensed in its range and as a separate tychautograph. In the process and hence in the result of the TAEXA, *the range of the panlogograph is automatically extended to include the panlogograph itself as its tychautograph*. A biune mental hypostasis (way of existence) of a panlogograph during its TAEXA is, not only harmless, but most often it is useful and even indispensable. Particularly, the TAEXA of the pertinent PLF's of $\mathbf{A}_1$ are indispensable in simultaneously stating formation, transformation, and decision rules of A_1 and $\mathbf{A}_1$ and in simultaneously solving decision problems for an infinite number of conspecific or congeneric ER's of A_1 by solving the decision problems for the pertinent PLR's of $\mathbf{A}_1$. If I use a xenograph, particularly a logograph or a panlogograph, *autonymously,* or, on the contrary, *xenonymously*, I say that I use it in an *autonymous*, or, correspondingly, *xenonymous*, *mental mode*. Accordingly, if I use a panlogograph (e.g) *autonymously* and *xenonymously intermittently but as if simultaneously* – briefly, *autoxenonymously* or *xenoautonymously,* I say that I use the panlogograph in the *autoxenonymous*, or *xenoautonymous*, *mental mode*, or alternatively in the *TAEXA mental mode* or, briefly, in the *TAEXA-mode*.

88) Besides TAEXA's, there are alternations of opposites of many other kinds, which are utilized in this treatise. Just as TAEXA's, these are involuntary but conscious mental processes. For instance, equivocal use of the noun "interpretation" for denoting both a concrete operation of interpreting and its result is also an instance of the Alternation of Opposites. Also, according to *the theory of the meaning content of a xenograph* that I adopt in this treatise, the sense of a non-idiomatic combined xenograph is a *biune mental entity* (*dynamic brain symbol, dynamic mental state*) of its creator (as me) or of its any interpreter, one *hypostasis* (*way of existence*, *aspect*) of which is a certain *mental process* (*operation*) *of coordinating the classes designated by relatively simple constituents of the xenograph*, while the other hypostasis is *the class resulted by that mental process*; the former hypostasis of the sense can be called *the sense-producing operation on the xenograph*, while the latter hypostasis of the sense can impartially be called *the designatum* (*designation value,* pl. "*designata*") *of the xenograph* or, alternatively, *the subject class of the sense* – as opposed to *the class-operata* (*operated classes*), which can be called *the object classes of the sense*. Thus, the sense of a xenogrpah (and generally of a xenonym) is a *mental process* – a *stream of thought* (cf. James [1890; 1950, pp. 224, 225]), and not a *memorized* (*as if*

static) *mental state* such as its designatum or its *denotatum* (*intended import value*). Various kinds of alternations of opposites are illustrated in the treatise by pointing to some of their live instances.

89) In the items 82–88, I have explained in general outline how various features of A_1 are incorporated into $\mathbf{A}_1$ owing to the interrelation between the two organons, according to which $\mathbf{A}_1$ is the *panlogographic interpretans* (*formalized metalanguage*) *of* A_1, while A_1 is the *euautographic interpretand of* $\mathbf{A}_1$. In this item, I shall explain in greater detail how the EAADM D_1 of A_1 is incorporated into the PLAADM $\mathbf{D}_1$ of $\mathbf{A}_1$ and how D_1 and $\mathbf{D}_1$ are fused into the EnSPGAADM (BUE&PLAADM) D_1 of A_1.

i) The set of *schematic panlogographic* and *metalinguistic rules* of inference and decision of A_1, in which all constituent *formulary* (*categorematic*) *elemental* (*primitive*, *atomic* or *molecular*) *panlogographs*, i.e. *elemental panlogographic formulas* (*ElPLF's*) *of* $\mathbf{A}_1$, are used *xenonymously*, i.e. as *eupanlogographs*, for mentioning common (general) *EF's of* A_1 of their ranges, is denoted by as 'D_1' and is called the *Advanced Algebraic Decision Method* (*AADM*) *of* A_1 or the *Euautographic AADM* (*EAADM*). *The same set of rules*, in which *the same constituent ElPLF's* are pescinded from their xenonymous denotata and are used *autonymously*, i.e. as *tychautographs*, for mentioning themselves or their homolographic (photographic) token-classes, is denoted by '$\mathbf{D}_1$' and is called the *exclusive panlogographic extension of* D_1 and also the *Advanced Algebraic Decision Method* (*AADM*) *of* $\mathbf{A}_1$ or the *Panlogographic AADM* (*PLAADM*).

ii) In accordance with the TAEXA between D_1, which is applicable to ER's of A_1, and $\mathbf{D}_1$, which is applicable to PLR's of $\mathbf{A}_1$, the union and superposition of D_1 and $\mathbf{D}_1$ is denoted by 'D_1' and is called the *inclusive endisemasipasigraphic* (*EnSPG*) *extension* of D_1 and also the *AADM of* A_1 or the *Biune Euautographic and Panlogographic AADM* (*BUE&PLAADM*) or the *Endosemasiopasigraphic AADM* (*EnSPGAADM*).

iii) By the above two points, D_1, $\mathbf{D}_1$, and D_1 are syntactically the same set of rules of inference and decision, which semantically differ from one another by the mental attitude of the interpreter (as me) towards their constituent ElPLF's and towards to the formulas, to which these rules apply. Hence, the above-mentioned two extensions of D_1, the exclusive one and the inclusive one, are *mental* (*psychical*, *imaginary*). Accordingly, the way, in which $\mathbf{A}_1$ or A_1 incorporates (condenses) the features of A_1, which have been indicated in the items 20 and 21 can most concisely be described by stating that those items apply, *mutatis mutandis*, verbatim with $\mathbf{A}$, $\mathbf{D}$, and "panlogographic" ("PL"), or with A, D, and "endosemasiopasigraphic" ("EnSPG"), in place of A, D, and "euautographic" ("E") respectively, while $\mathbf{D}_1$ or D_1 *is the same set of rules of inference and decision as* D_1 *subject to their TAEXA*. Thus, $\mathbf{A}_1$ or A_1 is, just as A_1, a single whole APO, every phase and every branch of which has *the same built-in panlogographic*, or, correspondingly, *endosemasiopasigraphic*, *algebraic*, and hence *analytical*, *decision method in common*, that is denoted by '$\mathbf{D}_1$' or 'D_1' respectively. In this case, the phasing and branching of $\mathbf{A}_1$ or A_1 are the same as those of A_1, while "*PLAPO*" ("*Panlogographic APO*") or "*EnSPGAPO*" ("*Endosemasiopasigraphic APO*") alone, without any modifiers, is the abbreviated generic name of every phase and every branch of respectively $\mathbf{A}_1$ or A_1, which comes instead of "EAPO" ("*Euautographic APO*") being the abbreviated generic name of every phase and every branch of A_1.

iv) In the metalinguistic description of the EADP of an ER **P** of A_1 that is given in the points v–xvi of item 21, '**P**' is the concrete *atomic panlogographic relation* (*APLR*) and '**i**' is the concrete *idempotent atomic panlogographic integron* (*IAPLI*), which are therein depicted between single quotation marks. In order to turn that description into a metalinguistic description of the *PLADP* (*panlogographic algebraic decision procedure*) for a certain *panlogographic slave-relatrion* (*PLSR*), or *PLR-slave*, *of* $\mathbf{A}_1$, the *panlogographic placeholders* (*PLPH's*) '**P**' and '**i**' should be replaced with appropriate *metalogographic* (*metalinguistic logographic*) *placeholders* (*MLPH's*), e.g. with 'P' and 'i', whose ranges are respectively the class of all PLR's of $\mathbf{A}_1$ and the class of all idempotent PLI's of $\mathbf{A}_1$, unless stated otherwise by some added words.

v) Thus, in analogy and in compliance with the respective arguments of the item 21, the *PLADP for a given PLR-slave* P *of* $\mathbf{A}_1$, *of academic or practical interest* (*API*), being primarily a *PLOR (panlogogographic ordinary relation*), is the *algebraic proof*, denoted by **'$\mathbf{D}_1(\mathsf{P})$'**, which proceeds from the identity

$$V(\mathbf{P}) \hat{=} V(\mathbf{P}), \tag{2.38}$$

analogous to (2.12), and which is based on the pertinent rules of $\mathbf{D}_1$ so as to end with the identity of one of the following three forms:

$$V(\mathbf{P}) \hat{=} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \mathbf{i}|\mathbf{P}\rangle & \text{(c)} \end{cases}, \tag{2.39}$$

analogous to (2.17). Just as in the case of the EADP for a given ER **P** of A_1, $V(\mathbf{P})$ is the *primary*, or *initial*, *validity-integron* (*PVI* or *IVI*) *of* **P**, whereas $\mathbf{i}|\mathbf{P}\rangle$ is a certain *irreducible*, or *ultimate*, *validity-integron* (*IRVI* or *UVI*) *of* **P** other than 0 or 1. In accordance with (2.13), $V(\mathbf{P})$ satisfies the *idempotent law*:

$$V(\mathbf{P}) \hat{\wedge} V(\mathbf{P}) \hat{=} V(\mathbf{P}) \tag{2.40}$$

and hence $\mathbf{i}|\mathbf{P}\rangle$ satisfies the similar law:

$$\mathbf{i}|\mathbf{P}\rangle \hat{\wedge} \mathbf{i}|\mathbf{P}\rangle \hat{=} \mathbf{i}|\mathbf{P}\rangle, \tag{2.41}$$

analogous to (2.18). The pertinent one of the three conditions (a), (b), and (c) of the scheme (2.39), which a given **P** satisfies, turns into an identity that is denoted by '$\mathbf{T}_{1+}(\mathbf{P})$', '$\mathbf{T}_{1-}(\mathbf{P})$', or '$\mathbf{T}_{1\sim}(\mathbf{P})$' respectively or indiscriminately by '$\mathbf{T}_1(\mathbf{P})$' and is called the *panlogographic master-theorem* (*PLMT*), or *panlogographic decision-theorem* (*PLDT*), *for* **P**, or more generally an *PLDT*, or *DT* (*decision-theorem*), *of* $\mathbf{A}_1$. Each *rule of inference* (*transformation*) that is used in PLADP's is alternatively and more specifically called a *rule of PLADP's* or a *PLADP rule* (*PLADPR*). A PLR of $\mathbf{A}_1$ is called a *decided PLR* (*DdPLR*) if it has a PLDT (PLMT). The metalinguistic three-fold scheme (2.39) is called the *panlogographic decision-theorem* (*EDT*) *scheme,* or *pattern, for* **P**.

vi) Just as in the case of EDT's (see the item 21x), in accordance with the distinctive form of an PLDT (PLMT), *its PLR-slave is decided* to be a PLR of exactly one of the three *decision classes* as stated in the following *decision rule* [*for PLR's*] *of* $\mathbf{A}_1$:

> A DdPLR **P** of $\mathbf{A}_1$ is said to be: (a) *valid* if its PLDT has the form (2.39a); (b) *antivalid* if its PLDT has the form (2.39b); or (c) *vav-neutral* (or *vav-indeterminate*), i.e. *neutral* (or *indeterminate*) *with respect to the validity-*

values validity and antivalidity or, in other words, to be *neither valid nor antivalid*, if its PLDT has the form (2.39c) subject to (2.41).

Therefore, in agreement with what was stated in the point iii of this item, the points v–xvi of the item 21, which are pertinent to the solution of vavn-decision problem for ER's of A_1, apply, *mutatis mutandis*, verbatim with **A**, **D**, and "panlogographic" ("PL"), or with A, D, and "endosemasiopasigraphic" ("EnSPG"), in place of A, D, and "euautographic" ("E") respectively.

vii) At the same time, the DdPLR's of $\mathbf{A}_1$ have the following peculiarities:

a) A PLR is valid if and only if every ER in its range is valid.

b) A PLR is antivalid if and only if every ER in its range is antivalid.

c) In the general case, the range of a vav-neutral PLR contains an indefinite (infinite) number of vavn-suspended ER's and also an indefinite (infinite) number of vavn-decided ER's of each of the three classes: valid, antivalid, and vav-neutral.

Consequently, a valid, or antivalid, PLR is a solution of the vavn-decision problem for every ER in its range. By contrast, a vav-neutral PLR is not, in the general case, a solution of the vavn-decision problem for every ER of its range. Therefore, a concrete ER of academic or practical interest, of the range of a vav-neutral PLR should be subjected to the EADP of its own in order to decide on its validity-value provided of course that this has not been done earlier. At the same time, any euautographic instance of a vav-neutral PLOR, which has the same pattern in terms of irreducible elementary (atomic or molecular) ER's as the pattern of the vav-neutral PLR in terms of irreducible elementary PLR's, is obviously a vav-neutral ER. Particularly, the so-called *analographic euautographic variant (isotoken) of a vav-neutral PLR is a vav-neutral ER*.

viii) In accordance with the previous point, in order to solve the vavn-decision problem for a given ER, it seems preferable to solve the vavn-decision problem for an adequately patterned PLR because the PLR condenses an infinite number of other ER's, for which the vavn-decision problem will be solved simultaneously with that for the given ER by the same work input.

3. Pragmatic aspects of the treatise

In contrast to the previous section of the Preface, the object of this subsection is to make explicit some *pragmatic aspects* of the treatise, i.e. some aspects of its *form* rather than those of its *subject matter*. I shall particularly explicate the most general features of organization of the treatise and also the most important auxiliary (especially abbreviative) devices, which facilitate wordings but which are not immediately relevant to their subject matter. Still, it is often impossible to separate form from matter, because the former is an aspect of the latter. Some general pragmatic devices employed in the treatise are conventional or self-explanatory ones, but some of these will also be explicated along with unconventional ones for the sake of completeness and clarity.

3.1. The layout of the treatise

1) The treatise is divided into three parts: *Preface* (this one), the *main text*, treating primarily the object logistic systems as outlined above, and the *Appendices*, treating primarily various aspects of the XML (exclusive metalanguage) of those systems. The main text is divided into five chapters. Sections 1 and 2 of Preface are in fact an introduction into the subject matter of the treatise *in general outline*. Therefore, Preface has the status of an additional chapter that can be called the C*hapter 0*. By contrast, Chapter I of the treatise, entitled "*Introduction*", is an introduction *in depth* both into Psychologistics and into the subject matter of the treatise. The totality of five *Appendices*, entitled "*Metalinguistic Themes*"¸ has the status of another additional chapter of the treatise that can be called C*hapter 6*. Separate appendices can be read or be selectively consulted in parallel with reading the main text in accordance with the pertinent references.

2) Each chapter is divided into sections, which are numbered by successive Arabic numerals relative to the beginning of a chapter and which are the *basic unit*s of the treatise. When necessary, a section is divided into subsections, while a subsection may, in turn, be divided into sub-subsections. Subsections are numbered with two Arabic numerals separated by a dot; the first numeral denotes the number of a section and the second one denotes the number of a subsection in the section. Such a two-component Arabic numeral is called a *double position-numeral*, whereas its *denotatum* (*denotation value*, pl. "*denotata*") is called a *double position-number*. Sub-subsections are numbered with *triple position-numerals*, the first two constituent

numerals of which form the double position-numeral of the subsection, in which a sub-subsection occurs, whereas the last constituent numeral denotes the ordinal number of the sub-subsection relative to the beginning of the subsection. Chapters, sections, subsections, and sub-subsections are called the *major articles* of the treatise. In addition to their identifying numerals, all major articles in the exclusion, perhaps, of some sub-subsections are provided with the appropriate wordy headings. A separate appendix has the status of a section. Accordingly, it is itemized like a section of the main text with the only difference that every double position-numeral occurring on an appendix is attached with the prepositive letter "A".

3) For convenience in cross-reference, and also for the sake of ordering and separating self-contained articles of the treatise ranking below sub-subsections, most of them, called *minor articles*, are provided with *bookmarks* in the form of *bold-faced logical heads* (*logical names*), which classify the articles by the pertinent *taxonyms* (*taxonomic names*) and which number the articles of the same class with successive postpositive double position-numerals similar to those numbering subsections. In forming the logical heads, I use the taxonomic abbreviations: "Ax" for "Axiom", "Cmt" for "Comment", "Cnv" for "Convention", "Crl" for "Corollary", "Df" for "Definition", "Dict" for "Dictionary", and "Th" for "Theorem". As usual, *a corollary is an instance of an axiom, theorem, or definition that is just demonstrated and that requires no additional proof*, whereas *a lemma* (pl. "lemmas" or "lemmata") *is an auxiliary theorem*. The logical heads carrying the above taxonyms are, as a rule, set *solid* (undisplayed, without surrounding white space) and *non-centered*. If, however, a logical head carrying one of the strings "Ax", "Df", "Crl", "Ex", and "Th" is immediately followed by a large group of separate *itemized interrelated* or *conspecific* articles of the given class (species) then the logical head can, from esthetic and pragmatic considerations, be *centered*.

4) The statement of an object or meta theorem or of an [object] lemma is separated from the proof following it by the heading flag "**Proof:**". Sometimes, however, the proof of a theorem may precede the statement of the theorem or be omitted because it is self-evident or because it is given informally prior the statement.

5) At the end of a minor article, I put a heavy dot, •, especially in the cases in which there might otherwise be doubt of the end. If a minor article is a theorem, lemma, or corollary, followed by its proof, then the heavy dot is put at the end of the

proof. In a single reference to several *congeneric* minor articles, the plural number form of any of the above abbreviations will be formed by suffixing it with “s” without an apostrophe.

6) The logical heads of some minor articles, which carry the taxonyms “Ax”, “Th”, “Df”, “Crl”, and “Lemma”, are preceded by the *flags* (*special marks*) such as: †, º, *, etc that indicate a certain subclass, to which a minor article thus marked is relegated. For instance, the dagger, †, preceding an article head signifies that the article includes one or more formation rules of the pertinent logistic system; º signifies that the article includes one or more concrete euautographic axioms or theorems of A_1; * signifies that the article includes one or more concrete panlogographic axioms or theorems of $\mathbf{A}_1$; ** signifies that the article includes one or more meta-axioms or meta-theorems of inference or decision of A_1, i.e. axioms or theorems of inference or decision of A_1, which belong to the IML of A_1 and not to A_1 itself. The meanings of any other flag will be explained as soon as it occurs for the first time. The absence of any flag before the head of a minor article evidences that that article is not immediately relevant to the setup and to the subject (intrinsic) axioms, theorems, lemmas, or corollaries of any one of the object logistic systems Also, the double position-numeral following the taxonym in a logical head is irrelevant to the flag (if any) preceding the head. For instance, the logical head “*Th 10.5” refers to the fifth one of *all* theorems occurring in section 10 in no connection with the asterisk, *. Accordingly, in cross-references, the logical heads of minor articles will be used without any prepositive flags.

7) In the main text, most of the displayed expressions are numbered relative to the beginning of the chapter, in which a displayed expression occurs, by successive *parenthesized* double position-numerals, which are put to the right of a displayed expression and which are similar to those of subsections. Namely, in this case, the first constituent of a double position-numeral denotes the number of the section, in which the displayed expression occurs, whereas its second constituent, separated from the first one by a dot, denotes the number of the displayed expression relative to the beginning of the section. In accordance with the item 2 of this subsection, displayed expressions occurring in an appendix are numbered likewise with the only difference that the first constituent of the double position numeral of a displayed expression has the letter “A” before the numeral denoting the ordinal number of the appendix.

8) If a displayed expression is a corollary of another displayed expression that occurs in the same section and is numbered with a certain double position numeral relative the section, the former will be numbered (bookmarked) by the same double position numeral suffixed either with an adscript letter “a”, “b”, etc or with an appropriate literal subscript, whose lexical and, when applicable, etymological sense will be explained when it is used for the first time. The separate successive displayed special identities that constitute the *algebraic decision procedure* (*ADP*) for a certain *slave relation*, i.e. the separate displayed identities that constitute the algebraic proof of the *master,* or *decision*, *theorem* (*MT* or *DT*) for the slave relation, will be numbered by the same double position numeral suffixed with the successive numeral subscripts ‘$_1$’, ‘$_2$’, etc.

9) If an item, which occurs in a certain *host* chapter and which is bookmarked with the pertinent double position numeral, is a *relatum* that *is referred to* in another chapter then the reference to the relatum is made by using the parenthesized *triple position numeral*, in which the original double position numeral of the relatum is preceded by the capital Roman numeral, which denotes the number of the host chapter of the relatum and which is separated from the double position numeral by a dot.

10) Several *interrelated* or *conspecific* (analogous or homologous) articles (as axioms, theorems, definitions, comments, etc) can be placed under a common logical head as separate items in two different ways. First, the items can be set *solid* (not displayed, without surrounding white space) and be numbered relative to their common logical head either by successive prepositive Arabic or Roman numerals or by successive prepositive small letters of the Latin or Greek alphabet, alone or with some labels (as primes). In this case, the individualizing logical head (a numeral or numeral equivalent) of an item is separated from it by a right parenthesis, whereas a cross-reference to a separate item of the minor article is made either descriptively after the manner of “the item 1 of Df 2.1” or “Df 2.1, the item 1” or briefly after the manner of “Df 2.1(1)”. Second, the items of a minor article can be displayed and be numbered by the corresponding parenthesized double position-numerals as indicated above in the item 7.

11) Besides numerous minor articles that are classified by using the taxonym “Definition”, abbreviated as “Df”, in their logical names, there are in the treatise some minor articles of various lengths that are classified by using the taxonym

“Dictionary”, abbreviated as “Dict”, in their logical names. The two kinds of articles differ in the following respects. Some dictionaries are *etymological* ones, while the others are *terminological* (*lexical*) ones. An *etymological dictionary* is a group of *etymological definitions* of Anglicized morphemes (usually combining forms), which are used in the appropriate Anglicized metaterms (metalinguistic terms) that are defined independently in the treatise or elsewhere. Each such dictionary is compiled primarily from Pring [1982] if it is an English-Greek one and from Simpson [1959] if it is an English-Latin one. Greek–English Dictionary (GED) and English–Greek Dictionary (EGD) of Pring [1982] or Latin–English Dictionary (LED) and English–Latin Dictionary (ELD) of Simpson [1959], are denoted as “Pring [1982, GED]” and “Pring [1982, EGD]” or as “Simpson [1959, LED]” and “Simpson [1959, ELD], respectively. At the same time, unless stated otherwise, “Pring [1982]” and “Simpson [1959]” will hereafter mean Pring [1982, GED] and “Simpson [1959, LED] A *terminological dictionary* is a group of *terminological explanatory definitions* of certain metaterms of the treatise. The *separate definitions* (*vocabulary entries*) of an etymological or terminological dictionary are relatively independent, while their *definienda* (*entries*) are *phonographic* (*wordy*, *verbal*) expressions, which are arranged *alphabetically*. By contrast, a “Df”-article is a single definition or a group of definitions of euautographs or panlogographs or metaterms. In the latter case, if the order, in which the separate wordy definienda are arranged, differs from the alphabetic one then this fact signifies that the definitions of the definienda are interrelated in that order so that the definitions may became incongruent if the order is changed.

12) Besides this treatise, being the principal part of Psychologistics, the latter is supposed to include an indefinite number of relatively independent essays, which will be called *Psychologistic Essays*, or briefly *Essays*, and which will form supplementary material to the subject matter of the treatise, to be included under the common heading “*Psychologistic themes*”, – the material to be treated primarily *egocentrically* (*like the treatise itself*) with one or another degree of thoroughness. This supplementary material is not however formally included into the treatise, because it is not related to the main object of the treatise as straightforwardly as the Appendices of the treatise. Consequently, in contrast to a separate Appendix, which has the same status as a *section* of the treatise, a separate Essay has the status of a

chapter of Psychologistic themes and hence the same status as a *chapter* of the treatise. Since separate Essays are relatively autonomous, therefore each of them is provided with a *list of references of its own*, which is however a certain part of *the list of references of the treatise* that is augmented by the reference to the treatise itself and perhaps by some other references when appropriate. In this case, the treatise will briefly be referred to in the Essays as the *Theory of Trial Logic* or as the *Treatise on Trial Logic* and also most briefly as the *TTL*. An Essay may, when convenient, contain some pertinent fragments of the treatise or of another Essay, repeated or cited.Separate Essays will be submitted singly or in groups as soon as they are completed and, for convenience in cross-reference, they will be numbered by ordinal numerals in that order, and not in the order, in which of the associated topics are used or mentioned of the treatise for the first time. For instance, various trends of psychology are briefly discussed in historical prospect in Essay 1 (E1); Essay 2 (E2) addresses the complete taxonomy of the senses of a man; native languages and their codes are discussed in Essay 3 (E3); Essay 4 (E4) addresses special quotations and some other relevant topics in greater detail as compared to their treatment below in this Preface; Essay 5 (E5) addresses my solution of the problem of univerals; etc.

3.2. References

I employ the British system of references. “James [1890; 1950, vol. 1, p. 1]”, “Church [1956]”, “Church [1956, chaps I, II]”, and “Church [1956, pp. 3–9, 23–31]” are typical self-explanatory examples of such references (see also the item 11 of the previous subsection).

The dictionaries A Merriam-Webster® [1978, 1979, 1981] and Allen (Consultant Editor) [2003] will as a rule be referred to by using the following abbreviations of their titles: “WNDS” for “Webster’s New Dictionary of Synonyms”, “WNCD” for “Webster’s New Collegiate Dictionary”, “WTNID” for “Webster’s Third New International Dictionary of the English Language Unabridged”, “APED” for “Allen’s Penguin English Dictionary” respectively. I have indicated the specific years of edition of the dictionaries simply because copies of these editions are in my library. The reader may of course consult copies of any other editions of the dictionaries.

The etymological senses of all new terms of my own and, when desirable, of some established terms or morphemes, which originate from Latin or Greek, are given according to Simpson [1959] or Pring [1982] respectively, unless stated otherwise. In spite of the last reservation, I shall most often refer to the above two dictionaries explicitly when I use them.

In citing a dictionary definition verbatim, I preserve the style and particularly all abbreviations that are used in the dictionary from which the definition is taken.

I shall have various occasions to refer to *Principia Mathematica* by Whitehead and Russell. As compared to the 1st edition of the monograph, Whitehead and Russell [1910–13, vols. 1–3], its 2nd edition: 1925, 1927, 1927, 674+742+491 pp., contains the following additional material: Introduction to the 2nd edition, pp. xiii–xlvi, and Appendices A, B, C in vol. 1, 34+15+9+8 pp. The abridged paperback edition of the 2nd ed. of vol. 1, namely, *Principia Mathematica to∗56*, 1962, Cambridge at the University Press, xlvi+410 pp., which has been repeatedly reprinted, includes: (a) Introduction to the 2nd ed. – pp. xiii–xlvi; (b) Introduction (comprising a brief introductory statement not in any numbered section and Chapters I–III) and Part I (to ∗43 inclusive) – pp. 1–326; (c) Section A (of A and B) of Part II, Appendices A, B, and C, and List of Definitions – pp. 327–410. The pagination of the portions (a) and (b) in the paperback edition is the same as in the full 2nd ed. of vol. 1. However, unlike the pagination of the portion (c) of the paperback edition, Part II, Appendices A, B, and C, and List of Definitions occupy pp. 327–674 of the full 2nd ed. of vol. 1. I shall have no occasion to refer either to Part II or to the appendices. Therefore, I shall refer to a specific passage of vol. 1 of Whitehead and Russell [1925–27] either after the manner of "Whitehead and Russell [1925; 1962, p. xvi]" if the passage does not occur in the 1st ed. of vol. 1 or after the manner of "Whitehead and Russell [1910; 1962, pp. 96, 97]" if otherwise.

3.3. Brackets

Df 3.1. 1) The pairs of marks ( ), [ ], { }, 〈 〉 of various sizes and thicknesses, which are used in writing and printing to enclose matter, are conventionally called *brackets*. Specifically, ( ) are called *round brackets* or *parentheses*, [ ] are called *square brackets*, { } are called *curly brackets* or *braces*, and 〈 〉 are called *angle brackets***.** Any of the marks (, [, {, 〈, is indiscriminately called a *left*, or *opening*, *bracket*, and also a *bra*, whereas any of the marks),], }, 〉 is indiscriminately called a *right*, or *closing*, *bracket*, and also a *ket*. Discriminately, the different bras, or kets, are called a *round*, *square*, *curly*, or *angle bra*, or *ket*, in that order. A single bra or a single ket is called an *atomic punctuation sign*, or *mark*, whereas a *bra-ket pair*, i.e. the pair of a bra and of the respective ket, is called a *paired molecular punctuation sign*, or *mark*. Any of the above brackets is qualified *decisive*.

2) Besides the decisive brackets, a pair of *forth-slashed* or *back-slashed virgules*, / / or \ \, and the pair of *upright strokes*. | |, are also used to enclose matter, whereas the *sole upright stroke*, |, is used (e.g. in Church [1956, pp. 72, 82]) to indicate the end of the scope of an operator of substitution. Any of these marks is called an *indecisive bracket*, whereas the left (opening), or right (closing), bracket of a pair of indecisive brackets is as before indiscriminately called a bra, or a ket, respectively, or, discriminately, a *forth-slashed*, or *back-slashed*, *virgule*, or *stroke*, *bra* or *ket*, in that order. The sole upright stroke restricting the scope of an operator of substitution is qualified a ket. I shall also use the *combined pair of stroke-angle brackets*, | 〉, which are naturally qualified a bra and a ket in that order. •

Cmt 3.1. I have adopted the words "bra" and "ket" from Dirac [1958, §5], who uses them as brief names of a left angle *bra*cket, 〈, and of a right angle brac*ket*, 〉, respectively. Dirac also uses the metaterms "*bra-vector*" and "*ket-vector*" as common verbal names of a row-vector and a column-vector in a Hilbert space, which are denoted by symbols such as '$\langle A|$' and '$|A\rangle$' and which are both associated with a quantum-mechanical state; '$\langle A|$' is the Hermitian conjugate of '$|A\rangle$'. My use of the pair | 〉 has nothing to do with its use by Dirac. Angle brackets are sometimes called *broken*, or *pointed*, *brackets*, but these metaterms will not be used in the treatise.•

The pairs of brackets [ ] and ( ) are *primary molecular euautographic ordinary*, or *logical*, *punctuation marks* (*PMEOPM's* or *PMELPM's*) of both A_1 and $\mathbf{A}_1$, while the pairs 〈 〉 and | 〉 are additional PMEOPM's (PMELPM's), which are employed in $\mathbf{A}_1$, but not in A_1. The ways of use of brackets of any given kind in A_1 or $\mathbf{A}_1$ are determined by the pertinent formation rules of A_1 or $\mathbf{A}_1$ and also by subsequent abbreviative conventions allowing to omit some occurrences the brackets, when desired, and to unambiguously recover them, when necessary. If the way of use of brackets of a given kind is temporarily changed, then that change and its scope are explicitly indicated by the corresponding statement (definition). Thus, brackets of any kind are used in A_1 or $\mathbf{A}_1$ *formally* (*technically*, *univocally*). The kinds of brackets that can be used in the XML are not restricted in advance. Also, any kind of brackets is or can be used in the XML either formally, i.e. under a certain formal definition, or informally. The most conspicuous informal functions that pairs of square, round, and angle brackets perform in the XML of the treatise can be summarized as follows.

i) Square brackets are used in forming names of references in the framework of the British system of references which is employed in the treatise (see the subsection 3.2 of this Preface for examples).

ii) If a part of a new or old verbal term is enclosed in square brackets then, in subsequent uses, the term can, for the sake of brevity, be abbreviated by omission of the [*square*-]*bracketed* expression, provided that the abbreviation cannot result in confusion.

iii) A certain double position-numeral enclosed in parentheses is a logical name (bookmark) of the respective displayed expression.

iv) Pairs of parentheses are used interchangeably with other appropriate punctuation marks (as commas or dashes) for indicating a linguistic form that is equivocally called a *parenthesis*, and that is defined in WTNID thus:

> «**parenthesis** … *n*, *pl* **parentheses** … **1 a :** an amplifying or explanatory comment inserted in a passage to which it may be grammatically unrelated and from which it is usually set off by punctuation (as curved lines, commas, or dashes) ...»

v) If a verbal term, – a new one, which is introduced for the first time by a formal definition or contextually, or an old one, which is already known either from an earlier definition or from another source and which is just used in a statement, – is followed by some one or some more parenthesized verbal terms then each of them is a synonym of the term immediately preceding the parenthesis. In this case, a parenthesized expression following the unparenthesized term can serve either as an *explicative definiens* or as a *synonymic definiens* of the latter.

vi) In accordance with the entry 4b in the article «**bracket**» of WTNID, a pair of angle brackets can be used in the XML «to enclose mutilated passage or the expression of an abbreviation in a text or to enclose quotations or verbal illustrations in a reference work such as a dictionary…».

3.4. Quotations

In this treatise, besides ordinary quotations that are used but occasionally, I widely use various so-called *special*, or *attitudinal*, *quotations* (*SQ’s*), which indicate the kind of a value, and hence the value itself, of the interior of a quotation, which is put forward as its accidental (circumstantial) *denotatum* (*denotation value*, pl. "*denotata*"). In order to state an *ordinary quotation* (as a repetition of the exact passage of another work or of the title of a book), I employ French double angle quotation marks, « », instead of *ordinary* English single or double quotations marks (as defined, e.g., under the vocabulary entry **quotation mark** in WTNID), whereas the latter are freed of their ordinary functions and are used only as *special quotation marks* (*SQ marks*). In this case, the light-faced or bold-faced single or double, straight (and curly) or slant English quotation marks are used differently. I shall also use a back-slashed and a forth-slashed virgule in the superscript line as another pair of SQ marks. The pair of quotation marks that is used for making an ordinary or special quotation will be called *the exterior of the quotation*, whereas the *graphonym* (*graphic expression*) quoted will be called *the interior of the quotation*. The senses of all SQ's will be defined in Introduction after introducing the pertinent terminology. Meanwhile, the following preliminary remarks will be useful.

I do not follow Frege [1893–1903, vol. 1, p. 4] and his followers either in *admitting only a single kind of SQ's, each of which is the name of its interior*, and which I call *Fregean*, or *Frege's*, *quotations* (*FQ's*), or in *obstinately attempting to indicate autonymy with the help of the appropriate SQ marks in all cases* simply because such an attempt is *impracticable*. For forming FQ's, which I also call *proper*, or *strict*, *autonymous quotations* or *kyrioautonymous quotations* (*KAQ's*) and which I shall use quite rarely, I shall employ *slant light-faced single quotation marks*, ʻ ʼ. Most often, I shall employ, – I have already started to, – *curly* (*decisive*) or *straight* (*indecisive*) *light-faced quotation marks*, *single ones*, ' ' or ' ', which I shall call *homoloautographic*, or *photoautographic*, *quotation marks* (*HAQ marks*), and *double ones*, " " or " ", which I shall call *iconoautographic*, or *pictoautographic*, *quotation marks* (*IAQ marks*). Also, I shall from time to time employ *slant light-faced double quotation marks*, ″ ″, which I shall call *phonoautographic quotation marks* (*PAQ marks*), and *light-faced virgule-like quotation marks*, \ /, which I shall call *enneoxenographic*, or *semantic*, or *sense*, *quotation marks* (*EXQ marks*). Accordingly, an SQ will be called: a *homoloautographic*, or *photoautographic*, *quotation* (*HAQ*) if it is formed by enclosing a *graphonym* between HAQ marks; an *iconoautographic*, or *pictoautographic*, *quotation* (*IAQ*) if it is formed by enclosing a graphonym between IAQ marks; a *phonoautographic*, *quotation* (*PAQ*) if it is formed by enclosing a *graphonym* between PAQ marks; a *enneoxenographic quotation* (*EXQ*) if it is formed by enclosing a *graphonym* between EXQ marks. HAQ's, IAQ's, and PAQ's are indiscriminately called *common*, or *lax*, *autographic quotations* and also *cenautographic quotations* (*CAQ's*). KAQ's and CAQ's are indiscriminately called *special autographic quotations* (*SAQ's*), whereas all quotation marks that are used for forming SAQ's are called *SAQ marks*.

I employ the exterior of an HAQ or IAQ for indicating my *ad hoc* (*epistemologically relativistic*) mental attitude, according to which its interior denotes the class of distinct recurrent recognizably same graphonyms, which occur in the treatise and which are called *isotokens of the interior*. The pair of quotation marks that is used for making an ordinary or special quotation is called *the exterior of the quotation*, whereas the graphonym quoted is called *the interior of the quotation*, or *percept-class*, *of the interior of the HAQ* or *IAQ* respectively. In this case, an HAQ denotes *the class of homolographic* (*photographic*), i.e. *proportional* or particularly *congruent*, *isotokens of its interior*, whereas an IAQ denotes *the class of iconographic* (*pictographic*), i.e. of both homolographic and *analographic* (*stylized*) *isotokens of its interior*. The interior of an IAQ may contain some constituent graphonyms that are known from a previous definition or definitions to be a *homolograph*, i.e. a graphonyms that have only homolographic isotokens. In this case, the isotoken-class of the interior of the IAQ is supposed to preserve this property. The interior of a PAQ is a *phonograph*, i.e. a graphonym that has phonic (vocal) values, which are produced when the graphonym is read orally and which are called *phononyms*. Accordingly, the PAQ denotes the class of distinct recurrent recognizably same phononyms, which are produced by orally reading the phonograph and which are called *its paratokens* and *isotokens of one another*. Accordingly, the class denoted by the PAQ is called *the paratoken-class of the interior of the PAQ*. An isotoken or a paratoken of a phonograph is indiscriminately called a *token* of the phonograph. Incidentally, if the interior of an IAQ is a phonograph then the IAQ may, depending on the mental attitude of its interpreter, denote either the isotoken-class or a paratoken-class of its interior or else the union of the two classes that is called the token-class of the interior.

The sense (*sense value*) *of*, or *expressed by, a complex* (*combined*) *linguistic graphonym*, – provided that the latter has the sense thus defined, – is a *biune mental* *process* (psychical entity, brain symbol) of the maker or interpreter of the graphonym (as me), which includes (i) a *sense operation of coordination of the classes* that are *designated* by the relevant simple constituent parts of the graphonyms and that are called *the object classes of the sense,* and which also includes (ii) *the class that is resulted by the sense operation and that is designated by the graphonym*. The latter class is called *the designatum* (pl. "*designata*") *of the graphonym* and alternatively *the subject class of the sense of the graphonym*. It is understood that if a graphonym is regarded as a simple one or is an idiom then its sense coincides with its designatum. Consequently, one of two given senses (sense values) of a glossonym is said to be broader, or narrower, than the other one if the subject class of the former is broader, or correspondingly narrower, then the latter.

The bold-faced quotation marks **′ ′**, **‘ ’** or **' '**, **“ ”** or **" "**, **″ ″**, and **\ /**, and quotations that they form will be qualified as *quasi-kyrioautographic* (*QKA*), *quasi-homoloautographic* (*QHA*), and *quasi-iconoautographic* (*QIA*), *quasi-phonoautographic* (*QPA*), and *quasi-enneoxenographic* (*QEX*) *quotation marks* (*Q marks*) and *quotations* (*Q's*), respectively. The interior of a QKAQ, QHAQ, QIAQ, QPAQ, or QEXQ is either entirely a placeholder or it contains some placeholders, upon replacing all of which with appropriate concrete graphonyms the bold-faced quotation marks should be replaced with the corresponding light-faced ones. That is to say, QKAQ's, QHAQ's, QIAQ's, QPAQ, and QEXQ are placeholders (place-holding variables) for KAQ's, HAQ's, IAQ's', PAQ, or EXQ respectively, whereas the latter are constants. QHAQ's, QIAQ's, and QPAQ's are indiscriminately called *common*, or *lax*, *quasi-autographic quotations* and also *quasi-cenautographic quotations* (*QCAQ's*). QKAQ's, QCAQ's, and QEXQ's are indiscriminately called *special*, or *attitudinal*, *quasi-autographic quotations* (*SQAQ's*), whereas all quotation marks that are used for forming SQAQ's are called *SQAQ marks*. The procedure of using SQ's (special quotations), which has preliminarily been described above, will be called *Special Quotation Method* (*SQM*).

Thus, the reader should remember that quotations marks of the different forms and shapes, which he encounters in the treatise, are not selected spontaneously and that therefore they are not interchangeable. At the same time, as I have already pointed out previously, no attempt will made to indicate autonymy with the help of autographic quotations in all cases because such an attempt is doomed to failure. I resort to the quotation device only where confusion between autonymous and xenonymous uses of xenographs might otherwise be harmful. In some cases, such confusion is harmless, while in many other cases, which will be made explicit in due course, it is productive and indispensable. For instance, in stating verbal definitions, I shall often use the defining predicate “is called” which should in principle be followed by the IAQ of a xenographic definiendum. In many cases, however, it will be, not only harmless, but useful to employ unquoted xenographic definienda after that predicate.

It is hoped that after the above preliminary remarks the reader will have no difficulties in understanding the text until the more detailed explanations concerning the senses of various special quotations and their uses are given

Chapter I: Introduction

1. Introduction to Psychologistics

1.1. “Psychologistics” and “mind”

Philosophy and logic as a part of it are products of individual *minds*, properly turned, trained, and meditated, and interpersonally verified by means of their intercommunication via exteroceptive signs (publications). That is to say, philosophy in general and logic in particular are pure theoretical disciplines that are, in the first place, based on *introspection* of each philosopher or each logician, and not on his extrospection, i.e. not on his observations and examinations (experiments). Therefore, there is no philosophy and no logic until the philosopher or the logician examines his own mind. «Γνῶθι σαυτόν!» \Γnóθi saftón!\ – «Gnothi seauton!» – «Know thyself!», said Socrates.

In this connection, I recall that (see Preface) the banner “*Psychologistics*”, under which this treatise is included, is an abbreviation of the description (descriptive name) “*Psychological foundations of logic and logical foundations of psychology*” (“*PFL & LFP*”). In this case, by “psychology”, I mean traditional *introspective psychology* (as opposed to various trends of modern *extrospective psychology*), or more precisely *cognitive* and *conative* aspects (as opposed to *affective* ones) *of introspection* (*introspective psychology*) *of my own*, along with the doctrine of *physicalistic monism* (relegated to *philosophical psychology*) that I have *adopted* and *interiorized* (*internalized*) with the minimum of abstractions and assumptions as follows.

Df 1.1. *My mind* is *my cerebral cortex* at any current moment of my *life* (*existence*) in the pertinent one of the two *states* (*hypostases*, *ways of existence*): *consciousness* (*awareness*) and *sleep*. Accordingly, my mind (cerebral cortex) is said to be *my conscious*, or *waking*, *mind* (*cerebral cortex*) when it wakes and *my subconscious*, or *sleeping*, *mind* (*cerebral cortex*) when it sleeps. Thus, particularly, *my conscious mind* is *my cerebral cortex* at any current moment of *my life* when I wake and hence when I am conscious and self-conscious, i.e. at any moment, which I mentally experience as my current present succeeding my current past and preceding my current future.•

By *transcendent extrapolation* and hence with certain additional abstractions and assumptions, the above definition can be paraphrased in the *third person singular form* as follows.

Df 1.1a. *The mind of a sapient subject* is *his cerebral cortex* at any current moment of *his life* (*existence*) in the pertinent one of the two *states* (*hypostases, ways of existence*): *consciousness* (*awareness*) and *sleep*. Accordingly, the mind (cerebral cortrx) of a sapient subject is said to be *the conscious*, or *waking*, *mind* of the subject when the latter wakes and *the subconscious*, or *sleeping*, *mind* of the subject when the latter sleeps. Thus, particularly, *the conscious mind of a sapient subject* is *his cerebral cortex* at any current moment of *his life* when he wakes and hence when he is conscious and self-conscious, i.e. at any moment that he mentally experiences as his current present succeeding his current past and preceding his current future. The mind, or cerebral cortex, of a sapient subject is briefly called *a mind* or *a cerebral cortex* [*in vivo*].•

Here follows a preliminary formal compound definition of "Psychologistics" that comprises the pertinent syntactic (synonymic), semantic, and nominal definitions, which will be explicated below in this section.•

Df 1.2. 1) "Psychologistics" is a *synecdoche* (from Greek "συν"- \sin\ *comb. form*, meaning *together, with,* or *completely*, and "εκδοχή" \ekdoxí, ekdochí\ *s.f.*, meaning *version* or *interpretation*) – a figure of speech, which is used *ad hoc* for referring to certain parts of its unbounded denotatum. For instance, for my purposes at hand, Psychologistics is this treatise, whereas generally this treatise is Psychologistics but not necessarily vice versa. In any case, Psychologistics is a *biune* field of study and discourse, so that the *psychology*, called the *psychologistic psychology* (*PLP*), and the *logic*, called the *psychologistic logic* (*PLL*) or *psycho-logic*, which are *complementary conceptual hypostases* (*ways of existence*, *aspects*) of Psychologistics, can be distinguished and contrasted, but they cannot be separated from each other, – like *matter* and *form* of a thing. As I have already indicated at the beginning of this section, the PLL is the *introspective psychology of mine*. It will, however, be objectified in a sense by Cnv 1.1 that is stated in the next subsection.

2) For convenience in description and study, the *psychologistic logic* (*PLL*) can in turn be divided into two parts, one of which is called the *principal*, or *first*, *PLL* (*PPLL*), and the other one is called the *auxiliary*, or *applied*, or *second*, *PLL* (*APLL*).

The PPLL is a certain *trial* (*three-valued*) *logic* (*TL*), so that it more specifically called the *trial PLL* (*TPLL*) or *psychologistic TL* (*PLTL*). The APLL comprises *three* rigorous self-subsistent systems (irrelevant to the *trial* PLL) of *terminological* and particularly *taxonomic onomatology* of Psychologistics in general and of the treatise in particular, and it is therefore alternatively called the *psychologistic onomatology* (*PLO*) and also the *onomastic PLL* (*OPLL*). Every *metaterm* (*metalinguistic term*) and particularly every *taxonym* (*taxonomic name*) of the PLO is a *description*, or more explicitly *description of the species, through a genus and the difference*, or *differences*, – briefly *DcTrG&D*, *DcSTrG&D*, *DcTrG&Ds*, or *DcSTrG&Ds* in that order, in Latin *descriptio*, or *descriptio species, per genus et differentiam*; or *differentias*, respectively. A *definition* whose definiens is a DcTrG&D or DcTrG&Ds is a traditional *definition through the genus and difference* (*differentia*), or *differences* (*differentiae*), – briefly a *DfTrG&D* or *DfTrG&Ds*, in Latin *definitio per genus et differentiam*, or *differentias*, which was introduced by Aristotle [350 BCE, *Posterior Analytics*] and which is often called a *real*, or *explicative*, *definition*. Therefore, I relegate the PLO to *applied logic* and term it by either of the abbreviated synonymous *descriptions* "APLL" and "OPLL", generic name (headword) of which is "logic" ("L"). In accordance with the above-said, the treatise can alternatively be called "*The psychologistic trial and onomastic logics*" in reference to both logics or as "*The psychologistic trial logic*" thus putting the APLL backward.

a) In accordance with Aristotelian *principle* (*doctrine*) *of opposition and unity of matter and form of a being*, which is callled *hylomorphism* (see Cmt 1.1(3) below in this section), the PLTL (TPLL, PPLL) has two *complementary conceptual hypostases* (*ways of existence*, *aspects*), namely, the *psychologistic trial formal logic* (*PLTFL*) and the *psychologistic trial material logic* (*PLTML*) *adjoint of the PLTFL*, the understanding being that the two can be distinguished and contrasted, but they cannot be separated from each other (cf. two aspects PLP and PLL of Psychologistics). The PLTFL is denoted by '$\mathcal{A}_1$' and is alternatively called the *Combined Algebraico-Predicate Organon* (*CAPO*) or the *Combined Advanced Algebraico-Logical Organon* (*CAALO*). Therefore, either expression "*A theory of the Combined Algebraico-Predicate Organon*" or "*A theory of the Combined Algebraico-Logical Organon*" could be used as another alternative title of this treatise. The word "organon" is used in this treatise in the sense of the description (descriptive name) “*master logical calculus having an inseparable associated trial* (*three-valued, three-fold*) *algebraic decision method*”, the understanding being that “*algebraic*” implies “*analytical*” (“*not tabular*”). $\mathcal{A}_1$ can be thought of as *the sequence of the four interrelated logistic systems* $\mathbf{A}_1$, A_1, I_1, *and* A_1 *in that order,* which will be described as I go along. Meanwhile, I shall remark that $\mathbf{A}_1$, A_1, and A_1 are organons that have similar built-in *trialistic algebraic decision methods* (*TADM’s*) $\mathbf{D}_1$, D_1, and D_1 respectively and that are interrelated as follows. $\mathbf{A}_1$ is the calculus of the so-called *panlogographic relations* (*PLR’s*), which are *panlogographic placeholders* (*PLPH’s*) of the so-called *euautographic* (*genuinely autographic, semantically uninterpreted*) *relations* (*ER’s*) of the calculus A_1, so that a PLR is the *panlogographic interpretans* (*anti-interpretands,* pl. “*interpretantia*”) of the ER’s condensed (comprised) in it’s range, while the latter are the *euautographic interpretands* of the PLR. I_1 is the so-called *conservative catlogographic* (*CCFCL*) *interpretation of* A_1, i.e. the *CCFCL interpretation* of some selective *valid* and *vav-neutral* (*vav-indeterminate, neither valid nor antivalid*) *decided ER’s,* in the result of which the latter are transduced into the respective *formally tautologous* (*f-tautologous, universally f-true*) and *f-ttatt-neutral* (*f-ttatt-indeterminate, neither f-tautologous nor f-antitautologous*) *catlogographic relations* (*CLR’s*). The totality of the rules of the CCFCL interpretation of the decided ER’s, denoted by ‘I_1’, is analogous to $\mathbf{D}_1$, D_1, and D_1. The CCFCL interpretation I_1 is the *interface between* A_1 *and* A_1, which provides A_1 with the input *f-ttatt-neutral CLR’s*, some of which can then be *postulated, permanently* or *ad hoc*, to be *f-veracious* (*accidentally f-true*), while some other can be decided with the help of D_1 to be either *f-veracious* (*accidentally f-true*) or *f-vravr-neutral* (*f-vravr-indeterminate, neither f-veracious nor f-antiveracious*).

b) The PLTML is the union of two sets of English *declarative sentences* (*DS’s*). One of the two sets comprises *assertive* and hence *materially true* (*m-true*), i.e. *m-tautologous* (*universally m-true*) *and m-veracious* (*accidentally m-true, fact-conformable*) *DS’s of the IML* (*inclusive metalanguage*) *of* $\mathcal{A}_1$, i.e. DS’s of this treatise, which *are used but not mentioned*, and which *latent* (*implicit*) *physical* (*substitutional*) *sentential interpretands of certain formally-true* (*f-true*), i.e. *f-tautologous* (*universally t-true*) *and f-veracious* (*accidentally f-true*) *CLR’s of* $\mathcal{A}_1$ (see the item 3 below in this definition for greater detail). The other set of DS’s of PLTML contains m-true and m-ttatt-neutral DS's that are explicitly used as examples illustrating material interpretations of certain f-true and f-ttatt-neutral CLR's.

c) In accordance with what is said in the paragraph preceding the above point a, the APLL (OPLL, PLO) comprises three self-subsistent *egocentric systems of psychologistic terminology*, i.e. systems, whose elements have *definite significations with respect to me* and, by transcendental extrapolation, *analogous significations with respect to you*. Two of the three systems are *egocentric terminological esperantos*, one which is called the *first psychologistic onomastics* and also "*onymology*" or "*nymology*", because any one of its elements is a *monomial description* of Greek origin, having either allomorph "*onym*" or "*nym*" *as its root* (*generic name*). The constituent graphonyms "*graphonym*" and "*phononym*" of *onymological* (*nymological*) *terms* are abbreviated respectively as "*graph*" and as "*phon*"¸ which are used as the pertinent *effective roots*. Another egocentric terminological esperanto is called the *second psychologistic onomastics* and also *onology*, because any one of its elements is a *monomial description* of Greek origin, having the morpheme "*on*" *as its root* (*generic name*). The third system of psychologistic terminology, called the *third psychologistic onomastics*, is an inhomogeneous system of *univocal* (*single-valued*, *monosemantic*) monomial and polynomial descriptions, involving chaste English or Anglicized Latin words, and hence it is not a terminological esperanto. For the above three parts of the APLL, see subsection 1.5 for greater detail.

3) The PLTFL, $\mathcal{A}_1$, involves a system of *euautographic* (*genuinely autographic, semantically uninterpreted*) kernel-signs (operators), including logical connectives, relational logical contractors (pseudo-quantifiers and pseudo-qualifiers), and substantival algebraic contractors (pseudo-multipliers), whose use is determined by the rules of formation, transformation (inference), and decision of A_1. At the same time, there are in the *exclusive metalanguage* (*XML*) *of* $\mathcal{A}_1$ some standard phonographic (wordy) operators (conjunctions and adverbs), which are associated with certain euautographic operators (kernel-signs) in the sense that they are supposed to apply to the appropriate declarative sentential clauses as their *operata* in accordance with the same rules, according to which their counterpart euautographic operators apply to the appropriate euautographic or logographic operata of $\mathcal{A}_1$. To be specific, I associate:

"not"¸ "it is not the case that", or "it is not the true that" with '$\neg$', “or” or “ior” (“inclusive or”), i.e. “vel” in Latin, with ‘$\vee$’,

“and” or “&” with ‘$\wedge$’,

“if … then –” or “… only if –” with ‘$\Rightarrow$’,

“if” with ‘$\Leftarrow$’,

‘if and only if” or “iff” with ‘$\Leftrightarrow$’,

“neither … nor –” with ‘$\not\vee$’ or ‘$\overline{\vee}$’,

“not both … and –” with ‘$\not\wedge$’ or ‘$\overline{\wedge}$’,

“but not” with ‘$\overline{\Rightarrow}$’¸

“not … but –” with ‘$\overline{\Leftarrow}$’,

“either … or – but not both” or “xor” (“exclusive or”), , i.e. “auf” in Latin, with ‘$\overline{\Leftrightarrow}$’,

“for some ∗:” or “for at least one ∗:” or “there exists at least one ∗ such that” with ‘$\bigvee_*$’,

“for all ∗:” or “for every ∗:” with ‘$\bigwedge_*$’,

“for some but not all ∗:” or “for strictly some ∗:” with ‘$\breve{\bigvee}_*$’,

“for at most one ∗:” or “there exists at most one ∗ such that” with ‘$\frown{\bigvee}_*^1$’,

“for exactly one ∗:” or “there exists exactly one ∗ such that” with ‘$\bigvee_*^1$’,

“the product of … over ∗” with ‘$\hat{\bullet}_*$...’

in all occurrences of the above-mentioned wordy operators (conjunctions and adverbs except the very last one). It is understood that alike ellipses that occur in a group of synonymous operators should be replaced alike by the appropriate concrete operata. In view of the analogy that exists between the binary disjunction operator ‘$\vee$’ and the existential quantifier ‘($\exists$∗)…’ and in view of the like analogy that exists between the binary conjunction operator ‘$\wedge$’ and the universal quantifier ‘($\forall$∗)…’, which are explicated in the treatise, I employ the kernel signs ‘$\bigvee_*$’ and ‘$\bigwedge_*$’ instead of ‘($\exists$∗)’ and ‘($\forall$∗)’ respectively.

Cmt 1.1. 1) According to the *English–Greek Dictionary* (*EGD*) of Pring [1982], the Greek parasynonym of the English noun “logistics” is “διοικητική μέριμνα” \dioikitikí mérimna\, which has nothing to do with the noun “λογική” \lojikí, loyikí\ *s.f.*, being the Greek parasynonym of the English noun “logic”. Therefore, the morphological construction “Psychologistics” is just a convenient English synonym for the descriptive name “Psychological foundations of logic and logical foundations of psychology” (“PFL & LFP”). That is to say, *psychological foundations of logic* (*PFL*) and *logical foundations of psychology* (*LFP*) are two inseparable aspects of a *biune* unbounded (immense) field of study and discourse, which I call Psychologistics, because this noun has the desired association with the above conjoined descriptive name and also because it has the convenient kindred adjective “psychologistic”.

2) It is impossible to demarcate a physical boundary between some parts of the treatise that could be regarded as belonging exclusively to PFL on the one hand and some other parts that could be regarded as belonging exclusively to LFP on the other hand. Sometimes, when I put forward certain aspects of the inclusive or exclusive metalanguage of the object logistic systems – such aspects, e.g., as a *theory of the meaning of its xenographs*, a *theory of definitions*, or *metalinguistic nomenclature* (*phonographic terminology* or *logographic notation*), which I attempt to formulate as rigorously as possible, I may temporarily regard these foreground aspects of Psychologistics as belonging primarily to LFP. By contrast, when I state some aspects of the setup of A_1 or $\mathbf{A}_1$ or some aspects of *conformal catlogographic* (*CFCL*) *interpretation of* A_1, I may temporarily regard these foreground aspects of Psychologistics as belonging primarily to PFL. However, the mental attitude that I temporarily take in each such case towards certain aspects of Psychologistics is an *ad hoc*, i.e. *epistemologically relativistic*, one.

3) There is a general *traditional philosophical doctrine* (*principle*) *of opposition and unity of form and matter*, credited to Aristotle, which is callled *hylomorphism*, The term “hylomorphism” originates from two Greek nouns: “ύλη” \íli\ (pl. “ύλαι” \íle\), meaning *matter*, and “μορφή” \morfí\ (dual “μορφά” \morfá\, pl. “μορφαί” \morfé\), meaning *form*. The English nouns “*matter*” and “*form*” are in turn derived respectively from the Latin nouns “*mātĕrĭa*”, meaning *matter, material, stuff of which anything is composed* (besides having some other meanings), and “*forma*”, meaning *form*, *figure*, *shape* (see Simpson [1968]). According to hylomorphism, every *thing*, i.e *real being* (in Greek: “*τό όν*” \to ón\, *s.n.*, pl. “*τά* όντα” \ta ónta\, meaning *a being* or *creature*; in Latin: *rēs*, both singular and plural, meaning *a thing, object, matter, affair, circumstance*, and espatially *the real thing, fact, truth, reality*),

is a *biune one* (*biune corporeal entity*) consists of two inherent principles, namely a *primordial* (*primary*) and *potential* one that is called *matter* and a *secondary* and *actual* (*entelechial*) one that is called *form*. Accordingly, matter and form are two conceptual epistemologically relativistic aspects of a thing, which can be distinguished and contrasted, but which cannot be separated from each other. The expressions “form and its matter” and “matter and its form” may have many different interpretations. In agreement with hylomorphism, it is impossible to demarcate a physical boundary between some parts of the treatise that could be regarded as belonging exclusively to the formal psycho-logic on the one hand and some other parts that could be regarded as belonging exclusively to the material psycho-logic. That is to say, $\mathcal{A}_1$ or any logistic system being a part of it is an *abstraction* that is prescinded from this tretise and hence from Psychologistics.

4) “όν”, meaning *a being* or *creature*, is an Aristotelian term, which is translated into English by either substantive “a thing” or “a real being” and which can be understood as anything that can be described in terms of some of the 10 *categories* (*basic classes*, or *kinds*, *of predicates*) of Aristotle [350, *Categories*], namely: «substance, quantity, quality, relation, place, time, position, state, action, or affection», – according to [ACE, Part 4], or «either Substance, or Quantity, or Quality, or Relation, or Where, or When, or Position, or Possession, or Action, or Passion», – according to [ACO, Chapter IV]); some scholars use the noun “doing” instead of “action” and “undergoing” instead of either “affection” or “passion”. In other words, a being (όν) is *anything* that can be *treated* (*spoken*) *of* as one that is located in the φύσις \físis\, i.e. in *the nature* or *physical world*. In this case, the Aristotelian term “πράγμα” \práγma\ (pl. “πράγματα” \práγmata\), which is also translated by the English noun “*thing*”, means *anything that can be treated (spoken) of*, including beings and also including anything *supra-natural* as the Aristotelian God, which is not located in the physical world and which is not, therefore, a being. Thus, “πράγμα” (“thing”) in Aristotle’s philosophy is a more general and vaguer term than “όν” (“being”). At the same time, in the Late Ancient Greek philosophy of Neo-Platonism founded by *Plotinus* (Πλωτῖνος \plotínos\, ca. AD 204/5–270), the noun “ὀντότης” meant *reality* (*Platonic Forms*). In contrast to the above ancient Greek terminology, in modern English, there is a tendency to use the noun “entity” for mentioning anything that can be treated (spoken) of, i.e. to use it in analogy with the Aristotelian term

“πράγμα”, and at the same time to use the noun “thing” in analogy with the Aristotelian term “όν” (“being”) and Plotinus’ term “ὀντότης” (“reality”). In this usage, the noun “thing” is parasynonym of the Latin noun “rēs” (pl. “rēs”), which, according to Simpson [1968], means *a thing, object, matter, circumstance,* and especially *the real thing, fact, truth,* as has been indicated in the previous item. The English words “reality” and “real” have been derived from that Latin etymon.

5) Besides this treatise, being the principal part of Psychologistics, the latter is supposed to include an indefinite number of relatively independent essays, which will be called *Psychologistic Essays*, or briefly *Essays*, and which will form supplementary material to the subject matter of the treatise, to be iincluded onder the common heading “*Psychologistic themes*”, – the material to be treated primarily *egocentrically* (like the treatise itself – see the next subsection) with one or another degree of thoroughness. This supplementary material is not however formally included into the treatise, because it is not related to the main object of the treatise as straightforwardly as the Appendices of the treatise. Consequently, in contrast to a separate Appendix, which has the same status as a *section* of the treatise, a separate Essay has the status of a *chapter of Psychologistic themes* and hence the same status as a *chapter* of the treatise. Since separate Essays are relatively autonomous, therefore each of them is provided with a *list of references of its own*, which is however a certain part of *the list of references of the treatise* that is augmented by the reference to the treatise itself and perhaps by some other references when appropriate. In this case, the treatise will briefly be referred to in the Essays as the *Theory of Trial Logic* or as the *Treatise on Trial Logic* and also most briefly as the *TTL*. An Essay may, when convenient, contain some pertinent fragments of the treatise or of another Essay, repeated or cited. Separate Essays will be submitted singly or in groups as soon as they are completed and, for convenience in cross-reference, they will be numbered by ordinal numerals in that order, and not in the order, in which of the associated topics are used or mentioned of the treatise for the first time. For instance, various trends of psychology are briefly discussed in historical prospect in Essay 1 (E1); Essay 2 (E2) addresses the complete taxonomy of the senses of a man; native languages and their codes are discussed in Essay 3 (E3); Essay 4 (E4) addresses special quotations and some other relevant topics in greater detail as compared to their treatment in Preface; Essay 5 (E5) addresses my solution of the problem of univerals; etc.•

1.2. Introspection and the egocentric phraseology: a game of "I"

In accordance with the above-mentioned interpretation of the noun "psychology", the adjective "*psychological*" means: «*of or relating to the introspective psychology of my own*». At the same time, the adjective "psychologistic" means: «*of or relating to Psychologistics*» and therefore it should, after all, be understood as: «*defined in relation to me and effective in my Universe*» or, by extrapolation, «*defined in relation to a certain sapient subject and effective in his Universe*». That is to say, the adjective "psychologistic" *does not mean*: «defined in relation to a group of sapient subjects and effective in the conjoined universe of the subjects», because the name "the conjoined universe of a group of sapient subjects" *has no denotatum*, i.e. it is a *nomen nudum* (*naked name*), *with respect to me.* Particularly, either of the names "*psychologistic terminology*" and "*psychologistic nomenclature*" should be understood in that way.

This treatise and the entire field of study and discourse that I call "Psychologistics" are results of *introspection of my own* and are therefore my self-expression. In this case, the conventional device when a single author of a discourse uses the pronoun "we" in place of "I" in order to emphasize *as if objective* (*impersonal* and *impartial*) character of his discourse is inappropriate. Accordingly, the metalinguistic phraseology in general and the metalinguistic terminology in particular, which I introduce and use in the treatise and which I qualify *psychologistic*, essentially differ from those of traditional writings on logic in two respect. Namely, the psychologistic phraseology is *egocentric*, or, more specifically, *my egocentric*, i.e. it has the *first person singular form*, and, at the same time, it is as rigorous, i.e. unambiguous and logically self-consistent, as possible. Therefore, the psychologistic phraseology is *my explicitly subjective phraseology*, according to which I am *the supient subject*, i.e. *the main sapient object of mine*, being *the master* of all relations that I treat of, including the *name relations* that I establish among *my coentities*, i.e. the entities, of which *I am conscious*, no matter whether or not somebody else is also conscious of them. In this way, I explicitly express the introspective and hence subjective character of this treatise. At the same time, in order to make the subject matter (sense) of the psychologistic phraseology immediately *interiorizable* (*internalizable*) by any concrete reader of the treatise and thus to make the phraseology *communicable* and *objective in this sense*, I invite the reader to adopt along with me the following *convention of introspective substitutions*.

Cnv 1.1: *The rule of a game of "I".* I ask of each individual reader of this treatise to interpret each of the singular first person pronouns "*I*", "*me*", "*mine*", and "*myself*" , and also the singular first person adjective "*my*" as applied to himself, whenever the assertive context in which that pronoun or that adjective occurs is agreeable to him. It is understood that I, on my part, interpret the same pronoun or adjective as applied to me. At the same time, I expect that the reader will interpret each of the singular second person pronouns "*you*", "*yours*", and "*yourself*", and also the singular second person adjective "*your*" as applied to me, whereas I shall naturally interpret the same pronouns and the same adjective as applied to each individual reader.•

Cmt 1.2. When I use the pronoun "I" as the subject of a statement, I mean that I am *the maker* of the statement and that I am *the thinker* (*sapient subject*), whose mental experience is expressed by the statement. If, particularly, the statement is a definition, then I am *the definer*. At the same time, by Cnv 1.1, I propose to each individual reader of the treatise to understand him by "I" and thus to take, if this is acceptable to him, the mental attitude according to which he is the subject and maker of that statement of mine and he is the definer of that definition of mine. Thus, Cnv 1.1 is, in accordance with its heading, the rule of a psychological game, which I propose to each reader of the treatise to play with me while he reads it. I call this game "*a game of* "I"" or "*an* "I"-game", – in analogy with common names such as "a game of chess", "a game of tennis", or "a game of ticktacktoe" ("a game of noughts-and-crosses"). In fact, Cnv 1.1 is *the rule of interpretation of the singular first person egocentric phraseology by the respective introspective substitutions*. This game of "I" is not the only one possible. Particularly, Cnv 1.1 applies to any part of Psychologistic that I may publish in the sequel as a continuation of this treatise as one of its foundations. Therefore, such an account of mine or any report that somebody else may present in the sequel under the pertinent token or version of Cnv 1.1 will be *a* game of "I". Accordingly, either of the synonymous count names "game of "I"" and ""I"-game" is a class-name like count names "game chess" or "game of tennis".

Cnv 1.1 allows me simplifying wordings and explicitly expressing, in the most natural and most concise way, the intimate relations, which a *sapient subject* establishes between him and his *coentities* – the entities of which he is conscious. At the same time, Cnv 1.1 prevents me from making any overt or covert transcendent but still subjective extrapolations from my mental experience to the mental experience of another sapient subject, as I have done in the previous sentence. Also, Cnv 1.1 is my *practical solution* of the intricate philosophical problem how some *conceptions* of a separate sapient subject, which are, in their very nature, *his mental* (*psychical*), i.e. *innermost* and *most intimate*, and hence *subjective entities,* can nevertheless be subjugated to interpersonal verification and interiorization (internalization), so as to become *socio-personal*, i.e. *public* and *objective* in this sense. The last role of Cnv 1.1 in this treatise and in the entire Psychologistics can figuratively be explained as follows.

Theoretical mechanics, classical or quantum, deals with problems of three kinds, namely, one-body problems, two-body problems, and many-body problems, where "body" means «point particle» and "many" means «three or more». A two-body problem reduces to a problem of motion of one particle of the reduced mass, which has an analytical solution in most cases of physical interest. For instance, if a two-body interaction force depends only on the distance between the particles, then the classical or quantum two-body problem reduces to the respective problem of motion of one particle of the reduced mass in the central force field – the problem that has an analytical solution in both cases. By contrast, no three-body problem, – to say nothing of a problem for a larger number of particles, – has any analytical solution. Therefore in order to solve a many-body problem, a physicist often proceeds from the assumption that, in the first approximation, the particles being objects of the problem do not interact with one another. In the result, the many-body problem turns into a set of alike one-body problems. After any of the latter problems is solved, the interaction among particles is mentally turned on. Then using the known independent solutions of the one-body problems as the zero approximation, an attempt can be made to solve the many-body problem asymptotically by the perturbation method, although there is always the question about convergence of the approximation. Thus, Cnv 1.1 plays the role of the *exchange interaction* between me and each specific reader of the exposition. In the result of this interaction, the readers who accept and internalize my concepts will *ipso facto objectify* them without any mental effort on my end to formulate explicitly the sense of the word "objective" when it applies to mental entities.

Still, any singular first person statement *of mine* relating *me*, *my objects, and the properties that those objects have with respect to me* essentially differs from an analogous singular third person statement *of mine* relating *a sapient subject*, *his objects, and the properties that those objects have with respect to himself* simply because the latter statement involves me, explicitly or not, as the detached onlooker and hence as another member of the relation stated. Therefore, instead of introspective statements of the former kind, I often make the corresponding extrapolative statements of the latter kind, which are in turn ones of two kinds. In extrapolative statements of one kind, the pronoun "I" is used as the *sentential subject*, while the appropriate one of the cpount names "*sapient subject*", "*thinker*", "*interpreter*", "*perceiver*", "*definer*", etc is used as a *sentential object*. Consequently, these statements are also subjugated to Cnv 1.1. In extrapolative statements of the other kind, the above count names are used as sentential subjects. In either case, a statement that explicitly express a relation among *a sapient subject, his objects*, *and the properties that those objects have with respect to himself* also express, explicitly or not, a relation of me to all the above entities. My use of the definite article before any of the above names should not be construed that I postulate *real existence* of an *abstract* (*ideal*, *Platonic*) individual carrying that name, but rather it implicitly asserts that I mentally fix a certain, i.e. concrete but not concretized, individual of the range of the count name used, e.g. me or you when applicable. In making statements based on the knowledge, which I have acquired from the literature, I shall give preference to one or another impersonal form of writing, although the egocentric (first person) form will sometimes be used in such cases as well. If a statement expresses a generally accepted concept or refers to a generally accepted nomenclature or method of a certain special field of study as psychology, logic, mathematics, physics, biology, etc then a member of the pertinent scientific community that is supposedly shares that concept or that nomenclature or that method will be referred to by using the respective one of the names "*psychologist*", "*logician*", "*mathematician*", "*physicist*", "*biologist*", etc instead of "sapient subject".•

Df 1.3. 1) Throughout the treatise, by a *sapient* (or *sage*) *subject* (or *individual*) I mean a waking normal adult member of species *Homo sapiens*, unless stated or obviously understood otherwise, – e.g. unless the question is about a sleeping sapient subject or about a sapient subject being deaf or deaf and dumb or blind. Accordingly, I shall, as usual, mention a particular but not particularized member, i.e. a *general member*, of the class denoted by the count name "*sapient subject*" by using the *limited common name* "*a* sapient subject" if I do not mentally fix that member or by using the *limited proper name* "*the* sapient subject" if I mentally fix that member. Just as the former common name, the latter one will always be used distributively, i.e. it will never be used as a class-name. A like remark applies with any of the names "*individual*", "*person*", "*thinker*", "*interpreter*", "*perceiver*", and "*definer*", and also with any of the individual occupation names "*psychologist*", "*logician*", "*mathematician*", etc in place of "sapient subject".

2) The sex a sapient subject that is mentioned by any of the above names is immaterial. Therefore, in repeatedly referring to the sapient subject that has been mentioned in a certain statement previously, I shall conventionally use the pronouns "he", "him", "his", and "himself" (see, e.g., the vocabulary entry «**[1]he 2**» in WTNID or «**he**[1] **2**» in APED).

3) I say that an animal is a *conscious*, or *higher*, one and that also, more generally, it is a *conscious subject*, if it has a *differentiated nervous system* (and not just a nervous organ), which includes the *central nervous system* (*CNS)* comprising the *brain* and the *spinal chord*. Hence, a sapient subject is a conscious subject, but not necessarily vice versa.•

Side by side with Cnv 1.1, I shall, in due course, adopt some other conventions, mostly abbreviative ones, which are designed to improve the readability of some statements without loss of rigor. Here follows one of them.

Cnv 1.2. In the course of the following discussion, I shall introduce some settled noun constructions, each of which consists of a head noun or noun construction and either of the adherent (prepositive) qualifier "*my*" or either of the adjoined (postpositive) qualifiers "*of mine*" and "*with respect to me*". In any subsequent occurrence of such a construction, I may, when desired, *abbreviate* it by omission of the qualifier. The abbreviated expression should not be interpreted as my implicit extrapolation of the concept, expressed by the full noun construction, from me to any other sapient subject, although the reader may, in accordance with Cnv 1.1, interpret the abbreviated expression as applied to him.•

1.3. "Mind" and "consciousness"

Df 1.4. A distinct *entity* (*being*), of which I am *conscious* (*aware*) at a *current moment* when I wake, is called *my coentity* [*at that moment*] or, more explicitly, *a coentity of mine* (to emphasize the fact that I may have more than one coentity simultaneously), no matter whether or not somebody else is also conscious of the entity. Thus, a coentity of mine is an *ingredient* of my *universe* at any current moment. The prefix "co", occurring in the noun "coentity" has, a double meaning. First, it is a conventional perfective, associative, and collective prefix meaning, in this case, *joint*. Second, it is an abbreviation of "*conscious*".•

Df 1.5. A *state* of a coentity of mine is a *form of existence*, or *hypostasis*, of the coentity with respect to me, so that the state is another, *purely immaterial* (*psychical*, *imaginary*, *abstract*) *coentity*. Putting it differently, one of two or more distinct but recognizably same coentities of mine, which come into existence in my mental realm or in the real world in place of one another at different times and which I regard as self-subsistent *modifications* (*allomorphs*) of one another, are called *hypostases of one another* and also *hypostases of the single whole synthetic coentity* that thus *manifests itself in the different states* (*forms of existence*). The synthetic coentity is called a *biune*, *triune*, *quadriune*, etc, or, generally, distrubitably *n-une* and indistributably *multiune*, *coentity* if it has two, three, four, etc, or, generally, distrubitably *n* and indistributably *many*, *hypostases* respectively. The quality of a multiune coentity to have many hypostases (states, ways of existence) is called the *hyperstasis* of the coentity. That is to say, the hyperstasis of a coentity of mine is the *class [of equivalence] of the hypostases* that the coentity has with respect to me.•

Cmt 1.3. 1) In its scientific uses, the noun "state" apparently assumes many different sense values depending on the contexts in which it occurs and hence depending on the situations to which it applies. However, the noun "state" alone, without any added words, is a generic name (class-name) that has the unique sense, which is defined by Df 1.5 subject to Df 1.4. At the same time, there are many different *kinds* (*species*, *specific classes*) *of states*, each of which is determined by the sense of the appropriate unlimited (particularly, not articled) *description through the genus, denoted by the generic name "state" and through the differentia* (*difference*), or *differentiae* (*differences*), *denoted by the pertinent qualifier*, or *qualifiers*, i.e. by [the sense of] the appropriate traditional *descriptio species per genus et differentiam*, or *differentias.*[3]

2) For instance, here follows a conventional definition through a genus and the differentia of WTNID:

> «**state of aggregation :** one of the three or more fundamental forms, conditions, or states of matter that are commonly considered to include the solid, liquid, and gaseous forms and often others (as the colloidal)»

From comparison of this definition and Df 1.5, it follows that *gaseousness*, *liquidness* (*liquidity*), and *solidness* (*solidity*) are three *forms of existence*, while *gas*, *liquid*, and *solid* are three respective *ways of existence*, of a concrete substance as nitrogen, carbon dioxide, water, mercury, gold, etc. At the same time, a *separate molecule* of gas, liquid, or solid itself is not gaseous, not liquid, and not solid, i.e. it has no state of aggregation simply because it is not an aggregate. A molecule, atom, or atomic nucleus has states (forms of existence) of its own, which are called *isomeric ones* or *isomerisms*, while the hypostases (ways of existence) of a particle in those states are called its *isomers*.

3) In accordance with Df 1.5, a state is necessarily an aspect of a certain coentity of mine – an aspect that is itself a coentity of mine. Consequently, a state is associated with *consciousness*. Moreover, in analogy with the above Webster's definition of "stste of aggregation", consciousness itself is phenomenologically defined below as the general state of aggregation of a certain matter. The following definition is just an explication of the definition of a conscious mind, i.e. of the conscious mind of a sapient subject, that has been given in Df 1.1a – the explication, which is based on the most general cytological facts of microscopic structure of the human cerebral cortex and on a certain analogy of that structure with molecular structure of continuous media.•

Df 1.6. The mind (cerebral cortex) of a sapient subject is a unique mass of *vesicular gray matter*, which consists of an enormous but finite number of

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[3] "differentiam" and "differentias" are the Latin accusative of the nominative "differentia" and "differentiae"; respectively, whereas the Latin accusative of the nominative "genus" coincides with the latter (see Cmt 1.6(2,3) below in this section for some further explanations regarding the pertinent traditional Latin terminology).

perikaryons (neuron bodies), dendrites, synapses, neuroglial cells, and some other biological structural ingredients. At any moment when the mind is *awake* (*waking*, *not sleeping*), the *consciousness* of the mind is its *state of aggregation* just as gaseousness, liquidness (liquidity), or solidness (solidity) is the state of aggregation of gas, liquid, or solid respectively. That is to say, the waking mind is conscious in the same sense as a body of gas, liquid, or solid is gaseous, liquid, or solid respectively; hence, *consciousness* is the way of existence of the waking mind. In this case, a separate neuron, perikaryon, dendrite, synapse, or neuroglial cell comprised in the mind is not conscious, i.e. it has no conscious state of aggregation simply because it is not an aggregate – just as a separate molecule has no physical state of aggregation because it is not an aggregate. In accordance with the above-said, consciousness can be prescinded from the conscious mind and be analyzed as such, – just as liquidness, e.g., can be prescinded from liquid and be analyzed as such. Stating that the consciousness of a sapient subject is the conscious state of aggregation of his mind (cerebral cortex), I mentally put forward *the conscious form of existence of the mind*. Under this mental attitude towards the consciousness, it is consistent to state that the *conscious mind* (*conscious cerebral cortex*) of the sapient subject is the *sole organ* (*effector*, *creator*) and *seat* (*receptacle and interpreter*) *of his entire consciousness* and of any parts of it, into which it is usually analyzed and which are called *mental* (or *psychical*) *states* (or *entities*), *states of consciousness*, *conscious modifications*, *brain*, or *cerebrocortical* (briefly, *cortical*) *symbols*, and also by some other names that will be introduced as required in order to express the pertinent connotations. It would, however, have been logically inconsistent to make the same statement with "mind" or "cerebral cortex" alone, without the qualifier "conscious". Indeed, the statement that the mind (cerebral cortex) of the sapient subject has the above properties of his conscious mind (conscious cerebral cortex) means that the entity that is, in this case, called "the mind" or "the cerebral cortex" is prescinded as a certain *unconscious matter* from the conscious mind (conscious cerebral cortex) by freeing the latter of its consciousness, i.e. of its conscious state of existence. This abstraction is as meaningless as prescinding a certain *non-liquid matter* from liquid by freeing the latter of its liquidness, i.e. of its liquid state of existence.•

Cmt 1.4. When *the conscious mind* of a sapient subject turns round and examines itself, it turns itself into one of its conceptual, i.e. imaginary, objects. In this case, *the conscious subject-mind* is unique, whereas there can be many different *object-minds*, each of which differs from the subject mind. In this connection, it is worthy to recall that, for instance, the set of natural numbers is not a natural number, and that the three-dimensional Euclidean affine space over the field of real numbers is not a point, not a vector, and not a real number. It would have been paradoxical to consider, e.g., the set of natural numbers, being the pertinent subject (master, receptacle) of its members, as a natural number, i.e. as an object (member) of itself. Analogously, the actual conscious mind of a sapient subject cannot be adequate to any mental image of it as a conceptual object of itself. The latter, whatever it is, is not a hypostasis of the former, so that the two belong to two different realms. The conceptual object-mind is in fact introduced for convenience in description and study of the actual conscious subject-mind, but identification of the two leads to many tantalizing paradoxes, because turning the conscious subject-mind into its object makes it impossible to distinguish clearly the latter object from the *consciousness*, being the state of the former. Absolute (unconditional) objectification of a conscious subject-mind, which amounts to idefication of it with the consciousness being its state of aggregation, is done in the framework of *Cartesian* (*Cartesius'*, *Decartes'*) *dualism* – an ontological doctrine, according to which *reality consists of two independent and fundamental principles (primary sources): mind and matter*. Cartesian dualism was adopted by all early introspectionistic psychologists, including William James, who described it with great clarity thus (James, [1890; 1950¸ vol. 1, p. 218):

> «*The psychologist's attitude towards cognition* will be so important in the sequel that we must not leave it until it is made perfectly clear. It is a *thoroughgoing dualism*. It supposes two elements, mind knowing and thing known, and treats them as irreducible. Neither gets out of itself or into the other, neither in any way *is* the other, neither *makes* the other. They just stand face to face in a common world, and one simply knows, or is known unto, its counterpart. This singular relation is not to be expressed in any lower term, or translated into any more intelligible name…»

By adopting dualism, James prescinded the mind of a man from his brain and thus caused a mental discomfort to his own mind – the discomfort, which he discussed under the heading: "DIFFICULTIES OF STATING THE CONNECTION

BETWEEN MIND AND BRAIN” (*ibidem*, pp. 176–179). Particularly, he says (*ibidem*, p. 177):

> «…The consciousness, which is itself an integral thing not made of parts, ‘corresponds’ to the entire activity of the brain, whatever that may be, at the moment. This is a way of expressing the relation of mind and brain from which I shall not depart during the reminder of the book…»

The above quotation explicitly indicates that *James equates mind and consciousnes, – just as all early psychologists do*. At the same time, the relation between mind (consciouness) and brain is blurred by using the verb “corresponds” in an undefined allegoric sense and also by using the vague description “the entire activity of the brain, whatever that may be, at the moment”, which may mean both conscious (mental, psychical) activity and unconscious (material, physical, physiological) activity. For avoidance paradoxes, I have, by Dfs 1.1 and 1.1a, defined the generic name “mind” in the framework of the doctrine of *physicalistic monism*. Unfortunately, the latter is an idea, and not a sensible thing.•

Cnv 1.3. By “mind” I shall hereafter understand “conscious mind” (“waking mind”), unless stated otherwise. However, in accordance with Df 1.6, I do not identify (equate) mind and consciousness, – in contrast to the early introspectionistic psychologists (see Cmt 1.4).•

1.4. The interpersonal verifiability principle

Voltaire (François Marie Arouet) said, «If you wish to converse with me, define your terms». This dictum is especially relevant to this treatise because, as I have already mentioned, it involves extensive and extremely ramified unconventional self-consistent *psychologistic, metalinguistic wordy (verbal) terminology and logographic nomenclature* (particularly, *special quotations*). In compliance with Cnv 1.1, I interpret Voltaire’s dictum both as a concrete requirement that should be met in order to make these explanations intelligible and as the following general condition that should be imposed on any rigorous discourse that concerns with logic or is based on logic, and especially if it is a psychologistic one. I call this condition “*the interpersonal verifiability principle*”, thus using the dictionary head name “*verifiability principle*” as a generic *nomen novum* (*new name*).

Cnv 1.4: *The interpersonal verifiability principle.* A discourse (as this treatise) can be subjected to comprehensive *interpersonal verification* if and only if any given *class-name* of fundamental importance that occurs in the discourse is defined and used effectively enough so as to enable any interested and properly learned reader (interpreter), – or, when applicable, debater or interlocutor, – deciding whether or not any given *object of him* belongs to the *class of him designated by the class-name with respect to him*; the class is a *mental entity* (*brain symbol*) of the reader. This means that no fundamental class-name of the discourse should be a *nomen nudum* (*naked name*, *mere name*) and that it should as far as possible have no homographs in the scope of its definition. If it is impossible to satisfy the latter condition then the ambiguity in using a homographic term should unambiguously be resolved by the contexts in which the term occurs. That is to say, all statements and all technical terms that are used in the discourse should be *syntaxic* (*mutually consistent*).•

Cmt 1.5. I use the qualifier "interpersonal" in the name "the interpersonal verifiability principle" in order to distinguish the principle that I attach to the name as its denotatum from the radical principle of *logical positivism*, or *logical empiricism*, which is known under the name "*the verifiability principle*", and which can, more explicitly, be called *the empiric*, or *empiriological*, *verifiability principle*. Two somewhat different formulations of the latter verifiability principle can be found in WTNID under the *vocabulary entries* (*head names*) "*verifiability principle*" and "*confirmability theory*". The verifiability principle without the qualifier "interpersonal" is effaced itself nowadays, and therefore I shall not discuss it in any detail. To be specific, in order to successfully subject this treatise to interpersonal verification, I should particularly unambiguously define the following fundamental epistemological [meta]terms, which have been mentioned in the items 18,x, 18,xi, 30, 38, 44–47, and 54 of Preface: (I) the *syntactic adjectives*: "valid", "antivalid", "vav-neutral" ("neutral with respect to the validity-values validity and antivalidity"), "invalid" ("antivalid or vav-neutral"), "non-antivalid" ("valid or vav-neutral"), "vav-unnutral" ("valid or antivalid"); (II) the *semantic adjectives*: (a) "tautologous" ("universally true"), "antitautologous" ("universally antitrue", "universally false", "contradictory"), "ttatt-neutral" ("neutral with respect to the tautologousness-values tautologousness and antitautologousness"), "untautologous" ("antitautologous or ttatt-neutral"), "non-antitautologous" ("tautologous or ttatt-neutral"), "ttatt-unnutral" ("tautologous or antitautologous"); (b) "veracious" ("accidentally true", "true ttatt-neutral”), “antiveracious” (“accidentally antitrue”, “accidentally false”, “antitrue ttatt-neutral”, “false ttatt-neutral”), “vravr-neutral” (“neutral with respect to the veracity-values veracity and antiveracity”), “unveracious” (“antiveracious or vravr-neutral”), “non-antiveracious” (“veracious or vravr-neutral”), “vravr-unnutral” (“veracious or antiveracious”); (c) “true” (“universally true or accidentally true”), “antitrue” (“false”, “universally anitrue or accidentally anitrue”, “universally false or accidentally false”), “tat-neutral” (“neutral with respect to the truth-values truth and antitruth”), “untrue” (“antitrue or tat-neutral”), “non-antitrue” (“non-false”, “true or tat-neutral”), “tat-unnutral” (“true or antitrue”¸ “true or false”); (III) *nounal derivatives* of the above adjectives such as “validity”, “antivalidity”, “vav-neutrality”, “invalidity”, etc; “tautologousness”, antitautologousness, “ttatt-neutrality”, “untautologousness”, etc; “veracity”, “antiveracity”, “vravr-neutrality”, “unveracity”, etc; “truth”, “antitruth” (“falsity”, falsehood”), “tat-neutrality”, “untruth”, etc.•

1.5. Object logistic systems versus their inclusive metalanguage

For convenience in the subsequent discussion, I shall summarize the *nominal definitions* of the main object logistic systems of this treatise, which have been made in Preface, and I shall also make explicit some general aspects of relationship of the treatise to those systems.

Df 1.7 (*Summary***).** 1) The *Comprehensive Endosemasiopasigraphic Algebraico-Predicate Organon* (*EnSPGAPO*) that is mentioned in the heading of the treatise is *logographically* denoted by ‘A_1’ and is alternatively called the *Comprehensive Endosemasiopasigraphic Advanced Algebraico-Logical Organon* (*EnSPGAALO*), the *Comprehensive Biune Euautographic and Panlogographic APO* (*CBUE&PLAPO*), and the *Comprehensive Biune Euautographic and Panlogographic Advanced ALO* (*CBUE&PLAALO*).

2) The *polynomial* qualifier “*Biune Euautographic and Panlogographic*” (“*BUE&PL*”), occurring in either of the pertinent two names of A_1 and being a synonym of the *monomial* qualifier “*Endosemasiopasigraphic*” (“*EnSPG*”), is explicitly descriptive of the fact that A_1 is the *union* and *superposition* two interrelated organons, which are denoted by ‘A_1’ and ‘$\mathbf{A}_1$’ and which are called the *Comprehensive Euautographic APO* (*CEAPO*) or the *Comprehensive Euautographic Advanced ALO* (*CEAALO*) and the *Comprehensive Panlogographic APO* (*CPLAPO*) or the *Comprehensive Panlogographic Advanced ALO* (*CPLAALO*), respectively. $\mathbf{A}_1$

is *a calculus of panlogographic placeholders* (*PLPH's*) *of euautographic* (*genuinely autographic*, *semantically uninterpreted*) *relations and terms of* A_1 and therefore it is alternatively called the *Logographic APO* (*LAPO*) *over the CEAPO*.

3) The class of *output*, or *sifted decided, euautographic relations* (*OptER's* or *SfdDdER's*) of A_1, denoted by '$\dot{\mathsf{R}}_1$', is by definition the *union* of the following *three disjoined classes*: (a) the class $\mathsf{R}^{\mathrm{O}}_{1+}$ of *valid euautographic ordinary relations* (*EOR's*), (b) the class $\mathsf{R}^{\mathrm{O}}_{1\sim}$ of *vav-neutral* (*vav-indeterminate*, *neither valid nor antivalid*) *EOR's*, (c) the class $\mathsf{R}^{\mathrm{O}\sim}_{1\oplus}$ of *euautographic master*, or *decision*, *theorems* (*EMT*'s or *EDT's*) *for the vav-neutral EOR's of the class* $\mathsf{R}^{\mathrm{O}}_{1\sim}$. The class of *conservative conformal catlogographic* (*CCFCL*) *interpretands of the OptER's of* A_1 is denoted by '$\dot{R}_1$' and also by '$I_1(\dot{\mathsf{R}}_1)$', so that by definition $\dot{R}_1 = I_1(\dot{\mathsf{R}}_1)$. The *class of CCFCL interpretations in extension of the OptER's of* A_1, i.e. the class of ordered pairs of an OptER in $\dot{\mathsf{R}}_1$ as a *euautographic interpretans* and of *its CCFCL interpretand* in $\dot{R}_1$, is denoted by 'I_1' and is alternatively called the *CCFCL interpretation of* A_1 *in extension*, whereas I_1 is the set of rules of I_1 that is called the *CCFCL interpretation of* A_1 *in intension*. The *progressive conformal catlogographic* (*PCFCL*) *interpretation of* A_1 is denoted by 'A_1' and is alternatively called the *Comprehensive Catlogographic Algebraico-Predicate Organon* (*CCLAPO*) or the *Comprehensive Catlogographic Advanced Algebraico-Logical Organon* (*CCLAALO*). Here and generally throughout the treatise, "*interpretand*" (pl. "*interpretands*") or "*interpretandum*" (pl. "*interpretanda*") means *one being the result of the act of interpretation*, while "*interpretans*" (pl. "*interpretantia*") means *one being interpreted*.

4) The qualifier "*Advanced*" ("*A*"), occurring in either of the pertinent two names of A_1, A_1, or $\mathbf{A}_1$ is relevant to the fact that each one of the three organons includes as its autonomous but inseparable part an organon, denoted by 'A_1^0', 'A_1^0', or '$\mathbf{A}_1^0$', respectively, which is qualified *Rich Basic*, whereas each of the three latter organons includes in turn as its autonomous and separable part the corresponding organon, denoted by 'A_0', 'A_0', or '$\mathbf{A}_0$' respectively, which is qualified *Basic* or *Depleted Basic*. To be specific, A_1^0 is called the *Comprehensive*

Endosemasiopasigraphic Binder-Free, or *Contractor-Free*, *Algebraico-Predicate Organon*” (*CEnSPGBFAPO* or *CEnSPGCFAPO*), and also the *Comprehensive Endosemasiopasigraphic Rich Basic Algebraico-Logical Organon*” (*CEnSPGRBALO*); whereas A_0 is called the *Endosemasiopasigraphic Predicate-Free*, or *Endosemasiopasigraphic* [*Depleted*] *Basic*, *Algebraico-Logical Organon* (*EnSPGPFALO* or *EnSPGRBALO*). The pertinent names of A_1^0 and A_0, or of $\mathbf{A}_1^0$ and $\mathbf{A}_0$, are variants of the above names of A_1^0, or A_0, with “*Euautographic*” (“*E*”), or “*Panlogographic*” (“*PL*”), in place of “*Endosemasiopasigraphic*” (“*EnSPG*”), respectively.

5) The CCFCL interpretations of A_1^0 and A_0 in extension are denoted by ‘I_1^0’ and ‘I_0’ respectively, the understanding being that I_1^0 is accomplished with the help of the same cumulative rule I_1 as that of I_1, whereas I_0 is accomplished with the help of the cumulative rule I_0, which is a strict part of I_1 and which is called the *CCFCL interpretation of* A_0 *in intension*.

6) Like A_1, the organon A_1 includes as its autonomous but inseparable part an organon, denoted by ‘A_1^0’, which includes in turn as its autonomous and separable part an organon, denoted by ‘A_0’. The names of A_1, A_1^0, and A_0 are variants of the names of A_1, A_1^0, and A_0 with “*Catlogographic*” (“*CL*”) in place of “*Euautographic*” (“*E*”). In general, A_1^0 and A_0 stand to A_1^0 and A_0 via I_1^0 and I_0 respectively in the same interpretational relations as that, in which A_1 stands to A_1 via I_1. Therefore, the qualifier “*Advanced*” (“*A*”), occurring in the pertinent name of A_1, has the same implication as that occurring in the respective name of A_1.

7) ‘A_n’, e.g., is a placeholder of both ‘A_1’ and ‘A_0’. Consequently, any statement involving ‘A_n’ is a schema of the two statements, one of which corresponds to ‘A_0’ and the other one to ‘A_1’ in place of ‘A_n’, the understanding being that the same statement remains also true with ‘A_1^0’ in place of ‘A_n’. The above remarks apply with ‘$\mathbf{A}$’, ‘A’, ‘I’, or ‘A’ in place of ‘A’. Indiscriminately, A_n or A_1^0 is called an *EALO*; $\mathbf{A}_n$ or $\mathbf{A}_1^0$ is called a *PLALO*; A_n or A_1^0 is called an *EnSPSG*, or *BUE&PL*, ALO; I_n or I_1^0 is called the *CCFCL interpretation of* A_n or A_1^0 respectively; A_n or A_1^0 is called a *CLALO* and also the *PCFCL of* A_n or A_1^0 respectively. As before, I use the

abbreviations: "E" for "euautographic", "PL" for "panlogographic", "EnSPSG" for "Endosemasiopasigraphic", "BUE&PL" for "Biune Euautographic and Panlogographic", "ALO" for "algebraico-logical organon", "CF" "for conformal", "CL" for "catlogographic", "C" or 'P" before "CFCL" for "conservative" and "progressive" respectively.

8) The system of interrelated logistic systems as described above is denoted by '$\mathcal{A}_1$' and is provided with the wordy proper names, which have, along with '$\mathcal{A}_1$', been introduced in Df 1.2(2).•

Df 1.8. 1) Any given object logistic system of this treatise is set up *by means of and within this treatise*, which will therefore be called the *inclusive metalanguage* (*IML*) *of the given system* and also more generally the *inclusive metalanguage* (without any postpositive qualifier). It goes without saying that the IML is the *graphic* (*written*) *English language*, because the *spoken* (*oral¸ phonic*) *English language* is *inexpressible* of any *aphonographic elements* such as *euautographs* and *logographs*, of which all object logistic systems of the treatise are made, or such as *punctuation marks*, some of which can be expressed by *variations of tone*, but *not phonemically*. An aphonograph of the IML can unambiguously be expressed orally only mediately by using its phonographic (wordy, verbal) name. This is particularly why all distinguished aphonographs of the IML should have unambiguous verbal names.

2) In accordance with the previous item, the names "*this treatise*", "*this theory*", "*this inclusive metalanguage*" ("*this IML*"), and "*the inclusive metalanguage*" ("*the IML*") will be used interchangeably as synonyms, while the qualifier "*metalinguistic*" means: «*of or relating to this metalanguage*», i.e. «*of or relating to this treatise*», – unless stated otherwise. This treatise is the *inclusive* metalanguage (*I*ML) not only of any of its object logistic systems, but it is also the *I*ML of itself, – just as a *grammar textbook in English* of the *object English written native language* (*WNL*) is an *I*ML both of the *object language* and of the grammar textbook itself. In this case, any object logistic system, – particularly A_n or $\mathbf{A}_n$, each taken individually and both in the aggregate, i.e. A_n, – is a self-subsistent abstraction from the IML, – just as the object English WNL is a self-subsistent abstraction from any of its grammar textbook or from a totality of such textbooks. In this case, a proof of a theorem of A_n or particularly an *algebraic proof of a master*, or *decision, theorem* (*MT* or *DT*), i.e. an *algebraic decision procedure* (*ADP*), for a certain *euautographic*,

or *panlogograph*, *slave-relation* (*ER-slave* or *PLR-slave*) *of* A_n or $\boldsymbol{\mathsf{A}}_n$ respectively is analogous to a discourse or statement in a WNL. Accordingly, once A_n is set up and learned, proofs of its theorems and particularly its ADP's can be executed without mentioning its theory (this theory), – just as a WNL is used in everyday practice without mentioning its grammar. A like remark applies to I_n and A_n.

3) The part of the IML in the exclusion of any one of its object logistic systems is called the *exclusive metalanguage* (*XML*) *of that system*. Since any object logistic system of the IML is an abstraction, the XML of that system is also an abstraction. Still, it is understood that upon replacing the *endosemasipasigraphs* (*euautographs* and *panlogographs*) of A_1 (e.g.), occurring in a certain statement of the IML, with blanks, that statement turns into an *incomplete syntactic construction* of the XML of A_1, which can be regarded as a *metalinguistic operator*. Hence, the XML of A_1, and generally the XML of any logistic system, is incomplete. Therefore, the name "*operator metalanguage*" ("*OML*") can be used interchangeably (and hence synonymously) with the name "*exclusive metalanguage*" ("*XML*"). Either of the two names "*the XML*" and "*the OML*" without any postpositive possessive qualifier means *the XML* (*OML*) *of all object logistic systems of the IML* (*this treatise*), unless specified.

4) From a somewhat different viewpoint, the XML is a part of the IML having a vocabulary that consists of a relatively small number of words and some other symbols, which are sufficient for developing the entire IML Therefore, the XML is alternatively called *the basic written native language* (*BWNL*) *of the IML* (*of this treatise*), whereas its spoken counterpart is called *the basic spoken native language* (*BSNL*) *of the IML* The pair of the BWNL and BSNL two languages is called *the basic native language* (*BNL*) *of the IML* Since the BNL of the IML is English, therefore the IML is called an *English-based IML.* When applicable, a like terminology applies with "French", "German", etc in place of "English".

5) In agreement with Df 1.2(2), the entire logic that is developed or just tacitly used in the IML (this treatise), including the formal psycho-logic (see Df 1.7(8)), is called the psychologistic logic or briefly psycho-logic.•

Cmt 1.6. 1) The IML (this treatise) is a complicated self-consistent linguistic construction which, in addition to the *nomenclature* of its object logistic systems and in addition to some *logographic notations* not belonging to any object logistic system, contains *extensive and extremely ramified unconventional self-consistent phonographic* (*wordy*, *verbal*) *terminology* concerning various items of the object logistic systems and also concerning *various aspects of the IML itself*. An element (member) of that terminology belongs to the XML, and it will therefore be called a *term of the XML*, a *metalinguistic term*, or a *metaterm*, and also a *taxonym* (*taxonomic name*). Either name "metalinguistic term" or "metaterm" can be abbreviated as "term" whenever the context, in which the name "term" occurs indicated unambiguously that this name is used for mentioning a term of the XML, and not a term of any one of the object logistic systems (as A_1 or $\mathbf{\mathsf{A}}_1$). It is understood that the verbal terminology of the treatise includes both *full terms* (*metaterms*) and *their abbreviations*. As indicated in Df 1.8(1), the *inclusive* metalaguage (*I*ML) is qualified so in the sense that not only the object logistic systems along with their nomenclatures, but also the whole of the verbal terminology and all logographic notations of the XML are created within and with the help of the IML (this treatise). The verbal terminology of the IML turns out to be highly ramified because it is necessary to distinguish phraseologically among many different classes of graphonyms of the IML, including those of which the object logistic systems are made, and also to distinguish among the different ways of using *isotokens* of a graphonym.

2) A metaterm (taxonym) of this treatise is either a separate *simple word* or most often a *predicate-free phrase*, i.e. a *phrase without predication*, that I shall call a *deduction from a genus and the differences* (*differentiae*), or particularly *difference* (*differentia*), – briefly "*DdFrG&Ds*" or "*DdFrG&D*" respectively. In this case, by a *deduction* I mean a *nominalistic deduction* in contrast to a *syllogistic deduction.* The basic kind of a DdFrG&Ds or DdFrG&D is a *descriptive* one (*DcDdFrG&Ds* or *DcDdFrG&D*), call also a *description*, or more explicitly *description of the species, through a genus and the difference*, or *differences*, – briefly *DcTrG&D*, *DcSTrG&D*, *DcTrG&Ds*, or *DcSTrG&Ds* in that order, in Latin *descriptio*, or *descriptio species, per genus et differentias*, or *differentiam*, respectively. A *definition* whose definiens is a DcTrG&D or DcTrG&Ds is a traditional *definition through the genus and difference* (*differentia*), or *differences* (*differentiae*), – briefly a *DfTrG&D* or *DfTrG&Ds*, in Latin *definitio per genus et differentiam*, or *differentias*, which was introduced by Aristotle [350 BCE, *Posterior Analytics*] and which is often called a *real*, or

explicative, *definition*. Any of the above terms that contain an occurrence of the word "differences" ("Ds") is supposed to be applicable also in the case, where there is a single difference to the genus, i.e. it is supposed to include its variant with "difference" ("D") in place of "differences" ("Ds"). The plural number forms of the terms and of their abbreviations will, when necessary, be made by replacing the nouns "deduction", "description", and "definition", abbreviated as "Dd", "Dc", and "Df", with "deductions", "descriptions", and "definitions", abbreviated as "Dds", "Dcs", and "Dfs", respectively; "DcDd" will be replaced with "DcDds". A DdFrG&Ds or DdFrG&D is briefly called a *deductive name* (*DdN*), whereas DcTrG&Ds (DcSTrG&Ds) or DcTrG&D (DcSTrG&D) is briefly called a *descriptive deductive name* (*DcDdN*) or a *descriptive name* (*DN*), and also a *description* if there is no dander of misunderstanding. In this case, the abbreviation "*Dd*" is used equivocally for "*deduction*" and "*deductive*" and "*Dc*" for "*description*" or "*descriptive*". A DdFrG&Ds that is not descriptive is called a *non-descriptive DdFrG&Ds* (*NonDcDdFrG&Ds*) or briefly a *non-descriptive DdN* (*NonDcDdN*). A DcSTrG&Ds (particularly a DcSTrG&D) is more precisely called a *description of the species through the intersection of the genus, designated by the pertinent generic name* (*GN), and through the differences* (correspondingly, *the difference*), *designated by the pertinent qualifiers* (correspondingly, *qualifier*). In contrast to a generic name (GN), a DcSTrG&Ds or DcSTrG&D is briefly called a *descriptive name* (*DN*) and also a *descriptive specific name* (*DSN*). A *qualifier* (*Ql*, pl. "*Ql's*") to a GN can be either prepositive (as a prefix, combining form, adjective, or adjective equivalent) or postpositive (as an adjective equivalent). However, every qualifier occurring in an *onymological* or *onological monomial DSN* (see subsection 1.6) is a prepositive one (namely an Anglicized prefix or combining form).

a) For instance, "man", "tree", or "water" is a GN; "adult man", "green tree", or "distilled water" is a DSN; "men", "adult men", "trees", or "green trees" is a NonDcDdN.

b) A DSN is either a *count DSN* (*CtDSN*), called also a *numeralable* (capable of being modified by a numeral or by the indefinite article) *DSN* (*NDSN*), or a *non-numeralable* (incapable of being modified either by a numeral or by the indefinite article) *DSN* (*NNDSN*). An NDSN is a DSN of a *multitudinous* (*many-member*) *class-species* (*specific class*), which is alternatively called a *multipleton-species* (*specific* *multipleton*), and therefore the NDSN is alternatively called a *descriptive specific multipleton-name* (*DSMN*). An NNDSN is either a DSN of a *one-member class-species* (*specific class*), which is alternatively called a *singleton-species* (*specific singleton*), or a DSN of a *mass-species* (*specific mass*). Accordingly, the former NNDSN is alternatively called a *descriptive specific singleton-name* (*DSSN*) and the latter NNDSN a *descriptive specific mass-name* (*DSMsN*). A DSMN or a DSSN is indiscriminately called a *descriptive specific class-name* (*DSCsN*). The GN of a DSCsN is a *count*, or *numeralable*, *GN* (*CtGN* or *NGN*), which is alternatively called a *generic class-name* (*GCsN*) or *generic multipleton-name* (*GMN*), because it necessarily denotes a *generic multitudinous class*, which is called a *class-genus* (*generic class*) or alternatively *multipleton-genus* (*generic multipleton*); and vice versa. Likewise, the GN of a DSMsN is a *mass GN* (*MsGN*), which is alternatively called a *generic mass-name* (*GMsN*), because it necessarily denotes a *mass-genus* (*generic mass*); and vice versa.

c) A DSN is either a complex *monomial* (*one-word*) *one* (*MDSN*) or a *polynomial* (*many-word*) *one* (*PDSN*), some constituent words of which can in turn be MDSN's. An MDSN is necessarily a *class one* (*CsMDSN*), i.e. *MDSCsN* (*MDSMN* or *MDSSN*), whereas a PDSN is either a *class one* (*CsPDSN)*, i.e. a *PDSCsN* (*PDSMN* or *PDSSN*), or a *mass one* (*MsPDSN)*, i.e. *PDSMsN*.

3) Regarding the above traditional Latin terms, the following remarks will be in order. Latin has no words corresponding to the English articles "a" and "the". Also, in Latin, the nouns "*genus*", "*species*", and "*differentiam*" are accusative singular of the nominative singular nouns "*genus*" ("*gĕnus*", neuter of the third declension), "*species*" ("*spĕciēs*", masculine of the third declension), and "*differentia*" ("*diffĕrentĭa*", feminine of the first declension), whereas the noun "*differentias*" is accusative plural of the nominative plural noun "*differentiae*"; nominative plural of the nominative singular feminine nouns "*decriptio*" ("*dēscriptĭo*") and "*definitio*" ("*dēfīnītĭo*")" are "*descriptiones*" ("*dēscriptiĭōnes*") and "*definitiones*" ("*dēfīnītĭōnes*") respectively.

4) A DSN is *self-explanatory* in the sense that *it denotes*, with respect to any interested interpreter (as me or you), *the species that it describes*, provided that the *lexical senses*, rather than the *etymological ones* (if applicable), of the generic name and of all qualifiers, which occur in the DSN, are defined univocally. A DSN can be

either a *formal* one (*FDSN*), such as any *Linnaean binomial* (see below) of a comprehensive *biological taxonomy of bionts* (*BTB*), or an *informal* one (*IDSN*). An IDSN is the widest and most fundamental kind of DSN's because it can immediately be turned into a description of a member of the species denoted by IDSN either by changing the mental attitude towards it or by adhering it with the appropriate *limiting modifier* as the indefinite or definite article. Also, if an IDSN, e.g., has several (two or more) qualifiers to its generic name (headword) then the intersection of two or more differentiae denoted by the respective qualifiers can be regarded as another differentia. Alternatively, the intersection of the genus denoted by the generic name and the differentia denoted by the qualifier immediately preceding or immediately succeeding the generic name is an *intermediate species* (*specific class, strict subclass*) of the genus, which can alternatively be regarded as *the genus* (*general class*, *strict superclass*) of the next consecutive description, intermediate or ultimate.

5) It is noteworthy that such an epistemologically relativistic use of the terms "species" and "genus" is impossible if these are used *formally* as *metataxonyms* (*metataxonomic names*, *rank-names*) of the *taxa* (*taxons*, *taxonomic classes*) of the two lowest *ranks* of the *hierarchy* of increasingly broad (inclusive) *taxa* of any comprehensive BTB. This hierarchy was suggested by Swedish botanist *Carolus Linnaeus* (the Latinized form of the name "*Carl von Linne*"), 1707–1778, who developed the first comprehensive BTB, which will be called the *Linnaean taxonomy* (*LT*). The LT is based on Aristotelian division of *bionts* (*living organisms)* into *two kingdoms*: **Plantae** (plants) and **Animalia (**animals**)**. The LT is described in general outline, e.g., in Villee [1957, chapter VI] and in Campbell [1990, pp. 484–486]. A review of the various modern BTB's can be found, e.g., in the article **Biological classification** of Wikipedia, <http://en.wikipedia.org/wiki/Biological_classification>. Among the modern BTB's, the most popular one is likely that based on the *five kingdoms of bionts*: **Monera** or **Prokaryotae**, **Protista**, **Fungi**, **Plantae**, and **Animalia**, which was suggested by Whittaker [1969], who revised the classical *two-kingdom LT* in the light of some modern concepts of genetic and evolution theories. This BTB, along with some important modifications by Lynn Margulis, which are discussed in detail in Margulis and Schwartz [1987], will be called the *Linnaeus-Whittaker taxonomy* (*LWT*). The LWT is substantiated and followed closely as the

general frame of reference in Campbell [1990, Unit Five, pp. 505-674, 518-520ff]. After the LT, any other BTB has the following three main features.

i) *The first main feature of a BTB* is that any *species* of the BTB is denoted by an *italicized two-word Latin name* that is called a *Linnaean* (or *Linnean*) *binomial* (or *binomen*, pl. "*binomona*"). The first, *capitalized*, word of a Linnaean binomial is a *generic name* (*class-name*) *denoting* the *genus* (*immediate general class*, *immediate strict superclass*), which the *species* (*specific class*, *strict subclass*) denoted by the binomial belongs to. The second, *uncapitalized*, word of the binomial is a specific *epithet* (*qualifier*) to the generic name, which denotes the *differentia* (*difference*), i.e. the *additional conceptual property*, by which a biont of the species is distinguished from a biont of any other species of the same genus. Accordingly, a Linnaean binomial is a *formal description of the species through the pertinent genus and pertinent differentia*. For instance, *Populus tremuloides* (American poplar), *Populus diversifolia* (Asiatic poplar), *Populus deloides* (berry-bearing poplar), *Populus nigra* (black poplar), etc; *Felis domesticus* (domestic cat), *Felis sylvestris* (European wild cat), *Felis leo* (lion), *Felis tigris* (tiger), *Felis pardus* (leopard), etc, and also *Pan troglodytes* (chimpanzee), *Simia satyrus* (orangutang), *Homo sapiens* (human being, man), etc are some species, whereas *Populus*, *Felidae* (cats), *Pan*, *Simia*, *Homo*, etc are some genera. Etymologically, "*species*" is a Latin noun that, among a great many of its meanings, takes on the same denotata as the English nounal names "*kind*", "*species*", "*division of* a genus" (see Simpson [1968]).

ii) *The second main feature of a BTB* is to file species into a *hierarchy* of increasingly broad (inclusive) categories (classes sensu lato), which are provided with the appropriate *capitalized proper Latin monomials* (*monomina*, *one-word names*); all the monomials except those of genera are set in *roman* (*upright*) *type*. The following count nouns are the main *hierarchal metataxonyms*, i.e. *rank-names, of increasingly broad taxa* of the LWT: "*species*", "*genus*", "*family*", "*order*", "*class*" [sensu stricto], "*division*" for plants and fungi or "*phylum*" for animals, "*kingdom*". The same rank-names are used in the LT with the only difference that the taxon Fungi of the LT is a *division* of the kingdom Plantae of the LT. Taxa of the *intermediate ranks* with respect to those denoted by the nouns on the above list are distinguished by adhering a noun of that list with the appropriate one of the combining forms "*sub*" (or "*infra*") and "*super*", i.e. by nouns such as "subspecies", "subgenus", "superfamily" or “suborder”, “superorder” or “subclass” (or “infraclass), “subphylum”, “subdivision”, “subkingdom”, etc. Incidentally, the fact that the taxonyms of genera, species, and subspecies are distinguished from the higher rank taxonyms by setting the former in italic means that a BTB *is not straightforwardly expressible orally*. Therefore, it is, in this case, etymologically more correct to use the term “*taxograph*” (“*taxographonym*”) instead “taxonym” and to replace the noun “taxonomy” in the name “biological taxonomy of bionts” with “*taxography*” (“*taxographonymy*”).

iii) *The third main feature of a BTB* is that *most* species of the BTB and *all* its higher taxa starting from genera are defined by their *morphological* and particularly *anatomical features*. Therefore, any species thus defined is called a *morphological species* or briefly a *morphospecies*, whereas all higher-rank taxa thus defined are called *morphological taxa* or briefly *morphotaxa*. By contrast, a species of *reproductively compatible sexual bionts* is called a *biological species* or briefly a *biospecies*. A biospecies may comprise many morphospecies. The morphospecies of a single biospicies of animals are called *races*. For instance, *Homo sapiens* is a biospecies and hence it is a race.

6) One of the objects of [the section 2 of] Preface and of this chapter is to explain what A_1 and $\mathbf{A}_1$, along with the principal interpretation of A_1, is as such and as compared to any of the *conventional axiomatic logical calculi* (*CALC’i*) and also to explain the most conspicuous peculiarities of the IML. General principles and some most general facts of the terminology of the treatise and relevance of the treatise to the new field of study that I call “Psychologistics” are also among the objects to be explained in this chapter. It is hoped that the explanations constituting Preface and this chapter will help the reader to see essentials for terminological and methodological niceties at the early stages of developing A_1 and $\mathbf{A}_1$. Unfortunately, explaining the essentials and explaining the terminology come into contradiction with each other – contradiction that amounts to a vicious circle. Namely, in order to intelligibly outline the most essential peculiarities of the IML and of its object logistic systems, at least a certain part of the setups A_1 and $\mathbf{A}_1$ and of the pertinent *specific nomenclature* (logographic notation and phonograpic terminology) should be available. On the other hand, there is a danger that an attempt to introduce particular elements of the specific terminology, which are necessary for explaining the essentials, in isolation from both the specific nomenclature and the setups of A_1 and

A$_1$, which provide *ostensive definientia for the specific metaterms as definienda*, can become counterproductive. Therefore, Preface and this chapter are an uneasy compromise between the above two aspects.

7) In order to exit the above-mentioned vicious circle, I follow a certain step-by-step method. Namely, when necessary, I replace a missing ostensive definiens by its description through the appropriate genus and differentia. Such a description or a concise term that is defined as definiendum in relation to the description as definiens by the pertinent definition through the genus and differentia is an adequate one although both should after all be interpreted (defined) ostensively. The preliminary definitions that seem to be rigorous enough are, for convenience in further references, included under numbered logical heads having the classifying abbreviation "Df" for the taxonym (taxonomic name) "Definition". Some other terms are introduced informally in narrative manner by passing as if they were *ad hoc* ones and not fundamental technical terms of the treatise as they actually are. All loosely defined fundamental terms will be redefined rigorously at the appropriate places later on. The reader is therefore advised to read the introduction through once and then return to the pertinent parts of it as he studies the subject matter of treatise concretely. The themes that are covered in the rest of this chapter and their order are selected with the purpose to allow explaining essentials of this treatise as soon as possible. Therefore, many themes underlying the essentials that do not meet the above criterion are put off in spite of the fact that discussing these themes does not require extensive special terminology and that clarifying them immediately would have been an easy task. These themes will be discussed elsewhere.•

1.6. The psychologistic onomatology: three pychologistic onomastics

Cmt 1.7. In the above heading, and generally in what follows, I use the word "onomastics" in accordance with the following definition of WNCD and WTNID:

> «**onomastics** … *n pl but sing or pl in constr* … **2 :** the system underlying the formation and use of words esp. for proper names or of words used in a specialized field».

According to both WNCD and WTNID, "*onomatology*" is a synonym of "*onomastics*". By contrast, I employ the name "*psychologistic onomatology*" or the name "*psycho-onamatology*", being its abbreviation, as a collective name of all onomastics of Psychologistics. Also, I regard the *psychologistic onomatology*, i.e. the denotatum of the above name, as a *part of applied logic* and I therefore alternatively call it *applied*, or *onomastic*, *psycholohistic logic* (*APLL* or *OPLL*), in accordance with Df 1.2(2).•

Df 1.9: *Onymology* (*nymology*), *the first psychologistic onomastics.* 1) The most conspicuous attribute (constituent part) of the *psychologistic terminology* is a certain *egocentric terminological esperanto*, which I call "*onymology*" or "*nymology*" and also the *first psychologistic onomastics*. In accordance with the two synonymous names, the nymology (to utilize the shorter one of the two names) is the main method that I from the very beginning informally practice in this treatise for forming *disyllabic and polysyllabic derivational monomina* (*monomials*), to be collectively called *onymological*, or *nymological, monomina* (or *terms*), by combining either of the Anglicized allomorphs "*onym*" and "*nym*" of Greek origin with some other Anglicized *morphemes* or *longer morphological units of Greek origin* and by abbreviating, when possible and desired, some of the polysyllabic monomina thus formed by omission of the base "onym". Theoretically, an original complex onomological monomen is a *description through the genus*, denoted by either of the two *new Anglicized allomorphic nouns* "*onym*" and "*nym*" of Greek origin, *and through the pertinent differentia* (*difference*) or *differentiae* (*differences*), denoted by one or more appropriate old (established) or new allomorphic prefixes or prepositive combining forms of Greek origin, – e.g., "ant" or "anto", "aut" or "auto", "dict" or "dicto", "gloss" or "glosso", "graph" or "grapho", "hom" or "homo", "icon" or "icono", "id" or "ido", "ide" or "ideo", "idi" or "idio", "phon" or "phono", "syn" or "syno", "tax" or "taxo", "xen" or "xeno", etc, and also, e.g., "autograph", "dictograph", "glossograph", "glossophon", "iconograph", "ideograph", "logograph", "pasigraph", "perigraph", "xenograph", "catlogograph", "endoiconograph", "exoiconograph", "euautograph", "eulogograph". "euxenograph", 'glossoideograph" (or "glossographoide"), "panlogograph", etc, etc. An English name will be called a *paleonym* or *nomen vetum* (pl. "*nomina veta*") if it is an established one both syntactically and semantically, and a *neonym* or *nomen novum* (pl. "*nomina nova*") if it is a *new* psychologistic term either from the standpoint of its [syntsctic] form or from the standpoint of its [semantic] matter or both.

2) Morphologically, the allomorphs "onym" and "nym" are *back-formations* from the established (old) English, primarily *disyllabic*, nouns such as "ananym"

(meaning a pseudonym formed by writing the real name backwards), "anonym", "antonym", "eponym", "homonym", "metonym", "hyponym", "paronym", "pseudonym", "synonym", "tautonym", etc. Etymologically, both allomorphs "onym" and "nym" originate from the Greek noun "όνομα" \ónoma\ that assumes (takes on) the same meanings as the English nouns "*name*" and (gram.) "*noun*". Accordingly, the *etymological sense* of either allomorph "onym" or "nym" can be expressed by the *nounal name* (*nounal construction*, *noun equivalent*) "*name sensu stricto*", i.e. "*name in a narrow sense*", which may have many different interpretains. Unless stated otherwise, I shall use the noun "*name*" as an abbreviation of the nounal name "*name sensu stricto*". Also, most generally, I assume that the nounal name "*name sensu stricto*" and hence the noun "*name*" is by definition a synonym of the nounal name "*linguistic form*". At the same time, in the framework of the egocentric psychologistic phraseology, any of the above three synonymous names "name", "name sensu stricto", and "linguistic form" is supposed to be followed eirher with the postpositive qualifier "*with respect to me*" (or "*in relation to me*") subject to Cnv 1.1 or with the postpositive qualifier "*with respect to a given* (*fixed*, *concrete and concretized*) *sapient subject as I or you*", by the corresponding transcendent extrapolation.

3) Under Cnv 1.1, in contrast to either of the synonymous descriptions (descriptive nounal names) "*name with respect to me*" and "*name sensu stricto with respect to me*", where either one he synonymous *generic names* "*name*" and "*name sensu stricto*" is *by interpretation* a synonym of the *generic name* "*linguistic form*", either one he allomorphs "onym" and "nym" is *by definition* a synonym of the description "*name sensu lato with respect to me*", where *generic name* "*name sensu lato*", i.e. "*name in a broad sense*", is *by interpretation* a synonym of the *generic name* "*sensible thing*". Hence, a name [sensu stricto] is an onym (nym), but not necessarily vice versa. Particularly, a *major form class* (*part of speech*) or *its equivalent* and a *sentence* are names [sensu stricto] and hence they are onyms (nyms). Since both "onym" and "nym" are nouns, therefore they are names.

4) A monomen (monomial) of the form **"**—onym**"**, including "onym" itself, will be called an "*onym*"-*based noun* or briefly an "*onym*"-*noun*, and similarly with "nym" in place of "onym". Thus, particularly, "onym" is both an "onym"-noun and a "nym"-noun, whereas "nym" is a "nym"-noun, but not an "onym"-noun. "Onym"-nouns containing two or more prefixes or combining forms, the last of which is either

combining form "*graph*" or "*phon*", – such "onym"-nouns, e.g., as "autographonym", "homographonym", "logographonym", "ideographonym", "xenographonym", "autophononym", "homophononym", "xenophononym", "euautographonym", "euxenographonym", etc, – will be abbreviated by omission of the morpheme "onym", thus becoming either conventional English nouns or their homonyms (homographs) or else new shorter well-formed (morphologically congruous) Anglicized nouns (monomina, monomials), – e.g. "autograph", "homograph", "logograph", "ideograph", "xenograph", "autophon", "homophon" (a synonym of the conventional noun "homophone"), "xenophon", "euautograph", "euxenograph", etc respectively. In accordance with the previous item, either of the allomorphs "onym" and "nym" denotes a *sensible thing with respect to a certain sapient subject as I or you*. Therefore, any "nym"-noun, i.e. any monomen, ends with "nym", *does so*.

5) Given a monomen of the form **"**—nym**"** (the undersanding being that "—" is an ellipsis, upon replacement of which with an appropriate prefix or combining form or juxtaposition of prefixes or combining forms primarily of Greek origin, the bold-faced double quotation marks should be replaced with light-faced ones), *paronyms* (*derivatives*) of **"**—nym**"** are universally formed as follows.

a) **"**—*nymous***"** or **"**—*nymic***"** is an adjective paronym (derivative) of **"**—nym**"**, meaning «*of, relating to, marked by the use of, consisting of, or characterized by —nyms*»;
b) **"**—*nymously***"** or **"**—*nymically***"** is an adverbial paronym of **"**—nym**"** meaning «*in the manner of a —nym* or *in a*, or *the*, *—nymous mental mode*»;
c) **"**—*nymy***"** or **"**—*nymity***"** is a kindred noun of **"**—nym**"**, meaning «*the quality or fact of being —nymous*».

For instance, "*autonymously*" means «*in an autonymous mental mode*» and "*xenonymously*" means «*in a xenonymous mental mode*».

6) In analogy with the regular transitive verb "*to name*", which is kindred to the count noun "*name*", the verb "*to nym*" will, when appropriate and especially in applications to *glossonyms* (*linguistic nyms*), be used as a regular transitive verb which is kindred to the count noun "nym". Particularly, the singular third person predicates "*nyms*" and "*is a nym of*", e.g., are synonyms – just as synonyms are the like predicates "*names*" and "*is a name of*". Accordingly, the verb "to nym" denotes

the class of mental acts of mine (or generally of a sapient subject), which is determined by the sense of any of the following expressions:

a) *to associate a physical or psychical entity as relatum with a nym as referent*;

b) *to use a nym for mentioning a physical or psychical entity, either a unique one or any one of a certain class*;

c) *to identify or classify a physical or psychical entity by its nym*.

Here, and generally in what follows, the verb "*to mention*" in any grammatical form has the same sense as the verbal construction "*to refer to*" in the respective grammatical form, whereas the kindred name "*mention of*" has the same sense as the name "*reference to*". Once I associate a nym as *referent* with a certain entity as *relatum*, I establish a mental (conceptual) relation between the nym and that entity – a relation, which will be called a *semantic*, or *xenonymous*, *relation* and also, more specifically, a *nym relation*, in analogy with "*name relation*".

7) A "nym"-noun, its kindred adjective, adverbial, nounal, and verbal derivatives, and also its abbreviation, which is obtained by omission of the root "onym" (when possible and desired), and all kindred adjective and adverbial derivatives of the abbreviation are called *onymological*, or *nymological*, *terms* or *words* or *monomina* (*monomials*), the understanding being that the names "*onymological noun*", "*onymological adjective*", and "*onymological adverb*" are self-explanatory. Some *disyllabic* onymological English monomina (monomials) are *paleonyms* (*nomina veta*), whereas the other *disyllabic* and all *polysyllabic* onymological English monomina that are used in the treatise are *neoonyms* (*nomina nova*).

8) A *morpheme* of an onymological term will be called an *onymological morpheme* or brieflt an *onymomorpheme*. Accordingly, the *root* (*base*) of an onymological term, namely "*onym*" or "*nym*", "*graph*", or "*phon*", will be called an *onymological root* (*base*).•

Cmt 1.8. 1) It would likely be etymologically more correct to introduce the Anglicized noun "onym", and not to introduce the noun "nym" at all. Still, I have introduced "nym" as well and I shall even give preference to it over "onym" because "nym" is a one-syllable word and is therefore more convenient in use than "onym". In this case, no problem of morphological identification of the letter "o" preceding "nym" in complex words arises. Indeed, "nym" and "onym" are by Df 1.9 allomorphs. In addition, the morpheme (prefix or combining form) preceding "nym" in a complex monomen (monomial) can also be regarded as an allomorph. Therefore, the above-mentioned occurrence of the letter "o" can always be regarded either as belonging to the morpheme "onym" or to the prepositive morpheme of "nym".

2) It is also noteworthy that the morphemes "*nom*" and "*onom*" occurring in the nouns "nomenclature", nomination", "onomastics", "onomatopoeia", "taxonomy", etc. should be regarded as two other allomorphs of "nym" and "onym". Still, since I shall not use either "nom" or "onom" in forming *neonyms* (*new nyms*), I do not provide them with the status of nouns.

3) Every "nym"-noun, including both allomorphs "nym" and "onym", belongs to the species of names [sensu stricto], which is called a *count*, or *numeralable*, *nounal name* in accordance with the following definition.•

Df 1.10. 1) A *noun* or a *predicate-free noun construction*, i.e. a noun together with all pertinent modifiers *except a predicate*, is called a *nounal*, or *substantival*, *name* (*NlN*) and also, briefly, a *substantive*. A nounal name is called:

a) a *count*, or *numeralable*, *nounal name* (*CtNlN* or *NNlN*) and also a *dimension* if it has both a singular and a plural number form and can therefore be used with a prepositive *numeral* or particularly in the singular with the numeral "one" or "1" or with the indefinite article "*a*" or "*an*";

b) an *unlimited non-numeralable nounal name* (*UnLtdNNNlN*), if it has, in such a use, only a singular form and if it has *no limiting modifier*.

It is undersood that an NNlN has no limiting modifier either, so that "*UnLtdCtNlN*" and "*UnLtdNNlN*" are synonyms of "CtNlN" and "NNlN, but the qualifier "unlimited" ("UnLtd") is redundant in this case. An NNlN is alternatively called an *unlimited proper multipleton-name* (*UnLtdPrMnN*), i.e. an *unlimited proper name* (*UnLtdPrN*) of a multipleton, whereas "*multipleton*" is by definition a synonym of the expression "*many-member class*", – in analogy with the conventional term "*singleton*", being a synonym of the expression "*one-member class*". An UnLtdNNNlN is either an *UnLtdPrN of a nonempty individual* or an *unlimited proper singleton-name* (*UnLtdPrSnN*), i.e an *UnLtdPrN of a singleton*, or else an *UnLtdPrN of a certain concept-mass* (*cmass*), i.e. an *unlimited proper cmass-name* (*UnLtdPrCmsN*), the understanding being that a cmass is a *universal*, which, ulike a class, has *parts* (*subcmasses*), and *not members*, as *its instances* – parts that have *mental projections* into the real world. In this case, the projection of an *indefinite* (*common*, *general*) *instance* of a given cmass can be referred to by using the *LtdPrCmsN*, which comprises the pertinent UnLtdPrCmsN and the prepositive indefinite adjective "*some*" as the added word, rather than "*a*" or "*an*". An UnLtdPrMnN or UnLtdPrSnN is indiscriminately called an *unlimited proper class-name* (*UnLtdPrCsN*). Also, in accordance with the above terminology, a *numeralable noun* is a noun that is commonly called a *count noun*, whereas a *non-numeralable noun* is a noun that is commonly called a *mass noun*.

2) The linguistic construction comprising a quantifier and a postpositive NNlN is called *a dimensional quantifier*, whereas the NNlN is called *the dimension of the dimensional quantifier*. The class designated by a dimensional quantifier is called a *dimensional quantity*, whereas the class designated by the dimension of the dimensional quantifier is called *the dimension of the dimensional quantity*. Particularly, the linguistic construction comprising a *numeral* and a postpositive numeralable nounal name is called *a dimensional numeral*, whereas the numeralable nounal name is called *the dimension of the dimensional numeral*. Consequently, the class designated by a dimensional numeral is called a *dimensional number*, whereas the class designated by the dimension of the dimensional numeral is called *the dimension of the dimensional number*. The word "dimension" alone, without any postpositive possessive qualifier (as "of the dimensional numeral" or of the dimensional number") is ambiguous because it can refer either to an NNlN or to the class designated by the name.•

Cmt 1.9. 1) A general effective *syntactic* device to indicate that one of two *homonyms* is used in a more inclusive sense than the other is to supplement the former with the Latin postpositive (adjoined, suffixed) attributive qualifier "*sensu lato*" or the latter with the *antonymous* postpositive qualifier "*sensu stricto*", or else to supplement both homonyms with the above respective qualifiers, although in this case one of the qualifiers will be redundant. I have employed this device in Df 1.9, and I shall repeatedly employ it in the sequel. WTNID defines the two qualifiers thus:

«**sensu lato** *adv* [NL] **:** in a broad sense – used esp with names of taxa to indicate that the name is used more inclusively than sanctioned by current practice ⟨*Pyrus sensu lato* includes pear, apple, quince, mountain ash and related forms⟩; compare SENSU STRICTO

sensu sricto *adv* [NL] **:** in a narrow sense – used esp with names of taxa to indicate that the name is used in a restricted manner ⟨*Pyrus sensu stricto* includes only the pears⟩; compare SENSU LATO**»**

In Latin (both Old L. and New L.), "sensu" is the ablative case of the nominative singular noun "*sensus*" meaning *sense*, "*lato*" is the ablative case of the adjective "*latus*" meaning *broad*, *wide*, and "*stricto*" is the ablative case of the participle "*strictus*" from the verb "*stringo*" meaning *to strip off*, *pluck*, *clip*, *prune* (see Simpson [1968]). At the same time, *Latin has no words corresponding to the English articles* "a" *and* "the". Therefore, depending on the context, in which the qualifier "sensu lato" (e.g.) occurs, the latter can be translated into English either as "in *a* broad sense" in an indefinite construction or as "in *the* broad sense" in a definite construction, and similarly, with "stricto" and "narrow" in place of "lato" and "broad" respectively.

2) In order to explicate the senses of the expressions "broad sense" and "narrow sense", I shall, in what follows, preliminarily explicate *the sense of the noun* "*sense*" in the above use, i.e. as a semantic term and not as a biological one, in application to a xenograph being an *ordinary* (*not onymological*) UnLtdPrMnN. In the next section, this explication will be extended to the case, where a xenograph is an UnLtdPrCmsN, and also to the various cases of limited names, including sentences.

i) An *unlimiting attributive modifier* to an UnLtdPrMnN is alternatively called a *qualifier*. A qualifier to UnLtdPrMnN can be either a prepositive one, which is usually an adjective, or a postpositive one, which is a combination of a *preposition*, being a *function word*, and a certain *semanteme*, i.e. a *full* (*notional*) *word*. An UnLtdPrMnN along with a qualifier to it is an UnLtdPrCsN; namely it is either another UnLtdPrMnN, i.e. UnLtdPrN of a *narrower* (*less inclusive*) *multipleton*, or an UnLtdPrSnN. Accordingly, I regard a qualifier to an UnLtdPrMnN as an UnLtdPrN of a certain *megaclass* whose intersection with the multipleton designated by that UnLtdPrMnN results in the class designated by the pertinent *descriptive* UnLtdPrCsN. Therefore, a qualifier to an UnLtdPrMnN is alternatively called an *unlimited proper megaclass-name* (*UnLtdPrMgCsN*), i.e. it is another UnLtdPrCsN. Consequently, in the following statements, by "xenograph" I mean an UnLtdPrCsN.

ii) A xenograph is called a *primary*, or *reference*, or *induced*, *xenograph* if it *designates* with respect to me a certain class, other than any one of its token-classes, which has been assigned to the xenograph by *nominalistic induction*, and not by *nominalistic deduction*, i.e. not by a *deduction from a genus and the differenses* (*DdFrG&Ds*). The above class is called the *designatum* (*designation value*, pl. "*designata*"), or, in contrast to the subsequent notion of an *autodesignatum* (*autodesignation value*, *isotoken-class*), the *xenodesignatum* (*xenodesignation value*), and also the *induced sense*, or *sense-value*, of the primary xenograph. That is to say, the xenodesignatum of a primry xenograph and its induced sense are one and the same mental coentity of mine. The term "*induced sense*" is hereafter abbreviated as "sense" if there is no danger of misunderstanding.

iii) If, particularly, a xenograph is an *idiograph* (*idiographonym*), i.e. a *graphic* (*written*) *idiom*, then I regard it as a primary (reference, induced) one.

iv) When I *consider* a given xenograph as a *secondary*, complex one that has or is supposed to have a certain xenodesignatum in a given domain (say, in a given field of study and discourse or in a given theory) and *analyze* (*divide*) it into primary (reference, induced) xenographs and perhaps into autographs (euautographs and tychautographs, if present), which I regard as *unit* graphonyms that have relevance to the subject matter of the given domain, a *deduced sense*, or *deduced sense-value*, *of the given secondary xenograph* is a *biune mental coentity entity of mine* that has the following two successive *hypostases* (*ways of existence*, *aspects*) with respect to me. The *first hypostasis of the deduced sense*, which is called *the xenodesignatum-producing operation*, or *sense-operation*, *on the secondary xenograph* and which is said to *be expressed by the latter*, is a *mental operation* (*process*) *of mine*, *of coordination* (*synthesis*) *of the xenodesignata* (*senses*) of the constituent primary xenographs and of the *autodesignata* (*isotoken-classes*) *of the constituent autographs* (if present), of the secondary xenograph, which are collectively called *the object-classes of the sense-operation*, *into a single whole class* that is called *the subject-class both of the sense-operation and of the deduced sense.* Once I complete the sense-operation, I mentally *substantivize* the subject-class being the final result of the sense-operation, thus taking another mental attitude towards the secondary xenograph. According to this mental attitude, the subject-class is *the second hypostasis of the deduced sense*, which is said to be *designated by* or to be *the designatum*, or more precisely *xenodesignatum*, *of the secondary xenograph*. A secondary xenograph is said *to express the sense-operation on it*, *to express* or *to have its sense*, and *to designate* or *to have its designatum*. The term "*deduced sense*" is hereafter abbreviated as "*sense*" if there is no danger of misunderstanding.

v) The subject-class of the sense-opeation *on* (*expressed by*) a secondary arithmetic logograph, as '[1+1]+2', i.e. the xenodesignatum, as [1+1]+2, of that *diffused* (*redundant*) *logograph*, can always be alternatively *designated* by some one of its *compendious* (*concise*) *designative synonyms*, as '4', which *express no sense-operation*. Therefore, the sense-operation on the secondary logograph and its subject-class can, in this case, be distinguished formally (syntactically). The difference between two hypostases of a deduced sense can formally be demarcated in a like way for some special secondary *linguistic phonographs* (*glossophonographs*, *glossoxenographs*, *glossophonoxenographonyms*). For instance, the secondary xenograph (glossophonograph) "founder of logic" is a DSN, which expresses the pertinent sense-operation and designates the subject-class of the latter, i.e. the designatum of the DSN. This designatum is the same *singleton* as that immediately designated by the primary xenograph (glossophonograph) "Aristotle". Likewise, the DSN expresses the pertinent sense-operation and designates, as the subject-class of the latter, the singleton of the known Scottish novelist as that immediately designated by the primary xenograph "Sir Walter Scott". The latter example, due to Whitehead and Russell [1910; 1962, p. 67], is based on the historical fact that Walter Scott published his twenty-nine *Waverley Novels* anonymously, and that he kept his authorship of *Waverley* secret. In these circumstances, the proper names "Sir Walter Scott" and "the author of *Waverley*" had different denotata and different senses with respect to any person who did not know the identity of the mysterious author of *Waverley*. Once the identity of the author of *Waverley* had become generally known, the above two proper names became denotative, but not connotative (not sense), synonyms. This example is also discussed by Church [1956, pp. 5–6], although his definition of "sense" differs from that adopted in this treatise. In the general case, however, a secondary glossoxenograph *has no compendious* (*concise*) *designative primary synonym* and therefore there is no way to demarcate the difference between the sense-operation on the glossoxenograph and its subject-class formally. In this case, the uniqueness of the sense of the glossoxenograph is predetermined by the pertinent grammatical rules and by the context, in which the glossoxenograph occurs.

vi) The [xeno]designatum-producing operation, or sense-operation, on a secondary xenograph is alternatively called the *ditto on the object-classes of the operation.* Therefore, depending on a context, in which a secondary xenograph occur, there can be more than one different sense-operation on the secondary xenograph – sense-operations, which differ from one another either by the induced object-classes designated by the constituent primary xenographs or by the order, in which the object-classes are coordinated, or else by both. Accordingly, in the different occurrences, the secondary xenograph can express more than one sense and to have more than one designatum.

vii) A xenograph, primary or secondary, is called a *disemantic* or *polysemantic homograph* if it has respectively two or more two or more different designata. A certain one of the *senses* (*sense-values*) *of a primary homograph* is said to be *narrower*, or on the contrary *broader*, than *another sense of the homograph* if it is a *strict subclass*, or correspondingly a *strict superclass*, *of the class identified with the latter sense*. A certain one of the *senses* (*sense-values*) *of a secondary homograph* is said to be *narrower*, or on the contrary *broader*, than *another sense of the homograph* if the subject-class of the former sense is *less inclusive*, or correspondingly *more inclusive*, than the subject-class of the latter sense.

3) In connection with Df 1.9, it is also noteworthy that, in accordance with the meaning of an IAQ (iconoautographic quotation), "*name*" *sensu stricto* and "*name*" *sensu lato*, e.g., are *two homographs*, or, from a different viewpoint, they are a *single homograph* "name", while "*name sensu stricto*" and "*name sensu lato*" are *two different xenographs* (*two different names*). In this case, the count noun "*name*" *sensu stricto*, or "*name*" *sensu lato*, can denote the same class of names as the count name "*name sensu stricto*", or correspondingly "*name sensu lato*".

4) The qualifiers "sensu lato" and "sensu stricto" to a given nym or to the class-denottum of the nym are epistemologically relativistic antonyms. That is to say, besides the interpretations of the names "name sensu stricto" and "name sensu lato" given in Df 1.9, these names can be interpreted in many other ways, depending on a.*domain*, in which the names are used. Particularly, *in the domain of glossonyms* (*linguistic nyms*), and not in the domain of *all nyms* (*sensible things*), the class of

glossonyms that I have denoted in Df 1.9(2) by the numeralable (count) name “name sensu stricto” and abbreviate as “name” should be *redenoted* (*denoted anew*) as “name sensu lato”, while the term “name sensu stricto” can be freed of its present denotatum and be *redefined* (*defined anew*) in one or another narrower sense (cf. various definitions of “name” in WTNID). For instance, the term “name sensu stricto”, abbreviated as “name”, can be defined either as a synonym of the IDSN “*nounal name*”, i.e. “*substantive name*”, or alternatively as a synonym of the IDSN “*major form class*”, i.e. “*part of speech*”. In this connection, it is noteworthy that in some languages other than English, *parasynonyms* (counterparts) of the English noun “name” are used in a sense close to that of “major form class”. For instance, the verbatim translations into English of the Russian parasynonyms of the English nouns “*noun*” and “*adjective*”, – namely, “имя существительное” \imya sushchestvitel’noe\ and “имя прилагательное” \imya prilagatel’noe\, – and of the Hebrew parasynonyms of the same English nouns “ *noun*“, “*adjective*” and of “*verb*”, – namely, “שם עצם” \shem etsem\, “שם תואר” \shem toar\, and “שם פעל” \shem poal\., – are “*noun name*”, “*adjective name*”, and “*verb name*”; “имя” \imya\ or “שם” \shem\ means «name».•

Cmt 1.10. 1) By transcendent extrapolation, the descriptive terms, which have been introduced in Df 1.9(3), can be objectified (externalized) by teplacing the qualifier “*with respect to me*” with the qualifier “*with respect to each given sapient subject as I or you*”. Either qualifier implies that any onymological term is defined *with respect* (*in relation*) *to each given participant of this game of* “I”, i.e. with respect to me and independently with respect to you, but not with respect to both of us and not with respect to all of us indiscriminately. Thus, most generally, either of the allomorphs (count nouns) “onym” and “nym” denotes the *class of sensible things with respect to a concrete sapient subject* (as *I* or *you*), *of which he is* (correspondingly, *I am* or *you are*) *conscious*. Consequently, any *numeralable* (*quantifiable with a numeral*, *count*) complex monomen that ends with “nym” denotes a certain *species* (*specific class*, *subclass*) of the above *egocentric class of egocentric sensible things*. That is to say, a *name* [*sensu stricto*] “onym” (“nym”), alone or together with some prepositive or postpositive qualifiers, is meaningless unless it has at least one *possessive* or *relative qualifier* (as “my”, “of mine”, “with respect to me”, “your”, “of yours, or “with respect to you”, “John’s”, “of John”, “with respect to John”) indicating who perceives the onym (nym) that is mentioned by using that name. It would be a nonsense to state that a concrete man, tree, or stone, – to say nothing of concrete head ache, hunger, or thirst, – is a nym without mentioning the concrete perceiver of the nym (sensible thing) thus named. This is the main principle of the entire phraseology and particularly of the entire terminology that I qualify *psychologistic*. Here follows an example illustrating psychologistic phraseology.

2) When you read a copy of my treatise, you read graphic isotokens of the graphonyms that I have created and used. Besides these isotokens, you perceive and comprehend some other *exteroceptive nyms* (*onyms*) and particularly some other graphonyms, none of which and no isotokens of which are available to my senses; and vice versa. To say nothing of all *your interoceptive nyms,* the latter exteroceptive nyms (sensible objects) of yours are not nyms (sensible objects) of mine, and conversely such nyms of mine are not nyms of yours. Therefore, in accordance with its defininition, the taxonym "nym" ("onym") and the taxonyms of various classes of nyms are defined in relation to a concrete sapient subject. Once prescinded from the sapient subject, all the taxonyms become equivocal. At the same time, as long as you read isotokens of my graphonyms and understand them as your own in accordance with Cnv 1.1, the above-mentioned equivocality of the taxonyms does not manifests itself and is harmless as if it does not exits, although it is fundamental – just as fundamental is Cnv 1.1. The mental substitutions that are made explicit in Cnv 1.1 are widely used in the entire practice of intercommunication of sapient subjects via their exteroceptive nyms.•

Cmt 1.11. 1) Besides *native languages* (*NL's*), there are *contrived languages* (*CL's*), written and spoken. *Esperanto* (from the Latin present indefinite verb "sperare" meaning «to hope») and *Ido* (from the Esperanto count noun "ido" meaning «descendant» or «offspring») are two most widespread contrived international *phonemic* languages. Esperanto was created by Polish oculist Ludwik Zamenhof who published his invention under pseudonym "Dr. Esperanto" in 1887. Esperanto is based on words common to the main European languages, whereas Ido is, as follows from its name, a modification of Esperanto. Like any one of the NL's, on which it is based, Esperanto is a pair of written Esperanto and spoken Esperanto, and the same is true of Ido.

2) There is a tendency to use the capitalized word "Esperanto" figuratively as a generic name for forming names of various means of communication among

members of an international scientific community. I shall employ the non-capitalized word "*esperanto*" for this purpose. Thus, speaking informally and figuratively, an international system of *nomenclature*, i.e. *notation* or *terminology*, which is used in a given branch of science (given field of study and discourse), or a part of an internationally spread native language (as English) augmented by such a system of nomenclature is an esperanto both of the given branch of science and of the community of people working in that branch. For instance, *the system of analytical and structural molecular formulas* or a certain part of English, augmented by that system of notation, is an esperanto of chemistry and of chemists. In this case, the *periodic table of chemical elements*, either in its *modern long-period form* or in its *original short-period form* (due to Russian chemist D. I. Mendeleev), is the *lexibary* (*xenographic syllabary*) *of the system of molecular formulas*. Analogously, a certain part of English, augmented with the appropriate nomenclature, is an esperanto of logicians, or mathematicians, or physicists, or oceanographers, or biologists, or physicians, etc. Under the above definition of the class denoted by the non-capitalized word "esperanto", there can be several different esperantos in a given branch of science. Any generally accepted system of notation (as a system of analytical or structural molecular formulas) is *logographic* (*aphonographic*) esperanto.

3) Either one of the allomorphic nouns "onym" and "nym", being the basic elements of nymology (onymology), meets lack of a most general impartial, unambiguous, and concise term which should allow conveniently treating of graphic objects (graph*onyms*) occurring in this treatise or in writings on logic or mathematics of other authors, and generally of any sensory objects, whose main property is to materialize both form and matter of logical reasining. Apart from being short, either of the allomorphs "onym" and "nym" has an advantage over either of their synonyms "sensible thing" and "name sensu lato" of allowing to form new *self-explanatory terms* by adhering the root "onym" or "nym" with one or more Anglicized Greek morphemes (prefixes or combining forms), having the appropriate etymological senses, and by attaching the *complex numeralable monomials* (*monomina*, *one-word names*, *nouns*) thus formed with the desired lexical senses (cf. the established English nouns of this kind mentioned in Df 1.9(2)). Particularly, the presence of the noun "nym" in the vocabulary of this IML allows conveniently introducing *polymorphemic* monomial terms expressible of the different statuses of interrelated nyms, – such

terms, e.g. as “*euautographonym*”, “*panlogographonym*”, “*endosemasiopasigrapho*┐*nym*”, and “*catlogographonym*”, abbreviated respectively as “*euautograph*”, “*panlogograph*”, “*endosemasiopasgraph*”, and “*catlogograph*”, – and also introducing polymorphemic monomial terms expressible of the different hypostases of the same nym, – such terms e.g. as “*xenographonym*”, “*euxenographonym*”, and “*tychautographonym*” abbreviated respectively as “*xenograph*”, “*euxenograph*”, and “*tychautograph*”. In fact, nymology is the most extensive and indispensable expressive means of f this treatise. Like any one of a great many systems of notation used in science, nymology is a kind of *esperanto* that has the following conspicuous properties.

4) All elements of a system of notation are *logographs*, i.e. *non-lettered graphonyms*. By contrast, all elements of nymology are *phonographs*, i.e. *lettered graphonyms* (*grammographonyms*, *grammographs*) that have therefore *graphic isotokens* and *phonic paratoken*s (*phonic values*). In this respect, nymology is similar to any other *verbal* system of terminology with the following essential difference, which allows qualifying it as an esperanto and which makes it similar to a *logographic* system of notation. Translation of any nymological term from one native language (as English in this case) into another can be made simply by *transliterating* the term in the target language in the exclusion of the case, where the target language is Greek. In this respect, nymology is independent of a specific WNL underlying the IML, in which it is developed.

5) Nymology is not the end in itself. A nymological term will be *introduced* only if the following two conditions are satisfied:

i) There is in Creek a prefix, combining form, or word that can be Anglicized so as to become an appropriate consonant and semantically expressive English prefix or combining form that can be adhered either to “onym” (“nym”) or to another “onym”-noun.

ii) Use of the nymological term is justified by considerations of necessity, brevity, or expressiveness especially in the case, where it is impossible to express the sense of the nymological term by a concise non-nymological term.

At the same time, I shall employ only a part of the nymological terms that I introduce. The others are just suggested as synonyms of longer native English descriptions through a genus and a differentia that will actually be used as terms. Some nymological terms will be used interchangeably with their native English descriptions. Subsequent intelligible use of the proposed nymological terms in practice will serve as the ultimate criterion of their efficiency and viability.•

Df 1.11: ***Onology, the second psychologistic onomastics.*** 1) In contrast to "όνομα" \όnoma\, meaning a *name* or *noun*, the Greek noun "ον" \on\ means *a being* or *creature*. Accordingly, replacing the base "onym" in a graphic (written) "onym"-noun with the base "*on*" results in the monomen (monomial), which is called an "*on*"-*based noun* or briefly an "*on*"-*noun* and which, depending on my mental attitude towards it, *designates* with respect to me, either *the class of tokens* of the "onym"-noun, called its *autovalues*, or *the class of some other coentities of mine*, called its *xenovalue*. I distinguish between the class of *isotokens of a graphonym*, i.e. tokens of the same genesis and hence of the same sensorial kind, and the class of its *paratokens*, i.e. tokens of another genesis and either of the same or of another sensorial kind, – if the graphonym has these. For instance, tokens of a graphonym are called its isotokens if they are graphic and paratokens if they are phonic; likewise *tokens of a phononym* are called its isotokens if they are phonic and paratokens if they are graphic. Thus, while an "onym"-noun designates a class of sensible (physical) coentities of a sapient subject, the respective "on"-noun designates a class of classes, i.e. a class of insensible (mental, psychical) coentities of the sapient subject. For instance, a *taxonym*, i.e. a *taxonomic name*, is a sensible thing, whereas a *taxon* is a *taxonomic class* denoted by a certain taxonym; a *graphonym* is any fragment of this treatise or the whole of it, whereas a *graphon* is a class of graphic tokens (isotokens) of the graphonym. "Phononym" and "phonon", "glossonym" and "glosson" are two other pairs of an "onym"-noun and of the respective "on"-noun. Still, in a field of study and discourse other than Psychologistics, some "on"-nouns may assume values that are irrelevant to values of their counterpart "onym"-nouns, while some other "on"-nouns may have no counterpart "onym"-nouns at all. For instance, "electron", "exciton", "meson", "nucleon", "phonon", "photon", "polaron", "positron", "proton", etc are *physical terms* of this kind.

2) Like a disyllabic or polysyllabic "onym"-noun, the respective or any like "on"-noun is an *informal description of the species* (*IDSN*) *through the pertinent genus designated by the generic name* "*on*" *and through the pertinent difference* or

differences designated by the appropriate qualifiers. An "on"-noun is alternatively called an *onological term* or *word* or *monomen* (*monomial*), while the system of onological terms is called "*onology*" and also the *second psychologistic onomastics*.

3) A *morpheme* of an onogical term will be called an *onological morpheme* or brieflt an *onomorpheme*. Accordingly, the *root* (*base*) "*on*" of an onymological term will be called an *onological root* (*base*). All other onological morphemes are the same as onymological ones.

4) Besides onymology, onology is another extensive *terminological esperanto* of the psychologistic terminology, but it is used in the treatise much less widely than onymology.•

Cmt 1.12. 1) Onymology and onology are not sufficient for making all necessary univocal psychologistic [meta]terms. In contriving *monomial* and *polynomial* psychologistic terms other than onymological and onological monomina, I use another *onomastic method*, which I call the ***third psychologistic onomastics*** and which has following three aspects. First, I modify some established but equivocal English (Anglicized) words of Latin origin either semantically or morphologically or both and use each one of the group of cognate English words thus obtained as a univocal psychologistic monomial term. Second, I adopt every relevant established univocal and etymologically correct English term, Latinized or chaste, without altering it with the proviso that if a Latinized term is incorporated as a taxonym into a certain taxonomy then its form should agree with the forms of other Latinized taxonyms of the same rank (to be illustrated in due course). Third, if it is impossible to contrive an appropriate new *monomial* English term or to utilize an existing one, I *formally* define (introduce) the appropriate English *binomial* or *polynomial IDSN* (*informal DSN*), i.e. an *informal binomial* or *polynomial description of the relevant species through a genus and the differentia or differentiae*. In this connection, it is noteworthy that, beyond the third psychologistic onomastics, the appropriate binomial or polynomial IDSN's are introduced and used *informally* and *routinely* without mentioning them. For instance, the binomial IDSN's "living organism", "green tree", and "graphic expression", and the polynomial IDSN's "autotrophic living organism", "leaf-bearing evergreen tree", and "genuinely xenonymous graphic expression" are introduced and used informally.

2) In forming complex monomial terms of Greek or Latin origin, I stick, as far as possible, to the following general *onomastic principle* (*word formation rule*), which I call the *principle of etymological homogeneity of complex monomials* or briefly the *Etymological Homogeneity Principle* (*EHP*):

> A new complex Anglicized *monomen* (*monomial*, *one-word name*) should, as far as possible, be *etymologically homogeneous* in the sense that any of its constituent morphemes should originate from the same language, particularly either from Greek or from Latin, unless of course it is a morpheme of both languages.

I shall also follow this principle in adopting established terms for use in the treatise. For instance, the established adjectives “endopsychic” (“endopsychical”) and “exopsychic” (“exopsychical”) of Greek origin and “intramental” and “extramental” of Latin origin satisfy the EHP, whereas the established adjectives “intrapsychic” (“intrapsychical”) and “extrapsychic” (“extrapsychical”) do not. At the same time, in no connection with the EHP, the adjectives “intramental”, “intrapsychic” (“intrapsychical”), and “endopsychic” (“endopsychical”) are *redundant* synonyms of “mental” and “psychical”.

3) The *etymological senses* of all new and some old (established) *morphological units*, which are used in the treatise as *prefixes*, *combining forms*, *roots* (*bases*), or *words* in forming Anglicized terms of Greek and Latin origins, are explained respectively in Dicts A1.1 and A1.2 that are given in Appendix 1 (A1) along with the pertinent comments. These two etymological dictionaries have been compiled mainly with the help of the Modern Greek–English and English–Modern Greek dictionary by Pring [1982] and the Latin–English and English–Latin dictionary by Simpson [1968]. In general, the latter two bilingual dictionaries are the main sources of all my excursions into Greek and Latin etymology throughout the treatise, although these sources may not be indicated explicitly every time when I use them.

4) The intended lexical sense of every new Anglicized *xenograph* (*xenographonym*, *significant graphonym*, *graphic name*, *graphic linguistic form*) of Greek or Latin origin, such as an Anglicized morphological unit or entire complex monomial or polynomial [meta]term, will after all be rigorously defined independently or as if independently of its etymological sense. Therefore, the lexical sense of an Anglicized xenograph may essentially differ from its etymological sense

(cf. Df 1.9(3)). Particularly, the lexical sense of a complex Anglicized monomial or polynomial term may deffer and often does differ from its etymological sense that is mentally obtained as intersection of the conjoined etymological senses of its simple constituent bound or free morphological units. Still, I usually attach the lexical sense to any new Anglicized xenograph either *by homology*, i.e. *by direct association*, or *by analogy*, i.e. *by oblique association*, with its etymological sense. For this reason, the etymological sense of a new Anglicized xenograph turns out to be suggestive of its intended lexical sense at least as its mnemonic justification. Therefore, the reader is advised, especially in reading preliminary explanations of this introduction, to consult Dict A1.1 or A1.2 every time when a new term of Greek or Latin origin is introduced.•

1.7. Basic taxonomy of onyms

Preliminary Remark 1.1. When I *consider* various aspects of a *nym* (*onym*) *but do not use it purposefully*, I shall use the following terminology and phraseology.

1) Any coentity of mine, which I associate with a nym, linguistic or not, including the nym itself, and which I can *put forward* (*mention*, *refer to*, *denote*, *mentally experience*) and communicate by using the nym properly or for my own purposes, is *impartially* called an *import value* and also, briefly, a *value*, or *import*, *of the nym with respect to me*. I shall say that a nym *exhibits* or *demonstrates itself* and *represents any other of its values* or that conversely the latter value *is represented* by the nym. In the general case, a nym may have or assume (take on) cognitive, conative, affective, syntactic, semantic, pragmatic, or monetary value with respect to me. The monetary value of a nym, if exists, is called its *worth*. A nym is said to be *worthful* if it has a worth and *worthless* if otherwise. Henceforth, a worthful nym is supposed to be devoid of (prescinded from) its worth.

2) When I use a nym as a *referent for mentioning* (*denoting*) one of its values as its *relatum*, the value that I thus put forward is called *the denotatum* (*denotation value*, pl. “*denotata*”) *of the nym*. Particularly, a nym may be used either for mentioning (denoting) itself, i.e. be used self-referentially, orit may be used for mentioning any one of an indefinite number of recognizably same nyms, which are called *isotokens of the former*, *prototypal nym* and also *isotokens*, or *coisotokens*, *of one another*. A prototypal nym itself, any one of its isotokens, or the class of its isotokens is called an *isoautonymous value*, or briefly *isoautovalue*, *of the prototypal*

nym. A prototypal nym may also have an indefinite number of counterpart nyms, which are called *paratokens of the prototypal nym* and also *isotokens*, or *coisotokens*, *of one another* in the sense that each of them has the following properties with respect to me:

a) A paratoken of the prototypal nym has isoautovalues of its own, which are distinct from the isoautovalues of the prototypal nym.

b) The values of the paratoken, which are not its isoautovalues, are the same as the values of the prototypal nym, which are not its isoautovalues.

An isotoken or paratoken of a prototypal nym is indiscriminately called a *token* of the nym. Any one of the paratokens of a prototypal nym or the class of the paratokens is called a *para-autonymous value*, or briefly *para-autovalue*, *of the prototypal nym*. Accordingly, the prototypal nym itself, any one of its tokens, or a certain class of its tokens is called an *autonymous value*, or briefly *autovalue*, *of the prototypal nym*. A value of a prototypal nym, other than any one of its autovalues, is called a *xenonymous value*, or briefly *xenovalue*, and also a *significand*, or equivocally *signification*, *of the prototypal nym*.

3) The fact that a nym (onym), i.e. a sensible thing, has values (imports) with respect to me justifies synonymously calling it *name sensu lato*. At the same time, since "nym" ("onym") and "sensible thing" are synonyms, classification of nyms (onyms) is practically inexhaustible. Particularly, nyms can be classified in accordance with one or more of the following criteria and, perhaps, in accordance with some other criteria that have no appropriate concise names:

i) physical properties of a nym, including its location relative to the *perceiver's* (*sapient subject's*) *body*, and hence including the type of a *sensor* (*sensory end organ*, *SEO*) of the *nervous system* (*NS*) or the type of the entire *sense organ* (*SO*), with the help of which the nym is perceived and recognized by *the perceiver* (*sapient subject*);

ii) psychical (semantic, syntactic, or pragmatic) properties of a nym relative to its perceiver;

iii) the genesis, i.e. the way or origin, particularly either natural or artificial, in which a nym has come into being.

Therefore, taxonomy of nyms turns out to be highly furcated, inhomogeneous, and unavoidably selective (fragmentary, incomplete). A part of this selective taxonomy, which is most immediately relevant to the subject matter of this treatise, is suggested below in this section. Relatively complete taxonomy of the senses of a man and the pertinent classification of nyms by adequate senses are given in Essay 2 (E2).•

Df 1.12. 1) A *sensory end organ* (*SEO*), or *sensor*, of the *nervous system* (*NS*) *of a sapient subject* is a *receptor and transducer* that is *inherently* (*innately*, *naturally*) *responsive* to the *energy* of the pertinent kind of any marked change (particularly, to the appearance or disappearance), of a certain material ingredient of the external or internal environment of the sapient subject – a change that is called a *stimulus* (pl. "*stimuli*"), or *input agent*, *of the SEO*. In response to a stimulus, the SEO *transduces* the stimulus into *the respective unique frequency-modulated sequence of similar nervous impulses in the associated afferent* (*centripetal*) *nerve fibers*, which is called an *output agent*, or *nervous signal*, *of the SEO*. Upon reaching the *specialized part* of the cerebral cortex of the sapient subject, called his *sensorium*, the nerve signal is transduced into the corresponding *sensation*. A sensation is a *projective* (or *polarized*) *mental* (*psychical*) *coentity* of the sapient subject, i.e. one, which is located within the physical limits of his sensorium and hence within the physical limits of his cerebral cortex and which is always mentally experienced by the sapient subject as a *nym* (*onym*), i.e. as an *extramental* (*exopsychical*) *real* (*physical*) *object of him*.

2) A specialized function (faculty, power) of a sapient subject to *perceive* (*receive*) *sensations* either by means of the pertinent *sense organ* (*SO*) or *organs* (*SO's*), containing the appropriate SEO's, or by means of separate SEO's or groups of SEO's is called a *sense function*, or, briefly, *sense*, *of the sapient subject*. A separate SEO or the entire SO of a sapient subject, which participates in converting the stimuli produced by a certain onym (nym) into the pertinent sensation of the sapient subject, is indiscriminately called an *onym-stimulus* (*nym-stimulus*) *receiver*, or briefly a *stimulus-receiver*, of the sapient subject.

3) In accordance with Df 1.3(3), the above two items apply also with "conscious subject" in place of "sapient subject".•

Cmt 1.13. In Aristotelianism, the term "*ενέργεια*" \enérgia\, conventionally Anglicized as "*energy*", is actually a synonym "God", which denotes the form-giving cause, or motive, of all changes of things, occurring in the world.•

Df 1.13: *The dichotomy of nyms in accordance with their locations relative to the interpreter's (sapient subject's) body.* A *nym* is said to be *exteroceptive* if it is located *outside the body of its interpreter* and *interoceptive*, or *proprioceptive*, if it is located *inside the body*. A *stimulus* is said to be *exteroceptive* or *interoceptive* (*proprioceptive*) if the nym producing it is one of the respective type. A *sensor* (*SEO*) is said to be an *exteroceptive* or *interoceptive* (*proprioceptive)* one, and also an *exteroceptor* or *interoceptor* (*proprioceptor*), if it is naturally responsive to stimuli of the respective type, i.e. to stimuli *arising outside the interpreter's body* or *within the body* respectively. A *sense* is said to be *exteroceptive* or *interoceptive* (*proprioceptive*) if its sensors (SEO's) are ones of the corresponding type.•

Cmt 1.14. The complete taxonomy of the senses of a man, both exteroceptive and interoceptive, and the pertinent taxonomy of nyms are presented in Essay 2 (E2). The taxonomy of nyms by adequate senses is essential in incorporating *semiotics* (*semiotic*) in general and *semeiotics* (*symptomatology*, *symptomology*) in particular into Psychologistics. At the same time, as opposed to nyms of various kinds that are *mentioned* in this treatise, the nyms that are *used* in the treatise are exclusively *graphonyms*, i.e *graphic* (*written*) and hence *exteroceptive* ones of a certain kind that have various semantic and syntactic properties with respect to me. These nyms form the nomenclature both of the object logistic systems of the IML and of the IML itself, including its terminology. Together with some other relevant kinds of exteroceptive nyms, the nyms comprised in this treatise are described below in this introduction. Relationship between Psychologistics and semiotics, including semeiotics, will be made explicit in due course later on.•

Df 1.14: ***The classification of exteroceptive nyms by adequate senses (nym-stimulus receivers).*** A sapient subject, i.e. a normal (healthy) adult man (unless stated otherwise), has the following *five exteroceptive senses* (*sense functions*): [*the senses of*] *sight*, *hearing*, *touch*, *taste*, and *smell,* called also the *visual*, *auditory, tactile*, *gustatory*, and *olfactory senses* in that order. Accordingly, a nym is said to be *visual* (*visible*), *auditory* (*audible*, *acoustic*), *tactile* (*tangible*, *palpable*), *gustatory,* or *olfactory* if it is perceived and identified by means of the corresponding sense. In accordance with Dict A1.1, a visual, auditory, tactile, or gustatory nym can alternatively be called an *optonym, acoustonym*, *aptonym*, or *gustonym* respectively. It is understood that if a nym can be perceived by means of two or more senses then it should be classified by the corresponding combined (compound) qualifier.•

Df 1.15: *The basic dichotomy of exteroceptive nyms by genesis.* All *exteroceptive* nyms are divided into two classes (kinds): *artificial* ones and *natural* ones. An *artificial*, or *man-made*, *nym* is one that is produced through the art, skill, and will (mental effort) of a man or group of men. By contrast, a *natural*, or *nature-made, nym* is one that is produced by *animate or inanimate nature*, without any *purposeful* agency of a man or group of men. An artificial nym can briefly be called a *technonym* and a natural nym a *physonym*.•

Df 1.16: *The basic taxonomy of artificial nyms by genesis.*

1) An artificial (man-made) nym that occurs in or on a physical surface and that has been created with an instrument or equipment is called a *written* [*sensu lato*], or *graphic*, *nym*, and also, in one word, a *graphonym*.

2) An articulated sequence of sounds, intonations, and pauses is called

a) a *spoken*, or *oral*, *nym*, and also, in one word, a *mylonym* if it is produced with the ordinary (non-musical) modulation of the human voice;

b) a *sung*, or *adoic*, *nym*, and also, in one word, an *adonym* if it is produced by the human voice in musical tones.

3) A mylonym or an adonym, each taken individually, is indiscriminately called a *vocal*, or *phonic*, or *phonetic*, *nym* and also in one word, a *phononym*. A reproduction of a mylonym or of an adonym or, generally, of a phononym, prerecorded or not, by means of technical equipments is also called a mylonym, adonym, phononym respectively.

4) When either of the morphological constructions “graphonym” and “phononym” is used as a postpositive (adjoined) combining form in forming a longer count noun that denotes a subclass of graphonyms or of phononyms respectively then “*graphonym*” will be abbreviated as “*graph*” and “*phononym*” as “*phon*”. For instance, “*homographonym*” is abbreviated as “*homograph*” and “*homophononym*” as “*homophon*”. A monomen (monomial) of the form **“**—graph**”**, or **“**—phon**”**, which is obtained by omission of the base “onym” from a certain “onym”-noun, will be called a “*graph*”*-based noun* or briefly a “*graph*”*-noun*, or, correspondingly, a “*phon*”*-based noun* or briefly a “*phon*”*-noun*.•

Cmt 1.15. 1) In accordance with kinds of nym geneses and kinds of nym-stimulus receivers (adequate senses), there are many different kinds of exteroceptive nyms, both artificial and natural, although some of them cannot be provided with concise unambiguous names owing to linguistic difficulties. For instance, an exteroceptive nym is said to be:s

i) a *light*, or, when applicable, a *light-flashed*, one, and also a *photonym*, if it is produced by means of light;

ii) a *sound*, or *sonic*, one, and also an *echonym*, if its produced by means of sound;

iii) a *wigwagged* one, and also a *kinonym* or *kunonym*, if it is produced by waving the hands or arms, naked or equipped with flags or portable lights,

iv) a *dactylological*, or *manual*, one, and also a *dactylonym*, if it is produced by the fingers of a hand (as in the manual alphabets of the deaf);

v) an *embossed*, or *relief*, one if it is produced by means of embossing (as in the *Braille* or *Moon code* of the blind);

etc. A photonym is either artificial (as any one formed with the help of the international light-flashed Morse code based on usage of short and long flashes of light) or natural (as a flash of lightening or a star). Likewise, an echonym is either artificial (as a phononym or as any one formed with the help of the international sonic Morse code based on usage of short and long sounds) or natural (as a peal of thunder). By contrast, a kinonym, a dactylonym, and an embossed nym are exclusively technonyms (artificial nyms). In the general case, a nym may belong to many different classes. For instance, a photonym, kinonym, and dactylonym is an optonym but not necessarily vice versa; a phononym is an echonym but not necessarily vice versa; and a embosed (relief) nym is by definition a *graphonym sensu lato*, i.e. a nym being *graphic sensu lato*, or *written sensu lato*. In the last case, whenever confusion can result, a *graphonym sensu stricto*, i.e. a *ordinary intangible* (*impalpable*) *graphonym*, which is *visually* perceived by a normal interpreter that can see and which is incapable of being felt by touch, is more specifically called an *optographonym* or briefly *optograph*, whereas a *Braille* or *Moon embosed* (*relief*) *graphonym* [*sensu lato*], which is perceived by a blind interpreter *tangibly*, is more specifically called an *aptographonym* or briefly *aptograph*. I shall also say that an optograph *is written* [*sensu stricto*], whereas an aptograph *is embossed* or *is written sensu lato.*

2) Writing, speaking, semaphoring (by dactylology, wigwag, or light), embossing, or sounding glossonyms of the respective kind are different ways of *signifying* certain entities as the intended *values* (*imports*) of the glossonyms and

exposing (*submitting*) the glossonyms along with the values for communication. Accordingly, the noun or gerund “writing”, “speaking”, “semaphoring”, “embossing”, or “sounding”, meaning the act or process or result of forming a sensible nym of the respective kind for communication, can be generalized as “*exposing*”, “*submitting*”, or “*transmitting*”, – depending on the situation or on the context or on both. The result of exposing an NL of a given name (as English) in trms of a code (as any one of those indicated above), is called an *encoded NL* (*ECNL*) of the same name.•

Cmt 1.16. In forming “nym”-based, “graph”-based, and “phon”-based nouns, introduced in Dfs 1.14–1.16 and Cmt 1.15, I employ the pertinent vocabulary entries of Dict A1.1. Here follow some relevant comments.

1) The prepositive or postpositive combining form “*graph*” originates from the following Greek etymons: the noun “γραφή” \grafí\ having the same sense as “writing”, its kindred verb “γράφω” \gráfo\ having the same sense as “*to write*”, and its adjective derivative “γραφικός” \grafikós\ having the same sense as “*written*” or “*drawn*”. By contrast, any of the prepositive or postpositive combining forms “*gramm*”-, “*grammo*”-, -“*gram*”, and -“*gramme*” originate from the Greek noun “γράμμα” \grámma\ having the same sense as “*letter*” both in the sense of “*primitive symbol*” and in the sense of “*message*”. The latter etymon is completely different from any of the former etymons of the combining form “graph”. This fact immediately implies the following two corollaries.

i) When the words such that “phonogram”, “ideogram”, “logogram”, etc are used as synonyms of the words “phonograph”, “ideograph”, “logograph”, etc, the former are misnomers from the standpoint of etymological analysis. Therefore, I shall not use such “gram”-words. At the same time, I shall use the new noun “*graphogram*” for denoting a *graphic* (*written*) *letter* of any given *alphabetic native language* (*AbNL*) and the new noun “*grammograph*” as an abbreviation of the “onym”-noun”*grammographonym*” for denoting a *lettered*, or *literal*, *graphonym*, i.e. a *juxtaposition* (*linear assemblage*) *of mutually articulated graphogramms of an AbNL*. Analogously, I shall use the new noun “*graphosyllable*” for denoting a *graphic* (*written*) *syllable* of any given *syllabic native language* (*SbNL*) and the new noun “*syllabograph*” as an abbreviation of the “onym”-noun”*syllabographonym*” for denoting a *syllabled graphonym*, i.e. a *juxtaposition* (*linear assemblage*) *of mutually articulated graphosyllables of an SbNL* (see the next definition).

ii) Letters of an AbNL can be encoded in many different ways, so that they are not necessarily graphic (written). Particularly, in accordance with Cmt 1.15, letters can also be *semaphored*, e.g., *wigwagged*, *Morse's light-flashed*, or *dactylological*, and also *Morse's sonic* and *Braille's* or *Moon's embossed* (*relief, tangible*) ones. Therefore, the qualifier "written" in the name "written letter" or the qualifying combining form "grapho" in the synonymous noun "graphogram" is not redundant.

2) In accordance with Simpson [1968], the Latin masc. adjective "grăphĭcus" (fem. -"a" and neut. -"um" in place of -"us") means «*concerned with painting*», so that its sense differs from the sense of *its Greek etymon* "γραφικός". Therefore, that Latin adjective has nothing to do with the etymological sense of the Anglicized combining form "graph" as explained in the above item 1.

3) The prepositive or postpositive combining form "*phon*" as used in this treatise originates from the noun "φωνή" \foní\ assuming the same senses as the nouns "*voice*", "*cry*", and "*shout*". In agreement with this etymon, WTNID defines the conventional Anglicized postpositive nounal combining form -"phone" thus:

> «**-phone** … *n comb form* -s …**:** sound **:** voice – in names of musical instruments and soundtransmitting devices ⟨saxo*phone*⟩ ⟨ear*phone*⟩ ⟨radio*phone*⟩».

This definition does not cover use of -"phone" in the established linguistic term "*homophone*". At the same time, the convenient and natural postpositive combining form -"phon" that has been introduced in Df 1.16(4) does not occur as an entry in WTNID. I regard "homophone" as an abbreviation of the new nymological noun "*homophononym*" and therefore I shall spell it as "*homophon*", – like many other new "phon"-nouns.•

1.8. Human languages

Df 1.17. 1) An *artificial nym* that is used, occasionally or systematically, for communication of a sapient subject with other sapient subjects or with himself is called a *communicative nym*. I say that, for instance, I use an artificial nym, primarily a graphonym, for communication with myself if I turn a certain *conception of mine* into a *concept of mine* by *assigning* the former to that nym as its intended value with the purpose to classify or memorize the conception or to repeatedly check its consistency.

2) A system of interrelated *artificial* nyms that is or was used, or is designed to be used, in systematic intercommunication among members of a group of people is called a *language* or, more specifically, a *human language* (*HL*) – in contrast, e.g., to a systematic method of programming a computer, which is called a *programming*, or *machine*, *language*, or *code*, and in contrast to a systematic means of intercommunication among members of a group of *socialized conscious animals*, which is called an *animal language* (*AL*).

3) The HL, which has been brought into being in a given speech community by many generations of people during a long historical process of development of the human civilization and which is therefore detached from the agency of distinguished individuals, is called a *native language of the speech community* or, less explicitly, a *native language* (*NL*). An HL, which has been invented and brought into being by a man or group of men, is called a *contrived*, or *invented*, *human language* (*CHL* or *IHL*). A CHL is called a *formalized language* if it is the result of a certain unambiguous interpretation of an uninterpreted logical calculus (logistic system) and an *unformalized language* otherwise. It is understood that a machine language is a necessarily contrived one. Therefore, for avoidance of confusion, by a *contrived*, or *invented*, *language* (*CL* or *IL*) I shall hereafter understand a *human* one (*CHL* or *IHL*).

4) With some exceptions when a primitive tribe has *no written language*, any of a great many of modern NL's comprises two interrelated languages. One of them, called a *written*, or *graphic*, *native language* (*WNL*, or *GNL*), is a systematic means of intercommunication of people by using *graphonyms*, – especially *written*, or *graphic*, *words*, – and the other, called a *spoken*, or *phonic*, *native language* (*SNL* or *PhNL*), is a systematic means of intercommunication of the people by using *phononyms* (*mylonyms*), – especially *spoken*, or *phonic*, *words*. A written language will be called a *phonographic*, or *phonetic*, *language* (*PhgL* or *PhtL*) if it has a spoken counterpart and an *aphonographic*, or *boobographic*, *language* (*APhgL* or *BgL*) if it has no spoken counterpart. A spoken language will be called a *graphophonic language* (*GphL*) if it has a written counterpart and an *agraphophonic language* (*AGphL*) if it has no written counterpart. Accordingly, a WNL is alternatively called a *phonographic*, or *phonetic*, *native language* (*PhgNL* or *PhtNL*), whereas an an SNL is alternatively called a *graphophonic native language* (*GphNL*). If to a given WNL

there exist two or more distinct and especially mutually unintelligible SNL's then the latter are said to be *vernacular*s, or *dialects*, of each other or of one another.

5) A pair of interrelated written and spoken NL's is conventionally denoted either by a *capitalized noun*, – e.g. "English", "French", "Mandarin", etc, – or by a proper name consisting of the *homonymous adjective* preceded by the definite article and followed by the appropriate one of the nouns "*language*", "*vernacular*", and "*dialect*". There is *no bijective* (*one-to-one*) correspondence between the WNL and the SNL, being the written and the spoken component of the NL of a given name. Moreover, it is impossible to establish a correspondence between the two components formally. From the standpoint of semantic analysis, every conception that can be expressed in the SNL can also be immediately and straightforwardly expressed in the paired WNL, but not vice versa. For instance, all punctuation marks, if present, are aphonographs of the WNL, some of which can be expressed by *variations of tone*, but *not phonemically*. All aphonographs of the WNL can unambiguously be expressed in the SNL only mediately and indirectly by using their verbal names. This is particularly why all aphonographs of the WNL should have univocal verbal names. Therefore, an IML is necessarily a graphic (written) language.

Cmt 1.17. 1) An HL that I qualify *native* is usually qualified *natural*, whereas a language that I qualify *contrived* is usually qualified *artificial*. I avoid applying the qualifiers "natural" and "artificial" to "language" for the following reason. An HL is neither a sensible thing nor a thing (real entity) at all. An HL is an entity that can, most generally, be called a *socio-personal psychophysical* (*psychico-physical*), or *physopsychical* (*physico-psychical*), *system*. This system is essentially *artificial*, i.e. *man-made*, and *not natural*, i.e. *not nature-made*. Accordingly, *all glossonyms*, which are employed in the HL, are *artificial* as well. In these circumstances, applying the qualifiers "natural" and "artificial" to "language" as antonyms synonymous with "native" and "contrived" respectively can result in terminological conflicts, and is therefore confusing. For instance, I may, in this case, state that *all nyms employed in a natural language are artificial*.

2) I qualify an HL as a *socio-personal psychophysical*, or *physopsychical*, system in the following sense. Any nym, artificial or natural, of the sapient subject perceiving (interpreting) it is a *sensible* and hence *extramental* (*exopsychic*) *thing* that *necessarily represents* one or more transient or lasting *insensible mental* (*psychical*)

entities, which are located within the physical limits of the cerebral cortex of the perceiver (interpreter) of the nym (as me) and which are therefore alternatively called *brain symbols of the perceiver*. The visual percept (sensation) of a graphonym or the audible percept of a phononym is such a brain symbol (mental entity). The *sense* that *is expressed* by a nym with respect to the perceiver or the *class* that *is designated* by the nym with respect to the perceiver and that coincides with the sense of the nym if the latter is *simple* (see Cmt 1.9(2)) is another brain symbol of the perceiver. A brain symbol and hence any portion of an HL can neither be exhibited on a material surface nor be uttered by phonic (vocal) sounds, but rather they can be represented by the pertinent graphic or phonic nyms. For instance, a natural number is a class and therefore it cannot be written on paper or printed on the screen of a computer monitor; it can only be represented by a graphic numeral, logographic (Arabic or Roman) or phonographic (wordy), which *denotes* the number. An HL cannot exist in isolation from at least one sapient subject who is capable of understanding (interpreting) nyms of the language. Therefore, the copula "*is*" in my statement that this treatise *is* the IML (inclusive metalanguage) of every one of its object logistic systems and of the IML itself should be understood as a *makeshift* for the predicate "*represents*".

3) Df 1.17(3) of the name "native language" ("NL") is a *persuasive* definition and *not a real definition*, i.e. *not a definition through a genus and differentiae*. Therefore, the name "*basic native language*" ("*BNL*") can be used interchangeably (synonymously) with the name "native language" ("NL"), subject to the following definition of WNTID, which is also a *persuasive* one:

> «**basic** … *adj* …**6** *of language* **:** having a vocabulary that consists of a very small number of words but that can be used to convey a wide range of ideas or information ⟨~ French⟩».

(cf. Df 1.8(4)). In fact, the names "NL" and "BNL" are epistemologically relativistic ones, which can be specified (interpreted) in many different ways, in general and in application to an NL of concrete nomenclature (as native English). They can be defined by a *real* definition so as to denote either the same NL of a given nomenclature or two NL's of a given nomenclature, of which the NL has more inclusive vocabulary than the pertinent BML.

4) Df 1.17 is tacitly subject to the following definition and to Cnv 5.1 supplementing it.•

Df 1.18. A nym, interpreted or not, is called a *linguistic nym* or *glossonym* (from the Greek noun "γλώσσα" \glóssa\ meaning «tongue» or «language») if it is a constituent part of an HL and a *nonlinguistic nym* or *aglossonym* if otherwise. Consequently, "*glossograph*" is an abbreviation of either of the names "*graphoglossonym*" ("*written linguistic nym*") and "*glossographonym*" ("*linguistic written nym*"), and similarly "*glossophon*" is an abbreviation of either of the names "*phonoglossonym*" ("*spoken linguistic nym*") and "*glossophononym*" ("*linguistic spoken nym*"). It is understood that a glossonym is either a *native linguistic form* of the HL or a new linguistic form that has been introduced into the HL by means of that same HL.•

Cnv 1.5. In accordance with Dfs 1.16(2) and 1.18, the noun "*mylonym*" is a synonym of the noun "*glossophon*" ("*glossophononym*", "*linguistic phononym*"), while adonyms and hence the noun "adonym" are irrelevant to languages. Therefore, in treating of glossonyms (linguistic nyms), I shall use the noun "*phononym*" synecdochically instead of "*mylonym*", which is tantamount to abbreviating "*glossophononym*" ("*glossophon*") as "*phononym*". Particularly, phonographs that are not glossogrsphs (as musical notes for human voice), i.e. *nonlinguistic phonographs*, which can be called *aglossophonographs* in one word, are not used in this treatise. Therefore, I shall use the noun "*phonograph*" ("*phonographonym*") synecdochically instead of "*mylograph*" ("*mylographonym*"), which is tantamount to abbreviating "*glossophonograph*" ("*glossophonographonym*") as "*phonograph*".•

Cmt 1.18. The main general properties that distinguishes the NL of a human speech community from the language of a community of any other socialized animals is its *abstractness*, i.e. *expressiveness of feelings*, or *thoughts*, *sensu lato* and its *recursiveness* (*recursivity*). The ability to use abstract recursive languages is in turn the property that distinguishes some members of the species *Homo sapiens* from all members of any other species of *conscious* animals, i.e. ones that have a CNS, but are *not reasonable* (*not sapient*, *not sage*). Statements of *definitions*, *axioms*, and *theorems*, and also statements of *proofs* of theorems are possible only because they are made by means and in the framework of a recursive language.

2) The general property of recursivity of an NL has many aspects, i.e. many *specific recursive properties*. One of the aspects is that an NL allows discussing (analyzing, synthesizing, and interrelating) nyms, linguistic or not, including those

constituting the NL itself. Another important recursive property of the NL is that it allows augmenting itself by new nyms without resorting to any visual or other sensory aids (as pictures, pictographs, drawings, or three-dimensional objects), which do not belong to the language. New graphonyms (e.g.) can be either *phonographic* (*alphabetic* or *syllabic,* i.e. *grammographic* or *syllabographic*) linguistic forms, bound or free, that are formed in accordance with the grammar rules and lexicon of the NL, and that are therefore similar to those already existing in the language, or they can be *pasigraphic* (*autographic* or *logographic*, *aphonographic*) nyms, e.g. new punctuation marks, atomic (indivisible, unanalyzable) characters, or assemblages of atomic characters that are used in symbolic logic (particularly, in this treatise) or in mathematics. Likewise, the NL allows assigning a new meaning to a nym already existing in the language, thus turning it into a neonym (nomen novum, new name), and it also allows replacing an untenable nym by a tenable one. There are two kinds of linguistic constructions, with the help of which neonyms, especially *neographonyms* (new graphonyms), are introduced into an NL: *axioms* and *linguistic definitions*. Henceforth, by "linguistic definition", I mean a written (graphic) linguistic definition, and not an oral one.

3) One of the purposes of this introduction is to elaborate, as belonging to the XML, a self-consistent taxonomy of graphonyms, which I need for setting up the object logistic systems. Still, the XML is a certain part of written English, being one of the WNL's. Therefore, some fleeting excursions into WNL's and and the associated SNL's are unavoidable. The following definition is one of such excursions.•

Df 1.19: ***A trichotomy of the PhgNL's and GphNL's***. 1) There are three kinds of PhgNL's (WNL's): *alphabetic native languages* (*AbNL's*) – e.g., written English, Greek, Hebrew, Latin, or Russian, *polysyllabic native languages* (*PSbNL's*) – e.g., Japanese Kana-Majiri or written Amharic, and *quasi-monosyllabic*, or *pasigraphic,* or *tonophonographic*, *native languages* (*QMSbNL's* or *PsgNL's* or *TPhgNL*'s) – e.g., written Chinese; QMSbNL's are often incorrectly qualified *monosyllabic.* An AbNL or a PSbNL is indiscriminately called a *phonemophonographic native language* (*PmPhgNL*). A PSbNL or a QMSbNL is indiscriminately called a *syllabic NL* (*SbNL*). A PhgNL is written in isotokes of certain prototypal *elemental* (*primitive*) *phonographs*, which are called *written letters* or *graphograms* if the PhgNL is an

AbNL and *written syllables* or *graphosyllables* if the PhgNL is a SbNL. The conventional set of graphograms of the AbHL, which are arranged in a customary order and provided with customary proper names, is called the *alphabet* of the AbNL. The conventional set of graphosyllables of the SbNL, which are arranged in a customary order and provided with customary proper names, is called the *syllabary* of the SbNL.

2) The smallest *free syntactico-semantic units* of an AbNL, which are capable of standing for *substance, quantity, quality, relation, place, time, position, state, action,* or *affection*, – to mention *ten Aristotelian categories* (see Aristotle [350 BCE, *Categories*, ACE]) for the sake of definiteness, – and hence of being used in intercommunication, are *written words* (cf. Df 1.15 and Cmt 1.16). Single *graphograms*, i.e. *written letters*, of the AbNL are *atomic autographs* so that they have *no xenovalues* when used in isolation from one another. They are just morphological (structural, not semantic) units of the AbNL, various finite sequences of which form different graphic referents to the respective prescribed relata. Some sole letters of the AbNL can be used as words. For instance, the letters "a" and "I" are used as English words. However, in any other English words as "man", "pan", "China", "India", "Israel", etc, in which either letter "a" or "I" occurs in juxtaposition with some other letters, that latter plays a pure morphological role and the sense that it has as word is not utilized. Consequently, a lettered graphic word stands as referent for its relatum, *not by means of any visual resemblance* and *not by means of any causal relationship* of the former to the latter, but by means of *the abstract association*, which exists between the two in the cerebral cortex of the interpreter of the word owing to an earlier arbitrary assignment. In other words, a lettered graphic word of an AbNL is an *ideograph* and *neither an iconoxenograph nor a dictograph*, i.e. it is, to use the pertinent semiotic terminology, a *graphic symbol* and *neither a graphic icon nor a graphic index*.

3) The above paragraph applies, *mutatis mutandis*, with "*PSbNL*", "*syllabic graphic word*", and either of the synonyms "*graphosyllable*" and "*written syllable*" in place of "*AbNL*", "*lettered graphic word*", and either of the synonyms "*graphogram*" and "*written letter*" respectively. Neither written letters of an AbNL nor written syllables of a PSbNL stand for separate objects or their properties, but written words

or combining forms, being modified words, do, – just as do so the spoken paratokens of the above phonographs.

4) Chinese is a group of related languages of the Sino-Tibetian (or Tibeto-Burman according to classification of some linguists) language family of the people of China, which have *mutually unintelligible spoken Chinese vernaculars,* called *tone*, or *tonal*, *NL's* (*TNL's*), and the same *pasigraphic*, i.e. *commonly intelligible, written Chinese language* that is shared by *all* Chinese speech communities; the prefix "pasi" originates from the Greek adjective "πᾶς" \pás\ meaning *all* or *every*. Written Chinese is based on 214 *aphonographic syllables*, called *Chinese radicals*, and on certain *phonetic symbol*s, called *Chinese phonetics*. Chinese radicals are combined with phonetics to form *phonographic syllables* or, in one word, *phonographosyllables*, most of which are *monosyllabic phonographic words*, i.e. the smallest ftee linguistic forms to be called briefly *lexigraphs* (*lexigraphonyms*), whose meanings are suggested by the respective constituent radicals, while the others are meaningless *autophonographic syllables* that have *no xenovalues in isolation*, – just as the letters of an AbNL or the syllables of a PSbNL. A phonetic is used with a radical to suggest pronounciation of the respective phonographosyllable, which depends however on a specific spoken Chinese dialect.

5) Thus, written Chinese is based on a syllabary of about 420 *written syllables*, most of which are lexigraphs, i.e. monosyllabic phonographic words Like a word of an AbNL or PSbNL, a lexigraph denotes a certain entity (as substance, quantity, quality, etc, – cf. the item 2). At the same time, in contrast to an AbNL or PSbNL, in forming Chinese polysyllabic words as juxtapositions of syllables, the senses of the pertinent lexigraphs (meaningful syllables) are coordinated or modified by the appropriate autographic (senseless) syllables so as to form the sense of a polysyllabic word by certain *mnemonic association* (cf. mnemonic associations of the lexical senses of Anglicized terms of this treatise with their etymological senses). English also has a lot of monosyllabic words (as "man", "pan", "pet", "pin", "pot", etc), but analogs of Chinese lexigraphs are the one-letter words "a" and "I" rather than monosyllabic words. However, in contrast to Chinese, the senses, which the letters "a" and "I" have as words, are irrelevant to the pure morphological role that they play in word formations (cf. the item 2). Still, some polysyllabic Chinese words are formed entirely of the senseless syllables and are attached with a certain sense by *abstract*

association, – just as many-letter words of an AbNL or polysyllabic words of a PSbNL. That is to say, written Chinese has *pronounced features of a hypothetic monosyllabic language* that was, probably, its ancestor.and at the same time it has some features of the modern *polysyllabic* languages. This is why it seems to be more correct to classify written Chinese as a *quasi-monosyllabic NL* (*QMSbNL*) rather than as a *monosyllabic* one (*MSbNL*). In this case, in order to emphasize the difference between the Chinese syllabary and the syllabary of a *eupolysyllabic*, i.e. *genuinely polysyllabic*, language (as Japanese Kana-Majiri or written Amharic), the former will be called a *lexibary*, although not all of its syllables are lexigraphs. Accordingly, unless stated otherwise, I shall hereafter employ the noun "syllabary" as an abbreviation of rhe name "*syllabary sensu stricto*", meaning a *syllabary of a PSbNL*. Since the written Chinese language is shared by *all* Chinese speech communities, it is alternatively called a *pasi*graphic NL (PsgNL). Written Chinese is often characterized as a *logographic* language. However, as indicated above, not all polysyllabic Chinese words consist exclusively of lexigraphs (atomic logographs). Therefore "pasigraphic" is more appropriate qualifier of this language than "logographic". In this case, since any language in general and written Chinese in particular is an interpreted system of nyms, use of either qualifier "xenopasigraphic" or "pasixenographic" instead of "pasigraphic" would be redundant.

6) When graphic tokens of a Chinese syllable are parts of different polysyllabic words having different meanings, the difference in meaning can be rendered orally only by reading the constituent syllables with certain modulations in *tone*. These tone modulations do the duty of phonemes of an alphabetic or polysyllabic language and are native to each one of a great many of Chinese speech communities. That is to say, any spoken Chinese dialect is based on *homophones* or *homophons* as spelled in this treatise in accordance with Df 1.16(3) and Cmt 1.16(3), which differ in sense only by nuances of tone. Because of a relatively small number of *vocables,* the number of homophons in each Chinese dialect is enormous. Therefore, Chinese spoken vernaculars are called *tone*, or *tonal*, *languages*. English speakers (e.g.) also employ homophons, which differ in sense by nuances of tone. Therefore, either of the qualifiers "*toneless*" and "*atonal*" that can be used for characterizing English in contrast to a Chinese dialect should not be understood literally. For instance, depending on variations in tone, the word "yes" may mean an emotionless

affirmation, irony or ironical agreement, boredom, excitement, impatience, interrogation, surprise, suspense, etc. However, in contrast to Chinese speech, which is based on systematic use of tonal homophons, use of tonal homophons in English speech is occasional and auxiliary. At the same time, there are AbNL's and PSbNL's, spoken counterparts of which are characterized as *polytonal languages*. For instance, spoken Greek is usually characterized as a polytonal language. Accordingly, written Greek, which is widely used in this treatise as an etymological source of the metalinguistic terminology, has a lot of *diacritics* (cf. Appendix 1) – phonetic marks that indicate the respective tonal variations of Greek speech. In order to express the peculiarity of a Chinese dialect as compared to a spoken language corresponding to an AbNL's or PSbNL's, I shall call the former a *eutonal* (*genuinely tonal*), or *eupolytonal* (*genuinely polytonal*), *language*. In order to emphasize that the spoken counterparts of written Chinese are tone (tonal, eutonal, eupolytonal) Chinese varniculars, I alternatively call written Chinese a *tonophonographic NL* (*TPhgNL*).

7) Thus, the people of China have a common (pasigraphic, commonly intelligible) medium of writing, shared by all Chinese speech communities, but they have no common medium of speech. There is no spoken language that could be called "*the* spoken Chinese language", but rather there are many mutually unintelligible tone (tonal) spoken (phonic) Chinese vernaculars, which are divided into four groups: the *Mandarin* dialects, of which the *North Chinese* dialects (especially *Pekingese*) are the most wide-spread ones; the *Kiangsi* dialects; the *Wu*, or *Central-Coastal Chinese*, dialects (*Shanghai*, *Ningpo*, *Hangkow*); and *South Chinese* dialects (*Amoy-Swatow*, *Cantonese-Hakka*, *Foochow*).

8) In contrast to written Chinese, which has many mutually unintelligible spoken vernaculars, written Serbian using the Cyrillic alphabet and written Croatian using the Latin alphabet are two different literary forms of the Serbo-Croatian language. At the same time, spoken Serbian, the native speech of the people of Serbia, and spoken Croatian, the native speech of the people of Croatia, are the same spoken Serbo-Croatian language. Accordingly, written Serbian and written Croatian can be regarded as *transliterations* of each other, rather than two different written languages.•

Cmt 1.19. 1) The acts or processes or instances of using *graph*onyms (*graphic*, or *written*, nyms) by a sapient subject for communication with other sapient subjects or for self-expression are collectively (undistributively) *writing*; and similarly with "*phon*", "*phonic*", "*spoken*", and "*speaking*" or "*speech*" in place of "graph", "graphic", "written", and "writing". In accordance with Df 1.16(3), reproduction of a speech, prerecorded or not, is also a speech. Writing in phonographs, i.e. in graphonyms that have *phonic* (*spoken*) *paratokens*, is called *phonetic writing*. Accordingly, a WNL, which is written in phonographs, is called a phonographic, or phonetic, native language (PhgNL). The act or process of *perception*, i.e. *sensation* and *understanding* (*comprehension*), of writing or speaking is called *reading*. The act or process of producing speech sounds either in talking of in reading writing aloud is called *phonation*.

2) In accordance with the English lexicon and common practice, the words "*writing*" and "*speaking*", or "*speech*", are conventionally used in this treatise homonymously both as *count* (*numeralable*) *nouns*, i.e. as ones capable of being modified with a numeral or with the indefinite article, and hence as ones having a plural number form, and as *mass* (*non- numeralable*) *nouns*, i.e. as ones incapable of being modified with a numeral and hence having no plural number form. Broadly speaking, the words "writing" and "speaking" (or "speech") are count nouns, when they are used as close synonyms of "graphonym" and "phononym" ("mylonym") respectively or of some of their subterms. Particularly, "a writing" may mean the same entity as "a piece of writing" or "a graphonym", whereas "a speaking" or "a speech" may mean the same entity as "a piece of speaking", "a piece of speech", "a mylonym", or "a phononym". At the same time, "writing" is a mass noun when it is used as an abbreviation of the description "one's act or process of forming a graphonym" or "an instance or the entire socio-personal phenomenon and practice of using graphonyms for communication or self-expression"; and similarly, *mutatis mutandis*, with "speaking" or "speech" in place of "writing" and with "phon" or "myl" in place "graph".

3) Like any human, especially as one not being a linguist, I am not familiar with properties of *all* WNL's. Therefore, I avoid stating that *all* WNL's are *divided* into AbNL's and SbNL's or that *all* SbNL's are *divided* into MSbNL's or QMSbNL's and PSbNL's, although this is likely the case. Such a statement would be an *axiom* which is not necessary for the purposes of this treatise. It is sufficient to state effective criteria, according to which some relevant WNL's can be recognized as AbNL's and

some other as SbNL's, and according to which the SbNL's can be partitioned further into MSbNL's or QMSbNL's and PSbNL's. All modern written European languages and also ancient Latin and Modern Hebrew are conventionally classified as AbNL's, although Hebrew, e.g., has some features of PSbNL's. Some details concerning alphabetic and polysyllabic NL's and their codes (cf Cmt 1.15(2)) are discussed in Essay 3 (E3). The interested reader can find many interesting facts of the history of writing and of modern NL's including written Chinese and spoken Chinese dialects in Bodmer [1944; 1981]. It should however be taken into account that the terminology of that book is ambiguous and that particularly the meanings of some terms used there differ from the meanings of their homonyms that are used in this treatise.•

Cmt 1.20. 1) The technical revolution of the nineteenth and twentieth centuries has allowed making *durable* records both of writings and speeches on various media and also *transiently* displaying recorded writing on screens of various kinds. Before all these technical metamorphoses, writing was enduring, and speech transient. Nowadays, the property of being enduring is not anymore a distinguishing property of writing and the property of being transient is not anymore a distinguishing property of speech. For instance, a written material presented on paper or that presented with the help of the Braille, or Moon, code is durable. By contrast, a written material introduced in the form of titles or subtitles into a motion picture or television program, especially subtitles representing a monologue or dialog, and also a written material appearing on the screen of a computer monitor, are transient.

2) Nevertheless, a written language preserves the following two important properties that it had before the above-mentioned technical revolution happened. First, a written language allows a sapient subject communicating, not only with contemporary members of the pertinent speech community, but also with members of past and future generations. Second, with the help of the pertinent immediate records in a written language, a thinker anchors down his conceptions, being *dynamic* and *subjective* (*mental, psychic*) entities, to graphonyms being *static* and *objective* (*exteroceptive* and *extramental, physical*) *entities*. These records enable the thinker to repeatedly check consistency of his conceptions and thereby to communicate with himself.•

1.9. "Isotoken", "paratoken", and "token", and the related onymalogical terms

Df 1.20. 1) A *conceptually indivisible graphonym*, topologically connected or not, that occurs in the treatise is called an *atomic graphonym* or briefly an *atomograph* (*atomographonym*) [*of the treatise*]. Accordingly, any *atomograph* or *any combination of atomographs* that occurs in this treatise, i.e. any segment (fragment) of the treatise or the whole of it, is a *graphonym*.

2) In agreement with Preliminary Remark 1.1(2), two or more (usually, an indefinite number of) distinct recurrent recognizably same graphonyms are called *graphic* (*written*) *isotokens*, or *endoiconographs*, *of one another*. An *isotoken of a graphonym* is called an *occurrence of the graphonym*, and vice versa. In contrast to numeralable (count) *neonym* (new name) "endoiconograph", the like neonym "*exoiconograph*" denotes by definition the same class of *xenographs* (*graphic signs* in the semiotic terminology) as the semiotic term "*graphic icon*". That is to say, unlike endoiconographs, i.e. graphic tokens (isotokens) of one another, which can be both autographs and xenographs, an exoiconograph is a *xenographic referent* (graphic sign) that is associated with its *nonlinguistic relatum by visual resemblance in form.* An exoiconograph is opposed both to an *ideograph* (*graphic symbol* in the semiotic terminology), i.e. a xenographic referent that refers to (stands for) its nonlinguistic relatum by means of *abstract association*, and to a *dictograph* (*graphic index* in the semiotic terminology), i.e. a xenographic referent that refers to its nonlinguistic relatum by means of some *causal relationship*.

3) In accordance with the above item, either combining form "endo" or "exo" is a qualifier to the noun "iconograph", i.e. in fact to the combining form "icono", and not to the base "graph". Therefore, either of the former two combining forms is not commutable with the latter combining form. That is to say, "endoiconograph", or "exoiconograph", is a description through the genus denoted by the generic name "iconograph" and the differentia denoted by the combining form "endo", or "exo", respectively. Consequently, an endoiconograph or an exoiconograph is indiscriminately called an *iconograph*.

4) *Mutually* (*pairwise*) *proportional* or particularly *congruent* graphic isotokens are called *homolographic isotokens*, or *homolographs*, *of one another*; the combining form "*homola*" can be used interchangeably with "*homolo*". Graphic isotokens that are *not* mutual homolographs are called *analographic* (*analogous graphic*), or *stylized*, *isotokens*, and also *analographs*, *of one another*. It is understood

that *an endoiconograph of a prototypal graphonym* is *either a homolograph of the latter or an analograph of the latter*. In contrast to “exoiconograph”, which is a univocal synonym of the equivocal semiotic term “graphic icon”, conventionally abbreviated as “icon”, I shall have no occasion to employ the morphological constructions “exohomolograph” and “exoanalograph”, although the last one could in principle be employed as a synonym of “exoiconograph”. Consequently, the morphological constructions “endohomolograph” and “endoanalograph” turn out to be redundant synonyms of “homolograph” and “analograph” respectively, and therefore I shall not employ them either.

5) In accordance with the above items 2–4, a homolographic or analographic (stylized grphic) isotoken of a given *prototypal* (*prototypic*) *graphonym* is [indiscriminately called] an *endoiconographic isotoken of the prototypal graphonym.* That is to say, either of the morphemes “homolo” and “analo” is a *hypotaxonym* (*subterm*) of “icono” and conversely the latter is a *hypertaxonym* (*superterm*) of either of the former two. Still, by way of emphatic comparison, the morphological construction “endoiconograph” can sometimes be used as an antonym of “homolograph”.

6) The combining form “*icono*” in any occurrence can be used interchangeably with “*picto*” without altering the sense of the xenograph, in which the former occurs. Likewise, the combining form “*homolo*” (or “*homola*”) in any occurrence can be used interchangeably with “*photo*”. That is to say, any one of the pair of nouns: “iconograph” and “pictograph”, “endoiconograph” and “endopictograph”, “exoiconograph” and “exopictograph”, and “homolograph” and “photograph” is a pair of synonyms.

7) A graphonym is called:

a) a *mylograph* (*mylographonym*) and also, by Cnv 1.5, a *phonograph* (*phonographonym*) if it has *phonic* (*phonetic, vocal, oral, spoken*) *values*, i.e. if it can be read orally in *articulated* manner and not only be mentioned by using (uttering) its name (if it has one);

b) an *aphonograph* (*aphonographonym*) or *boobograph* (*boobographonym*) if it has no phonic values.

It is understood that a graphonym that comprises both phonographs and aphonographs is an aphonograph.

8) The names “*atomic phonograph*” and “*phonic atomograph*”, abbreviated as “*atomophonograph*” and “*phonatomograph*” respectively, are synonyms.

9) In accordance with item 7a, a phonic value of a phonograph or mylograph is an articulated vocal sound that is produced by orally reading the phonograph. A phonic value of the phonograph that is prescinded from the latter is called a *phononym* or *mylonym*, in agreement with Df 1.16(2,3) and Cnv 1.5.

10) Any *smallest* phononym or any of its isotokens that is produced by the human voice is called an *atomic phononym* or briefly an *atomophon* (*atomophononym*) and also conventionally a *speech sound*.

11) Two or more (usually, an indefinite number of) distinct recurrent recognizably same phononyms are called *phonic* (*phonetic*, *spoken*, *oral*, *vocal*) *isotokens*, or *endophons*, *of one another* and also *phonic coisotokens* (in agreement with Preliminary Remark 1.1(2)). Phonic coisotokens that are produced by orally reading a given phonograph are called *phonic paratokens of the phonograph*. Conversely, a phonic paratoken of the phonograph is called a *graphophon* (*graphophononym*), whereas the given phonograph or any of its graphic isotokens is called a *graphic value*, or *graphic paratoken*, *of the graphophon*. Accordingly, graphic paratokens of a graphophon are called *graphic* (*written*) *isotokens*, or *endographs*, *of one another* and also *graphic coisotokens* (but again in agreement with Preliminary Remark 1.1(2)).

12) In general, in accordance with Preliminary Remark 1.1(2), a token of a given *prototypal* nym (onym) is said to be an *isotoken* of the prototypal nym if it is a nym of the *same genesis* and a *paratoken* if it is a nym of a *different genesis*, whereas the notions of isotoken and paratoken should be defined with respect to a concrete interpreter of the nyms in question. For instance, if an interpreter can see then an *optographic copy*, i.e. an *endoicono-optograph* (*endoicono-optographonym*), of a *prototypal optograph* is an isotoken of the latter with respect to the interpreter, while the following values of the prototypal optograph are its paratokens with respect to the interpreter: a phonic value if the interpreter can hear and also a dactylological, wigwagged, Morse light-flashed, or Morse sonic value if the interpreter can read the corresponding code fluently. If an interpreter cannot see and if he can read Braille or Moon code fluently then an *aptographic copy*, i.e. an *endoicono-aptograph* (*endoicono-aptographonym*), of a *prototypal aptograph* is an isotoken of the latter

with respect to the interpreter, while a phonic value and perhaps a Morse sonic value of the prototypal aptograph is a paratoken of the latter with respect to the interpreter. If an interpreter can hear then recognizable same recursive phononyms are phonic isotokens of one another or phonic coisotokens with respect to him. If at the same time the phonic isotokens are phonic values of a certain prototypal phonograph, which the interpreter can see, then the prototypal phonograph is a paratoken of each one of the above phonic isotokens, with respect to the interpreter. An isotoken or a paratoken of a nym is indiscriminately called a token of the nym.

13) Graphic coisotokens and their phonic paratokens, or phonic coisotokens and their graphic paratokens, are called *tokens of one another* or *cotokens*. Consequently, the names “*tokens of one another*” and “*cotokens*” are synonyms, so that the postpositive qualifier “of one another” to the word “token” and the prepositive qualifier “co” to the same word are synonyms. Therefore, “*conisotokens*” is a synonym of “*isotokens of one another*”, whereas “*coparatokens*” is a synonym of “*paratokens of one another*”. It is understood, that coisotokens, graphic or phonic, are cotokens, but cotokens are not necessarily coisotokens and not necessarily coparatokens.

14) A graphonym that has in a given discourse (as this treatise) *only homolographic isotokens*, and which hence has *neither analographic isotokens nor phonic paratokens*, is called a *homolograph of that discourse*. By contrast, a graphonym that has in the discourse *both homolographic isotokens and analographic isotokens and perhaps phonic paratokens* is called an *endoiconograph of the discourse*.•

Cmt 1.21. Sometimes the word “*phoneme*” is used in place of the metaterm “*speech sound*”, as defined in Df 1.20(10), but I shall not follow this usage. I shall stick to the conventional definition that a *phoneme* (from the Greek noun “φώνημα” \fónima\ having the same meaning as its English *parasynonym* “phoneme”) is a smallest *segmental* unit, which can particularly be a speech sound and which serves to differentiate otherwise similar spoken words in a native language or dialect. For instance, \b\, \k\, \f\, \h\, and \p\ are *consonant phonemes*, which differentiate phonic paratokens of the words “bat”, “cat”, “fat”, “hat”, and “pat”, whereas \æ\, \e\, \i\, \ɒ\, and \ʊ\ are *vowel phonemes*, which differentiate phonic paratokens of the words “pat”, “pet”, “pit”, “pot”, and “put”. A letter that stands for a consonant or vowel phoneme will be called a *consonant* or *vowel phonemic letter* respectively or, indiscriminately, a *phonemic letter*. In general, phonic paratokens of mono-, di-, and polysyllable phonographs of any AbNL or PSbNL are differentiated by the phonemes, which they involve. Therefore, a phonograph of an AbNL or PSbNL can be called a *phonemophonograph* (~*onym*). Accordingly, instead of the term "*phonetic language*", which apllies synecdochically to both a written alphabetic or polysyllabic language and to its spoken counterpart, and which is also applicable to QMSbNL, the terms "*phonemophonographic native language*" ("*PmPhgNL*") and "*phonemic native language*" ("*PmNL*") seem to be etymologically correct and lexically univocal *specific* names of an AbNL or PSbNL and of the spoken counterpart of the AbNL or PSbNL respectively. Also, the former two terms are expressive *antonyms* of either name "quasi-monosyllabic native language" ("QMSbNL") or "pasigraphic native language" ("PsgNL"), introduced in Df 1.17(6), and of either term "tone native language" or ""tonal native language" (abbreviated as "TNL" in both cases) respectively. It will be recalled that the spoken Chinese vernaculars (dialects) are called *tone*, or *tonal*, languages (from the Greek noun "τόνος" \tónos\, meaning *a tone,* and adjective "τονικός" \tonikós\, meaning *tonal*), because phononyms of each of them are composed of *homophones* (or *homophons* as spelled in this treatise in accordance with Df 1.16(4) and Cmt 1.16(3)), which differ in sense only by nuances of tone. In differentiating the phononyms of a spoken Chinese vernacular, tones of the constituting homophons of a phononym do the same duty as that done by phonemes in differentiating phononyms of a *phonemic spoken language*. Therefore, a graphonym, i.e. a *phonograph* and at the same time *pasigraph*, of the written Chinese language can be called a *tonal phonopasigraph* or, in one word, a *tonophonopasigraph* (~*onym*), and also, briefly, a *tonophonograph* (~*onym*). Consequently, the term "*tonophonographic native language*" ("*TPhgNL*"), morphologically analogous to the term "phonemophonographic native language" ("PmPhgNL"), can be used interchangeably (synonymoualy) with either term "quasi-monosyllabic native language" ("QMSbNL") or "pasigraphic native language" ("PsgNL"). At the same time, the terms "*atonophonographic native language*" ("*ATPhgNL*") and "*atonal native language*" ("*ATNL*") can be used interchangeably (synonymoualy) with the term "phonemophonographic native language" ("PmPhgNL") and "phonemic native language" ("PmNL") respectively.•

Cmt 1.22. In accordance with Dict A1.1, the Anglicized prepositive combining form “icon”- or “icono”- originates from the synonymous Greek nouns “εικών” \ikón\ and “εικόνα” \ikóna\ having the same sense as the English nouns “picture” or “image”. At the same time, in accordance with Dict A1.2, the combining form “picto”- as a synonym of the combining form “icono”- occurring in the morphological construction “iconograph” originates from the Latin etymon “pictūra” haviing in this case the same sense as “picture”. Therefore, both etymologically and lexically, the established noun “*pictograph*” is either an *etymologically inhomogeneous* synonym of “iconograph”.•

Cmt 1.23. It is necessary to distinguish between *endoiconographs* (*endopictographs*), i.e. *homolographic* (*photographic*) and *analographic tokens* (*isotokens*), *of one another* on the one hand and *exoiconographs* (*exopictographs*), i.e. *xenographs* associated with their *nonlinguistic relata* (*values*) *by resemblance in form*, on the other hand. In semiotics, the latter are briefly called *iconographs* (*pictographs*) or just *icons*.•

1.10. Token-classes and token-superclasses, and related onological terms

Df 1.21. 1) Given a graphonym, the *brain symbol* (*mental entity, mental process, memory image*), by means of which I recognize any other graphonym either as its isotoken or as a different graphonym, is called the *isotoken-class*, i.e. *class of isotokens*, or *autodesignatum, of the former graphonym* and also, more generally, *a graphic isotoken-class*, or *a graphon*, of *mine*.

2) The above item 1 applies with “*phon*” in place of “*graph*”. That is, given a phononym, the *brain symbol*, by means of which I recognize any other phononym either as its isotoken or as a different phononym, is called the *isotoken-class*, i.e. *class of isotokens, of the former phononym* and also, more generally, *a phonic isotoken-class*, or *a phonon*, of *mine*.

3) A graphic isotoken-class is called an *endoiconographic isotoken-class* or an *endoiconographon* if all its members are endoiconographs of one another and a *homolographic isotoken-class* or a *homolographon* if all its members are homolographs of one another. As before, “*icono*” can be used interchangeably with “*picto*” and “*homolo*” with “*photo*”.

4) If the graphonym is a phonograph then the brain symbol, by means of which I recognize any phononym either as its phonic paratoken or as a different phononym, is called the *paratoken-class*, i.e. *class of paratokens*, *of the graphonym*, the understanding being that this class is just a phonic isotoken-class that is associated with the given graphonym. The isotoken-class or the paratoken-class (if exists) of the graphonym, each taken individually, is indiscriminately called a token-class of the graphonym. The union of the two classes is called the *token-superclass*, i.e. *superclass of tokens*, *of the graphonym* or, more generally, *a* token-superclass.

5) The item 4 applies with "*phon*" and "*graph*" exchanged. Hence, in general, given a glossonym that has both the isotokens and the paratokens, the isotoken-class or paratoken-class of the glossonym is indiscriminately called a *token-class of the glossonym* or, more generally, *a* token-class. The union of the two classes is called the *token-superclass*, i.e. *superclass*, *of tokens*, *of the glossonym* or, more generally, *a* token -superclass.

6) In all terms introduced in the above items 1–5, the morpheme "*class*" can be used interchangeably with "*range*" or "*recept*". Consequently, the morphological constructions "token-class", "token-range", and "token-recept", e.g., are synonyms; and similarly with "isotoken" or "paratoken" in place of "token". Also, the name "*percept-class*" can be used interchangeably with "*isotoken-class*".

7) Any concrete token (member, specimen) of a given isotoken-class or paratoken-class or token-superclsss is called a *materialization*, or *individuation*, *of that class*.•

Cmt 1.24. 1) Hilbert and Ackermann [1950, p. 46, footnote 1] say: «In mathematics, the term "set" is used rather than "class"». However, such an indiscriminate use of the term "set" in no connection with any formal system of set theory is often incorrect, because *a set is a class but not necessarily vice versa*. In this treatise, a nonempty class being a set or the empty set is indiscriminately called a *regular class*, while a nonempty class not being a set is called an *irregular class.* It is understood that *the empty class is the empty set* and vice versa. Also, a denumerable nonempty class is a [denumerable] set. Hence, particularly, a one-member class, called also a *singleton*, is a [one-member] set. By contrast, a non-denumerable class can be either a set or an irregular class. In analogy with "singleton", a many-member class, denumerable or not, is called a *multipleton.* In the contemporary literature on logic and mathematics, an irregular class is called a *proper class*, whereas a set (regular class) is sometimes called a *small class* (see, e.g., Fraenkel et al [1973, pp. 128, 134–135, 167] for the former term or the article «**class**» in Wikipedia for both terms). The difference between a set and an irregular class will be made explicit in section 9. In accordance with the pertinent criteria, *any token-class is an irregular class, and not a set*. When there is no intention to utilize certain results of set theory, it is always saver to use the more general term "class" rather than "set".

2) Every graphonym of this treatise, and generally of *any given rigorous printed discourse*, has either a certain endoiconographic isotoken-class or a certain homolographic isotoken-class, which is determined by the fonts that are used in the treatise (or, correspondingly, in the given discourse). At the same time, the *universal endoiconographic isotoken-class* of any graphonym has an indefinite number of distinct recurrent materializations (isotokens) of an indefinite number of styles, printed or handwritten. For instance, in no connection with any specific printed or handwritten matter, the universal iconographic isotoken-class of "a" is the same class as that denoted by the count name: "*small first letter of the English alphabet*". This class has an indefinite number of distinct recurrent materializations (isotokens, specimens) of various typefaces – such materializations, e.g., as '*a*', '*a*', '**a**', 'a', '𝔞', etc. In this connection, it is worthy to recall that, in printing, a particular *style* (*design*) *of type* that preserves a due proportion of all pertinent primitive characters (as letters, punctuation marks, and accents) of different sizes is called a *typeface* or *face*, whereas a particular *style of type of one size and one typeface* is called a *font* or *fount*. Still, within a *point system* (as that of a word processor), in which the sizes of primitive characters and spaces of each printing type are measured as multiples of the point, a type is often called a font, so that each font (actually, type) exists in different sizes.•

Cmt 1.25. 1) The terms "graphon" and "phonon", introduced in Df 1.21(1,2) are, by Df 1.11, the appropriate onymological terms. In this treatise, I attach some words, which have the appropriate etymological senses and which are used as terms in other branches of science as physics, chemistry, or biology, with new lexical senses and thus turn them into psychologistic terms. The adjectives "*atomic*" and "*molecular*", and also the noun "*phonon*" are such words. The term "phonon" was introduced in theoretical physics in analogy with the earlier term "*photon*". To be recalled, the latter designates the class of quanta of radiant (electromagnetic-wave) energy in a wide range of cyclic frequencies from 10^{13} sec^{-1} to 10^{24} sec^{-1} by an order of magnitude – the range, which covers infrared, light (optic), ultraviolet, Roentgen, and gamma radiations. The term “phonon” designates the class of quanta of elastic-wave energy in special substances (as quanta of compression-wave or shear-wave energy in a single crystal or as quanta of compression-wave energy in a quantum liquid), but again in a wide range of cyclic frequencies from 10 to 10^{13} sec^{-1} by an order of magnitude. Therefore, from the standpoint of etymological analysis, both physical terms “photon” and “phonon” are *misnomers*. In this case, the new word “*echon*” of Greek origin or the new word “*sonon*” of Latin origin, which could be proposed instead of “phonon”, is not etymologically more correct than “phonon”. In contrast to its physical homonym, the psychologistic term “phonon” as defined in Df 1.21(2) is etymologically correct because it designates a class of *audible nyms* (*things*) that are produced by a human voice. Likewise, in agreement with their etymological sense (see Dicts A1.1 and A1.2), either of the new nouns “echon” and “sonon” can be utilized for designating a class of *audible nyms* produced by any *sonic* source, and not necessarily by a human voice, – e.g. a class of *Morse’s sonic symbols*, – whereas the existing noun “photon” or the new noun “*opton*” can be utilized for designatimg a class of *visible nyms* produced by any *light* (*optical*) source, – e.g. a class of *Morse’s light-flashed symbols*.

2) The nouns “singleton” and “multipleton”, which are, by Cmt 1.24(1), synonyms of the nounal names “one-member class” and “many-member class” respectively, can be analyzed morphologically as “singlete” + “on” and “multiplete” + “on”,. Therefore, these nouns are onological terms, although they are irrelevant to any onymological terms.•

Df 1.22. 1) A graphonym occurring in the treatise, which I select as one to exemplify certain properties that it has in common with any of its isotokens, is called *a prototypal* (*prototypic*, *prototypical*) *graphonym*, – either *ad hoc* (i.e. for the particular purpose at hand and without application in a wider scope) or universally. Therefore, *a prototypical graphonym represents the whole of its isotoken-class* (*percept-class*). Accordingly, an *isotoken of a prototypical graphonym* is alternatively called an *occurrence of the prototypical graphonym*.

2) A graphonym that is introduced in this treatise for the first time (as an element of the euautographic or panlogographic atomic basis) with the purpose to

serve as the prototypical one throughout the entire treatise is called an *archetypal* (*archetypical*) *graphonym* (correspondingly, an *archetypal euautograph* or *panlogograph*).•

Cmt 1.26. 1) A class in general and a set (regular class) in particular is an abstract (mental, psychical, imaginary) object of a sapient subject (as me or you) and therefore it cannot be written on paper; it can only be represented by its name. In this connection, there is a general philosophical principle, called *the prototype principle*, according to which a *concrete*, *i.e. most specific*, *instance of a class can represent the entire class* (cf. Hofstadter [1979, p. 352]). In this case, the name of the instance of the class is used as a name of the class. Consequently, my statement that a prototypical graphonym represents its isotoken-class is in agreement with the above prototype principle.

2) The indefinite phrase "*an isotoken of the graphonym*" (e.g.) should be understood as an abbreviation of any of the phrases such as: "given a prototypical graphonym, an isotoken of the graphonym" or "whatever graphonym I may select, an isotoken of the graphonym"; and similarly with "*occurrence*" in place of "*isotoken*".

3) By Df 1.11 and in analogy with a graphon or phonon, defined in Df 1.21(1,2), it can be asserted on the basis of the pertinent syntactic rule that a homolographon, endoiconographon or endopictographon, phonographon, graphophonon, euautographon, logographon, glossographon or graphoglasson, glossophonon or phonoglasson, etc is the *isotoken-class* of an appropriate *prototypal* graph (graphonym) or phon (phononym), namely, of a certain prototypal homolograph (homolographonym), endoiconograph (endoiconographonym) or endopictonograph (endopictographonym), phonograph (phonographonym), graphophon (graphophononym), euautograph (euautographonym), logograph (logographonym), glossograph (glossographonym) or graphoglassonym, glossophon (glossophononym) or phonoglassonym, etc, respectively.•

Df 1.23. 1) A given graphonym, which is prescinded from any context and hence from any *added words* not belonging to it, and which is *ad hoc* called *the prototypal graphonym*, is said to be used *autonymously* or in an *autonymous mental mode*, and also to be an *autograph* (*autographonym*), if I *use it as referent for mentioning* (*denoting*, *referring to*, *putting forward*) any one of the following entities as *its relatum*:

a) itself, i.e. the prototypal graphonym *sui generis* – the only concrete member of the *singleton* (*one-member class*) *of its own*;

b) a *certain*, i.e. a concrete (particular) but not concretized (not particularized) and hence *common* (*general*, *abstract*), member of one of the following *token-classes*:

b_1) the *endoiconographon* (*endoiconographic isotoken-class*);

b_2) the *homolographon* (*homolographic isotoken-class*);

b_3) the *phonon* (*phonic paratoken-class*).

The entity that is mentioned (denoted, referred to) by using the graphonym in a certain one of its autonymous mental modes is called an *autonymous value*, or briefly *autovalue* (cf. Df 1.11 and Preliminary Remark 1.1(2)). If particularly the prototypal graphonym is a phonograph that is used for mentioning its phonon (phonic paratoken-class) in accordance with the above point b_3 then that phonograph is called an *autophonograph* or *phonoautograph*. It is understood that along with the appropriate added words, an autograph can also denote some one or some more concrete tokens of it.

2) The autograph is called: a *proper*, or *strict*, *autograph* and also a *kyrioautograph* (*kyrioautographonym*) in the case a; an *iconoautograph* (*iconoautographonym*) in case b_1; a *homoloautograph* (*homoloautographonym*) in the case b_2; a *phonautograph* (*phonautographonym*) in the case b_3; a *common*, or *lax*, *autograph* and also a *cenoautograph* in the case b, i.e. in any of cases b_1–b_3 indiscriminately. As in Df 1.20(6), the combining form “*icono*” or “*homolo*” in any occurrence can be used interchangeably with “*picto*” or “*photo*” respectively without altering the sense of the xenograph, in which the former occurs.

3) In analogy with the item 1, a given phononym, which is prescinded from its context and hence from any possible added words and which is *ad hoc* called *the prototypal phononym*, is said to be used *autonymously* or in an *autonymous mental mode*, and also to be an *autophon* (*autophononym*) if I *use it for mentioning* (*denoting*, *referring to*, *putting forward*) any one of the following entities:

a) itself, i.e. the prototypal phononym *sui generis* – the only concrete member of the singleton (one-member class) of its own;

b) a *certain*, i.e. again a concrete (particular) but not concretized (not particularized) and hence *common* (*general, abstract*), member of one of the following *token-classes*:

b_1) the *phonon* (*phonic isotoken-class*);

b_2) the *graphon* (*graphic paratoken-class*).

The two last sentences of the item 1 apply with "phon" in place of "graph".

4) Consequently, an *autograph* or an *autophon*, each taken individually, is indiscriminately be called an *autonym*, – in agreement with the fact that the morphological constructions "autograph" and "autophon" are regarded as abbreviations of "*autographonym*" and "*autophononym*", and also of "*graphoautonym*" and "*phonoautonym*", respectively. However, an autonym is not necessarily either an autograph or an autophon because, when used self-referentially (circularily), the name "autonym" means that an autonym is a nym (onym), i.e. any sensible thing [with respect to me, e.g.], that is used autonymously.•

Cmt 1.27. 1) The qualifier "sui generis" (from the Latin etymon "sŭi gĕnĕris") is an adjective that is usually used predicatively or postpositively and that means *constituting a class alone, of its own kind, of the class of its own*, or *in a class of itself*. The class of a single object is conventionally called the *singleton of the object* or, more generally, *a* singleton, i.e. *a* one-member class or one-member set (see Cmt 1.24).

2) According to Pring [1982] (see also Dict A1.1), the combining form "*kyri*"- or "*kyrio*"-, also spelled in English as "*curi*"- or "*curio*"-, originates from the Greek noun "κΰριος" \kírios\ having the same sense as that of "lord", "master", "gentleman", "mister" (or, in the vocative case, "κΰριε" \kírie\, meaning «sir») and also from its kindred homonymous adjective having the same sense as that of "main", "principal", "chief"; the Greek set expression "κΰριον όνομα" means «proper name». The noun "curiologistics" means «hieroglyphic writing» and accordingly the adjective "curiologic" or "curiological" means «of or relating to hieroglyphic writing». The noun "kyrios" means «lord», especially in reference to Jesus Christ (as in the petitionary invocation "Kyrie, eleison!" meaning: «Lord, have mercy!»). In this treatise, I use the combining form "kyrio" exclusively in the sense of "proper" in the self-explanatory monomials: "kyrionym", "kyrioautonym", "kyrioxenonym",

“kyrioautograph” (“kyrioautographonym”), “kyrioxenograph” (“kyrioxenographo┐nym”), etc.

3) According to the same sources, the prefix “*cen*”- and also any one of its allomorphs: “*ceno*”-, “*coeno*”-, “*caen*”-, “*caeno*”, “*con*”--, and“*cono*”- originate from the Greek adjective. “κοινός” \kinós\ meaning *common* or [*held*] *in common* (cf. the perfective, associative, and collective prefixes of Latin origin: “*co*”-, “*col*”-, “*com*”-, “*con*”-, and “*cor*”-).

4) When I use a kyrioautograph for mentioning itself, I say that the latter is *denoted* (*put forward*) *by*, and is therefore the *denotatum* (*denotation value*, pl. “*denotata*”) *of*, the kyrioautograph [with respect to me], while *the singleton of the denotatum* is said to be *connoted by*, and be therefore the *connotatum* (*connotation value*, pl. “*connotata*”) *of*, the kyrioautograph [with respect to me]. In this case, I use the singleton of the denotatum, being a mental entity of mine, in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience it as my extramental* (*exopsychical*) *object* being the member of the singleton, i.e. the denotatum of the kyrioautograph. I do *so involuntarily but consciously* – just as I mentally experience the *percept* (*sensation*) *of any given nym* (*sensum*, *sensory object*) *as that nym*, particularly in the case when the nym is the pertinent kyrioautograph (see Df 1.12(1)). Accordingly, the kyrioautograph *represents its singleton*, thus being *just another hypostasis* (*way of existence*, *aspect*) *of the latter*. The name “connotatum” applies to the singlenon of a kyrioautograph only in reference to the mental model, in which the kyrioautograph is used as such, i.e. for mentioning (referring to) itself. Irrespectively to any mental mode of using a kyrioautograph, its singleton is impartially said to be *designated by*, or be the *designatum* (*designation value*, pl. “*designata*”) *of*, the kyrioautograph [with respect to me]. The above model of the meaning content of a kyrioautograph is generalized to a cenoautograph in the next item and to a *kyrioxenograph* (*proper xenograph*, *proper name*) in the next section.

5) In analogy with the previous item, when I use an cenoautograph for mentioning its *common* (*general*) isotoken, I say that the latter *is denoted* (*put forward*) *by*, and is therefore the *denotatum of*, the cenoautograph [with respect to me], while the isotoken-class of the cenoautograph is *connoted by*, and is therefore the *connotatum* (*connotation value*, pl. “*connotata*”) *of*, the cenoautograph [with respect

to me]. In this case, I use the isotoken-class, being a mental entity of mine, in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience it as my as if extramental* (*exopsychical*) *object* (other than the cenoautograph itself) that I call a *common* (*general*, *certain*, *concrete but not concretized*) *member*, or *element*, *of the isotoken-class*. I do so most often (but not always) and *habitually* and hence *involuntarily but consciously* – just as I *always* use and mentally experience the *percept* (*sensation*) of any nym (sensum, sensory object) and particularly that of the cenautograph itself (see Df 1.12(1)). Accordingly, the common element *represents the whole isotoken-class*, thus being *just another hypostasis* (*way of existence*, *aspect*) *of the latter*. The name "connotatum" applies to the isotoken-class of a cenoautograph only in reference to the mental model, in which the cenoautograph is used for mentioning (referring to) a common member (element) of its pertinent isotoken-class. Irrespectively to any mental mode of using the cenoautograph, its isotoken-class is impartially said to be *designated by*, or be the *designatum* (*designation value*, pl. "*designata*") *of*, the cenoautograph [with respect to me]. The above model of the meaning content of a cenoautograph is generalized to a *cenoxenograph* (*common xenograph*, *common name*) in the next section. •

Df 1.24. 1) An entity, which I associate with a graphonym and which I can mention (denote, refer to, put forward) by using the graphonym, but which is not any autovalue of the graphonym, is called a *xenonymous value*, or briefly *xenovalue*, and also a *significand* (*signification*), *of the graphonym* (cf. Df 1.11, Preliminary Remark 1.1(2), and Df 1.23(1)).

2) A graphonym is called a *euautograph* (*euautographonym*), i.e. a *genuinely autonymous graphonym,* if and only if it is a *homolograph that has no xenovalues*. Consequently, a euautograph has the following properties [with reespect to me].

i) A euautograph is always used autonymously and, throughout the treatise, it has *only homolographic* (*photographic*) *isotokens, each of which is a euautograph as well*. That is to say, within the scope of its definition, which stretches to the end of the treatise, a euautograph is a *graphic chip* (*fish*) that has or may have *certain functions* with respect to some other graphonyms or particularly with respect to some other euautographs but that *cannot assume* (*take on*) *any xenovalue.* In short, a euautograph is *functional but insignificant*. A euautograph can alternatively be called a *euauto-*

homolograph or, equivalently, *homolo-euautograph*, but the qualifier “homolo” (“photo”) is redundant in this case.

ii) The class of euautographic (and hence homolographic) isotokens that is designated by a euautograph is called the *isotoken-class*, or *class-percept*, *of the euautograph* and also, less explicitly, a *euautographic isotoken-class* or *euautographon*.

iii) A euautograph has *no phonic paratokens*, although it may have a phonographic (verbal) name, by which it can be mentioned orally.

iv) A functionally indivisible euautograph is called an *atomic euautograph* and vice versa. A combination (composition), i.e. either a juxtaposition or a superposition, of euautographs, particularly of atomic ones, that is made in accordance with certain *formation rules* is another euautograph, which is called a *combined euautograph*. In accordance with certain criteria, some simple combined euautograph are called *molecular euautograph*. An atomic or molecular euautograph is indiscriminately called an *elemental*, or *primitive*, *euautograph*. A euautograph is said to be a *complex*, or *compound*, one if it is not elemental.

v) In accordance with the above point i, a euautograph cannot be interpreted *psychically* (*mentally*) by assigning some xenovalue to it, because otherwise the euautograph would become a *xenograph*, in accordance with the next definition. Under certain conditions, a euautograph can, however, be interpreted *physopsychically* (*physico-psychically*), i.e. by *physically replacing* it with an appropriate *xenograph* as its *interpretand* in accordance with certain *formal rules of interpretational substitutions*; the understanding being that a *xenograph* is a *significant* (*psychically interpreted*) *graphonym.*

vi) A euautograph is a *euautonym*, but not necessarily vice versa. For instance, a chessboard, chessmen, and admissible positions of chessmen on the chessboard are euautonyms, but they are not euautographs, whereas figures of such positions, which occur in a textbook on chess (as Chernev [1958]), are euautographs.•

Df 1.25. 1) A graphonym that is prescinded from its context is called a *xenograph* (*xenographonym*, *graphoxenonym*) if, in addition to its autovalues, it has at least one xenovalue and also if it *is considered but is not used* for mentioning any of its values, autonymous or xenonymous. It is understood that an articulated graphonym that comprises one or more xenographs and one or more related autographs of various kinds, quoted or not (including some euautographs, particularly some *aphonic punctuation marks*), is also a xenograph. A xenograph is called a *pure*, or *chaste*, *xenograph* and also *agnoxenograph* if it contains no euautographs and a *mixed xenograph* or *mictoxenograph* if it contains at least one strict xenograph and at least one autograph.

2) An isotoken of the xenograph is said to be used *xenonymously* or in a *xenonymous mental mode*, and also to be a *euxenograph*, i.e. a *genuine* (*active*, *acting*) *xenograph*, if I *use it for mentioning* (*denoting*, *referring to*) any one of its xenovalues. Accordingly, a xenograph is said to be a *xenovalued*, or *significant*, *graphonym,* in contrast to a euautograph that is said to be an *insignificant graphonym* (see Df 1.24(2)).

3) An isotoken of the xenograph that I *use autonymously* is called a *tychautograph*, i.e. an *accidental,* or *circumstantial*, *autograph of the xenograph* or, more generally, a *tychautograph* (without the postpositive qualifier "of the xenograph").

4) When an autograph, i.e. either a euautograph or a tychautograph, is used together with some *added euxenographs* so as to form a single whole self-contained *euxenograph*, belonging to the IML, the latter euxenograph is called the [*pertinent*] *host euxenograph*, or *context*, *of the autograph*.

5) A tychautograph is called:

a) a *tychauto-endoiconograph* or *endoicono-tychautograph* if it is an endoiconograph,

b) a *tychauto-homolograph* or *homolo-tychautograph* if it is a homolograph,

c) a *tychauto-phonograph* or *phono-tychautograph* if it is a phonograph

(cf. "euauto-homolograph" and "homolo-euautograph" in Df 1.24(2)).•

Cmt 1.28. Dfs 1.24 and 1.25 apply, *mutatis mutandis,* with "*phon*" in place of "*graph*" and generally with "*nym*", "*autonym*", "*xenonym*", "*euautonym*", "*tychautonym*", and "*euxenonym*" in place of "graphonym", "autograph","xenograph", "euautograph", "tychautograph", and "euxenograph" respectively (cf. Df 1.23(3,4)).•

Cmt 1.29. 1) "*Eu*" is an established Anglicized combining form, which originates from the Greek adjective and combining form, meaning *well* and pronounced as \ev\ before voiced sounds or as \ef\ otherwise), whereas "*tych*" is a new

Anglicized prefix of my own, which originates from the Greek noun "τύχη" \tíxi, tíhi\, meaning *a chance*, and from the Greek adjective "τυχαῖος" \tixéos, tihéos\, meaning *accidental*. Thus, lexically, "eu" and "tych" is a pair of *antonyms*, of which "eu" means *genuinely* or *essentially* (*not accidentally*), whereas "tych" means *accidentally* or *circumstantially*.

2) A xenograph can be used either autonymously, thus becoming a *tychautograph*, or xenonymously thus becoming a *euxenograph*. A tychautograph, i.e. a xenograph in *the hypostasis of tychautograph*, is used for mentioning any one of its autonmymous values such as the tychutograph itself or such as a common member of a certain *class of its tokens* – a member that is another hypostasis of the latter class commonly called a *token-class*. A xenograph can not be turned into a euautograph and vice versa.•

Cmt 1.30. Musical notes are irrelevant to the object logistic systems of this treatise. Nevertheless, for avoidance terminological conflicts, it is noteworthy that musical notes are peculiar graphonyms that denote *melodies* – streams of sound of varying pitch (frequency), compositition, articulation, and intensity. That is to say, the sounds, for which musical notes stand, are, not just their *paratokens*, i.e. *para-autovalues*, but rather they are *intended ultimate xenovalues* and hence *principal interpretands* of the musical notes. Therefore, musical notes should be regarded as *xenographs*, while their xenovalues can be regarded as *euautoechonyms*. For instance, the melody produced by playing or singing musical notes of a wordless piece is the immediate xenovalue and immediate interpretand of the notes, and not only their immediate paratoken. In order to understand musical notes and to play or sing them, a musician or singer should learn the note symbolism just as he should learn a language in order to write and speak it. The difference between music and speech is that music is an autonym (euautoechonym), whereas speech is a xenonym (euxenophononym and hence euxenoechonym). If a WNL has diacritics (phonetic marks) these should also be treated as xenographs.•

Df 1.26. 1) A xenograph is called:

a) an *exoiconograph* (*exoiconographonym*), – by way of emphatic comparison with the term "*endoiconograph*" ("*endoiconographonym*") introduced in Df 19(2–6,14), – if it is associated with its intended relatum by *resemblance in form*;

b) a *dictograph* (*dictographonym*, *graphodictonym*) if it is associated with its intended relatum by *causal relationship*;

c) an *ideograph* (*ideographonym*, *graphoideonym*) if it is associated with its intended relatum by or else by *voluntary abstract assignment*.

Consequently, "*xenograph*", "*exoiconograph*", "*dictograph*", and "*ideograph*", and also the variants of the latter three with "picto" in place of "icono" are *synonyms* of the *semiotic* terms "*graphic sign*", "*graphic icon*", "*graphic index*", and "*graphic symbol*" respectively.

2) An exoiconograph (exopictograph) or an ideograph can serve as a dictograph. In this case, the dictograph is, more specifically, called an *exoiconodictograph* or an *ideodictograph*, respectively.

3) A euxenograph, i.e. a xenograph that is used xenonymously, is called:

a) a *euexoiconograph* if it is an exoiconograph,

b) a *eudictograph* if it is a dictograph,

c) a e*uideograph* if it is an ideograph.

4) A tychautograph, i.e. a xenograph that is used autonymously, is called:

a) an *autoexoiconograph* if it is an exoiconograph,

b) an *autodictograph* if it is a dictograph,

c) an *autoideograph* if it is an ideograph,

or, concurrently but redundantly, with "tychauto-" in place of "auto" (cf. Df 1.25(5)).

5) It is understood that an exoiconograph (exopictograph) is an *analographic* (*stylized graphic*) *image of its denotatum*. At the same time, like any graphonym, an exoiconograph has an indefinite number of *graphic isotokens*, which are, by Df 1.20(2), synonymously called endoiconographs (endopictographs). By Df 1.20(4), an endoiconograph (graphic isotoken) of a prototypal graphonym is either a *homolograph* or an *analograph* of the latter. Therefore, for avoidance of confusion, I assume that *an exoiconograph may have only homolographic* (*photographic*) *isotokens*.•

Cmt 1.31. In phonographic languages, there is the so-called *imitative*, or *echoing*, *method* of forming written words, according to which spoken paratokens of some written words are *imitations* (*stylized reproductions*) of the sounds associated with the things or actions being *acceptations* (commonly accepted meanings) of the words. This method is called *onomatopoeia*. Accordingly, a written or spoken word formed by *onomatopoeia* is called an *onomatopoetic word*, or, briefly, an *onomatope*. Use of onomatopes is also equivocally called *onomatopoeia*. Here follow some examples of written onomatopes: "*bottle*", "*buzz*", "*boing*", "*hiss*", "*mew*", "*moo*", "*roar*", "*tick*", "*ticktack*" (or "*tictac*"), etc. A spoken onamotope is analogous to an *iconograph* (*pictograph*) – a graphic sign whose form is descriptive of its meaning. Therefore, a spoken onomatope can alternatively be called a *phonic* (*vocal*, *phonetic*) *icon* and also an *exoiconophon* or *exopictophon*, whereas any of its graphic paratokens (pertinent written onomatopes) can alternatively be called an *exoiconophonic* (*exopictophonic*) *graphonym* or, briefly, *exoiconophonograph* (*exopictophonograph*). Still, the onomatopoetic sense of a written or spoken onomatope is just a *mnemonic* and hence *auxiliary* one and not the *intended* (*dominant*) one. In this respect, the onomatopoetic sense of an onomatope is analogous to the etymological sense of an Anglicized word. A written onomatope is formed of *autographic* letters or syllables like any non-onomatopoetic xenograph. Accordingly, when used in practice, an onamotope is usually disengaged (prescinded) from its onomatopoetic sense – just as is usually disengaged from its etymological sense an Anglicized word. Therefore, while a spoken onomatope is an exoiconophon (exopictophon), a written onomatope is in fact an ordinary phonograph.•

Df 1.27. 1) Two classes, particularly two *taxa* (*taxons*, *taxonomic classes*), are said to be *comparable* if and only if one of them is a *subclass* (*hypotaxon*, *part*) of the other or, equivalently, if and only if one of them is a *superclass* (*hypertaxon*, *whole*) of the other. Two classes are said to be *incomparable* if and only if they are *not comparable*. Two classes are said to be *compatible* or *conjoint* if and only if they *intersect*, and *incompatible* or *disjoint* if otherwise. *Comparable classes are compatible, but not necessarily vice versa*.

2) Two *taxonyms* (*taxonomic names*) are said to be *comparable*, *incomparable*, *compatible*, or *incompatible* and also *disjoint* if so are the taxa denoted by the taxonyms.

3) If two taxa are comparable then the taxonym of the subclass (hypotaxon) and the taxonym of the superclass (hypertaxon) are called a *hypotaxonym* and a *hypertaxonym*, or more generally a *subterm* and a *superterm*, *of* or *with respect to each other*.•

Cnv 1.6. A prefix, suffix, or infix is indiscriminately called an affix. In principle, I distinguish between the meanings of the names “affix” and “combining form” in accordance with the definition of the latter name in WTNID (see also A1). However, for simplifying wordings, I shall often use the noun “prefix” equivocally in a broad sense as a synonym of the name “prepositive qualifier” and, accordingly, I shall apply it to any appropriate linguistic form, bound or free.•

Df 1.28. A *phonograph* as defined by Df 1.20(7) is called an *autophonograph* (*autophonographonym*) or *phonoautograph* (*phonoautographonym*) if and only if it is simultaneously an *autograph* as defined by Df 1.23(1) and a *xenophonograph* (*xenophonographonym*) or *phonoxenograph* (*phonoxenographonym*) if and only if it is simultaneously a *xenograph* as defined by Df 1.25(1). A phonoxenograph (xenophonograph) is necessarily an *ideograph* as defined in Df 1.26(1c), i.e. it is an *ideophonograph* (*phonoideograph*), while some ideophonographs can serve as *dictographs*. It depends on on a phonograph and on its occurrence (context), whether the phonograph is an autophonograph or an ideophonograph. Particularly the occurrences of “phonograph” in the nouns “*phonemophonograph*” and “*tonophonograph*” introduced in Cmt 1.21 should be understood as occurrences of “ideophonograph”. Like a phonograph, an *aphonograph*, called also a *boobograph*, is either an autograph, i.e. an *autoaphonograph* (*aphonoautograph*), or *xenoaphonograph*, i.e. *xenoaphonograph* (*aphonoxenograph*); and similarly with “boobo” in place of “aphono”. A xenograph that comprises both phonographs and aphonographs (particularly, euautographs) is an aphonoxenograph.•

Df 1.29. 1) An *atomoxenograph*, i.e. *atomic* (*functionally indivisible*) *xenograph*, that is either a word of a certain TPhgNL (QMSbNL, PsgNL) or that is tantamount in sense to a word or to a phrase (articulated group of words) of a certain PmPhgNL (see Cmt 1.21), is called a *lexigraph* (*lexigraphonym*) or *atomic logograph* (*atomologographonym*).

2) A xenograph, being an *articulated juxtaposition* (*sequence*) *of lexigraphs* and, perhaps, of some *euautographs* (particularly *punctuation marks*) or *tychlogographs* (quoted or not), or *both*, is called a *combined logograph*.

3) An atomic logograph or a combined logograph is indiscriminately called a *logograph*.

4) A logograph is called a *pure*, or *chaste*, *logograph* and also *agnologograph* if it contains neither euautographs nor tychlogographs and a *mixed logograph* or *mictologograph* if otherwise.

5) An isotoken of the logograph is called a *eulogograph* if it is used xenonymously and a *tychlogograph* if it is used autonymously. That is to say, a euxenograph is called a eulogograph if it is a logograph, and similarly a tychautograph is called a tychlogograph if it is a logograph.

6) A logograph or particularly a lexigraph is either an exoiconograph, i.e. an *exoiconologograph* or *logoexoiconograph*, or an ideograph, i.e. an *ideologograph* or *logoideograph*, the understanding being that some logographs of either kind can serve as *dictographs* (to be examplifiied in Essay 3). At the same time, by Df 1.28, phonographs of WNL's are ideographs, whereas a great many of tonophonographs are logographs, i.e. *some logographs are phonographs* and hence conversely *some phonographs are logographs*. By Df 1.27(1,2), it follows from the above said tht the terms "logograph" and "ideograph" or "logograph" and "phonograph", e.g., are *compatible* but *incomparable*. Therefore, the nouns "logograph" and "ideograph", e.g., are not synonyms and they can be used nither synonymously nor synecdochically, – in contrast to what is often stated or tacitly assumed (cf. the article **ideogram** in WTNID subject to Cmt 1.16). On the other hand, the nouns "logograph" and "phonograph", e.g., are *not antonyms*, although these are often used so. Using these nouns antonomously is possible if their ranges are restricted properly, i.e. if these nouns are used as abbreviations of the terms "*logograph sensu stricto*" and "*phonograph sensu stricto*", defined properly. To be specific, "logograph" and "phonograph" becomes antonyms if *their domain*, i.e the *field of study and discourse*, in which they are employed, is restricted so as to exclude PsgNL's (QMSbNL's, TPhgNL's). Particularly, "logograph" and "phonograph" are antonyms when they apply to the logistic systems of this treatise and their XML that does not include irrelevant PsgNL's (see Df 1.31 below in this section).•

Cmt 1.32. In the light of Cmt 1.16(1i), it would, at first glance, have been consistent to propose the *neonym* (*nomen novum*, *new name*) "*grapholexis*" (pl. "*grapholexes*") in analogy with "graphogram" and "graphosyllable" instead of the neonym "lexigraph"as defined by Df 1.29(1) and to employ "lexigraph" instead of "logograph" in analogy with "grammograph" and "syllabograph". I have not done this for the following reason. According to Pring [1982] (see also Dict A1.1), the Greek singular noun "λε͂ξις" \léksis\ has the same sense as "word" when a word referred to by that noun is used *singly* – as contrasted to the plural noun "λόγια" \lójia\ ("sg "λόγος") having the same sense as "words" when words referred to by that noun are used *in connected speech*. Therefore, I use the combining forms "lexi" and "logo" in agreement with their etymological senses. In addition, this usage enables me to preserve the presently common sense of the noun "logograph", although I do not use the noun "ideograph" synonymously with "logograph". It is also worthy to recall that, for the reasons indicated in Cmt 1.16(1i), I do not use the nouns "ideogram", "logogram", "phonogram", etc as synonyms of the nouns "ideograph", "logograph", "phonograph", etc. At the same time, in accordance with the etymological senses of the combining forms "gram", "ideo", "logo", and "phono", it natural to assign the following straightforward lexical senses to the former three nouns. An *ideogram* is a *letter that is used as a symbol* and that is therefore a *one-letter word*, whereas a *logogram* is a *letter that is used as a word* and that is therefore a *one-letter word* as well. Hence, "ideogram" and "logogram" are synonyms. At the same time, a *phonogram* is a *letter that has a phonic value*, which should unavoidably be a *speech sound*. Any of the above letters is not necessarily a graphic (written) letter, i.e. not necessarily a *graphogram*, but rather it can be a letter of any one of the kinds indicated in Cmt 1.16(1ii). An *atomic word* can be called a *lexinym* or an *atomic logonym* (*atomologonym*) (cf. Df 1.29(1)). Consequently, an ideogram or logogram is a lexinym (atomologonym) but not necessarily vice versa.•

Cmt 1.33. In connection with Df 1.29(6), it is noteworthy that generally a xenograph can belong to two or more *compatible* species of those defined in Dfs 1.26, 1.28, and 1.29. When necessary or desired, a self-explanatory taxonym (taxonomic name) of such a xenograph will be formed by attaching the base "graph" with the pertinent prefixes in any order. For instance, a xenograph will be called:

i) a *dictoiconograph* or *iconodictograph*, and similarly with "picto" in place of "icono", if it is a dictograph and iconograph simultaneously,

ii) a *dictoideograph* or *ideodictograph* if it is a dictograph and ideograph simultaneously,

iii) an *iconologograph* or *logoiconograph* if it is an iconograph and logograph simultaneously,

iv) an *ideologograph* or *logoideograph* if it is an ideograph and logograph simultaneously,

v) an *ideophonograph* or *phonoideograph* if it is an ideograph and phonograph simultaneously,

vi) a *dictoiconologograph*, or *iconodictologograph*, or *logodictoiconograph*, etc if it is an dictograph, iconograph, and logograph simultaneously,

etc, where the combining form “icon” is, for convenience, used as an abbreviation of the combining form “*exoicon*”.•

Df 1.30. 1) A logograph or an autograph, each taken individually, is indiscriminately called a *pasigraph* but not necessarily vice versa because, for instance, some pasigraphs of the written Chinese language are not genuine logographs.

2) The noun “pasigraph”, being an abbreviation of “*pasigraphonym*”, can be generalized as “*pasinym*”. In accordance the pertinent entry of Dict A1.1, the prefix “pasi”- originates from the Greek adjective “πᾶς” \pás\ meaning *all* or *every*. Accordingly, the lexical sense of the new English noun “pasinym” can be expressed as follows. A *pasinym* is a nym that can be understood and therefore be used for intercommunication by all people who know its syntactic functions and its sense (if it has any), although they may speak different mutually unintelligible languages. Accordingly, “pasigraph” (“pasigraphonym”) is, at the same time, an abbreviation of “*graphopasinym*”, i.e. “*graphic pasinym*”, and similarly “*pasiphon*” (“*pasiphononym*”) is, at the same time, an abbreviation of “*phonopasinym*”, i.e. “*phonic pasinym*”.

3) A *pasinym*, which has only autovalues and which therefore has, in a given situation or universally¸ only autonymous tokens or no tokens at all, is called a *euautonym*, i.e. a *genuine autonym*. More specifically, the euautonym is called a *kyrioeuautonym*, i.e. a *proper*, or *strict*, *euautonym*, if it is used *self-referentially* and a *ceneuautonym*, i.e. a *common*, or *lax*, *euautonym*, if it is used in the *projective* (*polarized*, *connotative*) *autonymous mental mode* for mentioning a *common* (*general*) *member* of a certain token-class of the euautonym – a member being another hypostasis of the token-class. Accordingly, “euautograph” (“euautographonym”) is, at the same time, an abbreviation of “*grapho-euautonym*”, i.e. “*graphic euautonym*”, and

similarly "*euautophon*" ("*euautophononym*") is, at the same time, an abbreviation of "*phono-euautonym*", i.e. "*phonic euautonym*".

4) For instance, as was mentioned in Df 1.24(2vi), a chessboard, chessmen, and admissible positions of chessmen on the chessboard are euautonyms, and hence they are *pasinyms*. Therefore, any two chess-players can silently communicate (interact, enjoy in common) by playing a chess-game even if they speak two different languages. At the same time, figures of the chessboard and individual chessmen, and also figures of specific admissible positions of chessmen on the chessboard, which occur in a textbook on chess (as Chernev [1958]), are euautographs, and hence they are pasigraphs.

5) A pasinym that has xenovalues is called a *xenopasinym* or *pasixenonym*. Accordingly, a *graphic* xenopasinym or pasixenonym is briefly called a *graphoxenopasinym* or *graphopasixenonym*, and also, e.g. a *xenopasigraph* (*xenopasigraphonym*) or *pasixenograph* (*pasixenographonym*). Similarly, a *phonic* xenopasinym or pasixenonym is briefly called a *phonoxenopasinym* or *phonopasixenonym*, and also, e.g. a *xenopasiphon* (*xenopasiphononym*) or *pasixenophon* (*pasixenophononym*).•

Df 1.31. Unless stated otherwise, each one of the nouns "*logograph*", "*pasigraph*", and "*phonograph*" are hereafter used *in a narrow sense*, i.e. as "*logograph*" *sensu stricto*, "*pasigraph*" *sensu stricto*, and "*phonograph*" *sensu stricto* and hence as abbreviations of the pertinent monosemantic names "*logograph sensu stricto*", "*pasigraph sensu stricto*", and "*phonograph sensu stricto*"; the narrow senses (ranges) of the former three nouns are determined by the fact that their domain is hereafter assumed to be the theory indicated in the heading of the treatise. Accordingly, the pertinent logographs, pasigraphs, and phonographs satisfy the following additional conditions.

1) Like a euautograph, a logograph is a homolograph, i.e. it has only homolographic (photographic) isotokens and no paratokens. Particularly, a logograph has no phonic paratokens, i.e. it is an aphonoxenograph but not necessarily vice versa, – in accordance with Df 1.28. "Logograph" and "phonoxenograph" or "xenophonograph" are antonyms.

2) A *pasigraph* is a *euautograph* or a *logograph* or a *combination of euautographs and logographs*. A pasigraph is aphonograph but not necessarily vice versa, – in accordance with Df 1.20(7b).

3) Exoiconographs are not used in this treatise. Hence, by Df 1.29(6), a logograph is an ideograph but not necessarily vice versa. An ideograph is either a logograph or a phonoxenograph or a combination of logographs and phonoxenographs.

4) A logograph of $\mathbf{A}_1$ is called a *pan*logograph in the sense that its range is a certain class of euautographs of A_1. By contrast, a logograph that is used as an interpretand of a euautograph of A_1 is cllled a *cat*logograph. The antonymous combining forms "*pan*" and "*cat*" denote *above* or *over* and *below* or *under* respectively – in accordance with the senses of their Greek etymons: the adverb "πάνω" \páno\ or "απάνω" \apáno\, meaning *up*, *above*, or *on top*, and the adverb "κάτω" \káto\, meaning *down*, *below*, *beneath*, *under* (see, e.g., Pring [1982] or Dict A1.1)). Thus, the morphological consrtruction "*panlogograph*" means *over* (*up*, *above*) *with respect to euautographs*, whereas "*catlogograph*" means *under* (*down*, *below*) *with respect to* a *euautographs.* Accordingly, these two constructuins are used as combining forms in forming proper names of the pertinent object logistic systems of the treatise for connotatively describing their hierarchy with respect to A_1. Like any logograph, a panlogograph or a catlogograph may contain euautographs as its constituents.

5) A pasigraph of A_1, i.e. a euautograph of A_1 or a panlogograph of $\mathbf{A}_1$, is called an *endosemasiopasigraph* (*EnSPSG*) *of* A_1 if it *neither has nor assumes* (*takes on*) *any significands* (*significations*, *imports*, *values*) *beyond* A_1. Namely, a euautograph of A_1 has or assumes autovalues that belong to A_1, whereas a panlogograph of $\mathbf{A}_1$ assumes autovalues that belong to $\mathbf{A}_1$ and xenovalues that belong to A_1. Accordingly, a pasigraphic logistic system, as A_1 or A_1, is called an *endosemasiopasigraphic* (*EnSPSG'c* or equivocally *EnSPSG*) one if its every pasigraph is an endosemasiopasigraph (EnSPSG).

6) By contrast, a pasigrph of a logistic system is called an *exosemasiopasigraph* (*ExSPSG*) *of* the system if it *either has or assumes* (*takes on*) *some significands beyond the system*. Accordingly, a pasigraphic logistic system is

called an *exosemasiopasigraphikc* (*ExSPSG'c* or equivocally *ExSPSG*) one if *some* (*at least one*) of its pasigraphs is an exosemasiopasigraph (ExSPSG). •

Cmt 1.34. 1) The items 1 and 2 of Df 1.31 hold in the general case when the domain of the nouns "logograph", "pasigraph", and "phonograph" is any field of study and discourse, whose objects and whose metalanguage are irrelevant to any PsgNL. Particularly, any generally accepted system of logical, mathematical, physical, or chemical notation is a *logographic* and hence *pasigraphic* one in the above narrow senses of 'logographic" and "pasigraphic".

2) In the following definition, the noun "synonym" (as defined, e.g., in WTNID) is particularized for onyms of various kinds.•

Df 1.32. 1) Two or more xenonyms of the same language are said to be *mutually synonymous* and also to be *synonyms of one another* in a given *scope of their synonymity* if they have the same *denotatum* (*meaning*) in that scope, although they may have different senses. Therefore, if at least one of two denotative synomyms is a phonemophonograph or a phonemophon then these synonyms are not necessarily interchangeable in all occurrences in their scope. For instance, the root "onym" occurring in an "onym"-noun (as "xenonym") can not be used interchangeably with either one of the terms "name sensu lato" and "sensible thing", being its synonyms.

2) Two synonyms that can be used interchangeably in all occurrences within their scope are called *concurrent synonyms* or simply *concurrents.* Consequently, if two xenonyms are concurrents then they are synonyms but not necessarily vice versa.

3) In contrast to the adjective "*synonymous*" meaning «being or having the character of a synonym or synonyms», either of the adjectives "*synonymic*" and "*synonymical*" means «of, relating to, or dealing with synonyms».

4) Synonyms are called *synographs* if they are graphonyms and *synophons* if they are phononyms. Particularly, *synonymous logographs* or concurrently *logographic synonyms* are synographs that are called *synlogographs*.

5) Synlogographs are concurrents. In general, two or more pasigraphs, i.e. either euautographs or logographs, are said to be *synographs*, or *concurrents, of one another* if and only if they can be used interchangeably in any occurrences in the common scope of their definitions without altering the syntsctic and semantic identity of the entire host context. It is understood, that the scope of synonymity (concurrency)

of the pasigraphs does not include any of the definitions, by which the pasigraphs are defined to be synographs.

6) It is understood that two onyms (graphonyms or phononyms) are synonyms (synographs or synophons), or concurrents, if there is another onym (graphonym or phononym) being a synonym (synograph or synophon), or correspondingly, a concurrent, of each of the above two. •

Cmt 1.35. The noun "synograph" (cf. 'homograph") as defined in Df 1.32 means «*graphic synonym*», so that it can be regarded as an abbreviation of another new noun "*synographonym*" Accordingly, the new words "*synographic*" and "*synographically*", being the adjective and adverbial derivatives of "synograph", are hypotaxonyms of "synonymous" and "synonymously" respectively. Still, there are in this treatise a great many of new indispensable terms. Therefore, in this case, I shall usually employ the habitual words "synonym" (cf. "homonym"), "synonymous", and "synonymously" as synecdochical substituends for "synograph", "synographic", and "synographically". Nevertheless, I may occasionally employ any of the latter three in order to school the reader to the new more restricted terms. The self-explicative words "synophon" (cf. homophon"), "synophonic", and "synophonically" are three other new self-explicative hypotaxonyms of "synonym" (cf. "homonym"), "synonymous", and "synonymously" that I have introduced in analogy with the above "graph"-words. However, I shell have no occasion to use these words in the treatise.•

Cmt 1.36. In what follows I summarize the main modes of uses of the qualifying prefixes introduced in the above definitions starting from Df 1.20.

1) The prefixes "*icono*" and "*picto*" or "*homolo*" and "*photo*" can be used interchangeably in all occurrences, by Df 1.20(6).

2) The prefix "*endoicono*" ("*endopicto*") is a *hypertaxonym* (*superterm*) of the morpheme "*homolo*" ("*photo*"). For instance, an homolograph is an endoiconograph by not necessarily vice versa.

3) The pairs of antonymous prefixes "endo" and "exo", "eu" and "tych", and "auto" and "xeno" form the following hierarchy. Any of the first four prefixes qualifies another prefix, namely:

i) "endo" or "exo" is a qualifier to "icono" ("picto") or "semasio";

ii) "tych" is a qualifier to "auto". The qualifiercan be omitted whenever it is obviously understood that the combining form "auto" alone is used synecdochically instead of “tychauto” and not as the disjunction “euauto or tychauto”.

iii) “eu” is a qualifier to “auto” or “xeno”, and also to a subterm (restriction) of “xeno”, as “exoicono”, “dicto”, “ideo”, “logo”, “lexi”, “pasi”, etc, – a qualifier, which can, like “tych”, be omitted whenever it is obviously understood that the generic combining form alone, e.g. “auto” or “xeno”, is used synecdochically instead of the qualified combining form, e.g. instesd of “euauto” or “euxeno”.

4) The conbining form “auto” or “xeno” or a subterm of “xeno”, alone or together with the appropriate preceding qualifier “tych” or “eu”, is a qualifier to either of the bases “graph” and “phon” or generally to the base “nym” occurring in either of the postpositive combining forms “graphonym” and “phononym” if the latter are not abbreviated as “graph” and “phon”. Analogously, “icono” or “semasio”, alone or together with either preceding qualifier “endo” or “exo”, or “homolo” is a qualifier either to the base “graph” or to the base “nym” occurring in the postpositive combining form “graphonym” if the latter is not abbreviated as “graph”.•

1.11. “Numeral” vs. “number” and “taxonym” vs. “taxon”, and relevant terms

If the dominant xenovalue of a xenograph is a *class* (as a number, vector, or function), i.e. an *abstract* and hence *insensible* entity, then any given interpreter of the xenograph can easily confuse between the xenonymous use of the xenograph for mentioning its dominant *class-denotatum* and the autonymous use of the xenograph for mentioning either itself or its isotoken-class. At a certain moment, the interpreter can, e.g., use the xenograph xenonymously, i.e. as a euxenograph, for mentioning its class-denotatum, but at any subsequent moment he can *involuntarily but consciously* change his mental attitude towards the xenograph and use it autonymously, i.e. as a tychautograph (accidental autograph), without paying any heed to the change of his mental attitude. It is not therefore accidental that, for instance, the count noun “*number*” is equivocally used for mentioning both a *numeral* and *its class-denotatum*, i.e. both a *phonographic* (*wordy*) *or logographic* (*aphononographic*) *proper name of a number* and *the number itself*. Particularly, the *numerals* “one”, “two”, etc, or “1”, “2”, etc, and *their xenonymous class-denotata* are *equivocally called numbers*. When these numerals are naturally used for mentioning their denotata, they say that one, two, etc, or 1, 2, etc, are numbers, meaning the classes denoted by the numerals. However, such a statement may also mean that one, two, etc, or 1, 2, etc, are numerals. Like the equivocal use of the noun “number”, the count noun “*taxon*” is also used equivocally for mentioning both a *taxonomic name* and *the class denoted by the name*. Analogously, either noun “*vowel*” or “*consonant*” is equivocally used for mentioning a vowel or consonant speech sounds\ and the letter denoting that sound, respectively. Authoritative explanatory dictionaries of the English language and of other languages legitimize such equivocal usage of many names – usage that originates from confusion between autonymous and xenonymous uses of glossonyms (linguistic onyms) in general and of glossographs (glossographonyms) in particular. For instance, here follow a few pertinent definions of WTNID:

Dict 1.1.

«[1]**number** ... *n* -s ... **1 a :** an arithmetical total **:** sum of units involved **:** AGGREGATE ⟨~ of desks in the room⟩ ⟨~ of people in the hall⟩ ... **4 a :** an abstract unit in a numerical series ⟨seven is his lucky ~⟩ ⟨a ~ divisible by two⟩ ... **6 a :** a written word, symbol, or group of symbols representing a number ...

[2]**number** ... *vt* **1 a :** to ascertain a number of : COUNT ⟨~s his friends by the hundreds⟩ ... **4 :** to assign the number to esp. as a means of identification ⟨~ the pages of a book⟩ ...

[1]**numeral** ... *adj* ... **1 :** of, relating to, or expressing numbers ⟨~ adjective⟩ ⟨used the letters of their alphabets for ~ symbols – D.F.Smith⟩ **2 :** consisting of numbers or numerals ⟨~ cipher⟩ – **numerally** *adv*

[2]**numeral** ... *n* -s ... **1 :** NUMBER 6 ...

taxon *n, pl* **taxa** *also* **taxons 1 :** a taxonomic group or entity **2 :** the name applied to a taxonomic group in a formal system of nomenclature»

According to the first four of these definitions, the noun “number” and its homonymous kindred verb equivocally apply both to number-classes, i.e. to numbers, and to number-names, i.e. to numerals, and so does the word “numeral” both as an adjective and as a noun. According to the last definition, the noun “taxon” is equivocally used both as a synonym of the name “taxonomic name” and as a synonym of either of the synonymous names “taxonomic category” and “taxonomic class”. In the light of Aristotelian philosophy of *nominalism*, equivocal usage of the noun "taxon", which is legitimized by the pertinent Webster's definition, is explicable because, once any of a given group of entities (beings) is called by the same name, the entitiess become *ipso facto* members of the same class thus called. However, the ambiguity of the word "taxon" is confusing because this word is, as a rule, used in both senses in the same field of study and discourse. A like remark applies to the words "number" and "numeral". For instance, using the nouns "number" and "numeral" univocally, I can make the following statement:

> A numeral is a symbol whose denotatum is called a number. Accordingly, a *numeral is used for mentioning the number which it denotes,* provided of course that it is not used *autonymously,* i.e. *for mentioning either itself or its token-classs*.

With "number" in place of "numeral", the above italicized clause reduces to the nonsense: "*a number is used for mentioning the number which it denotes*".

In order to eliminate the ambiguity of the nouns "number" and "taxon", I shall stick to the following two stipulative definitions.

Df 1.33. A *numeral*, either *logographic,* e.g. "1", "2", etc, or *phonographic* (*grammographic*, *lettered written*, *alphabetic*), e.g. "one", "two", etc, is an *ideograph* (*graphic symbol* in the semiotic terminology) that denotes the corresponding *mental* (*psychical*) entity of the perceiver (interpreter) of the ideograph, which is called a *number* or, less explicitly, a *class*. A number cannot be exposed (depicted) on any material surface but it can only be represented by a numeral denoting it and be mentioned by using that numeral.•

Df 1.34. 1) A *hierarchical* (*orderly*) *system* of interrelated and unambiguously designated *classes* (*categories*) *of objects* of a given field of interest (field of study and discourse) is called a *taxonomic system* or briefly *taxonomy*.

2) A class being an element (unit object) of the taxonomy is called a *taxonomic class* or, briefly, a *taxon*. A *proper class-name*, i.e. a *proper name of a class* or, in one word, a *kyrioclasonym*, that *denotes a taxon* is called a *denotative taxonomic name* or, briefly, a *denotative taxonym*. A *common member-name*, or in one word a *melonym*, *connoting a taxon,* and at the same time *denoting a common* (*general, abstract*) *member* (*specimen*) *of the taxon and thus representing the entire taxon*, is called a *connotative taxonomic name* or, briefly, a *connotative taxonym*.

Either a denotative taxonym or a connotative taxonym, each taken individually, is indiscriminately called a *taxonym*, a *taxonomic name*. For instance, “Plantae”, “Animalia”, “Mammalia”, and “*Homo sapiens*”, and also the synonymous count nouns “plant”, “animal”, “mammal”, and “man”, without any modifier (particularly, without the indefinite article), are denotative taxonyms. The common individual names “a plant”, “an animal”, “a mammal”, and “a man” are connotative taxonyms, which are concurrent to the respective denotative taxonyms of each one of the above-mentioned two quadruples. Just as a number, a taxon cannot be exposed on any material surface; it can only be represented by a taxonym denoting it and be mentioned by using that taxonym.

3) A taxonomy, i.e. a taxonomic system, is necessarily a *graphic* (*written*) one because it can otherwise be ambiguous. For instance, it will be recalled that in a BTB (see Cmt 1.6**(**5)) item ii in sub-subsection 0.4.3), *genera* are denoted by *capitalized italicized Latin names*, e.g. *Acer* (maple) or *Canidae* (dogs); *species* are denoted by *Linnaean binomials* (*capitalized italicized two-word Latin names*), e.g. *Canis familiaris* (dog), *Canis lupus* (wolf), *Canis latrans* (praire dog), etc and also *Pan troglodytes* (chimpanzee), *Simia satyrus* (orangutan), etc; and the biological taxa broader than genera are denoted by capitalized Latin names in a current upright font. The above properties of *biological taxonyms*, particularly of *generic* and *specific ones*, are inexpressible orally. Therefore, “*taxograph*” (“*taxographonym*”) and “*taxography*” (“*taxographonymy*”) are etymologically more correct synonyms (synographs) of the nouns “*taxonym*” and “*taxonomy*” respectively.•

Cmt 1.37. Either of the allomorphic combining forms “*tax*”- and “*taxo*”- originates from the Greek noun “τάξις” \táxis\ that has¸ according to Pring [1982] (see also Dict A1.1), the same senses as “*order*” (the quality or state of being ordered or tidy) and as “*class*” or “*grade*”. At the same time, either of the allomorphs the “*nym*” and “*onym*” originates from the Greek noun “όνομα” \ónoma\ that has the same senses as “*name*” and as (gram.) “*noun*”, whereas the morpheme “*on*” originates from the Greek noun “ον” having the same sense as “*being*” or “*creature*”. At the same time, according to Df 1.34, any one of the nouns “taxonomy”, “taxonymy”, and “taxography” is a synonym of the description (descriptive name) “*hierarchical* (*orderly*) *classification*”, the noun “taxon” is a synonym of the name “*taxonomic class*”, the noun “taxonym” is a synonym of either name “*taxonomic name*” or “*name* *of the taxon*", and the noun "taxograph" is a synonym of either name "graphotaxonym" ("graphic taxonym") or "taxographonym". Therefore, the lexical senses that are attached to the above "taxo"-nouns are in agreement with their etymological senses.•

Df 1.35. The nouns "*hypotaxon*" and "*hypertaxon*" are synonyms of "*subclass of the taxon*" and "*superclass of the taxon*" respectively. Accordingly, "*hypotaxonym*" and "*hypertaxonym*" are synonyms of the expressions "*proper name of the hypotaxon*" and "*proper name of the hypertaxon*" respectively. •

Cmt 1.38. The combining forms "*hypo*"- and "*hyper*"- originate from the Greek etymons "ύπ(ο)"- \íp(o)\, having the same sense as "*under*", and "υπέρ"- \ipér\, having the same meaning as "*over*" or "*very much*" (Pring [1982]). The Latin preposition "*sŭb*" (with abl. and acc.), meaning *underneath* or *under*, and the Latin adverb and preposition and "*sŭper*", meaning *over* or *above*, are connected with the same Greek etymons (Simpson [1968]).•

f.12. "Word"

"*Word*" is one or the most fundamental terms of linguistics, which is widely used in this treatise. Still, it is not easy to define, especially concisely, the *sense* of the word "word" or, what is the same in this case, the *class of entities*, which is denoted by the count noun "word". For the sake of being specific, I shall adopt the following definition of the noun "word", which is in agreement with many presently common definitions (see, e.g., WTNID or APED) of that noun and which is applicable to the process of forming *new words* as technical terms in this treatise and in general.•

Df 1.36. 1) A *written word* of an AbNL or PSbNL is a graphonym, which occurs as an entry or within an entry of a dictionary of the given NL or which is formed anew within the pertinent assertive context of this treatise or elsewhere and which is set off by spaces on either side or is enclosed between *special autographic quotation* (*SAQ*) *marks* and is classified as a word or as a major class form (part of speech).

2) A spoken word is a phonic paratoken of a written word.•

Cmt 1.39. Regarding Df 1.36, the following remarks will be in order.

1) Bodmer [1944; 1981, p. 50] writes:

«…Even when a secular literature spread through the Greek and Roman world, the written language remained a highly artificial product remote from

> daily speech. Greek writing was never adapted to *rapid* reading, because Greek scribes never *consistently* separated words. The practice of doing so did not become universal among Roman writers. It became a general custom about the tenth century of our own era. When printing began, craftsmen took pride in the ready recognition of the written word, and punctuation marks, which individual writers had used sporadically without agreement, came into their own. Typographers first adopted an agreed system of punctuation, attributed to Aldus Manutius, in the sixteenth century. In the ancient world the reader had to be his own *paleographer*.»

Thus, strictly speaking, Df 1.36 immediately applies to this treatise (e.g.) and generally to writings in medieval and modern alphabetic and syllabic languages after those writings became punctuated, but it does not immediately apply to writings in ancient languages such as ancient Biblical Hebrew, ancient Greek, or ancient Latin. However, owing to the paleogrphers and lexicographers, ancient writings have afterwards been punctuated, so that Df 1.36 applies to the ancient languages as well.

2) One of the definitions the noun "word" in WTNID says:

> «[1]**word** … *n* –s … **2 a** (1) a speech sound or series of speech sounds that symbolizes and communicates a meaning without being divisible into smaller units capable of independent use **:** linguistic form that is a minimum free form ⟨the order of the ~s in a phrase⟩ ⟨the meaning of a ~⟩»

This definition is, however, applicable only to a *spoken word of a modern PmNL* (phonemic native language). It is not applicable to a spoken word of any ancient spoken language because there are no ancient speech communities nowadays. By contrast, Df 1.36(2) defines both a modern phonemic spoken word and to an ancient one.

3) In this treatise, e.g., I form new *graphic* (*written*) *words* first, while their phonic paratokens are by-side and unessential products of this word formation process. In this case, the special autographic quotations that I use particularly for representing new words or their constituent parts are not available in speech. One may define a word without either qualifier "written" or "spoken" as a *smallest free linguistic form*. But this definition defines *idem per idem* unless the term "free linguistic form" is defined independently of the term "word".

4) A graphonym that is called a graphic (written) word has an indefinite (infinite) number of graphic isotokens, which can be printed or handwritten in an indefinite number of styles and sizes, and it also has an infinite number of phonic paratokens. Analogously, a phononym that is called a phonic (spoken) word has an indefinite number of phonic isotokens, which can be produced by various speakers or be reproduced by various acoustical equipments, and it also has an indefinite number of graphic paratokens. Therefore, a written word is in fact *the isotoken-class of a certain graphonym being one of its members*, which is defined by Df 1.36(1) and which represents the entire isotoken-class with respect to an interpreter (perceiver) or creator of that graphonym. Likewise, a spoken word is *the isotoken-class of a certain one of a certain phononym being one of its members*, which is defined, e.g., by the above Webster's definition and which represents the entire isotoken-class with respect to an interpreter (perceiver) or creator of that phononym.

5) Complex onymological and onological mononina (monomials, one-word names), which I form in this treatise, have denotative polynomial synonyms consisting of two or more English words. This means that the Anglicized combining forms that I utilize or form are semantically concurrent to separate English words, – in agreement with a conventional definition of the name "combining form" (as that of WTNID, which is cited in Appendix A1). That is to say, a *complex written onymological or onological monomen is analogous to a continuous segment of ancient writing*. Hence, the fact that a written word, which is used but not mentioned, is set off by spaces on either side does not turn it into a more fundamental unit of the WNL than, say, an equivalent combining form.

In the following definition, I shall make explicit of the presently common taxonomy of the *written lettered words* of the English language, subject to Df 1.36 and to the above comments, which is followed in this exposition closely, but which has so far been implicit. It is understood that this taxonomy includes all new *written onymological and onological monomina* as new English words. Like taxonomy applies, *mutatis mutandis*, to the written lettered words of any AbNL.•

Df 1.37: *Kinds of English written lettered words.* In the following definitions, the noun "*word*" is used as an abbreviation of the name "*English lettered written word*".

1) A word is said to be a *monomorphemic* (*one-morpheme*), or *simple*, one if it is a single morpheme and a *polymorphemic*, or *complex*, one if it consists of two or more morphemes. For instance, “a”, “an”, “boy”, “bye”, “girl”, “sing”, “sang”, “sung”, “do”, “did”, “go”, and “went” are monomorpemic (one-morpheme) words; “boys”, “byes”, “girls”, “done”, and “gone” are dimorphemic (two-morpheme) words; “monomorphemic”, “dimorphemic”, and “trimorphemic” are trimorphemic (three-morpheme) words. Thus, a simple word does not have prefixes, suffixes, combining forms, and hyphenated or unhyphenated constituent words, but if can have an infix (as the letter “a” in the word ”sang” or “u” in “sung”).

2) A word is said to be *standard* or *basic* if it is an *entry* (*headword*) of one or more authoritative explanatory dictionaries of the language and if there is no indication that the word is an *inflectional form* (as defined in the next item) of another entry.

3) A change in the form of a basic word to indicate its case, gender, number, person, tense, mood, voice, or comparison, depending on *the major form class* (*part of speech*), to which the word belongs, is called an *inflectional change* or *inflection*. An inflection of a word is accomplished either (a) by attributing it with one of the suffixes, which are qualified *inflectional*, or (b) by means of internal change of the word (e.g., by replacing a monomorphemic basic word or a constituent morpheme of a polymorphemic basic word with the appropriate allomorph). A word is said to be an *inflectional form*, or *inflectional modification*, *of the pertinent basic word* if it is formed by the respective inflectional change of the latter. Less explicitly, an inflectional form of the pertinent basic word is said to be an *inflectional word* if it is abstracted from the basic word. An inflectional word is complex in case (a) and it can be either simple or complex in case (b). For instance, 〈“am”, “is”, “are”, “was”, “were”, and “been”〉, 〈“did” and “done”〉, 〈“worse” and “worst”〉, 〈“underwent” and “undergone”〉, or 〈“undertook” and undertaken”〉 are inflectional forms of “be”, “do”, “bad”, “undergo”, or “undertake” respectively.

4) A word is said to be *derivational* or *derivative* if it is formed by any of the following ways: (a) changing the function (major form class) of a certain basic word (e.g. by turning a noun into a verb), (b) back-formation of an earlier basic complex word (as in forming “percept” from “perception”); (c) fusing one or more combining forms, prefixes, or *derivational* (*non-inflectional*) suffixes to a basic word. A

derivational word is either simple or complex in cases (a) and (b) and it is complex in case (c).

5) A word is said to be *compound* if it consists of two or more words that are written either in one or with a hyphen between constituent words (e.g., “bedroom”, “newspaper”, “dining-room”, “by-side”, “passer-by”, “step-by-step”, man-of-war”, “commander-in-chief”, “mother-in-law”, etc). A compound word is a complex one, but not necessarily vice versa.

2. Introduction to Psychologistics (*continued*)

2.1. Historical and etymological notes on “organon”

The six works of Aristotle (384–322 BC) on logic, which are presently known under the English fheadings “*Categories*”, “*Prior Analytics*”, “*Posterior Analytics*”, “*On Interpretation*”, “*Topics*”, and “*On Sophistical Refutations*”, were collected and edited by the later Peripatetics[4] likely in about the early 1st century AD under the general title “*Organon*” (“Όργανον” \órganon\), i.e. «*Tool*» or «*Instrument*» in an alternative English translation, which meant *the tool, or instrument, of correct thinking*. By that title, the Peripatetics expressed their view that the study placed under it was not a part of philosophy (in contrast to what the Stoics maintained), and hence not a branch of science (as physics, metaphysics, or mathematics), but rather it was a *tool* of every inquiry. According to Pring [1982], “όργανον” (uncapitalized) means now *an organ*, *instrument*, or *agent*. About eight and half centuries after Aristotle’s death, his *Organon* was translated into Latin by Roman philosopher Anicius Manlius

[4]The school that Aristotle established in the fifty-third year of his age was the walk along the athletic field, on which he strolled up and down together with his scholars when teaching them. The athletic field was a part of the grounds of the temple of Apollo Luceus – the protector of flocks against wolves (from “λύκος” \lúkos\ meaning «wolf»). The walk was called “Peripatos” (from “περίπατος” meaning «walk», «ride», «drive», «trip»). Aristotle’s school took the Latinized name “the Luceum” from the name “Apollo Luceus”, and the name “Peripatetic School” from “Peripatos”. Accordingly, the scholars and later followers of Aristotle are called “Peripatetics”.

Severinus Boethius (AD ca475–ca524). Durant [1950, p. 99] writes of the Boethius work:

> «His translation of Aristotle's *Organon*, or logical treatises, and of Porphyry's *Introduction to the Categories of Aristotle* provided the leading texts and ideas of the next seven centuries in logic, and set the stage for a long dispute between realism and nominalism.»

Approximately one millennium later, Francis Bacon (1561–1626) published his most important treatise under the head "*Novum Organum*" ("*The New Organon*"), in which he flung a challenge to all medieval metaphysics, based on Aristotelian logic, by developing *inductive logic*. He says (Durant [1926, p. 100]):

> «To go beyond Aristotle by the light of Aristotle is to think that a borrowed light can increase the original light from which it is taken.»

Inductive logic was developed further by Mill [1843]. Nowadays, either of the two synonymous *metaterms* (*metalinguistic terms*) "*organon*" (in the Graecized spelling, pl. -"s") and "*organum*" (in the Latinized spelling, pl. -"s") is a well-established (dictionary) English noun, which, according to WTNID, means «an instrument for acquiring knowledge; *specif* **:** a body of methodological doctrine comprising principles for scientific or philosophical procedure or investigation».

In agreement with the above Webster's definition, I use the noun "organon" as a synonym of the noun "*master-calculus*", subject to the following three definitions, the first of which is one of the same Webster's dictionary, whereas the two others are interrelated definitions of Allen [2003]:

> «**calculus** … *n*, *pl* **calculi** … *also* **calculuses** … **3 :** a method or process of reasoning by computations of symbols: as … **b :** any one of the commonly distinguished divisions of symbolic logic».
>
> «**master**[1] … *noun* … **10** a mechanism or device that controls the operation of another: compare SLAVE[1] (3).
>
> **slave**[1] … *noun* … **3** a device that is directly controlled by and often copies the actions of the another: compare MASTER[1] (10).»

Thus, an organon *is a logical calculus, but not necessarily vice versa*. Accordingly, instead of the metaterm "organon", I shall, when convenient, synecdochically use the more inclusive metaterm "logical calculus" or just "calculus" as its abbreviation. At the same time, the statement that the nouns "organon" and "master-calculus" are synonyms is a *symmetric synonymic definition* (*SSD*), according to which the *intersection* of the two different broadest classes that are assigned to the two nouns by their dictionary definitions as their class-denotata becomes the class-denotatum of each of the nouns. The actual class-denotatum that the nouns "organon" and "master-calculus" have in common in application to any pertinent part or the whole of $\mathcal{A}_1$ is determined by certain unordinary features, which a logical calculus carrying either name has with respect to itself or with respect to some other subjects of formal logic. These features are described in the next subsection.

2.2. "Organon" sensu stricto as a term of Psychologistics

Df 2.1. 1) A *non-modal axiomatic sentential* (*propositional*) *calculus* and a *non-modal predicate* (*functional*) *calculus of first order*, which have been developed in symbolic logic so far, will be called a *conventional axiomatic sentential calculus* and a *conventional axiomatic predicate calculus* or briefly a *CASC* and a *CAPC*, respectively. A CASC or a CAPC will indiscriminately be called a *conventional axiomatic logical calculus* or briefly a *CALC*. In this case, the qualifier "conventional" can be used interchangeably with "*classical*"; the latter will be abbreviated as "*C*" so that all above abbreviations will remain unchanged. The plural number form of any of the above abbreviations will be formed by suffixing it with "'i", where the apostrophe should be understood as an operator of substitution of the ending "i" for the ending "us" in the word "calculus", for which the last letter "C" of an abbreviation stands.

2) Various systems of nomenclature of CALC'i are discussed, e.g., in Hilbert and Ackermann [1950, Editor's Notes, pp. 165, 166]. Unless stated otherwise, by a CASC I shall, for the sake of being specific, mean either *the Russell logistic system*, denoted by 'P_R', or *the* equivalent *Russell-Bernays logistic system*, denoted by 'P_{RB}', whereas by a CAPC I shall mean either the system F^1 of Church [1956, Chapters III and IV] or the first-order predicate calculus that is developed in Hilbert and Ackermann [1950, Chapter III] under the heading "*The restricted predicate calculus*" – as opposed to the higher-order predicate calculi that are developed there (*ibidem*, Chapter IV) under the heading "*The extended predicate calculus*". P_R is based on the five axioms, which were for the first time published in Russell [1908] and which were afterwards used in Whitehead and Russell [1910; 1962, pp. 96, 97]) as items **∗1·2–**

∗1·6. Bernays [1926] discovered the non-independence of Russell's axiom **∗1·5**, so that P_{RB} is based on the remaining four Russell's axioms. The calculi P_R and P_{RB} are discussed, e.g., in Hilbert and Ackermann [1950, §10, pp. 27–30] and in Church [1956, §25, pp. 136–138; §29, p. 157]. The axioms of P_{RB} are also used in Bourbaki [1960, Chapter I, §3, S1–S4]. When appropriate, I shall use the constants 'P_R', 'P_{RB}', and 'F^1' for mentioning the particular calculi, which they denote.•

Df 2.2. 1) Unless stated otherwise (as in the previous subsection), I use the name "*organon*" *in a narrow sense*, and hence as an abbreviation of the name "*organon sensu stricto*", as a synonym of the *description* (*descriptive name*) "*logical calculus having an inseparable associated algebraic decision method*", the understanding being that "*algebraic*" implies "*analytical*" ("*not tabular*").

2) A logistic system is said to be a *self-sufficient*, or *self-contained*, or *independent* one if and only if it is or can be set up and executed independently of any other logistic system and a *dependent* one if otherwise.

3) A logistic system is said to be *a logistic master-system of* or *in relation to* (*with respect to*) another logistic system, while the latter is said to be *a logistic slave-system of*, or *in relation to* (*with respect to*), the former, if the former *controls or can be regarded as one that controls, immediately or via some one or some more mediate logistic or systems*, the setup or execution or results of the latter. It is understood that a dependent logistic system is a logistic slave-system of at least one logistic system being its master-system, and vice versa. The relationship, in which a logistic master-system stands to any of its slave-systems, is said to be a *master-to-slave*, or *controlling*, *relationship*.

4) A logistic system is said to be an *autonomous* (*self-sufficient and self-controlled) part of another logistic system as its whole* if and only if the former is or can be set up and executed independently of the rest of the latter. If otherwise, the former logistic system is said to a *dependent*, or *subordinate*, *part* of the latter.•

Df 2.3. 1) In Df 1.2(2), the entire *formal psycho-logic*, $\mathcal{A}_1$, has alternatively been called the *Combined Algebraico-Predicate Organon* (*CAPO*) and also the *Combined Advanced Algebraico-Logical Organon* (*CAALO*), and has thus been classified as an *organon* in agreement with Df 2.2. Particularly, $\mathcal{A}_1$ has an associated combined decision method, which is *denoted* [*logographically*] by '$\mathcal{D}_1$' and be *called* (*denoted phonographically*) the *Combined Algebraic Decision Method* (*CADM*) *of* $\mathcal{A}_1$

or the *Combined Advanced Algebraic Decision Method* (*CAADM*), without the possessive qualifier "of $\mathcal{A}_1$". The occurrence of the qualifier "*Combined*" in either of the above two wordy name of $\mathcal{A}_1$ and hence the occurrences of the letter "*C*" in the abbreviations of the names, "CAPO" and "CAALO" are descriptive of the fact that $\mathcal{A}_1$ is a *hierarchical* logistic system that is analyzed, *physically* and *psychically* (*by abstraction*), into various constituent logistic systems, which stand with one another in various *relationships of succession* (as *phasing* or *juxtaposition*), *superposition, bundling* (*bunching*), and *comprehension* (*inclusion*). That is to say, "Combined" is a synonym of the conjunctive qualifier "*Sequential, Superimposed, Bundled, and Comprehensive*". The occurrences of the qualifier "Combined" in the wordy names of $\mathcal{D}_1$ and the occurrence of the letter "*C*" in the abbreviation "CAADM" have the same sense. It is understood that $\mathcal{D}_1$ is the totality of decision methods of the separate constituent logistic systems of $\mathcal{A}_1$ – the methods that actually belong to the inclusive metalanguage (IML) of $\mathcal{A}_1$, i.e. to the theory of $\mathcal{A}_1$ (this theory, this treatise), and not to $\mathcal{A}_1$, which *is prescinded from the IML* Therefore, $\mathcal{D}_1$ is in fact the interface between $\mathcal{A}_1$ and its IML. Consequently, the postpositive qualifier "*of* $\mathcal{A}_1$" to "$\mathcal{D}_1$" or "CADM" expresses *relationship of inseparable association of* $\mathcal{D}_1$ *with* $\mathcal{A}_1$, and *not relationship of inclusion of* $\mathcal{D}_1$ *in* $\mathcal{A}_1$.

2) One of the aspects of comprehensive properties of $\mathcal{A}_1$ is that it includes, as its *autonomous part*, i.e. as a part *capable of being set up and executed independently of the entire* $\mathcal{A}_1$, an organon, which is denoted logographically by '$\mathcal{A}_1^0$' and is called (denoted phonographically) the *Combined Binder-Free*, or *Contractor-Free, Algebraico-Predicate Organon*" (*CBFAPO* or *CCFAPO*), and also the *Combined Rich Basic Algebraico-Logical Organon*" (*CRBALO*). Accordingly, the associated decision method of $\mathcal{A}_1^0$, being an autonomous part of $\mathcal{D}_1$, is denoted by '$\mathcal{D}_1^0$' and is called the *Combined Rich Basic Algebraic Decision Method* (*CRBADM*) *of* $\mathcal{A}_1$. In these occurences, "Combined" means, as before, *Sequential, Superimposed, Bundled, and Comprehensive*. In turn, $\mathcal{A}_1^0$ includes, as an *autonomous part of its own* and hence as *that of* $\mathcal{A}_1$, another organon, which is denoted by '$\mathcal{A}_0$' and is called the *Combined Predicate-Free*, or *Combined [Depleted] Basic, Algebraico-Logical Organon* (*CPFALO* or *CBALO*). Accordingly, the associated decision method of $\mathcal{A}_0$, being an autonomous part of $\mathcal{D}_1^0$ and hence that of $\mathcal{D}_1$, is denoted by '$\mathcal{D}_0$' and is

called the *Combined Basic Algebraic Decision Method* (*CBADM*) *of both* $\mathcal{A}_1^0$ *and* $\mathcal{A}_1$. In these occurrences, "Combined" means *Sequential and Superimposed*, but *neither Bundled nor Comprehensive*. The above relationships among the combined organons $\mathcal{A}_1$, $\mathcal{A}_1^0$, and $\mathcal{A}_0$ and among their ADM's $\mathcal{D}_1$, $\mathcal{D}_1^0$, and $\mathcal{D}_0$ result from the like relationships among the initial euautographic organons A_1, A_1^0, and A_0 and among their ADM's D_1, D_1^0, and D_0, and also among the initial panlogographic organons $\mathbf{A}_1$, $\mathbf{A}_1^0$, and $\mathbf{A}_0$ and among their ADM's $\mathbf{D}_1$, $\mathbf{D}_1^0$, $\mathbf{D}_0$, (see the items 3–8 of the subsection 2.2 of Preface). Owing to its special simplicity, $\mathcal{A}_0$ can serve as an introduction into $\mathcal{A}_1$. Still, for saving room and labor, I have decided to start directly from $\mathcal{A}_1$ and to develop $\mathcal{A}_0$ as one of the restrictions of $\mathcal{A}_1$ (cf. the like remark regarding A_0 and A_1 in the above-mentioned item 8).•

Df 2.4. 1) All axioms and all theorems of any CASC, including those of the Russell-Bernays logistic system, P_{RB}, in the first place, and also all *sentential rules of inference of traditional logic* turn out to be *theorems of* $\mathcal{A}_0$ and hence *theorems of* $\mathcal{A}_1$ after all. Particularly, the above-mentioned traditional sentential rules of inference include the following ones: (a) *conditional syllogisms* (*modi*) of the *four kinds*, namely two kinds of *hypothetical syllogisms*: *modus ponendo ponens* and *modus tollendo tollens* and two kinds of *disjunctive syllogisms*: *modus tollendo ponens* and *modus ponendo tollens*; (b) *dilemmas* (*dilemmatic sentential syllogisms*):of four kinds: the *simple constructive dilemma*, the *simple destructive dilemma*, the *complex constructive dilemma*, and the *complex destructive dilemma*; (c) miscellaneous sentential syllogisms as: the *laws of double negation*, *excluded middle*, *reductio ad absurdum* (or *ad impossibile*), *simplification*, *addition*, *adjunction*, *contraposition*, *De Morgan's laws*, *commutative laws*, etc. All traditional sentential rules of inference, including De Morgan's laws, were invented partly by ancient Greek philosophers and partly by medieval Scholastics and were later deduced in CASC'i. At the same time, all axioms and all theorems of any CAPC and also the 19 categorical syllogisms turn out to be *theorems of* $\mathcal{A}_1$, but not of $\mathcal{A}_0$. Thus, owing to its CAADM, $\mathcal{A}_1$ stands in a master-to-slave (controlling) relationship, not only with itself, but also with any of the above-mentioned subjects of formal logic.•

2.3. Fundamental Latinized and chaste English psychologistic terms

As was mentioned previously in Cmt 1.12(1), onymology and onology are not sufficient for making psychologistic terminology as unambiguous as necessary. In contriving metaterms other then onomological and onological ones, *I modify some established but equivocal English (Anglicized) words of Latin origin either semantically or morphologically or both and use each one of the group of cognate English words thus obtained as a univocal psychologistic metaterm.* At the same time, I adopt every relevant established univocal and etymologically correct English term, Latinized or chaste, without altering it with the proviso that if a Latinized term is incorporated as a taxonym into a certain taxonomy then its form should agree with the forms of other Latinized taxonyms of the same rank (to be illustrated in due course).

Particularly, throughout the treatise and also throughout the entire Psychologistics, I shall stick to the next four definitions of a few most fundamental terms, the greater part of which are chaste Latin words, while the rest are chaste English words, although they are derived from certain Latin etymons. Most terms of the former group are *nomina nova* (sing. "*nomen novum*"), i.e. new names that have not been employed previously either at all or in the unique senses, which I attach to them, the term of the latter group are primarily *nomina veta* (sing. "*nomen vetum*"), i.e. established names that are utilized together with their acceptations.

2.3.1. "Operator" and relevant terms: "operandum" ("operand") versus "operatum"

Df 2.5. 1) Given a natural number $n \geq 1$, an *operator of weight n*, called also an *n-ary*, or *n-adic*, *operator*, is a graphonym that *applies to* (*is united with*, *operate*, or *act*, *upon*) an ordered multiple of *n* graphonyms of certain classes, called *elemental operata* (sing. "*operatum*") of the operator, in order to produce a new graphonym that is properly called the [pertinent] *operandum* (pl. "*operanda*"), or briefly *operand* (pl. "*operands*"), or alternatively *scope*, *of the n-ary operator*, and also less explicitly *an n-ary operandum* (without any postpositive qualifier) – in contrast to a graphonym that is called an *atomograph* (*atomic graphonym*) because it *involves no operators* and is therefore *functionally indivisible*. The ordered multiple of *n* elemental operata of the *n*-ary operator, i.e. the *ordered n-tuple* of the elemental operata, is called the *entire* (or *comprehensive*) *operatum* (or *argument*) of the operator.

2) Given an operand, an operator occurring in the operand is called the *principal operator of the operand* if it is either the only operator occurring in the operand or the one of two or more operators of the operand that is executed, i.e. is applied to its elemental operata, in the last place. An elemental operatum of the principal operator of the operand is either an atomic one or is in turn the operand of another principal operator.

3) The relatively complete act or process of purposefully working psychically (mentally) or physically, which has a distinct result, is called an *operation*, psychical or physical respectively. A *class*, or particularly a *rule*, *of operations* is called an *operation in intension*. By contrast, an operation being a member of the operation in intension is alternatively called an *operation in extension.*

4) Particularly, when I regard a given *n*-ary operand as the biune entity representing both the action (process) of application of its principal *n*-ary operator to the pertinent *n* elemental operata and the result of that action, I alternatively called the *n*-ary operand an *n-ary syntactic operation* [*in extension*]. Accordingly, the class of operands of an *n*-ary operator is called the *syntactic operation in intension of the n-ary operator* or, less explicitly, an *n-ary syntactic operation in intension*. Once the principal operator of an operand is prescinded from the operand, it is said *to denote its syntactic operation in intension*, whereas the latter is called the *denotatum* (*denotation value*, pl. "*denotata*") *of the operator.* The *n*-ary syntactic operation (*n*-ary operand) that results by application of a given *n*-ary operator to a given ordered *n*-tuple of elemental operata is alternatively called the *cut* of the operation in intension, denoted by the operator, at that ordered *n*-tuple. My mental attitude, under which I regard an *n*-ary operand as an *n*-ary operation, will be explicated in Cmt 2.4(2) below in the next subsection.

5) In the above occurrences, any one of the nouns "operator", "operandum" ("operand", "scope"), and "operatum" in either number form is supposed to be preceded with either qualifier "graphic" or "written". Also, the postpositive qualifier "in intension" can be used interchangeably the postpositive qualifier "*in potency*" and with either one of the prepositive qualifiers "*intensional*" and "*potential*", whereas the postpositive qualifiers "in extension" can be used interchangeably either of the postpositive qualifiers "*in actuality*" and "*in entelechy*" and with any one of the prepositive qualifiers "*extensional*", "*actual*", "*factual*", and "*entelechial*".•

Cmt 2.1. The terminology that has been introduced by Df 2.5 differs from the pertinent modern terminology, which is illustrated by the following interrelated definitions of WTNID.

Dict 2.1.

«**operand** ... *n* -s [L *operandum*, neut. of gerundive of *operari* to work, operate – more at OPERATE] **1 :** a quantity upon which a mathematical operation is performed or which arises from an operation **2** *logic* **a :** something that is operated by an operation **b :** the scope of an operator

operate ... *vb* -ED/-ING/-s [L *operatus*, past part. of *operari* to work …] *vi* **1 :** to perform a work or labor **:** exert power or influence **:** produce an effect …

operation ... *n* -s [ME *operacioun*, fr. MF *operation*, fr. L *operation-*, *operatio*, fr. *operatus* (past part. of *operari* to work) + *-ion, -io* -ion – more at OPERATE] … **7 a :** a process whereby one quantity or expression is derived from another or others **b** *logic* **:** (1) **:** TRANSFORMATION (2) **:** a function or correlation when conceived as a process of proceeding from one or more entities to another according to a definite rule …

operator ... *n* -s [...L *operatus* + *-or*] … **4 a :** a mathematical symbol denoting an operation to be performed ⟨$\frac{d}{dx}$ is the differentiating ~⟩ **b :** something that performs a logical operation or forms a symbol denoting such an operation (as a quantifier or a sentential connective) **c :** FUNCTION WORD ⟨a preposition, auxiliary, or conjunction is an ~⟩…»

Each one of the vocabulary entries **1** and **2** of the Webster's definition of "operand" contains two opposite definitions of that term. The following definitions of Church [1956, pp. 33, 39] support the first definition of the vocabulary entry **1** and the vocabulary entry **2a** of the Webster's definition:

«The constants or forms, united by means of a connective to produce a new constant or form, are called the *operands*.»

«An *operator* is a combination of improper symbols which may be used together with one or more variables – the *operator variables* (which must be fixed in number and all distinct) – and one or more constants or forms or both – the *operands* – to produce a new constant or form.»

By contrast, the following definition of Suppes [1957, p. 53] supports the second definition of the vocabulary entry **1** and the vocabulary entry **2b** of the Webster's definition of "operand":

> «The SCOPE of a quantifier occurring in a formula is the quantifier together with the smallest formula following the quantifier».

Under equivocal usage of the noun "operand", the formula that a quantifier acts upon is an operand, but the scope of the quantifier is also an operand. The formula being the scope of an operator may be used as an object of another operator. However, one cannot denote an object of an operation and the result of the operation by the same word, "operand". In order to resolve the ambiguity in using the word "operand", I have by Df 2.5 given the new name "*operatum*" (pl. "*operata*") to any element of the set of graphonyms that are acted upon by an operator to produce a new graphonym, which is in turn provided with the new name "*operandum*" (pl. "*operanda*"), abbreviated as "*operand*" (pl. "*operands*"). Etymologically, the English verb "*to operate*" originates from the Latin Present Infinitive deponent verb "ŏpĕrari" meaning *to work, labor, be busy, be occupied* (see "ŏpĕror" in Simpson [1968]); the kindred noun "ŏpus" (pl. "ŏpĕra") means *a work, labor* (*ibid.*). The new English noun "operatum" (pl. "operata") that I suggest as a term is derived from the Latin singular neuter past participle "ŏpĕrātum" of "ŏpĕrari" (*ibid.*), while the new English noun "operatum" (pl. "operata") that I suggest as a term is derived from the Latin neuter gerundive "ŏpĕrandum" of "ŏpĕrari". The Latin noun "effectĭo" (pl. "effectĭōnes"), meaning *a doing, practicing* (*ibid.*), is in fact a parasynonym of the English noun "operation" (*ibid.* ELD).•

Cmt 2.2. 1) Logistic systems and particularly logical calculi, uninterpreted or interpreted (formalized languages), of symbolic (mathematical) logic are constructed by synthesizing their *formulas* (*self-contained pasigraphs*), called also *categoremata* or *categorems* (sing. "*categorem*"), of certain *primary atomic* (*functionally indivisible*) *pasigraphs*. Therefore, given a synthetic (combined) pasigraph of such a system, its analysis into smaller formulas, which are called *operata* (*operatum-formulas*, *formula-operata*), and *operators, not being formulas,* can be made rigorously. An operator is, more generally, called a *syncategorem* (pl. "*syncategoremata*" or "*syncategorems*"), or *coformula* (my own term that is not in common use), *of the logistic system*.

2) A precise analysis into operata and operators is also possible for formulas of a mathematical calculus, i.e. a logistic system of mathematics, although it is not usually constructed as formally as a logical calculus.

a) For instance, in the *arithmetic operand-terms* (arithmetic combined terms) ‘1+3’, ‘3+1’, ‘[1+1]+2’, ‘[1+2]+1’, and ‘[[1+1]+1]+1’, all occurrences of ‘+’ are operators, whereas occurrences of ‘1’, ‘2’, ‘3’, ‘[1+1]’, ‘[1+2]’, and ‘[[1+1]+1]’ are operata of the pertinent occurrences of ‘+’. The operator ‘+’ is, by definition, *commutative*, e.g. 1+3=3+1, and *associative*, e.g. 1+1+2$\dot{\equiv}$[1+1]+2=1+[1+2], where ‘$\dot{\equiv}$’ is the rightward sign of equality by definition. Therefore, 1+3=3+1=1+1+2=… =1+1+1+1=4. Any one of the operations 1+3, 3+1, [1+1]+2, [1+2]], and [[1+1]+1]+1, e.g., is called the *designatum-producing operation*, or *sense-operation, on*, or *expressed by, the respective term* ‘1+3’, ‘3+1’, ‘[1+1]+2’, ‘1+[1+2]’, or ‘[[1+1]+1]+1’, whereas the class 4, resulted by that operation, is *impartially* called the *subject-class of the sense-operation* and also the *class designated by the term* or the *designatum* (*designation value,* pl. “*designata*”) *of the term.* By contrast, the class 1, 2, or 3, designated by the respective atomic operatum (constituent atomic term) ‘1’, ‘2’, or ‘3’ of an operand, is called an *object-class of the sense-operation*. In this case, I speak of the sense-operation or of the designatum of a term *with respect* (*in relation*) *to a particular interpreter of the term* (*as me*), which is supposed to be fixed, thus allowing to omit the latter qualifier. In the general case, an arithmetic operand-term *is expressed* with respect (in relation) to some (strictly some or all) of the *four arithmetic rules*, so that the *designatum-producing operation*, or *sense-operation*, *on the term* is a *mental operation of coordination* [*of the interpreter of the operand-term*] *of the object-classes of the sense-operation into its subject-class*, which is alternatively called the *designatum* of the operand.

b) Accordingly, in analogy with Cmt 1.9(2), I regard the *sense* (*sense-value*) of an arithmetic operand-term [with respect to me] as a *biune mental* (*psychical*) *coentity* (*process*) *of mine, one hypostasis* (*way of existence, aspect*) of which is the sense-operation on the operand[-term], i.e. on its object-classes, whereas *the other, dominant hypostasis of the sense* is the *designatum* of the operand that is prescinded from the sense-operation as its subject-class. The sense of a simple arithmetic term as ‘1’, ‘2’, ‘3’, etc coincides with its designatum. When I use an arithmetic term for mentioning its designatum, thus turning the latter into the *intended value* of the term, I say that the designatum *is denoted* by the term or that it is the *denotatum* (*denotation value*, pl. “*denotata*”), or *meaning*, of the term, while the singleton of the designatum sense-operation along with its subject-class of the term turns into its *connotatum* (*connotation value*, pl. “*connotata*”) or more precisely *sense-connotatum*.

c) Thus, the five different operands as mentioned in the point a) have different senses (sense-values) but the same designatum, 4. Accordingly, given an ambiguous arithmetic operand (expression) as ‘1+1+1+1+1’, in which the order of elemental binary addition operations is not indicated explicitly, *the* sense of the operand can be defined either as *the class of equivalence* of the senses of *all* pertinent unambiguous bracketed expressions or as the sense of the specific pertinent unambiguous bracketed expressions, in which all omitted pairs of square brackets are recovered in accordance with the general rule of associating them either to the left, ‘[[[[1+1]+1]+1]+1]’, or to the right , ‘[1+[1+[1+[1+1]]]]’).

d) When I use an arithmetic term for mentioning its designatum, thus turning the latter into the *intended value* of the term, I say that the designatum *is denoted* by the term or that it is the *denotatum* (*denotation value*, pl. “*denotata*”), or *meaning*, of the term, while the singleton of the designatum becomes the *connotatum* (*connotation value*, pl. “*connotata*”), or more precisely *sense-connotatum*, of the term.•

Cmt 2.3. 1) To say nothing of phonographic operators (as function words) of WNL’s, in writings on logic or mathematics, some pasigraphic, particularly logographic, operarors are often omitted. The *omitted operator* will alternatively be called a *latent operator* and also a *lanthanograph* (*lanthanographonym*) or *lanthanon* – from the Greek adjective “λανθάνων” \lanθánon, lanthánon\ meaning *latent*. For instance, in Hilbert and Ackermann [1950, p. 12], the object *inclusive disjunction formula* ‘*X*v*Y*’, where ‘*X*’ and ‘*Y*’ are atomic *relation-formulas* (atomic *formulas* in the terminology of that monograph), is abbreviated by omission of the *inclusive disjunction connective* (*operator*) ‘v’ (“*or*” or more precisely“*inclusive or*” in English and “*vel*” in Latin). Consequently, the latent (omitted) connective (lanthanograph) ‘v’ is represented in the abbreviated relation-formula ‘*XY*’ by the junction between the symbols ‘*X*’ and ‘*Y*’. At the same time, in Church [1956, p. 78, D5], the square-bracketed juxtaposition ‘[**AB**]’ of the *atomic placeholders* ‘**A**’ and ‘**B**’ of object *relation-formulas* (*well-formed formulas* or briefly *wffs* in the terminology of that monograph) is from the very beginning defined as a schema of object *conjunction* *formulas*, instead of '[**A**&**B**]',– to use the *conjunction connective* '&' of Hilbert and Ackermann (*ibid.*) for "*and*", – or instead of '[**A**∧**B**]', – to use the *conjunction connective* '∧' for "*and*". Consequently, in this case, the junction between the placeholders '**A**' and '**B**' in the abbreviated schema '[**AB**]' satands for the lanthanograph '&'. The pair of square brackets occurring in the abbreviated schema '[**AB**]' is another operator, namely *an operator of aggregation*. Therefore, once the schema '[**AB**]' is abbreviated further by omission of the pair of square brackets in accordance with a certain *effective convention*, which allows unambiguously recovering it when desired, the omitted pair of square brackets becomes another lanthanograph (lanthanon). Thus, the *molecular operator* '[&]' is the entire lanthanograph (lanthanon, latent operator) of the abbreviated schema '**AB**'. Analogously, once the symbol '*a*·*b*', or '[*a*·*b*]', of the product of two elements *a* and *b* of an algebraic system (as a group, ring, integral domain, or field) is abbreviated as '*ab*', the omitted operator '·', or '[·]', is the pertinent lanthanograph. In accordance with the above examples, *a lanthanograph* (*lanthanon, latent operator*) *can be the principal operator of an operandum*.

2) A phonographic (wordy) or pasigraphic (euautographic or logographic) *connective* will alternatively be called a *syndetograph* (*syndetographonym*) – from the Greek adjective "συνδετικός" \sindetikós\ meaning *connecting*, and also, more specifically, a *syndesmograph* (*syndesmographonym*) – from the Greek masculine noun "σύνδεσμος" \síndesmos\ meaning (*gram.*) *a conjunction*. Since "ά"- \á\ is a Greek privative prefix parasynonymous with English "*un*"-, "*in*"-, or -"*less*", an *omitted conjunction* will alternatively be called an *asyndesmograph* (*asyndesmographonym*), therefore from the standpoint of etymological analysis the the nomina nova (neonyms, new nouns) "*asyndetograph*" ("*asyndetographonym*") and "*asyndesmograph*" ("*asyndesmographonym*") can be interpretd as ones meaning *an omitted connevtive* and *an omitted conjunction* respectively.At the same time, aaccording to WNCD or WTNID, the established Anglicized noun "*asyndeton*" means «omission of the conjunctions that ordinarily join words or clauses (as in "I came, I saw, I concuered")». In order to simplify the terminology, I shall regard all nomina nova "*lanthanograph*", "*lanthanon*", "*asyndetograph*"¸ and "*asyndetograph*" and the presently common term "*asyndeton*" as synonyms meaning an *omitted operator*. Thus, in the previous item, the connective 'v', which is obviously

understood by the junction between ‘*X*’ and *Y*’ in the formula ‘*XY*’, the connective ‘&’, which is obviously understood by the junction between ‘**A**’ and **B**’ in the formula ‘[**AB**]’, and the operator ‘·’, which is obviously understood by the junction between ‘*a*’ and *b*’ in the formula ‘*ab*’, are asyndetons (lanthanons). •

2.3.2. “Relation” and relevant terms: “referent” (“referens”) versus “relatum”

Df 2.6. 1) A *referent* (pl. “referents”) or *referens* (pl. “*referntia*”) is a coentity of mine, which *refers to*, i.e. which I use *to refer to,* another coentity of mine that I call a *relatum* (pl. “*relata*”). The universal (generic) predicate “refers to” or any particular predicate (to be specified) that I use instead of it for establishing the mental (psychical) association (bond) between of the referent and the respective relatum or relata is called the *relater* or *relator*, and also more precisely the *object-relater* or *object-relator,* whereas the association (bond) itself is called a *relation* [*of mine* or *with respect to me*]. The sapient subject (as me) that relates, i.e. the one that establishes a relation, is equivocally called the *relater* or *relator*, and also more precisely the *subject-relater* or *subject-relater.* The referent and relata (or relatum) of a relation are indiscriminately called the *terms* of the relation, the understanding being that the referent is the first term of the relation, i.e. the one from which the relation proceeds, whereas the or a relatum is the second or any one of the succeeding terms of the relation. A relation is said to be an *n-ary* one, i.e. *binary*, *ternary*, *quaternary*, etc, or, alternatively, an *n-adic* one, i.e. *dyadic*, *triadic*, *tetradic*, etc, if it has *n* terms, i.e. 2, 3, 4, etc respectively. The relator of an *n*-ary (*n*-adic) relation is also qualified an *n*-ary (*n*-adic).

2) In accordance with the *meta-axiom of atomic basis* and *formation rules* of an object logistic system, some *atomic* (*functionally indivisible*) *graphonyms* of the logistic system are called *atomic*, or *degenerate*, *relations*. Accordingly, in a context, in which either metaterm (taxonym) “*atomic relation*” or “*degenerate relation*” occurs, a relation as defined in the previous item will be called a *non-degenerate relation*, while the noun “relation” is freed of its previous meaning and it indiscriminately denotes both a degenerate (atomic) relation and a non-degenerate relation. At the same time, beyond such contexts, i.e. whenever confusion cannot result, the noun “relation” is used *synecdochically* instead of the name “non-degenerate relation”.•

Cmt 2.4. 1) In this treatise and generally in Psychologistics, the noun "*term*" is used as an antonym of the term "*relation*". Particularly, a term or a relation of a logistic system is indiscriminately called a *formula,* or *categorem* (pl. "*categoremata*" or "*categorems*"), of the system. An antonym of "categorem" is "*syncategorem*" (pl. "*syncategoremata*" or "*syncategorems*"). Thus, a formula of a logistic system is by definition either a term, called also a *term-formula* or *formula-term*, or a relation, called also a *relation-formula* or *formula-relation*, and independently but by definition again, it is either an *atomic formula* or a *combined formula*, which is alternatively called an *operandum-formula* or *formula-operandum*. Hence, an atomic formula is either an *atomic term* or an *atomic relation* and likewise a combined formula is either a *combined term,* i.e. an *operandum-term* or *term-operandum*, or a *combined relation*, i.e. an *operandum-relation* or *relation-operandum*. In the general case, however, an operandum is not necessarily either a combined term or a combined relation, but rather it can also be an operator or its kernel-sign. That is to say, the classes of graphonyms denoted by the count names "term-operandum" and "relation-operandum" are not complementary of each other in the class denoted by the count noun "operandum". The taxonym "term" is used in the treatise in many different senses so that it cannot be defined by a single definition after the manner of Df 2.2.

2) When I say that an *n*-ary operand is a syntactic *n*-ary operation [in extension], as I have done in Df 2.1(4), I regard the latter operation as a syntactic (*n*+1)-ary *relation*, which I establish, via the principal operator of the *n*-ary operand, between the operand as *referent* and its *n* elemental operata as *relata*. Alternatively, I may regard the above (*n*+1)-ary relation as a *binary relation* between the *n*-ary operand as referent and the ordered *n*-tuple of elemental operata as the single whole *relatum.*•

Cmt 2.5. 1) As far as possible, the terminology that has been introduced by Df 2.6 is adjusted to the pertinent modern terminology, which is illustrated by the following interrelated definitions of WTNID.

Dict 2.2.

«[1]**referent** ... *n* -s [L *referent-*, *referens*, past. part. of *referre*] … **2 a :** a word or a term that refers to another **b** *logic* **:** the term (as *a* in the proposition *a* has the relation R to *b*) from which a relation proceeds **:** the first term of a relation (as *a* in R*a*,*b*,*c*) – compare RELATUM **3 :** that which is denoted or named by an expression or statement **:** a spatio-temporal object or event to which a term, sign, or symbol refers **:** the object of a relation

relation ... *n* -s … **7 b :** a logical bond; *specif* **:** a dyadic or polyadic predicate or propositional function …

relater ... *n* -s **:** one that relates; *esp* **:** NARRATOR

relator ... *n* -s [L, fr. *relatus* (superlative past. part. of *referre* to carry back, refer, relate) + *-or* – more at RELATE] **1 :** one that lelates **:** RELATER, NARRATOR …

relatum ... *n, pl* **relata**…[NL fr. L, neut. of *relatus*] **:** a thing or term related **:** one of a group of related things **:** CORRELATIVE; *specif* **:** one of the terms to which a logical relation proceeds **:** the second or one of the succeeding terms of a relation – compare REFERENT 2b»

This terminology is however inconsistent. Particularly, the vocabulary entries **2a** and **3** of the Webster's definition of "referent" are opposite to each other. Indeed, in accordance with **2a**, an entity «that refers to another» entity is called a referent. Hence, an expression or statement, which denotes or names a certain entity and which, hence, refers to that entity, is a referent. However, in accordance with **3**, an entity, «which is denoted or named by an expression or statement» and which, hence, is referred to by that expression or statement, is called a referent, although it should be called a relatum, in accordance with the Webster's definition of "relatum".

2) In order to explain the phenomenon of confusion in using the terms "referent" and "relatum" and to avoid the confusion in the psychologistic terminology, I proceed from the following definition of the two terms of Whitehead and Russell [1910; 1962, p. 33]):

«In the propositional function *xRy* we call *x* the *referent* and *y* the *relatum*. The class $\hat{x}(xRy)$, consisting of all the *x*'s which have the relation *R* to *y*, is called the class of referents of *y* with respect to *R*; the class $\hat{y}(xRy)$, consisting of all the *y*'s to which *x* has the relation *R*, is called the class of relata of *x* with respect to *R*. These two classes are denoted respectively by $\vec{R}$'*y* and $\overleftarrow{R}$'*x*. Thus,

$$\vec{R}\text{'}y = \hat{x}(xRy) \text{ Df},$$

$$\overleftarrow{R}\text{'}x = \hat{y}(xRy) \text{ Df}.$$

> The arrow runs towards *y* in the first case, to show that we are concerned with things having the relation *R* to *y*; it runs away from *x* in the second case, to show that the relation *R* goes from *x* to the members of $\overleftarrow{R}$*'x*. It runs in fact *from* a referent towards a *relatum*.»

The phraseology of this definition is not rigorous and is obscure here and there. For instance, the *adherent* (the first term) "propositional function" is not appropriate as the *appositive* (the second term) '*xRy*' of the apposition «the propositional function *xRy*», because '*xRy*' is a *placeholder* whose range is a class of *propositional* (*truth-functional*) *sentential forms*, and not a class of *propositional functions*. Therefore, *xRy is a certain* (*abstract, common, general, concrete but not concretized*) *propositional* (*truth-functional*) *sentential form* of the range of the placeholder '*xRy*', which *represents the entire range*. Nevertheless, the above-quoted definition gives a general idea that *the realization* (*instance, extension, cut*) *of a binary relation* (*operation*) *in intension, R, at a certain referent* (*argument*), *x*, is *the binary relation in extension, xRy, that goes from the referent x to the relatum, y, being the*, or *a*, *value of R at x.* At the same time, it is not mentioned in the above definition that to any binary relation (operation) in intension, *R*, functional or not, there is *the corresponding inverse binary relation* (*operation*) *in intension*, R^{-1}, such that, given *y*, there is at least one *x*, which is related to *y* by *the binary relation in extension* $yR^{-1}x$. This relation goes from *y* to *x*, so that *y is the referent of* $yR^{-1}x$, *while x is its relatum*. Supposing that *x* and *y* are the same in both relations *xRy* and $yR^{-1}x$, the names "the referent" and "the relatum" apply to *x* and *y* in that order in *xRy* and in the reverse order in $yR^{-1}x$.

3) The above considerations can be illustrated by the following simple examples. An affirmative simple declarative sentence that has a compound predicate consisting of a link-verb in the active voice and a *predicative* is a *syntactic relation* between *the grammatical subject* of the sentence, being the referent, and *the grammatical predicative* of the sentence, being the relatum. It will be recalled that *a predicative is the complementary part of the link-verb in a compound grammatical predicate*. For instance, in the sentence "All men are mammals", "men" is the referent and "mammals" the relatum. By contrast, in the inverse sentence "Some mammals are men", "mammals" is the referent and "men" the relatum. At the same time, the *proposition expressed by either sentence*, i.e. *the sense of that sentence*, is a *semantic relation* between the denotatum of the subject, which is the referent of the proposition,

and the denotatum of the predicative, which is the relatum of the proposition. Consequently, once the syntactic referent and the syntactic relatum exchanged, the respective semantic ones are also exchanged.

4) The referent and a relatum of a relation can be regarded as entities (beings) that belong to the following two of *ten Aristotelian categories* (see Aristotle [350 BCE, *Categories*]): *acting upon* [*the relatum*] and *is affected* (*is acted upon*) [*by the referent*]. Consequently, when a binary relation is replaced by its inverse, the two categories, to which the terms of the former relation belong, exchange.

5) In accordance with the above-said, in order to establish a *specific binary* relation, one should introduce two interrelated terms, provide them with the appropriate *specifc* taxonyms (common names), and define voluntarily, which one of the two terms will be called a *referent* and which one a *relatum*. In the case of a *binary relation*, at least one term of which is a graphonym, I shall use the terminology that is stated in the following definition.•

Df 2.7. If I use a given graphonym either in order to *define, explicate* (*explain*), *interpret,* or *justify* by means (in terms) of it another graphonym or in order to *denote* (*mention*), *connote*, *designate*, *imply,* or *signify* a certain entity, physical (as another graphonym) or psychical (mental), or if I *replace* given (particularly all) occurrences (isotokens) of the given graphonym throughout a certain scope with occurrences (isotokens) of another graphonym, then I regard the former graphonym as the *referent* (*referens*) of the pertinent *specific* relation and the latter entity (particularly the latter graphonym) as the *relatum* of that relation. In this case, I give the following respective names:

1) "*definition*" to the act of defining or to the relation (linguistic device) expressing that act; "*definiens*" (pl. "*definientia*") to the refere*nt* (refere*ns*) of the definition, i.e. to a graphonym that is used for defining another graphonym; "*definiendum*" (pl. "*definienda*") to the latter graphonym, which is the relatum and which is the one that is defined by the definition; "*definer*" or more precisely "*object-definer*" to the predicate, which relates the definiendum to the definiens; "*subject-definer*" to the sapient subject (as me) that defines (makes the definition);
2) "*denotation*" to the act of denoting or to the relation established by that act; "*denotans*" (pl. "*denotantia*") or "*denotant*" (pl. "*denotants*") to the

graphonym, which is the rerere*nt* (refere*ns*) of the denotation [relation] and which denotes another entity; "*denotatum*" (pl. "*denotata*") to the latter entity, which is the relatum of the denotation and which is the one denoted by the pertinent denotans (denotant);

3) "*connotation*" to the act of connoting or to the relation established by that act; "*connotans*" (pl. "*connotantia*") or "*connotant*" (pl. "*connotants*") to the graphonym, which is the rerere*nt* (refere*ns*) of the connotation [relation] and which connotes another entity; "*connotatum*" (pl. "*connotata*") to the latter entity, which is the relatum of the connotation and which is the one connoted by the pertinent connotans (connotant), the understanding being that a *connotans is at the same time the pertinent denotans*;

4) "*designation*" to the act of designating or to the relation established by that act; "*designans*" (pl. "*designantia*") or "*designant*" (pl. "*designants*") to the graphonym, which is the rerere*nt* (refere*ns*) of the designation [relation] and which designates another entity; "*designatum*" (pl. "*designata*") to the latter entity, which is the relatum of the designation and which is the one designated by the pertinent designans (designant);

5) "*explication*" to the act of explicating (explaining) or to the relation established by that act, "*explicans*" (pl. "*explicantia*") or "*explicant*" (pl. "*explicants*") to the graphonym, which is the rerere*nt* (refere*ns*) of the explication [relation] and which is to be explicated (explained) by another graphonym; "*explicandum*" (pl. "*explicanda*"), abbreviated as "*explicand*" (pl. "*explicands*"), to the latter graphonym, which is the relatum of the relation and which is the one that explicates (explains) the pertinent explicans (explicant);

6) "*implication*" to the act of implicating (implying) or to the relation established by that act; "*implicans*" (pl. "*implicantia*") or "*implicant*" (pl. "*implicants*") to the graphonym, which is the referent of the implication [relation] and which is used to imply another graphonym; "*implicatum*" (pl. "*implicata*") to the latter graphonym, which is the relatum of the relation and which is the one that is implicated (implplied) by the pertinent implicans (implicant);

7) “*interpretation*” to the act of interpreting or to the relation established by that act, “*interpretans*” (pl. “*interpretantia*”) or “*interpretant*” (pl. “*interpretants*”) to the graphonym, which is the referent of the interpretation [relation] and which is to be interpreted by another entity (particularly by another graphonym); “*interpretandum*” (pl. “*interpretanda*”), abbreviated as “*interpretand*” (pl. “*interpretands*”), to the latter entity (particularly to the latter graphonym), which is the relatum of the relation and which is the one that interprets the pertinent interpretans (interpretant); “*interpreter*” to the sapient subject (as me) that interprets;
8) “*justification*” to the act of justifying or to the relation established by that act, “*justificans*” (pl. “*justificantia*”) or “*justificant*” to the graphonym, which is the referent of the justification [relation] and which is to be justified by another graphonym; “*justificandum*” (pl. “*justificanda*”), abbreviated as “*justificand*” (pl. “*justificands*”), to the latter graphonym, which is the relatum of the relation and which the one that justifiess the pertinent justificans (justificant);
9) “*signification*” to the act of signifying or to the relation established by that act; “*significans*” (pl. “*significantia*”) or “*significant*” (pl. “*significants*”) to the graphonym, which is the rerere*nt* (refere*ns*) of the signification [relation] and which signifies another entity; “*significatum*” (pl. “*significata*”) to the latter entity, which is the relatum of the signification and which is the one significated by the prtinent significans (significant);
10) “*substitution*” to the act of substituting or to the relation established by that act; “*substituens*” (pl. “*substituentia*”) or “*substituent*” (pl. “*substituents*”) to the graphonym, which is the rerere*nt* (refere*ns*) of the substitution [relation] and whose given (particularly all) occurrences (isotokens) in a certain scope are to be replaced with occurrences (isotokens) of another graphonym, “*substituendum*” (pl. “*subtituenda*”), abbreviated as “*substituend*” (pl. “*substituends*”), to the graphonym, which is the relatum of the relation and which is the one whose occurrences (isotokens) are to be substituted for (are to replace) the given occurrences (isotokens) of the pertinent substituens (substituent).•

Cmt 2.6. The Latinized terminology that has been introduced in Dfs 2.6 and 2.7 is called the *basic psychologistic relational terminology* (*BPRT*). Some of the definitions comprised in Df 2.7 are conventional, some are stipulative, and the others are entirely new, so that the BPRT is *systematic* (*systemic*, *taxonomic*), *relatively monosemantic* (*univocal*), *relatively complete*, and *self-consistent*. By contrast, the pertinent *basic conventional* (*modern*, *presently common*) *relational terminology* (*BCRT*) is *unsystematic* (*non-systemic*, *non-taxonomic*), *incomplete*, *polysemantic*, and *not self-consistent.* The BCRT is illustrated by the following interrelated definitions of WTNID of practically all existing relevant cognate Latinized and chaste English substantives and verbs. The Webster's definition of the noun "gerundive" is included for convenience in the subsequent discussion.

Dict 2.3.

«**definiendum** … *n*, *pl* **definienda** … [L, neut. of *definiendus*, gerundive of *definire* to determine, bring to an end, **explain** – more at DEFINE] **:** whatever is being defined **:** the expression that precedes in a nominal definition the symbol of definitional equality – contrasted with *definiens*

definiens … *n*, *pl* **definientia** … [L, pres. part. of *definire*] **:** whatever serves to define **:** the expression that follows in a nominal definition the symbol of definitional equality – contrasted with *definiendum*

denotatum … *n*, *pl* **denotata** … [NL fr. L, neut. of *denotatus* 〈past part. of *denotare* to designate – Ya. I.〉] **:** an actually existing object is referred to by a word, or linguistic expression – contrasted with *designatum*

designatum … *n*, *pl* **designata** … [L, neut. of *designatus*, past part. of *designare* to designate, design, lit., to – more at DESIGN] **:** something that is referred to by a word, or linguistic expression whether actually existing or not **:** the class of objects referred to by a sign, including the null class – contrasted with *denotatum*

designation … *n* -s [ME *desygnacion*, fr. L *designation-*, *designatio*, fr. *designatus* + *-ion-*, *-io*, -ion] **1 :** the act of indicating or identifying by a mark, letter, or sign or by classification or specification … **6** *logic* **:** the relation between a sign, word, or linguistic expression and the object referred to; *also* **:** MEANING, CONNOTATION

explicandum … *n*, *pl* **explicanda** … [NL, fr. L, neut, of *explicandus*, gerundive of *explicare* to explain] **:** a word or an expression whose meaning is to be explicated – used chiefly in philosophy; contrasted with *explicans*

explicans … *n*, *pl* **explicantia** … [NL, fr. L, pres. part. of *explicare*] **:** the meaning of a word or an expression – used chiefly in philosophy; contrasted with *explicandum*

gerundive … *n* -s [ME, fr. LL *gerundivus*, fr. *gerundium* + *-ivus*, -ive – more at GERUND] **1 :** the Latin adjective that serves as the future passive participle, expresses necessity or fitness, and has the same suffix as the gerund **2 :** a verbal adjective in a language other than Latin analogous to the gerundive

implicant … *n* -s [L *implicant-*, *implicans*, pres. part. of *implicare*] **:** something that implies ⟨as a proposition⟩

[2]**implicate** … *vt* … [L *implicatus*, past part. of *implicare* …] … **2 :** to involve as a consequence, corollary, or natural inference **:** IMPLY …

implication … *n* -s [ME *implicacioun*, fr. L, *implication-*, *implicatio*, fr. *implicatus*, + *-ion-*, *-io*, *-ion*] … **2 a :** the act of implying or the state of being implied … **b :** one of several logical relationships or a statement containing propositions in such a relationship: (1) **:** a logical relationship of the form symbolically rendered "if *p* then *q*" in which *p* and *q* are propositions and in which *p* is false or *q* true or both; *also* **:** a statement in this form – called also *material implication* (2) **:** a logical relationship of the form symbolically rendered "if *p* then strictly *q*" in which *q* is deducible from *p*; *also* **:** a statement in this form – called also *logical implication, strict implication* **c :** the symbol used to indicate one of these two formal relationships rendered "if … then" or the logical operation implicit in one of them **3 :** something implied ⟨two propositions with a clear ~s⟩ **:** INFERENCE ⟨was aware of the ~ to be found in his remarks ⟩…

interpret …*vb* -ED/-ING/-s [ME *interpreten*, fr. MF&L; MF *interpreter*, fr. L *interpretari*, fr. *interpret-*, *interpres* broker, negotiator, expounder, interpreter, fr.. *inter-* + *-pret-* + *-pres* (prob. akin to L *pretium* value, price) – more at PRICE] *vt* **1 :** to explain or tell the meaning of **:** translate into intelligible or familiar language or terms **:** EXPOUND, ELUCIDATE, TRANSLATE … **2 :** to understand and appreciate in the light of individual belief, judgment, interest, or circumstance **:** CONSTRUE ⟨~ a law⟩ ⟨~ a contract⟩ **3 :** to represent and apprehend by means of art **:** show by illustrative representation understand and a ~ *vi* **:** to act as an interpreter **:** TRANSLATE

interpretant … *n* -s [L *interpretant-*, *interpretans*, pres. part. of *interpretari*] **1** … **b :** a sign or a set of signs that interprets another sign … **2 :** INTERPRETER

interpretation … *n* -s [… fr. L *interpretation-*, *interpretatio*, fr. *interpretatus* + *-ion-*, *-io* -ion]…**1 :** the act or the result of interpreting …

interpreter … *n* -s … **1 :** one that interprets, explains, or expounds …

justificandum …*n*, *pl* **justificanda** … [LL, neut, of *justificandus*, gerundive of *justificare* to justify] **:** something that is to be justify – compare with JUSTIFICANS

justificans …*n*, *pl* **justificantia** … [LL, pres. part. of *justificare* to justify] **:** something (as a principle) that serves to justify

significance ... *n* –s [ME *significaunce*, fr. L *significantia*, fr. *significant-*, *significans* + *-ia -y*] **1 a :** something signified … **b :** the quality of conveying or implying **:** SUGGESTIVENESS **2 a :** the quality of being important **:** CONSEQUENCE, MOMENT … **b :** the quality of being statistically significant **syn** see IMPORTANCE

significancy ... *n* –ES [L *significantia*] **1 s:** the quality or state of being significant **:** EXPRESSIVENESS **2 :** SIGNIFICANCE

[2]**significant** ... *n* -s **:** something that has or conveys significance **:** SIGN, TOKEN, SYMBOL

signification ... *n* –s [ME *significacioun*, fr. OF *signification*, fr. L *signification-*, *signification*, fr. *significatus* (past part. of *significare* to signify) + *-ion-*, *-io*, -ion – more at SIGNIFY] **1 a :** the act of signifying **:** a making known (as a choice, intent, decision) by signs or other means … **2 a :** IMPORT, SIGNIFICANCY … **b :** the meaning that a sign, character, or token is intended to convey **:** SENSE ⟨using the word in its ordinary ~⟩ … **4 a :** the connotation or comprehension of a term or the implication of a proposition **b :** the process of designating – compare DESIGNATION 6

significatum ... *n*, *pl* **significata** … [L – more at SIGNIFICATE] **:** something that a sign intentionally signifies **:** SIGNIFICATION

substituend ... *n* -s [NL *substituentdum*] **:** something that can be or is substituted in a logical relation

substituendum ... *n*, *pl* **substituenda** … [NL, fr. neut. of L *substituendus*, gerundive of *substituere* to substitute] **:** something that is to be substituted in a logical relation

substituent ... *n* -s [L *substituent-*, *substituens*, pres. part. of *substituere* to substitute] **:** something that is or may be substituted to be substituted; *usu* **:** an atom or group substituted for another or entering a molecule in place of some other part that is removed …

[1]**substitute** ... *n* -s [ME fr. L *substitutus*, past. part. of *substituere* to put under, put in the place of, substitute …] **1 :** a person who takes the place of or acts instead of another …

[2]**substitute** ... *vb* -ED/-ING/-s [L *substitutus*, past. part. of *substituere* to substitute] *vt* **1 a :** to put in place of another **:** EXCHANGE … ~ *vi* **:** to function, serve, or act as a substitute»

In accordance with Dict 2.3, the BCRT has the following peculiarities, in discussing of which I shall avoid using any new terms introduced in Df 2.7 without explicit references to that definition.

1) Etymologically, any Anglicized noun ending with “ndum” (e.g., “definiendum”, “explicandum”, “justificandum”, or “substituendum”) is a *Latin singular neuter gerundive* (see Dict 2.3); any Anglicized noun ending with “ns” (e.g., “referens”, “definiens”, “explicans”, or “justificans”) is a *Latin singular present participle*; any Anglicized noun ending with “tum” (e.g., “relatum”, “denotatum”, “designatum”, or “significatum”) is a *Latin singular neuter past participl*e; any «chaste» English noun ending with “nt” (e.g., “implicant”, “interpretant”, “significant”, or “substituent”) originates from the corresponding Latin “ns”-word, the understanding being that “s” in such an occurrence is an unstable letter, which changes for “t” when the “ns”-word becomes a part of another word. However, different Anglicized or chaste English nouns that are associated with one and the same Latin form class and also the denotata of the nouns are often used differently. Namely, a definiendum, e.g., is usually used to replace its definiens, – just as a substuendum is

used to replace its substituent. At the same time, one cannot posit that an explicandum replaces its explicans or that a justificandum replaces its justificans, but rather, on the contrary, the explicans or the justificans supplements and as if replaces the explicandum or the justificandum respectively.

2) It is naturally to consider a definition as a relation that goes from the definiens as referent to the definiendum as relatum. Indeed, the definiendum is a new term (neonym, nomen novum), while the definiens is an old term (paleonym, nomen vetum), i.e. one that is known from a previous definition or from another source. In his case, the definiens *acts upon* the definiendum, and hence the latter *is affected* (*is acted upon*) by the former, in the sense that the definiendum *is defined in terms of* (*with regard to, in relation to*) the definiens. Consequently, the maker or an interpreter of a definition establishes a *psychical* (*mental*) relation between the definiendum and its definiens, in which the former is referent and the latter relatum. If particularly a definition is an *asymmetric synonymic definition* (*ASD*) then in the *scope* of the definition, which is a certain part (continuous or broken) of the pertinent matter following but not including the definition, the definiendum is usually used instead of its definiens. That is to say, the definie*ns* is a substitue*ns* and the definien*dum* is a substituen*dum*. By contrast, the relation between an explican*dum* and its explica*ns*, or that between a justifican*dum* and its justifica*ns*, subject to the conventional meanings of the terms used therein (see Dict 2.3), goes from the former as refere*nt* (refere*ns*) to the latter as rela*tum*. That is to say, the explicandum or justificandum is the entity *acting upon* the explicans or the justificans respectively and hence the latter *is acted upon* (*is affected*) by the former.

3) There are in the pertinent modern terminology many glossonyms (words or descriptive names), each of which can be said of (predicated) as one that has a certain denotatum, designatum, or significatum. For instance, “name” and “declarative sentence” are two such glossonyms, However, except “*significant*”, which means *one that signifies,* there is no specific univocal word to mean *one that denotes* or *one that desinates.*

4) The noun “implicant” univocally means *one that implies* (cf. “[2]significant”). However, in the pertinent modern terminology there is no univocal word to be cognate of ‘imply” and to mean *an entity that is implied*. This duty is *equivocally* done by the noun “implication”. The established nouns “*antecedent*” and

“*consequent*” are akin to “implicant” and “implication” (in the above sense) respectively, but the former two nouns have very special meanings in logic and philosophy rather than to be general impartial terms.

5) One of the meanings which the equivocal noun “interpretant” has in the pertinent modern terminology is *a graphonym being a relatum that interprets another graphonym being its referent*. That is to say, “interpretant” belongs to the same Aristotelian category as, e.g., “definiendum”, “explicandum”, “implicant”, or “justificandum”, whereas from the standpoint of morphological and etymological analysis, “interpretant” (from Latin present participle “interpretans”) is analogous, e.g., to “definiens”. At the same time, there is in the pertinent modern terminology *no* kindred noun of the verb “to interpret” for *one that is* or *is to be interpreted*.

6) The entries of Dict 2.3 having the original Latin form, i.e. those ending with “ndum”, “tum”, and “ns” seem to be *conventional* philosophical or logical terms. However, the entire BCRT is, as was mentioned earlier and demonstrated above, unsystematic, equivocal, incomplete, and not self-consistent.•

2.2.3. Justification of the BPRT

1) A *definition concerning definitions* is called a *meta-definition*. Accordingly, the item 1 of Df 2.7 is one of the most general *meta-definitions* of the treatise. The first three terms (definienda) as defined in that item are in agreement with their definitions by Whitehead and Russell [1910; 1962, p. 11]). If a definition is a binary *antisymmetric synonymic definition* (*ASD*) then within its scope the definiendum of the definition is a *synograph* (*graphic synonym*) of its definiens. In this case both the definiendum and the definiens can be either xenographs or euautographs. If a definition is a *nominal* one (*ND*) and if its definiens is an *ostended autograph* that is to be named then the definiendum of the definition is a *graphic name* of the autograph and is hence a *phonoideograph*.

2) In treating of the series of interrelated trial decision problems that are successively solved in the treatise, the term “interpretation” and the relevant univocal cognate terms, which are introduced in item 7 of Df 2.3, turn out to be most important among the fundamental terms introduced in that definition, because it is necessary to distinguish between a graphonym interpreted and the graphonym interpreting it by special monosemantic (univocal) terms. In this case, it is, in principle, immaterial to a certain degree what different kindred substantive words of the root “interpret” are

used as the antonyms, one of which denotes a graphonym interpreted and the other one the graphonym or its denotatum, interpreting the former. A like remark applies, *mutatis mutandis*, with any one of the nine other roots that are used in the items 1–6 and 8–10 of Df 2.7 (as “definit”, “denot”, “connot”, etc to “substitut”) in place of “interpret”. My choice of the Latin words “interpretandum” (abbreviated as “interpretand”) and “interpretans” as the pertinent monosemantic psychologistic terms and consequently my choice of some other Latin words as relevant psychologistic terms have been stipulated by the following considerations.

3) In accordance with Df 2.3(7), an interpretation-operation, i.e. the act of interpreting, is the operation of assigning an interpretandum to the interpretans (interpreted nym), whereas an interpretation-relation is the relation, which the interpreter establishes in the result of that operation. Consequently, unless stated otherwise, I shall regard an interpretation as a *biune* entity, the two aspects of which I mentally experience by alternatively switching my attention from one aspect to the other – just as I do in perceiving any of Escher’s *Convex and Concave* pictures, e.g. «*Cube with Magic Ribbons* (see, for instance, Ernst [1985, p 85f]).

4) There are many different kinds of interpretation of graphonyms of object logistic systems of the treatise, which will be made explicit in due course. Most generally, all interpretations occurring in the treatise can be divided into *psychical* (*mental*) ones and *physical*, or *substitutional*, ones. As follows from its name, a substitutional interpretation of a formula of a given logistic system is the act (operation) or relation, or the two in one, of replacement of *the formula interpreted* with a certain *interpreting formula* either of the same or of another logistic system in accordance with certain *rules of substitution* being at the same time *rules of substitutional interpretation*. Thus, in this case, the interpreta*ns* is the substitue*ns* and the interpretan*dum* is the substituen*dum*, and vice versa. This *harmony* between the *form* and the *matter* (*meaning*) of the names is the result of two factors. First, I have adopted the univocal conventional term “substituendum” and supplemented it with the univocal antonymous term “substituens”, which is analogous to “definiens” in form and which means a *graphonym to be replaced with another graphonym*. Second, I have denoted an *interpreted graphonym* by the noun “interpretans”, in analogy with “substituens”, and an *interpreting entity* by the noun “interpretandum”, in analogy with “substituendum”.

5) The nouns "substituendum" and "substituens" are analogous to "definiendum" and "definiens", not only morphologically (formally), but also semantically (materially), because its definiendum is, in the scope of an ASD, usually used instead, i.e. as the substituendum, of its definiens (cf. the item 2 of Cmt 2.6). At the same time, "*to interpret*" often means *to explain,* while "*to justify*" sometimes also means *to explain*. Therefore, in order to preserve the agreement between the form and the matter of the pertinent kindred nouns of the latter three verbs, I have, by Df 2.7(5,8), exchanged the conventional meanings of the two antonyms "explicandum" and "explicans", or "justificandum" and "justificans" (cf. the item 2 of Cmt 2.6 again). Consequently, I may assert that, in some cases, an interpreta*ns* is an explica*ns* and that an interpretan*dum* is an explican*dum*.

6) The above-said implies that, in stating Df 2.7, I have tacitly postulated that any *operation* that belongs to the class denoted by any given one of the nouns: "definition", "denotation", "designation", "connotation", "explication", "implication", "interpretation", "justification", "signification", and "substitution", and hence the *associated relation of the same name*, i.e. the one that results by that operation, involves two terms (entities, beings) of the two Aristotelian categories, namely the *acting* one, called the referent, from which the relation proceeds, and the affected one (one being acted upon), called the relatum, to which the relation proceeds.

The referent of a relation of any given one of the above ten names is termed by the cognate substantive in the form of the Latin singular present participle, ending with the suffix "*ns*" (pl. "*ntia*"), – in analogy with the conventional term "definiens", whereas the relatum of the relation is termed by the cognate substantive either in the form of the Latin singular neuter gerundive, ending with the suffix "*ndum*" (pl. "*nda*"), – in analogy with the conventional terms "definiendum" and 'substituendum"), and also by its abbreviation ending with the suffix "*nd*" (pl. "*nds*"), or in the form of the Latin singular neuter past participle, ending with the suffix "*tum*" (pl. "*ta*"), – in analogy with the conventional terms "denotatum", "designatum", and "relatum".

7) Accordingly, in order to eliminate the shortcomings of the BCRT that are indicated in Cmt 2.6, I have, in stating Df 2.7, done the following things:

i) adopted the conventional terms "definiendum", "definiens", "denotatum", "designatum", "significatum", and "substituendum";

ii) introduced the term “connotatum” as a synonym of “connotation value”, – in analogy with the established “tum”-terms mentioned in the previous point;

iii) in analogy with the establishe term “definiens”, introduced the terms “denotans”, “connotans”, “designans”, “significans”, and “substituens” as antonyms of the terms “denotatum”, “connotatum”, “designatum”, “significatum”, and “ substituendum” respectively;

iv) exchanged the meanings of the conventional terms “explicandum” and “explicans” and of “justicandum” and “justificans”;

v) introduced the term “implicans” as a synonym of “implicant” to a graphonym that implies another graphonym and the term “implicatum” to an entity implied by the implicans (implicant), instead of the equivocal term “implication”;

vi) introduced the term “interpretans” to a graphonym to be interpreted and the term “interpretandum” to a graphonym interpreting the interpretans, instead of the equivocal term “interpretant”;

vii) introduced the term “substituens” to a graphonym being replaced with the substuendum;

viii) introduced synonyms of the nouns ending with the suffix “ndum” by abbreviating the latter as “nd” (cf. Cmt 2.1).

8) The Latin etymons of the terms introduced in Df 2.5–2.7, can be found in, or recovered by analogy (straightforwardly or with the help of the pertinent rules of the Latin grammar) from those found in, WTNID (cf. Dicts 2.1–2.3) or Simpson [1968]. For instance, the latter says:

Dict 2.4.

«**dēnŏto** -are (**1**) *to mark out for another, designate precisely* … Cic. (**2**) *to take note of*, for one’s own purposes Cic., Tac.

dēsigno -are *to mark out, trace out, plan* …

ŏpĕror –ari, dep. (opus), *to work, labour, be busy, be occupied*; esp. in perf. partic. **ŏpĕrātus** -a – um, *engaged, busy* (which may be derived directly from opus) …; with dat. of the occupation …; esp. *to be engaged in worship* …

rĕfĕro rĕferre retŭlli rĕlatum (rēlatum, Lucr.), (**1**) *to carry back*, *bring back*; … *to return*, *to go back*; … *to restore,* Verg. (**2**) *to bring again*, *restore*, *repeat*; … *to bring up again*; … *to echo*; … *to reproduce* by imagination, *to recall* (**3**) *to say back, answer, return*; …

rĕlātus -us m. (refero), *a bringing before,* (**1**) *a narrative*, *recital***:** Tac. (**2**) a report: Tac.»

In this case, the nomen novum (neonym, new term) "denotans", e.g., being analogous to the nomen vetum (paleonym, old, established term) "definiens" (see Dict 2.3 and Cmt 2.6(1)), is the present participle of the present indefinite verb "denotare", whereas the nomen vetum "denotatum" is the neuter of the singular masculine past participle "denotatus" of the same verb. Instead of "denotatum", I could optionally employ the nomen novum "*denotandum*" – the neuter of the masculine gerundive "*denotandus*" of the verb "dēnŏtare", which is analogous the nomina veta "definiendum" and "subsituendum". However, I have rejected the last option, because the word "denotatum" and some other similar "tum"-words have established usages in English, so that any attempt to alter those usages would be confusing and counterproductive. Like remarks apply, *mutatis mutandis*, with any root "design", "connot", or "signific" in place of "denot".

9) All relations that are defined in Df 2.7 are *direct* ones and therefore they should be distinguished from the respective *inverse* relation (see Cmt 2.5). For instance, in accordance with a [*direct*] *denotation relation,* defined in Df 2.7(2), the denotans of the relation is used as the referent for mentioning its denotatum as the rlatum. By contrast, the relation, according to which the entity serving as the denotatum was once provided with the name serving as the denotans, is the *inverse denotation relation* that can alternatively be called a *notation relation.* In this case, the denotatum is the referent of the notation relation, while the denotans is the relatum of the notation relation. If the denotans is a *proper name* then the [direct] denotation relation is called the *direct proper name relation*. Church [1956, pp. 4, 5] synecdochically calls such a relation "name relation" and defines it thus:

«The relation between a proper name and what it denotes will be called the *name relation,*[8] and the thing[9] denoted will be called the *denotation*.

[8]The name relation is properly a ternary relation, among a language, a word or phrase of the language, and a denotation. But it may be treated as binary by fixing the language in a particular context. Similarly one should speak of the denotation of a name *with respect to the language*, omitting the latter qualification only when the language has been fixed or when otherwise no misunderstanding can result.

[9]The word *thing* is here used in its widest sense, in short for anything namable»

A relation that is established by providing *a given entity as referent* with *a proper name as relatum* is an *inverse proper name relation*. Thus, the names "referent" and "relatum" of the respective terms of a direct name relation exchange in passing to the pertinent invrse name relation. Still, if the entity named is an insensible (imperceptible, mental, abstract) coentity of mine (e.g.) then my use of the coentity as a referent of its name as the relatum is incommunicable.●

2.4. An introduction into a psychologistic theory of the meaning content of xenographs

2.4.1. Primary divisions of the class of nounal (substantival) xenographs

Df 2.8. 1) A xenograph is called a *proper graphic name* or onymologically a *kyrioxenograph* (*kyrioxenographonym*), in analogy with "kyrioautograph" (see Df 1.23(2) and Cmt 1.27(2,4)), if it is used or is designed to be used as a referent to refer to (mention) a *unique distinct physical* (*real*) *or psychical* (*ideal*) *entity*, other than itself and other than any one of its token-classes, as *its intended xenovalue*. The term "proper graphic name", being a synonym of "kyrioxenograph", will be abbreviated as "*proper name*", unless stated otherwise. A kyrioxenograph (proper name) is said to be *unlimited* if does not involve any *limiting modifier* (as a predicate, the definite or indefinite article, a noun in the possessive case, a demonstrative or possessive pronoun, or any other to be indicated defined in Df 2.11 below in this subsection). For instance, "Aristotle", "London", "Mont Blanc", "Moscow", "Tahiti", and "Trafalgar Square" are *individual unlimited kyrioxenographs*; "the United States", "the Alps", "the Hawaii", "the West Indies", and "the Mediterranean" are *individual limited kyrioxenographs*; "chimpanzee" ("*Pan troglodytes*") "lion" (*Felis leo*), "man" ("*Homo sapiens*"), and "poplar" ("*Populus*") are *unlimited proper class-names* or onymologically *unlimited kyrioclassoxenographs* (*kyrioclassoxenographonyms*); and “dough”, “water”, “wood”, “heroism”, and “honesty” are *unlimited proper mass-names* or onymologically *unlimited kyriomazoxenographs* (*kyriomazoxeno*┐ └*graphonyms*) (see Dict A1.1 for “*maz*” or “*mazo*”). A kyrioxenograph that is limited by a predicate (predicate modifier) is called a *proper sentence* or *proper clause*, or onymologically *kyrioprotasograph* (*kyrioprotasographonym*). For instance, “Aristotle is the founder of logic” and “Abraham Lincoln was the 16th president of the USA in the years 1861–65” are *proper sentences*. An unlimited kyrioxenograph and a limited kyrioxenograph have the same semantic property, which has been stated at the beginning of this item and which is particularized in the next item.

2. a) Given a kyrioxenograph, the unique entity that is referred to by the kyrioxenograph as its intended value is said to be *denoted* by the kyrioxenograph and accordingly it is called the *denotatum* (*denotation value*, pl. “*denotata*”) of the kyrioxenograph [with respect to me]. In accordance with Cmt 1.27(1), the denotatum of a kyrioxenograph is an *object sui generis* and therefore, *ipso facto*, it automatically produces the *singleton of its own*, which becomes another value of the kyrioxenograph – the value that is said to be *connoted* by the latter and that is accordingly called the *singleton-connotatum* of the kyrioxenograph. In this case, I *use* the kyrioxenograph along with its singleton-connotatum *for mentioning* (*denoting*, *putting forward*) its denotatum, while both the kyrioxenograph and its singleton-connotatum *are used but not mentioned*. The above mental phenomenon of using a kyrioxenograph is theoretically substantiated as follows.

b) The singleton-connotatum (singleton-designatum) of a kyrioxenograph is a *mental* (*psychical*) *entity* of mine. However, when I use a kyrioxenograph for mentioning (denoting, putting forward) its denotatum, I use the singleton-connotatum in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* it as its member, i.e. as the denotatum of the kyrioxenograph, *in the hypostasis of my as if extramental* (*exopsychical*) *object* (other than the kyrioxenograph itself). I do so *habitually* and hence *involuntarily but consciously* – just as I most often but not always use a kyrioautograph (see Cmt 1.27(4)) and just as I *always* use the *percept* (*sensation*) of any nym (sensum, sensory object) and particularly that of the kyrioxenograph itself (see Df 1.12(1)). In this case, I *use* the kyrioxenograph along with its singleton-connotatum *for mentioning* (*denoting*, *putting forward*) its denotatum, while both the kyrioxenograph and its singleton-connotatum *are used but not mentioned*. Thus, the member of the singleton-connotatum is put forward as the intended import value, i.e. as the denotatum (meaning) of the kyrioxenograph, while the singleton-connotatum itself is *as if* put backward. In fact, however, the singleton-connotatum *is*, to use the appropriate *monistic phraseology*, involuntarily *mentally transduced into another hypostasis* (*way of existence*, *aspect*) in the form of its only member. In order to describe this mental phenomenon in the appropriate alternative *dualistic phraseology*, I say that the member of the singleton-connotatum of the kyrioxenograph *represents* the singleton-connotatum, so that the two entities as if coexist as a single biune entity.

c) When a kyrioxenograph is detached from the projective mental mode and is just *considered*, its singleton-connotatum is impartially called its *singleton-designatum* and also less explicitly its *class-designatum* or simply its *designatum* (*designation value*, pl. "*designata*").

d) A kyrioxenograph can *denote a singleton* so that it has a *singleton-denotatum*. In this case, the *singleton-connotatum* of the kyrioxenograph is the *singleton of its singleton-denotatum*, i.e. a *two-fold* (*repeated*) *singleton*. For instance, a DSN that describes and denotes a singleton is such a kyrioxenograph (see also Df 2.10(6) below).

e) In analogy with a one-member class that is called a *singleton*, a many-member class, i.e. a class that has two or more members, or particularly a non-denumerable multitude of members, is called a *multipleton*. A class that has no members is conventionally called *the empty class* or *the empty individual*. A multipleton or a singleton is indiscriminately called a *nonempty class*. Consequently, a *kyrioclassograph*, i.e. a *proper graphic class-name*, denotes *a multipleton* or *a singleton* or else *the empty class*.•

Df 2.9: *Unlimited numeralable nounal xenographs versus unlimited non-numeralable nounal xenographs.* 1) A xenograph is called an *unlimited singular nounal*, or *substantival*, *xenograph* (*UnLtdSgNlXG*) or an ~ (ditto, similarly qualified) *name* (*UnLtdSgNlN*) if it is a *noun* or *unlimited noun equivalent in the nominative case* (called also in English the *common case* in contrast to the *possessive case*) *of a singular number form that does not involve any limiting modifier* (to be defined in Df 2.11 below in this subsection). An UnLtdSgNlXG (UnLtdSgNlN) is said to be *primary* or *reference* or *induced* if its denotatum has been *induced* and *secondary* or *deduced* its denotatum has been *deduced*. Here, and generally in what follow, by "name" I synecdochically mean a *graphic* (*written*) *name*, i.e. a *xenograph*, unless stated otherwise. Therefore, the meaning of the above "name"-term and the meaning of any subsequent relevant or similar term remains unaltered if the occurrence of "name" ("N") in that term is replaced with an occurrence of "*xenograph*" ("*XG*").

2) An UnLtdSgNlN is called a *count singular name* (*CtSgN*) if it has a *plural number form*, which is called a *pluralized count singular name* (*PlzCtSgN*) or *count plural name* (*CtPlN*), and if hence it can be used with the prepositive *numeral* (*numeric quantifier*) "1" ("one") as a *limiting modifier,* while its plural number form can be used with any of the prepositive *numerals* (*numeric quantifiers*) "2" ("two"), "3" ("three"), etc as *limiting modifiers,* thus forming *dimensional numerals* that denote the corresponding *dimensional numbers*. I shall therefore use the qualifier "count" to "noun" or "name" ("xenograph") interchangeably with "*numeralable*" meaning *capable of being modified with a numeral* and also meaning *capable of serving as a dimension of a numeral*. Thus, "*count singular name*" ("*CtSgN*") and "*numeralable singular name*" ("*NSgN*") are synonyms and therefore "*pluralized count singular name*" ("*PlzCtSgN*"), "*pluralized numeralable singular name*" ("*PlzNSgN*"), "*count plural name*" ("*CtPlN*"), and "*numeralable plural name*" ("*NPlN*") are also synonyms. It is understood that an NSgN can also be modified with the *indefinite article* as another *limiting modifier* if it is available in the pertinent NL (as in English, but not in Greek, Latin, Hebrew, or Russian), whereas an NPlN (PlzNSgN) can also be *limited* (properly modified) with either of the prepositive *unspecific* quantifiers "*many*" and "*few*", denoting *unspecified numerable* (*numeric*) *quantities* and being two more limiting modifiers. A xenograph, which comprises an NSgN or NPlN and an appropriate limiting modifier, is called a *limited NSgN* (*LtdNSgN*) or a *limited NPlN* (*LtdNPlN*) respectively.

3) The fact that an NSgN has a plural number form signifies that the NSgN *designates* [with respect to me] a certain *multitudinous class,* i.e. a class that has strictly more than one member. This class is alternatively called a *multipleton,* in analogy with the conventional term "*singleton*", denoting a class of a single member. Irrespectively to the mental mode, in which I use the NSgN, I say that the multipleton that it designates is the *designatum,* or redundantly *multipleton-designatum* or *class-designatum, of the NSgN* [with respect to me]. Accordingly, the NPlN, i.e. PNSgN,

designates *the power class of the multipleton-designatum of the respective NSgN*. Once I put the multipleton forward as the intended value of the NSgN, I say that the multipleton is *denoted by* the NSgN and also that it is the *multipleton-denotatum*, or less explicitly *class-denotatum*, of the NSgN, while the NSgN is called an *unlimited proper name* (*UnLtdPrN*) *of the multipleton* or less explicitly an *unlimited proper multipleton-name* (*UnLtdPrMnN*). In this case, the NPlN, i.e. PNSgN, is said to *denote*, or to be an *unlimited proper name* (*UnLtdPrN*) *of*, *the power class of the multipleton-denotatum* (*class-denotatum*) *of the NSgN*. Incidentally, an *unlimited proper singleton-name* (*UnLtdPrSnN*), i.e. an *UnLtdPrN of a singleton*, is not an NSgN, but an *unlimited non-numeralable singular name* (*UnLtdNNSgN*), i.e. an UnLtdSgNlN that has no plural number form (see the item 6 below in this definition).

4) A *concept* is by definition a *conception* (*thought*, *idea*, *notion*), and hence a an *abstract* mental (psychical) coentity of mine, called also an *abstractum* (pl. "*abstracta*"), which is *represented*, particularly either *expressed* or *designated*, by an exteroceptive onym, i.e by a *concrete* exteroceptive sensible (physical) coentity of mine, called also a *concretum* (pl. "*concreta*"), especially by a xenograph (xenographonym).

5) Particularly, *a class is a concept* because it is a conception that necessarily has a proper name. Therefore, I shall use either apposition "*concept class*" or "*class concept*" in the sense of "*concept of the class*" by way of emphatic comparison with either apposition "*concept mass*", abbreviated as "*cmass*", or "*mass concept*" in the sense of "*concept of the mass*", which are explicated below in the item 7. The above four appositions will be hyphenated as "*concept-class*", "*class-concept*", "*concept-mass*", and "*mass-concept*" in that order. *A class is an abstractum, which has both members and parts*. Formally, a name of a member of a class is related to a name of the class by the *class-membership* (*class-belonging*) *predicate* '$\in$', whereas a name of a part of a class is related to a name of the class by the *class-inclusion predicate* '$\subseteq$'. A member of a class can be either a *nonempty individual* or *another class*. A member of a class is alternatively called an *element of the class* or less explicitly an *element*. Accordingly, a class being a member of another class is called an *element*. A class is called a *group*, and also *aggregate*, *collection*, or *totality*, *of elements* if it is thought of as one that contains those and only those particular elements from which it has been induced. A class that is a part of another class is alternatively called a *subclass* of

the latter class, while the latter is called a *whole*, or *superclass*, of the former. A class that is *a part but not the whole* of another class is called a *strict part*, or *strict subclass*, of the latter, while the latter is called a *strict whole*, or *strict superclass*, of the former. A class, denoted by '∅', which satisfies the axiom $\neg[x \in \varnothing]$ for every element x and which therefore has no elements (members), is called *the empty class* and also *the empty individual*. It follows from the above axiom as a theorem that $\varnothing \subseteq x$ for every element x, i.e. ∅ is a subclass (part) of every class, including itself ($\varnothing \subseteq \varnothing$). A class that is not empty is called a *nonempty class*. Thus, *a class is a multipleton or a singleton or else the empty class.* An entity *whose name cannot stand to the right of the predicate* '∈' *but can stand to the left of that predicate* is called a *nonempty individual.* As a consequence, a name of a nonempty individual *cannot stand to the right of the predicate* '⊆' either. A class is called a *class of individuals* if *all* its members are individuals and a *class of classes* if all its members are classes. An isolated *class of classes* or a *taxon* (*taxonomic class*), i.e. a class of classes or class of individuals that has a certain *rank* with respect to any other taxon of the pertinent *taxonomy*, will occasionally be called a *category*, which is an Anglicized Aristotelian term "κατηγορία" \kateγoría\ that Aristotle coined for any one of the ten kinds, into which he divided all beings (cf. Df 1.19(2)).

6) A xenograph is called an *unlimited non-numeralable singular xenograph* (*UnLtdNNSgXG*) or a *ditto name* (*UnLtdNNSgN*) if it is an *unpluralizable UnLtdSgNlN*, i.e. an *UnLtdSgNlN that has no plural number form* either universally or in a given circumstance, although in some other circumstances it may have a *numeralable homograph*, which can be pluralized. Accordingly, "non-numeralable" means *incapable of being modified either with the numeral* "1" ("*one*") *or with the indefinite article* (if the latter is available in the pertinent NL), and hence it also means *incapable of serving as a dimension of the numeral* "1".

7) The fact that an UnLtdNNSgN has no plural number form signifies that the UnLtdNNSgN *designates* [with respect to me] either *a certain singleton*, i.e. *the singleton of a certain entity*, or a *concept* (*abstractum*), which is, in accordance with the item 5, called a *concept mass* or a *mass concept*, i.e. a *concept of a mass*, and also a *concept-mass* or a *mass-concept* in the hyphenated form. The mental status of a concept-mass is analogous to that of a class (concept-class). In order to emphasize this analogy, the term "concept-mass" has been abbreviated as "*cmass*". Still in the sequel

I shall occasionally use the noun "mass" instead of "cmass" if confusion cannot result (cf. also Df 2.8(1), where the noun "mass" is used in the sense of the name "concept-mass"). In the case, where a given UnLtdNNSgN designates a singleton, once I put the singleton forward as the intended value of the UnLtdNNSgN, I say that the singleton is *denoted by* the UnLtdNNSgN or that the former is the *singleton-denotatum*, or less-explicitly, *class-denotatum*, of the latter, while the UnLtdNNSgN is called an *unlimited proper name* (*UnLtdPrN*) *of the singleton* or an *unlimited proper singleton-name* (*UnLtdPrSnN*) (cf. Df 2.8(2d)). Likewise, in the case, where a given UnLtdNNSgN designates a cmass, once I put the cmass forward as the intended value of the UnLtdNNSgN, I say that the cmass is *denoted by* the UnLtdNNSgN or that the former is the *cmass-denotatum* of the latter, while the UnLtdNNSgN is called an *unlimited proper name* (*UnLtdPrN*) *of the cmass* or an *unlimited proper cmass-name* (*UnLtdPrCmsN*).

8) In contrast to a class, *a cmass* (*concept-mass*) *is an abstractum that has parts but no members*. Formally, a name of a part of a cmass is related to a name of the cmass by the *cmass-inclusion predicate* '$\subseteq$', being a *homograph of the class-inclusion predicate*. Accordingly, a cmass that is a part of another cmass is alternatively called a *subcmass* of the latter, while the latter is called a *whole*, or *supercmass*, of the former. A cmass that is *a part but not the whole* of another cmass is called a *strict part*, or *strict subcmass*, of the latter, while the latter is called a *strict whole*, or *strict supercmass*, of the former. A cmass, denoted by '$\varnothing_m$', which satisfies the axiom $\varnothing_m \subseteq x$ for every cmass *x* and which is therefore a *part of every cmass and of itself* ($\varnothing_m \subseteq \varnothing_m$), is called *the empty cmass* (cf. the relations '$\varnothing \subseteq x$' and '$\varnothing \subseteq \varnothing$' for classes). A cmass that is not empty is called a *nonempty cmass*.

9) A part (subcmass) of a cmass has *mental projections* into the real world, each of which is called an *instance*, or *realization*, *of the cmass* and also less explicitly a *percept-mass* (*percept mass*) or simply a *mass*. Depending on its instances, a cmass is one of the two kinds: (i) a concept of an *abstract* mass, which is distinguished by one of the pertinent UnLtdPrCmsN's as: "*conscience*", "*courage*", "*energy*", "*heat*", "*heroism*", "*honesty*", "*light*", "*love*", "*sound*", "*time*", "*trouble*", etc, and also e.g. "*heat energy*", "*kinetic energy*", "*nuclear energy*", "*potential energy*", etc subject to [heat energy]$\subseteq$energy; [kinetic energy]$\subseteq$energy, etc, or e.g. "*daylight*", "*electric light*", "*moonlight*", '*sunlight*", etc subject to daylight$\subseteq$light, [electric light]$\subseteq$ light,

etc; (ii) a concept of a *material*, or *bulk*, *mass*, which is distinguished by one of the pertinent UnLtdPrCmsN's as: "*dough*", "*pastry*", "*sand*", "*soil*", "*water*", "*wood*", etc, and also e.g. "*fancy pastry*" ("*short pastry*") and "*flaky pastry*" ("*puff pastry*") subject to [fancy pastry]⊆pastry and [flaky pastry]⊆pastry, or e.g. "*cold water*", "*distilled water*", "*fresh water*", "*heavy water*", "*hot water*", "*mineral water*", "*sea-water*", etc subject to [cold water]⊆water, [distilled water]⊆water, etc. In order to refer to a common (general, indefinite) *instance* of a mass-concept, the pertinent UnLtdPrCmsN is limited by the prepositive limiting modifier "*some*" (if possible), so that e.g. [*some* courage]⊆courage, [*some* energy]⊆energy, [*some* dough]⊆dough, and [*some* water]⊆water. In this case, an absract mass (as *some* courage or *some* energy), i.e. an instance of a certain concept of abstract mass, is an *abstraction* from a certain sensible phenonomenon. By contrast, a material mass (as *some* dough or *some* water), i.e. an instance of a certain concept of material mass, is a *sensible homogeneous coherent* portion of substance (matter) making one body usually of indefinite sizes and of an indefinite shape of its own that can be divided into two or more *incoherent parts*, usually of indefinite sizes and indefinite shapes, each of which is not empty and is also a material mass of the same name because it has the same state of aggregation as the whole material mass. A material mass cannot be divided into any ultimate *individuals as its incoherent additive parts*, because such an individual has a completely different hypostasis, in which it lacks the characteristic aggregate properties of the material mass and is therefore distinguished by a different name. For instance, a grain of sand is a grain and not sand, and a glass of water is a glass and not water. Likewise, a molecule of water is a molecule and not water. A molecule is not liquid, not solid, and not gaseous; it has a completely different hypostasis. Consequently, no material mass is an *element* and conversely no *element* is a material mass. In contrast to a material mass that has neither definite sizes nor a definite shape, the notions of sizes and shape are not applicable to abstract masses at all. In any case, however, neither instances of a concept of abstract mass nor instances of a concept of material mass can be count by units. This is why a proper name of a cmas has no plural number form.

10) The noun "mass" itself is an NSgN, because there are different concepts of abstract masses that are distinguished by their UnLdtPrN's such as "courage", "heat", "heroism", "light", "love", "sound", etc and there are also different concepts of

material masses that are distinguished by their UnLdtPrN's such as "dough", "sand", "soil", "water", "wood", etc. I have used this property of the noun "mass" in the above wordings.

11) By definition, the *enneoxenographic quotation* (*EXQ*) *of a secondary xenograph*, which is formed by enclosing the xenograph between EXQ marks, \ /, *represents*, or *expresses*, *the sense of the interior of the EXQ*, i.e. *it expresses the designatum-producing operation* (*sense-operation*) *on the xenograph* and *denotes the designatum of the xenograph*, being the substantivized result of the operation. In this case, the EXQ marks, being *the exterior of the EXQ*, *are used for indicating the above mental attitude of mine towards the interior of the EXQ, but they are not mentioned*. Therefore, I may alternatively assert that the secondary xenograph, being the interior of the EXQ, expresses its own sense, i.e. it expresses the sense-operation on itself and denotes its designatum as the result of the sense-operation. The sense of the secondary xenograph and both the sense-operation on the latter and the designatum of the latter, being two hypostases of the sense, are concepts of mine, which are represented by that xenograph and which are therefore distinguished from one another only by the respective abstractions (mental attitudes). Accordingly, for instance, either of the appositions "*concept designatum*" or "*designatum concept*" in the sense of "*concept of the designatum*" or either of their hyphenated versions "*concept-designatum*" or "*designatum-concept*" is a denotative synonym of "*designatum*" (cf. the item 4). I shall also say that the designatum (the concept designatum, the designatum concept, a concept of the designatum) of a secondary xenograph, being the substantivized result of the sense-operation (the concept sense-operation, the sense-operation concept, a concept of the sense-operation) on the xenograph, *is determined* by that sense-operation. Depending on a secondary xenograph, its designatum, i.e. designatum-concept, is called either (a) its *class-designatum* or *class-concept* if it is a class or (b) its *cmass-designatum* or *mass-concept* if it is a cmass. Consequently, the class-designatum, or class-concept, of a secondary xenograph is called either (i) its *multipleton-designatum* or *multipleton-concept* if it is a multipleton or (ii) its *singleton-designatum* or *singleton-concept* if it is a singleton a *singleton-designatum*. Thus, in agreement with the items 5 and 7, "concept-class" and "class-concept", or "concept-mass" ("cmass") and "mass-concept", are denotative synonyms. A designatum-producing operation on a xenograph is *an operation of nominalistic* (as

distinguished from *syllogistic*) *deduction of its designatum from the known designata of its constituent xenographs.* No such operation exists if the designatum of a xenograph has been *induced*, i.e. has been assigned to it by *induction.* However, in any case, the EXQ of a xenograph *denotes* the designatum of the xenograph: it does this *immediately* if the designatum has been either *induced* or *deduced* earlier and *mediately*, via the sense-operation, if otherwise.

12) In accordance with the above-said, if the designatum of a xenograph is a *singleton* then the EXQ of the xenograph *denotes* this singleton, i.e. it is an *unlimited proper name* (*UnLtdPrN*) *of this singleton* and hence, less explicitly, it is an UnLtdPrSnN, which has the form of *the singleton of an UnLtdPrN of the member of the singleton that the UnLtdPrSnN denotes*. If the pair of EXQ marks, \ /, being the exterior of an EXQ, is replaced with the pair of braces (curly brackets), { }, then the EXQ turns into a conventional symbol of a singleton. For instance, like the conventional symbol "{Aristotle}", the EXQ "\Aristotle/" is an unlimited proper name of the singleton of Aristotle. Hence, \Aristotle/ or {Aristotle} *is* the singleton of Aristotle. In contrast to a conventional curly-bracketed symbol that denotes that denotes the singleton of its interior, in the general case an EXQ denotes the designatum of its interior, which can, depending on the interior, be a multipleton or a mass, and not only a singleton, and in addition it expresses the sense of its interior when applicable.

13) Since an UnLtdNNSgN, i.e. either an UnLtdPrSnN or an UnLtdPrCmsN, has no plural number form, therefore it cannot be limited by any modifier, which is associated with counting. In English, an UnLtdNNSgN can, depending on what it is, be limited by the definite article. Also, an UnLtdPrCmsN is limited in indefinite singular constructions by the prepositive limiting modifier "*some*" (cf. the item 9) – in contrast to the indefinite article "a" or "an" limiting an NSgN in the like constructions. Besides "some", an UnLtdPrCmsN can, when appropriate, be limited either by some other prepositive *unspecific mass quantifier* such as "*much*", "*a lot of*", "*a little of*", or "*plenty of*" (e.g. "plenty of time" or "plenty of trouble"), thus becoming a *limited common mass name* (*LtdCmnMsN*), or it can be limited by a prepositive *specific mass quantifier* (*possessive dimensional numeral*) such as "*a bottle of*", "*two bottles of*", "*three bottles of*", etc (applied, e.g., to "water", "juice", or "wine"), thus becoming a *limited proper name of a common member of the respective*

class. Consequently, the last case should more correctly be interpreted as follows. The string of an UnLtdPrCmsN and a preceding preposition in that order, which follows a limited or unlimited NSgN or NPlN, is a *postpositive qualifier* to the NSgN or NPlN, which has nothing to do with various roles that the UnLtdPrCmsN may play in some other occurrences.•

Df 2.10: *Descriptive specific names versus deductive names.* 1) In accordance with Cmt 1.6(2), a *descriptive specific name* (*DSN*), i.e. a *description of the species through a genus and the difference*, or *differences*, – briefly a *DcSTrG&D* or *DcSTrG&Ds*, in Latin *descriptio species per genus et differentiam*, or *differentias*; or, more precisely, a *description of the species through the intersection of the genus, designated by the pertinent generic name* (*GN), and through the differences* (particularly *the difference*), *designated by the pertinent prepositive or postpositive qualifiers* (*Ql's*) is either a complex *monomial* (*one-word*) *one* (*MDSN*) or a *polynomial* (*many-word*) *one* (*PDSN*). The *GN* (*generic name*) and *qualifiers* (*Ql's*) of an MDSN are *bound* (*not free*) *morphemes* (as combining forms, prefixes, suffixes, or infixes). The GN and qualifiers of a PDSN are *words*, *some of which can in turn be MDSN's*.

2) A DSN is called a *numeralable*, or *count*, *DSN* (*NDSN* or *CtDSN*) if it is an NSgN and a *non-numeralable DSN* (*NNDSN*) if it is an UnLtdNNSgN. An NDSN *describes and denotes a multitudinous* (*many-member*) *class-species* (*specific class*), which is alternatively called a *multipleton-species* (*specific multipleton*), and therefore the NDSN is alternatively called a *descriptive specific multipleton-name* (*DSMnN*). An NNDSN is called a *descriptive specific singleton-name* (*DSSnN*) if it is an UnLtdPrN of a singleton and a *descriptive specific mass-name* (*DSMsN*) if it is an UnLtdPrMsN (MsN). A DSSnN describes and denotes a *one-member class-species* (*specific class*), which is alternatively called a *singleton-species* (*specific singleton*), whereas a DSMsN describes and denotes a *mass-species* (*specific mass*). A DSMnN or a DSSnN is indiscriminately called a *descriptive specific class-name* (*DSCsN*).

3) The GN of a DSCsN (either DSMnN or DSSnN) is necessarily an NSgN and it is therefore called *numeralable*, or a *count*, *GN* (*NGN* or *CtGN*) and also alternatively a *generic multipleton-name* (*GMnN*), because it necessarily denotes a *generic multitudinous class*, which is called a *multipleton-genus* (*generic multipleton*). *An NGN (CtGN, GCsN) cannot be either a singleton or an MsN*. If the GN of a DSN

is a singleton name then all qualifiers of the DSN are redundant, so that the denotatum of the DSN is the singleton denoted by the GN. Such trivial cases are disregarded. The GN of a DSMsN is called a *generic mass-name* (*GMsN*), because it necessarily denotes a *mass-genus* (*generic mass*) An MsN (UnLtdPrMsN) is either a GMsN or a DSMsN and vice versa.

4) A DSN is either a complex *monomial* (*one-word*) *one* (*MDSN*) or a *polynomial* (*many-word*) *one* (*PDSN*), some constituent words of which can in turn be MDSN's. An MDSN is necessarily a *class one* (*CsMDSN*), i.e. *MDSCsN* (either *MDSMnN* or *MDSSnN*), whereas a PDSN is either a *class one* (*CsPDSN)*, i.e. a *PDSCsN* (either *PDSMnN* or *PDSSnN*), or a *mass one* (*MsPDSN)*, i.e. *PDSMsN*.

5) The MDSN's that are used as psychologistic metaterms of the treatise are primarily onymological and onological nouns. Every constituent morpheme of an onymological or onological MDSN is its *designative unit* in the sense that it designates a certain class other than any one of its token-classes. A *designative unit* of a PDSN is either a *semanteme* (*full*, or *notional*, *word*) *alone* or a *combination of a semanteme with the appropriate associated function word*, so that it designates a certain class other than any one of its token-classes, with the following proviso. If some constituent words of a PDSN are MDSN's, then each one of the latter is regarded as a *designative unit*, whose designatum is supposed to be deduced earlier by the general method to be described below in this section.

6) A pluralized NSgN (PlzNSgN), i.e. a numeralable plural name (NPlN), is either a *pluralized NGN* (*PlzGN*), i.e. a *plural number form of an NGN*, or a *pluralized NDSN* (*PlzNDSN*), i.e. a *plural number form of an NDSN*. In accordance with Df 2.9(3), a PlzNGN denotes the *power class* of the class-genus (generic class) denoted by the pertinent NGN and hence *the PlzNGN is not an NGN itself*. Likewise, a PlzNDSN denotes the *power class* of the class-species (specific class), or more precisely of the multipleton-species (specific multipleton), denoted by the pertinent NDSN and hence *the PlzNDSN is not an NDSN itself*. In order to pluralize an NDSN, i.e. to form the pertinent PlzNDSN, the constituent NGN of the NDSN should be pluralized, i.e. it should be replaced with the pertinent PlzNGN, while all qualifiers to the NGN remain unaltered.

7) By Cmt 1.6(2), it follows from the previous item that a DSN (i.e. an NDSN or an NNDSN), a PlzNGN, or a PlzNDSN is indiscriminately called a *deduction from*

the genus and differences (*DdFrG&Ds*) and also briefly a *deductive name* (*DdN*); a DSN is more specifically called a *descriptive DdFrG&Ds* (*DcDdFrG&Ds*) and also a *description*, or *description of the species, through the genus and differences* (*DcTrG&Ds* or *DcSTrG&Ds*); a PlzNGN or a PlzNDSN is called a *non-descriptive DdFrG&Ds* (*NonDcDdFrG&Ds*) and also briefly a *non-descriptive deductive name* (*NonDcDdN*). It is understood that a GN can also be a DcTrG&Ds, i.e. a DSN. In contrast to a *syllogistic deduction*, a DdFrG&Ds is called a *nominalistic deduction.*•

2.4.2. Taxonomy of modifiers

Preliminary Remark 2.1. The *self-consistent* taxonomy of glossoxenographs (linguistic xenographs), according to which some of them have, in the foregoing discussion, been qualified *limited* and some others *unlimited*, is related to the *self-consistent* taxonomy of the modifiers that are involved or not involved in the xenographs – the taxonomy, according to which some modifiers have been qualified *limiting* and some others *unlimiting*. These taxonomies are explicated in the next definition.•

Df 2.11. 1) In accordance with Df 2.10(7), a *deductive name* (*DdN*), i.e. a *deduction from the genus and differences* (*DdFrG&Ds*), is a *predicate-free proper graphic name*, being *self-explanatory* in the sense that *it denotes*, i.e. *puts forward as its intentional* (*intended*) *value, the entity that it deduces*. Accordingly, the entity that is deduced by a DdN is called the *denotatum of the DdN*, unless the latter is used *obliquely*. The name of a, or the, difference, which occurs in the DdN, is called a *non-predicate modifier* (*NPM*) – as opposed to a *predicate*, which is alternatively called a *predicate modifier* (*PM*). A name (linguistic form) that involves a predicate as its principal modifier is called a *sentence* or *clause*. An NPM can be one of the two kinds: an *attributive modifier* (*AM*), denoting an *attributive difference* (*AD*), or an *inflectional modifier* (*IM*), denoting an *inflectional difference* (*ID*). A PM denotes a *predicate difference* (*PD*), i.e. a *semantic predicate.*

2) A PM is by definition called a *limiting modifier,* i.e. it is a *limiting predicate modifier* (*LtgPM*) and vice versa. An IM is by definition called an *unlimiting modifier*, i.e. it is an *unlimiting inflectional modifier* (*UnLtgIM*) and vice versa. An AM can be either a *limiting attributive modifier* (*LtgAM*) or an *unlimiting* or *descriptive attributive modifier* (*UnLtgAM* or *DcAM*), called also a *descriptive*, or *qualitative, modifier* (*DcM* or *QlM*) or briefly a *qualifier*. By "*limiting modifier*"

(“*LtgM*”), I shall hereafter understand “*limiting attributive modifier*” (“*LtgAM*”), because a predicate modifier (predicate) cannot by definition be an unlimiting one.

3) A prefix or a combining form other than the root, of an onymological or onological term is a *qualifier,* i.e. *unlimiting attributive modifier,* either, most often, of the root or, occasionally, of another combining form of the term. Depending on a term, in which such a qualifier occurs, it is called either an *onymological qualifier* or an *onological qualifier*.

4) The English *limiting modifiers* (*limiting attributive modifiers*) can be divided into the following classes:

a) *quantifying modifiers*, called also *quantifiers*, that can be *numeral* (*numerical*), *mass*, or *logical* (*universal* or *existential*) ones;

b) *indicative modifiers* of the following subclasses:

i) *the possessive pronouns*, either of *the first form*: “my”, “his”, “her”, “its” “our”, “your”, and “their” or of *the second form*: “mine”, “his”, “hers”, “its”, “ours”, “yours”, and “theirs”;

ii) *the demonstrative pronouns*: “this”, “that”, these”, “those”, and “such”;

iii) *the relative pronouns*: “who” (in the nominative case), “whom” (in the objective case), “whose” (in the possessive case), “which”, and “that”;

iv) nouns in the possessive case;

v) the definite article: “the”;

c) the *indefinite article*: “a” or “an”.

A quantifier denotes a *quantity* – a *numerical*, *mass*, or *logical* one, called also a *quantitative difference* (briefly, *QnD*). The demonstrative and relative pronouns and the articles are *function words*, whereas the descriptive modifiers and all other limiting modifiers are *semantemes*, i.e. *full* (*notional*) *words*. An NL other than English may have no parasynonyms of some of the above limiting modifiers; particularly, it may not have either n indefinite article (e.g. Greek or Hebrew) or both a definite and indefinite article (e.g. Latin or Russian).

5) In this exposition, any occurrence of the [English] indefinite article before a substantive is called a *primary common* (or *general*) *projector*; an occurrence of the definite article replacing an occurrence of the indefinite article is called a *secondary common* (or *general*) *projector*; any occurrence of the definite article other than one of a secondary common projector is called a *proper projector*. A primary or

secondary common (general) projector is indiscriminately called a *common* (*general*) *projector*, whereas a common or proper projector is indiscriminately called a *projector*. Every indicative modifier other than the definite article, which can, however, be replaced with the definite article without altering the meaning of the host modified name, is qualified as a projector of the same kind as the substituend.

6) Given a *graphic name* (graphic linguistic form, graphoglossonym, glossographonym), the modifier, which is either the only one occurring in the name or the one of two or more modifiers occurring in the name that is identified as executed (applied to a certain constituent name) in the last place (as the pertinent indicative modifier or article or quantifier or predicate), is called *the principal modifier of the name.*

7) A *graphic name* (*xenograph*) is said to be:

a) *unmodified* or *unrestricted* if it has no modifier;

b) *modified* if it has at least one modifier,

8) A *modified graphic name* is said to be:

a) a *limited* name if its principal modifier is limiting;

b) an *unlimited*, or *descriptive*, or *qualified*, *name*, if its principal modifier is a descriptive one; i.e. a qualifier;

9) A *limited name* is said to be:

a) a *quantified name* if its principal modifier is a quantifier;

b) an *indicative name* if its principal modifier is an indicator;

c) a *commonly projected name* if its principal modifier is a common projector;

d) a *properly projected name* if its principal modifier is a proper projector;

e) a *projected name* if its principal modifier is a projector, proper or common.

10) A *deductive name* (*DdN*) is said to be:

a) an *attributal name* and also a *descriptive name*, i.e. a *description through a genus and attributive differences*, if its principal modifier is an attributive modifier;

b) a *predicated name*, and also, discriminately, either a *simple declarative sentence* or a *sentential clause*, if its principal modifier is a predicate.

An unmodified name is necessarily an unlimited name but not necessarily vice versa. Also, more specifically, an unmodified name is necessarily a GN that is not a DSN.

11) An NSgN or an ULtdNNSgN is indiscriminately called *an unlimited singular name* (*UnLtdSgN*) or *an unlimited singular xenograph* (*UnLtdSgXG*). A qualifier to an UnLtdSgN can be either a prepositive one, which is usually an *adjective*, or a postpositive one, which is a combination of a *preposition*, being a *function word*, and a certain *semanteme,* i.e. a *full* (*notional*) *word*. An UnLtdSgN along with a qualifier to it is another UnLtdSgN. More specifically, (a) if the former UnLtdSgN is an UnLtdPrMnN then the latter UnLtdSgN is an UnLtdPrCsN, namely it is either another UnLtdPrMnN, i.e. UnLtdPrN of a *narrower* (*less inclusive*) *multipleton*, or an UnLtdPrSnN; (b) if the former UnLtdSgN is an UnLtdPrSnN then the latter UnLtdSgN is either the same UnLtdPrSnN, if the qualifier is *redundant*, or a *contradictio in adjecto*, if otherwise; (c) if the former UnLtdSgN is an UnLtdPrCmsN then the latter UnLtdSgN is another UnLtdPrCmsN, i.e. UnLtdPrN of a *narrower* (*less inclusive*) *cmass*. Accordingly, I regard a qualifier to an UnLtdPrCsN, i.e. to an UnLtdPrMnN or to an UnLtdPrSnN, as an UnLtdPrN of a certain *megaclass* whose intersection with the class designated by that UnLtdPrCsN results in the class designated by the pertinent *descriptive* UnLtdPrCsN. Therefore, a qualifier to an UnLtdPrCsN is alternatively called an *unlimited proper megaclass-name* (*UnLtdPrMgCsN*), i.e. it is another UnLtdPrCsN. Analogously, I regard a qualifier to an UnLtdPrCmsN as an UnLtdPrN of a certain *megacmass* whose intersection with the cmass designated by that UnLtdPrCmsN results in the cmass designated by the pertinent *descriptive* UnLtdPrCmsN. Therefore, a qualifier to an UnLtdPrCmsN is alternatively called an *unlimited proper megacmass-name* (*UnLtdPrMgCmsN*), i.e. it is another UnLtdPrCmsN.

12) A megaclass or a megacmass is indiscriminately called a *megauniversal*. It can happen that a qualifier to an UnLtdPrCsN, which is an UnLtdPrMgCsN¸ and a qualifier to an UnLtdPrCmsN, which is an UnLtdPrMgCmsN¸ are *homographs*. In this case, the two homographs can be regarded as a single qualifier, which designates the *union* of the megaclass designated by one homograph and megacmass designated by the other homograph. This union is also called a *megauniversal*, while the *unified qualifier* designating the megauniversal is called an *unlimited proper megauniversal-name* (*UnLtdPrMgUlN*). The designatum of the unified qualifier (UnLtdPrMgUlN) is automatically restricted to its megaclass item, when the qualifier is applied to an UnLtdPrCsN, and to its megacmass item, when the qualifier is applied to an

UnLtdPrCmsN. For instance, the adjective “*green*” is supposed to designate the megauniversal whose every instance is a *green being* (*green entity*), namely either a *green individual*, such as a green tree, leaf, lizard, crocodile, etc, or a *green mass*, such as green grass, foliage, needles, dye, printer’s ink, water-color, color, chlorophyll, etc. By contrast, the adjective “*adult*”, is supposed to be the *megaclass* (*broadest class*) *of adult* (*fully developed*, *fully mature*) *bionts* (*living organisms*), as animals, plants, or bacteria, sharing the property of being adult.

13) An unlimiting or limiting postpositive qualifier can be treated as a megauniversal as well. For instance, the unlimiting postpositive qualifier “*of wood*” can be regarded as an UnLtdPrMgCsN that designates the megaclass whose every instance is a *being* [*made*] *of wood* such as a basket, box, construction, house, plane, plug, spoon, etc, *of wood*. Analogously, the limiting postpositive qualifier “*of mine*” is a LtdPrMgCsN that designates the megaclass whose every instance is a *being of mine*, e.g. a bicycle, book, computer, cup, etc, *of mine*.

14) By Df 2.9(2), is an unlimited NPlN (UnLtdPlN) that serves as a dimension of the dimensionless numeral used as its limititing attributive modifier (LtgAM). Accordingly, a dimensionless numeral limiting an NPlN can be regarded as semanteme that designates a *megaclass* (*broadest class*) *of all its dimensional instances and of all its dimensionless predecessors*. For instance, the megaclass designated by the dimensionless numeral “*five*” or “5” is supposed to be the *broadest class of beings*, whose instances are *dimensional natural numbers* such as 5 men, 5 trees, 5 books, etc and also the dimensionless natural numbers from 0 to 4, which it possesses both as its members and as its parts, in accordance with the conventional definition of the successor n+1 of a natural number n: $n+1=n\cup\{n\}$, subject to $0=\varnothing$, so that $1=\varnothing\cup\{\varnothing\}=\{\varnothing\}=\{0\}$, $2=1\cup\{1\}=\{0\}\cup\{1\}=\{0,1\}$, $3=2\cup\{2\}=\{0,1\}\cup\{2\}=\{0,1,2\}$, etc., $n+1=\{0,1,\ldots,n\}$.•

2.4.3. Nominalistic induction of classes and cmasses (concept-masses) and the basic hypostases of an UnLtdSgNlN

Preliminary Remark 2.2. 1) In order to deduce the species (specific class or specific mass) that is denoted and described by a DSN, one should know the genus (generic class or generic mass) denoted by the GN of the DSN and also the differences denoted by the qualifiers of the DSN. The GN itself can be a DSN. Therefore, a system of DSN’s is effective, if the *denotata* of all pertinent *primary*

GN's are known. A primary GN is a *primary* UnLtdSgNlN (UnLtdSgNlXG), so that the qualifier "primary" is, by Df 2.9(2), used as a synonym of "induced" and as an antonym of "deduced", being in turn a synonym of "secondary". In accordance with Dfs 2.8(1) and 2.9(1–8), an UnLtdSgNlN is either an UnLtdPrMnN (unlimited proper multipleton-name) or an UnLtdPrSnN (unlimited proper singleton-name) or else an UnLtdPrCmsN (unlimited proper cmass-name), each of which can be either a primary (induced) or secondary (deduced). However, by Df 2.10(2), if the GN of a DSN is an UnLtdPrSnN then all qualifiers of the DSN are redundant, so that the denotatum of the DSN is the singleton-denotatum of GN. Such trivial cases are disregarded.

2) To *induce* the denotatum of a primary UnLtdSgNlN and hence to induce the primary UnLtdSgNlN itself means to obtain the denotatum as the pertinent *universal*, *essential* or *accidental*, in the result of reasoning to it from the pertinent *particulars*, *essential* or *accidental*, – reasoning that can be called a *nominalistic induction* as opposed to a *mathematical induction*. Incidentally, the *four-fold classification* of beings into *essential universals*, *accidental universals*, *accidental particulars*, and *essential* (*non-accidental*) *particulars*, called also *nonempty individuals*, is equivalent to the *original four-fold classification of beings by Aristotle* [350 BCE, *Categories*, ACE]), who called essential (non-accidental) particulars "*primary substances*". Df 2.8 is in fact a description of a general process of induction of a singleton from an accidental or essential particular. An analogous process of induction of a multipleton, or of a cmass, from distinct accidental or essential particulars as the pertinent initial elements, or correspondingly parts, will be described Dfs 2.12(1) and 2.13(1) respectively.

3) In contrast to the denotataum of a primary UnLtdSgNlN, the denotatum of a *secondary* UnLtdSgNlN (as that of a DSN) is established by the pertinent *mental process of deduction*, which is called the *sense of the secondary UnLtdSgNlN* (particularly the *sense of the DSN*). However, once the denotatum of a secondary UnLtdSgNlN is established via its sense, it can be used in communication or for one's own purposes in the same way as a like primary UnLtdSgNlN. Therefore, in the following two definitions, I shall describe I shall also describe the hypostases of, i.e. the mental modes of using, an NSgN (UnLtdNSgN) and an UnLtdNNSgN, independent of whether they are primary or secondary. The notion of sense of a

secondary xenograph and particularly the notion of sense of a DSN will be discussed in the next two sub-subsections.•

Df 2.12. 1) In accordance with Df 2.9(1), in order to coin (create, contrive) a *primary*, or *induced*, *NSgXG* (*PrmNSgXG* or *IdNSgXG*), called also a *primary*, or *induced*, *NSgN* (*PrmNSgN* or *IdNSgN*), I associate its *prototypal autographic isotoken* as a *prospective xenographic referent*, with *every one* of the two or more (usually of an infinite number of) *distinct* (*distinguishable*) physical (real) or psychical (mental, ideal) entities, as its *prospective distributive related values* (*prospective relata*), which differ from tokens and token-classes of the autograph, but which have a certain *property in common*. *Ipso facto,* i.e. by the very fact, of providing them with the same PrmNSgN, *all prospective distributive values of the PrmNSgN are prescinded from* (*detached of*) *their mutual differentia and become conceptually indistinguishable members of a certain collective xenonymous value of the PrmNSgN* – a value, which is, in accordance with Df 2.9(3), called the *class* [*of equivalence*], or *multipleton, of those values* and also the *class-designatum*, or *multipleton-designatum*, *of the PrmNSgN*. Consequently, the PrmNSgN cannot be used *directly* for mentioning (denoting, referring to) any initial distinct entity *distributively*; it can only be used either *obliquely* for mentioning the entire class-designatum *undistributively* as a single whole or it can be used *directly* for mentioning the same class *as if distributively* in the hypostasis of *its common* (*general*) *member*. The former, *oblique homograph of the PrmNSgN* is called a *primary proper multipleton-name* (*PrmPrMnN*) and also synecdochically (more generally) a *primary proper class-name* (*PrmPrCsN*) or onymologically a *primary kyrioclassoxenograph*, – in accordance with Df 2.8(1) and in analogy with "kyrioautograph" (see Df 1.23(2) and Cmt 1.27(2,4)). The latter, *direct homograph of the PrmNSgN* is called a *primary name of a common member of the multipleton* or briefly a *primary common-member name* (*PrmCmnMrN*) and also onymologically a *primary cenomeloxenograph*, – in analogy with "cenoautograph" (see Df 1.23(2) and Cmt 1.27(3,5), and see also Dict A1.1 for "*melo*").

2) In accordance with Df 2.9(1–5), the above terminology applies, *mutatis mutandis*, in the general case, where the qualifier "primary" (Prm") is omitted. In principle, that qualifier should have been replaced with the qualifier "unlimited" ("UnLtd"). However, the latter qualifier is implied by the qualifier "numeralable" ("N") or by its synonym "count" ("Ct"), so that it is redundant in the presence (but

only in the presence) of either of the two last quantifiers in any pertinent term as “numeralable singular name” (”NSgN”) or “count singular name” (”NSgN”). Thus, like members of the multipleton-designatum of a PrmNSgN, members of the multipleton-designatum of an NSgN are *conceptually indistinguishable*, because they are prescinded from their differences. Consequently, just as a PrmNSgN, an NSgN cannot be used *directly* for mentioning (denoting, referring to) *initial* distinct prototypes of indistinguishable members of the multipleton-designatum of the NSgN *distributively*; the NSgN can only be used either *obliquely* for mentioning the entire multipleton-designatum *undistributively* as a single whole or it can be used *directly* for mentioning the same multipleton *as if distributively* in the hypostasis of *its common* (*general*) *member*. The former, *oblique homograph of the NSgN* is called an *unlimited proper multipleton-name* (*UnLtdPrMnN*) and also synecdochically (more generally) an *unlimited proper class-name* (*UnLtdPrCsN*) or onymologically an *unlimited kyrioclassoxenograph*. The latter, *direct homograph of the NSgN* is called an *unlimited common-member name* (*UnLtdCmnMrN*) or onymologically an *unlimited cenomeloxenograph*.

3) In English, the two homographs of an NSgN can be distinguished formally by replacing the *direct homograph of the NSgN*, i.e. the UnLtdCmnMrN, with the respective *limited name of a common member*, which is alternatively called a *limited common-member name* (*LtdCmnMrN*) or onymologically a *limited cenomeloxenograph* and which comprises the NSgN and the prepositive indefinite article “a” or “an” as the *pertinent added word*. In this case, the former, oblique homograph of the NSgN, which has been called an unlimited proper multipleton-name (UnLtdPrMnN) and also synecdochically an unlimited proper class-name (UnLtdPrCsN) or an unlimited kyrioclassoxenograph, remains unaltered. In any WNL that has no indefinite article, the two homographs of an NSgN can be distinguished formally by replacing the *oblique homograph of the NSgN* with its enneoxenographic quotation (EXQ), which is formed by enclosing an isotoken of the NSgN between the EXQ marks \ /. In this case, the *direct homograph of the NSgN* remains unaltered and is as before called an unlimited common-member name (UnLtdCmnMrN) or an unlimited cenomeloxenograph. The above EXQ method applies in English as well. Moreover, in this case the EXQ of a *limited cenomeloxenograph* and the EXQ of the pertinent *unlimited cenomeloxenograph*, and also the pertinent *homographic unlimited*

kyrioclassoxenograph designate the same class-designatum. Also, it is understood that, in the framework of any WNL, in order to switch from using an NSgN as an unlimited kyrioclassoxenograph (e.g.) to using it as a homolographic unlimited cenomeloxenograph or vice versa, the interpreter should just correspondingly change his mental attitude towards the NSgN. Therefore, an NSgN can be used as either one of its homographs equivocally if its hypostasis is obviously understood from the context, in which it occurs. The above two mental modes of using an NSgN are theoretically substantiated in the following two items. A further discussion of these modes and examples illustrating them will be given in Cmt 2.7 below in this subsection.

4) In what follows, an *unlimited cenomeloxenograph* or a *limited cenomeloxenograph* is indiscriminately called a *cenomeloxenograph*. Accordingly, the whole of the following discussion including the pertinent definitions, applies to an NSgN of any given WNL (particularly of English) – an NSgN, which I use in communication or for my own purposes as an *unlimited cenomeloxenograph*, and in English it also applies verbatim to a LtdNSgN, which comprises an NSgN and the prepositive indefinite article and which I use likewise as a *limited cenomeloxenograph*. The following two points a and b, and also the points a and b of the next item 5 are similar to the points a and b of the item 2 of Df 2.8.

a) A *cenomeloxenograph* that serves as a referent to a *common* (*general*, *abstract*, *certain*, *particular but not particularized*) *member* (*element*) *of its multipleton-designatum* (*class-designatum*) as the relatum is said to *denote the common member* and to *connote the multipleton-designatum*. Conversely, the common member is said to *be denoted*, while the multipleton-designatum is said to *be connoted*, *by* the cenomeloxenograph. Accordingly, the common member is called the *denotatum* (*denotation value*, pl. "*denotata*"), while the multipleton-designatum is called the *multipleton-connotatum* (*multipleton-connotatium value*, pl. "*multipleton-connotata*") or *class-connotatum*, of the cenomeloxenograph. In this case, I *use* the cenomeloxenograph along with its multipleton-connotatum (class-connotatum) *for mentioning* (*denoting*, *putting forward*) its denotatum, while both the cenomeloxenograph and its multipleton-connotatum *are used but not mentioned*. The whole of the above mental phenomenon of using the cenomeloxenograph can be explicated as follows.

b) The multipleton-designatum of a cenomeloxenograph is a *mental* (*psychical*) *entity* of mine. However, most often but not always, I use the latter in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* the multipleton-designatum as my *as if extramental* (*exopsychical*) *object* (other than the cenomeloxengraph itself) – the very one that I call a *common* (*general*, etc) *member of the class-designatum.* I do so *habitually* and hence *involuntarily but consciously* – just as I most often but not always use a cenoautograph (see Cmt 1.27(5)) and just as I *always* use the *percept* (*sensation*) of any nym (sensum, sensory object) and particularly that of the cenomeloxenograph itself (see Df 1.12(1)). In this case, the common member of the multipleton-designatum is put forward as the intended import value of the cenomeloxenograph and is therefore called its *denotatum*, while the multipleton-designatum is *as if* put backward and is therefore called the multipleton-*connotatum* of the cenomeloxenograph. At the same time, the multipleton-connotatum contains its common member *as if, and not actually*. Accordingly, the denotatum of the cenomeloxenograph, being the common member of its multipleton-connotatum, is in fact, to use the appropriate *monistic phraseology, another hypostasis* (*way of existence*, *aspect*) of its multipleton-designatum. In order to describe this mental phenomenon in the appropriate alternative *dualistic phraseology,* I say that the common member of the class-designatum of the cenomeloxenograph *represents the whole multipleton-designatum*, so that the former is the denotatum of the cenomeloxenograph, and the latter is the multipleton-connotatum of the cenomeloxenograph. In accordance with the above-said, an unlimited or limited cenomeloxenograph should be regarded as a *proper* (*not common*) *name of a common member of its multipleton-connotatum* (*multipleton-designatum*).

5. a) In analogy with the item 4a, if I use an NSgN of any given WNL (particularly of English) in communication or for my own purposes as an *unlimited kyrioclassoxenograph*, enneoxenographically quoted or not, i.e. as a referent to its multipleton-designatum (class-designatum) as its relatum, by mentally putting the latter forward as the intended value of the NSgN then I say that the multipleton-designatum is *denoted by* the kyrioclassoxenograph (NSgN) and that hence it is the *denotatum,* or more precisely *multipleton-denotatum*, or *class-denotatum, of the kyrioclassoxenograph* (*NSgN*) [with respect to me]. In accordance with Cmt 1.27(1),

the multipleton-denotatum of a kyrioclassoxenograph is an *object sui generis* and therefore, *ipso facto*, it automatically produces the *singleton of its own*, which becomes another value of the kyrioclassoxenograph – the value that is said to be *connoted* by the latter and that is accordingly called the *singleton-connotatum* of the kyrioclassoxenograph. In this case, I *use* the kyrioclassoxenograph along with its singleton-connotatum *for mentioning* (*denoting*, *putting forward*) its multipleton-denotatum, while both the kyrioclassoxenograph and its singleton-connotatum *are used but not mentioned*. The above mental phenomenon of using a kyrioclassoxenograph is analogous to that of using a cenomeloxenograph as described in the item 4a and therefore it can be explicated in analogy with the matter of the item 4b as follows.

b) The singleton-connotatum of a kyrioclassoxenograph is a *mental* (*psychical*) *entity* of mine. However, just as in the case of a kyrioautograph (cf. Df 1.12(1) and Cmt 1.27(2,4)), when I *habitually* and hence *involuntarily but consciously* use a kyrioclassoxenograph for mentioning (denoting, putting forward) its multipleton-denotatum, I use its singleton-connotatum in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* the singleton-connotatum as its member, i.e. as the multipleton-denotatum of the kyrioclassoxengraph, *in the hypostasis of my as if extramental* (*exopsychical*) *object* (other than the kyrioclassoxengraph itself). In this way, the multipleton-member of the singleton-connotatum is put forward as the intended import value, i.e. as the denotatum (meaning), of the kyrioclassoxenograph, while the singleton-connotatum is *as if* put backward, – in agreement with the pertinent mental status of the NSgN as kyrioclassoxenograph. In fact, however, the singleton-connotatum *is*, to use the appropriate *monistic phraseology*, *mentally transduced into another hypostasis* (*way of existence*, *aspect*) in the form of its only member. In order to describe this mental phenomenon in the appropriate alternative *dualistic phraseology*, I say that the multipleton-member of the singleton-connotatum of the kyrioclassoxenograph *represents* the singleton-connotatum, so that the two entities as if coexist as a single biune entity.

6) A *numeralable* (*count*) *noun* that is used or is supposed to be used as an unlimited cenomeloxenograph is conventionally called a *common noun* (see e.g. WNCD or WTNID). In the general case, it seems therefore to be natural to call an

unlimited cenomeloxenograph "*a common name*" by analogy. However, after all, both names "common noun" and "common name" turn out to be misnomers (contradictory ones) for the following reason. In accordance with the above items 4 and 5, a common member denoted by an *unlimited cenomeloxenograph* (and also in English by a cenomeloxenograph limited by the indefinite article) is just another hypostasis of the *multipleton* (*multitudinous class*) denoted by the *homographic unlimited kyrioclassoxenograph*. Since the kyrioclassoxenograph is a *proper* name of the multipleton, it is therefore natural to regard the cenomeloxenohraph as a *proper* name of the common member of the multipleton. When an unlimited cenomeloxenohraph is called "common noun" or in general "common name", the understanding is that the former is used as a *common* name of every *initial distinct prototype of the common member of the multipleton*. However, in order to mention a concrete initial prototype of the above common member, the latter should be provided with the respective differences. In order to do this explicitly, the unlimited cenomeloxenohraph should be supplemented with qualifiers denoting those differences, and therefore it i*s altered*. For instance, the unlimited xenograph "man" is a homograph that denotes the species (specific class) *man*, i.e. *Homo sapiens,* and that equivocally denotes a common member of the species, i.e. *a* man (to use the limited xenograph "a man" for more clarity) or *homo* (Latin has no articles). At the same time, the unlimited xenograph "man founded logic" is a homograph, one value of which is the singleton \Aristotle/ or {Aristotle} in the conventional notation, whereas its other value is Aristotle himself, i.e. *the* man founded logic (to use this limited proper name for clarity). *Man* (or *the man*) *founded logic* is one of the initial distinct prototypes of the common member *man* (or *a man*) of the species *man*, but it has the respective proper name of its own, namely "man founded logic", and not the noun "man" alone.

7) In what follows I summarize and generalize the most fundamental names of names that have been introduced earlier in Dfs 2.8, 2.9, and 2.12.

a) An *unlimited proper individual name* (*UnLtdPrIlN*), an *unlimited proper multipleton-name* (*UnLtdPrMnN*), an *unlimited proper singleton-name* (*UnLtdPrSnN*), and an *unlimited proper cmass name* (*UnLtdPrCmsN*) are *some unlimited kyrioxenographs*. An unlimited kyrioxenograph along with the appropriate prepositive *limiting modifier* (as the definite article in English) is called a *limited kyrioxenograph* or *limited proper* [*graphic*] *name*. In accordance with Df 2.8(2e), an

UnLtdPrMnN or an UnLtdPrSnN is indiscriminately called an *unlimited proper class-name* (*UnLtdPrCsN*) and also an *unlimited kyrioclassoxenograph*, whereas a *proper multipleton-name* (*PrMnN*), a *proper singleton-name* (*PrSnN*), or a *proper name of the empty class* is indiscriminately called an *proper class-name* (*PrCsN*) and also a *kyrioclassoxenograph*. A *proper cmas name* (*PrCmsN*) is alternatively called a *kyriomazoxenograph*.

b) A kyrioxenograph can be either a *primary*, or *induced*, one or a *secondary*, or *deduced*, one.

c) A *name of a common member of the multipleton* is briefly called a *common-member name* (*CmnMrN*) and also onymologically a *cenomeloxenograph*. A CmnMrN is either an *unlimited one* (*UnLtdCmnMrN*) or a *limited one* (*LtdCmnMrN*).•

Df 2.13. In accordance with the points a and b of Df 2.12(7), Df 2.12(1–5) applies, *mutatis mutandis*, with "NNSg" ('non-numeralable singular") in place of "NSg" ('numeralable singular"), with "Cms" ("cmass") in place of both "Mn" ("multipleton") and "Cs" ("class"), and with "part" ("Pt") in place or "member" ("Mr") and accordingly with "ceno*mero*xenograph" in place of "ceno*mazro*xenograph" (see Dict A1.1). In Df 2.12, in passing from the item 1 to the items 2–5, the qualifier "Prm" ("primary") in the abbreviation "PrmNSgN" (e.g.) from the

1) In order to coin (create, contrive) a *primary*, or *induced*, *NNSgXG* (*PrmNNSgXG* or *IdNNSgXG*), called also a *primary*, or *induced*, *NNSgN* (*PrmNNSgN* or *IdNNSgN*), I associate its prototypal *autographic isotoken* as a *prospective xenographic referent*, with *every one* of an indefinite number of *distinct* (*distinguishable*) physical (real) entities of indefinite sizes and shapes of their own or of distinct sizeless and shapeless psychical (mental, ideal) entities, as its *prospective distributive related values* (*prospective relata*), which differ therefore from tokens and token-classes of the autograph, but which have a certain *property in common*. *Ipso facto*, i.e. by the very fact, of providing them with the same NNSgN, *all prospective distributive values of the NNSgN are prescinded from* (*detached of*) *their mutual differentia and become conceptually indistinguishable parts of a certain conceptual xenonymous value of the NNSgNN* – a value, which is, in accordance with Df 2.9(7), called the *concept-mass* (*concept mass*), or *cmass*, *of those parts* and also the *cmass-*

designatum of the PrmNNSgN. Consequently, the PrmNNSgN cannot be used *directly* for mentioning (denoting, referring to) any initial distinct entity *distributively*; it can only be used either *obliquely* for mentioning the entire cmass-designatum *undistributively* as a single whole or it can be used *directly* for mentioning the same cmass *as if distributively* in the hypostasis of *its common* (*general*) *part*. The former, *oblique homograph of the PrmNNSgN* is called a *primary proper cmass-name* (*PrmPrCmsN*) or onymologically a *primary kyriomazooxenograph*, – in accordance with Df 2.8(1) and in analogy with "kyrioautograph" (see Df 1.23(2) and Cmt 1.27(2,4)). The latter, *direct homograph of the PrmNNSgN* is called a *primary name of a common part of the cmass* or briefly a *primary common-part name* (*PrmCmnPtN*) and also onymologically a *primary cenomeroxenograph*, – in analogy with "cenoautograph" (see Df 1.23(2) and Cmt 1.27(3,5), and see also Dict A1.1 for "*mero*").

2) In accordance with Df 2.9(6–8), the above terminology applies, *mutatis mutandis*, in the general case, where the qualifier "primary" (Prm") is replaced with "unlimited" ("UnLtd") (cf. the remark regarding this qualifier at the beginning of the item 2 of Df 2.12). In this case, like parts of the cmass-designatum of a PrmNNSgN, parts of the cmass-designatum of an UnLtdNNSgN are *conceptually indistinguishable*, because they are prescinded from their differences. Consequently, just as a PrmNNSgN, an UnLtdNNSgN cannot be used *directly* for mentioning (denoting, referring to) *initial* distinct prototypes of indistinguishable parts of the cmass-designatum of the UnLtdNNSgN *distributively*; the UnLtdNNSgN can only be used either *obliquely* for mentioning the entire cmass-designatum *undistributively* as a single whole or it can be used *directly* for mentioning the same cmass *as if distributively* in the hypostasis of *its common* (*general*) *part*. The former, *oblique homograph of the UnLtdNNSgN* is called an *unlimited proper cmass-name* (*UnLtdPrCmsN*) and also onymologically an *unlimited kyriomazoxenograph*. The latter, *direct homograph of the UnLtdNNSgN* is called an *unlimited common-part name* (*UnLtdCmnPtN*) or onymologically an *unlimited cenomeroxenograph*.

3) In English (or in any other WNL), the two homographs of an UnLtdNNSgN can be distinguished formally by replacing the *direct homograph of the UnLtdNNSgN*, i.e. the UnLtdCmnPtN, with the respective *limited name of a common part*, which is alternatively called a *limited common-part name* (*LtdCmnPtN*) or onymologically a

limited cenomeroxenograph and which comprises the UnLtdNNSgN and the prepositive indefinite adjective "*some*" (or its parosynonym in another WNL) as the *pertinent added word*. In this case, the former, oblique homograph of the UnLtdNNSgN, which has been called an unlimited proper cmass-name (UnLtdPrCmsN) and also an unlimited kyriomazoxenograph, remains unaltered. In any WNL including English, the two homographs of an UnLtdNNSgN can be distinguished formally by replacing the *oblique homograph of the UnLtdNNSgN* with its enneoxenographic quotation (EXQ), which is formed by enclosing an isotoken of the UnLtdNNSgN between the EXQ marks \ /. In this case, the *direct homograph of the UnLtdNNSgN* remains unaltered and is as before called an unlimited common-part name (UnLtdCmnPtN) or an unlimited cenomeroxenograph. The above EXQ method applies in English as well. In this case, the EXQ of a *limited cenomeroxenograph* and the EXQ of the pertinent *unlimited cenomeroxenograph*, and also the pertinent *homographic unlimited kyriomeroxenograph designate the same cmass-designatum*. Also, it is understood that, in the framework of any WNL, in order to switch from using an UnLtdNNSgN as an unlimited kyriomeroxenograph (e.g.) to using it as a homolographic unlimited cenomeroxenograph or vice versa, the interpreter should just correspondingly change his mental attitude towards the UnLtdNNSgN. Therefore, an UnLtdNNSgN can be used as either one of its homographs equivocally if its hypostasis is obviously understood from the context, in which it occurs. The above two mental modes of using an UnLtdNNSgN are theoretically substantiated in the following two items.

4) In what follows, an *unlimited cenomeroxenograph* or a *limited cenomeroxenograph* is indiscriminately called a *cenomeroxenograph*, Accordingly, the whole of the following discussion including the pertinent definitions, applies to both an UnLtdNNSgN of any given WNL (including English) – an UnLtdNNSgN, which I use in communication or for my own purposes as an *unlimited cenomeroxenograph*, and it also applies verbatim to the LtdNNSgN, which comprises an UnLtdNNSgN and the prepositive indefinite adjective "some" or its parosynonym in any other WNLarticle and which I use likewise as a *limited cenomeroxenograph*. The following two points, a and b, and also the points a and b of the next item 5 are similar to the points a and b of the item 2 of Df 2.8.

a) A *cenoxenograph* that serves as a referent to a *common* (*general*, *abstract*, *certain*, *particular but not particularized*) *part of its cmass-designatum* as the relatum is said to *denote the common part* and to *connote the cmass-designatum*. Conversely, the common part is said to *be denoted*, while the cmass-designatum is said to *be connoted*, *by* the cenomeroxenograph. Accordingly, the common part is called the *denotatum* (*denotation value*, pl. "*denotata*"), while the cmass-designatum is called the *cmass-connotatum* (*cmass-connotatium value*, pl. "*cmass-connotata*"), of the cenomeroxenograph. In this case, I *use* the cenomeroxenograph along with its cmass-connotatum (class-connotatum) *for mentioning* (*denoting*, *putting forward*) its denotatum, while both the cenomeroxenograph and its cmass-connotatum *are used but not mentioned*. The whole of the above mental phenomenon of using the cenomeroxenograph can be explicated as follows.

b) The cmass-designatum of a cenomeroxenograph is a *mental* (*psychical*) *entity* of mine. However, most often but not always, I use the latter in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* the cmass-designatum as my *as if extramental* (*exopsychical*) *object* (other than the cenomeroxengraph itself) – the very one that I call a *common* (*general*, etc) *part of the cmass-designatum.* I do so *habitually* and hence *involuntarily but consciously* – just as I most often but not always use a cenoautograph or a cenomeroxenograph and just as I *always* use the *percept* (*sensation*) of any nym (sensum, sensory object) and particularly that of the cenomeroxenograph itself (see Df 1.12(1)). In this case, the common part of the cmass-designatum is put forward as the intended import value of the cenomeroxenograph and is therefore called its *denotatum*, while the cmass-designatum is *as if* put backward and is therefore called the cmass-*connotatum* of the cenomeroxenograph. At the same time, the cmass-connotatum contains its common part *as if, and not actually*. Accordingly, the denotatum of the cenomeroxenograph, being the common part of its cmass-connotatum, is in fact, to use the appropriate *monistic phraseology*, *another hypostasis* (*way of existence*, *aspect*) of its cmass-designatum. In order to describe this mental phenomenon in the appropriate alternative *dualistic phraseology,* I say that the common part of the cmass-designatum of the cenomeroxenograph *represents the whole cmass-designatum*, so that the former is the denotatum of the cenomeroxenograph, and the latter is the cmass-connotatum of the cenomeroxenograph. In accordance with the above-said, an unlimited or limited cenomeroxenograph should be regarded as a *proper* (*not common*) *name of a common part of its cmass-connotatum* (*cmass-designatum*).

5. a) In analogy with the item 4a, if I use an UnLtdNNSgN of any given WNL (particularly of English) in communication or for my own purposes as an *unlimited kyriomazoxenograph*, enneoxenographically quoted or not, i.e. as a referent to its cmass-designatum as its relatum, by mentally putting the latter forward as the intended value of the UnLtdNNSgN then I say that the cmass-designatum is *denoted by* the kyriomazoxenograph (UnLtdNNSgN) and that hence it is the *denotatum*, or more precisely *cmass-denotatum of the kyriomazoxenograph* (*UnLtdNNSgN*) [with respect to me]. In accordance with Cmt 1.27(1), the cmass-denotatum of a kyriomazoxenograph is an *object sui generis* and therefore, *ipso facto*, it automatically produces the *singleton of its own*, which becomes another value of the kyriomazoxenograph – the value that is said to be *connoted* by the latter and that is accordingly called the *singleton-connotatum* of the kyriomazoxenograph. In this case, I *use* the kyriomazoxenograph along with its singleton-connotatum *for mentioning* (*denoting*, *putting forward*) its cmass-denotatum, while both the kyriomazoxenograph and its singleton-connotatum *are used but not mentioned*. The above mental phenomenon of using a kyriomazoxenograph is analogous to that of using a cenomeroxenograph as described in the item 4a and therefore it can be explicated in analogy with the matter of the item 4b as follows.

b) The singleton-connotatum of a kyriomazoxenograph is a *mental* (*psychical*) *entity* of mine. However, just as in the case of a kyrioautograph (cf. Df 1.12(1) and Cmt 1.27(2,4)), when I *habitually* and hence *involuntarily but consciously* use a kyriomazoxenograph for mentioning (denoting, putting forward) its cmass-denotatum, I use its singleton-connotatum in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* the singleton-connotatum as its member, i.e. as the cmass-denotatum of the kyriomazoxengraph, *in the hypostasis of my as if extramental* (*exopsychical*) *object* (other than the kyriomazoxengraph itself). In this way, the cmass-member of the singleton-connotatum is put forward as the intended import value, i.e. as the denotatum (meaning), of the kyriomazoxenograph, while the singleton-connotatum is *as if* put backward, – in agreement with the pertinent mental status of the UnLtdNNSgN as kyriomazoxenograph. In fact, however, the singleton-connotatum *is*, to use the appropriate *monistic phraseology*, *mentally transduced into anfother hypostasis* (*way of existence*, *aspect*) in the form of its only member. In order to describe this mental phenomenon in the appropriate alternative *dualistic phraseology*, I say that the cmass-member of the singleton-connotatum of the kyriomazoxenograph *represents* the singleton-connotatum, so that the two entities as if coexist as a single biune entity.•

Cmt 2.7. There is a doctrine due to Mill [1843] (mentioned in Church [1956, footnotes 6, 14, and 16]), according to which, not only a proper, or singular (in the terminology of Mill), name but also a common, or general, name has the property *to denote*, with the difference that the former denotes only one object, whereas the latter denotes many *distinct* objects *distributively* and *simultaneously*. In accordance with Mill's doctrine, the *indefinite-articled and hence limited common name* "a man", e.g., is said *to denote* Aristotle, Shakespeare, Einstein, etc and also a green-grocer, pedestrian, cyclist, etc, each taken individually and simultaneously. In Df 2.12, I have suggested a completely different interpretation of a numeralable (count) common nounal name that is mentioned by using the noun "*cenomeloxenograph*". In Df 2.13, that interpretation is generalized to a non-numeralable (mass) common nounal name that is mentioned by using the noun "*cenomeroxenograph*". My interpretation of numeralable (count) common names is illustrated below by an example of the name "man", which I have already occasionally mentioned in Df 2.12(6).

I have elaborated the class *man*, which is denoted by the [unlimited] NSgN (numeralable singular name) "man" by the pertinent informal act of reasoning that is indiscriminately called an *induction*, as opposed to a *deduction*, or more specifically a *nominalistic induction*, as opposed to a *mathematical induction*. Nominalistic induction of the class man can be thought of as the operation of providing some selected distinct individuals of the species *Homo sapiens*, alive or dead, with the same NSgN such as "man" in English, "άνθρωπος" \ánthropos\ in Greek, "אדם" \adam\ in Hebrew, "hŏmo" in Latin, or "человек" \chelovek\ in Russian, although in the absence of any pertinent ostended individuals this reasoning can be criticized for being a covert vicious circle. Such an operation is a *surjection*, i.e. a *surjective* (*many-to-one onto*) *mapping*, from the group of selected men onto the singleton of any one of the above names. In the result of each of these mappings, all selected distinct men are placed in one and the same class irrespectively of their mutual differences.

Consequently, the distinct men are devoid of their mutual differences. They preserve only the properties, which they have in common, and therefore they become indistinguishable members of *the class [of equivalence]* designated by the word “man” or by any one of their parasynonyms in other NL’s (native languages) as those mentioned above. Hence, the word “man” (e.g.) cannot be used *directly* for mentioning (denoting, referring to) any initial distinct man *distributively*; it can only be used either *obliquely* for mentioning the entire class *man unditributively* or it can be used *directly* for mentioning the same class *as if distributively in the hypostasis of its common member*. In English, in order to distinguish between the two *homographs* of the word “man”, the latter one is replaced with the *limited common member-name* (*limited cenomeloxenograph*) “a man”. However, Greek and Hebrew have no indefinite articles, whereas Latin and Russian have no articles at all. Therefore, in any one of these languages, equivocal use of the pertinent parasynonym of the word “man” is unavoidable. The only way to distinguish clearly between direct and oblique uses of an NSgN in such a language is to indicate an occurrence of the name that is used obliquely by enclosing it between some special quotation marks as \ /, which are employed in this exposition and which do not belong to any WNL. Thus, in Latin e.g., \hŏmo/ *is* the same species as *Homo sapiens.* A like device applies both to unlimited proper individual-names and to unlimited proper cmass-names in any WNL, having or not having an indefinite article. For instance, \Aristotle/, or {Aristotle} in the conventional notation, is the singleton of Aristotle, i.e. the singleton of the known man who carries the biographical name “Aristotle”. At the same time, \water/ (e.g.) is the cmass (concept-mass) water, while water or some water is an instance of that concept as or as if projected into the real world.

In accordance with the above-said, I cannot intelligibly mention (refer to) a concrete man, say Aristotle again, as my conceptual object, by using either name “man” or “a man” alone. In order to mention Aristotle, I should supplement a common member *a man* of the species *man* (*Homo sapiens*), which *is denoted* by the unlimited and unqualified proper class-name “man”, with a certain *differentia* – a characteristic property, by which I distinguish Aristotle from any other man. Formally, such a differentia can be supplemented to a man by adding some pertinent qualifiers (qualifying words) to the unlimited name “man”, so as to produce the corresponding unlimited DSN (descriptive specific name), e.g. “man who founded logic”, “man who founded biology”, “man who founded philosophy of nominalism”, “man who carries the biographical name “Aristotle””, etc, each of which uniquely determines Aristotle with the following proviso. Each of the above DSN’s can be used (but not mentioned) either directly for mentioning (referring to, denoting) Aristotle or obliquely for mentioning (referring to, denoting) the singleton \Aristotle/, which is a species sensu lato (specific class, strict subclass) of the class *man*. In order to resolve this ambiguity, the tokens (occurrences) of the DSN’s, which are used directly, are conventionally and habitually adhered with *the definite article* to become the *proper member-name*s “the man who founded logic”, “the man who founded biology”, etc, while the tokens of the DSN’s, which are used obliquely, remain unaltered. In this case, “the” plays the same role as that of “a” or “an” in the case of common member-names (as “a man”). In an article-free WNL, such a metamorphosis of a descriptive name of a singleton into a proper name of the member of the singleton is impossible. Therefore, use of certain paired punctuation marks, as the EXQ marks \ / or, conventionally, the braces { }, is the only way to indicate oblique use of a DSN designating a singleton.

A limited essential *common generic* member-name, e.g. “a man”, is qualified so, not because it is supposed to denote many distinct members of the species (specific class) man distributively and simultaneously, but because it can be used together with some added words (qualifiers), i.e. be included in the appropriate context, so as to *deduce* either another, less inclusive, limited essential *common specific* name, e.g. “an adult man”, “a male man” (“a he-man”), “a female man” (“a she-man”, “a woman”), etc, or an *accidental* (*circumstantial*, *as if*) *proper member-name*, e.g. “the man as mentioned above”, “the man crossing the street”, etc, or else a limited *quiditative* (*ultimate*, *essential proper*) individual-name as any one of the synonyms of “Atistotle” indicated in the previous paragraph. Any specification or individuation of the proper name “a man” is necessary accompanied by attaching *a man*, being its common member, with the pertinent differentiae, physical (perceptual) or psychical (mental), named or not. At the same time, added words do not change the object denoted by a personal proper name, unless the entire expression comprising the personal proper name and certain added words is a *contradictio in adjecto*. For instance, either of the appositions: “the philosopher Aristotle” and “Aristotle, the founder of logic” denotes the same conceptual object [of mine] as the personal proper name “Aristotle” alone, while the expression “the Greek ancient military leader Aristotle”, e.g., has no denotatum. Incidentally, the personal names as “John” or “Mary” are *common* personal names, which are converted into accidental (circumstantial) *proper* personal names by supplementing them with the appropriate differentiae, named or not. Even an outstanding personal name, as “George Washington” or “Abraham Lincoln”, can be used, not as a biographical name of the pertinent historical person, but as its homonym every time where there is a namesake of that person.

The above discussion can be recapitulated as follows.

1) The fact that there is an indefinite article in English allows immediately distinguishing between the class-species (specific class) *man* that is denoted by the homograph of the UnLtdNSgN “man”, which is used *obliquely*, and a common (general) member of that class-species that is denoted by the homograph of that same name, which is used *directly*, by replacing the latter homograph with the LtdNSgN “a man”. Accordingly, the occurrence of “a” in the name “a man” can be regarded as an indication (index) that the name “man” is used *directly*, whereas the absence of “a” in the name “man” can be regarded as an indication that this name is used *obliquely*. At the same time, there are NL’s, e.g. Greek and Hebrew, that have no indefinite article, and there are also NL’s, e.g. Latin and Russian, that have no articles at all. In any indefinite-article-free language, the parasynonym of the *oblique* homolograph of the UnLtdNSgN “man” is enclosed between the EXQ marks,\ /, whereas the parasynonym of the LtdNSgN “a man” remains unquoted. In the case of the English personal proper name “Aristotle”, no article is available. Therefore, an occurrence of the name “Aristotle”, in which it is used *obliquely* for denoting *its singleton-concept* (see Df 2.9(11)), being at the same time its sense (see Df 2.14 below), is also distinguished by enclosing this name between the EXQ marks,\ /, so that \Aristotle/ *is* the singleton of Aristotle. The same rule applies to transliterata (transliterations) of “Aristotle” in other WNL’s.

2) In contrast to any communicative *exteroceptive symbols*, as *graphic symbols*, which are *immutable* and *static*, *brain symbols*, i.e. mental entities, of mine are *mutable* and *dynamic*. Therefore, the *instantaneous spontaneous mental passage*, say, from a common member *a man* of the class *man* to the only concrete member *Aristotle* of the singleton \Aristotle/, being a strict subclass of the class *man*, by

supplementing the class man with some distinguishing differentia or differentiae can not be indicated by any graphic symbol. Such differentiae are called from the memory spontaneously and are not anchored down to any additional qualifiers to mention (denote) them formally. Only the initial and terminal brain symbols of the above mental process of specification of a man by Aristotle are fixed and indicated by anchoring the down to the names “a man” and “Aristotle”. This fact creates an impression that the former name denotes Aristotle and also an indefinite number of *concrete and concretized men*, rather than *concrete but not concretized*, persons simultaneously. Likely, such an impression underlies Mill’s doctrine of the meaning of common names. Aristotle *is associated with* the common member-name (cenomeloxenograph) “a man” as described above, but Aristotle *is not denoted by this name.* When the name “a man” is used but not mentioned, it *denotes* (*refers to*) *a man – a featherless biped* (by Russell’s definition), whereas when the name “Aristotle” or any one of its synonymous descriptive proper names, mentioned above, is used but not mentioned, it *denotes* (*refers to*) *Aristotle*. Like relationships exist between Aristotle and the parasynonym of the word “man” in any given NL, articled or article-free.•

2.4.4. A sense (sense-value) of a full (notional) xenograph

Preliminary Remark 2.3. Besides a noun, an onymological root (“onym”, “graph”, or “phon”), or the onological root (“on”), a DdFrG&Ds, i.e. a nominalistic deduction of a xenograph from a genus and the differences, may involve, depending on what it is, some other parts of speech (major form classes), onymological or onological qualifiers, some punctuation marks, and also some autographs (euautographs or tychautographs), quoted or not, which are conventionally called *citation forms* or *quotation nouns* or, equivocally, *hypostases*. Auxiliary verbs, demonstrative and relative pronouns, prepositions, and conjunctions, all being parts of speech, and also the articles, not being pats of speech, are *function words*, i.e. *syntactic linguistic operators*, which express primarily syntactic relationships among significant xenographs and particularly among other parts of speech called *full* (*notional*) *words*. The punctuation marks form another class of *syntactic linguistic operators*. In any case, a DdFrG&Ds necessarily involves one or more modifiers, wordy, onymological, or onological, as indicatedin Df 2.11. To be recalled, a wordy modifier is either a predicate one (PM), called also a predicate, or an attributive one

(AM) or else an inflectional one (IM). Every PM is a limiting modifier (LtgM), every IM is an unlimiting modifier (UnLtdM), some AM’s are unlimiting ones (UnLtdAM’s) and the other AM’s are limiting ones (LtdAM). An unlimiting AM is alternatively called a qualifying, or descriptive, AM and also a qualifier. Every onymological or onological modifier is a qualifier, i.e. an UnLtdAM. Of the LtgAM’s, the demonstrative and relative pronouns and articles are *function words*, whereas all other LtgAM’s and all qualifiers (UnLtgAM’s) are *semantemes*, i.e. particularly *full* (*notional*) *words*. An unlimiting or limiting AM is treated (interpreted) respectively as an unlimited or limited proper name of a certain megauniversal or, more specifically, as that of a certain megaclass, if it is applied to an UnLtdPrCsN, and that of a certain megacmass, if it is applied to an UnLtdPrCmsN.•

Df 2.14. 1) This definition is a generalization of Cmt 1.9(2). In this case, unless stated otherwise, by a *xenograph* I mean an UnLtdSgNlXG (UnLtdSgNlN), i.e. either an NSgN or an UnLtdNNSgN, as defined in Df 2.9.

2) A xenograph is called a *primary*, or *reference*, or *induced*, *xenograph* if it *designates* [with respect to me] a certain abstractum such as a class other than any token-class of the xenograph, or such as a cmass, megaclass, or megacmas, which has been assigned to the xenograph by nominalistic induction, and not by nominalistic deduction, i.e. not by any DdFrG&Ds. The above abstractum is called the *xenodesignatum* and also the *induced sense*, or *sense-value*, of the primary xenograph. That is to say, the [xeno]designatum of a primry xenograph and its induced sense are one and the same mental coentity of mine. The term “induced sense” is hereafter abbreviated as “sense” if there is no danger of misunderstanding.

3) A complex xenograph is called an *idiograph* (“*idiographonym*”) and also more generally an *idionym* or conventionaly an *idiom* if it can be dissected into simple designative parts but its [xeno]designatum cannot be derived by coordinating the designata of the constituent parts. The term “*idiograph*” should not be confused with the term “*ideograph*” (“*ideographonym*”), being a synonym of the semiotic term “*graphic symbol*”. An idiograph that is included into a larger xenograph as its grammatically congruent constituent part is regarded as a primary (reference, induced) xenograph, whose sense coincides with its designatum.

4) When I *consider* a given xenograph as a *secondary* and hence *complex* one that has or is supposed to have a certain [xeno]designatum in a given domain (say, in

a given field of study and discourse or in a given theory) and *analyze* (*divide*) it into primary (reference, induced) xenographs and perhaps into autographs (euautographs and tychautographs, if present), which I regard as *unit* graphonyms that have relevance to the subject matter of the given domain, a *deduced sense*, or *deduced sense-value, of the given secondary xenograph* is a *biune mental coentity entity of mine* that has the following two successive *hypostases* (*ways of existence*, *aspects*) with respect to me. The *first hypostasis of the deduced sense*, which is called the *xenodesignatum-producing operation*, or *sense-operation*, *on the secondary xenograph* and which is said to *be expressed by the latter, is* a *mental operation* (*process*) *of mine, of coordination* (*synthesis*) of the *xenodesignata* (*senses*) of the constituent primary xenographs and of the *autodesignata* (*isotoken-classes*) *of the constituent autographs* (if present), of the secondary xenograph, which are collectively called the *object-abstracta of the sense-operation*, *into a single whole abstractum* that is called *the subject-abstractum both of the sense-operation and of the deduced sense.* Once I complete the sense-operation, I mentally *substantivize* the subject-abstractum being the final result of the sense-operation, thus taking another mental attitude towards the secondary xenograph. According to this mental attitude, the subject-abstractum is *the second hypostasis of the deduced sense*, which is said to be *designated by* or to be *the designatum*, or more precisely *xenodesignatum*, *of the secondary xenograph*. The secondary xenograph is said *to express the sense-operation on it*, *to express* or *to have its sense*, and *to designate* or *to have its designatum*. The term “*deduced sense*” is hereafter afbbreviated as “*sense*” if there is no danger of misunderstanding.

5) The above items 2–4 are generalizations of the points ii–iv of Cmt 1.9(2) to the general case, where the term “designatum” is, in accordance with Df 2.9(11), understood in a broad sense (sensu lato) to apply, not only to a multipleton, but also to a singleton and to a cmass. To be recalled, depending on a secondary xenograph, its designatum is either a *class-designatum* or a *cmass-designatum*, while a class-designatum is either a *multipleton-designatum* or a *singleton-designatum*. Therefore, the points vi and vii of Cmt 1.9(2) can be generalized accordingly, while the point v retains.

6) When I perform the sense-operation on a secondary xenograph *fluently* or when I successfully complete the sense-operation whose performance requires some

mental efforts, the designatum of the xenograph is known to me. In either case, I often identify, *involuntarily but consciously*, the *sense* of the secondary xenograph with the subject-abstractum of the sense, i.e. with the designatum of the xenograph, because the designatum is the *final substantive entity* of the sense-operation and hence it is the *dominant hypostasis* (*aspect*) of the sense.

a) In accordance with Df 2.12(4), if a secondary xenograph is an UnLtdPrMnN and if I *use it along with its multipleton-designatum* in the projective (polarized, extensional, connotative) mental mode *as a cenomeloxenonograph for mentioning* (*denoting*, *referring to*, *putting forward*) *a common member of the multipleton-designatum* then the multipleton-designatum is automatically (involuntarily) mentally turned into *the multibleton-connotatum of the cenomeloxenonograph, both the cenomeloxenonograph and the multibleton-connotatum are used but not mentioned*. At the same time, the sense of the cenomeloxenonograph is mentally turned into its *sense-conotatum*, which is also used but not mentioned. Thus, Df 2.12(4) applies, *mutatis mutandis*, with "sense" in place of "multipleton-designatum" and with "sense-connotatum" in place of "multipleton-connotatum".

b) The above point applies, *mutatis mutandis*, with "Df 2.12(5)", "UnLtdPrCmsN", "cmass", and "cenomeroxenonograph"in place of "Df 2.12(4)", "UnLtdPrMnN", "multipleton", and "cenomeloxenonograph" respectively.

c) By the above points a) and b), the notion of sense is incorporated into the meaning content of a secondary xenograph.

7) A special quotation, which is called an *enneoxenographic quotation* (from the Greek noun "έννοια" \énnia\ meaning *an idea*, *concept*; *meaning*, or *sense*), or briefly *EXQ*, and which is formed by enclosing a xenograph between the special *EXQ* marks ${}^{\backslash}\ {}^{/}$, is a *proper ideograph* (*ideographonym*, *graphic symbol* in the semiotic terminology) or more precisely *ideodictograph* or *dictoideograph that denotes the sense of its interior* subject to the items 2, 4, and 6. Consequently, if the interior of an EXQ is a primary xenograph then the interior of the EXQ denotes the designatum of its interior, in accordance with the item 2. If the interior of an EXQ is a secondary xenograph then the EXQ is, in accordance with the items 4 and 6, *a biune ideograph* (*ideodictograph*, *dictoideograph*), *the first, interim denotatum of which is the sense-operation on its interior, whereas its second, ultimate, dominant denotatum is the*

subject-class that is precsinded from the sense-operation, i.e. *the designatum of the interior*.

8) A *substantive*, i.e. a *noun* or *noun equivalent*, is said to be a *paralogous* (*bare, mere, unsubstantial, absurd, paradoxical*) *one* or, more generally, a *paralogy*, if its designatum, i.e. the class (range) of its denotata is *empty*. Consequently, a paralogous substantive *has no denotata*, i.e. it is *meaningless*. A substantive that contains a paralogous substantive as its grammatically congruous constituent part that is used but not mentioned is also paralogous. Etymologically, the adjective "parlogous" and the noun "paralogy" are derived from the Greek adjective "παράλογος" \parálογos\ meaning *unreasonable* or *absurd*. Particularly, in the context of a *natural domain* such as a modern field of study and discourse or a concrete modern scientific treatise, a substantive is paralogous if it either belongs to a *supernatural domain,* – such a domain, e.g., as mythology, theology, or a concrete heroic or religious legend, or such as an antiquated geographic, political, or geopolitical reality or an antiquated scientific theory, – or if it is a *contradictio in adjecto*, or else, if it both peculiar properties. The *designatum* (*range of denotata*) of a paralogous substantive is the *empty class*, so that it has no denotatum. A paralogous substantive cannot be redefined to become meaningful one – in contrast to an *ill-defined* and hence *naked* but *redefinable* substantive of a natural domaain, which is conventionally called a *nomen nudum* (pl. "*nomina nuda*") in Latin, and which can alternatively be called a *gymnonym* (from the Greek adjective "γυμνός" \γimnós\ meaning *naked*) or more specifically a *gymnograh* (*gymnograhonym*). A substantive is said to be a *rational one* it is not *paralogous*.

9) Here follow some examples of paralogous substantives.

a) In [the domain of] Greek mythology, the following statement of Guirand [1959, p. 161] can be regarded as an axiom so that it is *ad hoc veracious* (*accidentally true*):

> «In addition to Satyrs and the Sileni, another kind of monstrous creatures formed part of the cortege of Dionysus: the Centaurs. Their torso and head were those of a man; the rest of their body belonged to a horse.»

Therefore in this domain, the class *centaur* that is designated by the count noun "centaur" is not empty. Just as any other class (as *man*, *tree*, etc) that is *designated* by the respective count name, the class *centaur*, along with its name, can be used *xenonymously* in a *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, indicated in English by the indefinite article, for mentioning an as if extramental (exopsychical) *common* (*general*) *element* (*member*) of that class, namely *a* centaur, which is *denoted* by its *common individual name* (*limited common name*) "a centaur" and which is another hypostasis of that same class. Accordingly in this domain, the name "a centaur" is *meaningful*, – just as the *proper class-name* (*unlimited common name*) "centaur". At the same time, "centaur" is not a biological term, i.e. any *natural* domain as *biology* and particularly as a *biological taxonomy of bionts* (*BTB*) is not a domain of definition of that noun. That is to say, in biology, both the unlimited common name "centaur" and the limited (indefinitely-articled) common name "a centaur" are *inadmissible* (*purposeless*, *functionless*) and hence *paralogous graphonyms*, which have *no xenonymous denotatum*, and which are not supposed to be attached with any. Any grammatically congruent linguistic construction that contains a paralogy as its constituent part is also a paralogy. For instance, from the standpoint of semantic analysis, the *sentence*: "A centaur is a mammal", its negation: "A centaur is not a mammal", and the inclusive disjunction of the two: "A centaur is a mammal or a centaur is not a mammal" are *paralogous* (*paradoxical*) *ones*, not only in the sense that they are *neither veracious* (*accidentally true*) *nor antiveracious* (*accidentally antitrue*, *accidentally false*), but in the sense that the notions of veracity (accidental truth) and antiveracity (accidental antitruth, accidental falsehood) are not applicable to them, although the last sentence is *valid* from the standpoint of syntactic analysis. However, the sentence: "A centaur does not exist" is veracious, because it is just a loose abbreviation of the veracious sentence: "The graphonym "a centaur" has no denotatum in the natural universe", in which the graphonym "a centaur" is used autonymously and is hence mentioned. Like remarks apply, *mutatis mutandis*, with the proper name "Pegasus", e.g., in place of both common names "centaur" and "a centaur".

b) Any one of the expressions (e.g.): "the 16th president of the USA", "the 16th president of the USA in the years 1861–65", "the president of the USA in the years 1913–21", and "the 28th president of the USA in the years 1913–21" is meaningful. Therefore, the expression "the 16th president of the USA in the years 1913–21" (e.g.) is a *contradictio in adjecto*, so that it is a paralogy. Consequently, each one of the sentences: "Abraham Lincoln was the 16th president of the USA in the years 1913–

21", "Abraham Lincoln was not the 16th president of the USA in the years 1913–21", "Abraham Lincoln was or was not 16th president of the USA in the years 1913–21" is also a paralogous one: it is neither veracious nor antiveracious. By contrast, any one of the sentences: "Abraham Lincoln was the 16th president of the USA", "Abraham Lincoln was the 16th president of the USA in the years 1861–65", "Woodrow Wilson was the president of the USA in the years 1913–21", and "Woodrow Wilson was the 28th president of the USA in the years 1913–21" is veracious, the negation of that senteence is antiveracious, and the inclusive disjunction of the two sentences is a tautologous sentence.

c) It will be recalled that on December 25, 1991, the USSR was officially dissolved and then consigned to oblivion by an agreement among the heads of its member republics to form Commonwealth of Independent States (CIS). Before that date, the name "the USSR" and hence the name "the capital of the USSR" were meaningful (had denotata) for approximately 70 years. In that time, the statement "The capital of the USSR is in Europe" was true, while the sentence "The capital of the USSR is not in Europe" was antiveracious. Nowadays, when the pertinent *geopolitical state of affairs* has changed, both sentences have become meaningless. Particularly, in the new geopolitical reality, the graphonym "the capital of the USSR in AD2000", e.g., is a *contradictio in adjecto* and therefore it is meaningless (paralogous), like "centaur", "a centaur", or "Pegasus" in biology. Consequently, any graphonym, containing that graphonym as its constituent part, is also meaningless. For instance, any one of the sentences: "The capital of the USSR in AD2000 was in Europe", "The capital of the USSR in AD2000 was not in Europe", and "The capital` of the USSR in AD2000 was in Europe or not in Europe" is paralogous and hence *inadmissible*. At the same time, like the graphonym "A centaur does not exist", the sentence "The capital of the USSR in AD2000 did not exist" is veracious, because it is a loose abbreviation of the veracious sentence "The graphonym "the capital of the USSR in AD2000" has no denotatum in the natural universe", in which the graphonym "the capital of the USSR in AD2000" is used autonymously. In general, any graphonym containing a *contradictio in adjecto* as its constituent part that is used xenonymously is also a *contradictio in adjecto*, whereas any *contradictio in adjecto*, which is used xenonymously, is meaningless. For instance, like "the capital of the USSR in AD2000", the graphonym "the king of Israel in AD2000" is *contradictio in* *adjecto* and hence sentence “The king of Israel in AD2000 was as wise as the king Solomon”, its negation, and the inclusive disjunction of the two are also *contradictiones in adjecto*.•

Cmt 2.8. 1) The main, nontrivial message of Df 2.14 is that the sense of a secondary (deduced) xenograph is a mental *process* and not a static mental substance (memory image). That is to say, *the sense is a thought*, while a *thought always streams either forward or back and forth*; *it cannot stay at rest*. In this connection, James [1890; 1950, vol. 1, pp. 224–225] says:

> «*The first fact for us, then, as psychologists, is that thinking of some sort goes on.* I use the word thinking, in accordance with what was said on p. 186, for every form of consciousness indiscriminately. If we could say in English ‘it thinks,’ as we say ‘it rains’ or ‘it flows,’ we should be stating the fact most simply and with the minimum of assumption. As we cannot, we must simply say that *thought goes on.*»•

2) The primary graphonyms that are comprised in a secondary xenograph as its constituent parts may belong to various syntactic and semantic categories. And even if all of them are of the same class, the sense-operation on the secondary xenograph cannot in the general case be expressed in terms of a single elemental (basic) operation of coordination of the abstracta designated by the constituent primary graphonyms. For instance, the senses of arithmetic expressions are expressed in terms of the *four* rules of arithmetic (cf. Cmt 2.2). If, however, a secondary xenograph is a DSN (descriptive specific name), i.e. a DcSTrG&Ds (description of the species through a genus and the differences), then its sense-operation can be expressed in terms of a single elemental binary coordination operation of the pertinent object-abstracta as explicated in the next sub-subsection.•

2.4.5. The sense of a sentence

Df 2.15. 1) According to grammarians (see, e.g., Lambuth [1964, pp. 10–26]), a *sentence* of a native language (NL) as English is a grammatically self-contained (independent) and self-consistent wordy *ideonym* (*symbol*) *expressing a complete thought* (*conception*, *sense*). Consequently, a graphic (written) sentence is an *ideograph* (*ideographonym*, *graphic symbol*), and a phonic (spoken, oral) sentence is an *ideophon* (*ideophononym*, *phonic symbol*), live or recorded. From the standpoint of both syntactic and semantic analysis, there are the following *four kinds* (i.e. *syntactico-semantic tetrachtomy*) *of English sentences*:

a) A declarative sentence that expresses a certain nonempty or empty class of state of affairs (facts)

b) An interrogative sentence that expresses a question.

c) An exclamatory sentence that expresses an emotion.

d) An imperative sentence that expresses a command or request with the help of a verb-predicate standing before a grammatical subject if present (e.g. “let us”, “let them” or “let … be”).

At the same time, from the standpoint of syntactic analysis, there are the following *five types* (i.e *syntactic pentachtomy*) *of primarily declarative English sentences*:

i) A simple unextended sentence, which consists of the principal parts only – the subject and the predicate.

ii) A simple extended sentence, which contains some secondary parts (as objects, attributes, and adverbial modifiers) besides the principal parts.

iii) A complex-subordinate sentence, alternatively called a complex sentence, which consists of a principal (independent) clause and one or more subordinate (dependent) clauses.

iv) A complex-coordinate sentence, alternatively called a compound se`ntence, which consists of two or more independent clauses, joined together by means of coordination.

v) A contracted sentence, which has two or more subjects to the same predicate or two or more predicates to the same subject and which can be developed into a complex-coordinate (compound) sentence.

2) A grammatical predicate is a *limiting modifier* to its grammatical subject. Therefore, a sentence is a *limited* glossonym (linguistic name), to which the notion of singular or plural number is not applicable. Nevertheless, I assume (postulate) that a sentence *expresses its sense* and *designates a certain class*, which is called *the designstum* (or more precicely *the xenodesignstum* in contrast to *the autodesignatum*, whenever there is a danger of misunderstanding) *of the sentence* and also *the subject-class of its sense* (in contrast to *the operation of coordination of the object-classes of the sense*, when applicable), – in analogy with a secondary unlimited xenograph as described in Df 2.14. The sense of a sentence will be called a *sententia* (pl.

"*sententiae*", adj. "*sententious*"). Conversely, a sententia is the sense of a certain sentence or, perhaps, the sense that several different sentences either of the same or of different NL's have in common. A sententia will be qualified by the same modifiers as those qualifying the English sentence expressing the sententia. Particularly, a *simple declarative affirmative* (*positive*) or *negative* (*privative*) *sententia* (briefly *DASa* or *DNSa*, pl. "*DASae*" or "*DNSae*"), *unextended* or *extended*, is the sense of a *simple declarative affirmative* or *negative sentence* (briefly *DAS* or *DNS*, pl. "*DAS's*" or "*DNS's*), *unextended* or *extended*, respectively and vice versa. Likewise, *complex-subordinate* (*complex*), *complex-coordinate* (*compound*), or *contracted declarative sententia* (*DSa*), and also an *interrogative*, *exclamatory*, or *imperative sententia* is the sense of a sentence of the respective nomenclature, and vice versa.

3) There is no one-word common name in English to denote *the sense of an arbitrary sentence*. Therefore, I have decided to employ the noun "sententia" as such a name, especially taking into account that the sense, which that noun acquires in this use, is very close to some dominant senses, which the homonymous etymon of the noun has in Latin. Namely, according to Simpson [1968, p. 547], the Latin etymon of the English noun "sententia" has (assumes) the following senses:

> «**sententĭa** -ae, f. (sentio), *a way of thinking*, *opinion*, *thought*, *meaning*, *purpose*...
>
> Esp., *an opinion formally expressed*, *a decision*, *a vote*...
>
> Transf., (**1**) *the meaning*, *signification*, *sense* of words, etc… (**2**) concrete, *a thought expressed in words*, *a sentence*, *period*, esp., *a maxim*, *aphorism*...»[5]

At the same time, according to WTNID, the senses the English common names "*a maxim*" and "*an aphorism*" are the only senses (sense-values) of the Latin noun "sententĭa" that have been carried over from Latin to English. Like the word "sentential" being a conventional kindred adjective of the noun "sentence", which most generally means *of or relating to a sentence*, the word "*sententious*" is by

[5]"(sentio)" is an etymological note; "Transf." is an abbreviation of "Transferred", which means: *used in an altered or metaphoric sense*. Latin has no words corresponding to the English articles '*the*' and '*a*'.

definition a kindred adjective of the noun “sententia”, which most generally means *of or relating to a sententia.*

4) In accordance with Df 2.14(4), the sense of a sentence is a *biune mental* (*psychical*) *coentity* (*process*) *of mine* (or, by extrapolation, of any sapient subject, being either a maker or an interpreter of the sentence) that has the following two successive *hypostases* (*ways of existence*, *aspects*) with respect to me (correspondingly, with respect to the sapient subject). The *first hypostasis of the sense*, which is called the *designatum-producing operation* or *sense-operation on the sentence* and which is also said to *be expressed by the latter, is* a *mental operation of mine*, *of coordination* (*synthesis*) *of the designata of*, i.e. *of the classes designated by*, the parts of the sentence, which are supposed to be *the* [*induced*] *senses of those parts* and which are called the *object-classes* of the sense of the sentence, *into a single whole class* that is called *the subject-class both of the sense-operation and of the* [*deduced*] *sense of the sentence* and also *the designatum of the sentence.*

5) In agreement with Df 2.14(8,9), a *declarative sentence* (*DS*) and *its negation* are said to be:

a) *paralogous* (*bare*, *mere*, *unsubstantial*, *absurd*, *paradoxical*) *DS’s* and also, more generally, *paralogies* if their *designata* are the *empty class* and if hence the notions of *truth* and *antitruth* (*falsity*) *are not applicable to them*;

b) *rational ones* (*RDS’s*) if the *designatum* of at least one of them is a *nonempty class* so that the notions of *truth* and *antitruth* (*falsity*) *are applicable to them*.

That is to say, the negation of a paralogous DS is another paralogous DS, whereas the negation of an RDS is another RDS. This treatise comprises primarily RDS’s of English WNL (written native language), some of which are enriched by pasigraphs – logographs, autographs, or both, or by special wordy terminology. I shall occasionally use (but not mention) imperative sentences of a form “Let us …” or “Let – be …”, but I shall have no occasion to *use* (but not to mention) interrogative and exclamatory sentences. *Formal logic* of declarative sentences of an enreached WNL is the subject matter of this treatise. *Material properties* of declarative sentences of enriched English (as the IML of this treatise) are explicated below. For the sake of brevity, I shall hereafter use the noun “*sentence*” alone synecdochically for mentioning an *English graphic* (*written*) *RDS*, unless stated or obviously understood otherwise. For more clarity, I may also use synecdochically the names "declarative sentence", "declarative affirmative sentence", and "declarative negative sentence" or their abbreviations "DS", "DAS", and "DNS" with the understanding that "sentence", abbreviated as "S", stands for "*English graphic sentence*", but again unless stated otherwise. *Material properties* of interrogative, exclamatory, and imperative sentences, graphic or phonic, will briefly be discussed in Cmt 2.9.

6) If in a given spatio-temporal situation or universally *the sense of a given DS conforms to* (*matches*) *a certain psychophysical* (*physopsychical*, *physicopsychical*) *complex object of mine*, which I know from another source, nonlinguistic (e.g. by acquaintance from my sensorial experience) or linguistic (e.g. from some other like sentences stated earlier), and which is called a *state of affairs* and also a *fact*, *case*, *relation*, *event*, *phenomenon, situation*, *circumstance*, etc, then that DS is said to be *veracious,* i.e. *accidentally true*, or more precisely *materially veracious* (*m-veracious*), i.e. *accidentally materially-true* (*accidentally m-true*), with respect to me in the given situation or universally respectively – in contrast to a *schema* (*sentential form*) of the DS, which is said to be *formally veracious* (*f-veracious*). The absence of a certain state of affairs is another state of affairs. For instance, if the *assertive* sentence "It is raining" denotes the respective state of affairs here and now then the *assertive* sentence "It is not raining" may, there and now, denote another state of affairs, which is the absence of the former one. If in the same circumstances the sense of the sentence *does not conform to any fact*, *but the negation of that sense*, i.e. *the sense of the negation of that sentence*, *does conform to a certain fact*, then that sentence is said to be *materially antiveracious* (*m-antiveracious*), i.e. *accidentally materially-antitrue* (*accidentally m-antitrue*, *accidentally m-false*). Hence, *the negation of an m-veracious sentence is an m-antiveracious sentence and vice versa*. If the sense of a sentence *neither conforms to nor contradicts any known relevant fact*, then that sentence is said to be *neither m-veracious nor m-antiveracious* and also to be *m-vravr-neutral* (*m-vravr-indeterminate*), but again in the given situation or universally with respect to me. Hence, *the negation of an m-vravr-neutral sentence is another m-vravr-neutral sentence*. For instance, if I do not know what are the weather conditions in Broadway of New York at this moment then either sentence "It is raining in Broadway" or "It is not raining in Broadway" is here and now m-vravr-neutral with respect to me. In the above definitions, the words "true" and "antitrue"

can be used instead of “veracious” and “antiveracious” provided that the former are understood as abbreviations of the expressions “accidentally m-true” and “accidentally m-antitrue” respectively. A DS is said to be: (a) *m-unveracious* if it is m-antiveracious or m-vravr-neutral (m-vravr-indeterminate); (b) *m-non-antiveracious* if it is m-veracious or m-vravr-neutral (m-vravr-indeterminate); (c) *m-vravr-unnutral* or *m-vravr-determinate* if it is m-veracious or m-antiveracious. It follows from the above-said that if the sense of a sentence conforms to a certain fact or if, on the contrary, it contradicts that fact and does not conform to any fact then the sentence itself does so.

7) In contrast to the syntactic *form* of an m-veracious DS, the state of affairs (fact) denoted by the sentence is the *matter* of the sentence. This fact has the following two implications.

a) The material property of a DS and of its sense to be m-veracious, m-antiveracious, or m-vravr-neutral (m-vravr-indeterminate) and that to be m-unveracious, m-non-antiveracious, or m-vravr-unneutral (m-vravr-determinate) are *semantic matter-of-fact properties*, i.e. semantic properties that are concerned with facts and not imaginative or fanciful ones. Accordingly, the kindred substantives (noun equivalents) of the above adjectival qualifiers, namely “m-veracity”, “m-antiveracity”, “m-vravr-neutrality” (“m-vravr-indeterminacy”), “m-unveracity”, “m-non-antiveracity”, and “m-vravr-unneutrality” (“m-vravr-determinacy”) carry the abbreviation “m” for the prepositive adjectival qualifier “material” – as opposed to “formal” abbreviated as “f”.

b) The act of interpreting of a DS as m-veracious, m-antiveracious, or m-vravr-neutral belongs to *material logic* and not to *formal logic*.

8) When I use an m-veracious DS for mentioning the state of affairs, to which it conforms, and thus turn the latter into the *intended import value* of the DS, I say that the state of affairs *is denoted* by the sentence or that it is the *denotatum* (*denotation value*, pl. “*denotata*”), or *meaning*, of the sentence. By contrast, an m-unveracious, i.e. m-antiveracious or m-vravr-neutral, DS denotes nothing, i.e. it has no denotatum, but rather it just expresses its own sense. An m-veracious DS is alternatively called a *meaningful DS*, whereas an m-unveracious DS is alternatively called a *meaningless DS*. In order to indicate that a DS is m-veracious and that hence it denotes the pertinent state of affairs (fact), the DS is put in a certain conventional or properly

defined unconventional format, which is called an *assertive format*. Such a DS is said to be *asserted* or *assertive*. An m-veracious DS is said to be *unasserted* or *unassertive* if it is *put* (*presented*, *exhibited*, *demonstrated*, *ostended*, *written* or *uttered*) *in an unassertive* (*not assertive*) *format within an assertive context*. An m-unveracious, i.e. m-antiveracious or m-vravr-neutral, DS cannot be asserted (be put in an assertive format). That is to say, an m-unveracious DS is an unassertive DS, i.e. a DS that is put in an unassertive format within an assertive context.

9) M-veracious, m-antiveracious, and m-vravr-neutral DS's are classified further as follows.

i) An m-veracious sentence is said to be:

a) an *enduringly*, or *permanently*, *m-veracious sentence* and also a *proper m-veracious*, or *m-veracious proper, sentence* if it confirms to a certain *enduring* (*lasting*, *permanent*) *unique fact* of nature or human society, e.g. an astronomic, historical, geographic, or geopolitical one;

b) a *transitorily*, or *temporarily*, *m-veracious sentence* and also a *common m-veracious*, or *m-veracious common*, *sentence* if the fact, to which it conforms in the given circumstances (spatio-temporal situation) with respect to me, is one of many similar *transitory* (*temporary*) states of affairs occurring occasionally here or there and now or then.

ii) An m-antiveracious sentence is said to be:

a) an *enduringly*, or *permanently*, *m-antiveracious sentence* and also a *proper m-antiveracious*, or *m-antiveracious proper*, *sentence* if its negation is a proper m-veracious sentence;

b) a *transitorily*, or *temporarily*, *m-antiveracious sentence* and also a *common m-antiveracious*, or *m-antiveracious common*, *sentence* if its negation is a common m-veracious sentence in the given circumstances with respect to me.

iii) An m-veracious or m-antiveracious proper sentence is indiscriminately called a *proper sentence*.

iv) An m-vravr-neutral sentence is alternatively called an *m-vravr common sentence* (in contrast to *m-tautologous common sentencess* to be defined before long), because in accordance with the points i.b and ii.b, whenever there is a fact (state of affairs), which an m-vravr-neutral (common) sentence either confirms to or contradicts, that sentence becomes a *common m-veracious* or *common m-antiveracious sentence* respectively.

10) Here follow some examples of DS's of the different kinds indicated in the previous item.

a) Proper m-veracious sentences.

"Sir Walter Scott is the author of *Waverley*", "Abraham Lincoln was the 16th president of the USA in the years 1861–65", "Woodrow Wilson was the 28th president of the USA in the years 1913–21", "Moscow is the capital of Russia", "Moscow is in Europe", "Chicago is North of New York".

b) Proper m-antiveracious sentences.

"Sir Walter Scott is not the author of *Waverley*", "Abraham Lincoln was the 28th president of the USA in the years 1913–21", "Abraham Lincoln was not the 16th president of the USA in the years 1861–65", "Moscow is not the capital of Russia", "Moscow is in Asia", "Chicago is South of New York".

c) Common (vravr-neutral) sentences.

"It is raining", "It is not raining", "The night is light", "The night is dark", "This water is cold", "This water is hot", "This meal is testy", "This meal is not testy", "I am hungry", "I am full up".

Any sentence of the point a) is an m-veracious (accidentally m-true) proper sentence with respect to me because it *conforms to* (*denotes* when asserted) the pertinent historical, geographical, or or present or present geopolitical fact. Any sentence of the point is an m-antiveracious (accidentally m-antitrue) proper sentence with respect to me because it contradicts the historical, geographical, or present geopolitical fact, which a certain m-veracious proper sentence of the point a) conforms to. Any possible state of affairs of the range of any sentence of the point c), which that sentence can conform to (denote when asserted), has the quality of *thisness* (*haecceity*), i.e. of *being here and now*, and hence it is local and transient. Incidentally, the first sentence of the point a) and the first sentence of the point b) are relevant to the historical fact that Walter Scott published his twenty-nine *Waverley Novels* anonymously, and that he kept his authorship of *Waverley* secret (see Cmt 1.9(2.v)). Therefore, either one of the two sentences was an m-vravr-neutral common sentence for any person who did not know the identity of the mysterious author of *Waverley*, particularly for the English King George IV. Once the identity of the author of *Waverley* had been revealed, the first sentence became m-veracious and the second one m-antiveracious. Likewise, any other sentence of the point a) and its negation or its contrary of the point b) are m-vravr-neutral ones with respect to every person who does not know the facts, which the two sentences of the point a) confirm to.

11) In accordance with the item 2 of this definition, the definition of the sense of a complex unlimited substantive as given in Df 2.14 applies also to a DS, although the latter is a limited xenograph. Particularly, Df 2.8 (especially in the points a and b of item 2 of it) applies, *mutatis mutandis*, with "*m-veracious proper DS*" in place of "*kyrioxenograph*" ("*proper graphic name*"), whereas the points a and b of item 4 of Df 2.12 apply, *mutatis mutandis,* with "*m-vravr-neutral common DS*" in place of "*cenomeloxenograph*" ("*common-member name*"). Here follows some specific comments on the above-said.

a) Just as the designatum of a kyrioxenograph (proper graphic substantive), described in the points a and b of item 2 of Df 2.8, *the designatum of an m-veracious proper DS*, i.e. *the subject-class of the sense of the DS*, is *the singleton of the enduring unique fact* (*state of affairs*), *which the DS conforms to via its sense*. From the standpoint of psychological analysis (introspection of my own), in order to produce the feeling of conformability of the sense of the DS to the fact in question, I use the DS along with its singleton-designatum in the *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* the designatum as *my as if extramental* (*exopsychical*) *object* (other than the DS itself), which I identify with the only member of the designatum and which I call the [*proper*] *denotatum of the sentence*. I do the above *habitually* and hence *involuntarily but consciously* – just as I *always* mentally experience the *percept* (*sensation*) of any given sensible entity (nym, sensum, sensory object) as that entity, particularly in the case when the sensible entity is a written (graphic) DS, which is used *self-referentially* (*kyrioautonymously*), as explicated in Df 1.12(1) (see also Cmt 1.27(4)). In other words, I *use* the DS along with its singleton-designatum *for mentioning* the fact (state of affairs), being the member of the designatum, while both the DS and its singleton-designatum *are used but not mentioned*. In this case, the singleton-designatum *is,* to use the appropriate *monistic phraseology,* involuntarily *mentally transduced into another hypostasis* (*way of existence, aspect*) in the form of its only member. In order to describe this mental phenomenon in the appropriate alternative *dualistic phraseology,* I say that the member of the singleton-designatum *is put forward by the DS as its intended import value* or that it *is denoted by DS* thus being its *denotatum* (*meaning*), while the singleton-designatum and the entire sense of the DS *are put backward by the DS* or that they are *connoted by the DS* thus being its *connotata*, i.e. the *singleton-connotatum* and the *sense-connotatum* respectively.

b) Just as the designatum of a cenomeloxenograph (common-member name) described in the points a and b of item 4 of Df 2.12, the designatum (range) of an *m-vravr-neutral common DS* is *the multipleton* (*many-member class*) *of an indefinite number of similar states of affairs*, which can occur here or there and now or then, and to any one of which the DS may conform via its sense when such a state of affairs actually occurs (comes into existence). In this case, in no connection with any actual state of affairs, I can use a token of the DS along with its multipleton-designatum in the *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I mentally experience the designatum as *my as if extramental* (*exopsychical*) *object* that I call a *common* (*general*, *certain*, *particular but not particularized*) *element* (*member*) *of the designatum* and also a *common denotatum of the sentence*. The common element of the designatum *represents the whole designatum*, thus being just *another hypostasis* (*way of existence*, *aspect*) *of the latter*. In this case, I also say that both *the sentence and its* [*original*, *unpolarized*] *designatum are used for mentioning* the common denotatum of the sentence, i.e. the common element of the designatum of the sentence, or that less explicitly *they are used but not mentioned*. In other words, in analogy with the pertinent dualistic phraseology of the previous point a, I say that the common member of the multipleton-designatum *is put forward by the DS as its intended import value* or thatit *is denoted by DS* thus being its *common denotatum* (*common meaning*), while the multipleton-designatum and the entire sense of the DS *are put backward by the DS* or that they are *connoted by the DS* thus being its *connotata*, i.e. the *multipleton-connotatum* and the *sense-connotatum* respectively. Once a certain state of affairs that matches the sense of the *vravr-neutral DS* actually occurs (comes to existence), I mentally identify that state of affairst with the common denotatum of the DS thus turning the common denotatum into the *concrete accidental denotatum* (*meaning*) of the DS, while the DS is turned into an *m-various common DS*. Just as in the case of an m-veracious proper DS discussed in the previous point a, I prform all the above mental operations, resulting in conforming an m veracious common DS to the pertinent state of affairs, *habitually* (*fluently*) and hence *involuntarily but consciously*, i.e. again in the same way as I *always* use the *percept* (*sensation*) of any sensible entity (nym, sensum, sensory object) and particularly the percept of the DS itself when I use the latter self-referentially (kyrioautonymously).

12) Besides m-veracious, i.e. *accidentally m-true*, DS's, there are in logic and mathematics *universally m-true* complex-coordinate DS's of various forms, such as '$p\vee\neg p$', i.e. "*p* or not *p*', or '$[p\Rightarrow q]\vee[q\Rightarrow r]$', i.e. "[if *p* then *q*] or [if *q* then *r*]" or "[*p* only if *q*] or [*q* only if *r*]", and of an indefinite number of some other forms, *unbound* (*uncontracted*, *unquantified*, *quantifier-free*) or *bound* (*contracted*, *quantified*, *quantifier-containing*), and hence *predicate-free* (as the two mentioned above) *predicate* (*predicate-containing*). *Sentential forms* (*schemata*), which are expressed in terms of some of the lexigraphs (atomic logographs)

$$\text{'}p\text{' to 's', '}p_1\text{' to '}s_1\text{', '}p_2\text{' to '}s_2\text{', etc,} \tag{2.1}$$

which are joined together by means of the logical (sentential) connectives, mentioned in Df 1.2(3), along with pairs of square brackets, are called *predicate-free catlogographic relations* (*PFCLR's*). Any one of the lexigraphs on the list (2.1) is a *logographic variable* that is called an *atomic*, or redundantly *atomic variable*, *catlogographic relation* (briefly *ACLR* or *AVCLR*), the understanding being that it is replaceable with a *rational simple declarative affirmative* (*positive*) *sentence* (*RSDAS*). The ACLR's on the list (2.1) are, in turn, *substituends* for the respective *atomic euautographic* (*genuinely autographic*, *semantically uninterpreted*), or *atomic pseudo-variable*, *relations* (*AER's* or *APVR's*) *of* A_1:

$$\mathsf{p} \text{ to } \mathsf{s},\ \mathsf{p}_1 \text{ to } \mathsf{s}_1,\ \mathsf{p}_2 \text{ to } \mathsf{s}_2\text{, etc,} \tag{2.2}$$

the understanding being that these are called *ordinary* (*not special*) *ones* (biefly *AEOR's* or *APVOR's*). The ACLR's on the list (2.1) are the so-called *conformal catlogographic* (*CFCL*), or *analohomolographic*, *interpretands* of the respective AER's on the list (2.2), whereas the latter are said to be *conformal euautographic* (*CFE*), or *analoeuautographic*, *interpretantia* (sing. "*interpretans*") of the former. A *euautographic axiom* of A_1 is by definition a *valid* ER of A_1. With the help of the pertinent *euautographic algebraic decision procedure* (*EADP's*), any given ER of A_1, other than an axiom, – *unbound* (*uncontracted*, *pseudo-unquantified*, *psudo-quantifier-free*) or *bound* (*contracted*, *pseudo-quantified*, *psudo-quantifier-containing*), and hence *predicate-free* or *predicate* (*predicate-containing*), – can be classified as an ER of exactly one of the following *three kinds*: a *valid one*, an *antivalid one*, or a *vav-neutral* (*vav-indeterminate*), i.e. *neutral* (or *indeterminate*) *with respect to the validity-values validity and antivalidity* or, in other words, *neither valid nor antivalid*. Any ER of A_1, thus classified, is called a *decided ER* (*DdER*) *of* A_1 or more precisely a *vavn-decided one* (*vavn-DdER*). An EADP for a given ER of A_1, called a *euautographic slave relation* (*ESR*), is an *algebraic proof* (*derivation*) of the so-called *euautographic master*, or *decision*, *theorem* (*EMT* or *EDT*) *of* A_1, which is a *valid ESpR* of one of the three forms, corresponding to the three different classes of ESR's. A DdER of A_1 is said to be: *invalid* if it is antivalid or vav-neutral, *non-antivalid* if it is valid or vav-neutral, and *vav-unneutral* (or *vav-determinate*) if it is valid or antivalid. In accordance with these definitions, the DdER's of A_1 are divided into *two complementary classes in three ways*, namely: (a) the *valid* ER's and the *invalid* ER's, (b) the *antivalid* ER's and the *non-antivalid* ER's, (c) the *vav-neutral* (*vav-indeterminate*) ER's and the *vav-unneutral* (*vav-determinate*) ER's. The valid and vav-neutral DdER's of A_1 and also the EDT's (EMT's) of the vav-neutral DdER's form the *least inclusive* class of ER's of A_1, whose *syntactico-semantic interpretation* by *xenographs* and particularly by *logographs* is equivalent to such an interpretation of *all* DdER's of A_1 and their EDT's. Therefore, the above class of ER's is called [the class of] the *output ER's* (*OptER's*) *of* A_1, whereas the totality of syntactico-semantic interpretations of the OptER's of A_1, which are made in accordance with the same rules, is called a *syntactico-semantic interpretation of* A_1. Accordingly, any PFCLR is the CFCL interperetand of the respective valid or vav-neutral PFER, being its CFE interpretans. Particularly, all AER's on the list (2.2) are by definition *vav-neutral* PFER's.

13) By the pertinent EADP's, it has been proved that the PFER's $p\vee\neg p$, which is called the *law of excluded middle* (*tertium non datur* in Latin), and $[p\Rightarrow q]\vee[q\Rightarrow r]$ are *valid*; the binary kernel-signs (logical connectives) $\vee$ and $\Rightarrow$ are ones of inclusive disjunction and of implication respectively (cf. Df 1.2(3)). By definition, the ER $[p\Rightarrow q]\vee[q\Rightarrow r]$ is equivalent to $[\neg p\vee q]\vee[\neg q\vee r]$. By the pertinent EADP's, it has been proved that the kenrnel-sign $\vee$ satisfies the *commutative* and *associative laws*:

$$[p_1\vee p_2]\Leftrightarrow[p_2\vee p_1] \text{ and } [p_1\vee[p_2\vee p_3]]\Leftrightarrow[[p_1\vee p_2]\vee p_3],$$

where $\Leftrightarrow$ is the kernel-sign of equivalence. Owing to these laws, $[[\neg p\vee q]\vee[\neg q\vee r]]$ can be written in various equivalent forms, differing from one another by the orders of $\neg p$,

q, $\neg q$, and r and by the arrangements of pairs of square brackets, e.g. as $[[q\vee\neg q]\vee[\neg p\vee r]]$, $[[[q\vee\neg q]\vee\neg p]\vee r]$, or $[[\neg p\vee r]\vee[q\vee\neg q]]$. There is an indefinite number of *valid* PFER's, each of which is reducible (equivalent) either to a certain *valid affirmative* (or *positive*) *n-fold* (or undistributively *repeated*) *binary disjunctive PFER* or to the *negation of such a PFER*, which is called a *valid negative n-fold binary disjunctive PFER*. In this case, the qualifier "*n*-fold" is descriptive the total number *n* of occurrences (tokens) of the *binary disjunctive operator* [$\vee$] in a PFER. In turn, the latter PFER is reducible (equivalent) to the respective *valid affirmative* (or *positive*) *n-fold binary conjunctive PFER,* whereas the former PFER is reducible (equivalent) to the respective *valid negative n-fold binary conjunctive PFER*. In this case, the qualifier "*n*-fold" is descriptive the total number *n* of occurrences (tokens) of the *binary conjunctive operator* [$\wedge$] in a PFER. Under the CFCL interpretations (substitutions)

$$p\mapsto \mathrm{p},\ q\mapsto \mathrm{q},\ r\mapsto \mathrm{r},\ s\mapsto \mathrm{s}, \tag{2.3}$$

a valid PFER of a certain one of the above names turns into the *f-tautologous* (*universally f-true*) *predicate-free CLR* (*PFCLR*) of the like variant name with "f-tautologous" (or "universally f-true") in place of "valid" and with "PFCLR" in place of "PFER". Consequently, a DS, having a certain one of the above-mentioned f-tautologous CLR's as its form (schema) and involving the wordy interpretands of the pertinent *euautographic kernel-signs* (*EKS's*) indicated in Df 1.2(3), is distinguished by the respective variant name with "*m*" for "*materially*" in place of "*f*" for "*formally*" and with "DS" in place of "PFCLR". Particularly, under the above interpretations, the valid PFER's $p\vee\neg p$ and $[p\Rightarrow q]\vee[q\Rightarrow r]$ turn into the f-tautologous (universally f-true) PFCLR's '$\mathrm{p}\vee\neg\mathrm{p}$', i.e. "p or not p', being an f-tautologous affirmative 1-fold binary disjunctive PFCLR, and '$[\mathrm{p}\Rightarrow\mathrm{q}]\vee[\mathrm{q}\Rightarrow\mathrm{r}]$', i.e. "[if p then q] or [if q then r]" or[p only if q] or [q only if r]", which are schemata (forms) of m-tautologous (universally m-true) complex-coordinate declarative sentences. The PFCLR "$[\mathrm{p}\Rightarrow\mathrm{q}]\vee[\mathrm{q}\Rightarrow\mathrm{r}]$' is equivalent to $[[\neg\mathrm{p}\vee\mathrm{q}]\vee[\neg\mathrm{q}\vee\mathrm{r}]]$', being an f-tautologous affirmative 3-fold binary disjunctive PFCLR, while the latter can be written in various equivalent forms such as '$[[\mathrm{q}\vee\neg\mathrm{q}]\vee[\neg\mathrm{p}\vee\mathrm{r}]]$', '$[[[\mathrm{q}\vee\neg\mathrm{q}]\vee\neg\mathrm{p}]\vee\mathrm{r}]$', or '$[\neg\mathrm{p}\vee\mathrm{r}]\vee[\mathrm{q}\vee\neg\mathrm{q}]$'. In reference to its *principal binary disjunctive operator* [$\vee$], a universally m-true DS of any of the above forms is alternatively called an *m-tautologous* (or *m-tautological*) *binary disjunctive DS*. The sentences "It is raining or it is not raining" and "Abraham Lincoln was the 16th president of the USA or Abraham Lincoln was not the 16th president of the USA" of the form '$p\vee\neg p$', which can be abbreviated as the respective *contracted sentences* "It is or is not raining" and as "Abraham Lincoln was or was not the 16th president of the USA", and the sentence "Abraham Lincoln *was* the 16th president of the USA only if it *is* raining, or it *is* raining only if Brutus loved not Caesar less but Rome more" of the form '$[p\Rightarrow q]\vee[q\Rightarrow r]$' and its variant with "*was not*" in place of "*was*" or that with "*is not*" in place of "*is*" or that with both are some examples of such DS's. In these examples, "raining" should be understood as *raining here* or *there* (e.g. in Broadway of New York) *and now*.

14) For the sake of being specific, let an m-tautologous *n*-fold binary disjunctive DS be given. No matter which one or more of the alternative states of affairs that are mentioned in that sentence are realized here and now, the DS is universally m-true by virtue solely of its *valid syntactic form* that has been imported in it via the pertinent f-tautologous PFCLR (as '$p\vee\neg p$' or '$[p\Rightarrow q]\vee[q\Rightarrow r]$') from the conformal euautographic interpretans of the latter (as $p\vee\neg p$ or $[p\Rightarrow q]\vee[q\Rightarrow r]$ respectively). Consequently, like the f-tautologous PFCLR, being the immediate catlogographic interpretans of the m-tautologous DS, and also like the valid PFER, being its immediate euautographic interpretans of the f-tautologous PFCLR, the m-tautologous DS in question can always be used *assertively*. However, when used assertively, that sentence denotes a certain abstract object (state of affairs), which differs from any one of the *n* separate states of affairs conformable to its *n*+1 disjunctive clauses, – an object that can be defined as follows. The *designatum* (*range*) of the given m-tautologous *n*-fold binary disjunctive sentence is *the union of the designata of its n disjuncts* (*clauses*). Consequently, when I assert the sentence, I use it, along with its designatum, in *the projective* (*polarized*, *extensional*, *connotative*) *mental mode* (cf., e.g., Df 2.8(2b)), in which I *mentally experience* the designatum as *my as if extramental* (*exopsychical*) *object* that I call a *common* (*general*, *certain*, *particular but not particularized*) *element* (*member*) *of the designatum* and also a *common denotatum of the sentence*. The common element of the designatum *represents the whole designatum*, thus being just *another hypostasis* (*way of existence*, *aspect*) *of the latter*. In this case, I also say that both *the sentence and its* [*original*, *unpolarized*] *designatum are used for mentioning* the common denotatum of the sentence, i.e. the common element of the designatum of the

sentence, or that, less explicitly, *they are used but not mentioned*, whereas the designatum is said to be *connoted by*, or to be *the connotatum* (*connotation value*, pl. "*connotata*") *of*, *the sentence*. Thus, an m-tautologous *n*-fold binary disjunctive sentence is a *common sentence* or more specifically an *m-tautologous common sentence* in contrast to a *vravr-neutral common sentence* (cf. the item 9.iv of this definition).

15) The negation of a valid PFER is an antivalid PFER and vice versa. Therefore, the negation of an f-tautologous PFCLR is an f-antitautologous (universally f-antitrue, universally f-antitrue, f-contradictory) PFCLR and vice versa. The last statement remains true with "m" in place of "f" and "DS" in place of "PFCLR". Since $\neg[p_1 \vee p_2] \to [p_2 \wedge p_1]$, therefore '¬[*p*∨¬*p*]' is equivalent to '*p*∧¬*p*', whereas

'¬[[*p*⇒*q*]∨[*q*⇒*r*]]', '¬[[¬*p*∨*q*]∨[¬*q*∨*r*]]', '¬[[*q*∨¬*q*]∨[¬*p*∨*r*]]',

'¬[[[*q*∨¬*q*]∨¬*p*]∨*r*]', and '¬[[¬*p*∨*r*]∨[*q*∨¬*q*]]',

e.g., are equivalent to

'[*p*⇒*q*]∧[*q*⇒*r*]', '[¬*p*∨*q*]∧[¬*q*∨*r*]', '¬[*q*∨¬*q*]∧[¬*p*∨*r*]',

'[[*q*∨¬*q*]∨¬*p*]∧*r*]', and '[[¬*p*∨*r*]∧[*q*∨¬*q*]]',

respectively, and to one another. Any one of the above PFCLR's having the conjunction ∧ as its *principal kernel-sign* is called an *f-antitautologous binary conjunctive PFCLR*. Making use the distributive law for ∧ over ∨:

$$[p_1 \wedge [p_2 \vee p_3]] \Leftrightarrow [[p_1 \wedge p_2] \vee [p_1 \wedge p_3]]$$

and the one that for ∨ over ∧:

$$[p_1 \vee [p_2 \wedge p_3]] \Leftrightarrow [[p_1 \vee p_2] \wedge [p_1 \vee p_3]]$$

in that order, any one of the above f-antitautologous binary conjunctive PFCLR's can be represented as equivalent f-antitautologous PFCLR's of various forms. Still, the main implications of the above-said are the following two. First, *the negation of an f-tautologous binary disjunctive PFCLR is an f-antitautologous binary conjunctive PFCLR and vice versa*. Second, up to the ACLR's used, any f-antitautologous binary *conjunctive* PFCLR other than '*p*∧¬*p*' and other than the variants of '*p*∧¬*p*' with any ACLR of the list (10) in place of '*p*' is reducible either to the f-antitautologous binary conjunctive PFCLR '[*p*∧¬*p*]∧***R***' or to its pertinent variant, ***R*** being a certain PFCLR.

The above two implications apply, *mutatis mutandis*, with “m” in place of “f” and “DS” in place of “PFCLR”. At the same time, no mutually exclusive states of affairs, as that of raining and that of not raining, can happen in the same place at the same time. Therefore, the designatum of an m-antitautologous binary conjunctive DS, being the intersection of the designata of its conjuncts, i.e. of the disjuncts of the respective m-tautologous binary disjunctive sentence, is the *empty class*. But, a sentence whose designatum is empty cannot be used in the projective (polarized, extensional, connotative) mental mode for denoting (mentioning) a common member of the designatum simply because the designatum has no members. That is to say, an m-antitautologous (m-contradictory) DS can *never be used assertively*, so that it can be classified as an *unassertive universally antitrue and hence meaningless sentence*, the understanding being that it is neither a proper sentence nor a common one. *In contrast to a paralogous DS, which is meaningless along with its negation*, the negation of an m-antitautologous DS is the respective *meaningful* m-tautologous DS. All m-antitautologous DS’s can be disregarded for being impracticable. The notion of m-antitautologous DS’s is however necessary for completeness of the following specific taxonomy of DS’s and also for completeness of the general taxonomy of DS’s that is established in the next item.

i) A DS that is *neither m-tautologous nor m-antitautologous* (*nor m-contradictory*) is said to be *neutral* (*indeterminate*) *with respect to the tautologousness-values m-tautologousness* (*universal m-truth*) *and m-antitautologousness* (*universal m-antitruth*, *universal m-falsehood*, *m-contradictoriness*) – briefly an *m-ttatt-neutral* (*m-ttatt-indeterminate*) *sentence*.

ii) An m-tautologous sentence is alternatively called an *m-tautologous common sentence* – in contrast to *m-vravr-neutral common sentences* (cf. the items 9.iv and 14).

iii) It is understood that *an m-veracious*, *m-antiveracious*, or *m-vravr-neutral sentence* is *a ttatt-neutral* (*ttatt-indeterminate*) *sentence* and vice versa. Hence, if a ttatt-neutral sentence is asserted then it is supposed to be an m-veracious one.

iv) A DS is said to be: (a) *m-atautologous* if is m-antitautologous or m-ttatt-neutral, (b) *m-non-antitautologous* if it is m-tautologous or m-ttatt-neutral, (c) *m-ttatt-unnutral* (*m-ttatt-determinate*) if it is m-tautologous or m-antitautologous.

v) The division of DS’s into three classes: m-tautologous, m-antitautologous (m-contradictory), and m-ttatt-neutral (m-ttatt-indeterminate) is called the *specific primary* (or *specific basic*) *trichotomy* (*trisection*, *trifurcation*) *of the DS’s*. The three divisions of the class of DS’s into two complementary classes each, namely: (a′) *m-tautologous* and *m-atautologous*, (b′) *m-antitautologous* and *m-non-antitautologous*, (c′) *ttatt-neutral* (*ttatt-inteterminate*) and *ttatt-unneutral* (*ttatt-determinate*) are called the *specific secondary* (or *specific subsidiary*) *dichotomies* (*bisections*, *bifurcations*) *of the DS’s*.

16) In this treatise, the notion of m-tautologies as statements that are universally m-true by virtue solely of the *abstract validity* (and hence by virtue of the *f-tautogousness*) *of their syntactic forms* applies, not only to a syntactically valid affirmative *n*-fold binary disjunctive or conjunctive sentence with a finite number *n*+1 of coordinated clauses as its *disjuncts* or *conjuncts*, but it also applies to *syntactically valid quantified* (*bound*, *contracted*) *statements*, e.g., of the forms

$$‘\bigvee_u \boldsymbol{P}\langle u\rangle’,\ ‘\bigwedge_v \boldsymbol{Q}\langle v\rangle’,\ ‘\breve{\bigvee}_w \boldsymbol{R}\langle w\rangle’,\ ‘\widehat{\bigvee}_x^1 \boldsymbol{S}\langle x\rangle’,\ ‘\bigvee_y^1 \boldsymbol{P}_1\langle y\rangle’ \tag{2.4}$$

(cf. Df 1.2(3)), whereas ‘$\bigvee_u \boldsymbol{P}\langle u\rangle$’ and ‘$\bigwedge_v \boldsymbol{Q}\langle v\rangle$’ are by definition equivalent to ‘$\neg\bigwedge_u \neg \boldsymbol{P}\langle u\rangle$’ and ‘$\neg\bigvee_v \neg \boldsymbol{Q}\langle v\rangle$’ respectively.

In the general case, each one of the *metalinguistic* relations (2.4) is a schema, whose range is a class of CFCL interpretands of the respective *valid and vav-neutral output ESR’s* (*OptESR’s*) *of* A_1, which are condensed into the range of the respective one of the schemata

$$‘\bigvee_{\mathsf{u}} \mathbf{P}\langle \mathsf{u}\rangle’,\ ‘\bigwedge_{\mathsf{v}} \mathbf{Q}\langle \mathsf{v}\rangle’,\ ‘\breve{\bigvee}_{\mathsf{w}} \mathbf{R}\langle \mathsf{w}\rangle’,\ ‘\widehat{\bigvee}_{\mathsf{x}}^1 \mathbf{S}\langle \mathsf{x}\rangle’,\ ‘\bigvee_{\mathsf{y}}^1 \mathbf{P}_1\langle \mathsf{y}\rangle’. \tag{2.5}$$

The CLR, being the CFLR interpretand of a certain *pseudo-quantified* (*bound, contracted*) *OptESR*, is obtained by the pertinent ones of the substitutions

$$\mathsf{u} \mapsto u,\ \mathsf{v} \mapsto v,\ \mathsf{w} \mapsto w,\ \mathsf{x} \mapsto x,\ \mathsf{y} \mapsto y,\ \mathsf{z} \mapsto z, \tag{2.6}$$

$$\mathsf{0} \mapsto \varnothing,\ \mathsf{0}' \mapsto \varnothing', \tag{2.7}$$

along with the pertinent ones of the substitutions of (2.3), when applicable, throughout the OptESR. The *substituentia* that are determined by the above substitutions are the following *primary atomic euautographic formulas* (*categoremata*) in those types:

a) *pseudo-variable ordinary terms* (*PVOT’s*):

$$\mathsf{u} \text{ to } \mathsf{z},\ \mathsf{u}_1 \text{ to } \mathsf{z}_1,\ \mathsf{u}_2 \text{ to } \mathsf{z}_2, \text{ etc;} \tag{2.8}$$

b) *pseudo-constant ordinary terms* (*PCOT’s*):

$$\emptyset \text{ and } \emptyset', \tag{2.9}$$

the first of which is a *systemic* (*permanent*) one, called the *euautographic ordinary zero,* or *pseudo-empty, term* (*EOZT* or *EOPET*), while the second one, called the *subsidiary* (*temporary*) *EOZT* or *EOPET*, is used exclusively for proving that $\emptyset = \emptyset'$, i.e. that $\emptyset$ is unique, and is disregarded after doing this duty.

A PVOT or a PCOT is indiscriminately called a *euautographic ordinary term* (*EOT*). An EOT or an AER is indiscriminately called an *atomic euautographic ordinary formula* (*AEOF*) or *atomic euautographic ordinary categorem*. The *substituenda* that are determined by the substitutions (2.8) and (2.7) are the following *primary atomic catlogographic formulas* (*categoremata*) in those types:

a′) *variable catlogographic ordinary terms* (*VCLOT's*):

$$\text{‘}u\text{’ to ‘}z\text{’, ‘}u_1\text{’ to ‘}z_1\text{’, ‘}u_2\text{’ to ‘}z_2\text{’, etc;} \tag{2.10}$$

b′) *constant catlogographic ordinary terms* (*CCLOT's*):

$$\text{‘}\varnothing\text{’ and ‘}\varnothing'\text{’}, \tag{2.11}$$

the first of which is called the *systemic catlogographic ordinary zero*, or *empty*, *term* (*CLOZT* or *CLOET*), whereas the second one is called the *subsidiary* CLOZT (CLOET); ‘∅′’ is introduced exclusively as the CFCL interpretand of $\emptyset'$, so that ∅′=∅, in accordance with the aabpve point b.

A VCLOT or a CCLOT is indiscriminately called an *catlogographic*, or redundantly *atomic catlogographic*, *ordinary term* (briefly *CLOT* or *ACLOT*). An ACLOT (CLOT) or an ACLR (*AVCLR*) is indiscriminately called an *ordinary catlexigraph* or *atomic ordinary catlogograph* and also an *atomic catlogographic ordinary formula* (*ACLOF*) or *atomic catlogographic ordinary categorem*. It is understood that, besides the dummy (bound) PVOT *u*, *v*, *w*, *x*, or *y*, every occurrence of which in the respective operata **P**⟨*u*⟩, **Q**⟨*v*⟩, **R**⟨*w*⟩, **S**⟨*x*⟩, or $\mathbf{P_1}$⟨*y*⟩ is bound to its first occurrence in the pertinent quantifier (binder, contractor), the operata may involve occurrences (tokens) of some other PVOT's of the list (2.8) (free, bound, or both), occurrences of $\emptyset$, and occurrences of some ACLR's of the list (2.2), – the occurrences, which are not indicated in the schemata (2.5). Consequently, it is supposed that the respective operata ***P***⟨‘*u*’⟩, ***Q***⟨‘*v*’⟩, ***R***⟨‘*w*’⟩, ***S***⟨‘*x*’⟩, or $\boldsymbol{P_1}$⟨‘*y*’⟩ involves occurrences (tokens) of the CFCL interpretands of the latent ACLOF's, which are determined by the substitutions (2.3), (2.6), and (2.7).

17) This is not the appropriate place for discussing any ESR's comprised in the ranges of the placeholders (2.5) or any CLR's comprised in the ranges of the placeholders (2.4) in detail. I shall only remark that, for instance, a CLR $\bigvee_u \boldsymbol{P}\langle u\rangle$ can be regarded as a *disjunction of an infinite number of disjuncts*, whereas $\bigwedge_v \boldsymbol{Q}\langle v\rangle$ can be regarded as a *conjunction of an infinite number of conjuncts*. Also, given a natural domain, the tokens of VCLOT's of the list (2.10), occurring in a given CLR of the range of a certain placeholder of the list (2.4), can be assigned with the appropriate classes as their designata (ranges), such as some taxa of a *biological taxonomy of bionts* or such as some sets of mathematical objects, e.g. the sets of various numbers, the underlying set of vectors of a linear space, the underlying set of points of an affine space, etc. At the same time, the tokens of CLKS's, occurring in the CLR can be provided with the wordy interpretands that have been given in Df 1.2(3). In the result, the CLR turns into a DS of *rich* written English or of another *rich* WNL, i.e. of a WNL that is augmented (enriched) by all necessary nomenclature (logographic notation and wordy terminology). In this case, the DS is said to be *m*-tautologous (*materially* tautologous) if the CLR is *f*-tautologous (*formally* tautologous) and *m*-ttatt-neutral (*m*-ttatt-indeterminate) if the CLR is *f*-ttatt-neutral (*f*-ttatt-indeterminate). The negation of an *m*-tautologous DS said to be *m*-antitautologous DS, whereas an *m*-ttatt-neutral DS is more specifically said to be an m-veracious, m-antiveracious, or m-vravr-neutral if the ttatt-neutral CLR, being its CFCL interpretans (form, schema), is f-veracious, f-antiveracious, or f-vravr-neutral respectively. Thus, the specific taxonomy of DS's that has been explicated in the previous item includes both *quantifier-free* and *quantifier-involving DS's.* Also, all specific taxonomies of DS's that have been discussed in the items 6–15 and above in this item are generalized (unified) as follows.

i) A DS is said to be:

a) *m-true* if it is either *m-tautologous* (*universally m-true*) or *m-veracious* (*accidentally m-true*);

b) *m-antitrue* or *m-false* if it is either *m-antitautologous* (*universally m-antitrue, universally m-false, m-contradictory*) or *m-antiveracious* (*accidentally m-antitrue*, *accidentally m-false*);

c) *neutral* (*indeterminate*) *with respect to the truth-values m-truth and m-antitruth*, i.e. *neither m-true nor m-antitrue,* – briefly *m-tat-neutral* (*m-tat-indeterminate*), if it is *m-vravr-neutral* (*m-vravr-indeterminate*).

In this case, the negation of an m-true sentence is an m-antitrue (m-false) sentence and vice versa, whereas the negation of an m-tat-neutral (m-tat-indeterminate) sentence is another m-tat-neutral (m-tat-indeterminate) sentence. The generic qualifiers "tat-neutral" ("tat-indeterminate") and "vravr-neutral" ("vravr-indeterminate") are synonyms.

ii) An m-tautologous sentence or a common m-veracious is indiscriminately called a *common m-true sentence* (cf. the items 9.iv and 15.ii).

iii) A DS is said to be: (a) *m-untrue* if it is m-antitrue or m-tat-neutral (m-tat-indeterminate); (b) *m-non-antitrue* or *m-non-false* if it is m-true or m-tat-neutral (m-tat-indeterminate); (c) *m-tat-unnutral* or *m-tat-determinate* if it is m-true or m-antitrue (m-false).

iv) The division of the class of DS's into three classes: *m-true*, *m-antitrue* (*m-false*), and *m-tat-neutral* (*m-tat-indeterminate*) is called the *general primary* (or *general basic*) *trichotomy* (*trisection*, *trifurcation*) *of the declarative sentences.* The three divisions of the class of DS's into two complementary classes each, namely: (a′) *m-true* and *m-untrue*, (b′) *m-antitrue* and *m-non-antitrue* (*m-non-false*), (c′) *m-tat-neutral* (*m-tat-inteterminate*) and *m-tat-unneutral* (*m-tat-determinate*) are called the *general secondary* (or *general subsidiary*) *dichotomies* (*bisections*, *bifurcations*) *of the DS's*.

18) The term "tautology" has arisen in logic after Wittgenstein [1921]. My use of the notion of m-tautologies as statements that are universally m-true by virtue solely of the *abstract validity* (and hence by virtue of *f-tautogousness*) *of their syntactic forms* agrees with his use of the notion of tautology in application to both quantifier-free and quantified truth-functional statements – the use that has been adopted in the modern dualistic truth-functional logic (cf. Quine [1951, p. 55]). At the same time, Wittgenstein suggested as a thesis the doctrine that all logic and all mathematics are tautological, which is of course wrong. For instance, 15 of 19 categorical syllogisms are tautological ones, while the remaining 4 are veracious ttatt-neutral ones (to be demonstrated in due course). Also, the class of ttatt-neutral CLR's is likely the main source of mathematical postulates (axioms and hypotheses) and

mathematical theorems, which are therefore veracious (accidentally true) and not tautological (not universally true).

19) In what follows, after the manner of the established (dictionary) Wittgenstein's *monomial synonym* "tauto*logy*" of the *binomial metaterm* (*metalinguistic term*) "tautologous *relation*" (or "tautological *relation*"), I shall, for the sake of brevity, introduce *monomial synonyms of the connotative binomial taxonyms of all decision classes of ER's*, *CLR's*, and *DS's*, which have been introduced previously. To this end, I shall adhere the appropriate Anglicized prefixes of Greek origin, existing (dictionary) ones or new ones of my own, to the English postpositive combining form "*logy*" of the same origin, which in this use means «oral or written expression 〈phraseo*logy*〉», – according to WTNID. The etymological senses of the pertinent prefixes are given in Dict A1.1, which has been compiled primarily on the basis of Pring [1982]. The hierarchy of the English privative prefixes, which I establish and employ in stating the basic decisional trichotomies and the associated decisional dichotomies of relations of various kinds is sxplicated in Appendix 2. The alternative Greacized decisional terminology is optional. In the sequel, I shall not use the whole of it. But certain, most expressive elements of it will be used interchangeably with the respective synonymous basic metaterms that have been introduced previously.

i) A *valid*, *antivalid*, *vav-neutral* (*vav-indeterminate*), *invalid*, *non-antivalid*, or *vav-unnutral* (*vav-determinate*) *ER* is synonymously) called a *kyrology*, *antikyrology*, *kak-udeterology* (*kak-anorismenology*), *akyrology*, *anantikyrology*, or *kak-anudeterology* (*kak-orismenology*) respectively. Consequently, a kyrology is either a euautographic axiom or a euautographic theorem, whereas an antikyrology is either a euautographic antiaxiom or a euautographic antitheorem. "*Kak*" is an abbreviation for "*kyrology-antikyrology*", so that "kak-udeterology" ("kak-anorismenology") means *neither a kyrology nor an antikyrology*.

ii) An *f-tautologous*, *f-antitautologous*, *f-ttatt-neutral* (*f-ttatt-indeterminate*), *f-untautologous*, *f-non-antitautologous*, or *f-ttatt-unnutral* (*f-ttatt-determinate*) *CLR* is synonymously called a *f-tautology*, *f-antitautology*, *f-ttatt-udeterology* (*f-ttatt-anorismenology*), *f-atautology*, *f-anantitautology*, or *f-ttatt-anudeterology* (*f-ttatt-orismenology*) respectively. "*Ttatt*" is an abbreviation for "*tautology-antitautology*", so that “ttatt-udeterology” (“ttatt-anorismenology”) means *neither a tautology nor an antitautology*.

iii) An *f-veracious*, *f-antiveracious*, *f-vravr-neutral* (*f-vravr-indeterminate*), *f-unveracious*, *f-non-antiveracious*, or *f-vravr-unnutral* (*f-vravr-determinate*) *CLR* is synonymously called an *f-philalythiology*, *f-antiphilalythiology*, *f-paapa-udeterology* (*f-paapa-anorismenology*), *f-aphilalythiology*, *f-anantiphilalythiology*, or *f-paapa-anudeterology* (*f-paapa-orismenology*) respectively. “*Paapa*” is an abbreviation for “*philalythiology-antiphilalythiology*”, so that “paapa-udeterology” (“paapa-anorismenology”) means *neither a philalythiology nor an antiphilalythiology*.

iv) An *f-true*, *f-antitrue*, *f-tat-neutral* (*f-tat-indeterminate*), *f-untrue*, *f-non-antitrue*, or *f-tat-unnutral* (*f-tat-determinate*) *CLR* is synonymously called an *f-alythiology*, *f-antialythiology*, *f-aaa-udeterology* (*f-aaa-anorismenology*), *f-analythiology*, *f-anantialythiology*, or *f-aaa-anudeterology* (*f-aaa-orismenology*) respectively. “*Aaa*” is an abbreviation for “*alythiology-antialythiology*”, so that “aaa-udeterology” (“aaa-anorismenology”) means *neither an alythiology nor an antialythiology*.

v) The above points a)–d) apply with “m” (“materially”) in place of “f” (“formally”) and with “DS” in place of “CLR”.

vi) In accordance with Pring [1982] (see also Dict A1.1), the prefix “*kyro*” is derived from the Greek noun “κῦρος” \kíros, kýros\, meaning *validity*; the prefix “*udetero*” is derived from the Greek adjective “ουδέτερος” \uðéteros, uthéteros\, meaning *neutral* or (gram.) *neuter* and from the homonymous pronoun, meaning *neither*; the prefix “*orismeno*” is derived from the Greek adjective “ωρισμένος” \orisménos\ meaning *determinate, determined*, or *certain*; the prefix “*tautolo*” is derived from the Greek noun “ταυτολογία’’ \taftolojía\, meaning *tautology*; the prefix “*philalythio*” is derived from the Greek noun “φιλαλήθεια” \filalíθia\, meaning *veracity*, and from the kindred adjective “φιλαλήθης” \filalíθis\, meaning *veracious*; the prefix “*alythio*” is derived from the Greek noun “αλήθειας” \alíthias\, meaning *truth*. Consequently, making use of the the first letters ‘κ’, ‘τ’, ‘φ’, and ‘α’ of the above Greek etymons, the hyphenated prepositive abbreviated qualifiers “kak”, “ttatt”, “paapa”, and “aaa” to any one of the generic names: “udeterology”, “anorismenology”, “unudeterology”, and “orismenology” can be used interchangeably with ‘κακ’, ‘τατ’, ‘φαφ’, and ‘ααα’ respectively; the middle letter ‘α’ in any of the latter four abbreviations stands for the Greek combining form “άντι” \ánti\ denoting *opposition, opposite situation,* or *negation.* In this case, in order not to violate the EHP (see Cmt 1.12(2)), the first letter ‘μ’ of the Greek noun “μορφή” \morfí\ (dual “μορφά” \morfá\, pl. “μορφαί” \morfé\), meaning *form,* should be used instead of the abbreviation “f” for “formally” and the first letter ‘υ’ of the Greek noun “ύλη” \íli\ (pl. “ύλαι” \íle\), meaning *matter*, should be used instead of the abbreviation “m” for “formally” (see. Cmt 1.1(3) for “hylomorphism”).

vii) The “*graph*”-terms: “*kyrograph*” “*antikyrograph*”, “*udeterograph*”, “*akyrograph*”, etc as abbreviations of “*kyrographonym*”, “*antikyrographonym*”, “*udeterographonym*”, “*akyrographonym*”, etc are the appropriate *onymological* synonyms of “valid relation”, “antivalid relation”, “neutral relation”, “invalid relation”, etc, respectively. I have however decided to form and use the respective “l*ogy*”-terms: “kyrology”, “antikyrology”, “udeterology”, “akyrology”, etc because these are morphologically similar to the established term “tautology”. If I had used the above abbreviated “graph”-terms then the similar metaterms: “*tautograph*”, “*antitautograph*”, “*atautograph*”, etc, being abbreviation of the respective full “*graphonym*”-terms: “*tautographonym*”, “*antitautographonym*”, “*atautographnym*”, etc, should have been used instead of the pertinent “logy”-terms.

20) In addition to the above *connotative monomial wordy taxonom*y of the various decisional classes of ER’s, CLR’s, and *DS’s*, it is ionstructive to introduce the following taxonomic system of logographic notations and wordy names of those classes – in contrast to the distributive common names of their members:

i) ‘κ_+’, ‘κ_-’, and ‘$\kappa_\sim$’ denote the classes of valid, antivalid, and vav-neutral (vav-indeterminate), i.e. of kyrologous, antikyrologous, and kak-udeterologous (kak-anorismenologous), ER’s of A_1 respectively. Consequently, $\kappa_- \cup \kappa_\sim$, $\kappa_+ \cup \kappa_\sim$, and a$\kappa_+ \cup \kappa_-$ are the classes of invalid, non-antivalid, and vav-unnutral (vav-determinate), i.e. of akyrologous, anantikyrologous, and kak-anudeterologous (kak-orismenologous), ER’s of A_1 respectively.

ii) ‘τ_+^μ’, ‘τ_-^μ’, and ‘$\tau_\sim^\mu$’ denote the classes of f-tautologous, f-antitautologous (f-contradictory), and f-ttatt-neutral (f-ttatt-indeterminate) CLR’s respectively. Consequently, $\tau_-^\mu \cup \tau_\sim^\mu$, $\tau_+^\mu \cup \tau_\sim^\mu$, and $\tau_+^\mu \cup \tau_-^\mu$ are the classes of f-atautologous, f-anantitautologous, and f-ttatt-unneutral (f-ttatt-determinate) CLR’s respectively.

iii) '$\tau^{\mu}_{\sim+}$', '$\tau^{\mu}_{\sim-}$', and '$\tau^{\mu}_{\sim\sim}$', or 'ϕ^{μ}_{+}', 'ϕ^{μ}_{-}', and '$\phi^{\mu}_{\sim}$' denote respectively the classes of f-veracious, f-antiveracious, and f-vravr-neutral (f-vravr-indeterminate) CLR's, the understanding being that $\phi^{\mu}_{+} = \tau^{\mu}_{\sim+}$. $\phi^{\mu}_{-} = \tau^{\mu}_{\sim-}$, and $\phi^{\mu}_{\sim} = \tau^{\mu}_{\sim\sim}$. Consequently, $\tau^{\mu}_{\sim-} \cup \tau^{\mu}_{\sim\sim}$, $\tau^{\mu}_{\sim+} \cup \tau^{\mu}_{\sim\sim}$, and $\tau^{\mu}_{\sim+} \cup \tau^{\mu}_{\sim-}$ are the classes of f-unveracious, f-non-antiveracious, and f-vravr-unneutral (f-vravr-determinate) CLR's respectively.

iv) 'α^{μ}_{+}', 'α^{μ}_{-}', and '$\alpha^{\mu}_{\sim}$' denote respectively the classes of f-true, f-antitrue (f-false), and f-tat-neutral (f-tat-indeterminate) CLR's, the understanding being that $\alpha^{\mu}_{\sim} = \phi^{\mu}_{\sim} = \tau^{\mu}_{\sim\sim}$. Consequently, $\alpha^{\mu}_{-} \cup \alpha^{\mu}_{\sim}$, $\alpha^{\mu}_{+} \cup \alpha^{\mu}_{\sim}$, and $\alpha^{\mu}_{+} \cup \alpha^{\mu}_{-}$ are the classes of f-untrue, f-non-antitrue (f-non-false), and f-tat-unneutral (f-tat-determinate) CLR's respectively.

v) 'τ^{υ}_{+}', 'τ^{υ}_{-}', and '$\tau^{\upsilon}_{\sim}$' denote the classes of m-tautologous, m-antitautologous (m-contradictory), and m-ttatt-neutral (m-ttatt-indeterminate) DS's respectively. Consequently, $\tau^{\upsilon}_{-} \cup \tau^{\upsilon}_{\sim}$, $\tau^{\upsilon}_{+} \cup \tau^{\upsilon}_{\sim}$, and $\tau^{\upsilon}_{+} \cup \tau^{\upsilon}_{-}$ are the classes of m-atautologous, m-anantitautologous, and m-ttatt-unneutral (m-ttatt-determinate) DS's respectively.

vi) '$\tau^{\upsilon}_{\sim+}$', '$\tau^{\upsilon}_{\sim-}$', and '$\tau^{\upsilon}_{\sim\sim}$', or '$\phi^{\upsilon}_{+}$', '$\phi^{\upsilon}_{-}$', and '$\phi^{\upsilon}_{\sim}$' denote respectively the classes of m-veracious, m-antiveracious, and m-vravr-neutral (m-vravr-indeterminate) DS's, the understanding being that $\phi^{\upsilon}_{+} = \tau^{\upsilon}_{\sim+}$. $\phi^{\upsilon}_{-} = \tau^{\upsilon}_{\sim-}$, and $\phi^{\upsilon}_{\sim} = \tau^{\upsilon}_{\sim\sim}$. Consequently, $\tau^{\upsilon}_{\sim-} \cup \tau^{\upsilon}_{\sim\sim}$, $\tau^{\upsilon}_{\sim+} \cup \tau^{\upsilon}_{\sim\sim}$, and $\tau^{\upsilon}_{\sim+} \cup \tau^{\upsilon}_{\sim-}$ are the classes of m-unveracious, m-non-antiveracious, and m-vravr-unneutral (m-vravr-determinate) DS's respectively.

vii) 'α^{υ}_{+}', 'α^{υ}_{-}', and '$\alpha^{\upsilon}_{\sim}$' denote respectively the classes of m-true, m-antitrue (m-false), and m-tat-neutral (m-tat-indeterminate) DS's, the understanding being that $\alpha^{\upsilon}_{\sim} = \phi^{\upsilon}_{\sim} = \tau^{\upsilon}_{\sim\sim}$. Consequently, $\alpha^{\upsilon}_{-} \cup \alpha^{\upsilon}_{\sim}$, $\alpha^{\upsilon}_{+} \cup \alpha^{\upsilon}_{\sim}$, and $\alpha^{\upsilon}_{+} \cup \alpha^{\upsilon}_{-}$ are the classes of m-untrue, m-non-antitrue, and m-tat-unneutral (m-tat-determinate) DS's respectively.

viii) In the above notations, the letters 'κ', 'τ', 'ϕ', and 'α' and also the superscripts 'μ' and 'υ' on them are used in accordance with their etymological senses explicated in the item 19.vi.

21) Here follow alternative wordy names of the above classes:

i) The classes κ_{+}, κ_{-}, $\kappa_{\sim}$, $\kappa_{-} \cup \kappa_{\sim}$, $\kappa_{+} \cup \kappa_{\sim}$, and $\kappa_{+} \cup \kappa_{-}$ are alternatively called (in that order) the *validity-values validity*, *antivalidity*, *vav-neutrality* (*vav-indeterminacy*), *unvalidity*, *non-antivalidity*, and *vav-unneutrality* (*vav-determinacy*), and also the *kyrologousness-values kyrologousness*, *antikyrologousness*, *udeterologousness* (*anorismenlogousness*), *akyrologousness, anantikyrologousness,* and *anudeterologousness* (*orismenologousness*), respectively.

ii) The classes τ_{+}^{μ}, τ_{-}^{μ}, $\tau_{\sim}^{\mu}$, $\tau_{-}^{\mu} \cup \tau_{\sim}^{\mu}$, $\tau_{+}^{\mu} \cup \tau_{\sim}^{\mu}$, and $\tau_{+}^{\mu} \cup \tau_{-}^{\mu}$ are alternatively called (in that order) the *f-tautologousness-value f-tautologousness, f-antitautologousness* (*f-contradctoriness*), *f-ttatt-neutrality* (*f-ttatt-indeterminacy*), *f-atautologousness, f-anantitautologousness,* and *f-ttatt-unneutrality* (*f-ttatt-determinacy*) respectively.

iii) The classes $\tau_{\sim+}^{\mu}$, $\tau_{\sim-}^{\mu}$, $\tau_{\sim\sim}^{\mu}$, $\tau_{\sim-}^{\mu} \cup \tau_{\sim\sim}^{\mu}$, $\tau_{\sim+}^{\mu} \cup \tau_{\sim\sim}^{\mu}$, and $\tau_{\sim+}^{\mu} \cup \tau_{\sim-}^{\mu}$ are alternatively called (in that order) the *f-veracity-values f-veracity, f-antiveracity, f-vravr-neutrality* (*f-vravr-indeterminacy*), *f-unveracity, f-non-antiveracity,* and *f-vravr-unneutrality* (*f-vravr-determinacy*).

iv) The classes α_{+}^{μ}, α_{-}^{μ}, $\alpha_{\sim}^{\mu}$, $\alpha_{-}^{\mu} \cup \alpha_{\sim}^{\mu}$, $\alpha_{+}^{\mu} \cup \alpha_{\sim}^{\mu}$, and $\alpha_{+}^{\mu} \cup \alpha_{-}^{\mu}$ are alternatively called (in that order) the *f-truth-values f-t```ruth, f-antitruth* (*f-falsity, f-falsehood*), *f-tat-neutrality* (*f-tat-indeterminacy*), *f-untruth, f-non-antitruth* (*f-non-falsity, f-non-falsehood*), and *f-tat-unneutrality* (*f-tat-determinacy*) respectively.

v) The classes τ_{+}^{υ}, τ_{-}^{υ}, $\tau_{\sim}^{\upsilon}$, $\tau_{-}^{\upsilon} \cup \tau_{\sim}^{\upsilon}$, $\tau_{+}^{\upsilon} \cup \tau_{\sim}^{\upsilon}$, and $\tau_{+}^{\upsilon} \cup \tau_{-}^{\upsilon}$ are alternatively called (in that order) the *m-tautologousness-values m-tautologousness, m-antitautologousness* (*m-contradctoriness*), *m-ttatt-neutrality* (*m-ttatt-indeterminacy*), *m-atautologousness, m-anantitautologousness,* and *m-ttatt-unneutrality* (*m-ttatt-determinacy*) respectively.

vi) The classes $\tau_{\sim+}^{\upsilon}$, $\tau_{\sim-}^{\upsilon}$, ‘$\tau_{\sim\sim}^{\upsilon}$, $\tau_{\sim-}^{\upsilon} \cup \tau_{\sim\sim}^{\upsilon}$, $\tau_{\sim+}^{\upsilon} \cup \tau_{\sim\sim}^{\upsilon}$, and $\tau_{\sim+}^{\upsilon} \cup \tau_{\sim-}^{\upsilon}$ are alternatively called (in that order) the *m-veracity-values m-veracity, m-antiveracity, m-vravr-neutrality* (*m-vravr-indeterminacy*), *m-unveracity, m-non-antiveracity,* and *m-vravr-unneutrality* (*m-vravr-determinacy*) respectively.

vii) The classes α_{+}^{υ}, α_{-}^{υ}, $\alpha_{\sim}^{\upsilon}$, $\alpha_{-}^{\upsilon} \cup \alpha_{\sim}^{\upsilon}$, $\alpha_{+}^{\upsilon} \cup \alpha_{\sim}^{\upsilon}$, and $\alpha_{+}^{\upsilon} \cup \alpha_{-}^{\upsilon}$ are alternatively called (in that order) the *m-truth-values m-truth, m-antitruth* (*m-falsity, m-falsehood*)), *m-tat-neutrality* (*m-tat-indeterminacy*), *m-untruth, m-non-antitruth* (*m-non-falsity, m-non-falsehood*), and *m-tat-unneutrality* (*m-tat-determinacy*) respectively.

viii) The four generic appositive names: “validity-value” (“kyrologousness-value”), “tautologousness-value”, “veracity-value”, and “truth-value” are used interchangeably (synonymously) with their variants with “class” in place of “value” in analogy with the generic name “*numeric value*”, which is conventionally used for mentioning, e.g., any natural number, i.e. any natural *number-class* 0, 1, 2, etc.

22) A ttatt-neutral DS sentence, which is or can be *either veracious* (*accidentally true*) *or antiveracious* (*accidentally antitrue*), and not only vravr-neutral, is called a *veracity-valued* or *veracity-functional* or *propositional sentence*, while the sense of a propositional sentence is called a *propositional sententia* or conventionally a *proposition*. In Aristotelianism, however, *a proposition is a veracity-valued* (*veracity-functional*) *sentence and vice versa*. Accordingly, the adjective “*propositional*” most generally means *of or relating to a proposition*, subject to the respective sense of “proposition”.•

Cmt 2.9. 1) The semantic property of a ttatt-neutral DS to be, with respect to its interpreter (its sender or its receiver), either veracious (accidentally true), – in a case where and when there exists a state of affairs, which the sentence conforms to, or to be tat-neutral (vravr-neutral), – in a case where and when the interpreter knows no state of affairs, which either conforms to or contradicts the sense of the sentence, is inherent in a interrogative, exclamatory, or imperative sentence as well. For instance, I may *imagine* a situation that involves me and another person, in which I *utter and address to that person*, for instance, the following five sentences, in the appropriate moments of our conversation:

«It is raining.» (2.12)

«What nasty weather we are having today! » (2.13)

«Please sit down. » (2.14)

«Are you hungry?» (2.15)

«What are you going to eat?» (2.16)

The sentence (2.12) is a simple unextended affirmative DS that has been quoted on the list 10.c of Df 2.15 and that has been discussed in detail in that definition, starting from its item 6; (2.13) is a simple extended exclamatory sentence; (2.14) is a simple extended imperative sentence; (2.15) and (2.16) are simple interrogative sentences, an unextended one and an extended one respectively. Instead of (2.15), I may say:

«I am hungry. Are you?», (2.15a)

thus using another DS of the list 10.c of Df 2.15 along with the appropriate abbreviation of the sentence (2.15). Just as the sentence (2.12) or the first sentence (2.15a), I *associate* any one of the sentences (2.13)–(2.16) with a certain class of states of affairs or, alternatively but equivalently, with a common (general, certain, concrete but not realized) member of the class, any realized instance of which becomes *the current concrete state of affairs, which the sense* (*sententia*) of *the sentence in question and hence the sentence itself conforms to*. The above class of states of affairs is said to be *designated by*, or to be *the designatum*, *of that sentence.*

2) Thus, there is an indefinite number of *transient psychophysical* (*physopsychical*) *circumstances*, in which I can *utter* (*produce an assertive spoken paratoken of*) some one of the sentences (2.12)–(2.16) and (2.15a) in order to put forward and to communicate the pertinent current state of affairs, which that sentence conforms to, to the pertinent person or persons. In such circumstances, the state of affairs that I put into correspondence to the sense of the uttered sentence is said to be *denoted by*, and also to be the *denotatum* or *meaning*, of the sentence, whereas the sentence itself is said to be *meaningful*, – with respect to me. By extrapolation, if some one or some more of the sentences (2.12)–(2.16) and (2.15a) are either *transmitted* (*uttered*) or *received* by an interpreter other than me then that sentence denotes the corresponding state of affairs with respect to that interpreter. In this respect, each one of the sentences in question is analogous to an ordinary phonographic (wordy) limited common name as ‘a man’ or ‘a tree’, and it is also analogous to a logographic variable such as ‘*n*’ that takes on numbers of a certain set as its accidental values. Therefore, I say that each such sentence is a *common sentence*, no matter of which kind it is. At the same time, the assertive imperative sentences of a form, e.g., “Let us …” or “Let – be …” are equivalent to certain assertive veracious (accidentally true) declarative sentences, proper or common. Consequently, an imperative sentence of this or of any other form should be regarded as a veracious *proper* one if it is equivalent to a certain veracious proper declarative sentence and as a veracious *common* one if it is equivalent to a certain veracious common declarative sentence. For instance, the sentence (2.14) can be restated, e.g., as the veracious common declarative sentence: «I invite you to take a seat.»

3) It follows from the above items 1 and 2 that the subject matter of Df 2.15(11.b), which is relevant to *m-vravr-neutral common DS’s*, applies, *mutatis mutandis,* also to interrogative, exclamatory, and imperative sentences In this case, “*mutatis mutandis*”, i.e. “*with the corresponding changes*” means that some terminology that has been introduced for DS’s should be changed properly, while

some other should be disregarded. For instance, in accordance with the points b–d of Df 2.15(1), the state of affairs that is denoted by an assertive sentence is a *question* or an *emotion* or a *command* or *request* of the maker of the sentence if the sentence is respectively interrogative or exclamatory or imperative. The questions are of two kinds: *general questions* and *special questions*. An assertive interrogative sentence (as (2.15)), which requires a *general answer* «yes» or «not», denotes a *general question*, and it is therefore called a *general-question-valued sentence*. An assertive interrogative sentence (as (2.16)), which requires a *special* (*concrete*) *answer*, denotes a *special question* and it is therefore called a *special-question-valued sentence*. Also, it would be owkward to say that an asserted (purposeful) written or spoken interrogative sentence, which denotes a raised question requiring answer, is a veracious (accidentally true) one, because in this case there is no anticipation of denial of the question. Such a sentence should naturally be characterized as *active* or *effective*. A like remark applies to an exclamatory sentence. However, in the case of imperative sentences, a certain part of the taxonomy of declarative sentences may be retained. For instance, in order to make statements about a large number (usually an infinite number) of DS's, I use homolographic tokens of the letters '*p*', '*q*', '*r*', and '*s*' in this typeface (alone or furnished with Arabic numerals '$_1$', '$_2$', etc) as atomic placeholders whose range is the class of *simple declarative affirmative sentences* (*SDAS*) of the written English language or of any other given WNL. Consequently, using for instance '*p*' for mentioning a common member of its range, I can assert that *p is* an SDAS. I can then restrict the range of '*p*', e.g., to veracious (accidentally true) SDAS's by asserting the *imperative sentence-schema*: "Let *p* be veracious". This sentence-schema is equivalent to the veracious declatative sentence-schema "*p* is a veracious SDAS", and therefore it can also be regarded as veracious. In this case, the declarative sentence-schema "*p* is not a veracious SDAS" is an antiveracious one, which is equivalent to the imperative sentence-schema: "Let *p* not to be veracious". Therefore, the latter can also be regarded as antiveracious, the understanding being that both sentence schemata are *unassertive*.

4) In accordance with Df 2.15(3), the semantic property of a sentence of any kind to conform or to contradict a certain state of affairs or else to be distracted from any state of affairs existing here and now is a *matter-of-fact property*, i.e. a property concerned with facts and *not an imaginative or fanciful one.* However, the entire

discussion, being the subject matter of the previous items 1–3 of this comment, is based on some *imaginative* situations and particularly on some *imaginative states of affairs*, which the sentences (2.12)–(2.16) and (2.15a) can conform to, contradict, or be distracted from. That discussion is possible owing to the fact that thought of a sapient subject is *intensional* and not *extensional* in the sense that it can go on without being anchored down to any specific extramental entities, being its objects (cf. Hofstadter [1980, p. 338]). Thoughts can be and most often are anchored down to exteroceptive symbols, graphic (ideographs) or phonic (ideophons), and especially to graphic or phonic sentences, – symbols, with the help of which thoughts are framed and intercommunicated. The intensionality of thought allows people to create abstract axiomatic theories and works of fictions, but at the same time it is a source of paradoxes and means of deceptions.

5) In accordance with the above discussion. the sentences of all four kinds are objects of *material logic*. However, the main difference of principle between DS's on the one hand and interrogative, exclamatory, and imperative sentences on the other hand is that there is a certain class of formalized DS's, which can, in accordance with the *forms* of its members, be *trisected* (*trifurcated*) *into the classes of m-tautologous* (*universally m-true*), *m-antitautologous* (*m-contradictory, universally m-antitrue, universally m-false*), and *m-ttatt-neutral* (*m-ttatt-indeterminate*) *DS's* as the *ultimate interpretands* of the respective *underlying valid, antivalid,* and *vav-neutral* (*vav-indeterminate*) *ER's of the organon* A_1, belonging entirely to *formal logic.* The organon A_1 does not underlie interrogative, exclamatory, and imperative sentences. However, all these can, without loss of generality, be classified as *ttatt-neutral* (*ttatt-indeterminate*) in analogy with *ttatt-neutral* (*ttatt-indeterminate*) DS's.•

Cmt 2.10. 1) I have adopted the verbs "to *denote*" and "to *express*" from Church [1956, pp. 4, 6, footnotes 7, 16], who in turn uses the verbs "to *denote*" and "to *name*" as two synonymous translations of the Frege [1892] verb "*bedeuten*" and who also uses the verb "to *express*" both as a translation of Frege's verb "*drückten aus*" (*ibid*.) and as a close synonym of the Mill [1843] verb "to *connote*" in Mill's original meaning of the verb – the meaning that differs from all other meanings which the verb has since acquired in common English usage.

2) In *the Frege-Church theory of the meaning of proper names and proper declarotive sentences* (Frege [1892], Church [1956, pp. 3–9, 25–28]), *unassertive*

proper declarative sentences are regarded as *proper names* such that a true sentence *denotes* (*names*) *the truth-value truth* and a false sentence *denotes the truth-value falsity* (*falsehood*). The theory of meaning, which I have developed above in this sectiom, radically differs from the Frege-Church theory.

3) In connection with his use of the singular third-person verbs “denotes” and “names” as two synonymous predicates of a proper name, Church particularly says (*ibid*, footnote 7):

> «We thus translate Frege’s *bedeuten* by *denote* and *name*. The verb to *mean* we reserve for general use, in reference to possible different kinds of meaning.»

In accordance with the presently common practice, I use the noun “*meaning*” and its kindred verb “to *mean*” somewhat differently from what Church declares to do. Namely, on the one hand, I use “meaning” as a synonym of “denotatum” (“denotation value”) and “to mean” as a synonym of “to denote”, i.e. “to put forward” or “to mention”. On the other hand, any import value of a xenograph (as its sense-operation, xenodesignatum, or sense) can mentally be put forward to become its denotatum. Therefore, *a* meaning of the xenograph is *any one* of its import values, which is *supposedly* turned into its denotatum, while *the* meaning of the xenograph is a *certain one* of its import values, which is *actually* turned into its denotatum. Accordingly, the antonymous adjective derivatives “*meaningful*” and “*meaningless*” of the noun “meaning” *mean* «*having a denotatum*» and «*not having a denotatum*» respectively As an example, see the discussion of the sentences “It is raining” and “It is not raining” in Df 2.15(6).

4) As used in this treatise, the verbs “to express” and “to connote” are not synonyms either. The verb “expresses” expresses the general relation between a glossonym (linguistic name) as referent and its sense as the pertinent relatum, – no matter whether the glossonym is used purposefully as a euxenonym or whether it is just considered. By contrast, the verb “connotes” expresses the relation between the glossonym and its sense (including its designatum) in case when the glossonym is used together with its sense as a direct euxenonym for denoting (mentioning, referring to, putting forward) the pertinent import value of the glossonym. Thus, the sense that I attach to the verb “to express” is broader than that attached to it by Church, whereas my use of the verb “to connote” agrees with its use by Mill and Church.

5) Church [1956, p. 6, footnote 17] uses the term "*concept of*" as a close synonym of the term "*class-concept of*" of Russell [1903, §69]. Particularly, he says: «Of the sense we say that it *determines* the denotation, or *is a concept*[17] of the denotation.» This statement applies to the denotatation (denotatum) of a proper name and it is also supposed to be applicable to the denotation of a proper declarative propositional sentence, in accordance with the Frege-Church theory of the meaning that is mentioned above in the item 2 of this comment. My terms "concept" as defined in Df 2.9(4) and "class-concept" as defined in Df 2.9(5,11) are *homographs* of those of Church and Russell. Also, my definition of "sense" as given in Cmt 1.9(2) and Dfs 2.14 and 2.15 differs from that of Church.•

2.4.6. Explication of the binary operation of concatenation underlying a descriptive specific name (DSN)

Df 2.16. 1) The morphemes forming a monomial DSN (MDSN) or the words forming a polynomial DSN (PDSN) are *juxtaposed syntactically* and *concatenated semantically*. However, the *elemental mental designatum-producing (semantic) operations of concatenation of the designative units of a DSN* have so far been implicit. In other words, whatever those operations are, a DSN involves *no operators* to indicate them, so that the above operations are *latent*. In order to explicate them, I assume that the *entire designatum-producing operation of concatenation in extension on the designative units of a DSN*, i.e. on the pertinent *GN* and pertient *qualifiers*, – the operation that is also called a *sense-operation on the DSN*, is a *sequence of properly associated* (*grouped*) *binary sense-operations of concatenation in extension*. *The elemental binary designatum-producing operation of concatenation in intension* will be denoted by '$^\wedge$' and be called the [*latent*] *concatenation operator*. The designative units that are concatenated by a given occurrence (token) of '$^\wedge$' are called the *operata* (singular "*operatum*") of the latter or, more explicitly, the *percept-operata* or *operatum-percepts* – as opposed to their denotata to be called the *concept-operata* or *operatum-concepts* and also the *class-operata* or *operatum-classes*; the latter are the *operata of the operation* $^\wedge$ denoted by the given occurrence of '$^\wedge$'.

2) Under the assumption that, in the general case, a given DSN involves two or more qualifiers to its constituent GN, some of which are prepositive and the other ones are postpositive, the consecutive binary designatum-producing operations of concatenation in extension on the designative units of the given DSN are supposed to

be performed in the following order, unless stated otherwise. The operations involving the prepositive qualifiers are performed in the first place in the order of *association* (*grouping*) *to the right of the GN*. That is to say, the GN is at first concatenated with the prepositive qualifier immediately preceding it, thus resulting *the first interim rightward description*, which can alternatively be called either *the first interim rightward DSN* or *the first interim rightward GN*; then this interim description is concatenated with the prepositive qualifier immediately preceding it, thus resulting in *the second interim rightward description*; which can alternatively be called either *the second interim rightward DSN* or *the second interim rightward GN*; etc until the leftmost (first) prepositive qualifier is concatenated with the interim rightward description succeeding it, thus resulting in *the final description*, which can alternatively be called either *the final rightward DSN* or *the final rightward GN*. If the given DSN involves no postpositive qualifiers then it is identical with its final rightward DSN. If, however, the given DSN involves some postpositive qualifiers then its final rightward DSN, which is, in this case, alternatively called *the first leftward interim description*, or *DSN*, or *GN*, is successively concatenated with those qualifiers in the order of *association to the left of it*, in analogy with the rightward association of the prepositive qualifiers.

3) The above order of consecutive binary concatenation operations will be called the *default*, or *standard*, one, i.e. the one that is obviously understood as *pre-selected*. Any other order will be called a *nonstandard*, or *post-selected*, one. A nonstandard order and, when desired, the standard one will be indicated with the help of *square brackets*. Most generally, a pair of square brackets can be regarded as an inseparable part of the single whole operator '[$^\wedge$]', in which the 'n'-spaces on both sides of '$^\wedge$' are ellipses for any percept-operata to be concatenated. In this case, '$^\wedge$' is called the *kernel-sign*, or briefly *kernel*, of the operator '[$^\wedge$]'. The occurrence of '[$^\wedge$]' in the DSN, which is executed last, is called the *principal* binary concatenation operator of the DSN, whereas the DSN is said to be the *operandum* (pl. "*operanda*"), or *operand* (pl. "*operands*"), and also the *scope*, of that occurrence.

4) In accordance with the foregoing items of this definition, an NDSN (DSMnN) determines the class-species (specific class), being its *denotatum* (*intended value, meaning*), through the *intersection*, either in the standard order or in a nonstandard order, of the class-genus (generic class), denoted by its constituent NGN,

and of the megaclasses that are included into the megauniversals, denoted by its separate qualifiers, to produce the pertinent class-species (specific class). In accordance with the same items, a DSMsN determines the mass-species (specific mass), being its denotatum, through the *intersection*, but again either in the standard order or in a nonstandard order (as indicated), of the mass-genus (generic mass), denoted by its constituent GMsN, and of the megamasses included into the megauniversals, denoted by its separate qualifiers, to produce the pertinent mass-species (specific mass). That is to say, the elemental binary designatum-producing operation of concatenation in intension that has been denoted by '$^\wedge$' is *the binary operation of intersection of classes in the case of an NDSN* (*DSMnN*) and *the binary operation of intersection of masses in the case of a DSMsN*, subject to the following formal definitions:

[the intersection of classes *u* and *v*]

$$\to[uv]\to[u^\wedge v]\to[u\cap v]\to\{z\|[z\in u]\wedge[z\in v]\}, \qquad (2.17)$$

[the intersection of masses *u* and *v*]

$$\to[uv]\to[u^\wedge v]\to[u\cap v]\to\{z\|[z\subseteq u]\wedge[z\subseteq v]\}. \qquad (2.18)$$

In this case, the arrow $\to$ is a *rightward sign of ASD* (see subsection 2.5 for greater detail) and $\{\,|\,\}$ is an *abstraction operator from a relation to a class or to a mass*.

5) In accordance with the above-said, a *deduced sense*, or *sense-value*, of a given *NDSN*, e.g., is by definition a *biune mental coentity of mine* having the following two successive *hypostases* (*ways of existence*, *aspects*). The *first hypostasis of the sense* is an *entire mental operation* (*process*) *of mine*, *of intersection in a certain order of the classes*, which are designated by the designative units of the NDSN and which are collectively called the *object-classes of that operation* and also ones *of the sense*, *into the pertinent class-species* (*specific class*) – the one that is called the *class-designatum of the NDSN* and also the *subject-class both of the above operation and of the sense.* The above operation is called a *designatum-producing operation*, or *sense-operation, on the NDSN*. The *second hypostasis of the sense* is *the subject-class alone,* i.e. *the class-designatum of the NDSN*, *prescinded from that sense-operation.*

Thus, an NDSN may have several sense-operations and hence several senses that have the same subject-class but differ from one another by the orders, in which their constituent binary concatenation operations are performed (cf. Cmt 2.2(2)). Therefore, by *the* sense-operation on an NDSN, I shall mean that one, whose all constituent binary concatenation operations are performed in the default (standard) order, unless stated otherwise. Consequently, *the* sense of the NSDN is that one whose first hypostasis is *the* sense-operation on the NSDN. *The* sense of an NSDN could in principle be defined as the *class of equivalence* of all its senses with respect to their sense operations having the same subject-class. However, *this sense of* "*the* sense" is too abstract and is therefore impracticable.

6) The above item applies with "*DSMsN*" in place of "*NDSN*" and "*mass*" in place of "*class*".

7) Here follow some examples illustrating the matter of the foregoing items.

a) The NPDSN "leaf-bearing evergreen tree" is formalized as "leaf-bearing$^\wedge$evergreen$^\wedge$tree" or, more explicitly, as "[leaf-bearing$^\wedge$[evergreen$^\wedge$tree]]" under the standard order of consecutive binary concatenation operations and as "[[leaf-bearing$^\wedge$evergreen]$^\wedge$tree]" under the only possible nonstandard order of consecutive binary concatenation operations not involving the permutation of the qualifiers. The latter case illustrates the following general property of DSN's. If a DSN has only prepositive adjective qualifiers whose grouping is immaterial then they can be grouped in a nonstandard way separately from the GN so as to form a single whole *conjoined qualifier*. This qualifier denotes a single whole difference being the intersection of the partial differences denoted by the separate constituent qualifiers. In this case, the DSN can be called in full a description of the species through a genus and the *difference* rather than the *differences* – just as in the case where there is a single qualifier. Thus, for instance, "leaf-bearing$^\wedge$evergreen" is a single whole conjoined qualifier to "tree", so that the DSN "[[leaf-bearing$^\wedge$evergreen]$^\wedge$tree]" is a description of the species through the genus *tree* and the difference *leaf-bearing$^\wedge$evergreen* rather than the differences *leaf-bearing* and *evergreen*. It goes without saying that the conjoined qualifier is denotatively concurrent to "evergreen$^\wedge$leaf-bearing", in which the conjuncts have been permuted. Incidentally, the qualifiers 'leaf-bearing" and "evergreen" are complex monomial DSN's, which can be formalized as 'leaf$^\wedge$bearing" and "ever$^\wedge$green", but I do not analyze them and regard their class-designata as known, in accordance with Df 2.10(5)

b) Under the standard order of consecutive binary concatenations, the DSN "leaf-bearing evergreen tree of Mediterranean", e.g., is formalized thus: "[leaf-bearing^[evergreen^tree]]^[of Mediterranean]", where the outermost pair of square brackets has been omitted for the sake of brevity. Likewise, the polynomial DSN "elemental binary designatum-producing operation of concatenation in intension", which has been used but not mentioned earlier in the items 1 and 4 of this definition, is formalized thus:

"[[elemental^[binary^[designatum-producing^operation]]]
^[of concatenation]]^[in intension]".

c) Likewise, the DSMsN "cold distilled water" is formalized as "cold^distilled^water" or, more explicitly, as "[cold^[distilled^water]]" under the standard order of consecutive binary concatenation operations and as "[[cold^distilled]^water]" under the only possible nonstandard order of consecutive binary concatenation operations not involving the permutation of the qualifiers. In either case, the two pertinent formalized DSN's, the standard one and the nonstandard one, are *denotatively concurrent*, i.e. they *have the same class-denotatum*, but they are *not connotatively concurrent* or, in other words, *not sense-concurrent*, i.e. the DSN's. have *different senses* that are used (but not mentioned) as their *sense-connotata*.

d) It is understood that any binomial DSN can be formalized in only one way; e.g., the description "green tree" is formalized as "green^tree" and the description "cold water" as "cold^water".

9) The fact that some qualifiers are prepositive with respect to a given substantive and some others are postpositive is determined by rules of the English grammar. At the same time, the binary operation of intersection of classes or masses satisfies *the law of commutativity* and *the law of associativity*. Therefore, one could expect that the species being the denotatum of a DSN should not depend on the grouping of the qualifiers and GN and on the grammatically congruent orders of the separate prepositive or postpositive qualifiers. However, some constituent words of a polynomial DSN or some prefixes of a complex monomial DSN can be multisemantic, so that their senses are fixed by the contexts, in which they occur. Therefore, the order of the qualifiers of a DSN or their grouping can be essential in some cases and unessential in some others. If, particularly, a DSN has only prepositive adjective qualifiers, whose grouping is immaterial then they can be

grouped in a nonstandard way separately from the GN so as to form a single whole conjoined qualifier. This qualifier denotes a single whole difference being the intersection of the partial differences denoted by the separate constituent qualifiers. In this case, the DSN can be called in full a description of the species through a genus and the *difference* rather than *differences* – just as in the case where there is a single qualifier. For instance, the DSN “leaf-bearing evergreen tree” can alternatively be formalized as “[leaf-bearing$^{\wedge}$evergreen]$^{\wedge}$tree”, so that “leaf-bearing$^{\wedge}$evergreen” is a single whole *conjoined* qualifier to “tree”. This qualifier is denotatively concurrent to “evergreen$^{\wedge}$leaf-bearing”.•

Cmt 2.11. At first glance, the subterm “*green trees*” of the superterm “*trees*”, e.g., is a description of the *species* (*strict subclass*) *green trees* through the *genus* (*strict superclass*) *trees* and the *differentia* (*difference*) *green*. This is not, however, the case. In fact, the description “green trees” is just the plural number form of the NDSN “green tree” – the form that is not, however, an NDSN (DcSTrG&Ds) itself. In other words, in forming the plural number form of “green tree”, the latter is regarded as the *compound word* “green-tree” or “green$^{\wedge}$tree”, so that “green trees” should be understood as “green-trees” or, more explicitly, as “[green$^{\wedge}$tree]$^{\wedge}$s”. The paradoxical character of the name “green trees” is caused by the fact that the word “*trees*” is in this context regarded as a class-name that denotes a certain class, being a *common member of the power class*, i.e. *of the class of parts*, of the class *tree*. At the same time, neither a class nor any part of it can be qualified *green*, because *sensible* members of some classes are the only beings that can be green.•

Ax 2.1: ***An analysis of the EXQ of a descriptio per genus et differentiam.*** Let ‘Γ’ be a placeholder for a GN (generic name), while ‘E_1’ and ‘E_2’ are placeholder for two commutative prepositive qualifiers to the GN. To be recalled, the quotation that is formed by enclosing a placeholder between bold-faced quotation marks **“ ”** or **\ /** is called a *quasi-iconoautographic quotation* (*QIAQ*) or a *quasi-enneoxenographic* (*QEXQ*) respectively. Accordingly, the above quotation marks are called *QAIQ marks* and *QEXQ marks* in that order. Upon replacing the place-holding interior of a QIAQ or QEXQ with an appropriate concrete graphonym, the bold-faced quotation marks should be replaced with the corresponding light-faced ones, so that the QIAQ turns into an IAQ and the QEXQ into an EXQ. Under the above notation, **“**$E_1E_2\Gamma$**”** *is* a description through the genus ****Γ**/** denoted by the generic name **“**Γ**”** and through the differentiae ${}^{\backslash}E_1{}^{/}$ and ${}^{\backslash}E_2{}^{/}$, denoted by the commutable qualifiers **“**E_1**”** and **“**E_2**”**, so that each of the two qualifiers applies to **“**Γ**”**. Let also ‘↔’ be a binary *synonymity*, or *concurrency, sign* (for greater detail, see Df 2.19 below in this section). Then

$$ {}^{\backslash}E_1E_2\Gamma^{/}\leftrightarrow{}^{\backslash}E_1{}^{/\wedge\backslash}E_2{}^{/\wedge\backslash}\Gamma^{/}\leftrightarrow[{}^{\backslash}E_1{}^{/\wedge}[{}^{\backslash}E_2{}^{/\wedge\backslash}\Gamma^{/}]], \qquad (2.19)$$

where the sign **∧** is formed by the QEXQ marks **/** and ****. Once the interiors of the QEXQ’s in the above train of equivalences are replaced with concrete graphonyms, all bold-faced forth-slashed and back-slashed virgules in the superscript line should be replaced with light-faced one. In the result, the sign **∧** turns into ∧, the operator denoting the binary operation of intersection of the classes or cmasses that are denoted by the EXQ’s standing on both sides of ∧. The form and size of the sign ∧ and locations of its tokens in the superscript line serve as a mnemonic justification of the analysis, which is represented by the train of equivalence relations (2.19) and which reminds Principle of Juxtaposition of *special autographic quotations* (*SAQ’s*) or *special quasi-autographic quotations* (*SQAQ’s*).•

2.5. Formal methods of stating definitions

In order to set up $\mathcal{A}_1$, I shall require some *secondary* (*derivational*) *subject formulas of* A_1 to stand as abbreviations for some *primary* (*original, initial*) *subject formulas of* A_1, and I shall also require, as belonging to the IML, various proper and common names both of the above-mentioned formulas and of the formulas of other constituent logistic systems of $\mathcal{A}_1$. The required graphonyms are introduced with the help of various linguistic constructions (as a statement, sentential clause, turn of speech, or parenthesis), belonging to the IML, which are called *linguistic definitions*. So-called *synonymic definitions* (*SD’s*) and *nominal definitions* (*ND’s*) are linguistic definitions that will be used most often. Most linguistic definitions of this treatise are stated verbally and informally. However, for the sake of brevity and rigor, some SD’s and ND’s will be formalized as specified below.

Df 2.17. 1) In order to state a *binary asymmetric synonymic definition* (*BASD*) conveniently and formally, I shall make use of either one of the horizontal arrows → and ←, which belong to the IML and which are indiscriminately called a *universal asymmetric*, or *one-sided, synonymic definition signs* or, discriminately, the *universal rightward synonymic definition sign* and the *universal leftward one* respectively. At the head of an arrow I shall write the *material definiens* – the graphonym, which is already known either from a previous definition or from another source. At the base of the arrow I shall write the *material definiendum* – the new graphonym, which is being introduced by the definition and which is designed to be used instead of or interchangeably with the definiens. Accordingly, the arrow → is rendered into ordinary language thus: "*is to stand as a synonym for*" or straightforwardly "*is the synonymous definiendum of*", and ← thus: "*can be used instead of or interchangeably with*" or straightforwardly "*is the synonymous definiens of*". The [material] definiendum and [material] definiens of a BASD are indiscriminately called the *terms* of the definition. A BASD, which is made with the help of → or ←, is said to be a *formal BASD* or briefly an *FBASD*. Neither the definiendum nor the definiens of an FBASD should involve any *function symbols*, particularly any outermost (enclosing) quotation marks, that are not their constituent parts and that are therefore used but not mentioned with the following proviso. If it is necessary to indicate the integrity of the definiendum or of the definiens then that term of the definition can be enclosed in *square brackets as metalinguistic punctuation marks*, which do not, by definition, belong to the enclosed term and which are therefore used but not mentioned. If an arrow stands between a definiendum schema and a definiens schema then the arrow is supposed to apply simultaneously to the schemata and to every pair of interrelated instances (denotata, interpretands) of the schemata, unless stated otherwise.•

Df 2.18. An FBASD is said to be an *abbreviative* FBASD or simply a *formal abbreviative definition* (*FAD*) if it prescribes that the definiendum is to stand as an abbreviation for the definiens. In this case, the arrow → can more specifically be rendered into ordinary language thus: "*is to stand as an abbreviation for*" and ← thus: "*is to be abbreviated as*".•

Df 2.19. In order to state formally that two old or two new graphonyms are or are to be used interchangeably (synonymously), I shall write the graphonyms, without any quotation marks that are not their constituent parts, in either order on both sides of the two-sided arrow ↔ belonging to the IML. Such a relation is called a *formal binary symmetric synonymity*, or *concurrency*, *relation* (FBSSR), whereas ↔ is accordingly called a *synonymity*, or *concurrency, sign*. The two graphonyms standing on both sides of ↔ are called the *terms* of the FBSSR. If an FBSSR is a *corollary* from the pertinent FBASD stated previously then ↔ is read as "*is concurrent to*" or, alternatively, **"**— ↔ …**"** is read as **"**— *and … are concurrent***"** or as **"**— *and … are* *synonyms***”**, where alike ellipses should be replaced alike and then the bold-faced double quotation marks should be replaced with the light-faced ones. If an FBSSR is stated in no connection with any previous FBASD then the FBSSR is said to be a *formal binary symmetric synonymic definition* (*FBSSD*), whereas $\leftrightarrow$ is called the *symmetric*, or *two-sided*, *synonymic definition sign*. In this case $\leftrightarrow$ is read as “*is to be concurrent to*” or, alternatively, **“**— $\leftrightarrow$ …**”** is read as **“**— *and … are to be concurrent***”** or as **“**— *and … are to be synonyms***”**, where alike ellipses should, as before, be replaced alike, while the bold-faced double quotation marks are placeholders for the light-faced ones. Just as in the case of $\rightarrow$ or $\leftarrow$, if $\leftrightarrow$ stands between schemata then the arrow is supposed to aplly simultaneously to the schemata and to every pair of interrelated instances (denotata, interpretands) of the schemata, unless stated otherwise.•

Cmt 2.12. 1) In order to make explicit the main properties of FBASD’s and FBSSD’s, let any of the letters ‘Γ’, ‘Δ’, and ‘Z’, alone or with some labels (as primes or alphanumeric subscript, be a placeholder (ellipsis) for any appropriate concrete [material] definiendum or definiens.

2) It is understood that a definiendum cannot be a definiens of itself and vice versa. Also, the definiendum and the definiens of an FBASD are not exchangeable, because their roles in the definition are different. Therefore, the signs $\rightarrow$ and $\leftarrow$ satisfy *neither the reflexive law nor the symmetric law*. At the same time,

$$\text{if } \Gamma\rightarrow\Delta \text{ and } \Delta\rightarrow Z \text{ then } \Gamma\rightarrow Z \tag{2.20}$$

or, equivalently,

$$\text{if } \Delta\leftarrow\Gamma \text{ and } Z\leftarrow\Delta \text{ then } Z\leftarrow\Gamma, \tag{2.21}$$

i.e. $\rightarrow$ and $\leftarrow$ satisfy the *transitive law*. In this case, since ‘Γ’, ‘Δ’, and ‘Z’ are placeholders, therefore either sign $\rightarrow$ or $\leftarrow$ is supposed to be applied, not to the placeholders between which they are put, but to the specific graphonyms which are supposed to be substituted for the placeholders; that is to say, a definition sign is supposed to be applied formally or slidingly, and not materially or not contactually.

3) Though the definiendum and the definiens are not exchangeable in the FBASD defining the former in terms of the later, after the FBASD is stated a token of the definiendum can be used *instead of* or *interchangeably with* a token of the definiens, provided that the definition is stated as a secondary formation rule of a certain logistic system or provided that a concrete use of the definiendum instead of

the definiens is congruent with the pertinent grammar rules or lexicon of the IML. This property of a definiendum Γ and its definiens Δ is expressed symbolically by stating that $\Gamma\leftrightarrow\Delta$. In contrast to $\rightarrow$ and $\leftarrow$, the sign $\leftrightarrow$ satisfies all the three above-mentioned laws, i.e.

$$\Gamma\leftrightarrow\Gamma \text{ (\textit{Reflexive law})}, \tag{2.22}$$

$$\text{if } \Gamma\leftrightarrow\Delta \text{ then } \Delta\leftrightarrow\Gamma \text{ (\textit{Symmetric law})}, \tag{2.23}$$

$$\text{if } \Gamma\leftrightarrow\Delta \text{ and } \Delta\leftrightarrow Z \text{ then } \Gamma\leftrightarrow Z \text{ (\textit{Transitive law})}. \tag{2.24}$$

Hence, $\leftrightarrow$ is a *sign of equivalence relation*.

4) To a given definiens Γ, there can exist many definienda, say, Δ, Z, Δ′, Z′, Δ″, Z″, etc, which can be defined in terms of Γ independently of one another thus:

$$\Delta\rightarrow\Gamma,\ Z\rightarrow\Gamma,\ \Delta'\rightarrow\Gamma,\ Z'\rightarrow\Gamma, \text{ etc.} \tag{2.25}$$

In accordance with the transitive law versions (2.20) and (2.21), the same definienda can be introduced successively, for instance, thus:

$$\Delta\rightarrow\Gamma,\ Z\rightarrow\Delta,\ \Delta'\rightarrow Z,\ Z'\rightarrow\Delta', \text{ etc..} \tag{2.26}$$

The latter series of definitions can, in turn, be written in continuation, for instance, thus:

$$Z'\rightarrow\Delta'\rightarrow Z\rightarrow\Delta\rightarrow\Gamma \tag{2.27}$$

or thus:

$$Z\rightarrow\Delta\rightarrow\Gamma\leftarrow\Delta'\leftarrow Z'. \tag{2.28}$$

In the result of any one of the four series of definitions (2.25)-(2.28), the terms of the definitions, Γ, Δ, Z, Δ′, Z′, etc, become mutually (pairwise) concurrent, i.e.

$$\begin{gathered}\Delta\leftrightarrow\Gamma,\ \Delta\leftrightarrow Z,\ \Delta\leftrightarrow\Delta',\ \Delta\leftrightarrow Z',\ Z\leftrightarrow\Gamma,\ Z\leftrightarrow\Delta',\ Z\leftrightarrow Z',\\ \Delta'\leftrightarrow\Gamma,\ \Delta'\leftrightarrow Z',\ Z'\leftrightarrow\Gamma.\end{gathered} \tag{2.29}$$

By the transitive law (2.24), this series of concurrency relations (2.29) can be written in continuation, for instance, thus:

$$Z'\leftrightarrow\Delta'\leftrightarrow Z\leftrightarrow\Delta\leftrightarrow\Gamma \tag{2.30}$$

or thus:

$$Z\leftrightarrow\Delta\leftrightarrow\Gamma\leftrightarrow\Delta'\leftrightarrow Z'. \tag{2.31}$$

Either of the *series* of definitions (2.25) and (2.26) and also the *series* of synonymity (concurrency) relations (2.29) are said to be written in the *staccato style*, i.e. *interruptedly* (*discontinuously*), one by one. Either of the *trains* of definitions (2.27) and (2.28) and also either of the *trains* of concurrency relations (2.30) or (2.31) are said to be written in the *legato style*, i.e. *uninterruptedly* (*continuously*). In this case, the trains (2.30) or (2.31) can, when applicable, be interpreted in analogy with (2.27) and (2.28) as trains of FBSSD's, and not as trains of FBSSR's. The term of a train of FBASD's that stands either at the head of the last arrow $\rightarrow$ or at the head of the first arrow $\leftarrow$, or else between the heads of the arrows $\rightarrow$ and $\leftarrow$ or is called the *principal definiens* of the definition train. A term of the train that stands between the head of one arrow and the base of the other arrow is called an *intermediate term*, i.e. an *intermediate definiendum* and at the same time an *intermediate definiens*, of the train. In the sequel, iterative (repeated) definitions and iterative concurrency relations, and also all other iterative binary relations, whose predicates (as ~, =, $\hat{=}$, $\Leftrightarrow$, $\Rightarrow$, $\Leftarrow$, <, >, $\geq$, $\leq$, etc) satisfy the transitive law, will often be written in the legato stale for the sake of brevity.•

Df 2.20. In accordance with Dfs 2.17–2.19 and Cmt 2.12(3), within the scope of an FBASD, which is stated by means of either sign $\rightarrow$ or $\leftarrow$, or within the scope of an FBSSD, which is stated by means of the $\leftrightarrow$, *isotokens of the terms of the respective definition* can be related by:

a) the sign $\hat{=}$ if the isotokens belong to *the class of terms* (*as opposed to the classs of relations*) *of a logistic system*, on which that sign is defined;

b) the sign = if the isotokens belong to *the class of terms of a logistic system*, on which that sign is defined;

c) the sign $\Leftrightarrow$ if the isotokens belong to *the class of relations of a logistic system*, on which that sign is defined;

the scope of a defininition does not includes the definition itself. Accordingly, in a definition itself, which is stated by means of a certain one of the signs $\rightarrow$, $\leftarrow$, and $\leftrightarrow$, that sign can be replaced with the respective one of the signs $\overset{\rightarrow}{\hat{=}}$, $\overset{\leftarrow}{\hat{=}}$, and $\overset{\leftrightarrow}{\hat{=}}$ in the case a, with the respective one of the signs $\overset{\rightarrow}{=}$, $\overset{\leftarrow}{=}$, and $\overset{\leftrightarrow}{=}$ in the case b, and with the respective one of the signs $\overset{\rightarrow}{\Leftrightarrow}$, $\overset{\leftarrow}{\Leftrightarrow}$, and $\overset{\leftrightarrow}{\Leftrightarrow}$. The signs $\overset{\rightarrow}{\hat{=}}$, $\overset{\leftarrow}{\hat{=}}$, and $\overset{\leftrightarrow}{\hat{=}}$ are called *the special rightward, leftward, and two-sided signs of equality by definition*; the signs $\overset{\rightarrow}{=}$, $\overset{\leftarrow}{=}$, and $\overset{\leftrightarrow}{=}$ are called *the ordinary rightward, leftward, and two-sided signs of equality by definition*; the signs $\overset{\rightarrow}{\Leftrightarrow}$, $\overset{\leftarrow}{\Leftrightarrow}$, and $\overset{\leftrightarrow}{\Leftrightarrow}$ are called *the [ordinary] rightward, leftward, and two-sided signs of equivalence by definition.*•

Df 2.21. In order to state an *ostensive nominal definition* (*OND*) conveniently, I shall make use of either of the slant arrows ↘ and ↙, which belong to the IML and which are called *the nominal definition signs, the rightward one* and *the leftward one* respectively. At the head of an arrow I shall write the *definiens* – the graphonym which is already known either from a previous definition or from another source. At the base of the arrow I shall write the *definiendum* – the metalinguistic name which is being introduced by the definition and which is designed to be used for mentioning the definiens. Accordingly, the arrow ↘ is to be read thus: "*is a name of*" or "*is a nominal definiendum of*", whereas ↙ thus: "*is the ostensive definiens of*" or thus: "*is called,* or *denoted, by*". The definiendum and the definiens of an OND are indiscriminately called the *terms* of the definition. An OND, which is made with the help of ↘ and ↙, is said to be a *formal OND* or briefly an *FOND*. Just as in the case of FBASD's, no metalinguistic signs (operators), particularly no quotation marks, that are used but not mentioned are allowable in the terms of an FOND, in the exclusion of square brackets that can be used as metalinguistic signs of association.

3. An introduction in depth to A_1

Df 3.1: *A cumulative outline definition of* A_1. 1) The calculus, which is denoted by 'A_1' and is called the *Comprehensive Euautographic Algebraico-Predicate Organon* (*CEAPO*), and also the *Comprehensive Euautographic Advanced Algebraico-Logical Organon* (*CEAALO*), it is a *tree-like, phased and branched, euautographic* (*essentially uninterpreted) algebraico-predicate calculus of first order*, whose structure remotely reminds both the structure of a *conventional axiomatic predicate calculus of first order* (*CAPC*, pl. "*CAPC'i*"), especially the structure of the calculus F^1 of Church [1956, chaps. III and IV], and the structure of an abstract *integral domain* (as framed, e.g., in Birkhoff & Mac Lane [1965, pp. 1, 2] or Mac Lane & Birkhoff [1967, pp. 132–134]). A_1 includes, as its self-subsistent and self-contained but inseparable part, another calculus, which is denoted by 'A_1^0' and is called the *Comprehensive Euautographic Binder-Free,* or *Contractor-Free, Algebraico-Predicate Organon* (*CEBFAPO* or *CECFAPO*) and also the *Comprehensive Euautographic Rich Basic Algebraico-Logical Organon* (*CERBALO*). Either of the synonymous qualifiers "binder-free" and "contractor-free" means *pseudo-qualifier-free, pseudo-quantifier-free, and pseudo-multiplier-free*. For the sake of brevity, A_1^0 is set up as a constituent part of A_1, but it is unambiguously identifiable within A_1. In turn, A_1^0 includes, as its self-subsistent constituent part, a calculus that is denoted by ‘A_0’ and is called the *Euautographic Predicate-Free*, or *Euautographic Basic* (or *Euautographic Depleted Basic*, in contrast to *Euautographic Rich Basic*) *Algebraico-Logical Organon* (*EPFALO* or *EBALO* or *EBALO*). A_0 is an *unbranched* (*indivisible, single whole*) *euautographic algebraico-logical calculus*, whose structure remotely reminds both the structure of a *conventional axiomatic sentential calculus* (*CASC*, pl. “*CASC’i*”), especially that of the Russell-Bernays logistic system (as framed, e.g., in Hilbert and Ackermann [1950, §10, pp. 27–30], Church [1956, §25, pp. 136–138; §29, p. 157], or Bourbaki [1960, chap. I, §3, S1–S4]), and the structure of the abstract integral domain as mentioned above. A_0 is set up and executed within A_1^0, but it is self-subsistent (and hence self-contained) in the sense that it can be set up and executed independently of A_1^0.

2) In accordance with the verbal names of A_1, the graphonyms dealt with in A_1 and also in any of its constituent organons, are called *euautographs*. There is in A_1 an indefinite number of *euautographs* of different *kinds* (*subclasses*, *specific classes*, *species*). Particularly, a euautograph can be *primary* (*postulated*, *undefined*) or *secondary* (*defined* or *deduced*), *atomic* (*functionally indivisible*) or *combined* (*composite*), *elemental* (*primitive*, *atomic* or *molecular)* or *complex* (*compound*), *categorematic* (*formulary*, a *term* or a *relation*) or *syncategorematic* (a *kernel-sign* or a *punctuation mark*), *ordinary* (*non-special*) or *special* (*unordinary*), *logical* or *algebraic* (*special algebraic*), etc. The pairs of qualifiers as mentioned above (e.g. “primary” and “secondary”, “atomic” and “combined”, etc) are pairs of *complimentary antonymous technical metaterms* (*metalinguistic terms*) of this treatise to be defined in due course. I shall use the appropriate abbreviations of the pertinent *descriptive taxonyms* (*taxonomic names*, *names of taxa*, *names of taxonomic classes*) of euautograph – such abbreviations, e.g., as: “*AE*” for “*atomic euautograph*”, “*PAE*” for “*primary atomic euautograph*”, “*SAE*” for “*secondary atomic euautograph*”, “*PAOE*” for “*primary atomic ordinary euautograph*”, “*PASpE*” for “*primary atomic special euautograph*”, etc or such as the variants of those abbreviations with “Cb” for the qualifier “*combined*” in place of “A” for “*atomic*” or with “*S*” for the qualifier “*secondary*” in place of “*P*” for “*primary*”, and also such abbreviations as the variants

of the above ones with any one of the following abbreviations for *specific* names in place of the abbreviation "*E*" for the *generic* name "*euautograph*": "*EF*" for the *specific* name "*euautographic formula*" or "*EC*" (pl. "*EC'ta*") for the synonymous *specific* name "*euautographic categorem*" (pl. "~ *categoremata*"), "*ESC*" (pl. "*ESC'ta*") for "*euautographic syncategorem*" (pl. "~ *syncategoremata*"), "*ER*" for "*euautographic relation*", "*ET*" for "*euautographic term*", "*EKS*" for "*euautographic kernel-sign*", and many other similar abbreviations that will be defined as I go along. It is understood that given a generic name as any one of those mentioned above, the description of a would-be species of euautographs through the genus denoted by that generic name and the differentiae (differences) denoted by some of the above-mentioned qualifiers can turn out be *insignificant*, i.e. having no class-*denotatum* (pl. "*denotata*"). For instance, there are in A_1 *neither* CbOT's (combined ordinary terms) and *nor* CbLT's (combined logical terms), both primary and secondary, and *no* secondary punctuation marks. Also, it is understood that the class-denotatum of a description of the species or subspecies of euautographs through the genus denoted by a certain generic name (as "euautograph", "formula" or "categorem", "syncategorem", etc) and through the differences denoted by two or more qualifiers that apply to the generic name and not to one another remain unaltered if the qualifiers are permuted. For instance, the anterior qualifier "euautographic" ("E") and any one of the posterior qualifiers "ordinary" ("O"), "special" ("Sp"), "logical" ("L"), and "algebraic" ("Al") to either of the generic names "relation" ("R") and "term" ("T") can be *permuted* without altering the meaning of the pertinent taxonym. That is to say, the abbreviated taxonyms of the following groups and the corresponding full taxonyms are synonyms: "*EOR*" and "*OER*"; "*ESpR*" and "*SpER*"; "*ELR*" and "*LER*"; "*EAlR*" and "*AlER*"; "*EOT*", "*OET*", "*ELT*", and "*LET*"; "*ESpT*", "*SpET*", "*EAlT*", "*AlET*", and "*EI*" ("*euautographic integron*"). The plural number form of an abbreviation of a descriptive taxonym, which ends with a capital letter abbreviating the pertinent generic head noun of the taxonym in the singular number form, will be formed by suffixing that letter with "'s" if the noun is a chaste English one or with the appropriate apostrophized ending if the noun is a Latinized or Graecized one (e.g. "CAPC'i" or "EC'ta"). Some abbreviations will be used ad hoc, whereas some others may be used throughout the treatise or throughout some large parts of it. In the latter case, the abbreviations will be deciphered contextually from time to time.

3) Any *admissible* (*operant*, *busy*) *unit* (*self-contained*) *euautograph of* A_1 is indiscriminately called a *categorematic euautograph* or a *euautographic categorem* (*EC*), and also a *euautographic formula* (*EF*) or, more specifically, a *subject* (*intrinsic*) *EF* (*EC*) *of* A_1 or an *object EF* (*EC*) *of the inclusive metalanguage* (*IML*) *of* A_1. Euautographs of A_1, which are not EF's but which are used together with EF's to form longer EF's, are called *syncategorematic euautographs*, or *euautographic syncategoremata* (*ESC'ta*), *of* A_1. In accordance with both the *axiom of atomic basis of* A_1 and the *formation rules of EF's of* A_1, the entire class of EF's is divided into two subclasses in the following different ways. An EF is called:

a) a *primary* one (*PEF*), if it is *postulated* to be an EF, and a *secondary* one (*SEF*), if it is either *defined* in terms of (with respect to) a certain PEF or is a combination of some SEF's, defined earlier, and, perhaps, of some PEF's;

b) an *atomic* one (*AEF*), if it has no other EF's as its constituents, and a *combined*, or *composite*, one (*CbEF* or *CpEF*) if otherwise;

c) an *elemental*, or *primitive*, one (*ElEF*), if is an AEF or a *molecular EF* (*MEF*) by definition, and a *complex*, or *compound*, one (*CxEF* or *CdEF*), if it is a CbEF other than an MEF;

d) a *special* one (*ESpF*), if it is either an AEF that is a *special* one (*AESpF*) by definition or is a CbEF that has at least one AESpF as its constituent, and an *ordinary* one (*EOF*) if otherwise;

e) an *algebraic* one (*EAlF*) or a *logical* one (*ELF*), in accordance with certain criteria to be explicated as I go along;

f) a *euautographic relation* (*ER*) or a *euautographic term* (*ET*) if it is postulated or defined to be so, the understanding being that the above items a)–e) apply with "R" ("relation") or "T" ("term") separately in place of "F" ("formula").

An ER standing alone is analogous to a declarative sentence of a *written native language* (*WNL*), whereas an ER being a constituent part of a longer ER is analogous to a clause of a complex-coordinate or complex-subordinate declarative sentence. Within A_1, an ET can occur only as a constituent part of one or another ER so that it is, in this respect, analogous the subject or an object of a simple declarative sentence of a WNL. Thus, A_1 is in fact a *calculus of ER's*. The class of ESC'ta of A_1 is divided into two subclasses: the *euautographic kernel-signs* (*EKS's*) and the *euautographic* *punctuation marks* (*EPM's*). The above items a)–e) apply with "KS" ("kernel-sign") in place of "F" ("formula"), whereas any EPM of A_1 is a *primary ordinary euautograph* (*POE*), i.e. a *primary euautograph* (*PE*) and an *ordinary euautograph* (*OE*) simultaneously.

4) Within A_1, a euautograph (categorematic or syncatergorematic, atomic or combined) is *functional but insignificant*, i.e. it is a *graphic chip* (*fish*) or a *pattern group* (*combination*) *of such chips*, which has a *certain syntactic function* or *functions* in itself or, when applicable, with respect to the *pasigraphs* (*euautographs* or *logographs*) of its immediate surrounding, but which has *no psychical (mental) signification* (*import*, *xenovalue*). Therefore, a euautograph of any kind (class) *is incapable either of having or of assuming* (*taking on*) *any denotatum* (*denotation value,* pl. "*denotata*"). Particularly, within A_1, a *euautographic relation* (*ER*) *is incapable either of having or assuming psychically* (*mentally*) *any truth-value,* or of being *physically replaced* with any significant graphic relation such as a *propositional* (*truth-valued*) *functional form* or a *propositional declarative sentence* of any *written native language* (*WNL*). That is to say, a euautograph can be *neither a variable nor a constant* and *neither a qualifier nor a quantifier*. Consequently, for the purpose of description or reference, a euautograph is, when applicable, called a *pseudo-variable* or a *pseudo-constant*, in accordance with its function or in accordance with its subsequent semantic interpretation by a variable or by a constant respectively, or in accordance with both reasons. An EKS's of A_1, whose function is *to bind every occurrence of an APVEOT in its scope to the first occurrence of that APVEOT in the EKS itself*, is impartially called a *euautographic binder* (*EB*) or *euautographic contractor* (*ECt*), whereas the APVEOT that it binds is called a *bound*, or *dummy*, *APVEOT.* An EB (ECt), which is united with an ER (euautographic relation) to produce another ER, and whose logical status is therefore similar to that of a *logical quantifier*, is called a *euautographic logical*, or *ordinary*, *pseudo-qualifier* (*EPQl*, pl "*EPQl's*") if it is utilized in $A_{1\subseteq}$ and a *euautographic logical*, or *ordinary, pseudo-quantifier* (*EPQn*, pl "*EPQn's*") if it is utilized in $A_{1\in}$. An EB (ECt), which is united with an EI (euautographic integron), i.e. with ESpT (euautographic special term), to produce another EI (ESpT), and whose logical status is therefore similar to that of an *algebraic multiplier* (*multiplication operator*) over occurrences of a dummy variable, is called a *euautographic algebraic*, or *special*, *binder* (*EAlB* or *ESpB*) or a

euautographic algebraic, or *special*, *contractor* (*EAlCt* or *ESpCt*), and also a *euautographic pseudo-multiplier* (*EPM*). The only values that a euautograph can have or assume within its domain are its autonymous values such as the *class of its homolographic* (*photographic*, *congruous* or *proportional*) *isotokens*, i.e. *tokens* of the same genesis and hence of the same sensorial properties (visual ones in this case), or a certain member of the above class as the euautograph itself or any one of its homolographic isotokens, concrete or common (general); a common isotoken of a euautograph (or that of any graphonym in general) is another *hypostasis* (*way of existence*, *aspect*) of its isotoken-class. At the same time, a euautograph has *no phonic* (*oral*) or any other *paratokens*, i.e. tokens of another genesis and hence of another sensorial properties (as audible ones). By the complete absence of any semantic properties, a euautograph is analogous to a chessman or to a position of chessmen on the chessboard or, more precisely, it is analogous to a figure of either of the above objects in a textbook on chess (as Chernev [1958]).

5) All *primary atomic euautographs* (*PAE's*) of A_1 will be introduced in Ax 1.1, called the *axiom of atomic basis*, or *euautographic atomic basis axiom* (*EsABA*), *of* A_1. The PAE's are divided into two complementary classes in a few different ways, particularly in theses two:

a) the *ordinary*, or *logical*, ones (*PAOE's* or *PALE's*) and the *special*, or *algebraic*, ones (*PASpE's* or *PAAlE's*);

b) the *categorematic*, or *formulary*, ones (*PACE* or *PAFE*), called also the *primary atomic euautographic categoremata* (*PAEC'ta*), or *formulas* (*PAEF's*), and the *syncategorematic* ones (*PASCE*), called also the *primary atomic euautographic syncategoremata* (*PAESC'ta*).

The *ordinary*, or *logical*, *PAEF's* (*PAEOF's* or *PAELF's*) are divided into three classes:

i) the *atomic pseudo-variable ordinary*, or *logical*, *relations* (*APVOR's* or *APVLR's*), called also the a*tomic euautographic ordinary*, or *logical*, *relations* (*AEOR's* or *AELR*'s) or, briefly, the *atomic euautographic relations* (*AER's*), because there are in A_1 no AER's of any other kind;

ii) the *euautographic ordinary*, or *logical*, *terms* (*EOT's* or *ELT's*) of two subclasses (specific classes, species): the *pseudo-variable* ones (*PVOT's* or *PVLT's*) and the *pseudo-constant* ones (*PCOT's* or *PCLT's*), the understanding being that the prepositive qualifiers “primary” (“P”) and “atomic” (“A”) to any of the above metaterms would be redundant because there are in A_1 *no* EOT’s (ELT’s) that could be qualified either *secondary* or *combined*;

iii) the two Arabic digits 0 and 1 in this light-faced Roman (upright) narrow Gothic (sans serif) type, called the Light-Faced Roman Arial Narrow Type (LFRANT) – the digits that are collectively called the *primary atomic euautographic special*, or *algebraic*, *terms* (*PAESpT’s* or *PAEAlT’s*) and also the *primary atomic euautographic integrons* (*PAEI’s*) or briefly the *idempotent digital integrons* (*IDI’s*); occurrences of the qualifier “euautographic” (“E”) in the above metaterms can be replaced with occurrences of the qualifier “*pseudo-constant*” (“*PC*”) without altering the denotatum of the metaterms.

The qualifier “algebraic” both to *euautographs* and to their *placeholders* (*place-holding variables*), called *panlogographs* (*PL’s*) or *panlogographic placeholders* (*PLPH’s*), is used as an abbreviation of the combined qualifier “*special algebraic*”. This abbreviation is unambiguous because there are in the treatise no euautographs and no panlogographs that could be qualified *ordinary algebraic*. The qualifiers “ordinary” and “logical”, or “special” and “algebraic”, are accidental synonyms when they apply to PAE’s, but they are not necessarily so when they apply to linear combinations (sequences) of PAE’s (to be specified).

6) A finite linear sequence of homolographic tokens of PAE’s of A_1 *without blanks* (*empty spaces*) *and without blank-signs* is called a *primary assemblage* (*PA*) *of* A_1 and also, synecdochically, a *primary euautographic assemblage* (*PEA*). A PEA is said to be a *primary euautographic ordinary*, or *ordinary euautographic*, *assemblage* (*PEOA* or *POEA*) if it comprises [homolographic tokens of] PAOE’s and a *primary euautographic special assemblage* (*PESpA*), if it contains at least one [homolographic token of a] PAOE and some or no [homolographic tokens of] PAOE’s. Then certain *axiomatic rules*, called the *primary formation*, or *composition*, *rules* (*PFR’s*) *of* A_1, are given, by which certain PEA’s are designated as *admissible* (*operant, busy*) ones of various kinds that are collectively called *primary euautographic formulas* (*PEF’s*) or *primary euautographic categoremata* (*PEC’ta*), and also, more specifically, *subject* (*intrinsic*) *PEF’s of* A_1 or *object PEF’s of the IML of* A_1, – in agreement with the pertinent general definition of the item 3. A PEF is said to be an *ordinary* one (*PEOF*)

if it is a PEOA and a *special* one (*PESpF*) if it is a PESpA. Some PEF's are *atomic ones* (*PAEF's*), while the others are *combined*, or *composite*, *ones* (*PCbEF's* or *PCpEF's*), i.e. ones that have some other PEF's as their constituent parts.

7) In accordance with the PFR's, the class of PEF's is divided into two subclasses: the [class of] *primary euautographic relations* (*PER's*) and the *primary euautographic terms* (*PET's*). Consequently, a PER is either an *ordinary PER* (*PEOR*) or a *special PER* (*PESpR*) but not both at one time; and similarly a PET is either an *ordinary PET* (*PEOT*) or a *special PET* (*PESpT*), called also a *primary euautographic integron* (*PEI*). Independently, a PER is either a *primary atomic euautographic relation* (*PAER*) or a *primary combined euautographic relation* (*PCbER*), and similarly a PET is either a *primary atomic euautographic term* (*PAET*) or a *primary combined euautographic term* (*PCbET*). In this case, by the item 5i, a PAER is an AER and vice versa; whereas, by the items 5ii and 5iii, a PAET is either an EOT or an IDI. At the same time, a PCbER is either an *ordinary* one (*PCbEOR*) or a *special* one (*PCbESpR*), whereas a PCbET is necessarily a *special* one (*PCbESpT*), i.e. a *primary combined euautographic integron* (*PCbEI*).

8) In compliance with the pertinent property of any ER as indicated in the item 3, a PER can either stand alone, like a declarative sentence of a WNL or like an admissible chess position, or be a constituent in a longer PER, like a clause of a complex-coordinate or complex-subordinate declarative sentence. Just as a PER, a PET is a self-contained euautograph of A_1, but, in contrast to the former, the latter cannot ever stand alone; it is always a constituent part of one or another PER, like the subject or an object of a simple declarative sentence. A PEOR and a PESpR are *akin* in many respects so that the two are treated alike in some cases and differently in the others. It is therefore natural that a PEOR or a PESpR is indiscriminately called a PER. By contrast, a PEOT and a PEI (PESpT) are essentially different so that they are always treated differently. Still, in order to set the classes of PEOT's and of PEI's together off the class of PER's, a PEOT or a PEI (PESpT) is indiscriminately called a PET.

9) Some PCbEF's are used as *definientia* of the appropriate *asymmetric synonymic definitions* (*ASD's*), which are called *secondary formation rules* (*SFR's*) *of* A_1, because each of these definitions prescribes that its *definiendum*, called a *secondary euautographic formula* (*SEF*) or *secondary euautographic categorem*

(*SEC*, pl. "*SEC'ta*"), will stand as an *abbreviation* for the PCbEF being its definiens, no matter whether this abbreviation stands alone or as a constituent in a longer *combined SEF* (*SCbEF*). An SEF is called a *secondary euautographic relation* (*SER*) or a *secondary euautographic term* (*SET*) if the PEF serving as its definiens is a PER or a PET respectively. Consequently, if a certain SEF, SER, or SET exists then it is qualified with the same qualifier as that qualifying the PEF, PER, or PET serving as its definiens. For instance, an SEF is said to be an *ordinary* one (*SEOF*), or a *special* one (*SESpF*), if the PEF serving as its definiens is an ordinary one (PEOF), or a special one (PESpF), respectively. Also, like a PEF, an SEF is either an *atomic SEF* (*SAEF*) or a *combined SEF* (*SCbEF*), i.e. one that has some other PEF's or SEF's as its constituent parts. At the same time, in accordance with the items 5i and 5ii, there are in A_1 *no secondary AER's and no combined EOT's* (*ELT's*), *either primary or secondary*. Therefore particularly, a SET is necessarily a *special*, or *algebraic*, one (*SESpT* or *SEAlT*), i.e. a *secondary euautographic integron* (*SEI*); this can be either an *atomic* one (*SAEI*, i.e. *SAESpT* or *SAEAlT*) or a *combined* one (*SCbEI*, i.e. *SCbESpT* or *SCbEALT*), whereas an SAEF is necessarily an SAEI. To be specific, an SAEI (SAESpT, SAEAlT) is by definition any one of the eight digits 2 to 9 in this LFRANT. The ten digits 0, 1, and 2 to 9 altogether are called the *atomic euautographic special*, or *algebraic*, *terms* (*AESpT's* or *AEAlT's*) and also the *atomic euautographic integrons* (*AEI's*). All the digits are *pseudo-constant* euautographs. Therefore, the occurrence of the qualifier "euautographic" ("E") in any of the above taxonyms of these digits or of the digits 2 to 9 can be replaced with an occurrence of the qualifier "*pseudo-constant*" ("*PC*") without altering the denotatum of the taxonym (cf. the item 5iii). An AER, EOT (ELT), or AEI is indiscriminately called an *atomic euautograpnic formula* (*AEF*) or an *atomic euautographic categorem* (*AEC*, pl. "*AEC'ta*") and also an *atomic formulary*, or *categorematic*, *euautograph* (*AFE* or *ACE*).

10) Like a PCbEF, which is either a PCbER or a PCbEI (PCbESpT), an SCbEF is either a *secondary combined euautographic relation* (*SCbER*) or a *secondary combined euautographic integron* (*SCbEI*), i.e. a *secondary combined euautographic special term* (*SCbESpT*). Like a PCbER, which is either a PCbEOR or a PCbESpR, an SCbER is either a *secondary combined euautographic ordinary relation* (*SCbEOR*) or a *secondary combined euautographic special relation* (*SCbESpR*). The class of SCbEI's is furcated into two subclasses: the [subclass of]

secondary combined pseudo-variable integrons (*SCbPVI's*) and the *secondary combined pseudo-constant integrons* (*SCbPCI's*). The former subclass will be illustrated as I go along. An SCbPCI is, e.g., any one of the Arabic numerals from 10 ad infinitum in this LFRANT, which are called the *combined decimal digital integrons* (*CbDDI's*). The AEI's 0, 1, and 2 to 9 and the CbDDI's altogether are called the *decimal digital integrons* (*DDI's*), the understanding being that these constitute the conventional *decimal system of numeration*. The whole infinite set of DDI's has been defined with the help of certain recursive formation rules of A_1, which determine the *primary combined pseudo-constant integrons* (*PCbPCI's*). However, in executing A_1, no DDI's strictly larger than 2 are actually used. The *binary system of numeration* seems to be more natural as a part of A_1 than the decimal one. Still, I have decided to employ the latter because, owing to the force of habit, it is more convenient to employ in algebraic computations, e.g., the numeral 2 of the decimal system and not the concurrent numeral 10 of the binary system.

11) The facts of the EF's (EC'ta) indicated in the items 2–10 can be summarized thus.

a) An EF (EC) is either an ER or an ET and, independently, an EF (EC) is either a PEF or an SEF. Therefore, discriminately, an ER is either a PER or an SER and an ET is either a PET or an SET. Also, an ER is either an EOR or an ESpR and likewise an ET is either an EOT (ELT) or an EI (ESpT, EAlT). Consequently, an EOR is either a PEOR or an SEOR and similarly an ESpR is either a PESpR or an SESpR. Like an ESpR, an EI (ESpT, EAlT) is either a PEI (PESpT, PEAlT) or an SEI (SESpT, SEAlT), but unlike an EOR, an EOT (ELT) is a *primary atomic euautographic ordinary,* or *logical¸ term* (*PAEOT* or *PAELT*) and vice versa.

b) An EF is said to be an *atomic* one (*AEF*) if it has no other EF's as its constituents and a *combined*, or *composite*, one (*CbEF* or *CpEF*) if otherwise. Accordingly, an EF is either an AEF or a CbEF. A CbEF is alternatively called a *euautographic operandum* (pl. **"**~*operanda*") or a *euautographic operand* (pl. **"**~ *operands***"**) and also, more precisely, a *euautographic operand-formula* or a *euautographic formula-operand.* A constituent EF of a euautographic operand, i.e. an EF *acted upon* by a certain EKS, is called a *euautographic operatum* (pl. **"**~ *operata***"**), or, more precisely, an *operatum-formula*, or *formula-operatum*, of the euautographic operand, the understanding being that the operatum can be either an

AEF or a CbEF, i.e. another euautographic operand. In analogy with the linguistic terms "*free* linguistic form" and "*bound* linguistic form", a subject EF of A_1 is said to be a *disjoined* (*disconnected*, *unrelated, free*) one if it occurs *singly* (*separately*) and a *joined* (*connected*, *related*, *bound*) one if it occurs as a constituent part of another subject EF. In this case, some subject *combined ER's* (*CbER's*) have free (disconnected) occurrences, whereas homolographic (photographic) tokens of some of these or of some other subject ER's, atomic or combined, and also *all* subject ET's, ordinary and special, occur only as bound (connected) constituent pats of free (disconnected) connected subject ER's. Therefore, A_1 is, as indicated in the item 3, a *calculus of ER's,* both *pseudo-predicate-free* ones and *pseudo-predicate-containing* ones, whereas some of the latter ER's are *pseudo-quantified* (*logically contracted, logically bound*) and the other are not. In this case, all ER's are, as was indicated in the item 4, functional but insignificant (uninterpreted) graphic chips or pattern groups of such chips.

12) An SCbEF, or, more precisely, an SCbEOR or an SCbPVI, may contain, as a constituent and as the pertinent *effective definiendum*, a *secondary euautographic kernel-sign* (*SEKS*), called also, synecdochically, a *secondary syncategorematic euautograph* (*SSCE*) or a *secondary euautographic syncategorem* (*SESC*), the understanding being that *no secondary punctuation marks are introduced in* A_1. Any given SEKS (SSCE) can be either an *atomic* one (*SAEKS* or *SASCE)*) or a *combined* one (*SCbEKS* or *SCbSCE)*) and independently either *pseudo-constant* one (briefly, *SPCKS*) or a *pseudo-variable* one (briefly, *SPVKS*). An SAEKS is necessarily a *pseudo-constant* one (briefly, *SAPCKS*), whereas an SCbEKS can be either a *pseudo-constant* one (briefly, *SCbPCKS*) or a *pseudo-variable* one (briefly, *SCbPVKS*). It is understood that the *primary euautographic kernel-sign* (*PEKS*), being the *effective definiens* of a given SEKS, is always a *combined* one (*PCbEKS*) and that it is a *pseudo-constant* one (*PCbPCKS*) or a *pseudo-variable* one (*PCbPVKS*) if and only if the pertinent SEKS is an SPCKS (i.e. either an SAPCKS or an SCbPCKS) or an SCbPVKS respectively. Thus, the class of EKS's of A_1 is the union of two complementary subclasses: the [class of] PEKS's and the [class of] SEKS's. The class of PEKS's is bifurcated into the class of PEAKS's that are introduced in the EABA and the class of PCbEKS's that are introduced as by-side products of some PFR's of A_1; certain simplest PCbEKS's are called *primary molecular EKS's* (*PMEKS's*).

Likewise, the class of SEKS's, all of which are introduced as by-side products of the SFR's of A_1, is bifurcated into of the class of SEAKS's and the class of SCbEKS's; certain simplest SCbEKS's are called *secondary molecular EKS's* (*SMEKS's*).

13) Any *continuous fragment*, without blanks (empty spaces) and without blank-signs, of an EF, primary or secondary, including the EF itself, is called a *purposeful*, or *working*, *euautographic assemblage* (*EA*), or briefly a *euautograph*, *of* A_1, a *primary* one (*PE*) or a *secondary* one (*SE*) respectively. An EA of A_1, which is not purposeful, is called a *purposeless*, or *idle*, *EA of* A_1. A *secondary purposeful* (*working*) *EA of* A_1 is called a *secondary euautographic assemblage* (*SEA*) *of* A_1 or, in accordance with the previous definition, a secondary euautograph (SE), and vice versa. That is to say, there is, by definition, *no secondary idle assemblage of* A_1. An EA of A_1 is said to be an *atomic* one (*AEA*) if it comprises a single *atomic euautograph* (*AE*) of A_1 and a *combined*, or *juxtapositional*, one (*CbEA*) if otherwise. Accordingly, a euautograph, i.e. a purposeful EA, is called a *combined euautograph* (*CbE*) if it is not atomic A combination (juxtaposition) of a PEA and a SEA of A_1 is a SEA of A_1. Likewise, a combination (juxtaposition) of a PE and an SE of A_1 is an SE of A_1. Therefore, *any EA of* A_1 *is either a PAE or an SAE* .and *any euautograph of* A_1 *is either a PE or an SE*. An EA is said to be a *euautographic ordinary assemblage* (*EOA*) if it comprises [homolographic tokens of] AOE's and a *euautographic special assemblage* (*ESpA*) if it contains at least one [homolographic token of a] ASpE and some or no [homolographic tokens of] AOE's.

13) A euautographic operand necessarily involves exactly one operator, called *the principal associated operator* (*PAO*) *of the operand*, such that it either *applies* to a single operatum to transform it into that operand or unites two or more operata to produce that operand. Consequently, the PAO of an operand is either the only operator occurring in the operand or the one of two or more operators occurring in the operand, which is executed in the last place, so that *the operand is the scope of its PAO* in either case. The PAO of an operand is said to be:

a) discriminately, an operator *of weight* 1, 2, 3, etc or a *singulary*, *binary*, *ternary*, etc one, and also generally an operator of weight *n* or an *n-ary* one, if it has 1,2,3, etc, or generally *n* operata respectively;

b) indiscriminately, a *multiary operator* if it has two or more operata.

An *n-ary* PAO consists of a certain kernel-sign followed by a pair of parentheses that encloses *n* tokens (occurrences) of the ‘n’-Space, which are separated by *n*–1 commas. Alternatively, a *singulary* or *binary PAO* may consist of a pair of square brackets enclosing a certain kernel-sign either along with an ‘n’-Space to the right of it, if it is singulary, or along with two ‘n’-Spaces on both sides of it, if it is binary. The kernel-sign of the PAO of a euautographic operand is qualified by the same prepositive or postpositive qualifiers as those of the PAO. Particularly, the kernel-sign of the PAO of a euautographic operand called *the principal kernel-sign* (*PKS*) *of the operand*. Since the PAO is determined by its kernel-sign, therefore the latter is sometimes equivocally called an operator.

14) The class of EKS’s of A_1 as defined in the item 12 is divided into the following two complementary subclasses:

a) The *euautographic ordinary*, or *logical*, *kernel-signs* (*EOKS’s* or *ELKS’s*), including (i) *euautographic ordinary*, or *logical*, *connectives* (*EOCv’s* or *ELCv’s*); (ii) *predicate-signs* (*EOPS’s* or *ELPS’s*), both *pseudo-variable* ones (*PVOPS’s* or *PVLPS’s*) and *pseudo-constant* ones (*PCOPS’s* or *PCLPS’s*) such as =, $\subseteq$, $\in$, and $\subset$; (iii) *euautographic ordinary*, or *logical*, *contractors* (*EOCt’s* or *ELCt’s*), or *binders* (*ELB’s* or *EOB’s*) and particularly *euautographic pseudo-quantifiers* (*EPQ’s*) – atomic, molecular, and complex (compound).

b) The *euautographic special*, or *algebraic* (in full, *special algebraic*), *kernel-signs* (*ESpKS’s* or *EAlKS’s*), including: $\hat{-}$, called the *singulary sign of additive inversion*; $\hat{+}$, called the *binary sign of addition*; $\hat{\cdot}$, called the *binary sign of multiplication*, $\hat{-}$, defined as an abbreviation of the assemblage $\hat{+}\hat{-}$ and called the *binary sign of subtraction*; $\hat{=}$, called the *binary special sign of equality*; $(\hat{\cdot}\,\mathbf{x})$ or $\hat{\cdot}_{\mathbf{x}}$, called the *euautographic special*, or *algebraic*, *contractor* (*ESpCt* or *EAlCt*) *over* **x** or, less explicitly, *an* ESpCt or EAlCt, and also, synonymously, with “*binder*” (“*B*”) in place of “*contractor*” (“*Ct*”); *V*, called the *validity-sign* or, when regarded as an abbreviation of *V*( ), *the validity-operator*, because its function is to «*termize*» (*substantivize*) an ER, i.e. to convert it into an integron (special term), which is called the *primary*, or *initial*, *validity-integron* (*PVI* or *IVI*) *of that ER* an also, less explicitly, a *primary*, or *initial*, *euautographic validity-integron* (*PEVI* or *IEVI*); $\overline{V}$, called the *antivalidity-sign* or, when regarded as an abbreviation of $\overline{V}(\,)$, *the validity-operator* – dual of *V* and defined in respect to *V*. In the above occurrences, ‘**x**’ is an *atomic* *panlogograph* (*APL*) or *atomic panlogographic placeholder* (*APLPH*), whose range is the class of PVOT's and which is therefore called an *atomic panlogographic ordinary term* (APLOT).

15) An EKS is called:

a) a *euautographic relational kernel-sign* (*ERlKS*) if it is either an EOKS (ELKS) or the ESpKS (EAlKS) $\hat{=}$;

b) a *euautographic substantival kernel-sign* (*ESlKS*) if it is an ESpKS (EAlKS) other than $\hat{=}$.

Thus, besides its dichotomy indicated in the item 14, the class of EKS's is independently divided into another two complementary subclasses: the ERlKS's and ESlKS's. Consequently, a *euautographic operand*, i.e. a *combined euautographic formula* (*CbEF*), is a *combined*, or *composite, euautographic relation* (*CbER* or *CpER*) if and only if its PKS (principal kernel-sign) is a *relational* one and a *combined*, or *composite, euautographic term* (*CbET* or *CpET*), i.e. a *combined*, or *composite*, *euautographic integron* (*CbEI* or *CpEI*), if and only if its PKS is a *substantival* one. A CbER is called a *connected ER* if it is an operatum or the operatum of an ELCv (EOCv) and a *bound*, or *contracted*, or *pseudo-quantified, ER* if it is the operatum of an ELCt (EOCt). Analogously, a CbEI is called a *connected EI* if it is an operatum or the operatum of a substantival EAlKS other than $\hat{\cdot}_{\mathbf{x}}$ (or $(\hat{\cdot}\,\mathbf{x})$) and a *bound*, or *contracted*, *EI* if it is the operatum of $\hat{\cdot}_{\mathbf{x}}$.

16) The pairs of qualifiers "logical" and "algebraic" and "ordinary" and "special" are used in the treatise as pairs of *complementary antonyms*. At the same time, in accordance with the items 5 and 15, when these qualifiers apply to PAE's or to EKS's, "logical" and "ordinary" or "algebraic" and "special" are used interchangeably, i.e. as *accidental* synonyms, although their lexical senses are distinct. *Neither secondary atomic euautographic logical* (*ordinary*) *terms nor combined euautographic logical* (*ordinary*) *terms, both primary and secondary,* are introduced in the treatise. Therefore, the descriptive *taxonyms* (*taxonomic names*, *names of taxa*, *names of taxonomic classes*) introduced in the item 5ii do not involve the prepositive qualifier "primary" ("P") – in contrast to the taxonyms introduced in the item 5iii. For the same reason, the former metaterms can be abbreviated further by omission of the prepositive qualifier "atomic" ("A"). Hence, the following three groups of four abbreviations each, of the corresponding full names are groups of synonyms:

(a) "EOT", "ELT", "AEOT", "AELT"; (b) "PVOT", "PVLT", "APVOT", "APVLT"; (c) "PCOT", "PCLT", "APCOT", "APCLT" .

In accordance with the above-said, a *combined euautographic term* (*CbET*) is a *combined euautographic integron* (*CbEI*) and vice versa. At the same time, a PAEI or a CbEI is indiscriminately called an EI (euautographic integron), while a PAEI is alternatively (synonymously) called a PAESpT or PAEAlT, by the item 5iii. Consequently, a PAESpT (PAEAlT) or a CbET is indiscriminately called a *euautographic special*, or *algebraic*, *term* (*ESpT* or *EAlT*) and also, by the previous definition, a *euautographic integron* (*EI*). Thus, "ESpT", "EAlT", and "EI" are synonyms and hence "*combined ESpT*" ("*CbESpT*"), "*combined EAlT*" ("*CbEAlT*"), "*CbET*", and "*CbEI*" are synonyms as well. Thus, when either pair of qualifiers "logical" and "algebraic" or "ordinary" and "special" applies to ET's, it remains a pair of *accidental synonyms*. The difference in senses of the qualifiers of each pair becomes essential when they apply to *combined euautographic relations* (*CbER's*), as specified in the next item.

17) The class of CbER's of A_1 (see the item 15) can be divided into two subclasses in two ways. In accordance with one of the two dichotomies of that class, a CbER is called:

a) a *combined euautographic ordinary relation* (*CbEOR*) if it involves *no EI* – or, what comes to the same thing, if it involves *no occurrence of* $\hat{=}$.

b) a *euautographic special relation* (*ESpR*) if it involves *at least one EI* – or, what comes to the same thing, if it involves *at least one occurrence of* $\hat{=}$.

In accordance with the other dichotomy, a CbER is called:

a′) a *combined euautographic logical relation* (*CbELR*) if and only if its PKS (principal kernel-sign) is an ELKS (EOKS);

b′) a *euautographic algebraic relation* (*EAlR*) or a *euautographic algebraic equality* (*EAlE*) if and only if its PKS is $\hat{=}$.

The prepositive qualifier "combined" ("Cb") to any of the taxonyms introduced in the items b) and b′) would have been redundant because there are in A_1 neither ESpR's nor EAlR's that could be qualified *atomic*.

The above two definitions have the following implications.

i) It follows from the items a) and a′) that *every CbEOR is a CbELR but not necessarily vice versa*. Specifically, a CbELR is called:

a_1) a *combined euautographic ordinary logical relation* (*CbEOLR*) or, simply, a *combined euautographic ordinary relation* (*CbEOR*) if and only if it involves *no occurrence* of $\hat{=}$;

a_2) a *combined euautographic special logical relation* (*CbESpLR*) or, simply, a *euautographic special logical relation* (*ESpLR*) if and only if it involves at least one occurrence of $\hat{=}$, not being the principal one.

At the same time, an AEOR (atomic EOR) or a CbEOR (combined EOR) is indiscriminately called an EOR (euautographic ordinary relation) and likewise an AELR (atomic ELR) or a CbELR (combined ELR) is indiscriminately called an ELR (euautographic ordinary relation), while "AEOR" and "AELR" are synonyms. Hence, *every EOR is an ELR but not necessarily vice versa.*

ii) By the item b′), an EAlR involves at least one occurrence of $\hat{=}$ and hence it involves at least two EI's standing on both sides of that occurrence of $\hat{=}$. Therefore, it follows from the items b) and b′) that *every EAlR* is an *ESpR but not necessarily vice versa*. Specifically, an ESpR is called:

b_1) a *euautographic algebraic special relation* (*EAlSpR*) or, simply, a *euautographic algebraic relation* (*EAlR*) and also a *euautographic algebraic equality* (*EAlE*) if and only if it involves at least one occurrence of $\hat{=}$ as its PKS;

b_2) a *euautographic logical special relation* (*ELSpR*) if and only if no token of $\hat{=}$ occurs in it as its PKS.

An EAlR (EAlE) is a called a *euautographic algebraic identity* (*EAlI*) if it is *valid*. and a *euautographic algebraic anti-identity* (*EAlAntI*) if it is *antivalid*.

iii) By the items i) and ii), it follows from the items a), b), and a′) that some ELR's are EOR's, i.e. *EOLR's* (*euautographic ordinary logical relations*), while the other ELR's are ESpR's, i.e. *ESpLR's* (*euautographic special logical relations*). Since an EOR involves, as its constituent parts, no EI's and hence no ESpR's, therefore it involves no EAlR's either. Therefore, an EOR (EOLR) can alternatively be called a *euautographic chaste logical relation* (*EChLR*). Consequently, an ESpLR can alternatively be a *euautographic mixed logical relation* (*EMxLR*) in the sense that it involves at least one EAlR as its constituent part – or, what comes to the same thing, at least one occurrence of $\hat{=}$ not being its principal operator. Thus, the combined qualifiers "*ordinary logical*" ("*OL*") and "*chaste logical*" ("*ChL*"), or "*special* *logical*” (“*SpL*”) and “*mixed logical*” (“*MxL*”), are concurrent (exchangeable). Incidentally, since an EOT (ELT’) is exclusively a primary atomic one, it can also be alternatively called a *euautographic chaste* (*pure*) *logical term* (*EChLT*). In analogy with the above terminology, an EAlR (EAlE) as defined by the item b′), called also an EAlSpR by the item b_1), will discriminately be called:

$b_{1'}$) a *euautographic chaste algebraic relation* (*EChAlR*) or a *euautographic chaste algebraic equality* (EChAlE) if it involves, as its constituent parts, *neither AEOR’s nor EOT’s*;

$b_{2'}$) a *euautographic mixed* algebraic *relation* (*EMxAlR*) or a *euautographic mixed algebraic equality* (*EMxAlE*) if otherwise.

iv) In the item 16, I have indicated that there are in A_1 no terms that could be qualified either as *combined euautographic ordinary* or as *combined euautographic logical*. Therefore, in the versions of the above two bifurcations of the class of CbER’s with “CbET” in place of “CbER” and “term” (“T”) in place of “relation” (“R”), the pertinent versions of the items a) and a′) should be disregarded, whereas the rest of those definitions can be restated thus. A *CbET* is called:

b″) a CbESpT , i.e. a CbEI, if it is neither EOT nor AEI;

b‴) a CbEAlT if its PKS is an ESlKS.

At the same time, by the item 16, “CbET”, “CbEI”, “CbESpT”, and “CbEAlT” are synonyms. Therefore, the items b″) and b‴) are just two different explicative definitions of a CbET (CbEI). Hence, there are *no logical EI’s* (*logical ESpT’s*) – just as there are *no algebraic EOT’s*.

v) In agreement with the pertinent remark made at the end of the item 2, it is tacitly assumed that in any of the taxonyms (metaterms), which have been introduced above in this and in the previous items of the definitions the anterior qualifier “euautographic” (“E”) and any one of the posterior qualifier: such as: “ordinary” (“O”), “special” (“Sp”), “logical” (“L”), “algebraic” (“Al”), “ordinary logical” or “chaste logical” (“OL” or “ChL”), “special logical” or “mixed logical” (SpL” or “MxL”), “logical special” (“LSp”), “chaste algebraic” (“ChAl”), and “mixed algebraic” (“MxAl”), the two qualifiers can be permuted without altering the meaning of the taxonym.

18) A CbER (CpER) is either a *molecular ER* (*MER*) or a *complex*, or *compound*, *ER* (*CxER* or *CdER*). The class of MER’s is united with the class of

AER's to form the class of *elemental*, or *primitive*, *ER's* (*ElER's* or *PvER's*). Consequently, an ER is either an ElER (PvER) or a CxER (CdER), – in addition to the fact that an ER is either an AER or a CbER. The above statements apply with "I" for "integron" or w with "KS" for "kernel-sign" in place of "R" for "relation". In this case, every MER turns out to be *ordinary and logical simultaneously* – just as every AME, every *MEI* turns out to be *special and algebraic simultaneously* because so is every EI, whereas an *MEKS* is either an *ordinary*, or *logical*, one *special and algebraic simultaneously* (*MEOKS* or *MELKS*) or a *special*, or *algebraic*, one (*MESpKS* or *MEAlKS*), because so is any EKS. In accordance with the item 15, a CbEF is either a CbER or a CbEI, and therefore an MER or an MEI is indiscriminately called a *molecular EF* (*MEF*). At the same time, a pair of square or round brackets, [ ] or ( ), is called a *molecular euautographic punctuation mark* (*MEPM*), whereas a single bracket or a comma "," is an *atomic euautographic punctuation mark* (*AEPM*). An AEPM or an MEPM is indiscriminately called a *euautographic punctuation mark* (*EPM*), – without either of the qualifiers "primary" and "elemental", because there are no EPM's in A_1 that could be qualified *secondary* or *complex*. An EPM or an EKS is indiscriminately called a *syncategorematic euautograph* (*SCE*) or a *euautographic syncategorem* (*ESC*), the understanding being that an EPM is an *euautographic ordinary*, or *logical*, *syncategorem* (*EOSC* or *ELSC*). Thus, when applied to an MEF or to an EKS, the qualifiers "ordinary" and "logical" or "special" and "algebraic" are used synonymously – just as in the case of PAE's (cf. the item 16).

19) A_1 is a *bunch* (*bundle*) of an infinite number of EAPO's, called *branches of* A_1, which are intermixed for conveniently treating them simultaneously as a single whole. All branches of A_1 have as an inseparable part of each of them and of the entire A_1 *one and the same built-in euautographic algebraic*, and hence *analytical*, *decision method*, that will be denoted by 'D_1' and be called the *Advanced Algebraic Decision Method* (*AADM*) *of* A_1 and also the *Euautographic AADM* (*EAADM*). D_1 is a system of *intrinsic* (*subject*) *and extrinsic* (*metalinguistic*) *rules of inference* (*transformation*) *and decision of* A_1 in progress, which consists of two parts to be denoted by '$\mathsf{D}_1^{\mathrm{a}}$' and '$\mathsf{D}_1^{\mathrm{t}}$'. In the beginning, D_1 is the totality (list), $\mathsf{D}_1^{\mathrm{a}}$, of the *general* (*universal*, *typical*) *intrinsic* (*subject*) *axioms of* A_1 *and extrinsic* (*meta*) *axioms* (*primary rules*) *of inference and decision of* A_1, all of which are stated in *the*

very first, *unbranched but a posteriori branchable* (*analyzable into branches*), *phase of* A_1, called verbally the *Primordial*, or *Root*, *EAPO* (briefly *PEAPO* or *REAPO*) and denoted logographically by 'A_{1P}' or 'A_{1R}'. But as I go along, D_1^a is supplemented with an increasingly broad totality, D_1^t, of the *general intrinsic* (*subject*) *theorems of* A_1 *and extrinsic* (*meta*) *theorems* (*secondary rules*) *of inference and decision of* A_1, which are deduced from and with the help of D_1 and which are used in subsequent inferences and decisions for the sake of brevity. Thus, 'D_1' will be used for mentioning the EAADM of A_1 in any current state of it and particularly for mentioning D_1^a. That is to say, D_1 (the denotatum 'D_1') is augmented with *abbreviative rules* of inference and decision as A_1 is executed.

20) D_1 includes the *ADM of* A_1^0, which is denoted by 'D_1^0' and is alternatively called the *Rich Basic Algebraic Decision Method* (*RBADM*) *of* A_1 and also the *Euautographic RBADM* (*ERBADM*). In turn, D_1^0 and hence D_1 includes the *ADM of* A_0, which is denoted by 'D_0' and is alternatively called the *Basic*, or *Depleted Basic*, *Algebraic Decision Method* (*BADM* or *DBADM*) *of* A_1^0 *and* A_1. Thus, A_1^0 and A_1 are two increasingly broad regions of applicability of D_1^0, whereas A_0, A_1^0, and A_1 are three increasingly broad regions of applicability of D_0. Particularly, within A_1, D_1^0 is a strict part of D_1, which concerns exclusively with occurrences of the *logical* (*ordinary*) *connectives* and of the *algebraic* (*special*) *sign of equality* $\hat{=}$ in *euautographic relations* (*ER*'s), and neither with any predicate-signs and any EOT's nor with any binders (contractors), if present. That is to say, in the framework of A_1, every largest (most inclusive) *operand* (*scope*) *of a binder* (including the latter) that is not an *operatum* (pl. "*operata*") *of* another binder, i.e. every operand of a binder that is not an *ER operated upon by* another binder, is treated by D_1^0 as a single whole ER – just as *elemental* (*atomic and molecular*) ER. In contrast to D_1^0, D_0 does not apply to any ER that involves at least one occurrence of a predicate-sign or of EOT. Like D_1, D_1^0 consists of two parts, D_1^{0a} and D_1^{0t}, being the pertinent parts of D_1^a and D_1^t respectively. Consequently, D_0 consists of two parts, D_0^a and D_0^t, being the pertinent parts of D_1^{0a} and D_1^{0t}, and hence of D_1^a and D_1^t, respectively. It is in contrast to the ADM's D_1^0 and D_0, which are qualified *rich basic* and [*depleted*] *basic* respectively,

the ADM D_1 is qualified *advanced.* Since both D_1^0 and D_0 differ from D_1, therefore A_1^0 and A_0 are *neither phases nor branches of* A_1, while A_0 is *not an EAPO* at all (see the item 1 for its two descriptive proper names and their abbreviations). Owing to its simplicity, A_0 can be regarded as an introduction into A_1, so that, in reference to this role, A_0 can be called *the first zero quasi-phase of* A_1, whereas A_1^0 having nearly the same *EBADM*, D_1^0, can be called *the second zero quasi-phase of* A_1.

21) A_1 is set up as a single whole CEAPO in such a way that it has the unbranched *self-contained* (*self-sufficient*) *and self-controlled* Root EAPO, A_{1R}, and an indefinite number of a *self-contained and self-controlled* branch EAPO's both within A_{1R} and over A_{1R} as their extensions. In this connection, it is noteworthy that once a certain part of A_1 is called an EAPO, it is necessarily a *self-contained* (*self-sufficient*) *and self-controlled* calculus in the sense that it can be executed independently both of any other partial EAPO and of the entire A_1. Moreover, any branch of A_1 can, in principle, be regarded as an *autonomous* EAPO, i.e. as one that is *capable of being set up alone independently of the entire* A_1. Accordingly, any branch of A_1 that is developed from A_{1R} has some phases of its own, the first of which is a certain part of A_{1R}. It is however impractical to set up D_1 for a particular one of an infinite number of potential branches of A_1, while it is possible to set up that same universal and unaltered D_1 for the entire A_1 and hence for all potential branches simultaneously by the same work input. A phase or a branch, or a phase of the branch, of A_1 will be commonly (indiscriminately) called *an EAPO*. Also, each conspicuous partial EAPO of A_1 will be distinguished by a deductive proper name describing it through the genus, denoted by the generic name "EAPO", and the difference or differences, denoted by the appropriate prepositive qualifier or qualifiers; the name has the definite article as the limiting modifier. In addition, each conspicuous EAPO will be denoted by a logographic constant comprising 'A_1' and the appropriate subscript or superscript or both on it.

22) I say that an ER of A_1 is one of *academic or practical interest* (*API*) if it is a *comprehensible*, i.e. not unreasonably long and complex, ER of at least one of the following kinds:

a) an illustration of certain aspects of A_1;

b) an illustration of the effectiveness of D_1;

c) a master relation of a certain subject of logic;

d) a general formal solution of one of the logical paradoxes;

e) formal groundwork upon which a system of reasoning is erected in the treatise or can be erected in logic or mathematics in the sequel;

f) an instructive example of mental experience.

EOR's of A_1 have analogues or interpretands among relations of *conventional axiomatic logical calculi* (*CALC'i*), *sentential* ones (*CASC'i*) or *predicate* ones (*CAPC'i*), while EAlR's (EAlE's) and ESpLR's are either *tools* or *by-side products* of D_1 that have no analogues in any CALC. Therefore, an EOR can have either practical interest or academic interest or both, whereas some EAlR's and ESpLR's can have *academic interest* only.

23) Given an ER (primarily an EOR) **P** of A_1 *of academic or practical interest*, an *algebraic proof*, which begins with application of the appropriate rule of D_1 to the *euautographic algebraic identity* (*EAlI*)

$$V(\mathbf{P}) \hat{=} V(\mathbf{P}) \tag{3.1}$$

as the *initial premise* and which ends with the pertinent *ultimate concluding identity* of one of the following three forms:

$$V(\mathbf{P}) \hat{=} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \mathbf{i}|\mathbf{P}\rangle & \text{(c)} \end{cases} \tag{3.2}$$

as the *pertinent theorem thus proved* is denoted by '$\mathsf{D}_1(\mathbf{P})$' and is called a *euautographic algebraic decision procedure* (*EADP*) *for* **P** or less explicitly an *EADP of* A_1. In this case, the following nomenclature (notation and terminology) and informal phraseology are used.

When used xenonymously as above, '**P**' is an *AtPLPH* (*atomic panlogographic placeholder*), whose range is the class of all ER's of A_1, unless it is restricted, as done above by the qualifier "of academic or practical interest". When used autonymously, '**P**' is an *atomic panlogograph* (*AtPL*) or, more specifically, an *analytical atomic panlogographic relation* (*AnAPLR*). The ER **P**, *proceeded* by the EADP $\mathsf{D}_1(\mathbf{P})$, is called the *euautographic slave-relation* (*ESR*), or *euautographic relation-slave* (*ER-slave*), and also the *object ER, of the* $\mathsf{D}_1(\mathbf{P})$. $V(\mathbf{P})$ is the so-called *primary*, or *initial*, *validity-integron* (briefly *PVI* or *IVI*) *of* **P**, whereas $\mathbf{i}|\mathbf{P}\rangle$ is, when applicable, a certain *irreducible*, or *ultimate*, *validity-integron* (*IRVI* or *UVI*) *of* **P** other than 0 or 1, which is *commonly* (less explicitly) called a *non-digital* or *pseudo-variable*, *irreducible* or *ultimate*, *euautographic validity-integron* (briefly, *NDIREVI*, *PVIREVI*, *NDUEVI*, or *PVUEVI)*, without the qualifier "*of* **P**". It is postulated that for any ER **P** of A_1, *V*(**P**) satisfies the *idempotent law*:

$$V(\mathbf{P})\hat{\cdot}V(\mathbf{P})\hat{=}V(\mathbf{P}), \tag{3.3}$$

and hence $\mathbf{i}|\mathbf{P}\rangle$ satisfies the similar law:

$$\mathbf{i}|\mathbf{P}\rangle\hat{\cdot}\mathbf{i}|\mathbf{P}\rangle\hat{=}\mathbf{i}|\mathbf{P}\rangle, \tag{3.4}$$

– just as 0 and 1 do.

When '**P**' is mentally used *xenonymously* as above, i.e. in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode* for mentioning any *particular* (*concrete*) *but not particularized* (*not concretized*) *ER of the range of* '**P**', the identity (3.3) is called either, subjectively, *a euautographic axiom* (*EA*) *of* (*belonging to*) A_1 or, objectively, a *panlogographic schema* (*PLS*) *of EA's of* A_1. When '**P**' is mentally used *autonymously*, i.e. as a *tychautograph* (*accidental autograph*), either for mentioning itself or for mentioning its any *homolographic* (*photographic*, *congruent* or *proportional*) token, the identity (3.3) is called a *panlogographic axiom* (*PLA*) *of* $\mathbf{A}_1$. Thus, the qualifier "*panlogographic*" means «*of*, i.e. *belonging to*, $\mathbf{A}_1$», whereas $\mathbf{A}_1$ is the *background calculus of logographic placeholders of foreground euautographic formulas of* A_1 – the calculus, which will be described in the next section. Like remarks apply *verbatim* to the identities (3.1) and (3.4). By contrast, any of the three equalities (a), (b), and (c) of the *metalinguistic scheme* (*pattern*) (3.2), i.e. a scheme that belongs to the *exclusive metalanguage* (*XML*) of both A_1 and $\mathbf{A}_1$, is *not an identity*, so that it cannot be used assertively as a valid tychautographic relation. These equalities are used here xenonymously as three mutually independent *ad hoc conditions on* **P**, i.e. on *accidental euautographic denotata of* '**P**'. However, the pertinent one of the three conditions (a), (b), and (c) of (3.2), which a given ER **P** satisfies, turns *ipso facto* into an *identity*, which will be denoted by '$\mathsf{T}_{1+}(\mathbf{P})$', '$\mathsf{T}_{1-}(\mathbf{P})$', or '$\mathsf{T}_{1\sim}(\mathbf{P})$' respectively or indiscriminately by '$\mathsf{T}_{1}(\mathbf{P})$' and which will be called the *euautographic master-theorem* (*EMT*), or *euautographic decision theorem* (*EDT*), *for* **P**, or, more generally, an *EDT*, or *DT (decision theorem*), *of* A_1. The ER **P**, which has been called the *euautographic slave-relation* (*ESR*), or *euautographic* *relation-slave* (*ER-slave*), or *object ER, of the algebraic proof* $\mathsf{D}_1(\mathbf{P})$, is also said to be so of $\mathsf{T}_1(\mathbf{P})$ itself. $\mathsf{D}_1(\mathbf{P})$, being an *algebraic and hence analytical* (*computational, not tabular*) *proof of* $\mathsf{T}_1(\mathbf{P})$, can schematically be written in the *staccato style* as either one of the following two *sequences of euautographic algebraic identities* (*EAlI's*), i.e. *valid* EAlE's, which are interrelated by certain rules of inference comprised in D_1:

$$V(\mathbf{P}) \hat{=} V(\mathbf{P}), V(\mathbf{P}) \hat{=} \mathbf{i}_1|\mathbf{P}\rangle, \mathbf{i}_1|\mathbf{P}\rangle \hat{=} \mathbf{i}_2|\mathbf{P}\rangle, ..., \mathbf{i}_{n-1}|\mathbf{P}\rangle \hat{=} \mathbf{i}_n|\mathbf{P}\rangle,$$

$$\mathbf{i}_n|\mathbf{P}\rangle \vec{\hat{=}} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \mathbf{i}|\mathbf{P}\rangle & \text{(c)} \end{cases} \tag{3.2$_1$}$$

or

$$V(\mathbf{P}) \hat{=} V(\mathbf{P}), V(\mathbf{P}) \hat{=} \mathbf{i}_1|\mathbf{P}\rangle, V(\mathbf{P}) \hat{=} \mathbf{i}_2|\mathbf{P}\rangle, ..., V(\mathbf{P}) \hat{=} \mathbf{i}_{n-1}|\mathbf{P}\rangle,$$

$$V(\mathbf{P}) \hat{=} \mathbf{i}_n|\mathbf{P}\rangle \vec{\hat{=}} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)}, \\ \mathbf{i}|\mathbf{P}\rangle & \text{(c)} \end{cases} \tag{3.2$_2$}$$

In the *legato style*, either of the two sequences can be written as:

$$V(\mathbf{P}) \hat{=} \mathbf{i}_1|\mathbf{P}\rangle \hat{=} \mathbf{i}_2|\mathbf{P}\rangle \hat{=} ... \hat{=} \mathbf{i}_{n-1}|\mathbf{P}\rangle \hat{=} \mathbf{i}_n|\mathbf{P}\rangle \vec{\hat{=}} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)}, \\ \mathbf{i}|\mathbf{P}\rangle & \text{(c)} \end{cases} \tag{3.2$_3$}$$

In this case, $\mathbf{i}_1|\mathbf{P}\rangle$ to $\mathbf{i}_{n\text{-}1}|\mathbf{P}\rangle$ are *reducible*, or *intermediate*, *secondary euautographic validity-integrons* (*RSEVI*) *of* **P**, each of which satisfies the respective variant of the idempotent law (3.4). The sign $\vec{\hat{=}}$ in (3.2$_1$)–(3.2$_3$), and generally in what follows, is a metalinguistic *sign of equality by definition*. Unlike '**P**', which is an *A*PL (*atomic* panlogograph) or *A*PLPH (*atomic* panlogographic placeholder), '$V(\mathbf{P})$', '$\mathbf{i}_1|\mathbf{P}\rangle$', '$\mathbf{i}_2|\mathbf{P}\rangle$', etc are *molecular panlogographs* (*MPL's*) or *molecular panlogographic placeholders* (*MPLPH's*), while '$\mathbf{i}_{n\text{-}1}|\mathbf{P}\rangle$' and '$\mathbf{i}_{n\text{-}}|\mathbf{P}\rangle$' are *metalogographic* (i.e. *metalinguistic logographic*) *placeholders* (*MLPH's*) because the *numeral-valued subscripts* '$_{n-1}$' and '$_n$' belong to the XML of A_1 and $\mathbf{A}_1$. The difference between the PLPH's '**P**' and '$V(\mathbf{P})$' on the one hand and the *logographic placeholders* (*LPH's*) '$\mathbf{i}_1|\mathbf{P}\rangle$', '$\mathbf{i}_2|\mathbf{P}\rangle$', etc, '$\mathbf{i}_{n\text{-}1}|\mathbf{P}\rangle$', and '$\mathbf{i}_{n\text{-}}|\mathbf{P}\rangle$' on the other hand is that the former will used in *subject panlogographic formulas of* $\mathbf{A}_1$, particularly in *panlogographic algebraic decision procedures* (*PLADP's*), whereas the latter are subsidiary ones that are and will be used only in the *aboutness* of A_1 and $\mathbf{A}_1$ and their ADM's D_1 and $\mathbf{D}_1$, and particularly in the aboutness of EADP's and PLADP's. To summarize the above-said, all EADP's are *analytical* (*computational*, *transformative*) decision procedures – as opposed to both *tabular* decision procedures (as those based on truth-tables) and *conformal interpretational* (*substitutional*) ones to be described in due course. The same is true of all PLADP's, which will be discussed in the next section. An EADP is called a *basic one* (*BEADP*) if it is performed by means of D_0, a *rich basic one* (*RBEADP*) if it is performed by means of D_1^0, and an *advanced* one (*AEADP*) if it involves applications of at least one rule of D_1 not belonging either to D_0 or to D_1^0. A BEADP of $\mathbf{P}$ is denoted by '$\mathsf{D}_0(\mathbf{P})$', whereas the pertinent EDT $\mathsf{T}_{1+}(\mathbf{P})$, $\mathsf{T}_{1-}(\mathbf{P})$, or $\mathsf{T}_{1\sim}(\mathbf{P})$ will, when desired, be denoted more specifically by '$\mathsf{T}_{0+}(\mathbf{P})$', '$\mathsf{T}_{0-}(\mathbf{P})$', or '$\mathsf{T}_{0\sim}(\mathbf{P})$' respectively or indiscriminately by '$\mathsf{T}_0(\mathbf{P})$' instead of '$\mathsf{T}_1(\mathbf{P})$'. An RBEADP of $\mathbf{P}$ is denoted by '$\mathsf{D}_1^0(\mathbf{P})$', whereas the pertinent EDT $\mathsf{T}_{1+}(\mathbf{P})$, $\mathsf{T}_{1-}(\mathbf{P})$, or $\mathsf{T}_{1\sim}(\mathbf{P})$ will, when desired, be denoted more specifically by '$\mathsf{T}_{1+}^0(\mathbf{P})$', '$\mathsf{T}_{1-}^0(\mathbf{P})$', or '$\mathsf{T}_{1\sim}^0(\mathbf{P})$' respectively or indiscriminately by '$\mathsf{T}_1^0(\mathbf{P})$' instead of '$\mathsf{T}_1(\mathbf{P})$'.

24) An ER $\mathbf{P}$ of A_1 is said to be *valid* if its DT has the form (3.2a), *antivalid* if its DT has the form (3.2b), and *vav-neutral* (or *vav-indeterminate*), i.e. *neutral* (or *indeterminate*) *with respect to validity and antivalidity* or, in other words, *neither valid nor antivalid*, if its DT has the form (3.2c) subject to (3.4). Thus, the *form* of the EDT allows unambiguously attributing the processed relation to one of the following three kinds: *valid*, *antivalid*, or *vav-neutral*. Therefore, the *scheme* (3.2) of three possible forms of the EDT for an ER, $\mathbf{P}$, is called the *EDT* (*euautographic decision theorem*) *scheme*, or *pattern*, *for* $\mathbf{P}$. An ER of A_1 that has been subjected to a successful EADP, in the result of which it is relegated to one of the above tree classes, is called a *decided ER* (briefly, *DdER*) or, more precisely, a *vavn-decided ER*, i.e. decided with respect to validity, anivalidity, or vav-neutrality (vav-indeterminacy). Accordingly, in reference to a relation of A_1, the noun "*decision*", kindred of the adjective "*decided*", should be understood as *decision with respect to validity, antivalidity, and vav-neutrality* or briefly as *vavn-decision*. Particularly, the abbreviations "EADP", "EDT", and "DT", introduced above", should, more precisely, be replaced with the abbreviations "*vavn-EADP*", "*vavn-EDT*", and "*vavn-DT*" respectively. The division of the vavn-decided relations of A_1 into the three classes:

valid, antivalid, and vav-neutral (vav-indeterminate) is called the *basic decisional trichotomy* (*trisection*, *trifurcation*) *of the vavn-decided ER's*. A vavn-decided ER of A_1 is said to be: *invalid* if it is either antivalid or vav-neutral, *non-antivalid* if it is either valid or vav-neutral, and *vav-unneutral* if it is either valid or antivalid. In all above-mentioned terms, the words "*neutral*", "*unneutral*", "*neutrality*", and "*unneutrality*" can be used interchangeably with "*indeterminate*", "*determinate*", "*indeterminacy*", and "*determinacy*" respectively. The latter three divisions of the vavn-decided ER's into two complementary classes each, namely: (a) *valid* and *invalid*, (b) *antivalid* and *non-antivalid*, (c) *vav-neutral* (*vav-indeterminate*) and *vav-unneutral* (*vav-determinate*) are called the *subsidiary decisional dichotomies* (*bisections, bifurcations*) *of the vavn-decided ER's*. *Orismological* (term-formation) aspects of these dichotomies are made explicit in Appendix 2 (A2).

25) It is proved (inferred) by the pertinent rule of D_1 that

$$V(\neg\mathbf{P}) \hat{=} 1 \hat{-} V(\mathbf{P}), \tag{3.5}$$

which is the EDT for $\neg\mathbf{P}$, $\neg$ being the *kernel-sign* (*logical connective*) *of negation*. Therefore, any one of the three identity schemata (3.2,a–c) subject to (3.4) holds if and only if the respective one of the following three identity schemata holds:

$$V(\neg\mathbf{P}) \hat{=} \begin{cases} 1 & \text{(a)} \\ 0 & \text{(b)} \\ \mathbf{i}|\neg\mathbf{P}\rangle & \text{(c)} \end{cases} \tag{3.6}$$

where, in accordance with (3.5),

$$\mathbf{i}|\neg\mathbf{P}\rangle \hat{=} 1 \hat{-} \mathbf{i}|\mathbf{P}\rangle, \tag{3.7}$$

In this case, it follows from (3.3) and (3.4) by (3.5) and (3.7) that

$$V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{P}) \hat{=} V(\neg\mathbf{P}), \tag{3.8}$$

$$\mathbf{i}|\neg\mathbf{P}\rangle \hat{\cdot} \mathbf{i}|\neg\mathbf{P}\rangle \hat{=} \mathbf{i}|\neg\mathbf{P}\rangle, \tag{3.9}$$

In accordance with (3.2) and (3.6), *the negation of a valid ER*, **P**, *is an antivalid ER*, $\neg$**P**, *and vice versa*, whereas *the negation of a vav-neutral* (*vav-indeterminate*) *ER*, **P**, *is another vav-neutral* (*vav-indeterminate*) *ER*, $\neg$**P**. At the same time, under the definitions

$$\overline{V}(\mathbf{P}) \vec{\hat{=}} V(\neg\mathbf{P}) \hat{=} 1 \hat{-} V(\mathbf{P}), \tag{3.10}$$

$$\bar{\mathbf{i}}|\mathbf{P}\rangle \vec{\hat{=}} \mathbf{i}|\neg\mathbf{P}\rangle \hat{=} 1 \hat{-} \mathbf{i}|\mathbf{P}\rangle, \tag{3.11}$$

which are based on (3.5) and (3.7), the decision theorem pattern (scheme) (3.6) for $\neg\mathbf{P}$ becomes the *dual euautographic decision theorem* (*DEDT*) *pattern* (*scheme*) *for* **P**:

$$\bar{V}(\mathbf{P}) \hat{=} \begin{cases} 1 & \text{(a)} \\ 0 & \text{(b)} \\ \bar{\mathbf{i}}|\mathbf{P}\rangle & \text{(c)} \end{cases}, \tag{3.12}$$

which are *dual* of (3.2), while the identities (3.8) and (3.9) turn into

$$\bar{V}(\mathbf{P}) \hat{\cdot} \bar{V}(\mathbf{P}) \hat{=} \bar{V}(\mathbf{P}), \tag{3.13}$$

$$\bar{\mathbf{i}}|\mathbf{P}\rangle \hat{\cdot} \bar{\mathbf{i}}|\mathbf{P}\rangle \hat{=} \bar{\mathbf{i}}|\mathbf{P}\rangle, \tag{3.14}$$

which are *dual* of (3.3) and (3.4) respectively. It goes without saying that the identities (3.11) and (3.14) apply with any of the logographs '$\bar{\mathbf{i}}_1$' to '$\bar{\mathbf{i}}_n$' in place of '$\bar{\mathbf{i}}$'. The theorem (a), (b), or (c) of (3.72) is denoted by '$\bar{\mathsf{T}}_{1+}(\mathbf{P})$', '$\bar{\mathsf{T}}_{1-}(\mathbf{P})$', or '$\bar{\mathsf{T}}_{1\sim}(\mathbf{P})$' respectively or indiscriminately by '$\bar{\mathsf{T}}_1(\mathbf{P})$', so that $\bar{\mathsf{T}}_{1+}(\mathbf{P})$, $\bar{\mathsf{T}}_{1-}(\mathbf{P})$, or $\bar{\mathsf{T}}_{1\sim}(\mathbf{P})$ is the EDT for **P**, which is *dual* of $\mathsf{T}_{1+}(\mathbf{P})$, $\mathsf{T}_{1-}(\mathbf{P})$, or $\mathsf{T}_{1\sim}(\mathbf{P})$ respectively.

26) Any *euautographic special* (*algebraic) term* is alternatively called a *euautographic integron* (*EI*). In accordance with the items 23 and 24, an EI as $V(\mathbf{P})$, $\mathbf{i}_1|\mathbf{P}\rangle$ to $\mathbf{i}_n|\mathbf{P}\rangle$, $\mathbf{i}|\mathbf{P}\rangle$), 0, or 1 is called a *euautographic validity-integron* (*EVI).* Consequently, the pertinent one of the three terms 0, 1, and $\mathbf{i}|\mathbf{P}\rangle$ on the right hand side of the identity pattern (3.2) is called, indiscriminately, the *irreducible*, or *ultimate*, *validity-integron* (briefly *IRVI* or *UVI*), and also *validity-identifier* or *validity-index* (briefly *VID* in both cases), *of* **P** and, discriminately and less explicitly, the *validity-integron* (*VI*) *validity*, the *VI antivalidity*, or *a euautographic VI* (*EVI*) *neutrality* (and also *an EVI indeterminacy*) in that order, – or, alternatively, with "*VID*" in place of "*VI*". At the same time, in accordance with the item 25, any one of the EI's: $\bar{V}(\mathbf{P})$, $\bar{\mathbf{i}}_1|\mathbf{P}\rangle$ to $\bar{\mathbf{i}}_n|\mathbf{P}\rangle$ (i.e. $\bar{\mathbf{i}}|\mathbf{P}\rangle$), 0 or 1 is called a *euautographic antivalidity-integron* (*EAVI).* Consequently, the pertinent one of the three terms 1, 0, and $\bar{\mathbf{i}}|\mathbf{P}\rangle$ or on the right hand side of the identity scheme (pattern) (3.12) is called, indiscriminately, the *irreducible*, or *ultimate*, *antivalidity-integron* (briefly *IRAVI* or *UAVI*), and also *antivalidity-identifier* or *antivalidity-index* (briefly *AVID* in both cases), *of* **P** and, discriminately and less explicitly, the *antivalidity-integron* (*AVI*) *validity*, the *AVI antivalidity*, or *a euautographic AVI* (*EAVI*) *neutrality* (and also *an EAVI*

indeterminacy) in that, – or, alternatively, with “*AVID*” in place of “*AVI*”. Since both EVI’s and EAVI’s satisfy the idempotent law, therefore an EVI or an EAVI is *impartially* called a *idempotent euautographic integron* (*IEI*). In agreement with the above terminology, the special (algebraic) singular kernel-sign V is called the *validity-operator,* because it converts an ELR its *primary*, or *initial, validity-intrgron* (*PVI* or *IVI*) of an ELR, whereas the *dual* kernel-sign $\bar{V}$ is called the *antivalidity-operator,* because it converts the same ELR into its *primary*, or *initial, antivalidity-intrgron* (*PAVI* or *IAVI*). Unlike the IEI’s, the Arabic numerals as 0, 1, 2, etc ad infinitum in this light-faced Roman (upright) narrow Gothic (sans serif) type, called Light-Faced Roman Arial Narrow Type, – the numeral, which constitute the conventional *decimal system of numeration* and which are used autonymously, are called the *decimal digital integrons* (briefly, *DDI’s*). Hence, an EI is an IEI (EVI or EAVI) or a DDI, the understanding being that 0 and 1 are IEI’s and DDI’s simultaneously.

27) *A posteriori*, I introduce the following *secondary* PLPH’s:

a) ‘ $\mathbf{P}_*$ ’ is an *atomic panlogographic relation* (*APLR*) whose range is the class of DdER’s (vavn-decided ER’s); ‘ $\mathbf{P}_+$ ’, ‘ $\mathbf{P}_-$ ’, or ‘ $\mathbf{P}_\sim$ ’ is an APLR whose range is the class of valid, antivalid, or vav–neutral ER’s respectively.

b) ‘ $\mathbf{i}|\mathbf{P}_\sim\rangle$ ’ is a *molecular panlogographic integron* (*MPLI*) whose range is *the class of euautographic validity-integrons* (*EVI’s*) *neutrality*.

c) ‘ $\bar{\mathbf{i}}|\mathbf{P}_\sim\rangle$ ’ is an *MPLI* whose range is *the class of euautographic antivalidity-integrons* (*EAVI’s*) *neutrality*.

In accordance with the item 26, in the above definitions b and c, either name “validity-identifier” or “validity-index” (briefly “VID” in both cases) can be used instead of “validity-integron” (“VI”) and either name “antivalidity-identifier” or “antivalidity-index” (briefly “AVID” in both cases) can be used instead of “antivalidity-integron” (“AVI”). By the above definitions a–c, (3.2) and (3.12) reduce to

$$V(\mathbf{P}_+)\hat{=}0,\ V(\mathbf{P}_-)\hat{=}1,\ V(\mathbf{P}_\sim)\hat{=}\mathbf{i}|\mathbf{P}_\sim\rangle, \tag{3.15}$$

$$\bar{V}(\mathbf{P}_+)\hat{=}1,\ \bar{V}(\mathbf{P}_-)\hat{=}0,\ \bar{V}(\mathbf{P}_\sim)\hat{=}\bar{\mathbf{i}}|\mathbf{P}_\sim\rangle, \tag{3.16}$$

respectively.

28) In accordance with the item 26, *validity*, *antivalidit*y, or *vav-neutrality* (*vav-indeterminacy*) of a DdER, i.e. the *quality* of the DdER to be valid, antivalid, or

vav-neutral (vav-indeterminate), can conveniently be regarded as the *state of membership* in the respective *decision class* which will be called indiscriminately a *validity-class* and discriminately *the validity-class validity* (or *validness*) or *the validity-class antivalidity* (or *antivalidness*) or *the validity-class vav-neutrality* (*or vav-indeterminacy*), i.e. *neither validity nor antivalidity,* in that order. Also, the generic name "validity-class" will be used synonymously (interchangeably) with the name "*validity-value*" in analogy with the generic name "*numeric value*", which conventionally used for mentioning, e.g., any of the natural numbers, i.e. *number-classes*, 0, 1, 2, etc. Logographically, the above three validity-values (validity-classes) will be denoted by 'v_+', 'v_-', and '$\mathsf{v}_\sim$' , the understanding being that these constants belong to the *exclusive metalanguage* (*XML*) *of* A_1, and not to A_1, Still, in accordance with the item 26, there are two mutually dual metalinguistic functions, to be denoted by 'V' and '$\overline{\mathsf{V}}$', such that

$$\mathsf{V}(0)=\mathsf{v}_+,\ \mathsf{V}(1)=\mathsf{v}_-,\ \mathsf{V}(\mathbf{i}|\mathbf{P}_\sim\rangle)=\mathsf{v}_\sim, \tag{3.17}$$

$$\overline{\mathsf{V}}(1)=\mathsf{v}_+,\ \overline{\mathsf{V}}(0)=\mathsf{v}_-,\ \overline{\mathsf{V}}(\bar{\mathbf{i}}|\mathbf{P}_\sim\rangle)=\mathsf{v}_\sim. \tag{3.18}$$

In this case, the validity-operator V and the EDA scheme (3.5) are associated with the mapping V defined by (3.17), whereas the antivalidity-operator $\overline{V}$ and the EDTS'ta (3.12) are associated with the mapping $\overline{\mathsf{V}}$ defined by (3.18). It would be counterproductive (cumbersome and confusing), if possible at all, to set up and execute A_1 under both mappings (3.17) and (3.18) simultaneously. Therefore, for the sake of being specific, I have set up A_1 under the mapping (3.17). But I shall occasionally enter into minor digressions (as this one) in order to demonstrate the relative character of the current setup of A_1.

29) A relation of A_1 that is taken for granted to be valid is called an *axiom*, or, more precisely, *subject axiom*, *of* A_1. Consequently, the fact that an ER $\mathbf{P}^{\mathrm{a}}$ is taken for granted to be an axiom of A_1 means that $V(\mathbf{P}^{\mathrm{a}})\hat{=}0$. An ER of A_1 that *is proved* to be valid either by inference or in the result of the appropriate EADP is called a *theorem* of A_1. The negation of an axiom is called an *antiaxiom*. The negation of a theorem is called an *antitheorem*. Consequently, the main properties of the EAADM, D_1, can be recapitulated thus.

a) To any given ER of A_1, of academic or practical interest practical interest there is an EDT, according to the form of which the ER is classified as a valid, antivalid, or vav-neutral one. The main tool of any EADP is the validity-operator V, which converts ER's (euautographic relations) into EI's (euautographic integrons) and which also converts relational logical (ordinary) kernel-signs (logical connectives and pseudo-quantifiers) into substantival algebraic kernel-signs that are marked with a caret. In addition, the operator V is also capable of converting the euautographic algebraic relation between two validity-integrons into a certain EI in accordance with the following PLA (EAS of $\mathbf{A}_1$):

$$V(V(\mathbf{Q}) \hat{=} V(\mathbf{R})) \hat{=} [V(\mathbf{Q}) \hat{-} V(\mathbf{R})]^2, \qquad (3.19)$$

where '**Q**' and '**R**' are APLPHs, whose range is the class of all ER's of A_1.

b) In the case, when *the object ER*, **P**, *of an EADP*, $\mathsf{D}_1(\mathbf{P})$, *is an EOR*, any *valid sequent* (*sequential identity*) of $\mathsf{D}_1(\mathbf{P})$, including both (3.1), being the initial one, and $\mathsf{T}_1(\mathbf{P})$, being the final one, is a *subject* (*and not object*) *relation of* $\mathsf{D}_1(\mathbf{P})$ in the sense that it is determined by the pertinent *rules of inference* of D_1, so that its *deduction* from D_1 or from some previous sequents of $\mathsf{D}_1(\mathbf{P})$ *does not require proving EDT's for that sequent* and hence the deduction does not require any act of decision. That is to say, none of the sequents of $\mathsf{D}_1(\mathbf{P})$:

$$\mathbf{i}_i|\mathbf{P}\rangle \hat{=} \mathbf{i}_j|\mathbf{P}\rangle,\ 1 \le i < j \le n;\ \text{or}\ V(\mathbf{P}) \hat{=} \mathbf{i}_i|\mathbf{P}\rangle,\ 1 \le i \le n, \qquad (3.20)$$

has the form $V(\mathbf{Q}) \hat{=} V(\mathbf{R})$, where **Q** and **R** are some ER's other than **P**. A reflexive EAlI (euautogaphic algebraic identity), as $V(\mathbf{P}) \hat{=} V(\mathbf{P})$, or any EAlI that is deduced from another EAlI in accordance with the pertinent rules of inference of D_1 is called a s*elf-decided*, or *auto-decided*, *EAlI* (*SfDdEAlI*) and also a *valid self-decided*, or *auto-decided*, *EAlE* (*EAlR*). Consequently, any of the identities (3.18) is an SfDdEAlI, so that it i*mmediately implies* the respective *euautographic decision corollary* (*EDC*) such as:

$$V(\mathbf{i}_i|\mathbf{P}\rangle \hat{=} \mathbf{i}_j|\mathbf{P}\rangle) \hat{=} 0,\ 1 \le i < j \le n;\ \text{or}\ V(V(\mathbf{P}) \hat{=} \mathbf{i}_i|\mathbf{P}\rangle) \hat{=} 0,\ 1 \le i \le n. \qquad (3.20_1)$$

By way of emphatic comparison with any *self-decided* (*auto-decided*) sequent of the EADP, $\mathsf{D}_1(\mathbf{P})$, for the pertinent *decided euautographic ordinary relation* (*DdEOR*), **P**, can alternatively be qualified a *xeno-decided EOR* (*XDdEOR*). Still, an EOR that is not an axiom can be vavn-decided only as the object relation of a certain EADP. Therefore, the prefix "xeno" in the qualifier "xeno-decided" to "EOR" is redundant, so that it will not be used in the sequel. At the same time, every EAlI being a sequent of the EADP for an EOR is *necessarily* a self-decided one. Therefore, the prefix "self' in the qualifier "self-decided", or "auto" in "auto-decided", to any "EAlI" of the EADP for EOR is also redundant, so that it will not be used in the sequel either.

30) If the object ER, **P**, of an EADP, $\mathsf{D}_1(\mathbf{P})$, is an ESpR then some of the constituent parts of **P** can, as pointed in the previous item, be EAlE's of the form $V(\mathbf{Q}) \hat{=} V(\mathbf{R})$, where **Q** and **R** are some ER's other than **P**. If this happens then a reducible EVI of the form $V(V(\mathbf{Q}) \hat{=} V(\mathbf{R}))$ unavoidably appears in a certain sequent of $\mathsf{D}_1(\mathbf{P})$. In order to compute $V(V(\mathbf{Q}) \hat{=} V(\mathbf{R}))$, it is necessary, in accordance with (3.19), to compute $V(\mathbf{Q})$ and $V(\mathbf{R})$, and hence to perform $\mathsf{D}_1(\mathbf{Q})$ and $\mathsf{D}_1(\mathbf{R})$, i.e. the EADP's for **Q** and **R**, separately. Upon completing $\mathsf{D}_1(\mathbf{Q})$ and $\mathsf{D}_1(\mathbf{R})$, $\mathsf{D}_1(\mathbf{P})$ can be continued until another irreducible EVI of the similar form is encountered. This EVI should be treated in the same way as the previous one. If either of the above-mentioned ER's **Q** and **R** or both involve a constituent EAlE of the like form, $V(\mathbf{Q}_1) \hat{=} V(\mathbf{R}_1)$, then $\mathsf{D}_1(\mathbf{Q})$ or $\mathsf{D}_1(\mathbf{R})$ unavoidably involves $\mathsf{D}_1(\mathbf{Q}_1)$ and $\mathsf{D}_1(\mathbf{R}_1)$ at certain stage. And so on. It is understood $V(\mathbf{Q}) \hat{=} 0$ and $V(\mathbf{Q}) \hat{=} 1$, e.g., are particular cases of $V(\mathbf{Q}) \hat{=} V(\mathbf{R})$. Thus, if **P** is an ESpR of the above structure then its EADP, $\mathsf{D}_1(\mathbf{P})$, reduces to a series of EADP's for all CbEOR's that it involves.

31) An ER of A_1 may have several EADP's, which differ in orders of the elementary algebraic operations constituting the EADP's. The different EADP's for a given ER result in the same EDT, and hence in the same decision. However, one of the EADP's may turn out to be shorter and simpler than another one. Therefore, in spite of the fact that any EADP is mechanical, choice of the optimal EADP for a given ER is a kind of art that is acquired by experience – just as in the case of arithmetical calculations with natural numbers.

32) Although it has not happened so far, should it happens in the sequel that a certain ER of A_1 is subjected to all conceivable would-be EADP's, all of which fail because the relation is too complicated and too long so that any one of the EADP's cannot be completed or comprehended or because some unknown rules of inference are missing, the relation will be called a *vavn-undecided*, or simply *undecided*, *ER.* In addition to the vavn-decided ER's and some supposedly vavn-undecided ER's, A_1 has an infinite number of ER's, which t are determined by the formation rules of A_1, but which are not subjected to any EADP's or are not even written down. These relations will be called *vavn-suspended* ones. Accordingly, the vavn-decided and, if detected,

vavn-undecided ER's are collectively called the *vavn-unsuspended ER's*. Vavn-undecided ER's (if detected some) and vavn-suspended ER's will collectively be called *vavn-nondecided*, or simply *nondecided*, *ER's of* A_1.•

Cmt 3.1. 1) A_0 is parallel to a CASC, but it has certain features of a CAPC. Particularly, just as in the case of a CAPC, all formulas of A_0 are, in accordance with their formation rules, divided into two complementary classes: *terms* and *relations*. Still, all formulas of A_0 are euautographic (insignificant) ones, so that the qualifiers "variable" and "constant" are inapplicable to them. Therefore, they are divided into pseudo-variable ones and pseudo-constant ones in relation to their subsequent interpretational replacements with the appropriate conventional variables and constants respectively. Also, all terms of A_0 are *special ones*, called also *integrons*, i.e. ones, which are relevant to the EBADM. All *pseudo-variable integrons* A_0 are *combined* (*not atomic*) and therefore they have only *free occurrences*. Accordingly, A_0 has no operators analogous to quantifiers.

2) In accordance with Df 3.1(1) the occurrence of the adjective "Algebraic", along with the suffixed connective vowel "o" followed by the hyphen, in any verbal name of A_1, A_1^0, or A_0, means «*involving the laws of algebra and*», while the post positive occurrence of the adjective "Predicate" in the pertinent verbal name of A_1 or A_1^0 should be understood as an abbreviation of the adjective equivalent "*concerned in predicate-containing and predicate-free relations*". At the same time, the occurrence of the prepositive adjective equivalent "Predicate-Free" in pertinent verbal name of A_0 evidently means «*concerned in predicate-free relations*». In spite of the fact that A_0 is parallel to a CASC, in forming a verbal name of A_0, I utilize one of the compound qualifiers "*Predicate-Free Algebraico-Logical*" and "*Restricted Basic Algebraico-Logical*" instead of either of the conventional qualifiers "*sentential*" and "*propositional*" (cf. Hilbert and Ackerman 1950, pp. 27, 165, 166], Church [1956, pp. 27, 28, 69, 119], Suppes [1957, p. 3], Lyndon [1966, pp. 20, 35]), because the latter two are, in accordance with Df 3.1(2), incompatible with the adjective "euautographic", which I use as the most essential qualifier of both A_1 and A_0 and also of A_1^0. The adjectives "sentential" and "propositional" are also incompatible with the adjective "panlogographic", which I use as the most essential qualifier of $\mathbf{A}_1$ and $\mathbf{A}_0$, and with the adjective "*endosemasiopasigraphic*", which I use as the most

essential qualifier of both the *biune organon* A_1 *and* $\mathbf{A}_1$, denoted by 'A_1', and of the *biune organon* A_0 *and* $\mathbf{A}_0$, denoted by 'A_0' (to be described in section 4).•

Df 3.2: ***An alternative vavn-decisional terminology*.** After the manner of the established term "tautology" and its derivatives, I introduce the following *monomial synonyms* of the taxonyms of the decision classes of ER's of A_1. A *valid*, *antivalid*, *vav-neutral* (*vav-indeterminate*), *invalid*, *non-antivalid*, or *vav-unnutral* (*vav-determinate*) *ER* is alternatively (synonymously) called a *kyrology*, *antikyrology*, *kak-udeterology* (*kak-anorismenology*), *akyrology*, *anantikyrology*, or *kak-anudeterology* (*kak-orismenology*) respectively. Consequently, a kyrology is either a euautographic axiom or a euautographic theorem, whereas an antikyrology is either a euautographic antiaxiom or a euautographic antitheorem. It goes without saying that "*kak*" is an abbreviation for "*kyrology-antikyrology*", so that "kak-udeterology" ("kak-anorismenology") means *neither a kyrology nor an antikyrology*. Just as their original counterparts, the alternative taxonyms (metaterms) that are introduced above are formed in accordance with the principles that are explicated in Appendix 2.•

Cmt 3.2. 1) In Df 3.2, and generally in the sequel, I introduce the appropriate monomial "logy"-synonym of a binomial metaterm consisting of the *headword* (*generic name*) "*relation*" and a prepositive *epithet* (*qualifier*) to it after the manner of the established synonym "tauto*logy*" (due to Wittgenstein [1921]) of the metaterm "tautologous *relation*". The relevance of "tautology" to "kyrololy" will be explicated as I go along. Meanwhile, it will be sufficient to notice that a tautology is a *truth-valued interpretatand* of a certain kyrology The prefix "*kyro*"- occurring in the *neonym* (*new name*) "kyrology", which I have suggested in Df 3.2 as a synonym of "valid relation", originates from the Greek noun "κῦρος" \kíros, kýros\ meaning *validity* and also *weight* or *authority*. (*Gravity weight* and also *heaviness* or *burden* is denoted in Greek by the noun "βάρος" \báros\.) The above prefix should not be confused with the similar combining form "*kyri*"- or "*kyrio*"-, spelled also as "*curi*"- or "*curio*"-, which I utilize in this treatise in the neonyms "*kyrionym*", meaning *a proper name,* and "*kyriograph*", meaning *a proper graphic name*. The latter combining form originates from the Greek noun "κΰριος" \kírios\ having the same sense as "lord", "master", "gentleman", "mister" (or, in the vocative case, "κΰριε" /kírie/, meaning *sir*) and also from its kindred homonymous adjective having the same meaning as "main", "principal", "chief"; the Greek set expression "κΰριον όνομα" means *a proper name*. This combining form occurs, e.g., in following established Anglicized expressions of the Greek origin: the noun “curiologistics” having the same sense as “hieroglyphic writing”, the adjective “curiologic” or “curiological” meaning «of or relating to hieroglyphic writing», the noun “kyrios” meaning the same as “lord”, especially in reference to Jesus Christ as in the petitionary invocation «Kyrie, eleison!» («Lord, have mercy!»).

2) The prefix “*udetero*”- occurring in the neonym “udeterology”, which I have also suggested in Df 3.2, is derived from the Greek adjective “ουδέτερος” \uðéteros, uthéteros\ meaning *neutral* or (gram.) *neuter* and from the homonymous pronoun meaning *neither*.

3) My new term “*orismenology*” (from the Greek adjective “ωρισμένος” \orisménos\ meaning *determinate*, *determined*, or *certain*) should not be confused the established English noun “*orismology*” (from the Greek noun “ορισμός” \orismós\ meaning *definition*), which denotes the science of defining technical terms (cf. WTNID). The word “anudeterology” (“anoudeterology”), being a synonym of “orismenology”, is morphologically similar, e.g., to the well-established Anglicized adjective “anurous” (or “anourous”), which means “having no tail”; the root “urous” (or “ourous”) originates from the Greek noun “ουρά” \urá\ meaning *a tail*. Instead of either of the two synonyms “orismenology” and “anudeterology”, I might alternatively have used in the same sense the neonym “*bebeology*”, which originates from the Greek adjective “βέβαιος” \bébeos\ having the same sense as “sure” or “certain” and from the kindred noun “βεβαιότης” \bebeótis\ having the same sense as “certainty”. Accordingly, the neonym “*abebeology*” that originate from the Greek adjective “αβέβαιος” having the same sense as “uncertain” might have been used as a synonym of “indeterminate relation” instead of “anorismenology”.

4) In order to incorporate the notation as introduced in Df 3.1(28) into the relevant uniform system of Greek-related notation that will be introduced in the sequel in regard to semantic interpretands of kyrologies, antikyrologies, or udeterologies, I introduce the following logographic synonyms:

$$\kappa_+ \to \vee_+, \kappa_- \to \vee_-, \kappa_\sim \to \vee_\sim, \mathrm{K} \to \vee, \overline{\mathrm{K}} \to \overline{\vee}, \tag{3.21}$$

where ‘κ’ or ‘K’ is, mnemonically, the first letter of the Greek noun “κῦρος” meaning «validity» (see item 1 of this comment). In accordance with (3.21), either of the synonymous prepositive abbreviations “vav” and “kak” can be used interchangeably with ‘καϰ’, where the middle letter ‘α’ stands for the Greek combining form “άντι” \ánti\ denoting *opposition, opposite situation,* or *negation.* Also, relations (3.17) and (3.18) can synonymously be rewritten as:

$$\mathrm{K}(0)=\kappa_{+},\ \mathrm{K}(1)=\kappa_{-},\ \mathrm{K}\left(\mathbf{i}\middle|\mathbf{P}_{\sim}\right\rangle)=\kappa_{\sim}, \tag{3.17a}$$

$$\overline{\mathrm{K}}(1)=\kappa_{+},\ \overline{\mathrm{K}}(0)=\kappa_{-},\ \overline{\mathrm{K}}\left(\bar{\mathbf{i}}\middle|\mathbf{P}_{\sim}\right\rangle)=\kappa_{\sim}, \tag{3.18a}$$

respectively.•

4. An introduction in depth to $\mathbf{A}_1$ and A_1

4.1. A description of $\mathbf{A}_1$ and A_1

Df 4.1: ***Relationship between*** A_1 ***and*** $\mathbf{A}_1$**.** 1) The calculus, which is denoted by ‘$\mathbf{A}_1$’ and is called the *Comprehensive Panlogographic Algebraico-Predicate Organon* (*CPLAPO*), and also the *Comprehensive Panlogographic Advanced Algebraico-Logical Organon* (*CPLAALO*), is a *calculus of panlogographic placeholders* (*PLPH’s*) *of euautographic formulas* (*EF’s*), i.e. *euautographic relations* (*ER’s*) and *euautographic terms* (*ET’s*), *of* A_1. Consequently, $\mathbf{A}_1$ can alternatively be called the *Logographic APO* (*LAPO*) *over the CEAPO*. In this case, A_1 and $\mathbf{A}_1$ are mentally *superimposed and united* to form a single whole organon, which is denoted by ‘A_1’ and is called the *Comprehensive Biune Euautographic and Panlogographic*, or *Comprehensive Endosemasiopasigraphic*, *APO* (*CBUE&PLAPO* or *CEnSPGAPO*); the occurrence of the qualifier “*Biune*” (“*BU*”) in the former name means: «*being the union and at the same time a superposition of the two pertinent APO’s*”. In accordance with the above-said, Df 3.1(1) formally applies with “*panlogographic*” (“*PL*”) and ‘$\mathbf{A}$’ in place of “*euautographic*” (“*E*”) and ‘A’, and it also applies with “*biune euautographic and panlogographic*” (“*BUE&PL*”) or “*endosemasiopasigraphic*” (“*EnSPG*”) in place of “*euautographic*” (“*E*”) and with ‘A’ in place of ‘A’, respectively. Consequently, the organons A_1^0, $\mathbf{A}_1^0$, and A_1^0, or A_0, $\mathbf{A}_0$, and A_0, are interrelated in the same way as A_1, $\mathbf{A}_1$, and A_1, whereas the organons $\mathbf{A}_1^0$, $\mathbf{A}_0$, and $\mathbf{A}_1$, or A_1^0, A_0, and A_1, and also their verbal names, full and abbreviated, are interrelated in the same way as A_1^0, A_0, and A_1, and as their verbal names, respectively.

2) It will be recalled that a *logographic placeholder* (*LPH*), or *place-holding variable*, is a variable condensing a large number (commonly an infinite number) of graphonyms of a certain class called the *range* of the LPH. In no connection with any specific *mental mode* of using an LPH, the range of an LPH is *impartially* said to be *designated* or to be a *designatum* (pl. "*designata*") of the LPH. An LPH of $\mathbf{A}_1$ is called a *panlogographic LPH* (*PLPH*) or briefly a *panlogograph* (*PL*) – in contrast to an LPH of the *exclusive metalanguage* (*XML*) of A_1 and $\mathbf{A}_1$, which is called a *metalinguistic logographic placeholder* or briefly a *metalogographic placeholder* (*MLPH*). It is understood the range of a PL (PLPH) is necessarily a certain class of euautographs, whereas the range of a MLPH can be either a certain class of euautographs or a certain class of logographs, particularly of panlogographs. *Ad hoc*, it is possible to distinguish between the metaterms "*panlogograph*" ("*PL*") and "*panlogographic placeholder*" ("*PLPH*") or between the metaterms "*metalogograph*" ("*ML*") and "*metalogographic placeholder*" ("*MLPH*") as follows. A PLPH is called a PL if it is used *autonymously*, and conversely a PL is called a PLPH if it is used autonymously; and similarly with "ML" in place of "PL". Sometimes, in using the metaterms "panlogograph" ("PL") and "PLPH" or "metalogograph" ("ML") and "MLPH", I shall follow the above definition. In practice, however, it is as a rule counterproductive or even impossible to fix the xenonymous or autonymous mental mode, in which I use a concrete token of a certain panlogograph or metalogograph and to indicate that mental mode symbolically (see the item 6 below in this definition for greater detail). Therefore, I shall use the metaterms "panlogograph" ("PL") and "PLPH", or "panlogograph" ("ML") and "MLPH", synonymously (interchangeably), unless stated otherwise. It will be also recalled that a euautograph or a logograph is indiscriminately called a *pasigraph* and conversely a pasigraph is either a euautograph or a logograph.

3) A *pasigraph* (*pasigraphonym*) *of* A_1, i.e. a *euautograph* (*euautographonym*) *of* A_1 or a *panlogograph* (*panlogographonym*) *of* $\mathbf{A}_1$, is called an *endosemasiopasigraph* (*endosemasiopasigraphonym*) in the sense that it *has certain syntactic functions* with respect to some other *pasigraphs of* A_1, each of which is also called *endosemasiopasigraph* (*EnSPG*, pl. "*EnSPG's*"), and that at the same time it *neither has nor assumes* (*takes on*) *any significations* (*imports, values*) *beyond* A_1.

Accordingly, A_1 is a *semantically closed* logistic system of EnSPG's (euautographs and panlogographs), and therefore it is qualified *endosemasiopasigraphic* (equivocally abbreviated as "*EnSPG*"). In contrast to A_1, a logistic system of logographs, which have or assume significations beyond it, is qualified *exosemasiologographic* (*ExSPG*). The EnSPG properties of A_1 are explicated below.

i) Within A_1 and hence within A_1, a euautograph is always used autonymously, so that the only values that an EF can have or assume are, as was indicated in Df 3.1(4), its *autonymous values* such as a *concrete member* of the *class of its homolographic* (*photographic*) *isotokens*, as the euautograph itself, or a *common* (*general*, *certain*) *member* of that class, which is just another *hypostasis* (*way of existence*, *aspect*) of that same class.

ii) Like a euautograph, a panlogograph of $\mathbf{A}_1$, and hence of A_1, has only *homolographic*, i.e. *photographic* (*congruous or proportional*), *isotokens*; it does not have either *analographic*, i.e. *stylized* (*not photographic*) *iconographic* (*pictographic*) *isotokens*, or *phonic* (*oral*, *spoken*) *paratokens*. The range of a panlogograph of $\mathbf{A}_1$ is a *certain class of euautographs* of A_1 so that it is one of *my psychical* (*mental*, *imaginary*, *ideal*, *abstract*) *entities*. A panlogograph is called a *panlogographic formula* (*PLF*), *panlogographic ordinary term* (*PLOT*), a *panlogographic special term* (*PLSpT*) or *panlogographic integron* (*PLI*), or a *panlogographic relation* (*PLR*) if its range is a class of EF's, a class of EOT's, a class of ESpT's, i.e. of EI's, or a class of ER's respectively.

In general, a *logograph* (*L*) is called an *atomic logograph* (*AL*) or a *lexigraph* if it is functionally indivisible and a *combined logograph* (*CbL*) if it is not atomic, i.e. if it is a combination of two or more *atomic pasigraphs*, at least one of which is a lexigraph. The above definition applies with "*metalogograph*" ("*ML*") and "*metalexigraph*" in place of "logograph" ("L") and "lexigraph" respectively, and it also applies with "*panlogograph*" ("*PL*"), "*panlexigraph*", and "*endosemasiopasigraph*" ("*EnSPG*") in place of "logograph" ("L"), "lexigraph", and "pasigraph" respectively. To be specific, in the latter case, a *panlogograph* (*PL*) is called an *atomic panlogograph* (*APL*) or a *panlexigraph* if it is functionally indivisible and a *combined panlogograph* (*CbPL*) if it is not atomic, i.e. if it is a combination of two or more *atomic EnSPG's*, at least one of which is a panlexigraph.

Likewise, in reference to the former case, I shall, when necessary, use the abbreviations "*AML*" and "*CbML*" without any further comments. Also, in accordance with the previous item, I shall use the pairs of abbreviations "APL" and "APLPH", "CbPL" and "CbPLPH", "AML" and "AMLPH", and "CbML" and "CbMLPH" synonymously (interchangeably), unless stated otherwise.

A logograph and particularly a panlogograph may contain euautographs as its constituent parts. Particularly, the PKS (principal kernel-sign) of a *combined PLR* (*CbPLR*) or of a *combined PLI* (*CbPLI*) can be either an EKS (euautographic kernel-sign) or a *PLKS* (*panlogographic kernel-sign*), whereas the latter can, in turn, contain some euautographs.

A panlogograph of $\mathbf{A}_1$ condenses a large number (usually an infinite or indefinite number) of euautographs of A_1 in its range. Therefore, I can *mentally* (*psychically*) use a panlogograph, or, more precisely, a homolohraphic (photographic) isotoken of a certain *prototypal panlogograph*, either (a) *xenonymously*, i.e. in a *xenonymous mental mode,* as a *genuine* (*active*, *assertive*) *panlogograph*, called a *eupanlogograph*, for mentioning all euautographs of its range simultaneously or (b) I can use the same or another isotoken of the prototypal panlogograph *autonymously*, i.e. in an *autonymous mental mode*, as a *tychautograph* (*accidental*, or *circumstantial*, *autograph*) for mentioning either any member of its homolographic token-class or for mentioning itself. The two mental modes of using a panlogograph are explicated below.

a) When I *prescind* a panlogograph from its context (graphic surrounding) and hence from any possible *added words*, which may effectively alter the panlogograph and thus alter its range, and when I use the panlogograph *xenonymously*, I can use the panlogograph, like any xenograph, along with its range in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* the range as *my as if extramental* (*exopsychical*) *object*, which I call a *common* (*general*, *certain*, *particular but not particularized*) *euautographic denotatum of the panlogograph* and also a *common element* (*member*) *of its range*; the common element *represents the whole range*, thus being just *another hypostasis* (*way of existence*, *aspect*) *of the latter*. In this case, I also say that both *the panlogograph and its* [*original*, *unpolarized*] *range are used for mentioning a common element of the range* or that, less explicitly, *they are used but not mentioned*, whereas the range is said to be *connoted by*, or to be *the connotatum* (*connotation value*, pl. "*connotata*") *of*, *the panlogograph*. Alternatively, I can, when necessary or desired, *physically replace* the panlogograph with any concrete euautograph of its range without any special rules of substitution. The act or process of psychically (mentally) using a panlogograph of $\mathbf{A}_1$ for mentioning a *common* (*certain*) euautograph of A_1 of its range is said to be a *psychical* (*mental*, *imaginary*) *euautographic interpretation of that panlogograph*. The act of presenting (printing or writing) a *concrete* (*particular*) euautograph of the range of a panlogograph of $\mathbf{A}_1$ is called a *physical* (*real*, *material*) *euautographic interpretation of that panlogograph*. Accordingly, a common euautograph of A_1 of the range of a panlogograph of $\mathbf{A}_1$ is said to be a *psychical* (*mental*, *imaginary*) *euautographic interpretand* (*interpretation value*) *of that panlogograph*, whereas a concrete euautograph of A_1 of that range is said to be a *physical* (*real*, *material*, *substitutional*) *euautographic interpretand*, or a *concrete* (*particular*) *euautographic instance*, *of that panlogograph*. Conversely, the panlogograph is said to be a *physical panlogographic interpretans* (*anti-interpretand*) both of its psychical euautographic interpretand and of each one its concrete euautographic interpretands (instances). It will be recalled that a psychical interpretand of a panlogograph of $\mathbf{A}_1$ is just *another mental hypostasis* (*form of existence*) *of its range* and is therefore tantamount to the latter. Thus, A_1 can be called the *psychophysical*, or *physopsychical*, *euautographic interpretand of* $\mathbf{A}_1$ and, conversely, $\mathbf{A}_1$ can be called the *physical panlogographic interpretans* (pl. "*interpretantia*") *of* A_1.

b) When I use a panlogograph *autonymously*, i.e. as a tychautograph, the properties that the panlogograph has *ad hoc* with respect to me are analogous to the permanent (intrinsic) properties of a euautograph as outlined in Df 3.1(4). Namely, a tychautograph is, like a euautograph, a functional but insignificant graphic chip that has a certain syntactic function or functions in itself or with respect to other pasigraphs (euautographs or panlogographs or both), especially those of its immediate surrounding (when applicable), but which has no psychical (mental) significations (imports, values) except autonymous ones. In this case, like an autonymous value of a euautograph, an autonymous value of a panlogograph is either its homolographic token-class or one of its homolographic isotokens, a concrete one or a common (general, certain) one, being another hypostasis of the isotoken-class.

4) In accordance with the previous item, $\mathbf{A}_1$ can be regarded as a *formalized logographic* and hence essentially *graphic* (*written*) *metalanguage*, which is designed for stating and processing infinite numbers of conspecific or congeneric euautographs of A_1 simultaneously and which also allows, when necessary or desired, interpreting (illustrating) a panlogograph of $\mathbf{A}_1$ by concrete euautographs of A_1. Particularly, $\mathbf{A}_1$ allows setting up A_1 in the general form, while being itself a by-side product of the setup of A_1. Consequently, the setup of $\mathbf{A}_1$ is, to a great extent, a by-side product of the setup of A_1. At the same time, the setup of $\mathbf{A}_1$ has some remarkable digressions from the setup of A_1. Namely, while any FR of A_1 either is or can be restated so as to be simultaneously an FR of $\mathbf{A}_1$, *some FR's of* $\mathbf{A}_1$ do not introduce any new EF's of A_1 and are either *generalization rules* or *sortation rules of EF's of* A_1. A mapping from ER's of A_1 to PLF's of $\mathbf{A}_1$ is a *surjection* (*onto-mapping*, *onto-function*), not being an *epimorphism* (for morphisms of algebraic systems, see, e.g., Mac Lane & Birkhoff [1967, pp. pp. 56–58]). In this case, $\mathbf{A}_1$ can be regarded as an *extension and generalization of* A_1 and, conversely, A_1 is a *restriction and specification of* $\mathbf{A}_{1.}$ Also, A_1 is the *psychophysical*, or *physopsychical*, *euautographic interpretand of* $\mathbf{A}_1$ and, conversely, $\mathbf{A}_1$ is the *physical panlogographic interpretans of* A_1.

5) Alternatively, $\mathbf{A}_1$ can be regarded as an *axiomatic quasi-algebraic system*, whose objects are EF's, or, more precisely, ER's, of A_1, whereas A_1 is the pertinent *biune axiomatic quasi-algebraic system*, which concerns both with the EF's of A_1 and with the *panlogographic formulas* (*PLF's*) *of* $\mathbf{A}_1$ and which also concerns with interrelations of formulas of the two classes. Letting aside the fact that there are in algebra no algebraic systems analogous to $\mathbf{A}_1$, the character of the *endosemasiopasigraphs*, i.e. *euautographs* and *panlogographs*, that are employed in $\mathbf{A}_1$ and A_1 essentially differ from the character of *logographs*, i.e. *variables and constants*, that are employed in a *conventional axiomatic algebraic system* (*CAAS*), and therefore the mental modes, in which I use the above pasigraphs, differ from the mental modes of using the logographs of a CAAS. To be specific, a CAAS is a logographic nomenclatural system that is used for representing and mentioning certain abstract (imaginary) objects (as points, vectors, or numbers) and their interrelations – entities, which cannot be exhibited on a material surface (as that of a sheet of paper or that of the screen of a computer monitor). The logographic symbols themselves are used but not mentioned – just as xenographic symbols of an *alphabetic* or *syllabic*

WNL (*AbWNL* or *SbWNL)* are used but not mentioned in everyday life. Accordingly, *logographic variables* and *logographic constants* that are used in a CAAS are *purely representing and not place-holding ones*. By contrast, formulas of both mentally superimposed systems of A_1, viz. EF's of A_1 and PLF's of $\boldsymbol{\mathsf{A}}_1$, occur on the same material surface. In this case, an EF of A_1 is always used autonymously, because it is a euautograph, whereas a PLF of $\boldsymbol{\mathsf{A}}_1$ can be used either xenonymously as a eupanlogograph or autonymously as a tychautograph, because it is a panlogograph and is therefore significant.

6) In order to indicate syntactically that I use a panlogograph autonymously for mentioning *itself*, I use instead of the panlogograph its *proper name*, which is formed by enclosing the panlogograph in *slant light-faced single quotation marks*, ‘ ’, and which is called a *kyrioautographic*, or *proper autographic*, *quotation* (*KAQ*). That is to say, a KAQ denotes *its interior* prescinded from all its xenonymous values and from all its autonymous value except the interior itself, while the pair of *KAQ marks*, being *its exterior*, indicates (denotes) the above mental attitude of me towards the interior of the KAQ, and it also indicates the analogous mental attitude, which any interpreter of the KAQ should take towards its interior. By contrast, in order to indicate syntactically that I use a panlogograph autonymously for mentioning a *common* (*general*, *certain*) *member of its homolographic isotoken-class*, which is just another hypostasis (way of existence, aspect) of that class, I use instead of the panlogograph it its *common name*, which is formed by enclosing the panlogograph in *curly* or *upright straight light-faced single quotation marks*, ‘ ’ or ' ', and which is called a *homoloautographic*, or *photoautographic*, *quotation* (*HAQ*). The pair of *HAQ marks*, being the *exterior* of the HAQ, indicates (denotes) the above mental attitude of me towards the interior of the HAQ, and it also indicates the analogous mental attitude, which any interpreter of the HAQ should take towards its interior. It is understood that a KAQ, or an HAQ, has the denotatum of the above kind if the quotation is prescinded from any added words that may alter its meaning. If a panlogograph is not enclosed in any *special quotation marks* to indicate explicitly that it is used autonymously then I can, in accordance with the previous item, use it either xenonymously or autonymously.

7) Most often, however, I use an *unquoted* panlogograph that is prescinded from its symbolic surrounding, especially from the added words (if any), in both

opposite mental modes, *xenonymous and autonymous, as if simultaneously but actually equivocally and intermittently by repeatedly switching, involuntary but consciously, from one mental attitude towards the panlogograph to the other* – just as I perceive any one of Escher's *Convex and Concave* pictures, e.g. "*Cube with Magic Ribbons*" (see, for instance, Ernst [1985, p. 85f]). The class of conceptional or sensational mental phenomena of perceiving *graphonyms as having two opposite alternating hypostases* will be properly called by the *count name* "*alternation of opposites*" (without any article) – or by the limited (articled) and capitalized version of that name "*the Alternation of Opposites*" as the *intended proper class-name of the phenomena*, whereas a concrete instance (member, phenomenon) of this class will be commonly called "*an alternation of opposites*". I regard an alternation of opposites as a concrete manifestation of *the general dialectic principle of unity of opposites* due to Hegel. The class of involuntary but conscious alternations between the autonymous and xenonymous perceptions of a xenograph in general and of a logograph in particular, being a subclass of the Alternation of Opposites, is properly called by the count name "*tychautograph and euxenograph alternation*" or briefly "*tychauto-euxenograph alternation*" ("*TAEXA*"). The two count names can be limited and capitalized as "*the Tychautograph and Euxenograph Alternation*" and "*the Tychauto-Euxenograph Alternation*" to become thus the pertinent intended proper class-names. The TAEXA is a wide class of mental phenomena of perceptions of xenographs including representing and place-holding logographs and particularly including panlogographs, i.e. panlogographic placeholders (PLPH's). A concrete instance (member, phenomenon) of this class will be commonly called "*a tychauto-euxenograph alternation*" ("*a TAEXA*"), the understanding being that this name can, when necessary, be attributed with an appropriate postpositive qualifier such as "*of a xenograph*", "*of a logograph*", or "*of a panlogograph*". Likewise, the set of Escher's *Convex and Concave* pictures, being another subclass of Alternation of Opposites, will be called the count name "*Escher convex and concave alternation*" ("*ECCA*") or by the limited and capitalized intended proper class-name "*the Escher Convex and Concave Alternation*". Either of the latter two names can also be used as an *allegorical name* of the entire class of alternations of opposites, although the mental processes underlying ECCA's, being pure *sensational* (*sensorial*) *alternations of* *opposites*, differ from the mental processes underlying TAEXA's, being *conceptional ones*.

8) The TAEXA of a panlogograph is a *spontaneous* (*involuntary*) *but conscious mental process* of treating the panlogograph simultaneously as a large number (usually an indefinite or infinite number) of euautographs condensed in its range and as a separate tychautograph. In the process and hence in the result of the TAEXA, *the range of the panlogograph is automatically extended to include the panlogograph itself as its tychautograph*. A biune mental hypostasis (way of existence) of a panlogograph during its TAEXA is, not only harmless, but most often it is useful and even indispensable. Particularly, the TAEXA of the pertinent PLF's of $\mathbf{A}_1$ are indispensable in simultaneously stating formation, transformation, and decision rules of A_1 and $\mathbf{A}_1$ and in simultaneously solving decision problems for an infinite number of conspecific or congeneric ER's of A_1 by solving the decision problems for the pertinent PLR's of $\mathbf{A}_1$. If I use a xenograph, particularly a logograph or a panlogograph, *autonymously*, or, on the contrary, *xenonymously*, I say that I use it in an *autonymous*, or, correspondingly, *xenonymous*, *mental mode*. Accordingly, if I use a panlogograph (e.g) *autonymously* and *xenonymously intermittently but as if simultaneously* – briefly, *autoxenonymously* or *xenoautonymously*, I say that I use the panlogograph in the *autoxenonymous*, or *xenoautonymous*, *mental mode*, or alternatively in the *TAEXA mental mode* or, briefly, in the *TAEXA-mode*.

9) Since a panlogograph can be used for mentioning any one of a large number (usually an infinite or indefinite number) of euautographs condensed in its range, the *morphism* (see, e.g., Mac Lane & Birkhoff [1967, pp. pp. 56–58]), i.e. a certain *function* (*mapping*), called often "*homomorphism*", from the class of euautographs of A_1 to the class of panlogographs of $\mathbf{A}_1$ is an *epimorphism* (*onto-mapping*, *onto-function*, *surjection*), and *not an isomorphism* (*not a bijection*). That is to say, A_1 and $\mathbf{A}_1$ are *not isomorphic*. Accordingly, the setup of $\mathbf{A}_1$ has some remarkable digressions from the setup of A_1. In this case, $\mathbf{A}_1$ can be regarded as an *extension and generalization of* A_1 and, conversely, A_1 is a *restriction and specification of* $\mathbf{A}_1$. Also, by the above item 3a, A_1 is the *psychophysical, or physopsychical, euautographic interpretand of* $\mathbf{A}_1$ and, conversely, $\mathbf{A}_1$ is the physical panlogographic interpretans of A_1.•

Cmt 4.1. 1) No matter what a *brain symbol*, i.e. *mental state*, is from the standpoint of biophysical and biochemical processes in the aggregate of perikaryons constituting the cerebral cortex of a sapient subject, it is a *dynamic and mutable* (*varying*) *entity*, – in contrast, e.g., to a graphic (written) symbol (ideograph), which is *static and immutable* (*invariable*, *unchangeable*), or in contrast to a vocal (spoken) symbol (ideophon), which is just *transient but immutable* as well. Therefore, any *functional*, i.e. *single-valued*, *correspondence* (*mapping*) from brain symbols to graphic ones (e.g.) is, in the general case, *many-to-one*, i.e. *surjective*, *and not bijective*. Consequently, the inverse correspondence is *many-valued*, i.e. *not functional*. For instance, an interpreter of any given ideograph (graphic symbol), – a logograph or a phonohraph (a word or word group), – in a given occurrence can, depending on his mental attitude towards the ideograph, use it either in any of many *autonymous mental modes* or in any of many of *xenonymous mental modes* and he can repeatedly change his mental attitude. The involuntary alternation between two different autonymous uses or between two different xenonymous uses or between a certain autonymous use and one or more xenonymous uses of the ideograph is an alternation of opposites. A TAEXA is an alternation of opposites of the last kind.

2) Besides TAEXA's, there are alternations of opposites of many other kinds, which are utilized in this treatise. Just as TAEXA's, these are involuntary but conscious mental processes. For instance, equivocal use of the noun "interpretation" for denoting both a concrete operation of interpreting and its result is also an instance of the Alternation of Opposites. Also, according to *the theory of the meaning content of a xenograph* that I adopt in this treatise, the sense of a non-idiomatic combined xenograph is a *biune mental entity* (*dynamic brain symbol*, *dynamic mental state*) of its creator (as me) or of its any interpreter, one *hypostasis* (*way of existence*, *aspect*) of which is a certain *mental process* (*operation*) *of coordinating the classes designated by relatively simple constituents of the xenograph*, while the other hypostasis is *the class resulted by that mental process*; the former hypostasis of the sense can be called *the sense-producing operation on the xenograph*, while the latter hypostasis of the sense can impartially be called *the designatum* (*designation value*, pl. "*designata*") *of the xenograph* or, alternatively, *the subject class of the sense* – as opposed to *the class-operata* (*operated classes*), which can alternatively be called *the object classes of the sense*. Thus, the sense of a xenogrpah (and generally of a xenonym) is a *mental*

process – a *stream of thought* (cf. James [1890; 1950, pp. 224, 225]), and not a *memorized* (*as if static*) *mental state* such as its designatum or its *denotatum* (*intended import value*). Various kinds of alternations of opposites will be illustrated by pointing to some of their live instances.•

4.2. Taxonomy of panlogographs of A_1

Df 4.2: ***Taxonomy of panlogographs – the first supplement to Df 4.1.*** 1) A PL (PLPH) is called an *ER-valued*, *EOT-valued*, *EI-valued*, or *EKS-valued panlogograph* (*PL*, *PLPH*), and also a *panlogographic relation* (*PLR*), *panlogographic ordinary term* (*PLOT*), *panlogographic integron* (*PLI*), or *panlogographic kernel-sign* (*PLKS*) in that order, if and only if every element (member) of its range is a *euautographic relation* (*ER*), *euautographic ordinary term* (*EOT*), *euautographic integron* (*EI*), or *euautographic kernel-sign* (*EKS*) respectively. In this case, the *taxonyms* (*taxonomic names*, *metaterms*) "*panlogographic logical term*" ("*PLLT*") and "panlogographic ordinary term" ("PLOT") or the taxonyms "*ESpT-valued panlogograph*", "*panlogographic algebraic term*" ("*PLAlT*"), "*panlogographic special term*" ("*PLSpT*"), and "*panlogographic integron*" ("*PLI*") are synonyms because synonyms are the taxonyms "euautographic logical term" ("ELT") and "euautographic ordinary term" ("EOT") or the taxonyms "euautographic algebraic term" ("EAlT"), "euautographic special term" ("ESpT"), and 'euautographic integron" ("EI") respectively. Here follow further furcations of the above *taxa* (*taxons, taxonomic classes*) of panlogographs.

i) A PLR is called either (a) a *panlogographic special logical relation* (*PLSpLR*) or (b) a *panlogographic algebraic relation (PLAlR*) and also a *panlogographic algebraic equality* (*PLAlE*) if every element of its range is either (a′) an ESpLR or (b′) an EAlR (EAlE) respectively.

ii) A PLKS is called: (a) a *panlogographic ordinary*, or *logical, kernel-sign* (*PLOKS* or *PLLKS*), (b) a *panlogographic special*, or *algebraic*, *kernel-sign* (*PLSpKS* or *PLAlKS*), (c) a *panlogographic relational kernel-sign* (*PLRlKS*), or (d) a *panlogographic substantival kernel-sign* (*PLSlKS*) if every element of its range is respectively either (a′) an EOKS (ELKS), (b′) an ESpKS (EAlKS), (c′) an ERlKS, or (d′) an ESlKS respectively (for "ERlKS" and "ERlKS" see Df 3.1(15)).

iii) A PLOKS (PLLKS) is called (a) a *panlogographic ordinary,* or *logical, connective* (*PLOCv* or *PLLCv*) or (b) a *panlogographic pseudo-quantifier-sign* (*PLPQS*) if every element of its range is either (a′) an EOCv (ELCv) or (b′) an EPQS respectively.

It will be recalled that a panlogograph can contain euautographs. Particularly, the PKS (principal kernel-sign) of a *combined PLR* (*CbPLR*) or of a *combined PLI* (*CbPLI*) can be either an EKS or a PLKS, whereas the latter can, in turn, contain some euautographs. All pertinent definitions apply with "operator" ("O"), 'predicate-sign" ("PS"), or 'predicate-operator" ("PO") in place of "kernel-sign" ("KS"), the understanding being that "operator" may sometimes be used loosely instead of "kernel-sign". In any of the above definienda involving the adjective "panlogographic" ("PL") and any one of the adjectives "ordinary" ("O"), "special" ("Sp"), "logical" ("L"), "algebraic" ("Al"), "special logical" ("SpL"), the former can be permuted with the latter without altering the meaning of the definiendum, because the respective definiens has the like property with "euautographic" in place of "panlogographic", – in accordance with the items 2 and 17iv of Df 3.1. Hence, the following pairs of abbreviated taxonyms and their full counterparts are pairs of synonyms: "*PLOT*" and "*OPLT*", "*PLLT*" and "*LPLT*", "*PLAlT*" and "*AlPLT*", "*PLSpT*" and "*SpPLT*", "*PLSpLR*" and "*SpLPLR*", etc. Also, the meaning of any of the above taxonyms of panlogographs is not altered if the prepositive qualifier "panlogographic" occurring in a taxonym is replaced with the postpositive qualifier "of $\mathbf{A}_1$" or if both qualifiers are used simultaneously.

2) The above definitions can be summarized as the following *metalinguistic syntactico-semantic orismological* (*terminological*) relation, which will be called the *panlogograph-to-euautographs relation schema* (*PLERS*):

A *panlogograph* (*PL*, *PLPH*) is a *panlogographic* — if every element (member) of its range is a *euautographic* —; "—" is an ellipsis for an appropriate specific descriptive head name, both occurrences of which should be replaced alike.

According to the PLERS, a panlogograph of any *taxon* (*taxonomic class,* pl. "*taxa*" or "*taxons*") as defined in the item 1, is coined with the taxonym that results by replacing the qualifier "euautographic" with "panlogographic" in the taxonym coining every euautograph of the range of the panlogograph – *the range that is determined by the* *original definition of the panlogograph*. Therefore, in spite of its apparent generality and clarity, if the PLERS is regarded, not as a summary of the concrete definitions, which have already been made in the previous item, but as a pattern for making some other concrete definitions by concretizing the ellipsis "—" then that schema has the following two pitfalls.

a) An apparently appropriate concretization of the ellipsis "—" in the name schema "a panlogographic —" may result, not in a common metalinguistic name of a panlogograph, but rather in a common metalinguistic name of some other metalinguistic names of panlogographs. That is to say, *a panlogographic — is not necessarily a panlogograph*. For instance, in accordance with Df 3.1(11), an EOT (ELT's) or an ESpT (EAlT, EI) is indiscriminately called an ET (euautographic term), while an ET or an ER is indiscriminately called an EF (euautographic formula). However, the instance of the above definition schema of "panlogographic —" with "term" ("T") or "formula" ("F") in place of the ellipsis "—" is not correct, because there is neither a panlogograph (PL, PLPH) whose range contains both the EOT's and the ESpT's nor a panlogograph whose range contains both the ET's and the ER's. Consequently, I define the metaterms "*panlogographic term*" ("*PLT*") and "*panlogographic formula*" ("*PLF*") in analogy with the metaterms "*euautographic term*" ("*ET*") and "*euautographic formula*" ("*EF*") as follows.

i) A PLT is either a PLOT or a PLSpT (PLI) and vice versa.

ii) A PLF is either a PLT or a PLR and vice versa. More precisely, a PLF is a panlogograph of any of the following four kinds: a PLOR, PLSpR, PLOT, or PLSpT (PLI).

Thus, the metaterm "PLT" is a *hypertaxonym* of its *hypotaxonyms* "PLOT" and "PLSpT" ("PLI"), whereas the metaterm "PLF" is a hypertaxonym of its hypotaxonyms "PLT" and "PLR" or, alternatively, or its hypotaxonyms "PLOR", "PLSpR", "PLOT", and "PLSpT" ("PLI"). Hence, the taxonyms "PLT" and "PLF" are ones of a *higher taxonomic rank* (*higher logical type*) as compared to the taxonomic ranks of their hypotaxonyms. At the same time, all the taxonyms are instances of the same general taxonym schema "*panlogographic* —", so that the latter turns out to be *epistemologically relativistic*. Therefore, for avoidance of confusion, each such taxonym is scrutinized before letting it pass and it is used then in accordance with its definition.

b) Even if a given LPH (logographic placeholder) is defined as a panlogograph (PLPH), whose taxonym is an instance of the schema "a panlogographic —" and whose range is a certain class of euautographs, the panlogograph, its taxonym, and its range *may not satisfy* the PLERS. Particularly, the taxonyms (metaterms) "*panlogographic ordinary relation*" ("*PLOR*"), "*panlogographic special relation*" ("*PLSpR*"), "*panlogographic logical relation*" ("*PLLR*"), and "*panlogographic algebraic relation*" ("*PLAlR*") are defined *syntactically in analogy* with the taxonyms "EOR", "ESpR", "ELR", and "EAlR" (which are defined concisely in Df 3.1(11)) and hence not necessarily semantically in terms of EOR's, ESpR's, ELR's, and EAlR's respectively. In this case, the synonymous taxonyms "PLSpT" and "PLAlT", as defined in the previous item semantically, can alternatively be defined syntactically in the same way as the above four taxonyms of PLR's. The meaning of the above four metaterms will be explained below in this definition.

3) If *a panlogographic* — and *a euautographic* — satisfy the PLERS then *the panlogographic* — or *any euautographic* — of its range is indiscriminately called *an endosemasiopasigraphic* (*EnSPSG*) —, – in accordance with the definition of the taxonym "endosemasiopasigraph" as given in Df 4.1(3). As before, "—" is an ellipsis for a specific descriptive head name, all occurrences of which should be replaced alike. Thus, by the item 1, either of the prepositive qualifiers "panlogographic" ("PL") and "euautographic" ("E") to any of the generic names "relation" ("R"), "ordinary term" ("OT"), "integron" ("I"), "kernel-sign" ("KS"), etc, "logical connective" ("LCv"), and "pseudo-quantifier-sign" ("PQS") can synecdochically, be replaced with "endosemasiopasigraphic" ("EnSPG"). For instance, a PLR or an ER' is indiscriminately called an *endosemasiopasigraphic relation* (*EnSPGR*), and a PLKS or an EKS is indiscriminately called an *endosemasiopasigraphic kernel-sign* (*EnSPGKS*).

4) The most general kinds (classes) of PLPH's are given below.

a) An APL (APLPH), whose range is a certain class of AE's (atomic euautographs), is called a *structural panlexigraph* or *structural atomic panlogograph* (*StAPL*, pl. "*StAPL's*") or *structural APLPH* (*StAPLPH*) or *structural atomic panlogographic schema* (*StAPLS*, pl. "*StAPLS'ta*"). A StAPL is called a *structural panlogographic ordinary,* or *logical, term* (*StPLOT* or *StPLLT*) and also, redundantly, a *structural atomic panlogographic ordinary*, or *logical, term* (*StAPLOT* or *StAPLLT*) if its range is the set of EOT's (ELT's, PEOT's, PELT's, PAEOT's, PAELT's). A StAPL is called a *structural atomic panlogographic special*, or *algebraic*, *term* (*StAPLSpT* or *StAPLAlT*) and also, a *structural atomic panlogographic integron* (*StAPLI*) if its range is the set of AEI's (AESpT's, AEAlT's). A StAPL is called a *structural atomic panlogographic relation* (*StAPLR*) and also, redundantly, a *structural atomic panlogographic ordinary*, or *logical, relation* (*StAPLOR* or *StAPLLR*) if its range is the set of AER's (PAPVOR's, PAPVLR's). For instance, '**u**' to '**z**' are StPLOT's (StPLLT's, StAPLOT's, StAPLLT's); 'i', whose range is the set of nine digits 1 to 9, and '**j**', whose range is the set of ten digits 0 to 9, are StAPLI's (StPLSpT's, StPLAl'T's); and '**p**' to '**s**' are StAPLR's (StAPLOR's, StAPLLR's). There are no *structural atomic panlogographic special*, or *algebraic, relations.*

b) A CbPL (CbPLPH) is called a *structural CbPL* (*StCbPL*) or *structural CbPLPH* (*StCbPLPH*) or *structural combined panlogographic schema* (*StCbPLS*, pl. "*StCbAPLS'ta*") if it consists of some StAPL's (StAPLPH's, StAPLS'ta) and, perhaps, of some euautographs. A StAPL (StAPLPH) or a StCbPL (StCbPLPH, StPLS) is indiscriminately called a *structural PL* (*StPL*) or a *structural PLPH* (*StPLPH*).

c) An APL (APLPH) is called an *analytical atomic panlogograph* (*AnAPL*) or *analytical APLPH* (*AnAPLPH*), and also an *analytical atomic panlogographic description* or *analytical atomic panlogographic mnemonic* (*AnAPLD* or *AnAPLM*), if *it is not structural*. Thus, in contrast to a StAPL (StAPLPH), which is a StAPLS and vice versa, and which is hence a *panlogographic schema* (*PLS*, pl. "*PLS'ta*") (to be defined), an AnAPL (AnAPLPH) is *not a PLS*. Therefore, the taxonym "structural atomic panlogographic schema" ("StAPLS") can be abbreviated as "*atomic panlogographic schema*" ("*APLS*"). An AnAPL is called an *analytical atomic panlogographic special*, or *algebraic*, *term* (*AnAPLSpT* or *AnAPLAlT*) and also an *analytical atomic panlogographic integron* (*AnAPLI*) if its range is the class of *some* or, particularly, *all* EI's of A_1; an AnAPL is called an *analytical atomic panlogographic ordinary*, or *logical, relation* (*AnAPLOR* or *AnAPLLR*) if its range is the class of *some* or, particularly, *all* ER's of A_1. For instance, **I**' to '**N**' are AnAPLI's (AnAPLSpT's, AnAPLAlT's), whose range is the class of all EI's of A_1, whereas '**P**' to '**S**' are AnAPLOR's (AnAPLLR's), whose range is the class of all ER's of A_1.

d) There are *neither analytical atomic nor any combined*, *panlogographic ordinary*, or *logical*, *terms*. Therefore, a StPLOT (StPLLT, StAPLOT, StAPLLT) as defined in the above point a) is alternatively called an *atomic panlogographic ordinary*, or *logical*, *term* (*APLOT* or *APLLT*). At the same time, every *combined PLT* (*CbPLT*) is necessarily a *combined PLI* (*CbPLI*), i.e. a *combined special*, or *algebraic*, *term* (*CbPLSpT* or *CbPLAlT*).

e) A StAPLI (StAPLSpT, StAPLAlT) or an AnAPLI (AnAPLSpT, AnAPLAlT) is indiscriminately called an *atomic panlogographic integron* (*APLI*) and also an *atomic panlogographic special*, or *algebraic*, *term* (*APLSpT* or *APLAlT*), and vice versa. A StAPLOR (StAPLLR) or an AnAPLOR (AnAPLLR) is indiscriminately called an *atomic panlogographic ordinary,* or *logical*, *relation* (*APLOR* or *APLLR*) and vice versa. An APLOT (APLLT) or an APLI (APLSpT, APLAlT) or an APLOR (APLLR) is indiscriminately called an *atomic PLF* (*APLF*),

f) A CbPL is called an *analytical combined panlogograph* (*AnCbPL*) or an *analytical combined panlogographic placeholder* (*AnCbPLPH*) if it contains at least one AnAPL (AnAPLPH) as its constituent part. Hence, a CbPL (CbPLPH) is either a StCbPL (StCbPLPH) or an AnCbPL (AnCbPLPH) and vice versa. Unlike a StCbPL that is always a StPLS and vice versa, an AnCbPL is called

- an *analytical combined panlogographic description* (*AnCbPLD*) or *analytical combined panlogographic mnemonic* (*AnCbPLM*) if and only if it has *no PKS* (*principal kernel-sign*), so that it is not an operand;
- an *analytical combined panlogographic schema* (*AnCbPLS*, pl "*AnCbPLS*") if and only if it has a certain *PKS*, euautographic or panlogographic, so that it is the operand (scope) of that PKS or, less explicitly, an operand.

An AnAPL or an AnCbPL is indiscriminately called an *analytical PL* (*AnPL*) or an *analytical PLPH* (*AnPLPH*). A StPL or an AnPL is indiscriminately called a PL (panlogograph) or a PLPH (panlogographic placeholder) and vice versa.

g) A CbPL is called a *combined PLI* (*CbPLI*) if it is a PLI and a *combined PLR* (*CbPLR*) if it is a PLR, whereas there are *no combined PLOT's*. Consequently, an AnCbPL is called an *analytical CbPLI* (*AnCbPLI*) if it is a PLI and an *analytical CbPLR* (*AnCbPLR*) if it is a PLR. Likewise, an AnCbPLD is called an *AnCbPLI-description* (*AnCbPLID*) if it is a PLI and an *AnCbPLR-description* (*AnCbPLRD*) if it is a PLR; an AnCbPLS is called an *AnCbPLI-schema* (*AnCbPLIS*) if it is a PLI and an

AnCbPLR-schema (*AnCbPLRS*). It is understood that every element of the range of a CbPLI is a CbEI and that every element of the range of a CbPLR is a CbER. A PLI is either an APLI or a CbPLI and a PLR is either an APLR or a CbPLR, whereas PLOT is an APLOT and vice versa. In accordance with the items 2a and 2ii of this definition, a CbPLI or a CbPLR is indiscriminately called a *combined PLF* (*CbPLF*). An APLF or a CbPLF is indiscriminately called a PLF.

h) Df 3.1(11b) applies, *mutatis mutandis*, with "panlogographic" ("PL") in place of "euautographic" ("E"). Consequently, a CbPLF is alternatively called a *panlogographic operand* (or *operandum*), whereas a *constituent formula* of the panlogographic operand is called an *operatum*, or, more precisely, an *operatum-formula* or *formula-operatum*, of the panlogographic operand. In this case, a CbPL in general and therefore a panlogographic operand (CbPLF) in particular can contain some euautographs as its constituent parts. For example, as was already indicated in the item 1, the PKS (principal kernel-sign) of a CbPLR or CbPLI, i.e. of a CbPLF, can be either an EKS or a PLKS, whereas the latter can contain some euautographs. Consequently, in analogy with the last statement of Df 3.1(15), a CbPLF (panlogographic operand) is (a) a CbPLR if and only if its PKS (principal kernel-sign) is either an ERlKS or a PLRlKS or (b) a CbPLI if and only if its PKS is either an ESlKS or a PLSlKS. At the same time, an operatum of a panlogographic operand can be either (i) a PLF, atomic or combined, which is called a *panlogographic operatum*, or, more precisely, a *panlogographic operatum-formula* or a *panlogographic formula-operatum*; or (ii) an EF, atomic or combined, which is, in accordance with Df 3.1(11b), called a *euautographic operatum*, or, more precisely, an *euautographic operatum-formula* or *eautographic formula-operatum*. In accordance with the item 3, either qualifier "panlogographic" or "euautographic" to any of the generic names "operatum", "operatum-formula", and "formula-operatum" can, synecdochically, be replaced with "endosemasiopasigraphic" ("EnSPG").

5) Just as in the case of euautographs (see Df 3.1(16)), the qualifiers "logical" and "ordinary" or "algebraic" and "special" are accidental synonyms when they apply to PLKS's, APLR's, or PLT's, but they are used differently when applied to CbPLR's. This fact is explicated in the next item, which is, *mutatis mutandis*, word for word the same as Df 3.1(17). In this case, the expression "mutatis mutandis", which is translated in English as "with the corresponding changes", means with the

following replacements: '$\mathbf{A}_1$' in place of 'A_1', "(see the items 1, 4g, and 4h of Df 4.2)" in place of "(see the item 15)", "item 4d of Df 4.2" in place of "item 16", "panlogographic" in place of "euautographic", and "PL" for "panlogographic" in place of "E" for "euautographic" in all abbreviations except the occurrences of "EI" in the points a, b, ii, and iii (but not in the point iv) of Df 3.1(17). The latter occurrences should be replaced with occurrences of "EnSPGI". For more clarity, the above variant of Df 3.1(17) is stated below with minor explanatory reservations regarding the exact meaning of particular occurrences of "EnSPGI". Since the above-mentioned replacements do not affect the point v of Df 3.1(17), the latter is not restated here, although it remains effective.

6) The class of CbPLR's of $\mathbf{A}_1$ (see the items 1, 4g, and 4h of this definition) can be divided into two subclasses in two ways. In accordance with one of the two dichotomies of that class, a CbPLR is called:

a) a *combined panlogographic ordinary relation* (*CbPLOR*) if it involves *no EnSPGI*, i.e. *neither EI's nor PLI's*, – or, what comes to the same thing, if it involves *no occurrence of* $\hat{=}$.

b) a *panlogographic special relation* (*PLSpR*) if it involves *at least one EnSPGI*, *at least one EI or at least one PLI*, – or, what comes to the same thing, if it involves *at least one occurrence of* $\hat{=}$.

In accordance with the other dichotomy, a CbPLR is called:

a′) a *combined panlogographic logical relation* (*CbPLLR*) if and only if its PKS (principal kernel-sign) is either an ELKS (EOKS) or a PLLKS (PLOKS);

b′) a *panlogographic algebraic relation* (*PLAlR*) or *panlogographic algebraic equality* (*PLAlE*) if and only if its PKS is $\hat{=}$.

The prepositive qualifier "combined" ("Cb") to any of the taxonyms introduced in the points b) and b′) would have been redundant because there are in $\mathbf{A}_1$ neither PLSpR's nor PLAlR's that could be qualified *atomic*.

The above two definitions have the following implications.

i) It follows from the points a) and a′) that *every CbPLOR is a CbPLLR but not necessarily vice versa*. Specifically, a CbPLLR is called:

a_1) a *combined panlogographic ordinary logical relation* (*CbPLLOR*) or, simply, a *combined panlogographic ordinary relation* (*CbPLOR*) if and only if it involves *no occurrence* of ≙;

a_2) a *combined panlogographic special logical relation* (*CbPLSpLR*) or, simply, a *panlogographic special logical relation* (*PLSpLR*) if and only if it involves at least one occurrence of ≙, not being the principal one.

At the same time, an APLOR (atomic PLOR) or a CbPLOR (combined PLOR) is indiscriminately called a PLOR (palogographic ordinary relation) and likewise an APLLR (atomic PLLR) or a CbPLLR (combined PLLR) is indiscriminately called a PLLR (panlogographic ordinary relation), while "APLOR" and "APLLR" are synonyms. Hence, *every PLOR is a PLLR but not necessarily vice versa.*

ii) By the point b′), a PLAlR involves at least one occurrence of ≙ and hence it involves at least two EnSPGI's standing on both sides of that occurrence of ≙. Therefore, it follows from the points b) and b′) that *every PLAlR* is a *PLSpR but not necessarily vice versa*. Specifically, a PLSpR is called:

b_1) a *panlogographic algebraic special relation* (*PLAlSpR*) or, simply, a *panlogographic algebraic relation* (*PLAlR*) and also a *panlogographic algebraic equality* (*PLAlE*) if and only if it involves at least one occurrence of ≙ as its PKS;

b_2) a *panlogographic logical special relation* (*PLLSpR*) if and only if no token of ≙ occurs in it as its PKS.

A PLAlR (PLAlE) is a called a *panlogographic algebraic identity* (*PLAlI*) if it is *valid*. and a *panlogographic algebraic anti-identity* (*PLAlAntI*) if it is *antivalid*.

iii) By the points i) and ii), it follows from the points a), b), and a′) that some PLLR's are PLOR's, i.e. *PLOLR's* (*panlogographic ordinary logical relations*), while the other PLLR's are PLSpR's, i.e., more explicitly, *PLSpLR's* (*panlogographic special logical relations*). Since a PLOR involves, as its constituent parts, no EnSPGI's and hence no PLSpR's, therefore it involves no PLAlR's either. Therefore, a PLOR (PLOLR) can alternatively be called a *panlogographic chaste logical relation* (*PLChLR*). Consequently, a PLSpLR can alternatively be a *panlogographic mixed logical relation* (*PLMxLR*) in the sense that it involves at least one PLAlR as its constituent part – or, what comes to the same thing, at least one occurrence of ≙ not being its principal operator. Thus, the combined qualifiers "*ordinary logical*" ("*OL*") and "*chaste logical*" ("*ChL*"), or "*special logical*" ("*SpL*") and "*mixed logical*" ("*MxL*"), are concurrent (exchangeable). Incidentally, since a PLOT (PLLT') is exclusively a primary atomic one, it can also be alternatively called a *panlogographic chaste* (*pure*) *logical term* (*PLChLT*). In analogy with the above terminology, a PLAlR (PLAlE) as defined by the point b′), called also a PLAlSpR by the point b_1), will discriminately be called:

$b_{1'}$) a *panlogographic chaste algebraic relation* (*PLChAlR*) or a *panlogographic chaste algebraic equality* (PLChAlE) if it involves, as its constituent parts, *neither APLOR's nor PLOT's*;

$b_{2'}$) a *panlogographic mixed* algebraic *relation* (*PLMxAlR*) or a *panlogographic mixed algebraic equality* (*PLMxAlE*) if otherwise.

iv) In the item 4d of this definition, I have indicated that there are in $\mathbf{A}_1$ no terms that could be qualified either as *combined panlogographic ordinary* or as *combined panlogographic logical*. Therefore, in the versions of the above two bifurcations of the class of CbPLR's with "CbPLT" in place of "CbPLR" and "term" ("T") in place of "relation" ("R"), the pertinent versions of the points a) and a′) should be disregarded, whereas the rest of those definitions can be restated thus. A *CbPLT* is called:

b″) a CbPLSpT , i.e. a CbPLI, if it is neither PLOT nor APLI;

b‴) a CbPLAlT if its PKS is an PLSlKS.

At the same time, by Df 3.1(16), "CbPLT", "CbPLI", "CbPLSpT", and "CbPLAlT" are synonyms. Therefore, the points b″) and b‴) are just two different explicative definitions of a CbPLT (CbPLI). Hence, there are *no logical PLI's* (*logical PLSpT's*) – just as there are *no algebraic PLOT's*.

7) Df 3.1(18) applies, *mutatis mutandis,* with "PL" for "panlogographic" in place of "E" for "euautographic". In accordance with the items 2a, 2ii, and 4g of this definition, a CbPLF is either a CbPLR or a CbPLI and therefore an MPLR or an MPLI is indiscriminately called a *molecular PLF* (*MPLF*) – in analogy with MEF. However, the range of an MPLF does not necessarily consist of MEF's and likewise the range of an MPLKS does not necessarily consist of MES's. All EPM's (euautographic punctuation marks) of A_1 are EPM's of $\mathbf{A}_1$, but not vice versa. In addition to the former, $\mathbf{A}_1$ has two paired (molecular) EPM's, $\langle\ \rangle$ and $|\ \rangle$.A pair of angle brackets, $\langle\ \rangle$, is used in certain *molecular PLOR's* (*MPLOR's*) and *molecular*

PLI's (*MPLI's*), which turn out to be effective and indispensable in laying down *inference* (*transformation*) *rules* of both A_1 and $\mathbf{A}_1$ and in solving vavn-decision problems for *bound* (*pseudo-quantified*) *PLLR's*, each of which condenses an infinite number of *bound* (*pseudo-quantified*) *ELR's*.

8) I distinguish between a PLPH and an MLPH by the types, in which they are set up, by the punctuation marks, which they involve, or by their ranges. For instance, an LPH (as '**Γ**' or '**Δ**'), whose range is the class of *assemblages* of PAE's of A_1, or an LPH (as '**Φ**' or '**Ψ**'), whose range is the class of *formulas* (*terms and relations*) of A_1, is necessarily an MLPH, because neither an assemblage in general nor a formula in general is a *specific operant unit* of A_1. The only specific operant units of A_1 are *ER's* (*euautographic relations*), *EOT's* (*euautographic ordinary terms*), and *ESpT's* (*euautographic special terms*), i.e. *EI's* (*euautographic integrons*), – the euautographs, which are collectively called *euautographic formulas* (*EF's*). An LPH, whose range is either the entire or a restricted class of ER's, EOT's, or EI's of A_1, and which does not involve any peculiar signs that are not associated with $\mathbf{A}_1$, is a PLPH.•

4.2. The panlogographic algebraic decision method of A_1

Df 4.3: ***The panlogographic algebraic decision method of*** A_1 ***– the second supplement to Df 4.1.*** 1) Dfs 4.1 and 4.2 informally describe how the features of A_1, which have been indicated primarily in the items 1–18 of Df 3.1, are incorporated (condensed) into the organon $\mathbf{A}_1$ being the *panlogographic interpretans* (*formalized metalanguage*) *of* A_1. The way, in which $\mathbf{A}_1$ incorporates (condenses) the features of A_1, which have been indicated in the items 19–32 of Df 3.1, are explicated below in this definition.

1) As indicated in with Df 3.1(19), the set of *schematic panlogographic* and *metalinguistic rules* of inference and decision of A_1, in which all constituent *formulary* (*categorematic*) *elemental* (*primitive*, *atomic* or *molecular*) *panlogographs*, i.e. *elemental panlogographic formulas* (*ElPLF's*) *of* $\mathbf{A}_1$, are used *xenonymously*, i.e. as *eupanlogographs*, for mentioning common (general) *EF's of* A_1 of their ranges, is denoted by as 'D_1' and is called the *Advanced Algebraic Decision Method* (*AADM*) *of* A_1 or the *Euautographic AADM* (*EAADM*). *The same set of rules*, in which *the same constituent ElPLF's* are pescinded from their xenonymous denotata and are used *autonymously,* i.e. as *tychautographs*, for mentioning themselves or their homolographic (photographic) token-classes, is denoted by '$\mathbf{D}_1$' and is called the

exclusive panlogographic extension of D_1 and also the *Advanced Algebraic Decision Method* (*AADM*) *of* $\mathbf{A}_1$ or the *Panlogographic AADM* (*PLAADM*). D_0 and $\mathbf{D}_0$ and their verbal names are interrelated likewise. Particularly, while D_0 is called the *Basic Algebraic Decision Method* (*BADM*) *of* A_1 or the *Euautographic BADM* (*EBADM*), $\mathbf{D}_0$ is called the *exclusive panlogographic extension of* D_0 and also the *Basic Algebraic Decision Method* (*BADM*) *of* $\mathbf{A}_1$ or the *Panlogographic BADM* (*PLBADM*).

2) In accordance with the TAEXA between D_1, which is applicable to ER's of A_1, and $\mathbf{D}_1$, which is applicable to PLR's of $\mathbf{A}_1$, the union and superposition of D_1 and $\mathbf{D}_1$ is denoted by 'D_1' and is called the *inclusive endisemasipasigrphic extension* of D_1 and also the *AADM of* A_1 or the *Biune Euautographic and Panlogographic AADM* (*BUE&PLAADM*) or the *Endosemasiopasigraphic AADM* (*EnSPGAADM*); A_1 is the union and superposition of A_1 and $\mathbf{A}_1$, as indicated in Df 4.1(1). Accordingly, the union and superposition of D_0 and $\mathbf{D}_0$ is called the *inclusive endisemasipasigrphic extension* of D_0 and also the *BADM of* A_1, the *Biune Euautographic and Panlogographic AADM* (*BUE&PLAADM*), or the *Endosemasiopasigraphic AADM* (*EnSPGAADM*); A_0 is the union and superposition of A_0 and $\mathbf{A}_0$, as indicated in Df 4.1(1).

3) In accordance with the above two items, D_1, $\mathbf{D}_1$, and D_1 are syntactically the same set of rules of inference and decision, which semantically differ from one another by the mental attitude of the interpreter (as me) towards their constituent ElPLF's and towards the formulas, to which these rules apply. Hence, the above-mentioned two extensions of D_1 or D_0, the exclusive one and the inclusive one, are *mental* (*psychical*, *imaginary*). Accordingly, the way, in which $\mathbf{A}_1$ or A_1 incorporates (condenses) the features of A_1, which have been indicated in the items 19–21 of Df 3.1, can most concisely be described by stating that those items apply verbatim with $\mathbf{A}$, $\mathbf{D}$, and "panlogographic" ("PL"), – and also, alternatively, with A, D, and "endosemasiopasigraphic" ("EnSPG") , – in place of A, D, and "euautographic" ("E") respectively, while $\mathbf{D}_1$ or D_1 *is the same set of rules of inference and decision as* D_1 *subject to their TAEXA*. Thus, $\mathbf{A}_1$ or A_1 is, just as A_1, a single whole APO, every phase and every branch of which has *the same built-in panlogographic*, or, correspondingly, *endosemasiopasigraphic*, *algebraic*, and hence *analytical*, *decision*

method in common, that is denoted by ‘$\mathbf{D}_1$’ or ‘D_1’ respectively. In this case, the phasing and branching of $\mathbf{A}_1$ or A_1 are the same as those of A_1, while “*PLAPO*” (“*Panlogographic APO*”) or “*EnSPGAPO*” (“*Endosemasiopasigraphic APO*”) alone, without any modifiers, is the abbreviated generic name of every phase and every branch of respectively $\mathbf{A}_1$ or A_1, which comes instead of “EAPO” (“*Euautographic APO*”) being the abbreviated generic name of every phase and every branch of A_1.

4) The above two sets of replacement of the nomenclature of the items 19–21 of Df 3.1 apply also to the item 22 of Df 3.1 with the proviso that the phrase “EOR’s of A_1” following the item 22f should be replaced with the phrase “EOR’s in the ranges of PLOR’s of $\mathbf{A}_1$” along with the former set of replacements or with the phrase “EOR’s of A_1” along with the latter set of replacements. Thus, either of the postpositive qualifiers “*of academic or practical interest*” (“*of API*”) and “*of academic interest*” to a PLR or EnSPGR has the same sense as that it has when applied to an ER. Accordingly, a PLR that is qualified as one of API is a PLOR but not necessarily vice versa, whereas an EnSPGR that is qualified as one of API is an *EnSPGOR* (*EnSPG ordinary relation*), i.e. either a PLOR or an EOR, but not necessarily vice versa.

5) In accordance with the previous items 1–4, the extension of the properties of D_1, which have been explicated in the items 23–32 of Df 3.1, to $\mathbf{D}_1$, and hence to D_1, can be described in general outline as follows. The instance (act or process) of application of $\mathbf{D}_1$ to a PLR (primarily PLOR) of $\mathbf{A}_1$ of API is called a *panlogographic algebraic decision procedure* (*PLADP*) *for that relation* or less explicitly a *PLADP of* $\mathbf{A}_1$. A PLADP is called a *basic* one (*BPLADP*) if it is performed by means of $\mathbf{D}_0$ and an *advanced* one (*APLADP*) if it involves applications of at least one rule of $\mathbf{D}_1$ not belonging to $\mathbf{D}_0$. It is understood that a PLADP is performed in accordance with the same rules as an EADP, provided that all elemental panlogographs involved in the PLADP are used autonymously, i.e. as tychautographs. However, the panlogographs ‘**P**’, ‘**Q**’, and ‘**R**’ that is used in the general metalinguistic description of EADP’s in Df 3.1 are concrete APLR’s (atomic panlogographic relations) therein depicted between single quotation marks. Therefore, in order to turn that description into a metalinguistic description of PLADP’s, the PLPH’s ‘**P**’, ‘**Q**’, and ‘**R**’ should be replaced with appropriate MLPH’s, say ‘**P**’, ‘**Q**’, and ‘**R**’, whose range is the class of

all PLR's of A_1, unless it is restricted somehow, e.g. by attributing the qualifier "of academic or practical interest" to the taxonym (count name) "PLR of A_1" that is used for mentioning a given PLR. Also, the PLPH's '$V(\mathbf{P})$', '$\mathbf{i}_1|\mathbf{P}\rangle$', '$\mathbf{i}_2|\mathbf{P}\rangle$', etc and the MLPH to '$\mathbf{i}_{n`-1}|\mathbf{P}\rangle$', whose values are certain successive RSEVI (reducible secondary euautographic validity-integrons) of **P** that have been described in Df 3.1(23), should be replaced with appropriate MLPH's, say '$V(\mathsf{P})$', '$\mathsf{i}_1|\mathsf{P}\rangle$', '$\mathsf{i}_2|\mathsf{P}\rangle$', etc, and '$\mathsf{i}_{n-1}|\mathsf{P}\rangle$', whose values are certain successive *RPLVI's* (*reducible panlogographic validity-integrons*) of P. Likewise, the MLPH '$\mathbf{i}_{n`}|\mathbf{P}\rangle$' or the PLPH '$\mathbf{i}_{`\sim}|\mathbf{P}\rangle$', whose value is a certain irreducible, or ultimate, EVI (IREVI or UEVI) of **P**, should be replaced with the MLPH '$\mathsf{i}_n|\mathsf{P}\rangle$' or '$\mathsf{i}_\sim|\mathsf{P}\rangle$' respectively, whose value is a certain *irreducible*, or *ultimate*, *PLVI* (*IRPLVI* or *ULVI*) of P. Accordingly, '$\bar{V}(\mathbf{P})$', '$\bar{\mathbf{i}}_1|\mathbf{P}\rangle$' to '$\bar{\mathbf{i}}_{n`}|\mathbf{P}\rangle$', and '$\bar{\mathbf{i}}_\sim|\mathbf{P}\rangle$', whose values are EAVI's (euautographic antivalidity integrons), should be replaced with '$\bar{V}(\mathsf{P})$', '$\bar{\mathsf{i}}_1|\mathsf{P}\rangle$' to '$\bar{\mathsf{i}}_n|\mathsf{P}\rangle$' and '$\bar{\mathsf{i}}_\sim|\mathsf{P}\rangle$' respectively, whose values are *panlogographic antivalidity integrons* (*PLAVI's*). In addition to the above replacements, the replacements that have been indicated earlier in the item 3 should also be made throughout the items 23–32 of Df 3.1 except for the passages dealing with relations between panlogographs and euautographs of their ranges. At the same time, the occurrences of the DDI's 0, 1, and 2 and of the EKS's as V, $\hat{=}$, $\hat{\cdot}$, $\hat{-}$, and $\neg$ in all relevant metalinguistic statements and MLPH's remain unaltered.

6) Thus, in analogy and in compliance with the respective arguments of Df 3.1(23), the PLADP for a given PLR (primarily PLOR) P of $\mathsf{A}_{1,}$ *of academic or practical interest* (*API*) of API, denoted by **'$\mathsf{D}_1(\mathsf{P})$'**, is a *single whole sequence of panlogographic algebraic identities* (*PLAlI's*), i.e. *valid panlogographic algebraic equalities* (*PLAlE's*), which are interrelated by the appropriate rules of inference comprised in D_1. This sequence proceeds from the identity

$$V(\mathsf{P}) \hat{=} V(\mathsf{P}), \tag{4.1}$$

analogous to (3.1), and ends with the identity of one of the following three forms:

$$V(\mathsf{P}) \hat{=} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \mathsf{i}_\sim|\mathsf{P}\rangle & \text{(c)} \end{cases}, \tag{4.2}$$

analogous to (3.2). Just as in the case of the EADP for a given ER **P** of A_1, $V(\mathbf{P})$ is the *primary*, or *initial*, *validity-integron* (*PVI* or *IVI*) *of* **P**, whereas $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ is a certain *irreducible,* or *ultimate, validity-integron* (*IRVI* or *UVI*) *of* **P** other than 0 or 1. In accordance with (3.3), $V(\mathbf{P})$ satisfies the *idempotent law*:

$$V(\mathbf{P})\hat{\cdot}V(\mathbf{P})\hat{=}V(\mathbf{P}) \tag{4.3}$$

and hence $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ satisfies the similar law:

$$\mathbf{i}_{\sim}|\mathbf{P}\rangle\hat{\cdot}\mathbf{i}_{\sim}|\mathbf{P}\rangle\hat{=}\mathbf{i}_{\sim}|\mathbf{P}\rangle, \tag{4.4}$$

which is analogous to (3.4). The pertinent one of the three conditions (a), (b), and (c) of the scheme (4.2), which a given **P** satisfies, turns into an identity that is denoted by '$\mathbf{T}_{1+}(\mathbf{P})$', '$\mathbf{T}_{1-}(\mathbf{P})$', or '$\mathbf{T}_{1\sim}(\mathbf{P})$' respectively or indiscriminately by '$\mathbf{T}_1(\mathbf{P})$' and is called the *panlogographic master-theorem* (*PLMT*), or *panlogographic decision theorem* (*PLDT*), *for* **P**, or more generally an *PLDT*, or *DT* (*decision theorem*), *of* $\mathbf{A}_1$. Each *rule of inference* (*transformation*) that is used in PLADP's is alternatively and more specifically called a *rule of PLADP's* or a *PLADP rule* (*PLADPR*). A PLR of $\mathbf{A}_1$ is called a *decided PLR* (*DdPLR*) if it has a PLDT (PLMT). The metalinguistic three-fold scheme (4.2) is called the *panlogographic decision-theorem* (*PLDT*) *scheme,* or *pattern*, *for* **P**. The PLR **P** proceeded, is called the *panlogographic slave-relation* (*PLSR*), or *panlogographic relation-slave* (*PLR-slave*), and also *panlogographic object relation of both the EADP* $\mathbf{D}_1(\mathbf{P})$ *and the PLMT* (*PLDT*) $\mathbf{T}_1(\mathbf{P})$.

7) Just as in the case of EDT's (see Df 3.1(24)), in accordance with the distinctive form of an PLDT (PLMT), *its PLR-slave is decided* to be a PLR of exactly one of the three *decision classes* as stated in the following *decision rule* [*for PLR's*] *of* $\mathbf{A}_1$:

> A DdPLR **P** of $\mathbf{A}_1$ is said to be: (a) *valid* if its PLDT has the form (4.2a); (b) *antivalid* if its PLDT has the form (4.2b); or (c) *vav-neutral* (or *vav-indeterminate*), i.e. *neutral* (or *indeterminate*) *with respect to the validity-values validity and antivalidity* or, in other words, to be *neither valid nor antivalid*, if its PLDT has the form (4.2c) subject to (4.4).

Therefore, in agreement with what was stated in the item 3 of this definition, the items 24–32 of Df 3.1, which are pertinent to the solution of vavn-decision problem for

ER’s of A_1, apply, *mutatis mutandis*, verbatim with $\mathbf{A}$, $\mathbf{D}$, and “panlogographic” (“PL”), or with A, D, and “endosemasiopasigraphic” (“EnSPG”), in place of A, D, and “euautographic” (“E”) respectively.

In analogy with the basic decisional trichotomy of the vavn-decided ER’s, indicated in Df 3.1(24), a PLR $\mathbf{P}$ of $\mathbf{A}_1$ is said to be *valid* if its DT has the form (4.2a), *antivalid* if its DT has the form (4.2b), and *vav-neutral* (or *vav-indeterminate*), i.e. *neutral* (or *indeterminate*) *with respect to validity and antivalidity* or, in other words, *neither valid nor antivalid*, if its DT has the form (4.2c) subject to (4.4). Consequently, the items 24–32 of Df 3.1 apply, *mutatis mutandis,* under the pertinent substitutions indicated in the items 2–5 of this definition.

8) The vavn-PLDT, i.e. the solution of the vavn-decision problem, for a PLOR, e.g., has the following peculiarities:

a) A PLOR is valid if and only if every EOR in its range is valid.

b) A PLOR is antivalid if and only if every EOR in its range is antivalid.

c) In the general case, the range of a vav-neutral PLOR contains an indefinite (infinite) number of vavn-suspended EOR and also an indefinite (infinite) number of vavn-decided EOR of each of the three classes: valid, antivalid, and vav-neutral.

Consequently, a valid, or antivalid, PLOR is a solution of the vavn-decision problem for every EOR in its range. By contrast, a vav-neutral PLOR is not, in the general case, a solution of the vavn-decision problem for every EOR of its range. Therefore, a concrete EOR of academic or practical interest, of the range of the vav-neutral PLOR should be subjected to the EADP of its own in order to decide on its validity-value provided of course that this has not been done earlier. At the same time, any euautographic instance of a vav-neutral PLOR, which has the same pattern in terms of irreducible elementary (atomic or molecular) EOR’s as the pattern of the vav-neutral PLOR in terms of irreducible elementary PLOR’s, is obviously a vav-neutral EOR. Particularly, the so-called *analographic euautographic variant (isotoken) of a vav-neutral PLOR is a vav-neutral EOR.*

9) In accordance with the previous item, in order to solve the vavn-decision problem for a given EOR, it seems preferable to solve the vavn-decision problem for an adequately patterned PLOR, an analytical one or at least a structural one, because the PLOR condenses an infinite number of other EOR’s, for which the vavn-decision

problem will be solved simultaneously with that for the given EOR by the same work input.

10) In compliance with the above items 6–9, Df 3.2 applies with "panlogographic" in place of "euautographic" and with "PL" for "panlogographic" in place of "E" for "euautographic".•

4.4. A summary of the main properties of A_1 and $\mathbf{A}_1$

1. In principle, D_1 is applicable to any comprehensible relation of A_1. In practice, however, I apply D_1 only to certain *preselected* EnSPGR's of A_1, primarily *ordinary* ones (EnSPGOR's), which I regard as ones having academic or practical interest in accordance with Dfs 3.1(22) and 4.3(4).

2. To any given relation of A_1 or $\mathbf{A}_1$ of academic or practical interest, there is respectively an EADP (euautographic algebraic decision procedure) or PLADP (panlogographic algebraic decision procedure), whose final identity is the pertinent DT (decision theorem), according to the form of which the processed relation is unambiguously classified either as a valid one (kyrology) or as an antivalid one (antikyrology), or else as a vav-neutral, or vav-indeterminate, one (kak-udeterology, kak-anorismenology). My using the adjectives "indeterminate" and "neutral" synonymously (interchangeably) should not mislead the reader. *There is no indeterminacy* (*uncertainty*) *in attributing a relation of* A_1 or $\mathbf{A}_1$ *to the class of vav-neutral* (*vav-indeterminate*) *relations if it is so*. A vav-neutral relation of A_1 or $\mathbf{A}_1$ is *not an improvable relation of the Gödelian type*, because it *is proved* to be vav-neutral – just as a valid relation, other than a subject axiom of A_1 or $\mathbf{A}_1$, is proved to be valid and just as an antivalid relation is proved to be antivalid. For instance, under the self-evident meta-axiom that: p and q are vav-neutral AEOR's (atomic euautographic ordinary relations) of A_1, it is elementarily proved that (a) the EOR's $\mathsf{p}\vee\neg\mathsf{p}$ and $\neg[\mathsf{p}\wedge\neg\mathsf{p}]$ are theorems of A_1 (cf. "2·2=4", which is a theorem of any axiomatic theory of numbers); (b) the EOR's $\neg[\mathsf{p}\vee\neg\mathsf{p}]$ and $\mathsf{p}\wedge\neg\mathsf{p}$ are antitheorems of A_1; (c) the EOR's $\neg\mathsf{p}$, $\neg\mathsf{q}$, $\mathsf{p}\vee\mathsf{q}$, $\mathsf{p}\wedge\mathsf{q}$, $\neg[\mathsf{p}\vee\mathsf{q}]$, and $\neg[\mathsf{p}\wedge\mathsf{q}]$ are vav-neutral (vav-indeterminate) relations of A_1. Likewise, under the self-evident meta-axiom that '**P**' and '**Q**' are vav-neutral APLR's (atomic panlogographic relations) of $\mathbf{A}_1$, it is proved in the same elementary way that: (a') the graphonyms '**P**∨¬**P**' and '¬[**P**∧¬**P**]' are *panlogographic theorems*, and hence *valid PLR's*, of $\mathbf{A}_1$, which can univocally be written as '**P**'∨¬'**P**' and

¬[‘**P**’∧¬‘**P**’], when both occurrences of ‘**P**’ in either graphonym are mentally used autonymously, and they are *valid PLS’ta* (*panlogographic schemata*) *of euautographic theorems* (*valid ER’s*) of A_1, namely **P**∨¬**P** and ¬[**P**∧¬**P**], when the occurrences of ‘**P**’ are used xenonymously; (b') the graphonyms ‘¬[**P**∨¬**P**]’ and ‘**P**∧¬**P**’ are *panlogographic antitheorems,* and hence *antivalid PLR’s,* of $\mathbf{A}_1$, namely ¬[‘**P**’∨¬‘**P**’] and ‘**P**’∧¬‘**P**’, when ‘**P**’ is used autonymously, or they are *antivalid PLS’ta of euautographic antitheorems* (*antivalid ER’s*) of A_1, namely ¬[**P**∨¬**P**] and **P**∧¬**P**, when ‘**P**’ is used xenonymously; (c') the graphonyms ‘¬**P**’, ‘**P**∨**Q**’, ‘**P**∧**Q’**, ‘¬[**P**∨**Q**]’, and ‘¬[**P**∧**Q**]’ are *vav-neutral* (*vav-indeterminate*) *PLR’s* of $\mathbf{A}_1$, namely ¬‘**P**’, ‘**P**’∨‘**Q**’, ‘**P**’∧‘**Q**’, ¬[‘**P**∨‘**Q**’], and ¬[‘**P**’∧‘**Q**’], when ‘**P**’ and ‘**Q**’ are mentally used autonymously, and any of the graphonyms is a *vav-neutral* (*vav-indeterminate*) *AnPLS* (*analytical PLS*) *of ER’s of* A_1 *of various decisional classes* (to be immediately specified below), namely ¬**P**, **P**∨**Q**, **P**∧**Q**, ¬[**P**∨**Q**], and ¬[**P**∧**Q**], when ‘**P**’ and ‘**Q**’ are mentally used xenonymously. In agreement with Df 3.5(8c), the range of a vav-neutral AnPLS contains ER’s of A_1 of three following three types:

a) vav-neutral ER’s, which are determined to be so by the form of the AnPLS;

b) ER’s of all the three classes: valid, antivalid, and vav-neutral, which have been decided to be so in the result of appropriate EADP’s or PLADP’s accomplished earlier and the forms of which are compatible with (fit into), but not necessarily adequate to, the form of the AnPLS;

c) vavn-suspended ER’s, the form of which is compatible with the form of the AnPLS.

For instance, ¬*p*, *p*∨*q*, *p*∧*q*, ¬[*p*∨*q*], and ¬[*p*∧*p*] are vav-neutral ER’s because these are *analographic variants* (*tokens*), namely the variants with *p* in place of ‘**P**’ and *q* in place of ‘**Q**’, of the vav-neutral PLR’s ¬‘**P**’, ‘**P**’∨‘**Q**’, ‘**P**’∧‘**Q**’, ¬[‘**P**∨‘**Q**’], and ¬[‘**P**’∧‘**Q**’] respectively. At the same time, the valid ER ¬[*p*∧¬*p*] is an instance of either of the vav-neutral PLS’ta ‘¬**P**’, and ‘¬[**P**∧**Q**]’, the valid ER *p*∨¬*p* is an instance of either of the vav-neutral PLS’ta ‘**P**∨**Q**’ and ‘**P**∨¬**Q**’, the antivalid ER ¬[*p*∨¬*p*] is an instance of either of the vav-neutral PLS’ta ‘¬[**P**∨**Q**]’ and ‘¬[**P**∨¬**Q**]’, the antivalid ER *p*∧¬*p* is an instance of either of the vav-neutral PLS’ta ‘**P**∧**Q**’ and ‘**P**∧¬**Q**’, etc.

3. The calculus $\mathbf{A}_1$ will be developed *semi-formally* as a *schematic pattern of* A_1, i.e. as a system of PLPH's (panlogographic placeholders) of functional euautographs of A_1 of various classes. In the cases, where a PLPH of $\mathbf{A}_1$ is mentally used autonymously and where there might otherwise be doubt regarding the mental mode of using the placeholder, it will be enclosed in a pair of the appropriate quotation marks, KAQ ones or HAQ ones, – in the framework of the *special quotation method* (S*QM*) that is adopted in this treatise. To be recalled a KAQ is a proper name of its interior, whereas a HAQ, is a proper name of the homolographic token-class of its interior. In the general case, however, I shall use a concrete PLF in the mental *TAEXA-mode*, i.e. autonymously and xenonymously intermittently and as if simultaneously. At the same time, many (usually an infinite number of) different atomic PLPH's (APLPH's) are defined so as to have the same range. Any two or more different atomic or combined panlogograhs (PL's, PLPH's) that have the same pattern and the same range are called *congeneric*, or *conspecific*, ones. Therefore, when a large number (usually an infinite number) of euautographs of A_1 of a certain class is represented by an atomic or combined panlogograph, designating that class, the same euautographs can be represented by any other congeneric panlogograph. In order to express explicitly this property of *intercommutability* (*mutual commutability*) of congeneric, or conspecific, panlogographs (PL's, PLPH's), I shall *occasionally* introduce MLPH's (metalogographic placeholders), e.g. ''**P**', '**Q**', and '**R**' introduced and employed in Df 4.3(5,6), whose ranges are classes (genera or species) of congeneric or conspecific panlogograps (as PLR's, PLI's, or PLOT's). However, I shall not attempt to do this systematically, because a *system of MLPH's of PLPH's* would have been tantamount to a certain *pan-panlogographic organon*, and therefore an attempt to create such an organon seems to be counterproductive and impractical. Instead of such a system, I tacitly assume that, besides being used either xenonymously or autonymously or else in the TAEXA-mode, a *concrete* panlogograph *represents* the entire class (genus) of its *congeners*, unless stated otherwise. This mental mode of using a panlogograph is called a *genus-representative*, or *meta-autonymous*, one, whereas a panlogograph that is used in this mode is said to be used *genus-representatively* or *meta-autonymously*. Accordingly, the property of *concrete* panlogographs to be used in this way is called their *meta-autonymity* or *meta-autonymy*. The meta-autonymy f a *concrete* panlogograph is a realization (instance) of a general *philosophical principle*, called the *prototype principle*, according to which a *concrete*, i.e. *most specific, instance* (*member*) *of a class can represent the entire class* (cf. Hofstadter [1979, p. 352]). In this case, *a proper name of the concrete instance* (*member*) *of the class is used as a proper name of the class*. The genus-representative (meta-autonymous) use of a panlogograph is illustrated by the following example.

4. Each of the four letters '**P**', '**Q**', '**R**', '**S**' is by definition a *primary analytical* (*not structural*) *atomic panlogographic relation* (*PAnAPLR*) *of* $\mathbf{A}_1$, i.e. an AnAPL (analytical atomic panlogograph), whose *initial range* is the class of *primary euautographic relations* (*PER's*). Any of the four letters can, when needed, be furnished with any of the light-faced digital subscripts $_1$, $_2$, etc of the decimal numeric system in the current type, thus becoming another PAnAPLR of the same name and of the same range. Consequently, there is the ordered infinite (open) list of PAnAPLR's that is called the *alphabet of PAnAPLR's of* $\mathbf{A}_1$. With this notation, '**P**$\veebar$**Q**' or '**P**'$\veebar$'**Q**', e.g., is a *primary analytical panlogographic relation* (*PAnPLR*), which can be used either *xenonymously* for mentioning *a certain* (*concrete but not concretized, common, general, any*) *euautographic relation* **P**$\veebar$**Q** or *autonymously* for mentioning *itself*, i.e. the *concrete eupanlogographic relation* '**P**'$\veebar$'**Q**', or *its homolographic* (*photographic*) *token-class*, or *the concrete eupanlogographic relation* '**P**'$\veebar$'**Q**'. In the latter case, the pairs of light-faced single quotation marks are used for indicating my *instanteous* mental attitude regarding their interiors and they are written down here for more clarity. In fact, however, the concrete relation '**P**'$\veebar$'**Q**' is one of the *distinguished member* of the *extended range* of the schematic PLR '**P**$\veebar$**Q**', when the latter is used in the TAEXA-mode. At the same time, if '**P**' and '**Q**' are used xenonymously, any PLR, which is obtained from '**P**$\veebar$**Q**' by an alphabetic change of the PAnAPLR's and which is therefore called an *alphabetic variant of* '**P**$\veebar$**Q**', has the same range and hence the same semantic properties with respect to PER's belonging to this range. That is to say, from the standpoint of semantic analysis, any two alphabetic variants of '**P**$\veebar$**Q**', say '**P**$\veebar$**Q**' itself and '**R**$\veebar$**S**', are distinguishable only by their autonymous denotata, i.e., correspondingly, by '**P**'$\veebar$'**Q**' and '**R**'$\veebar$'S'. Consequently, besides using '**P**$\veebar$**Q**' either xenonymously or autonymously or else in the TAEXA-mode, I tacitly assume that '**P**$\veebar$**Q**' or '**P**'$\veebar$'**Q**' *represents* the entire genus of its alphabetic variants, unless stated otherwise. This mental mode of using

‘**P**∨̸**Q**’ or ‘**P**’∨̸’**Q**’, and generally any *panlogograph* or, respectively, *eupanlogograph*, is the very one that is called a *genus-representative*, or *meta-autonymous*, one, whereas ‘**P**∨̸**Q**’ or ‘**P**’∨̸’**Q**’, and generally any other panlogograph or, respectively, any other eupanlogograph, which is used in this mode, is said to be used *genus-representatively* or *meta-autonymously*. *Primary analytical atomic panlogographic integrons* (*PAnAPLI’s*) *of* $\mathbf{A}_1$, i.e. the letters ‘**I**’ to ‘**N**’, alone or indexed, have the similar property.

5. The meta-autonymy of panlogographs has the following fundamental implication. Until any *secondary panlogographic formation rule* (*SPLFR*) *of* A_1 is laid down, the *current range* of every PAnPLR (e.g) is by definition its *initial range*, i.e. the class of PER’s – the ER’s of A_1, which are determined by the *primary* and only by the *primary panlogographic formation rules* (*PPLFR’s*) *of* A_1. After stating the first or any subsequent SPLFR of A_1 in terms of PAnAPL’s of $\mathbf{A}_1$, the *current range* of any earlier occurrence of any PAnPLR and also *the same current range of any earlier occurrence of any alphabetic variant of the* PAnPLR are supposed to be automatically, retroactively and recursively, augmented (updated) with all new SER’s of A_1 introduced by the pertinent SPLFR. For instance, once I lay down the definitions:

$$[\neg\mathbf{Q}]\to[\mathbf{Q}\barwedge\mathbf{Q}], \tag{4.5}$$

$$[\mathbf{Q}\vee\mathbf{R}]\leftrightarrow[\mathbf{Q}\,\overline{\barwedge}\,\mathbf{R}]\to[\neg[\mathbf{Q}\barwedge\mathbf{R}]]\leftrightarrow[[\mathbf{Q}\barwedge\mathbf{R}]\barwedge[\mathbf{Q}\barwedge\mathbf{R}]], \tag{4.6}$$

the range of the tokens of ‘**P**’ and ‘**Q**’ occurring in ‘**P**∨̸**Q**’ is supposed to be augmented (updated) with all ER’s determined by the above definitions. This property of PAnPLR’s and the similar property of PAnPLI’s is called their *retroactivity*.

6. As mentioned in the item **3**, genus-representative use of a PLF can, when desired, be indicated explicitly, for instance thus. Let either of the letters ‘**P**’ and ‘**Q**’ be an MLPH, whose range is the alphabet of AnAPLR’s. Then ‘**P**∨**Q**’ is an MLPH, i.e. a metalinguistic schema, whose range is the set that contains ‘**P**∨**Q**’ and all its alphabetic variants (as ‘**P**∨**R**’¸‘**P**∨**S**’, ‘**O**∨**P**’, etc). That is to say, asserting **P**∨**Q** is tantamount to asserting, e.g., ‘**P**∨**Q**’ in the TAEXA-mode or ‘**P**’∨’**Q**’ in the autonymous mode, subject to the assumption that either of the latter two relations represents the genus of all its alphabetic variants. Since such a genus-representative use of ‘**P**∨**Q**’ or ‘**P**’∨’**Q**’ is understood in accordance with the respective convention, introducing ‘$\mathbf{\mathsf{P}}\vee\mathbf{\mathsf{Q}}$’ turns out to be unnecessary and counterproductive.

7. In accordance with Dfs 3.1(1) and 4.1(1), A_1 and $\mathbf{\mathsf{A}}_1$ include as their constituent parts two respective *self-contained* (capable of being set up independently of setting up A_1 and $\mathbf{\mathsf{A}}_1$) and hence *self-subsistent* (having independent existences), *quantifier-free* organons that are denoted by ‘A_1^0’ and ‘$\mathbf{\mathsf{A}}_1^0$’, while the latter include as their constituent parts two respective self-contained *predicate-free* organons that are denoted by ‘A_0’ and ‘$\mathbf{\mathsf{A}}_0$’. As indicated in Df 3.1(20), the organons A_1^0 and A_0 have the same EBADM (euautographic basic ADM) that is denoted by ‘D_0’, and accordingly $\mathbf{\mathsf{A}}_1^0$ and $\mathbf{\mathsf{A}}_0$ have the same PLBADM (panlogographic basic ADM) that is denoted by ‘$\mathbf{\mathsf{D}}_0$’.

8. Once the organons A_1 and $\mathbf{\mathsf{A}}_1$, i.e. the single whole organon A_1, are set up and learned, they can be executed without mentioning their theory – just as a native language is used in everyday communication without mentioning its grammar. Particularly, all inference and decision procedures of A_1, i.e. all executions of its AADM, D_1, turn out to be almost as simple as computational procedures with natural integers of primary school arithmetic. Especially simple are executions of D_0, the BADM of A_0, As compared to D_0, D_1, contains some additional, more sophisticated rules. However, AEADP’s (advanced euautographic algebraic decision procedures) and APLADP’s (advanced panlogographic algebraic decision procedures), i.e. concrete applications of D_1 and $\mathbf{\mathsf{D}}_1$, are as straightforward and intelligible as BEADP’s (basic euautographic algebraic decision procedures) and BPLADP’s (basic panlogographic algebraic decision procedures), i.e. concrete applications of D_0 and $\mathbf{\mathsf{D}}_0$. The most difficult problems concerning the organons A_1 and $\mathbf{\mathsf{A}}_1$ are setting them up and explicating various *epistemological aspects* of them, including significant (semantic) interpretations of A_1, – the problems, which lie far beyond the scope of the primary school arithmetic. In order to solve these problems and to instruct the reader how to execute D_1 and $\mathbf{\mathsf{D}}_1$, I have set up A_1 and $\mathbf{\mathsf{A}}_1$ within their IML (inclusive metalanguage) that is identical with *the theory of* A_1, i.e. with *this treatise* (see the next subsection for greater detail). The IML is a complicated self-consistent linguistic construction which, in addition to the pasigraphic nomenclature of A_1 and $\mathbf{\mathsf{A}}_1$ and of the other relevant object logistic systems, contains extensive and extremely ramified unconventional self-consistent syntaxic phonographic (wordy, verbal) terminology concerning various aspects both of all object systems and of the IML itself .

9. A relation of A_1 or $\mathbf{A}_1$ may have respectively several EADP's or several PLADP's, which differ in orders of the elementary algebraic operations constituting the EADP's or PLADP's. The different EADP's, or PLADP's, for a given ER, or PLR, result in the same EDT, or PLDT, respectively, and hence in the same decision. However, one of the procedures may turn out to be shorter and simpler than another one. Therefore, in spite of the fact that any EADP or PLADP is mechanical, choice of the optimal EADP or PLADP for a given complex euautographic or panlogographic relation is a kind of art that is acquired by experience – just as in the case of arithmetical calculations with natural numbers

5. The atomic basis of A_1 and the atomic bases of $\mathbf{A}_1$ and A_1

5.1. The atomic basis of A_1

†**Ax 5.1:** ***The axiom of atomic basis of*** A_1. In order to set up A_1 as an *algebraico-logical* organon, the *euautographic atomic basis* (*EAB*) of A_1, denoted by 'B_1', is composed of two parts: the *ordinary* (*non-special*), or *logical*, *EAB* (*OEAB* or *LEAB*), denoted by 'B_{1O}', and the *special* (*unordinary*), or *algebraic*, *EAB* (*SpEAB* or *AlEAB*), denoted by 'B_{1Sp}'. In order to set up A_1 as a *branching tree-like* organon, B_{1O} is composed of two parts: *the mandatory*, or *obligatory*, *ordinary basis*, denoted by 'B_{1OM}', and *the selective ordinary basis*, denoted by 'B_{1OS}'. The union of B_{1OM} and B_{1Sp} is called (denoted phonographically) *the mandatory*, or *obligatory*, *basis of* A_1 and id is denoted [logographically] by 'B_{1M}'. An element of B_1 is called a *basic*, or *primary atomic*, *euautograph* (*BscE* or *PAE*), "*primary*" meaning *postulated* or, concurrently, *undefined*. Consequently, an element of B_{1O} is called a *primary atomic ordinary*, or *logical*, *euautograph* (*PAOE* or *PALE*) and an element of B_{1Sp} is called a *primary atomic special*, or *logical*, *euautograph* (*PASpE* or *PAAlE*). The qualifiers "ordinary" and "logical", or "special" and "algebraic", can be used interchangeably (synonymously) when they apply either to the respective part of B_1 or to a PAE and generally to *any euautographic term*s. When, however, they apply to *combined euautographic relations* (*ER's*), they remain synonyms in some cases and cease to be

synonyms in some other cases. A branch of A_1 is determined by the PAOE's, which are included into its basis besides B_{1M}.

A) The ordinary (logical) basis, B_{1O}

i) The mandatory (obligatory) ordinary (logical) basis, B_{1OM}

1) The square and round brackets: [ ] ( )

2) The comma: ,

3) The *primary universal logical connective* (*connective sign*): $\veebar$

4) The *primary logical sign of contraction* (*binding*): $\exists$

5) The open list of *atomic pseudo-variable ordinary terms* (*APVOT's*):

$$u, v, w, x, y, z, u_1, v_1, w_1, x_1, y_1, z_1, u_2, v_2, w_2, x_2, y_2, z_2, \ldots \qquad (5.1)$$

ii) The selective (optional) ordinary (logical) basis, B_{1OS}

6) The open list of *atomic pseudo-variable ordinary relations* (*APVOR's*):

$$p, q, r, s, p_1, q_1, r_1, s_1, p_2, q_2, r_2, s_2, \ldots, \qquad (5.2)$$

7) The open lists of *atomic pseudo-variable ordinary predicate-signs* (*APVOPS's*), *singulary* ones:

$$f^1, g^1, h^1, f_1^1, g_1^1, h_1^1, f_2^1, g_2^1, h_2^1, \ldots, \qquad (5.3^1)$$

binary ones:

$$f^2, g^2, h^2, f_1^2, g_1^2, h_1^2, f_2^2, g_2^2, h_2^2, \ldots, \qquad (5.3^2)$$

ternary ones:

$$f^3, g^3, h^3, f_1^3, g_1^3, h_1^3, f_2^3, g_2^3, h_2^3, \ldots, \qquad (5.3^3)$$

and so on.

8) Any one and only one of the three *primary atomic pseudo-constant ordinary predicate signs* (briefly, *PBAPCOPS's* or *primary BAPCOPS's* or *primary binary APCOPS's*):

a) =, called the *ordinary equality sign*;

b) $\subseteq$, called the *rightward mass-inclusion predicate-sign*;

c) $\in$, called the *rightward class-membership predicate-sign*.

9) In the presence of $\subseteq$ or $\in$, two *atomic pseudo-constant ordinary terms* (*APCOT's*) $\emptyset$ and $\emptyset'$, the first of which is a *systemic* (*permanent*) one, called the *euautographic ordinary zero-term* (*EOZT*) or *euautographic ordinary pseudo-empty term* (*EOPET*), while the second one, called the

subsidiary EOZT or *EOPET*, is used exclusively for proving that $\emptyset = \emptyset'$, i.e. that $\emptyset$ is unique, and is disregarded after doing this duty.

B) The special (algebraic) basis, B_{1Sp}

10) The *special* (*algebraic*) *kernel-signs*: $\hat{-}$, the *singulary sign of additive inversion*; $\hat{+}$, the *binary sign of addition*; $\hat{\cdot}$, the *binary sign of multiplication* and at the same time the *base transcendental sign of multiplication*, called also (in this hypostasis) the *base sign of multiplicative contraction* (*binding*); $\hat{=}$, the *binary sign of equality*.

11) The *singulary special* (*algebraic*) *kernel-sign*: V, which is called *the validity-sign*, or, when regarded as an abbreviation of $V(\ )$, *the validity-operator*, *of termizing* (*substantivating*, *substantivizing*) *an ER*, because its function is *converting an ER into a computable special term* (*substantive*), which is called *the primary*, or *initial*, *validity-integron of that ER*.

12) The two *primary atomic special* (*algebraic*) *terms*: 0, called the *zero integron* or the *special* (*algebraic*) *zero-term* (*SpZT*), and 1, called the *unity integron* or the *special* (*algebraic*) *unity-term* (*SpUT*). Collectively, the two terms are called the *primary atomic euautographic integrons* (*PAEI's*) or the *digital idempotent euautographic integrons* (*IEI's*) or *digital euautographic validity-integrons* (*DEVI's*). In order to connote certain *dual properties* of 0 and 1 in the decision method D_1, 0 is called the *validity-integron validity* or alternatively the *antivalidity-integron antivalidity*, while 1 is called the *validity-integron antivalidity* or alternatively the *antivalidity-integron validity*. Accordingly, 0 and 1 are collectively called the *digital validity-integrons* (*DVI's*) or alternatively the *digital antivalidity-integrons* (*DAVI's*).

C) Associated definitions

i) Here and generally in what follows, either of the qualifiers "primary" ("P") and "undefined" is often omitted from a *descriptive proper or common name of a specific primary* (*undefined*) *atomic euautograph through the genus denoted by the pertinent generic name and the differentiae* (*differences*) *denoted by the pertinent qualifiers* if no euautograph of the same genus or of the same species that could be qualified as a *secondary* ("*S*") one, or *defined*, *atomic euautograph of* A_1, is intended to be defined in terms of some PAE's in the sequel. A like remark applies to any other qualifier in use as “atomic”, “pseudo-variable”, “pseudo-constant”, “ordinary”, “special”, etc. For instance, the abbreviated names of the following groups are synonyms: “*PAPVOT*’, “*APVOT*”, and “*PVOT*”; “*PAPCOT*’, “*APCOT*”, and “*PCOT*”; “*PAPVOR*”, “*APVOR*”, “*AEOR*”, and “*AER*”; “*PAPVOPS*” and ““*APVOPS*”; and some other to be specified in due course.

ii) A *branch of* A_1 is equivocally denoted by ‘a_1’ and is commonly called an *EAPO*. The *euautographic atomic basis* (*EAB*) of any given branch a_1, equivocally denoted by ‘b_1’, comprises *all* PAE’s of B_{1M} and *strictly some,* i.e. *some but not all*, PAOE’s of B_{1OS} with the following proviso. The complement of B_{1M} in b_1, denoted by ‘b_{1OS}’, *necessarily includes either at least one of the infinite lists* (5.3[1])–(5.3[3]), etc *or at least exactly one of the three PBAPCOPS’s*, indicated in the item 8. In the latter case, b_{1OS} may also include $\emptyset$ and $\emptyset'$ subject to the sub-item ix of the item 6. In all other respects, the choice of b_{1OS} out of B_{1OS} is unrestricted. At the same time, A_1 will be set up as a single whole calculus in such a universal way that no formal partition it into branches will be required.

iii) Any of the atomic euautographs that are indicated in the items 5, 6, 9, and 12 will be called a *primary* s*ubject atomic categorematic*, or *formulary*, *euautograph of* A_1 or a *primary subject atomic euautographic categorem* (pl. “*categoremata*”), or *formula*, *of* A_1, and also, alternatively, a *primary object one of the inclusive metalanguage* (*IML*) *of* A_1, *which belongs to* A_1. The atomic euautographs that are indicated in all other items will be called the *primary subject atomic syncategorematic euautographs of* A_1 or the *primary subject atomic syncategoremata* (singular “*syncategorem*”) *of* A_1, and also, alternatively, the *primary object ones of the inclusive metalanguage* (*IML*) *of* A_1, *which belong to* A_1.

iv) The atomic euautographs indicated in the items 1 and 2 are collectively called the [*subject*] *auxiliary*, or *punctuational*, or *punctuation*, *atomic signs*, or *marks*, *of* A_1. The pair of a token of the square or round bra and of the corresponding token of the respective ket, [ ] or ( ), is called a *paired molecular sign of aggregation*; the comma is called the *atomic sign of separation.* The atomic signs that are explicitly introduced in the items 3, 4, 7, and 8, along with those that are obviously understood in the item 7, and also the sign $\hat{=}$ that is introduced in the item 10 are called the *primary atomic relational kernel-signs of* A_1, whereas the rest of the atomic signs that are introduced in the items 10 and 11 are called the *primary atomic substantival* *kernel-signs of* A_1. The above *primary atomic kernel-signs of* A_1, both *relational* and *substantival*, are alternatively called the *primary main*, or *principal*, *atomic signs of* A_1.

v) The PAE's occurring on either one of the lists (5.1) and (5.2) are called *congeneric ones*, whereas the PAE's occurring on any one of the lists (5.3^1)–(5.3^3) etc are called *conspecific* ones. The order, in which the congeneric or conspecific PAE's are presented on any one of the above lists, is called the *alphabetic order* of those PAE's, whereas the list itself is called the *alphabet* of the PAE's listed.

vi) An APVOT of the list (5.1) or an APCOT indicated in the item 9 is indiscriminately called an *atomic euautographic ordinary term* (*AEOT*), the understanding being that the abbreviated names "*PAEOT*', "*AEOT*", and "*EOT*'' are synonyms, in accordance with the pertinent example in the item i. An APVOPS of any of the lists (5.3^1)–(5.3^3) etc or a PBAPCOPS indicated in the item 8 is indiscriminately called a *primary atomic euautographic ordinary predicate-sign* (*PAEOPS*).

vii) The totality of atomic euautographs indicated in the items 1, 3, 6, and 10–12 is the *atomic basis of* A_0, which will be denoted by 'B_0'.

viii) In the exclusion of ∃, indicated in the item 4, the rest of B_1 is the *atomic basis of* A_1^0, which will be denoted by 'B_1^0'.•

Cmt 5.1. 1) The qualifiers "*special*" ("*unordinary*") and "*ordinary*" ("*non-special*") to an endosemasiopasigraph, i.e. to a euautograph or a panlogograph, in general or to one of a specific class as an endosemasiopasigraphic formula, term, relation, or sign are antonymous technical metaterms (metalinguistic terms) of the treatise, which have the following meanings:

a) "Special" ("unordinary") means «specially designed for setting up D_1, the AADM of A_1, or serving as a tool of D_1, or being a by-side product of D_1, and having therefore no analogues in any CALC and in its metalanguage».

b) "Ordinary" ("non-special") means «having none of the above features», i.e. «not specially designed for setting up D_1, not serving as a tool of D_1, and not being a by-side product of D_1, but being exclusively an object of the pertinent *ADP* (*algebraic decision procedure*) and having therefore an analogue or an interpretand in some CALC or in its metalanguage».

2) When the qualifier “ordinary” applies to a euautograph of A_1, it does not necessarily mean that the euautogaph can be interpreted directly by a certain logograph of a CALC. A euautograph of A_1 can, also, be qualified as an ordinary one if it is used for defining some other, secondary euatographs that have direct interpretands in a CALC. For instance, the universal logical connective $\not\vee$, which will be called the *former*, or *primary, antidisjunction sign*; has no *direct* interpretand in any CALC. Nevertheless, it is qualified as an ordinary one, because I shall use it as the definiens for defining another *twelve secondary elemental euautographic logical connectives* of the following cumulative list:

$$\not\vee,\ \neg,\ \vee,\ \wedge,\ \Rightarrow,\ \Leftarrow,\ \Leftrightarrow,\ \not\wedge,\ \overline{\vee},\ \overline{\wedge},\ \overline{\Rightarrow},\ \overline{\Leftarrow},\ \overline{\Leftrightarrow}. \qquad (5.4)$$

In the exclusion of $\neg$, which is the only *singulary* logical connective, the rest of logical connectives on the list (5.4) are *binary* ones. The secondary connectives will be distinguished by the following proper names (not quoted for the sake of brevity): $\neg$, the *negation*, or *denial*, *sign*; $\vee$, the *inclusive disjunction sign*; $\wedge$, the *conjunction sign*; $\Rightarrow$, the *rightward implication sign*; $\Leftarrow$, the *leftward implication sign*; $\Leftrightarrow$, the *biimplication*, or *equivalence*, *sign*; $\not\wedge$, the *former anticonjunction sign*; $\overline{\vee}$, the *latter antidisjunction sign*; $\overline{\wedge}$, the *latter anticonjunction sign*; $\overline{\Rightarrow}$, the *rightward antiimplication sign*; $\overline{\Leftarrow}$, the *leftward antiimplication sign*; $\overline{\Leftrightarrow}$, the *anti-biimplication*, or *antiequivalence*, or *exclusive disjunction*, *sign*. The occurrence of the word “sign” in any of the above metaterms should be understood as an abbreviation of the compound noun “kernel-sign” as opposed to the name “punctuation sign” or “punctuation mark”. Also, any of the above metaterms has been abbreviated by omission of the prepositive qualifier “*formal*” (as opposed to “*material*”) that should immediately follow the definite article occurring in the metaterm. The first seven binary logical connectives on the list (5.4) are called *positive* ones, whereas the remaining five are called *negative* ones. The former are *atomic*, whereas the latter are *molecular*, because the *overbar of an adjustable length*, $\bar{\ }$, can be regarded as an *overscript synonym of the adscript negation sign* $\neg$. In order to make this fact evident, let ‘**Q**’ and ‘**R**’ be, as before, APLPH’s (atomic panlogographic placeholders), whose range is the class of all ER’s of A_1, and let ‘$\boldsymbol{\eta}$’ be an APLPH, whose range is the set of positive logical connectives, whereas $[\mathbf{Q}\,\overline{\boldsymbol{\eta}}\,\mathbf{R}]\to\neg[\mathbf{Q}\boldsymbol{\eta}\mathbf{R}]$. In this case, effectively, $\vee\to\overline{\not\vee}$ and $\wedge\to\overline{\not\wedge}$, while the five remaining values of ‘$\overline{\boldsymbol{\eta}}$’ are the five negative signs given on the list (5.4). The first eight sign on the list (5.4) are *atomic*, whereas the last five are *molecular*, because an overbar of *an adjustable length*, $\overline{}$, can be regarded as *an overscript synonym of the adscript negation sign* ¬. This fact becomes evident if binary ER's [$\mathbf{Q}\boldsymbol{\eta}\mathbf{R}$] and [$\mathbf{Q}\bar{\boldsymbol{\eta}}\mathbf{R}$] are represented in the *Clairaut-Euler form* in accordance with the definitions $\boldsymbol{\eta}(\mathbf{R},\mathbf{Q})\to[\mathbf{Q}\boldsymbol{\eta}\mathbf{R}]$ and $\bar{\boldsymbol{\eta}}(\mathbf{R},\mathbf{Q})\to[\mathbf{Q}\bar{\boldsymbol{\eta}}\mathbf{R}]$, so that effectively $\bar{\boldsymbol{\eta}}\to\neg\boldsymbol{\eta}$. To be recalled, an atomic or molecular endosemasiopasigraph is indiscriminately called an *elemental*, or *primitive*, one. Up to their faces (figures) and proper names, the secondary ELCv's have counterparts (analogues, interpretands) among the sentential logical connectives of conventional axiomatic logical calculi (CALC'i) – the sentential connectives that are also qualified *conventional* from the standpoint of their *functions* but not necessarily from the standpoint of their appearances.

3) Throughout the treatise, taxonyms (taxonomic names) of species of graphonyms in general (including statements) and of pasigraphs (euautographs and logographs) or endosemasiopasigraphs (euautographs and panlogographs) in particular are usually formed as traditional *descriptiones species per genus et differentias,* i.e. *descriptions of the species through a genus and the differentiae* (*differences*). In this case, a genus is denoted by the appropriate generic noun or noun equivalent, while the differentiae are denoted by the appropriate qualifiers selected out of certain pairs of epistemologically relativistic antonymous adjectives such as: "primary" and "secondary", "atomic" and "combined", "atomic" and "molecular", "elemental" ('primitive", "atomic or molecular") and "complex" ("compound"), "basic" ("elementary") and "advanced", etc. The pertinent restricted meanings of the adjectives are discussed in Appendix 3 (A3).

4) Owing to the *law of double negation,* according to which $\neg\neg\mathbf{P}$ is equivalent to **P** (to be proved in due course by the pertinent EADP or PLADP), the signs ⩝ and $\overline{\vee}$, or ⩜ and $\overline{\wedge}$, are functionally concurrent, so that they can be used interchangeably: In this treatise, I use ⩝ as the only primary (postulated, undefined) atomic euautographic logical connective, while all other logical connectives are secondary ones, which are defined in terms of ⩝. At the same time, I shall show that the signs ⩝ and ⩜ are *dual* in the sense that ⩜ can be used instead of ⩝ as an *effective definiens* of the rest of the corresponding remaining twelve logical

connectives on the list (5.4). I shall also show that, alternatively, the corresponding extra eleven logical connective can be defined either in terms of $\neg$ and $\vee$ or in terms of $\neg$ and $\wedge$, in agreement with the principle of *duality* of $\vee$ and $\wedge$. The sign $\not\wedge$ is a euautographic *parasynonym* (*analogue*) of any of the synonymous logographic signs: '|' of Whitehead and Russell [1925; 1962, pp. xii, xvi], '/' of Hilbert and Ackermann [1950, pp. 11, 29], and '**|**' of Church [1956, p. 37], which are indiscriminately called *Sheffer's stroke* after Sheffer [1913] who suggested that, to use the notation (5.4), the formerly undefined sentential forms '$\neg{,}p$' ("not *p*") and '$p\vee q$' ("*p* or *q*") could be defined as instances of a single indefinable binary sentential form '$p|q$' ("*p* and *q* are incompatible", "*p* if and only if not *q*"), – namely $[\neg{,}p]\to[p|p]$ and $[p\vee q]\to[\neg p|\neg q]\to[p|p]|[q|q]$, – and that hence the same held for the rest of major sentential forms in use, particularly for '$p\wedge q$' ("*p* and *q*"), '$p\Rightarrow q$' ("if *p* then *q*"), and '$p\Leftrightarrow q$' ("*p* if and only *q*"). Consequentially, Nicod [1917] set forth an axiomatic sentential calculus, which was based on a single object axiom, instead of the five *Propositions* **∗1·2–∗1·6** of Whitehead and Russell [1910; 1962, pp. 96, 97]), and on *moduus ponendo ponens* (*MPP*) as a single rule of inference. Both the object axiom, known as *Nicod's postulate*, and the rule of inference were expressed in terms of Sheffer's stroke as the only sentential connective (see also Hilbert and Ackermann [1950, pp. 28, 29]).

5) Either of the two functionally synonymous relations '$\left[p \not\wedge q\right]$' and '$\left[p \barwedge q\right]$' can be rendered into ordinary language either as "*not both p and q*". At the same time, '$\left[p \not\wedge q\right]$' can by definition be rewritten as '$\left[p \Rightarrow \neg q\right]$', which can be read as: "*p so that not q*" if both '$\left[p \Rightarrow \neg q\right]$' and '*p*' are assumed to be *formally veracious* (*f-veracious*), i.e. *accidentally f-true*. According to Simpson [1968], the phrase "*so that not*" is a translation into English of the Latin word "**quōmĭnŭs**" \quominus\, which was particularly employed in this sense by Cicero and Livius. Since the signs $\not\wedge$ and $\Rightarrow\neg$ are by definition synonyms ($\not\wedge \leftrightarrow \Rightarrow\neg$), therefore either of the two can be called the *quominus sign* (*kernel-sign*, *logical connective*).•

Cmt 5.2. 1) Ax 5.1 is a meta-axiom that postulates which PAE's will be employed in A_1 and which ones in A_0. At the same time, Ax 5.1 it is an *ostensive definition* of various proper and common names of those euautographs – names that belong to the *exclusive metalanguage* (*XML*) *of* A_1.

2) It goes without saying that a comma between two atomic euautographs and the ellipsis (omission points) at the end of each of the lists (5.1)–(5.3[3]) etc are punctuation marks of the XML which have nothing to do with the atomic euautographs introduced herewith. In general, a punctuation mark of the XML is an *operator* that denotes the mental attitude of an interpreter of the *scope* (*operand*) of the operator towards the *operata* (*operated objects*) of the operator.•

Cmt 5.3. 1) All euautographs that have been introduced in Ax 5.1 are *atomic* in the sense that they are *functionally indivisible* and at the same time they are *primary* in the sense that they are *postulated* to be atomic and hence *undefined* (*original*) euautographs, i.e. that they are neither defined in terms of nor derived from any euautographs introduced earlier. Still, in accordance with Ax 5.1(i), the qualifier "primary" is omitted from the descriptive names of some of the PAE's and is preserved in the names of the others. A like remark applies to some other qualifiers, e.g. "pseudo-constant" and "pseudo-variable". For instance, the punctuation marks of A_1, which are introduced in items 1 and 2 of Ax 5.1, or those of A_0, which are introduced in the item 1, are entitled to be qualified both as primary and as pseudo-constant. I do not, however, intend to introduce in the sequel any secondary (defined) punctuation marks of A_1 or A_0, – such punctuation marks, e.g., as heavy dots that are employed in many writings on symbolic logic (cf. Whitehead and Russell [1910; 1962, pp. 9–11], Church [1956, pp. 74-76], and Quine [1976, §7]). Nor I intend to introduce any punctuation marks that could be qualified as pseudo-variable ones. Therefore, both qualifiers "primary" and "pseudo-constant" are redundant in this case. Likewise, the APVOPS's have not been qualified explicitly as primary ones because no predicate-signs that could be called "secondary APVOPS's" will be defined in terms of them in the sequel. By contract, exactly one of the atomic euautographs $\in$, $\subseteq$, and = that is *selected* (*activated*) as a *primary* one is explicitly qualified so for the following reasons.

2) In the case, when I selected $\in$ as the *primary* BAPCOPS (*P*BAPCOPS's), I define, in terms of it, another *thirteen predicate-signs* of the following cumulative list:

$$\in, \subseteq, =, \subset, \overline{\in}, \overline{\subseteq}, \overline{=}, \overline{\subset}, \ni, \supseteq, \supset, \overline{\ni}, \overline{\supseteq}, \overline{\supset}, \qquad (5.5)$$

which will be called the *secondary binary primitive* (*elemental) pseudo-constant ordinary predicate-signs* (briefly, *secondary BPvPCOPS's* or *SBPvPCOPS's*) or, along with $\in$, the *BPvPCOPS's* (without the qualifier "secondary"). The six predicate-signs signs $\in$, $\subseteq$, $\subset$, $\overline{\in}$, $\overline{\subseteq}$, $\overline{\subset}$ are called the *rightward*, or *direct*, ones, while $\ni$, $\supseteq$, $\supset$, $\overline{\ni}$, $\overline{\supseteq}$, $\overline{\supset}$ are called the *leftward*, or *converse*, ones, the understanding being that the latter are *subsidiary versions* of the former. The predicate-signs without an overbar are called *positive*, while those with an overbar are called *negative*. As was pointed out in Cmt 5.1(2)**,** the overbar of an adjustable length, $\overline{\ }$, is an overscript synonym of the adscript negation sign $\neg$., so that, e.g., $\overline{\in} \rightarrow \neg\in$, $\overline{\subseteq} \rightarrow \neg\subseteq$, $\equiv \rightarrow \neg=$, etc. Accordingly, like the negative logical connectives, i.e. ones having an overbar, the seven negative predicate-signs are qualified *molecular*, while the seven positive predicate-sign having no overbar are qualified *atomic*. More specifically, the predicate-signs $\subseteq$, $=$, $\subset$, $\ni$, $\supseteq$, $\supset$ are called the *secondary* BAPCOPS's (*S*BAPCOPS's) or, along with $\in$, BAPCOPS's (without the qualifier "secondary"), whereas the predicate-signs $\overline{\in}$, $\overline{\subseteq}$, $\equiv$, $\overline{\subset}$, $\overline{\ni}$, $\overline{\supseteq}$, $\overline{\supset}$ are called *binary molecular pseudo-constant ordinary predicate-signs* (*BMPCOPS's*). At the same time, the APVOPS's on the lists (5.3^1)–(5.3^3), etc and the predicate-signs on the list (5.5) will collectively be called the *primitive (elemental) euautographic ordinary predicate-signs* (*PvEOPS's*) *of* A_1.

3) Instead of $\in$, I may select $\subseteq$ as the *primary* BAPCOPS (*P*BAPCOPS). In this case, $\in$, $\overline{\in}$, $\ni$, and $\overline{\ni}$ are disregarded, $\subseteq$ that was previously qualified secondary becomes primary, while the remaining nine secondary BPvPCOPS's are now defined in terms of $\subseteq$ and therefore they preserve their status of being *secondary* (*defined*) ones. Lastly, I may select = as the *primary* BAPCOPS. In this case, the signs $\subseteq$, $\subset$, $\overline{\subseteq}$, $\overline{\subset}$, $\supseteq$, $\supset$, $\overline{\supseteq}$, $\overline{\supset}$ are disregarded, the sign = that was previously qualified secondary becomes primary, while the only remaining secondary BPvPCOPS $\equiv$ preserves its status of being *secondary* (*defined*) because it is defined in terms of =.

4) Instead of qualifying the predicate-signs on the lists (5.3^1)–(5.3^3) etc as *pseudo-variable* ones, I might have qualified them as *typical* or as *undistinguished*. Accordingly, instead of qualifying the predicate-signs on the list (5.5) as *pseudo-constant* ones, I might have qualified them as *typical* or *distinguished*. However, from mnemonic considerations, I have decided to use the unified qualifiers "pseudo-variable" and "pseudo-constant", conveniently abbreviated as "PV" and "PC", in all appropriate cases.•

Cmt 5.4. 1) All atomic euautographs introduced in Ax 5.1 are *archetypes* of their *homolographic* tokens which will be used throughout the treatise. Particularly, all tokens of the special atomic terms 0 and 1 will be set in this light-faced narrow Roman (upward) Gothic (sans serif) type, called the Light-Faced Roman Arial Narrow Type, whereas all tokens of the sign *V* and of the ordinary atomic euautographs of the list (5.1)–(5.3^3) etc will be set in the *light-faced narrow italic* (*slant upward to the right*) *Gothic* (*sans serif*) *type* called *the Light-Faced Italic Arial Narrow Type*. It is understood that homolographic tokens of the archetypal euautographs, which occur in a subscript or superscript line, will have *proportionally* smaller sizes. In addition, tokens of the archetypal brackets may have various sizes and thicknesses because brackets enclosing some other brackets are conventionally larger and thicker.

2) The Light-Faced Roman and Italic Arial Narrow Types, in which all alphanumeric primary special atomic euautographs are set, serve for distinguishing these graphonyms from similar graphonyms that do not belong to A_1, although some headings belonging to the XML of the logistic systems of the treatise may also be set in these types. At the same time, the object square and round brackets and the object comma of the logistic systems are set in the same type as the analogous punctuation marks of the XML. This ambiguity is immediately resolved by the context or by the immediate symbolic surrounding, in which the equivocal punctuation marks occur.

3) In the sequel, I shall, as a rule, customarily use the expressions such as “square brackets” instead of “a homolographic token of the square brackets”, “a comma” instead of “a homolographic token of the comma”, “an atomic euautograph” instead of “a homolographic token token of the atomic euautograph”, etc.•

Cmt 5.5. A caret in any of the special atomic euautographs introduced in Ax 5.1(10) is designed to distinguish that euautograph from a similar conventional graphonym without a caret, which can informally be used in the metalanguage, and from a similar graphonym with some other label or without any label, which may be introduced in the sequel either in A_1 or in some of its extrinsic *interpretands*. In mathematics, the ordinary multiplication sign, ·, is often omitted, whereas the long minus sign, –, to be called the En-Minus-Sign or ‘n’-Minus-Sign, is equivocally used both as the singulary sign of additive inversion and as the binary sign of subtraction. By contrast, in this treatise, the special careted multiplication sign, $\hat{\cdot}$, will never be omitted, whereas the special careted short minus-sign, $\hat{-}$, to be called the Careted

Half-En-Minus Sign or Careted ½‘n’-Minus-Sign, will be used exclusively as the singulary sign of additive inversion. As the binary sign of subtraction, I shall employ the special long minus-sign, $\hat{-}$, which is defined as an abbreviation of the assemblage $\hat{+}\,\hat{\text{-}}$, and which will be called the Careted En-Minus-Sign or Careted ‘n’-Minus-Sign. •

Cmt 5.6. The prepositive distributive quantifiers “*singulary*”, “*binary*”, “*ternary*”, etc, or generally “*n-ary*” – of Latin origin”, and also “*monadic*”, “*dyadic*”, “*triadic*”, etc, or generally “*n-adic*” – of Greek origin, which occur in the descriptive names involving any of the nouns “*predicate*”, “*predicate-sign*”, and “*kernel-sign*” as the headword (generic name) and which mean *one-placed*, *two-placed*, *three-placed*, etc, or generally “*n-placed*”, can be used interchangeably with the postpositive distributive quantifiers “*of weight* 1”, “*of weight* 2”, “*of weight* 3”, etc, or generally “*of weight n*”, respectively, whereas either of the prepositive collective quantifiers “*multiary*” and “*polyadic*” has the same sense as the postpositive collective quantifier “*of weight equal or greater than* 2”.•

Cmt 5.7. 1) The digital subscript or superscript occurring in an atomic euautograph of the lists (5.1)–(5.3^3) etc. is an *inseparable part* of the euautograph. In this case, the qualifier “atomic” is descriptive of this very inseparability property. At the same time, by force of habit, the Arabic numerals are unavoidably associated with the natural numbers which they denote. Particularly, the former are ordered by the latter. The list of a few first sequential Arabic numerals followed by an ellipsis in the form of omission points, viz. 0, 1, 2, ..., will be called the *natural infinite sequence*, or *natural infinite ordered multiple*, or *alphabet*, *of the Arabic numerals*, while the order which is indicated by the alphabet will be called the *natural,* or *alphabetic*, *order of the numerals*. As a consequence, the atomic euautographs on each of the lists (5.1)–(5.3^3) etc. are ordered by their numeral subscripts, while the entire lists (5.1)–(5.3^3) etc. in the given order are ordered by the superscript on any of the atomic euautographs occurring on a list. Like remarks apply, *mutatis mutandis*, to the infinite *alphabets of atomic placeholders*, belonging either to $\mathbf{A}_1$ or to the XML, which will be introduced in the sequel. It is understood that the property of an Arabic numeral index to be an inseparable distinguishing visual attribute of an object atomic euautograph or of an atomic placeholder, in which it occurs, does not contradict the property of the index to be associated with the natural number which it habitually denotes. Still, regarding this association, the following remark should be made.

2) According to the *SQM* (*special quotation method*) of naming tychautographs, i.e. of xenographs that are used autonymously, a numeral or its homolographic (photographic) token is, when appropriate, mentioned by using its homolographix (photographic) quotation, while the number that is denoted by the numeral is mentioned by using an unquoted token of the numeral. At the same time, all numeral indices of the indexed atomic euautographs are used for mentioning themselves and not for mentioning the natural numbers which they denote. Consequently, in the framework of the SQM, those indices should have been replaced by their homolographic quotations. For instance, one should have written $u_{‘1’}$, $u_{‘2’}$, etc., instead of u_1, u_2, etc. Needless to say that such a modified system of notation is unacceptable. Therefore, one should either replace the convenient infinite alphabets of atomic euautographs, which are ordered by unquoted numeral indices, with some other alphabets, in which no numeral labels are employed or, alternatively, one should not use the SQM thus admitting that the latter is inappropriate in this case. I have already mentioned previously that the SQM is an *ad hoc*, i.e. *epistemologically relativistic, device* and that it cannot therefore be used systematically in principle. In many cases, use of autonymous (photographic or pictographic) quotations or quasi-quotations may obscure fundamentals. For instance, occurrences of such quotations in a sentence may obscure the logical form of the sense of the sentence, which repeats the form of the sentence. The symbols $u_{‘1’}$, $u_{‘2’}$, etc. serve as another example where use of photographic quotations is counterproductive.•

Cmt 5.8. It is clear that only a small part of the atomic euautographs which either occur on the lists (5.1)–(5.3^3) etc or are obviously understood by the ellipsis at the end of any of the lists will be used in the sequel. However, I introduce the infinite alphabets of atomic euautographs, – or, in fact, I just leave each of the lists open, – in order to have any of the euautographs available if needed. Moreover, it is convenient to leave the following option open. Any atomic euautograph of any of the lists (5.1)–(5.3^3) etc can, if desired, be furnished with any number of primes thus becoming another atomic euautograph of A_1 of the same class. Once such primed euautographs are used, they are supposed to be ordered as follows. For instance, any of the euautographs u, u', u'', etc. (thus ordered) precedes v; any of the euautographs v, v', v'', etc. precedes w; etc.; any of the euautographs u_1, u_1', u_1'', etc. precedes v_1; etc. The imaginary infinite list of unprimed and primed atomic relations thus ordered is called *the hyper-alphabet of APVOT's*, whereas the order itself is called the *hyper-alphabetic order of APVOT's*. Like remarks apply to the infinite alphabets (5.1)–(5.3[3]) etc and to the *infinite alphabets of the placeholders* belonging to $\mathbf{A}_1$ or to the XML, which will be introduced below in this section and in the sequel.•

Cmt 5.9. The formation rules of A_1 and its rules of inference and decision, D_1, are stated with allowance for all elements of its atomic basis, which have been declared in Ax 5.1. I fact, however, an EAPO, whose atomic basis involves any one of the three APCOPS's: =, ⊆., and ∈ as a primary atomic one differs both from the organon involving another one of the three signs and from an EAPO that does not involve none of them. Likewise, an EAPO with APCOT's $\emptyset$ and $\emptyset'$ differs from an EAPO without them. Fortunately, it has turned out to be possible to incorporate all those different EAPO's into a single one EAPO, A_1, which has a universal AADM, D_1, as branches or phases of A_1. At the same time, adding some new elements to the selective ordinary basis, B_{1OS}, which are supposed to have properties distinct from the properties of the elements of B_{1OS}, will, unavoidably, change the above rules and therefore the new elements will be harmful for D_1 if the later turns out not to be adjustable to them. Any extension of A_1, which results in altering D_1, is another EAPO that requires a different treatment and a different interpretation. Therefore, I have avoided including into the statement of B_{1OS} any items to allow introducing some additional PAE's without specifying them in advance after the manner of stating the atomic basis of the calculus F^1 in Church [1956, p. 169]. Some options of extending of A_1 by introducing additional APCOT's, to be indiscriminately called *primary atomic pseudo-constant extraordinary terms* (*PAPCXOT's*), will be discussed in due course, but no attempt will be made to develop any thus extended full-scale EAPO in this treatise. •

5.2. The primary structural atomic panlogographic basis of $\mathbf{A}_1$ and A_1

†**Df 5.1:** ***The atomic bases of*** $\mathbf{A}_1$ ***and*** A_1**.** 1) The elements of B_1 are employed, not only in A_1, but also in $\mathbf{A}_1$. In addition to B_1, $\mathbf{A}_1$ and hence $f\,\mathsf{A}_1$ involve two paired *undefined* (*euautographic*) *auxiliary* (*punctuation*) *marks* $\langle\ \rangle$ and $|\ \rangle$, the totality (set) of which is denoted by '$\Delta\mathsf{B}_1$', and they also involve certain *atomic panlogographs*

(*APL's*) *of* $\mathbf{A}_1$, the set of which is denoted by '$\mathbf{B}_1$', and which are called *elements*, or *members*, *of* $\mathbf{B}_1$. The union of B_1 and $\Delta\mathsf{B}_1$ is logographically denoted by 'B_{1+}' and is called the *euautographic atomic basis* (*EAB*) *of* $\mathbf{A}_1$, or *of* A_1. The set $\mathbf{B}_1$ is called the *panlogographic atomic basis* (*PLAB*) *of* $\mathbf{A}_1$, or *of* A_1.

2) Unlike B_1, $\Delta\mathsf{B}_1$, or B_{1+}, all elements (atomic members) of which are introduced *axiomatically* and are therefore *undefined* in this sense, *all elements of* $\mathbf{B}_1$ are *defined* by the appropriate *semantic definitions* in terms of certain atomic or combined euautographs of A_1. Therefore, any element of $\mathbf{B}_1$ can intelligibly be introduced only after introducing some (strictly some or all) euasutographs that constitute its initial or permanent range. Consequently, in contrast to B_1, $\Delta\mathsf{B}_1$, or B_{1+}, the whole of $\mathbf{B}_1$ cannot be specified compactly in advance under a single head, various semantic definitions or groups of semantic definitions defining the respective elements of $\mathbf{B}_1$ are made singly or in groups in the appropriate places of the treatise as needed.

3) *A posteriori*, $\mathbf{B}_1$ is conveniently divided into two subsets (parts) in three ways:

a) the *primary PLAB* (PPLAB), denoted by '$\mathbf{B}_{1\mathrm{P}}$', which comprises *primary atomic panlogographs* (*PAPL's*) *of* $\mathbf{A}_1$, and the *secondary PLAB* (*SPLAB*), denoted by '$\mathbf{B}_{1\mathrm{S}}$', which comprises *secondary atomic panlogographs* (*PAPL's*) *of* $\mathbf{A}_1$;

b) the *structural StPLAB* (*SPLAB*), denoted by '$\mathbf{B}_{1\mathrm{St}}$', which comprises *structural atomic panlogographs* (*StAPL's*), *of* $\mathbf{A}_1$, and the *analytical PLAB* (*AnPLAB*), denoted by '$\mathbf{B}_{1\mathrm{An}}$, which comprises *analytical atomic panlogographs* (*AnAPL's*) *of* $\mathbf{A}_1$;

c) the *categorematic*, or *formulary*, *PLAB* (*CtgPLAB* or *FPLAB*), denoted by '$\mathbf{B}_{1\mathrm{Ctg}}$' or '$\mathbf{B}_{1\mathrm{F}}$', which comprises *categorematic*, or *formulary*, *atomic panlogographs* (*CtgAPL's* or *FAPL's*), i.e. *atomic panlogographic formulas* (*APLF's*), *of* $\mathbf{A}_1$, and the *syncategorematic PLAB* (*SctgPLAB*), denoted by '$\mathbf{B}_{1\mathrm{Sctg}}$', which comprises *syncategorematic atomic panlogographs* (*SctgAPL's*) *of* $\mathbf{A}_1$.

Furcated taxonomy of elements of $\mathbf{B}_1$ can formally be obtained by combining any two or all the three dichotomies a)–c) in accordance with the following syntactico-semantic rules. The *logograph* that is obtained by attributing the juxtaposition of some two or three qualifying subscripts (in any order), which are selected by one from certain two or all the three pairs of antonymous: subscripts: '$_{\mathrm{P}}$' and '$_{\mathrm{S}}$', '$_{\mathrm{St}}$' and '$_{\mathrm{An}}$', and '$_{\mathrm{Ctg}}$', or '$_{\mathrm{F}}$', and '$_{\mathrm{Sctg}}$', to '$\mathbf{B}_1$' denotes the same subset of $\mathbf{B}_1$ as that denoted by the abbreviated verbal (phonographic) taxonym obtained by adhering the juxtaposition of the respective two or three qualifiers (in any order) that are selected by one from the respective two or from all the three pairs of antonyms: "primary" ("P") and "secondary" ("S"), "structural" ("St") and "analytical" ("An"), and "categorematic" ("Sctg"), or "formulary" ("F"), and "syncategorematic" ("Sctg"), as defined by the items a)–c), to "PLAB" with the proviso that some specific logographs or phonographs involving different qualifiers may denote the same set and that some specific logographs or phonographs thus formed are *empty* in the sense .that they denote the empty set.

4) For instance, syntactically, i.e. from the standpoint of syntactic analysis, $\mathbf{B}_1$ can be divided into four subsets in the following three ways:

a) the *structural PPLAB* (*StPPLAB*), the *analytical PPLAB* (*AnPPLAB*), the *structural SPLAB* (*StSPLAB*), and the *analytical SPLAB* (*AnSPLAB*), denoted by '$\mathbf{B}_{1\mathrm{PSt}}$', '$\mathbf{B}_{1\mathrm{PAn}}$','$\mathbf{B}_{1\mathrm{SSt}}$', and '$\mathbf{B}_{1\mathrm{SAn}}$' respectively

– in accordance with the dichotomies 3a and 3b;

b) the *categorematic*, or *formulary*, *PPLAB* (*CtgPPLAB* or *FPPLAB*), the *syncategorematic PPLAB* (*SctgPPLAB*), the *categorematic*, or *formulary*, *SPLAB* (*CtgSPLAB* or *FSPLAB*), the *syncategorematic SPLAB* (*SctgSPLAB*), denoted by '$\mathbf{B}_{1\mathrm{CtgP}}$' or '$\mathbf{B}_{1\mathrm{FP}}$', '$\mathbf{B}_{1\mathrm{SctgP}}$', '$\mathbf{B}_{1\mathrm{CtgS}}$ or '$\mathbf{B}_{1\mathrm{FS}}$, and '$\mathbf{B}_{1\mathrm{SctgS}}$ respectively

– in accordance with the dichotomies 3a and 3c;

c) the *structural categorematic*, or *formulary*, *PLAB* (*StCtgPLAB* or *StFPLAB*), the *structural syncategorematic PLAB* (*StSctgPLAB*), the *analytical categorematic*, or *formulary*, *PLAB* (*AnCtgPLAB* or *AnFPLAB*), and the *analytical syncategorematic PLAB* (*AnSctgPLAB*), denoted by '$\mathbf{B}_{1\mathrm{StCtg}}$' or '$\mathbf{B}_{1\mathrm{StF}}$', '$\mathbf{B}_{1\mathrm{StSctg}}$', '$\mathbf{B}_{1\mathrm{AnCtg}}$' or '$\mathbf{B}_{1\mathrm{AnF}}$', and '$\mathbf{B}_{1\mathrm{AnSctg}}$' respectively

– in accordance with the dichotomies 3b and 3c. In this case, it follows 3a–3c by the above items a)–c) that

α) $\mathbf{B}_{1\mathrm{P}}=\mathbf{B}_{1\mathrm{PSt}}\cup\mathbf{B}_{1\mathrm{PAn}}=\mathbf{B}_{1\mathrm{PF}}\cup\mathbf{B}_{1\mathrm{PSctg}}$, $\mathbf{B}_{1\mathrm{S}}=\mathbf{B}_{1\mathrm{SSt}}\cup\mathbf{B}_{1\mathrm{SAn}}=\mathbf{B}_{1\mathrm{SF}}\cup\mathbf{B}_{1\mathrm{SSctg}}$;

β) $\mathbf{B}_{1St}=\mathbf{B}_{1StP}\cup\mathbf{B}_{1StS}=\mathbf{B}_{1StF}\cup B_{1StSctg}$, $\mathbf{B}_{1An}=\mathbf{B}_{1AnP}\cup\mathbf{B}_{1AnS}=\mathbf{B}_{1AnF}\cup\mathbf{B}_{1AnSctg}$;

γ) $\mathbf{B}_{1F}=\mathbf{B}_{1FP}\cup\mathbf{B}_{1FS}=\mathbf{B}_{1FSt}\cup\mathbf{B}_{1FAn}$, $\mathbf{B}_{1Sctg}=\mathbf{B}_{1SctgP}\cup\mathbf{B}_{1SctgS}=\mathbf{B}_{1SctgSt}\cup\mathbf{B}_{1SctgAn}$.

Syntactically again, combination of any one of the three tetrachotomies a)–c) and the respective one of the dichotomies 3c, 3b, and 3a (in that order) results in the division of $\mathbf{B}_1$ into eight subsets. However, semantically, i.e. from the standpoint of semantic analysis, by the pertinent subsequent definition (Df II.1.1), $\mathbf{B}_{1PAn}$ contains only formulary (categorematic) elements, so that it equals $\mathbf{B}_{1PAnF}$, whereas $\mathbf{B}_{1PAnSctg}$ is empty, i.e. logographically

δ) $\mathbf{B}_{1PAn}=\mathbf{B}_{1PAnF}=\mathbf{B}_{1PAnCtg}$, $\mathbf{B}_{1PAnSctg}=\varnothing$.

5) The bases B_1 and $\mathbf{B}_{1P}$ are needed from the very beginning in laying down the formation rules of A_1 and $\mathbf{A}_1$. The basis $\mathbf{B}_{1S}$, along with B_1 and $\mathbf{B}_{1P}$, is needed in laying down and executing D_1 and $\mathbf{D}_1$, i.e. the rules of inference and decision of A_1 and $\mathbf{A}_1$. $\mathbf{B}_{1PSt}$ is specified in the next *semantic definition*. $\mathbf{B}_{1PAn}$ will be outlined in the subsection 5.5 and it will be specified in full in the subsection II.1.1 by the pertinent *semantic definition* that is made immediately after stating Ax II.1.1 – the meta-axiom called the *system of restricted primary formation rule*s (*RPFR-system*) of A_1, and before laying down any formation rules of A_1 and $\mathbf{A}_1$, in which some elements of $\mathbf{B}_{1PAn}$ are utilized. Various elements of $\mathbf{B}_{1S}$ are specified as needed in the course of laying down and executing D_1 and $\mathbf{D}_1$.

6) An element of $\mathbf{B}_1$ is an *APL* (*atomic panlogograph*) *of* $\mathbf{A}_1$, whose range is a certain *taxonomic class* (or, particularly, *taxonomic set*), i.e. a certain *taxon* (pl. "*taxons*" or "*taxa*"), of categorematic (formulary) or syncategorematic euautographs of A_1. Therefore, an element of $\mathbf{B}_1$ is called an *APL*, or a *connotative atomic panlogographic taxonym* (*CntAPLTxm*), of $\mathbf{A}_1$. Consequently, a *semantic definition* of one or more elements of $\mathbf{B}_1$ is called a *panlogographic generalization rule* (*PLGR*) [*of euautographs*] *of* A_1 – in contrast to both a *panlogographic specification*, or *sortation*, *rule* (*PLSR*, *PLSpcR*, or *PLSrtR*) [*of euautographs*] *of* A_1 and a *panlogographic formation rule* (*PLFR*) [*of euautographs*] *of* A_1, which will be defined formally and illustrated in due course. Meanwhile, it is noteworthy that I understand the generic taxonyms "*panlogographic generalization rule*" ("*PLGR*"), "*panlogographic specification rule*" or "*panlogographic sortation rule*" ("*PLSR*"), and "*panlogographic formation rule*" ("*PLFR*") as synonyms of "*rule of panlogographic generalization*", "*rule of panlogographic specification*" or "*rule of panlogographic

sortation”, and “*rule of panlogographic formation*”, subject to the following informal (descriptive) definitions.

a) The taxonym “*panlogographic generalization*” (“*PLG*”) means the act or process of classifying of euautographs by *assigning* to every one of the euautographs, having the pertinent classifying property, a *common name* in the form of an appropriate *APL* , which is to serve as a *CntAPLTxm* of the taxon (taxonomic class) of all those euautographs; assigning the CntAPLTxm to the euautographs is done by way of *abstract association*, i.e. in abstraction from both their specific forms (if they are combined, i.e. not atomic) and their specific compositions. That is to say, “panlogographic generalization” means *panlogographic abstract generalization* in the sense that the CntAPLTxm, being a result of the generalization, is an *ideograph* (*graphic symbol* in the semiotic terminology), in the exclusion of the case where all members of the taxon, being the range of the CntAPLTxm, are *atomic* euautographs, so that the CntAPLTxm is an *ideoiconograph*, i.e. an *ideograph* and an *iconograph* (*graphic icon* in the semiotic terminology) *simultaneously*. An CntAPLTxm of the latter kind is evidently a *StAPL* (*structural atomic panlogograph*).

b) The taxonym “*panlogographic specification*” means the act or process of *qualifying* the euautographs of a *given genus* (*generic taxon*), which have a certain general *compositional peculiarity* (*peculiar compositional property*) prescinded from and hence independent of their forms, by assigning to each of them a *common name* in the form of a *panlogographic description* (*descriptive name*) *of the species of euautographs through the given genus, designated by a certain generic APLTxm, and through the differentia* (*difference*) or *differentiae* (*differences*), *designated by a certain combined panlogograph* (*CbPL*) or, correspondingly, *certain CbPL’s as the pertinent panlogographic qualifier* (*epithet*) or *qualifiers.* The *species* of euautographs that is designated by this panlogographic description is alternatively called a *sort* of euautographs. Accordingly, the taxonym “*panlogographic sortation*” is used synonymously (interchangeably) with the taxonym “panlogographic specification”. I regard a panlogographic description of the sort of euautographs of A_1 as a panlogographic instance of the traditional *descriptio species per genus et differentiam* (or *differentias* in the plural), which is, in turn, the *panlogographic definiendum* of a certain semi-verbal semantic traditional (Aristotelian) *definitio [species] per genus et differentiam* (or *differentias*), whose *definiens* is a semi-verbal *descriptio [species]* *per genus et differentiam* (or *differentias*). At the same time, such a panlogographic description is by definition called a *[analytical] molecular panlogographic description* (*AnMPLD* or *MPLD*), or *connotative molecular panlogographic taxonym* (*CntDPLTxm*), *of* $\mathbf{A}_1$. The set of all AnMPLD's of $\mathbf{A}_1$ in progress is denoted by '$\mathbf{B}_1'$' and is called the *panlogographic descriptive molecular basis* (*PLDMB*) *of* $\mathbf{A}_1$ – as contrasted to the PLAB of $\mathbf{A}_1$ (see the atem 1), the *euautographic molecular basis* (*EMB*) *of* A_1 (denoted by '$\overline{\mathrm{B}}_1$'), and the *panlogographic schematic molecular basis* (*PLSchMB*) *of* $\mathbf{A}_1$ (denoted by '$\overline{\mathbf{B}}_1$'). The basis $\mathbf{B}_1'$ is not formally partitioned in advance in analogy with $\mathbf{B}_1$, because only few parts of $\mathbf{B}_1'$ are nonempty. Therefore, I shall distinguish any conspicuous nonempty part of $\mathbf{B}_1'$ by the symbol '$\mathbf{B}_1'$' together with the appropriate additional subscripts only if I actually define its elements to be used.

c) I understood the noun "*formation*" as one that means the act or process of *giving a form and classifying in accordance with that form*. Therefore, the taxonym "*panlogographic formation*" means the act or process of *giving a panlogographic form to euautographs and classifying them in accordance with that form*.

7) In accordance with the above definitions, any of the taxonyms "*panlogographic generalization*" ("*PLG*"), "*panlogographic specification*" or "*panlogographic sortation*" ("*PLS*"), and "*panlogographic formation*" ("*PLF*") is a *hypotaxonym* of *hypertaxonym* "*panlogographic classification* ("*PLC*"); i.e. the taxon (taxonomic class), designated by any of the former taxonyms (hypotaxonyms), is a *hypotaxon* (*taxonomic subclass*) of the *hypertaxon* (*taxonomic superclass*), designated by the latter taxonym (hypertaxonym). Hence, *a PLGR, PLSR, or PLFR* [*of euautographs*] *of* A_1 *is a PLCR* [*of euautographs*] *of* A_1. The nouns "classification" and "sortation" are used in English as close synonyms. However, in order to distinguish terminologically among the above three kinds of PLCR's, I assign two distinct meanings to occurrences of the nouns "classification" and "sortation" in the pertinent taxonyms of the rules.

8) For the obvious reasons, a *PLGR of* A_1 is alternatively called an *atomic PLFR* (*APLFR*) *of* $\mathbf{A}_1$ and vice versa, whereas a *PLSR of* A_1 is alternatively called a *descriptive molecular PLFR* (*DMPLFR*), or *molecular PLFR-description* (*MPLFRD*),

of $\mathbf{A}_1$ and vice versa. In accordance with the items 3a–3c, a PLGR of A_1, i.e. an APLFR of $\mathbf{A}_1$, is called:

a) a *primary PLGR* (*PPLGR*) *of* A_1 or a *primary APLFR* (*PAPLFR* or *APPLFR*) *of* $\mathbf{A}_1$ if it defines one or more elements of $\mathbf{B}_{1P}$ and a *secondary PLGR* (*SPLGR*) *of* A_1 or a *secondary APLFR* (*SAPLFR* or *ASPLFR*) *of* $\mathbf{A}_1$ if it defines one or more elements of $\mathbf{B}_{1S}$;

b) a *structural PLGR* (*StPLGR*) *of* A_1 or a *structural APLFR* (*StAPLFR*) *of* $\mathbf{A}_1$ if it defines one or more elements of $\mathbf{B}_{1St}$ and an *analytical PLGR* (*AnPLGR*) *of* A_1 or an *analytical APLFR* (*AnAPLFR*) *of* $\mathbf{A}_1$ if it defines one or more elements of $\mathbf{B}_{1An}$.

c) a *categorematic*, or *formulary*, *PLGR* (*CtgPLGR* or *FPLGR* or *PLFGR*) *of* A_1 or a *categorematic*, or *formulary*, *APLFR* (*CtgAPLFR* or *FAPLFR* or *APLFFR*) *of* $\mathbf{A}_1$ if it defines one or more elements of $\mathbf{B}_{1Ctg}$, or $\mathbf{B}_{1F}$, and a *syncategorematic PLGR* (*SctgPLGR*) *of* A_1 or a *syncategorematic APLFR* (*SctgAPLFR*) *of* $\mathbf{A}_1$ if it defines one or more elements of $\mathbf{B}_{1Sctg}$.

Consequently, in analogy with pertinent formal rule the item 3 and with the same proviso, the standard abbreviated specific verbal taxonym of the PLGR of A_1, i.e. of the APLFR of $\mathbf{A}_1$, that defines a certain part of $\mathbf{B}_1$, which is denoted by '$\mathbf{B}_1$' along with one, two, or three pertinent qualifying subscripts on it, is formed by attaching the respective full qualifiers or their abbreviations to either one of the abbreviated synonymous generic verbal taxonyms "PLGR of A_1" and "APLFR of $\mathbf{A}_1$". Particularly, in accordance with the above rule, the following definition, which explicitly defines $\mathbf{B}_{1PSt}$, is called the *primary structural PLFR* (*PStPLFR*, *StPPLFR*, *StPLPFR*) [*of PStAPL's*] *of* $\mathbf{A}_1$ or the *primary structural PLGR* (*PStPLGR*, *StPPLGR*, *StPLPGR*) [*of PAE's*] *of* A_1, so that it is simultaneously a PLFR of PStAPL's of $\mathbf{A}_1$ and a PLGR of PAE's of A_1.•

†**Df 5.2:** ***The primary structural atomic panlogographic basis of*** $\mathbf{A}_1$. In accordance with Df 4.2(4a), every element of $\mathbf{B}_{1PSt}$ is called a *primary structural panlexigraph* (*PStPLxg) of* $\mathbf{A}_1$ and also synonymously (interchangeably) by a variant of the above name with "*atomic panlogograph*" ("*APL*"), "*atomic panlogographic placeholder*" ("*APLPH*"), or "*atomic panlogographic schema*" ("*APLS*", pl. "*APLS'ta*") in place of "*panlexigraph*" ("*PLxg*"). Each of these names is descriptive of the fact that every panlogograph thus called is defined in terms of certain elements

of B_1, which are condensed in its range. The different species of PStAPL's (PStPLxg's, PStAPLPH's, PStAPLS'ta) of $\mathbf{A}_1$ are given below along with their taxonyms (taxonomic names, specific names), which are abbreviated by omission of the postpositive qualifier "of $\mathbf{A}_1$".

1) Each of the graphonyms:

$$\text{'}\mathbf{u}\text{', '}\mathbf{v}\text{', '}\mathbf{w}\text{', '}\mathbf{x}\text{', '}\mathbf{y}\text{', '}\mathbf{z}\text{', '}\mathbf{u}_1\text{', '}\mathbf{v}_1\text{', '}\mathbf{w}_1\text{', '}\mathbf{x}_1\text{', '}\mathbf{y}_1\text{', '}\mathbf{z}_1\text{', '}\mathbf{u}_2\text{', '}\mathbf{v}_2\text{', } \ldots \quad (5.6)$$

is a PStAPL, whose range, to be denoted by 'τ', is the set of all pertinent AEOT's, which necessarily includes the set of all APVOT's on the list (5.1), to be denoted by 'τ^{pv}', and which optionally includes the set $\{\emptyset, \emptyset'\}$ of two APCOT's indicated in Ax 5.1(9), to be alternatively denoted also by 'τ^{pc}', i.e. $\tau^{\text{pv}} \to \{\emptyset, \emptyset'\}$. Once it is proved that $\emptyset = \emptyset'$, τ^{pc} turns into the singleton $\{\emptyset\}$, i.e. $\tau^{\text{pv}} = \{\emptyset\}$. The above PStAPL's are called the *primary structural atomic panlogographic ordinary terms* (*PStAPLOT's*) or just the *panlogographic ordinary terms* (*PLOT's*), because there are no PLOT's that can be qualified "secondary" ("S"), "analytical" ("An"), or "combined" ("”Cb").

2) Each of the graphonyms:

$$\text{'}\mathbf{p}\text{', '}\mathbf{q}\text{', '}\mathbf{r}\text{', '}\mathbf{s}\text{', '}\mathbf{p}_1\text{', '}\mathbf{q}_1\text{', '}\mathbf{r}_1\text{', '}\mathbf{s}_1\text{', '}\mathbf{p}_2\text{', '}\mathbf{q}_2\text{', '}\mathbf{r}_2\text{', '}\mathbf{s}_2\text{', } \ldots \quad (5.7)$$

is a PStAPL, whose range, to be denoted by 'σ' (the first letter of the Greek noun "σχέσις" \sçésis, shésis\ meaning *a relation*), is the set of all APVOR's on the list (5.2). These PStAPL's are called the *primary structural atomic panlogographic ordinary relations* (*PStAPLOR's*) or just the *structural atomic panlogographic ordinary relations* (*StAPLOR's*), because there are *no secondary* ones.

3) Each graphonym on each one of the following lists:

$$\text{'}\mathbf{f}^1\text{', '}\mathbf{g}^1\text{', '}\mathbf{h}^1\text{', '}\mathbf{f}_1^1\text{', '}\mathbf{g}_1^1\text{', '}\mathbf{h}_1^1\text{', '}\mathbf{f}_2^1\text{', '}\mathbf{g}_2^1\text{', '}\mathbf{h}_2^1\text{', } \ldots, \quad (5.8^1)$$

$$\text{'}\mathbf{f}^2\text{', '}\mathbf{g}^2\text{', '}\mathbf{h}^2\text{', '}\mathbf{f}_1^2\text{', '}\mathbf{g}_1^2\text{', '}\mathbf{h}_1^2\text{', '}\mathbf{f}_2^2\text{', '}\mathbf{g}_2^2\text{', '}\mathbf{h}_2^2\text{', } \ldots, \quad (5.8^2)$$

$$\text{'}\mathbf{f}^3\text{', '}\mathbf{g}^3\text{', '}\mathbf{h}^3\text{', '}\mathbf{f}_1^3\text{', '}\mathbf{g}_1^3\text{', '}\mathbf{h}_1^3\text{', '}\mathbf{f}_2^3\text{', '}\mathbf{g}_2^3\text{', '}\mathbf{h}_2^3\text{', } \ldots, \quad (5.8^3)$$

etc is a PStAPL, whose range, to be denoted respectively by 'κ^1', 'κ^2', 'κ^3', etc or in general 'κ^m' ('κ' being the first letter of the Greek noun "κατηγόρημα" \kateγórema\ meaning *a predicate*), is the set of all primary AEOPS's of the corresponding weight. That is to say, if '$\kappa^{1\text{pv}}$', '$\kappa^{2\text{pv}}$', '$\kappa^{3\text{pv}}$', etc denote the sets of all singulary, binary, ternary, etc APVOPS's on the respective one of the lists (5.3^1)–(5.3^3), etc then κ^1, κ^3, etc, are respectively $\kappa^{1\text{pv}}$, $\kappa^{3\text{pv}}$, etc, whereas κ^2 includes $\kappa^{2\text{pv}}$ and at most one of the singletons $\{\in\}$, $\{\subseteq\}$, and $\{=\}$ as specified, which will indiscriminately (equivocally) be denoted by ‘κ^{2pc}’ or ‘κ^{pc}’ and discriminately by ‘$\kappa^{pc}_{\in}$’, ‘$\kappa^{pc}_{\subseteq}$’, and ‘$\kappa^{pc}_{=}$’ in that order, i.e. $\kappa^{pc}_{\in} \to \{\in\}$’. ‘$\kappa^{pc}_{\subseteq} \to \{\subseteq\}$’, and ‘$\kappa^{pc}_{=} \to \{=\}$’. The PStAPL’s on the lists (5.8^1)–(5.8^3) etc are called, discriminately, the *primary structural atomic panlogographic predicate-signs* (*PStAPLPS’s*).*of weight* 1, 2, etc, of $\mathbf{A}_1$ or, indiscriminately, the *weighed* ones (*WPStAPLPS’s*). Incidentally, the Greek parasynonyms of the English noun “term” is “όρος” \óros\. However, the first letter of the latter is indistinguishable from the English letter ‘o’, and therefore use of Greek ‘ο’ instead of ‘τ’ introduced in the item 1, after the manner of ‘σ’ and ‘κ’, would be confusing in the sequel, particularly in discussing Aristotelian syllogistics, where English “o” is used as the formal predicate a negative universal judgment.

4) Each graphonym on the following list:

$$\text{‘}\mathbf{f}\text{’}, \text{‘}\mathbf{g}\text{’}, \text{‘}\mathbf{h}\text{’}, \text{‘}\mathbf{f}_1\text{’}, \text{‘}\mathbf{g}_1\text{’}, \text{‘}\mathbf{h}_1\text{’}, \text{‘}\mathbf{f}_2\text{’}, \text{‘}\mathbf{g}_2\text{’}, \text{‘}\mathbf{h}_2\text{’}, \ldots \tag{5.8}$$

is a PStAPL, whose range, to be denoted respectively by ‘κ’, is the set of all APVOPS’s and of an APCOPS if present, i.e. the union of the sets ‘κ^1’, ‘κ^2’, ‘κ^3’, etc, being the ranges, e.g., of ‘$\mathbf{f}^1$’, ‘$\mathbf{f}^2$’, ‘$\mathbf{f}^3$’, etc. Thus,

$$\kappa = \bigcup_{m=1}^{\infty} \kappa^m = \kappa^{pv} \cup \kappa^{pc}, \kappa^{pv} \doteqdot \bigcup_{m=1}^{\infty} \kappa^{mpv}, \kappa^{pc} \doteqdot \kappa^{2pc}. \tag{5.9}$$

The panlogographs on the list (5.8) are called the *unweighed PStAPLPS’s* (*UWPStAPLPS’s*) of $\mathbf{A}_1$.

5) The PStAPL’s occurring on any one of the lists (5.6) and (5.7) are called the *primary structural atomic formulary* (*categorematic*) *panlogographs* (*PStAFPL’s*, *PStACtgPL’s*), and also the *primary structural atomic panlogographic formulas*, or *categoremata* (*PStAPLF’s* or *PStAPLC’ta*) – specifically, *ordinary terms* (*PStAPLOT’s*) and *ordinary relations* (*PStAPLOR’s*) respectively. The PStAPL’s occurring on the lists (5.8^1)–(5.8^3), etc and (5.8) are called the *primary structural atomic syncategorematic panlogographs* (*PStASctgPL’s*), and also the *primary structural atomic panlogographic syncategoremata*, or, more specifically, *predicate-signs* (*PStAPLSC’ta*, *PStAPLPS’s*). The PStAPL’s occurring on any one of the lists (5.6)–(5.8) are called *congeneric ones*, whereas the PStAPL’s occurring on any one of the lists (5.8^1)–(5.8^3), etc are called *conspecific* ones. The order, in which the PStAPL’s are presented on any one of the above lists, is called the *alphabetic order* of those PStAPL’s, whereas the list itself is called the *alphabet* of the PStAPL’s listed

(cf. Ax 5.1(v)). In accordance with Df 5.1(4c), the above items 1 and 2 determine the set $\mathbf{B}_{1StCtg}$, i.e. $\mathbf{B}_{1StF}$, whereas the items 3 and 4 determine the set $\mathbf{B}_{1StSctg}$.

6) In the presence of $\emptyset$ and $\emptyset'$, the range τ of any given PStAPLOT (StAPLOT) of the list (5.6) may, at any place, be restricted either to τ^{pv} or to τ^{pc} either by the appropriate statement in the metalanguage or by providing the given PStAPLOT with the corresponding superscript '^{pv}' (meaning "pseudo-variable") or '^{pc}' (meaning "pseudo-constant"). For instance, the range of each of the PStAPLOT's '$\mathbf{u}^{pv}$', '$\mathbf{u}_1^{pv}$', '$\mathbf{u}_2^{pv}$', etc is τ^{pv}, while the range of each of the PStAPLOT's '$\mathbf{u}^{pc}$', '$\mathbf{u}_1^{pc}$', '$\mathbf{u}_2^{pc}$', etc is τ^{pc} subject to $\tau^{pv} = \{\emptyset, \emptyset'\}$; and similarly with any of the letters '**v**', '**w**', '**x**', '**y**', '**z**' in place of '**u**'. In this case, '$\mathbf{u}^{pv}$' or any one of its conspecific specimens (variants) is called a *PVOT-valued* (*APVOT-valued*) *panlogograph*, whereas and '$\mathbf{u}^{pc}$' or any one of its conspecific specimens are called *PCOT-valued* (*APCOT-valued*) *panlogograph*.

7) Likewise, in the presence both of the *binary APVOPS's* (*BAPVOPS's*) and of a primary binary APCOPS (PBAPCOPS), the range κ^2 of any given StAPLOPS on the list (5.8^2), or the range κ of any given PStAPLOPS on the list (5.8), may at any place be restricted either to κ^{2pv} or to κ^{2pc}, or, correspondingly, either to κ^{pv} or to κ^{pc}, but again either by the appropriate statement in the metalanguage or by furnishing the given PStAPLOPS with the corresponding superscript '^{pv}' or '^{pc}', which is put after the digital superscript if present. For instance, the range of any PStAPLOPS of the list:

$$\text{‘}\mathbf{f}^{2pv}\text{’}, \text{‘}\mathbf{g}^{2pv}\text{’}, \text{‘}\mathbf{h}^{2pv}\text{’}, \text{‘}\mathbf{f}_1^{2pv}\text{’}, \text{‘}\mathbf{g}_1^{2pv}\text{’}, \text{‘}\mathbf{h}_1^{2pv}\text{’}, \text{‘}\mathbf{f}_2^{2pv}\text{’}, \ldots \qquad (5.8^{2pv})$$

is κ^{2pv}, while the range of any PStAPLOPS of the list:

$$\text{‘}\mathbf{f}^{2pc}\text{’}, \text{‘}\mathbf{g}^{2pc}\text{’}, \text{‘}\mathbf{h}^{2pc}\text{’}, \text{‘}\mathbf{f}_1^{2pc}\text{’}, \text{‘}\mathbf{g}_1^{2pc}\text{’}, \text{‘}\mathbf{h}_1^{2pc}\text{’}, \text{‘}\mathbf{f}_2^{2pc}\text{’}, \ldots \qquad (5.8^{2pc})$$

is κ^{2pc}, i.e. some one of the three: $\kappa_{\in}^{pc}$. $\kappa_{\subseteq}^{pc}$, or $\kappa_{=}^{pc}$, subject to $\kappa_{\in}^{pc} = \{\in\}$. $\kappa_{\subseteq}^{pc} = \{\subseteq\}$, or $\kappa_{=}^{pc} = \{=\}$. By (5.9), $\kappa^{pc} \doteqdot \kappa^{2pc}$ and hence $\mathbf{f}^{pc} \doteqdot \mathbf{f}^{2pc}$, and similarly with any other logograph of the list (5.8) in place of '**f**'.

8) Once a branch of A_1 is fixed, the respective ranges of all the above PStAPL's are specified automatically.•

5.3. Numeral indices of atomic euautographs and atomic panlogographs

Preliminary Remark 5.1. I have already pointed out in Cmt 5.7(2) that an Arabic numeral index (subscript or superscript), occurring in an atomic pasigraph being an element of any one of the alphabets of A_1 and $\mathbf{A}_1$ is an inseparable distinguishing visual attribute of the pasigraph, which is used autonymously. In order to analyze and fix the mental attitude that I take in relation to the Arabic numeral indices, I shall make the following two analogous preliminary definitions in the framework .of the SQM (special quotation method).•

Df 5.3. 1) Each of the six small light-faced italic (slant upward to the right) English letters '*i*', '*j*', '*k*', '*l*', '*m*', '*n*' therein depicted between light-faced single quotation marks or any of its homolographic tokens of the full, subscript, or sub-subscript size, is an *AMLPH* (*atomic metalogographic placeholder*), whose *range* (*class-connotatum*) is the *set* of digital (Arabic) numerals 1, 2, etc, without any quotation marks, in the current Roman type. Since the numerals are not quoted, the set of these numerals *is denoted by* 'ω_1' or '{1,2,…}' (and not, say, by '$\omega_{‘1’}$' or '{'1','2',…}'), i.e. ω_1 or {1, 2,…} *is* this set, so that $i \in \omega_1$ and similarly with '*j*', '*k*', '*l*', '*m*', or '*n*' in place of '*i*'. In this case, $\in$ is the conventional set-membership predicate, which is informally used in the XML as a congruent homograph of the PBAPCOPS $\in$ introduced in Ax 5.1(8c). By the *principle of juxtaposition of photographic quotations*, 'ω_1' is the same as 'ω''$_1$' or 'ω'$_{‘1’}$, which reduces to 'ω'$_1$, because '1' is used autonymously. Any of the above six letters can, when necessary, be furnished either with any of the upright Arabic numeral subscripts '$_1$', '$_2$', etc of the appropriate size or with any number of primes or with both labels simultaneously, thus becoming another *digital-numeral-valued MLPH* (*DNVMLPH*) with the same range.

2) It is understood that none of the DNVMLPH's is an element (member) of the set ω_1. Therefore, the relation '$i \in \omega_1$', e.g., is a metalinguistic schema, which represents (condenses) an infinite number of the concrete relations: $1 \in \omega_1$, $2 \in \omega_1$, etc. Accordingly, when I use '*i*' for mentioning a certain (concrete but not concretized) numeral, I say that *i is* a *common* (*general*) *element* (*member*) *of* ω_1. In this case, I use the range of '*i*", being my mental entity, in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I mentally experience that range as *my as-if extramental* (*exopsychical*) *object representing the whole set*. Thus, a *common* (*general*) *element* (*member*) *of* ω_1 (as *i* to *n*, i_1 to n_1, etc) *is just another hypostasis* (*way of existence*, *aspect*) *of* ω_1.

3) In general, ω_m or $\{m,m+1,\ldots\}$ is the set of digital (Arabic) numerals, without any quotation marks, in the current Roman type from a given numeral *m* (as 1, 2, etc) to infinity. Analogously, $\omega_{m,n}$ or $\{m,m+1,\ldots,n-1,n\}$ is the set of digital (Arabic) numerals fom a given numeral *m* (as 1, 2, etc) to a given numeral *n* subject to $m<n$. It is understood that

$$\omega_{m,m}=\{m\},\ \omega_{m,\infty}=\omega_m,\ \omega_{m,n}=\varnothing \text{ if } m\geq n. \tag{5.10}$$ •

Df 5.4. 1) 'u_i' is a *molecular MLPH* (*MMLPH*), whose *immediate* range is the set of APVOT's: u_1, u_2, u_3, etc. Similarly, '$\mathbf{u}_i$' is an MMLPH, whose *immediate* range is the set of StAPLOT's: '$\mathbf{u}_1$', '$\mathbf{u}_2$', '$\mathbf{u}_3$', etc. Therefore, the *ultimate* range of '$\mathbf{u}_i$' is the range of any of the latter StAPLOT's, and hence that of any StAPLOT on the list (5.6). Unless stated otherwise, I shall use '$\mathbf{u}_i$' for mentioning any StAPLOT of its immediate range.

2) The above item applies with any of the subscripts: '*j*' to '*n*', 'i_1' to 'n_1', 'i_2' to 'n_2', et) in place of '*i*'

3) The item 1 and all its variants subject to the item 2 apply with any of the letters: *v*, *w*, *x*, *y*, *z* in place of *u* and with any of the letters **v**, **w**, **x**, **y**, **z** in place of **u.**

4) The item 1 and all its variants subject to the item 2 apply with "APVOR's" and "StAPLOR's" in place of "APVOT's" and "StAPVOT's" respectively,: with any of the letters: *p*, *q*, *r*, *s* in place of *u* and with any of the letters **p**, **q**, **r**, **s** in place of **u.** •

Cmt 5.10. In accordance with Df 5.4 the graphonyms '$\mathbf{u}_1,\mathbf{u}_2,\ldots,\mathbf{u}_n$' and '$\mathbf{u}_m,\mathbf{u}_{m+1},\ldots,\mathbf{u}_{n-1},\mathbf{u}_n$', e.g., and their variants, or instances, with *u* in place of '**u**' (see also Df 5.4(2,4)) .are complex placeholders belonging to the XML of A_1 and $\mathbf{A}_1$. Particularly, '$\mathbf{u}_1,\mathbf{u}_2,\ldots,\mathbf{u}_n$' is a *complex MLPH* (*CxMLPH*) for any string of the open list:

$$'\mathbf{u}_1',\ '\mathbf{u}_1,\mathbf{u}_2',\ '\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3',\ \ldots \tag{5.11}$$

At the same time, '$\mathbf{u}_m,\mathbf{u}_{m+1},\ldots,\mathbf{u}_{n-1},\mathbf{u}_n$' can be regarded as a CxMLPH for any CxMLPH of either of the following two open lists:

$$'\mathbf{u}_m,\mathbf{u}_{m+1},\ldots,\mathbf{u}_1',\ '\mathbf{u}_m,\mathbf{u}_{m+1},\ldots,\mathbf{u}_2',\ '\mathbf{u}_m,\mathbf{u}_{m+1},\ldots,\mathbf{u}_3'\ldots \tag{5.12}$$

$$'\mathbf{u}_1,\mathbf{u}_2,\ldots,\mathbf{u}_n',\ '\mathbf{u}_2,\mathbf{u}_3,\ldots,\mathbf{u}_n',\ '\mathbf{u}_3,\mathbf{u}_4,\ldots,\mathbf{u}_n',\ \ldots \tag{5.13}$$

Under the natural condition that '$\mathbf{u}_m,\mathbf{u}_{m+1},...,\mathbf{u}_{n-1},\mathbf{u}_n$' is vacuous if $m>n$, the above two lists turn into these two:

$$'\mathbf{u}_1'; '\mathbf{u}_1,\mathbf{u}_2', '\mathbf{u}_2'; '\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3', '\mathbf{u}_2,\mathbf{u}_3', '\mathbf{u}_3'; \ldots, \tag{5.14}$$

$$'\mathbf{u}_1', '\mathbf{u}_1,\mathbf{u}_2', '\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3', \ldots; '\mathbf{u}_2', '\mathbf{u}_2,\mathbf{u}_3', '\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4', \ldots;$$
$$'\mathbf{u}_3', '\mathbf{u}_3,\mathbf{u}_4', '\mathbf{u}_3,\mathbf{u}_4,\mathbf{u}_5', \ldots; \ldots \tag{5.15}$$

respectively. Either of the lists (5.14) and (5.15) *represents* the set strings being *the ultimate range of the CxMLPH* '$\mathbf{u}_m,\mathbf{u}_{m+1},...,\mathbf{u}_{n-1},\mathbf{u}_n$'. In this connection the following remark should be made.

The set of Arabic numerals and the set of natural numbers denoted by those numerals stand in a bijective (one-to-one) correspondence with each other. In this case, binary arithmetical operations of addition, subtraction, and multiplication of natural numbers are performed at the syntactic level with the help of the corresponding tables for one-digit and two-digit Arabic numerals. Also, either of the direct order predicates ≤ and < and either of the respective converse order predicates ≥ and > contactually apply to Arabic numerals, and not to natural numbers. At the same time, owing to the above-mentioned bijective correspondence, confusion between an Arabic numeral, or generally any numeral (as Roman or wordy), and the natural number denoted by the numeral is harmless, – in contrast to the confusion between the nouns "numeral" and "number", i.e. in contrast to usage of either noun interchangeably or instead of the other. Maintaining the *terminological difference* between "numeral" and "number" is the must. For instance, this subsection would have been unreadable if I had used either of the two nouns equivocally. At the same tie, no harm is done if an Arabic numeral index is involuntarily but consciously associated with the natural number, which it habitually denotes.•

5.4. Operators of specification and particularization

Preliminary Remark 5.2. In writings on theoretical physics, mathematics, and mathematical logic, the sign of equality for denotata (denotation values), =, is customarily used for indicating the act of assigning a specific value to a variable. Let, for instance, '*n*' be an DNVMLPH as defined in Df 5.3 or a *natural-number-valued metalogographic* (*metalinguistic logographic*) *abstract* (*not place-holding*) *variable* (briefly *NNVMLAV*), while either of the constants '$\{1,2,...\}$' and 'ω_1' denotes the range of ‘*n*’, which is, depending on the mental attitude of the interpreter, either the set of all strictly positive Arabic numerals in the current type or the set of strictly positive natural numbers denoted by those numerals. In such cases, in order to indicate that ‘*n*’ assumes denotata in its range, it is generally accepted to state a barbarism such as “$n = 1,2,3,\ldots$” instead of stating that $n \in \{1,2,...\}$ or $n \in \omega_1$. Barbarisms of the above kind can be encountered even in the most rigorous and exquisite writings on mathematical logic (see, e.g., Church [1956, p. 171]). The sign = satisfies *the reflexive*, *symmetric*, and *transitive laws*. Therefore, it is incorrect to use this sign either for particularizing a single accidental denotatum of a variable, place-holding or not, or for specifying the range [of denotata] of a variable – just as it is incorrect to use it for stating definitions.•

Df 5.5. 1) A statement of the act of assigning a *range*, i.e. *class-connotatum*, to a variable, place-holding or not, is a *linguistic definition* that is called a *specification of the range of the variable* or a *specification of the variable* and also, less explicitly, a *specification*. The variable, whose range is specified, is called the definiendum of the definition, whereas the proper class-name (class-constant), whose class-denotatum is assigned to the variable as its range (class-connotatum) is the definiens of the definition. The variable specified can be either an entirely new one that has not occurred in the discourse previously or an old one, whose range is being either restricted or extended in the specification. In either of the two latter cases, the variable specified should be regarded as a *homograph* (*graphic homonym*) of any one of its isotokens occurred earlier.

2) Like any other definition, a specification of the variable has *its scope* that is either an *ad hoc* one or a certain *unbroken* (*whole*) or *broken* (*scrappy*) *part* of the discourse, which begins immediately *after* the specification and hence does not includes the latter, and which stretches, continuously or interruptedly, either to the end of the discourse or up to the statement disregarding or revising the specification, particularly up to another specification of the variable.

3) An object that is distributively mentioned (denoted) by using a variable is called a *general*, or *common, element of its range*. In contrast to a concrete element of the range, being an instance of the range, a general element of the range is tantamount to the range (cf. Df 5.3(2)). •

Df 5.6. 1) In order to state the act of specification of a variable conveniently and formally, I shall make use of either one of the signs $\vec{\in}$ and $\overleftarrow{\ni}$, which belong to the XML of A_1 and $\boldsymbol{\mathsf{A}}_1$ and which are called *the rightward universal sign* and *leftward universal sign*, *of mental* (*psychical*) *specification of the range of a variable.* The occurrence of the generic name "sign" in either of the above two metaterms can be used interchangeably with any of the following generic names: "*operator*", "*predicate*", "*predicate-sign*", "*predicate-operator*", etc. Either sign $\vec{\in}$ or $\overleftarrow{\ni}$ is a *biune sign of class-membership*, or particularly, when applicable, of *set-membership*, *and of definition* (cf. $\vec{\equiv}$ and $\overleftarrow{\equiv}$). At the rear (back) of either sign (and hence at the base of its over-arrow), I shall write the *definiendum* – the variable, whose *range* (*class-connotatum*) is to be specified. At the front (fork) of the sign (and hence of the head of its over-arrow), I shall write the *definiens* – the proper class-name (class-constant), whose class-denotatum is designed to be the range (class-connotatum) – an *ad hoc one or the permanent one within a certain broad scope* of the specification. Accordingly, the sign $\vec{\in}$ and $\overleftarrow{\ni}$ are rendered into English by the expressions: "*has the range*" and "*is the range of*", respectively. A definition, which is stated with the help of either sign $\vec{\in}$ or $\overleftarrow{\ni}$, is called a *formal specification of its definiendum* or less explicitly a *formal specification.* Neither the definiendum nor the definiens of a specification should be enclosed in any quotation marks that are supposed to be used but not mentioned.

2) The fact whether the definiendum of a specification is a placeholder or a not place-holding variable depends on the character of the definiens. For instance, '*i*' occurring in either of the concurrent specifications '$i\,\vec{\in}\,\omega_1$' and '$i\,\vec{\in}\,\{1,2,...\}$' is is a DNVMLPH if '$\omega_1$' denotes the set of unquoted Arabic numerals 1,2,… and an NNVMLAV if 'ω_1' denotes the set of natural numbers 1,2,… In ambiguous cases as the above one, I shall assume that the variable specified is a placeholder, unless stated otherwise.

3) After stating a specification, the specification sign $\vec{\in}$ or $\overleftarrow{\ni}$ used can in any place within the scope of the specification, be replaced with the respective membership-sign $\in'$ or $\ni'$ with the proviso that the latter signs are homographs of those used as APCOPS's of A_1.•

Df 5.7 (*A supplement to Df 5.6***).** 1) In a formal specification of a placeholder as described in Df 5.6, I shall, when possible and convenient, form the self-explanatory ostensive definiens by writing a list of graphonyms, which are supposed to be elements of its set-denotatum, between braces, the graphonyms being separated from each other by commas. If the class to be denoted by the definiens contains a relatively small and definite number of graphonyms then all of them are listed in braces. If that class contains an indefinite or infinite number of graphonyms then a few typical of them are exhibited, whereas all others, which are obviously understood by analogy, are represented by omission points (cf. the lists (5.11)–(5.15)). The graphonyms embraced and the entire definiens thus formed is supposed to have the following properties:

a) The graphonyms embraced can be either pasigraphic (euautographic or logographic) or phonographic (verbal) expressions, or else mixed pasigraphic and phonographic expressions, i.e. non-phonographic ones after all.

b) The graphonyms can be either unquoted or quoted expressions, each of which can, in turn, contain quotations as its constituents. However, in any case, the graphonyms embraced must stand for themselves as they are, i.e. they must be *used autonymously* and be thus either euautographs of tychautographs. In other words, the graphonyms cannot be mentioned by using their xenographic proper names, because such names are some other graphonyms, which are not intended to be in the range of the placeholder being specified. Accordingly, the graphonyms cannot contain any quotation marks that are used but not mentioned.

c) As usually in set-theory, the set being the denotatum of the embraced list of graphonyms remains unchanged if the order of the graphonyms is changed or if any of them is listed repeatedly any number of times in any order relative to the other graphonyms. Still, for the sake of definiteness and simplicity, I shall, in the sequel, use only non-redundant and, when possible, alphabetically ordered lists.

2) If the number of elements in the range of the placeholder to be specified is infinite then a compact constant (as 'ω_1') denoting the pertinent class can be defined beforehand and be used as definiens in the specification of the placeholder.•

Cmt 5.11. 1) With the help of Dfs 5.6 and 5.7, Df 5.2, e.g., can formally be restated as follows.

a)

$$\mathbf{u} \vec{\in} \tau, \mathbf{u}^{\mathrm{pv}} \vec{\in} \tau^{\mathrm{pv}}, \mathbf{u}^{\mathrm{pc}} \vec{\in} \tau^{\mathrm{pc}}, \tag{5.16}$$

subject to

$$\tau \dot{=} \tau^{\mathrm{pv}} \cup \tau^{\mathrm{pc}},$$
$$\tau^{\mathrm{pv}} \dot{=} \{u,v,w,x,y,z,u_1,v_1,w_1,x_1,y_1,z_1,u_2,v_2,w_2,\ldots\}, \tau^{\mathrm{pc}} \dot{=} \{\emptyset,\emptyset'\}; \quad (5.17)$$

and similarly with ‘**v**’ to ‘**y**’, ‘$\mathbf{u}_1$’ to ‘$\mathbf{y}_1$’, ‘$\mathbf{u}_2$’ to ‘$\mathbf{y}_2$’, etc in place of ‘**u**’.

b)

$$\mathbf{p} \vec{\in} \sigma, \quad (5.18)$$

subject to

$$\sigma \dot{=} \{p,q,r,s,p_1,q_1,r_1,s_1,p_2,q_2,r_2,s_2,\ldots\}; \quad (5.19)$$

and similarly with ‘**q**’ to ‘**s**’, ‘$\mathbf{p}_1$’ to ‘$\mathbf{s}_1$’, ‘$\mathbf{p}_2$’ to ‘$\mathbf{s}_2$’, etc in place of ‘**p**’.

c)

$$\mathbf{f} \vec{\in} \kappa; \mathbf{f}^m \vec{\in} \kappa^m \text{ for each } m \vec{\in} \omega_1, \quad (5.20)$$

subject to

$$\kappa \dot{=} \bigcup_{m=1}^{\infty} \kappa^m, \kappa^1 \dot{=} \kappa^{1\mathrm{pv}}, \kappa^2 \dot{=} \kappa^{2\mathrm{pv}} \cup \kappa^{2\mathrm{pc}}, \kappa^m \dot{=} \kappa^{m\mathrm{pv}} \text{ for each } m \vec{\in} \omega_3,$$
$$\kappa^{m\mathrm{pv}} \dot{=} \{f^m, g^m, h^m, f_1^m, g_1^m, h_1^m, f_2^m, g_2^m, h_2^m, \ldots\} \text{for each } m \vec{\in} \omega_1; \quad (5.21)$$

while $\kappa^{2\mathrm{pc}}$ can, depending on a branch A_1, equal {=}, {⊆), or {∈}. Hence,

$$\mathbf{f}^{m\mathrm{pv}} \dot{=} \mathbf{f}^m \text{ for each } m \vec{\in} \{1\} \cup \omega_3 \text{ and } \mathbf{f}^{2\mathrm{pc}} \vec{\in} \kappa^{2\mathrm{pc}}. \quad (5.22)$$

Relations (5.20)–(5.22) apply with ‘**g**’ or ‘**h**’, ‘$\mathbf{f}_1$’ to ‘$\mathbf{h}_1$’, ‘$\mathbf{f}_2$’ to ‘$\mathbf{h}_2$’, etc in place of ‘**f**’.

2) At any place, I may, for instance, state that $\mathbf{u} \vec{\in} \{u,v,w\}$ or $\mathbf{p} \vec{\in} \{p,q,r,s\}$ or $\mathbf{f} \vec{\in} \{\in\}$ and then use any of the placeholders ‘**u**’, ‘**p**’, and ‘**f**’ in accordance with its new specification until it will be disregarded and replaced with another one, particularly with the initial one.

3) It will be recalled that the set-theoretic predicate ‘∈’ is a stylized Greek letter “ε”, which is mnemonically an abbreviation of the Greek transitive verb “εστί” \estí\ meaning as *is*. Beginning with “ε” are also the personal intransitive verbs “είμαι” \íme\ and “είναι” \íne\ meaning *am* and *is* or *are* respectively; the kindred noun “τὸ είναι” means *the being* or *existence*. At the same time, in accordance with Df 5.6, the sign ‘$\vec{\in}$’ can be regarded as one suggesting pictographically that the objects (particularly, graphonyms) forming the class, which is denoted by the name standing to the right of ‘$\vec{\in}$’ (i.e. at its fork), are condensed into a single general element of the

class, being the denotatum of the variable (correspondingly, the placeholder) standing to the left of '$\vec{\in}$'. In fact, general element of the class is not any one of the genuine concrete elements of the class, but rather it is an object, which is tantamount to the class and which at the same time is used distributively.

5) The sign '$\vec{\in}$' is a *predicate of psychical* (*mental*) *specification* of the range of the variable, standing to the left of it as definiendum, by the class denoted by the constant, standing to the right of it as definiens. This particularly means that if the variable being specified , e.g. is a placeholder of $\mathbf{A}_1$, the act of its specification expressed by '$\vec{\in}$' together with the pertinent definiens does not imply any replacement of a token of the placeholder occurring in any formula of $\mathbf{A}_1$ with any euautograph belonging to the specified range of the placeholder. This remark remains in force also in the case where the range assigned to the placeholder is a singleton. For instance, the statement that $\mathbf{u}\vec{\in}\{u\}$ does not immediately imply that '**u**' should somewhere be replaced with *u*.•

Df 5.8. 1) The operation of *replacement* of an occurrence of a prototypal placeholder either with an occurrence of a more restricted placeholder or with an occurrence of a concrete member of the range of the former placeholder is called a *substitutional*, or *physical*, *specification*, *of that placeholder* and also, less explicitly, a *specifying replacement*. The name "*particularization*" can be used interchangeably (synonymously) with "*specification*". A substitutional (physical) specification of a placeholder by a concrete member of its range is called a *substitutional*, or *physical*, *concretization*, *of that placeholder* and also, less explicitly, a *concretizing substitution.* A substitutional (physical) specification (particularization) of a placeholder is a *substitutional interpretation* of the placeholder, but not necessarily vice versa. For instance, an analogomolographic substitution is, also, a substitutional interpretation of the substituens.

2) An operation of substitutional specification or particularly of substitutional concretization of a placeholder will be expressed by putting the substituens (interpretans, placeholder) to the left and the substituend (interpretand) to the right of the sign '$\triangleright$'; both the substituens and the substituend are written without any special quotation marks. The sign will be called the *sign* (and also *operator*, *predicate*, *predicate-sign*, *predicate-operator*, etc – cf. Df 5.6(1)) *of substitutional*, or *physical*, *specification of a placeholder*. It is understood that a statement of the substitutional

specification of a placeholder should be accompanied by a statement indicating the scope of the specification, i.e. the occurrence or occurrences of the placeholder, which should be replaced with occurrences of the substituend.•

Cmt 5.12. 1) The sign '$\triangleright$' is an operator of substitution which applies only in cases where the range of the substituend is narrower than the range of the substituens, i.e. where the two ranges are *comparable*. In order to indicate, the operation of replacement of one graphonym with another one in no connection to their semantic properties, I shall use a token of the barred arrow, '$\mapsto$', which is placed *between the substituend and the substituens in this order* and whose direction is opposite to that of '$\triangleright$'. With the help of '$\mapsto$', I may, for instance, make the statement $\langle\,\rangle \mapsto (\,)$, which means that angle brackes in certain occurrences should be replaced with round ones. It is understood that a specification that is indicated with the help of '$\triangleright$' can also be indicated with the help of '$\mapsto$' as a pure physical substitution, but not necessarily vice versa.

2) The function of the specification sign '$\triangleright$' differs from the function of the definition sign '$\rightarrow$'. Indeed, unless stated otherwise, the statement that $\mathbf{f}\rightarrow\in$, e.g., is meaningless, because it means that any concrete primary atomic euautographic ordinary predicate-sign of A_1 of the range of the placeholder '**f**' is *redenoted* as $\in$. By contrast, the statement that $\mathbf{f}\triangleright\in$, e.g., means that certain occurrences of '**f**' to be specified separately should be replaced with occurrences of $\in$. Likewise, the statement that $n\triangleright 1$, e.g., means that certain occurrences of the placeholder '*n*' should be is specified (concretized) by replacing them with occurrences of the digit 1.•

5.5. Primary assemblages of A_1 and their metalogographic placeholders

Df 5.9: *Primary euautographic assemblages (PEA's) of* A_1. 1) A homolographic (photographic) token of a PAE (primary atomic euautograph) of A_1, standing alone or mentally isolated from its surrounding, is called a *primary atomic assemblage* (*PAA*) *of* A_1 or, more precisely but redundantly, a *primary atomic euautographic assemblage* (*PAEA*) *of* A_1. A finite juxtaposition (linear sequence) of homolographic tokens of PAE's of A_1 *without blanks* is called a *combined*, or *compound*, or *juxtapositional, assemblage* (*CbA*) of *PAE's of* A_1 or, more precisely but redundantly again, a *primary combined euautographic assemblage* (*PCbEA*) *of* A_1. A PAA (PAEA) or PCbA (PCbEA) of A_1 is indiscriminately called a *primary* *assemblage* (*PA*) *of* A_1 and also a *primary euautographic assemblage* (*PEA*) *of* A_1. The above definitions apply with 'A_1^0' or 'A_0' in place of 'A_1', the understanding being that a PEA of A_0 is a PEA of both A_1^0 and A_1 and that a PEA of A_1^0 is a PEA of A_1, but not necessarily vice versa.

2) A PEA is called a *primary euautographic ordinary*, or *ordinary euautographic, assemblage* (*PEOA* or *POEA*) if it comprises [homolographic tokens of] PAOE's and a *primary euautographic special*, or *special euautographic, assemblage* (*PESpA* or *PSpEA*) if it contains at least one [homolographic token of a] PASpE and some or no [homolographic tokens of] PAOE's.•

Df 5.10: ***Primary [analytical] atomic metalogographic placeholders (PAnAMLPH's, PAMLPH's) of PEA's of*** A_1**.** 1) Each of the bold-faced upright capital Greek letters '**Γ**' and '**Δ**', taken alone or furnished either with any of the upright Arabic numeral subscripts '$_1$', '$_2$', etc in the current type or with any number of primes or with both, is a *primary analytical atomic metalogograph* (*PAnAML*), called also a *primary analytical atomic metalogographic* (*metalinguistic logographic*) *placeholder* (*PAnAMLPH*), whose *range* is the class of PEA's of A_1 (without any quotation marks). These placeholders are qualified *metalogographic* (*ML*), and not panlogographic (PL), because they belong, *not to* $\mathbf{A}_1$, but to the *XML* (*exclusive metalanguage*) *of* A_1 *and* $\mathbf{A}_1$ when they are used *autonymously* and to the *IML* (*inclusive metalanguage*) *of* A_1 *and* $\mathbf{A}_1$ when they are used *xenonymously* or *autoxenonymously* (*xenoautonymously*), i.e. *in the TAEXA-mode*. I use the qualifier "*analytical*" ("*An*") to "ML" or "MLPH" in analogy with its use to 'PL" or "PLPH" as an antonym of "*structural*" ("St"). However, I shall have no occasion to introduce any ML's (MLPH's) that could be qualified structural. Therefore, the above taxomyms of AML's (AMLPH's) and all subsequent taxonyms of *combined ML's* (*CbML's*), i.e. *combined MLPH's* (*CbMLPH's*), *combined ML's* (*CbML's*), *of PEA's of* A_1, can in principle be abbreviated by omission of the qualifier "analytical" ("An") without altering their ranges. Particularly, I can use the abbreviations "*PAMLPH*" and "*PAMLPH*" instead of "*PAnAMLPH*" and "*PAnAMLPH*". Nevertheless, I shall preserve occurrences of that qualifier in many taxonyms of ML's (MLPH's) for convenience in introducing analogous taxonyms of PL's (PLPH's) by substitution of "pan" for "meta" and "P" for "M" and vice versa.

2) The set of PAnAML's (PAML's) introduced above in the item 1 is called the *primary [analytical] metagographic atomic basis* (*PAnMLAB, PMLAB*) *of the IML of* A_1 *and* $\boldsymbol{\mathsf{A}}_1$, whereas that item is called the *primary [analytical] metalogographic generalization rule* (*PAnMLGR, PMLGR*) *of PEA's of* A_1, – in analogy with the respective taxonyms with "*pan*" in place of "*meta*" and 'A_1' in place of "*the IML of* A_1 *and* $\boldsymbol{\mathsf{A}}_1$" that have been introduced in the items 1, 3, and 8 of Df 5.1.•

Df 5.11: ***Descriptive primary analytical molecular metalogographic placeholders (DPAnMMLPH's) of PEA's of*** A_1**.** 1) '$\mathbf{\Gamma}\langle\mathbf{u}\rangle$', e.g., is a *descriptive primary analytical molecular metalogograph* (*DPAnMML*), called also a *descriptive primary analytical molecular metalogographic placeholder* (*DPAnMMLPH*) or *primary analytical molecular metalogographic description* (*PAnMMLD*), whose *range* is the class of PEA's of A_1 (without any quotation marks), any given member of which, $\mathbf{\Gamma}\langle\mathbf{u}\rangle$, is a PEA of A_1, which involves certain occurrences of a given (selected) AEOT $\mathbf{u}$ of A_1 and perhaps occurrences of some other AEOT's $\mathbf{v}$, $\mathbf{w}$, etc that are not mentioned by using the metalogograph '$\mathbf{\Gamma}\langle\mathbf{u}\rangle$', but which *does not involve either of the PEA's* $(\exists\mathbf{u})$ *and* $(\hat{\cdot}\,\mathbf{u})$. Such an occurrence of $\mathbf{u}$ in $\mathbf{\Gamma}\langle\mathbf{u}\rangle$ is called a *free* occurrence of $\mathbf{u}$ in $\mathbf{\Gamma}\langle\mathbf{u}\rangle$.

2) Let '$\mathbf{\Delta}\langle\mathbf{u}\rangle$' be a DPAnMML (DPAnMMLPH) of PEA's of A_1, which involves one or more *free* occurrences of a given A*PV*OT $\mathbf{u}$. If $\mathbf{\Gamma}$ contains either of the PEA's $[(\exists\mathbf{u})\mathbf{\Delta}\langle\mathbf{u}\rangle]$ and $[(\hat{\cdot}\,\mathbf{u})V(\mathbf{\Delta}\langle\mathbf{u}\rangle)]$ as its constituent part, i.e. if $\mathbf{\Gamma}$ has a form $\mathbf{\Gamma}\langle(\exists\mathbf{u})\mathbf{\Delta}\langle\mathbf{u}\rangle\rangle$ or $\mathbf{\Gamma}\langle(\hat{\cdot}\,\mathbf{u})V(\mathbf{\Delta}\langle\mathbf{u}\rangle)\rangle$, then an occurrence of $\mathbf{u}$ in $\mathbf{\Gamma}$ is called a *bound occurrence* of $\mathbf{u}$ in $\mathbf{\Gamma}$. It goes without saying that $[(\exists\mathbf{u})\mathbf{\Delta}\langle\mathbf{u}\rangle]$ and $[(\hat{\cdot}\,\mathbf{u})V(\mathbf{\Delta}\langle\mathbf{u}\rangle)]$ are themselves PEA's of A_1, which have a bound occurrence of the APVOT $\mathbf{u}$.

3) Under the assumption that $\mathbf{\Gamma}\langle\mathbf{u}\rangle$ does not involve, e.g., $\mathbf{v}$ in any occurrence, free or bound, $\mathbf{\Gamma}\langle\mathbf{v}\rangle$ is defined as:

$$\mathbf{\Gamma}\langle\mathbf{v}\rangle \rightarrow \mathrm{S}^{\mathbf{u}}_{\mathbf{v}}\mathbf{\Gamma}\langle\mathbf{u}\rangle\big|, \tag{5.23}$$

where '$\mathrm{S}^{\mathbf{u}}_{\mathbf{v}}\ \big|$' is the *metalinguistic operator of substitution* such that $\mathrm{S}^{\mathbf{u}}_{\mathbf{v}}\mathbf{\Gamma}\langle\mathbf{u}\rangle\big|$ is the PEA of A_1 resulting by substitution of $\mathbf{v}$ for $\mathbf{u}$ throughout $\mathbf{\Gamma}\langle\mathbf{u}\rangle$.

4) Like ‘$\mathbf{\Gamma}\langle\mathbf{u}\rangle$’ or ‘$\mathbf{\Delta}\langle\mathbf{u}\rangle$’, ‘$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle$’, e.g., is a DPAnMML (DPAnMMLPH), whose range is the class of PEA’s of A_1, any given member $\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle$ of which is a PEA of A_1 that involves certain *free* occurrences of two given different AEOT’s **u** and **v** and perhaps free or bound occurrences of some other AEOT’s **w**, **x**, etc, which are not mentioned by using the metalogograph ‘$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle$’. If **x** and **y** are two different AEOT’s, other than **u** and **v**, which do not occur in $\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle$ either as free AEOT’s or as bound APVOT’s, then

$$\begin{gathered}\mathbf{\Gamma}\langle\mathbf{u},\mathbf{x}\rangle \to \mathrm{S}^{\mathbf{x}}_{\mathbf{v}}\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle|, \mathbf{\Gamma}\langle\mathbf{x},\mathbf{v}\rangle \to \mathrm{S}^{\mathbf{x}}_{\mathbf{u}}\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle|, \mathbf{\Gamma}\langle\mathbf{x},\mathbf{y}\rangle \to \mathrm{S}^{\mathbf{y}}_{\mathbf{v}}\mathbf{\Gamma}\langle\mathbf{x},\mathbf{v}\rangle|,\\ \mathbf{\Gamma}\langle\mathbf{x},\mathbf{x}\rangle \to \mathrm{S}^{\mathbf{x}}_{\mathbf{y}}\mathbf{\Gamma}\langle\mathbf{x},\mathbf{y}\rangle|, \mathbf{\Gamma}\langle\mathbf{y},\mathbf{y}\rangle \to \mathrm{S}^{\mathbf{y}}_{\mathbf{x}}\mathbf{\Gamma}\langle\mathbf{x},\mathbf{y}\rangle|, \text{etc.}\end{gathered} \tag{5.24}$$

The DPAnMML’s (DPAnMMLPH) ‘$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$’, ‘$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{x}\rangle$’, etc and their any congeneric variants are defined in analogy with ‘$\mathbf{\Gamma}\langle\mathbf{u}\rangle$’ and ‘$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle$’.

5) If it is necessary to indicate that, for instance, the occurrences of **u** or of **v** or of both in **Γ** are bound, without indicating a specific character of the bondages, then the following *indexed metalogographs,* called also *contracted metalogographs,* can be used: ‘$\mathbf{\Gamma}_{\langle\mathbf{u}\rangle}\langle\mathbf{u}\rangle$’, ‘$\mathbf{\Gamma}_{\langle\mathbf{u}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$’, ‘$\mathbf{\Gamma}_{\langle\mathbf{u}\rangle}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$’, etc, or ‘$\mathbf{\Gamma}_{\langle\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$’, ‘$\mathbf{\Gamma}_{\langle\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$’, etc; or else ‘$\mathbf{\Gamma}_{\langle\mathbf{u},\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$’, ‘$\mathbf{\Gamma}_{\langle\mathbf{u},\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$’, etc, respectively. To avoid serious terminological conflicts, such an indexed (contracted) ML (MLPH) is qualified *molecular,* so that it is commonly called a *DPAnMML* (*DPAnMMLPH*) or *PAnMMLD* (*PAnMMLPHD*) – just as its *unindexed* (*index-free*) counterpart. When an *indexed* PAn*A*ML, e.g. ‘$\mathbf{\Gamma}_1$’, is used in place of the *base index-free letter* ‘**Γ**’, an angle-bracketed subscript should be written after the numeral subscript, e.g. ‘$\mathbf{\Gamma}_{1\langle\mathbf{u}\rangle}$’, ‘$\mathbf{\Gamma}_{1\langle\mathbf{v}\rangle}$’, ‘$\mathbf{\Gamma}_{1\langle\mathbf{u},\mathbf{v}\rangle}$’, etc.

6) In the above items 1, 2, 4, and 5, I use the noun “description” in the same sense, in which I use it in Df 5.1(6b). For instance, ‘$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle$’ *is the metalogographic description of the species of PEA’s of* A_1 *through the genus* **Γ**, *denoted by the generic name* ‘**Γ**’, *and through the differenntia (difference)* $\langle\mathbf{u},\mathbf{v}\rangle$, *denoted by the qualifier* ‘$\langle\mathbf{u},\mathbf{v}\rangle$’. Likewise, ‘$\mathbf{\Gamma}_{\langle\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$’, e.g., is the metalogographic description of the species of PEA’s of A_1 through the genus **Γ**, denoted by the generic name ‘**Γ**’, and through the

differentiae (differences) $\langle\mathbf{v}\rangle$ and $\langle\mathbf{u},\mathbf{v}\rangle$, denoted by the qualifiers '$\langle\mathbf{v}\rangle$' and '$\langle\mathbf{u},\mathbf{v}\rangle$' respectively. Accordingly, given a genus $\mathbf{\Gamma}$ of PEA's of A_1, the *species* $\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle$ and $\mathbf{\Gamma}_{\langle\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$ of PEA's of A_1 are alternatively called *sorts* of PEA's of the genus $\mathbf{\Gamma}$, whereas either of the PAnMMLD's (DPAnMML's) '$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle$' and '$\mathbf{\Gamma}_{\langle\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$ is called a *sorting*, or *descriptive*, *primary analytical molecular metalogograph* (*StgPAnMML* or *DPAnMML*) *of PEA's of* $\mathbf{\Gamma}$, – to use the qualifier "*sorting*" as an antonym of both "*generalizing*" and "*forming*" or "*formative*", i.e. "*form giving*". Accordingly, a *semantic definition* of one or more StgPAnMML's, e.g. the above item 1 or 4 or the totality of the above items 1–5, is called a *primary analytical metalogographic sortation rule* (briefly *PAnMLStnR* or *PAnMLSR* whenever confusion cannot result) *of PEA's of* $\mathsf{A}_{1,}$ – in contrast to both a *primary analytical metalogographic generalization rule* (*PAnMLGR*) *of* A_1 (as Df 5.10(1)) and a *primary analytical metalogographic formation rule* (*PAnMLFR*) *of* A_1 (to be defined and illustrated in due course). A PAnMMLD (DPAnMML, StgPAnMML), index-free (not contracted) or indexed (contracted), can be interpreted by a *concrete* PEA of A_1 of its range only *as a single whole in accordance with its semantic definition*, i.e. *in accordance with the pertinent PAnMLSR of PEA's of* A_1, *and not as a schema by interpreting its separate constituent PAnAML's*. This is the very reason for qualifying a PAnMMLD of PEA's of A_1 *descriptive* (or a *description*), and *not schematic* (or *not a schema*) and also for qualifying it *molecular*.

7) If in a certain statement, the range of $\mathbf{\Gamma}$, e.g., is restricted *ad hoc* to a certain class PEA's of A_1 as that the class of *PEI's* (*primary euautographic integrons*) or *PER's* (*primary euautographic relations*) *of* A_1 then any of the PAnMMLD's (PMML's) introduced above in the items 1–5, which involves the letter '$\mathbf{\Gamma}$', determines a certain *sort of the PEI's or PER's of* A_1 and it will therefore be called *ad hoc* a *StgPAnMML of the PEI's* or *PER's of* A_1, respectively.

8) As was already indicated in Df 5.10(1), in contrast to StPL's (see Df 4.2(4b)), I do not introduce any structural matalogographs (StML's). Therefore, the occurrences of the qualifier "analytical" ("An") in the taxonyms (metaterms) that have been introduced in Df 5.10 and in the items 1, 5, and 6 of this definition are redundant. However, I preserve those occurrences for convenience in incorporating the above taxonyms into the comprehensive taxonomy of metalogographic and

panlogographic placeholders, which is used in this treatise and which contains the variants of those taxonyms with “pan” (“P”) in place of “meta” (“M”).

9) The qualifier “molecular” (“M”) to a pasigraph (euautograph or logograph) is a subterm (restriction) of the qualifier “combined” (“Cb”) – a subterm, any use of which should be subject to the pertinent definition, as the items 1,4, and 5 is of this definition. However, owing to its special integrity, *any combined metalogographic* or *panlogographic description* can specifically be qualified *molecular* (“*M*”). Therefore, the qualifier “molecular” and its abbreviation “M” can particularly be omitted from all above taxonyms of *molecular metalogographic descriptions* (*MMLD’s*) without altering their meanings. Meanwhile, I have preserved occurrences of that qualifier and its abbreviation for more clarity.

10) The above items 1, 3, and 4 can in principle apply with any PStAPLPH’s introduced in the items 2-4 of Df 5.2 in place of those introduced in the item 1 of that definition. However, such variants of ‘$\mathbf{\Gamma}\langle\mathbf{u}\rangle$’, ‘$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle$’, ‘$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$’, etc are not usable

6. An introduction into the global system of classification rules of A_1 and formation rules of $\mathbf{A}_1$

6.1. An introduction into the formation, generalization, and sortation rules of A_1 and into the formation rules of $\mathbf{A}_1$

Df 6.1: ***A general outline of the formation, generalization, and sortation rules of*** A_1**.** 1) The *formation rules* (*FR’s*) *of euautographs of* A_1 are divided into two overlapping classes: the *FR’s of euautographic formulas* (*EF’s*), or *euautographic categoremata* (*ECtg’ta*) *of* A_1 and the *FR’s of euautographic syncategoremata* (*ESctg’ta*) *of* A_1. The FR’s of EF’s (ECtg’ta) of A_1, called also *formulary*, or *categorematic*, *FR’s* (*FFR’s* or *CtgFR’s*) *of* A_1, determine *EF’s of* A_1 *of each one of following three genera* (*hypertaxons*, *hypertaxa*, *general taxonomic classes*, *taxonomic superclasses*) *or of some of their species* (*hypotaxons*, *hypotaxa*, *specific taxonomic classes*, *taxonomic subclasses*): the *euautographic ordinary terms* (*EOT’s*), the *euautographic integrons* (*EI’s*), called also *euautographic special terms* (*ESpT’s*), and the *euautographic relations* (*ER’s*). The FR’s of ESctg’ta of A_1, called also *syncategorematic FR’s* (*SctgFR’s*) *of* A_1, determine *secondary euautographic operators* (*SEO’s*) or *just secondary euautographic kernel-signs* (*SEKS’s*) *of* A_1. The four generic taxonyms "*euautographic formula*" ("*EF*"), "*euautographic categorem*" ("*ECtg*", pl "*ECtg'ta*"), "*formulary euautograph*" ("*FE*"), and "*categorematic euautograph*" ("*CtgE*") or the two generic taxonyms "*euautographic syncategorem*" ("*ESctg,* pl "*ESctg'ta*") and "*syncategorematic euautograph*" ("*SctgE*") are synonyms. For more clarity, I may occasionally use either of the redundant synonymous taxonyms "FFR of EF's of A_1" and "CtgFR of EF's of A_1" instead of "FFR of A_1" or "CtgFR of A_1" respectively. A CtgFR (FFR) or SctgFR of A_1 is indiscriminately called a *formation rule* (*FR*) *of* A_1. Accordingly, the meaning of the generic taxonym "formation rule" ("FR") as used in this treatise differs from the meaning of its homonym, which is used in Church [1956, p. 50]. A SctgFR of A_1 is, most often, contextually implied by a certain CtgFR of A_1, Therefore, I shall as a rule use the taxonym "formation rule" ("FR") *synecdochically* as an *abbreviation* of either of the synonymous taxonyms "categorematic formation rule" ("CtgFR") and "formulary formation rule" ("FFR") if there is no danger of misunderstanding. The above definitions and conventions apply with "*panlogographic*" and "*panlogograph*" (abbreviated as "*PL*" both) in place of "*euautographic*" and "*euautographic*" (abbreviated as "*E*" both) and with '$\mathbf{A}_1$' in place of 'A_1'.

2) There are many different kinds of FFR's (FR's) of EF's of A_1 and many different kinds of FFR's (FR's) of PLF's of $\mathbf{A}_1$, which will be specified as I go along. In this case, the generic *taxonym* (*taxonomic name*, *metaterm*) "*formulary formation rule*" ("*FFR*") is *semantically* distinguished from each any of the three generic taxonyms: "*formulary generalization rule*" ("*FGR*"), "*formulary sortation rule*" ("*FSR*"), "*formulary classification rule*" ("*FCR*"), and "*formulary unification rule*" ("*FUR*"), subject to the following definitions (cf. Df 5.1(6–8)).

a) In agreement with the items 6a an 8c of Df 5.1, a *formulary panlogographic,* or *panlogographic formulary, generalization rule* (*FPLGR* or *PLFGR*), or briefly (synecdochically) *panlogographic generalization rule* (*PLGR*), *of EF's of* A_1 is a *semantic definition,* by which one or more *formulary atomic panlogographs* (*FAPL's*), i.e. *atomic panlogographic formulas* (*APLF's*), *of* $\mathbf{A}_1$ are specified as *elements of* $\mathbf{B}_{1F}$ ($\mathbf{B}_{1Ctg}$), called *formulary atomic panlogographs* (*FAPL's*), or *connotative formulary atomic panlogographic taxonym*s (*CntFAPLTxms*), *of* (*belonging to*) $\mathbf{A}_1$, whose *range* (*designatum)* is either a certain one of the three genera of EF's of A_1: the EOT's, the EI's (ESpT's), and the ER's or a certain species of a given genus. In the general case, a PLGR of EF's of A_1 can be *compound* (*complex-coordinate*), i.e. one that introduces several different sets of elements of $\mathbf{B}_{1F}$.

b) In agreement with the items 6b of Df 5.1, a *formulary panlogographic*, or *panlogographic formulary*, *sortation*, or *specification, rule* (*FPLSR* or *PLFSR*), or briefly (synecdochically) *panlogographic sortation*, or *specification*, *rule* (*PLSR*), *of EF's of* A_1 is a *semi-verbal semantic definition*, or, more specifically, a *definitio [species] per genus et differentiam* (or *differentias*), by which one or more *descriptive [analytical molecular] panlogographic formulas* (*DAnMPLF's*, *DPLF's*), or *[analytical molecular] formulary panlogographic descriptions* (*AnMFPLD's, FPLD's*), *of* $\mathbf{A}_1$ are specified as *elements of* $\mathbf{B}'_{1F}$ ($\mathbf{B}'_{1Ctg}$), called also *connotative descriptive panlogographic taxonyms* (*CntDPLTxms*), *of* (*belonging to*) $\mathbf{A}_1$. The *range* (*class-connotatum*), called impartially the *designatum,* of an FPLD (AnMFPLD) is is a certain *species*, called a *sort*, either of the genus of EI's (ESpT's) of A_1 or of the genus of ER's of A_1 – a sort of EF's (either EI's or ER's) of A_1, which is distinguished from any other sort of EF's of the same genus by a certain general *compositional* peculiar property of its specimens (members) that is prescinded from, and hence independent of, the specific *form* of any concrete specimen. Accordingly, I regard the FPL*D*, i.e. FPL-*description*, itself as a *panlogographic descriptio [species] per genus et differentiam* (or *differentias*), i.e. as a *panlogographic description of the given species* (*sort*) *of EI's or ER's of* A_1 *through the pertinent one of the two genera of EF's of* A_1 *and through the pertinent difference* (or *differences*) *designated by the appropriate panlogographic qualifiers* (to be specified in due course by properly interpreting Df 5.11). Accordingly, when a BscAnMPL (BscMPL) of $\mathbf{A}_1$ is used *xenonymously*, it is called a *BscAnMPLPH* (*BscMPLPH*) *of EF's* (*EI's* or *ER's*) *of* A_1. Incidentally, the generic name "*composition rule*" is sometimes used in logic as a synonym of "formation rule". However, in agreement with the above remark, the former can alternatively be used as a synonym of both "sortation rule" and "specification rule", subject to the pertinent sense of the noun "composition". Just as a PLGR, a PLSR of EF's of A_1 can, in the general case, be compound (complex-coordinate), i.e. one that introduces several different sets of *elements of* $\mathbf{B}'_{1F}$ (CntCbPLTxms of $\mathbf{A}_1$) such that all elements of $\mathbf{B}'_{1F}$ being members of each given set designate the same sort of EF's of A_1 of the same genus. That is to say, a compound

PLSR of EF's of A_1 introduces several different sorts of EF's of A_1 of one or more different genera.

c) In accordance with Df 5.1(7), an *FPLGR* (*PLFGR*), *FPLSR* (*PLFSR*), or *FPLFR* (*PLFFR*) *of EF's of* A_1 is indiscriminately called a *formulary panlogographic*, or *panlogographic formulary*, *classification rule* (*FPLCR* or *PLFCR*) *of EF's of* A_1.

d) A *formulary metalogographic*, or *metalogographic formulary*, *unification rule* (*FMLUR* or *MLFUR*), or briefly (synecdochically) *metalogographic unification rule* (*MLUR*), *of EF's of* A_1 is a *semantic definition*, by which one or more *formulary atomic metalogographs* (*FAML's*), i.e. *atomic metalogographic formulas* (*AMLF's*), *of* (*belonging to*) *the IML* (*inclusive metalanguage*) *of* A_1 *and* $\mathbf{A}_1$, are specified as *analytical atomic metalogographic* (*metalinguistic logographic*) *placeholders* (*AnAMLPH's*), i.e. as *connotative atomic metalogographic taxonyms* (*CntAMLTxms*), *of the IML*, whose *range* (*designatum*) is either the *union* of some two or all of the three genera of EF's of A_1: the EOT's, the EI's (ESpT's), and the ER's or the *union* of some one of the three genera and of a certain species of another genus. Such an AnAMLPH is called a *unifying AnAMLPH* (*UAnAMLPH*), or *unifying CntAMLTxm* (*UCntAMLTxm*), *of EF's of* A_1. For convenience in description (making general statements) and study of EF's of A_1, I may occasionally introduce some UAnAMLPH's of EF's of A_1 by laying down the appropriate MLUR's. However, all such UAnAMLPH's and hence the MLUR's introducing them are irrelevant to the setup of A_1; they are auxiliary and dispensable. That is to say, a UAnAMLPH of EF's of A_1 is a *classification rule* (*CR*) *of the IML of* A_1 *and* $\mathbf{A}_1$, and not one of either of the organons A_1 and $\mathbf{A}_1$ (cf. Df 5.1(6)). For instance, each of the bold-faced upright capital Greek letters '**Φ**', '**Ψ**', and '**Ω**', alone or furnished with any one of the upright Arabic digital subscripts '$_1$', '$_2$', etc in this font, is utilized in the treatise as a *universal* (*comprehensive*) *UAnAMLPH* (*UUAnAMLPH*, *UUCntAMLTxm*), whose range is the *class of all EF's of* A_1, i.e. of all EOT,s, EI's, and ER's, primary and secondary, which are determined by any given stage of development of A_1. However, the above UUAnAMLPH's are usable only in the IML of A_1 and $\mathbf{A}_1$ and not in A_1 and $\mathbf{A}_1$ themselves.

e) *Explanatorily (for more clarity) and hence redundantly*, a PLGR of EF's of A_1 is called an *abstract PLGR* (*AbPLGR*), or *PLGR-abstractum* (*PLGRA*, pl. "*PLGRA'ta*"), *of EF's of* A_1; a *PLSR of EF's of* A_1 is called a *descriptive PLSR*

(*DPLGR*), or *PLSR-description* (*PLGRD*), *of EF’s of* A_1; and an *MLUR of EF’s of* A_1 is called an *abstract MLUR* (*AbMLUR*), or *MLUR-abstractum* (*MLURA*, pl. “*MLURA’ta*”), *of EF’s of* A_1.

f) The above items a) and b) are, in principle, applicable word for word with “meta” in place of “pan”, “PL” in place of “E”, ‘$\boldsymbol{\mathsf{A}}_1$’ in place of ‘$\mathsf{A}_1$’, and “the IML of A_1 and $\boldsymbol{\mathsf{A}}_1$ in place of ‘$\boldsymbol{\mathsf{A}}_1$’, whereas the item c) in the exclusion of the specific example given there is, in principle, applicable verbatim with the first three of the four substitutions just indicated. In accordance with the above variant of the item a), I may occasionally introduce some *AnAMLPH’s* (*CntAMLTxms*), *of* (*belonging to*) *the IML of* $\boldsymbol{\mathsf{A}}_1$, e.g. ‘‘$\boldsymbol{\mathsf{P}}$’, ‘$\boldsymbol{\mathsf{Q}}$’, and ‘$\boldsymbol{\mathsf{R}}$’ (see Df 4.3(5,6) and subsection 4.4(3)), by stating the pertinent *metalogographic generalization rule* (*MLGR*) *of PLF’s of* $\boldsymbol{\mathsf{A}}_1$, but I do this occasionally (not systematically) and informally, and I do not utilize such AnAMLPH’s in any formal ADP’s (algebraic decision procedures) of $\boldsymbol{\mathsf{A}}_1$. At the same time, *I do not lay down either any metalogographic sortation rules* (*MLSR’s*) *of PLF’s of* $\boldsymbol{\mathsf{A}}_1$ *or any MLUR’s of PLF’s of* $\boldsymbol{\mathsf{A}}_1$.

3) Unless stated otherwise, I shall use the generic taxonyms “*formation rule*” (“*FR*”), “*generalization rule*” (“*GR*”), and “*sortation rule*” (“*SR*” or “*StnR*”), synecdochically, not only as an abbreviation of the generic taxonyms “*formulary formation rule*” (“*FFR*”), “*formulary generalization rule*” (“*FGR*”), and “*formulary sortation rule*” (“*FSR*”), but also as abbreviations of the generic taxonyms “*systemic formation rule*” (“*SysFR*”), “*systemic generalization rule*” (“*SysGR*”), and “*systemic sortation rule*” (“*SysSR*”), whereas the qualifier “systemic” means: *belonging to the global system of formulary classification rules*, i.e. *formulary formation, generalization, and sortation rules* (briefly *GbFCR-system* or *GbFFGSR-system*) *of* A_1, i.e. *of* A_1 *and* $\boldsymbol{\mathsf{A}}_1$, which will be specified as I go along. Thus, “FR”, “GR”, and “SR” are after all abbreviations the self-explanatory abbreviations “*SysFFR*”, “*SysFGR*”, and “*SysFSR*” respectively. A *classification rule* (briefly *ClsR* or *CR* whenever confusion cannot result), i.e. an *FR*, *GR*, or *SR*, [*of EF’s*], *of* A_1 or an *FR* [*of PEF’s*] *of* $\boldsymbol{\mathsf{A}}_1$ (cf. Df 5.1(6)), which is not systemic, is called an *extra systemic CR* (*ExSysCR*), i.e. an *ExSysFR*, *ExSysGR*, or *ExSysSR*, *of* A_1 or $\boldsymbol{\mathsf{A}}_1$ with the proviso that I shall likely have no occasion to make any ExSysSR’s. Until the GFCSR-system is specified, the qualifier “systemic” can be understood as “*mandatory*” (“*obligatory*”) or “*indispensable*”, while “extra systemic” can be understood as “*alternative*”

(“*optional*”) or “*auxiliary*”. Henceforth, by an FR, GR, or SR, [of EF’s] of A_1 or by an FR [of PEF’s] of $\mathbf{A}_1$, I mean a *systemic* one, unless stated otherwise.

4) All FR’s of A_1 and all FR of $\mathbf{A}_1$ are *metalinguistic statements* in the sense that they are *made in the IML* (*inclusive metalanguage*) *of* A_1, i.e. of A_1 and $\mathbf{A}_1$, and that therefore they belong to the IML Consequently, the possessive postpositive qualifiers “of A_1” and “of $\mathbf{A}_1$” to “formation rule” (“FR”) should be understood as abbreviations of the *transitive* possessive postpositive qualifiers “*of euautographic formulas of* A_1” (“*of EF’s of* A_1”) and “*of panlogographic formulas of* $\mathbf{A}_1$” (“*of PLF’s of* $\mathbf{A}_1$”) respectively, while the hypertaxonym (generic taxonym) “*formula*” (“*F*”) should be understood as a placeholder of any one of the hypotaxonyms (specific taxonyms) “*ordinary term*” (“*OT*”), “*integron*” (“*I*”) or “*special term*” (“*SpT*”), and “*relation*” (“*R*”) and also of any appropriate hypotaxonyms of the latter such as “*OR*” (“*ordinary relation*”), “*SpR*” (“*special relation*”), “*LR*” (“*logical relation*”), or “*AlR*” (“*algebraic relation*”), or such as any hypotaxonym involving either of the additional prepositive qualifiers “*pseudo-variable*” (“*PV*”) and “*pseudo-constant*” (“*PC*”) in the presence of the postpositive qualifier “*of* A_1” or either of the additional prepositive qualifiers “*variable*” (“*V*”) and “*constant*” (“*C*” or “*Cst*”) in the presence of the postpositive qualifier “*of* $\mathbf{A}_1$”.

5) Most generally, any given FR (SysFFR) [of EF’s] of A_1 is one of the following three kinds (taxa, taxonomic classes):

a) a *euautographic FR* (*EFR*) [*of EF’s*] *of* A_1, which is *explanatorily (for more clarity) and hence redundantly* called also a *concrete EFR* (*CEFR*), or *euautographic FR-concretuma* (*EFRC*, pl. “*EFRC’ta*”), [*of EF’s*] *of* A_1;

b) a *panlogographic FR* (*PLFR*) [*of EF’s*] *of* A_1, which is *explanatorily and redundantly* called also a *schematic PLFR* (*SchPLFR*), or *PLFR-schema* (*PLFRS*, pl. “*PLFRS’ta*”), [*of EF’s*] *of* A_1;

c) a *metalogographic FR* (*MLFR*) [*of EF’s*] *of* A_1, which is *explanatorily and redundantly* called also a *schematic MLFR* (*SchMLFR*), or *MLFR-schema* (*MLFRS*, pl. “*MLFRS’ta*”), [*of EF’s*] *of* A_1,

subject to the following definitions.

a′) An EFR (CEFR, EFRC) [of EF’s] of A_1 is an FR of some *concrete decimal digital EI’s* (*DDEI’s*) of A_1 and vice versa. Since the DDEI’s belong to both A_1 and

$\mathbf{A}_1$, therefore an *EFR of DDEI's of* A_1 is at the same time an *EFR of DDEI's of* $\mathbf{A}_1$ and also an *EFR of DDEI's of* A_1.

b′) A PLFR (SchPLFR, PLFRS) [of EF's] of A_1 introduces a large number (usually an infinite number) of EF's of A_1 as members of the range of a *concrete PLF* (*panlogographic formula*), i.e. *FPLPH* (*formulary panlogographic placeholder*), *of* $\mathbf{A}_1$, being the, or a, *syntactic subject* of the PLFRS that is used *xenonymously*. Using all its syntactic subjects *autonymously*, the PLFRS can be *mentally metamorphosed* (*be regarded*) and, when desired, be explicitly restated, in terms of the pertinent *HAQ's* (*homolographic autonymous quotations*) and, if appropriate, *QHAQ's* (*quasi-homolographic autonymous quotations*), as the respective *PLFR* [*of PLF's*] *of* $\mathbf{A}_1$, which is *explanatorily and redundantly* called also a *concrete PLFR* (*CPLFR*), or *PLFR-concretum* (*PLFRC*, pl. "*PLFRC'ta*"), [*of PLF's*] *of* $\mathbf{A}_1$; either of the latter three taxonyms is a taxonym of the PLFRS of EF's of A_1 in its metamorphosed hypostasis. Using all its syntactic subjects in the *TAEXA-mode*, the two hypostases of the PLFR [of EF's] of A_1 can be regarded as a *single whole PLFR of* A_1, i.e. *of* A_1 and $\mathbf{A}_1$ [as if] simultaneously. Therefore, in making most general statements about PLFR's of A_1 and $\mathbf{A}_1$, it is often convenient to treat of PLFR's of A_1, rather than of PLFR's of A_1 and PLFR's of $\mathbf{A}_1$ separately. Consequently, with allowance for the previous item a′), *an FR of* A_1 *is either an EFR's of* A_1 *or a PLFR of* A_1.

c′) Like a PLFR [of EF's] of A_1, an MLFR (SchMLFR, MLFRS) [of EF's] of of A_1 introduces a large number (usually an infinite number) of EF's of A_1 as members of the range of a *concrete MLF* (*metalogographic formula*), i.e. *FMLPH* (*formulary panlogographic placeholder*), *of the IML of* A_1 *and* $\mathbf{A}_1$, being the, or a, *syntactic subject* of the MLFRS. However, in contrast to a PLF of $\mathbf{A}_1$, which is the, or a, syntactic subject of a certain PLFR, which is universal in the sense that it preserves its recognizable semantic identity in the widest scope throughout the treatise, the above-mentioned MLF is an *ad hoc* one and it is not therefore a formula of any organon. Consequently, an MLFRS of EF's of A_1 cannot be metamorphosed as a concrete FR's of PLF's of $\mathbf{A}_1$. PLFR's A_1 are stated in the treatise systematically, whereas I lay down seven and only seven systemic MLFR's of A_1 at the very beginning of the setup of A_1 and $\mathbf{A}_1$. All the rest of systemic *schematic FR's*

(*SchFR's*), i.e. *FR-schemata* (*FRS'ta*), [*of EF's*] *of* A_1 that are laid down in the treatise are panlogographic ones (SchPLFR's, PLFRS'ta.).

6) The synonymous taxonyms that have been introduced in the previous item are interrelated as follows.

a) The prepositive qualifier "concrete" ("C") or the appositive (postpositive) qualifying noun "concretum" to "EFR" is redundant independent of the fact, which one of the possessive postpositive qualifiers '*of* A_1', '*of* $\mathbf{A}_1$', and '*of* A_1' or of their variants with 'A_1^0', '$\mathbf{A}_1^0$', and 'A_1^0', or 'A_0', '$\mathbf{A}_0$', and 'A_0', in place of 'A_1', '$\mathbf{A}_1$', and 'A_1' respectively is attributed to "EFR".

b) The prepositive qualifier "schematic" ("Sch") or the appositive (postpositive) noun "schema" ("S") to "PLFR" is redundant if and only if "PLFR" is followed by either of the possessive postpositive qualifier "of EF's of A_1" and "of A_1" or by its variant with 'A_1^0' or 'A_0' in place of 'A_1'; "EF" is, as before, a placeholder for "EOT", "EI" ("ESpT"") or "ER" or for any appropriate hypotaxonym of any of the three latter taxonyms. A like statement applies with "MLFR" in place of "PLFR". Accordingly, a PLFR or MLFR of EF's of A_1 is indiscriminately called a *logographic FR* (*LgFR* or *LFR* whenever confusion cannot result) *of EF's of* A_1, and also *explanatorily and redundantly* a *schematic LFR* (*SchLFR*), or *LFR-schema* (*LFRS*, pl. "*LFRS'ta*"), *of EF's of* A_1.

c) The prepositive qualifier "concrete" ("C") or the appositive (postpositive) qualifying noun "concretum" ("C") to "PLFR" is redundant if and only if "PLFR" is followed by either of the possessive postpositive qualifier "of PLF's of $\mathbf{A}_1$" and "of $\mathbf{A}_1$" or by its variant with '$\mathbf{A}_1^0$' or '$\mathbf{A}_0$' in place of '$\mathbf{A}_1$'; "PLF" is, as before, a placeholder for "PLOT", "PLI" ("PLSpT"") or "PLR" or for any appropriate hypotaxonym of any of the three latter taxonyms.

d) Either of the synonymous generic taxomyms "SchPLFR" and "PLFRS", or "SchMLFR" and "MLFRS", should explicitly be followed or be obviously understood as followed by either of the possessive postpositive qualifier "of EF's of A_1" and "of A_1" or by its variant with 'A_1^0' or 'A_0' in place of 'A_1',

e) Either of the synonymous generic taxomyms "CPLFR" and "PLFRC" should explicitly be followed or be obviously understood as followed by either of the possessive postpositive qualifier “of PLF’s of $\mathbf{A}_1$” and “of $\mathbf{A}_1$” or by its variant with ‘$\mathbf{A}_1^0$’ or ‘$\mathbf{A}_0$’ in place of ‘$\mathbf{A}_1$’.

7) The adjective “concrete” (“C”) and of the kindred noun “concretum” (“C”, pl. “concreta” – “C’ta”), or the adjective “schematic” (“Sch”) and the kindred noun “schema” (“S”, pl. “schemata” – “S’ta”), which have been used as qualifiers in the item 5, are *antonyms* respectively of the adjective “abstract” (“Ab”) and the kindred noun “abstractum” (“A”, pl. “abstracta” – “A’ta”), or the adjective “descriptive” (“D”) and of the kindred noun “description” (“D”), which have been used as qualifiers in the item 2d. At the same time, the synonymous taxonyms that have been introduced in the item 2d are interrelated as follows.

a) The prepositive qualifier “abstract” (“Ab”) or the appositive (postpositive) qualifying noun “abstractum” (“A”) to “PLGR” is redundant if and only if “PLGR” is followed by either of the possessive postpositive qualifier “of EF’s of A_1” and “of A_1” or by its variant with ‘A_1^0’ or ‘A_0’ in place of ‘A_1’; “EF” is, as before, a placeholder for “EOT”, “EI” (“ESpT””) or “ER” or for any appropriate hypotaxonym of any of the three latter taxonyms. A like statement applies with “MLUR” in place of “PLGR” with the proviso that in this case “EF” is used a count name and not a placeholder of another count name. Therefore, it is impossible to unite the set of PLGR’s of *discriminate EF’s* (*EOT’s or EI’s or ER’s*) *of* A_1 and the set of MLUR’s *indiscriminate EF’s* (*EOT’s and EI’s and ER’s*) of A_1 into a single whole semantically homonymous set under the same generic taxonym “*LGR’s*” (“*AbLGR’s*”, “*LGRA’ta*”) after the manner of “*LFR’s*” (“*SchLFR’s*” , “*LFRS’ta*”) as done in the item 6b.

b) The prepositive qualifier “descriptive” (“D”) or the appositive (postpositive) qualifying noun “description” (“D”) to “PLSR” is redundant if and only if “PLSR” is followed by either of the possessive postpositive qualifier “of EF’s of A_1” and “of A_1” or by its variant with ‘A_1^0’ (but not ‘A_0’) in place of ‘A_1’; “EF” is a placeholder for “EI” (“ESpT”) or “ER” (but not for “EOT”) or for any appropriate hypotaxonym of any of the two latter taxonyms.

c) Either of the synonymous generic taxomyms “AbPLGR” and “PLGRA”, or “AbMLGR” and “MLGRA”, should explicitly follow or be obviously understood as followed by either of the possessive postpositive qualifier “of EF’s of A_1” and “of A_1”

or by its variant with ‘A_1^0’ or ‘A_0’ in place of ‘A_1’, subject to the respective sense of “EF” as indicated above in the item a).

d) Either of the synonymous generic taxomyms “DPLSR” and “PLSRD” should explicitly be followed or be obviously understood as followed by either of the possessive postpositive qualifier “of EF’s of A_1” and “of A_1” or by its variant with ‘A_1^0’ (but not ‘A_0’) in place of ‘A_1’, subject to the pertinent sense of “EF” as indicated above in the item b).

8) In analogy with a PLFR of EF’s of A_1, both a PLGR of EF’s of A_1 and a PL*S*R of EF’s of A_1 can be mentally metamorphosed and, when desired, be explicitly restated in terms the pertinent HAQ’s and, if appropriate, QHAQ’s, as the corresponding *BscAPLFR* (*basic atomic PLFR*) [*of* $\mathbf{B}_{1F}$] *of* $\mathbf{A}_1$ and as the corresponding *BscMPLFR* (*basic molecular PLFR*) [*of* $\mathbf{B}'_{1F}$] *of* $\mathbf{A}_1$ respectively (see Df 5.1(6–8)).

9) There are many different syntactic forms of FR’s of A_1. Broadly speaking, an FR of A_1, is an *effective* (*monosemantic*) *veracious* (*accidentally*, or *circumstantially*, *true*) or *tautological* (*universally true*) *semi-verbal* (*mixed phonographic and pasigraphic*,) or *chaste pasigraphic statement in the IML of* A_1, which has one or more (often an infinite list of) *syntactic subjects* as the *syntactic definiendum* or *definienda* respectively to a *single predicate* as the *syntactic definiens*. Each (or particularly the, when applicable) syntactic subject of an FR of A_1 is a *pasigraphic formula* (*PSGF*), i.e. either a *euautographic formula* (*EF*) or a *logographic formula* (*LF*), a *meta* one (*MLF*) or a *pan* one (*PLF*), – a *PSGF* whose status in A_1 or $\mathbf{A}_1$ is predicated by the FR. If an FR of A_1 has many syntactic subjects (definienda) then these are either synonyms or (especially if they form an infinite list) congeners. Particularly, if an FR of A_1 has a finite or infinite (open) list of congeneric (not synonymous) PLF’s as its syntactic subjects then each of these has the same class of EF’s as its range, which is predicated (usually *connotatively*) by the FR. In accordance with the above said, in subsequent statements about FR’s, any name such as “the syntactic subjects”, “a syntactic subject”, “each syntactic subject”, etc or such as “the syntactic definienda”, “a syntactic definiendum”, “each syntactic definiendum”, which is followed by the postpositive qualifier “of FR” alone or with

some qualifiers (usually abbreviated) of its own, should be understood as "the syntactic subject" and as "the definiendum" respectively if they apply to an FR that has exactly one syntactic subject as its definiendum. At the same time, a semi-verbal FR of A_1 that has a conjunction or, what comes to the same thing, a list of many congeneric (not synonymous) pasigraphic (euautographic or logographic) syntactic subjects is a *contracted FR* in the sense that it is analogous to an ordinary contracted declarative sentence that has two or more subjects to the same predicate and that can be developed into a *compound* (*complex-coordinate*) declarative sentence. Accordingly, the use of the definite article in either of the singular names "*the* syntactic subject" and "*the* definiendum", which may occur in the sequel in a statement about an FR, should be construed either that I tacitly assume that the FR in question has exactly one syntactic subject (definiendum) or that the FR is a contracted one, which I mentally develop into the respective *compound FR* and refer to a certain one of the simple conjoined (coordinated) FR's for convenience.

10) Epistemologically, an FR of EF's of A_1 or an FR of PLF's of $\mathbf{A}_1$ is either an *inseparable collateral semantic definition of a meta-axiom* or *meta-theorem* or a *chaste semantic* (*basal*) or *syntactic* (*synonymic*) *definition*, – a definition that, in any case, introduces either a number (one or more) of *concrete* EF's (EOT's, EI's, or ER's) of A_1 or a number of *concrete* PLF's (PLOT's, PLI's, or PLR's) of $\mathbf{A}_1$, or else a *certain class* of EF's of A_1 (as the class of EOT's, EI's, or ER's or as a certain subclass of any of the above three classes), which is usually introduced *connotatively* as the range of a *concrete MLF* (*metalogographic formula*) *of the IML of* A_1 or as the range of a concrete PLF of $\mathbf{A}_1$.

11) The following definitions are in agreement with the item 3ff and also with all other relevant items of this definition:

a) An EFR of EF's of A_1 is [alternatively called] a *euautographic classification rule* (*ECR*, *EClsR*) *of EF's of* A_1 and vice versa.

b) A PLFR or a PLGR or a PLSR, of EF's of A_1 is [indiscriminately called] a *panlogographic classification rule* (*PLCR*, *PLClsR*) *of EF's of* A_1 and vice versa.

c) An MLFR of EF's of A_1 is [alternatively called] *metalogographic classification rule* (*MLCR*, *MLClsR*) *of EF's of* A_1 and vice versa,

d) A PLFR or an MLGR, of EF's of A_1 is [indiscriminately called] a *logographic classification rule* (*LCR*, *LgClsR*) *of EF's of* A_1 and vice versa.

e) A PLFR of PLF's of $\mathbf{A}_1$ is [alternatively called] a *panlogographic classification rule* (*PLCR*, *PLClsR*) *of PLF's of* $\mathbf{A}_1$ and vice versa.

In accordance with the item 3, rules of all kinds that are mentioned in the above definitions, i.e. *FR's* (*formation rules*), *GR's* (*generalization rules*), and *SR's* (*sortation rules*, *specification rules*), and hence generally *CR's* (*classification rules*) are *systemic* (*Sys*) and *formulary* (*F*) ones, i.e. *SysFFR's*, *SysFGR's*, *SysFSR's*, and *SysFCR's* respectively

12) Depending on the intrinsic syntactico-semantic properties of separate CR's (FR's, GR's, and SR's) of A_1 or $\mathbf{A}_1$ and also depending on the relative order, in which they are laid down, the specific taxonyms of CR's and of their syntactic subjects (syntactic definienda) are provided either with appropriate mutually antonymous prepositive qualifying adjectives (the same ones to both substantives when appropriate) selected, e.g., out of these: "primary" ("P") vs. "secondary" ("S"); "concrete" ("C") vs. "abstract" ("Ab"); "first kind" ("FK")", synonymous with both "semantic" ("Smn") and "basal" ("Bsl"), vs. "second kind" ("SK"), synonymous with "syntactic" ("Snt") and, depending on the context, either with "synonymic" ("Snmc") or with "synonymous" ("Snms", or equivocally "Snm" in both cases); both "descriptive" ("D") vs. both "schematic" ("Sch") and "operative" ("O"); "euautographic" ("E") vs. "logographic" ("L") and hence vs. both "metalogographic" ("ML"), and "panlogographic" ("PL"); "structural" ("St") vs. "analytical" ("An"), etc, or with kindred mutually antonymous postpositive (appositive) qualifying nouns (when available), e.g. "concretum" ("C") vs. "abstractum" ("A") or both "description" ("D") vs. both "schema" ("S") and "operand" ("O"). The meanings of the above qualifiers and of any other qualifiers in use will be explained as I go along. Depending on the mental attitude that I take towards an FC of A_1 and its syntactic subjects, i.e. depending on the specific mental mode, namely, a xenonymous, autonymous, or TAEXA one, in which I use the syntactic subjects and all relevant placeholders occurring in the FC, *I may call the FC or their syntactic subjects differently*.

13) The previous and subsequent items of this definition and also any subsequent statements, in which any of the logographs 'A_1', 'A_1', and '$\mathbf{A}_1$' occur, apply, with the corresponding changes (mutatis mutandis) or without any changes,

with ‘ A_1^0 ’, ‘ $\mathbf{A}_1^0$ ’, and ‘ A_1^0 ’ or, but not always (see, e.g. the items 7b and 7d above), with ‘A_0’, ‘A_0’, and ‘$\mathbf{A}_0$’ in place of the respective former, unless stated otherwise. Therefore, the postpositive possessive qualifiers “of A_1”, “of A_1”, and “of $\mathbf{A}_1$”, which occur in the pertinent taxonyms, are not redundant, although they can be omitted if they are obviously understood.•

Df 6.2: ***Basic taxonomy of FR’s of*** A_1 ***and*** $\mathbf{A}_1$**.** 1) The setup of A_1 is organized in such a way that the set of FR’s of A_1 can be divided into two sets in three independent ways such that, to put it *connotatively*, an *FR* [*of EF’s*] *of* A_1 is:

a) either a *primary FR* (*PFR*) or a *secondary FR* (*SFR*), [*of EF’s*] *of* A_1;

b) either a *euautographic FR* (*EFR*) [*of EF’s*] *of* A_1, called also, explanatorily and redundantly, a *concrete* (*particular*) *EFR* (*CEFR*), or an *EFR-concretum* (*EFRC*, pl. “*FRC’ta*”), [*of EF’s*] *of* A_1 (by Df 6.1(5a)) or a *logographic FR* (*LFR*) [*of EF’s*] *of* A_1, called also, explanatorily and redundantly, a *schematic LFR* (*SchLFR*), or an *LFR-schema* (*LFRS*, pl. “*LFRS’ta*”), [*of EF’s*] *of* A_1 **(**by Df 6.1(6b);

c) either a *semantic*, or *basal*, *FR* (*SmnFR* or *BslFR*), called also an *FR of first kind* (*FK*), i.e. an *FKFR*, [*of EF’s*] *of* A_1 or a *syntactic*, or *synonymic*, *FR* (*SntFR* or *SnmFR*), called also an *FR of second kind* (*SK*), i.e. an *SKFR*, [*of EF’s*] *of* A_1;

whereas, in accordance with Df 6.1(6b),

b′) an *LFR* (*SchLFR*, *LFRS*) [*of EF’s*] *of* A_1 is either a *PLFR* (*SchPLFR*, *PLFRS*) or an *MLFR* (*SchMLFR*, *MLFRS*), [*of EF’s*] *of* A_1.

In the item c) the postpositive qualifier “of FK” or the concurrent prepositive qualifier “FK” to “FR” is a connotatively impartial denotative synonym of both “*semantic*” (“*Smn*”) and “*basal*” (“*Bsl*”), whereas either of the like qualifiers “of SK” and “SK” to “FR” is a connotatively impartial denotative synonym of both “*syntactic*” (“*Snt*”) and “*synonymic*” (‘*Snm*”). To be recalled, in accordance with Df 6.1(6), in the presence of the prepositive qualifier “euautographic” (“E”), the prepositive qualifier “concrete” or the postpositive (appositive) noun “concretum” (abbreviated as “C” both) to the generic taxonym “formation rule” (“FR”) is redundant. Likewise, in the presence of any of the prepositive qualifiers “logographic” (“Lg” or “L”), “panlogographic” (“PL”), “metalogographic” (“ML”) and of either postpositive qualifier “of EF’s of A_1”

or “of A_1”, the prepositive qualifier “schematic” (“Sch”) or the postpositive (appositive) noun “schema” to the generic taxonym “formation rule” (“FR”) is redundant. Therefore, either taxonym (metaterm) “CEFR” or “EFRC” is a redundant synonym of “EFR”, whereas either taxonym “SchLFR [of EF’s] of A_1” or “LFRS [of EF’s] of A_1” is a redundant synonym of “LFR [of EF’s] of A_1”, and similarly with “ML” or “PL” in place of “L”. Still, use of some redundant taxonyms makes no harm and it may even be useful in making some comparative statemets about FR’s of different kinds.

2) Either of the two antonymous qualifiers occurring in any one of the dichotomies 1a–1c is not necessarily compatible with either of the two antonymous qualifiers occurring in another one of those dichotomies. Therefore, the graphonym obtained by attaching the juxtaposition of two or three qualifiers, which are selected by one from certain two or three of the above dichotomies, to the generic name “*FR of EF’s of* A_1”, or to “*FR of* A_1” as its abbreviation, can be empty, i.e. it cannot be a name of any existing FR of EF’s of A_1. However, either of the two antonymous qualifiers occurring in the dichotomy 1a and either of the two antonymous qualifiers occurring in the dichotomy 1b are *mutually compatible*, so that combination of the two dichotomies yields:

a) A *PFR* [*of EF’s*] *of* A_1 is either a *euautographic PFR* (*EPFR*), called also a *primary EFR* (*PEFR*), or a *logographic PFR* (*LPFR*), called also a *primary LFR* (*PLgFR*), [*of EF’s*] *of* A_1; and similarly “*secondary*” (“*S*”) in place of “*primary*” (“*P*”).

By the items 1c and 1b′, the FR’s of the above four hypotaxa (taxonomic subclasses) are classified further as follows:

b) A *PEFR* [*of EF’s*] *of* A_1 is an *FKEFR* (*SmnEFR*, *BslEFR*) [*of EF’s*] *of* A_1 *and vice versa*, whereas an *SEFR* [*of EF’s*] *of* A_1 is an *SKEFR* (*SntEFR*, *SnmEFR*) [*of EF’s*] *of* A_1 *and vice versa*.

c) A *PLgFR* [*of EF’s*] *of* A_1 is either an *MLFR* or a *primary PLFR* (*PPLFR*, *PLPFR*), [*of EF’s*] *of* A_1 *and vice versa*, whereas an *SLgFR* [*of EF’s*] *of* A_1 is a *secondary PLFR* (*SPLFR*, *PLSFR*) [*of EF’s*] *of* A_1 *and vice versa.*

The hypotaxa (taxonomic subclasses) of FR’s that are mentioned in the predicatives of the statement c) satisfy the following relations:

d) An *MLFR* [*of EF's*] *of* A_1 is a *primary MLFR* (*PMLFR*, *MLPFR*) *of FK*, i.e. an *FKPMLFR* (*SnmPMLFR*, *BslPMLFR*), [*of EF's*] *of* A_1 and *vice versa*.

e) A *PLFR* [*of EF's*] *of* A_1 is either a *PLPFR* or a *PLSFR*, [*of EF's*] *of* A_1 *and vice versa*.

The statements c) and d) are implied by the fact that, in the setup of A_1, there is *no* MLFR [of EF's] of A_1 that could be qualified synonymic (syntactic, SK) and hence secondary ones, whereas the PLFR's [of EF's] of A_1 have no such restrictions.

In addition to the above relations among taxa (taxonomic classes) of FR's of A_1, there many others such as:

f) A PFR is an FKFR (SmnFR, BslFR) but not necessarily vice versa.

g) An SFR is either an FKFR or an SKFR (SntFR, SnmFR).

h) A SchFR is either an FKFR or an SKFR.

The taxonomy of the FR's of A_1 that has been indicated in the previous and this item, is explicated, developed further, and adjusted to $\mathbf{A}_1$ below in this definition and in the subsequent definitions of this subsection.

3) An FR of EF's of A_1 is called a *euautographic FR* (*EFR*) *of EF's of* A_1 (see Dfs 5.10(5a) and 6.2(1b) for two redundant synonyms of this taxonym) if and only if it *ostensively introduces* one or more *digital euautographic integrons* (*DEI's*), which are called the *syntactic subjects of the EFR*. That is to say, an EFR [of EF's] of A_1 is an FR of *concrete* (*ostended*, *demonstrated*) *EF's* (*CEF's*) *of* A_1 and vice versa with the proviso that every concrete EF is, in this case, a concrete DEI. Hence, each syntactic subject (syntactic definiendum) of a EFR of A_1 is a *CEF of* A_1, *but not necessarily vice versa*, because there is an indefinite (infinite) number of CEF's of A_1 of various kinds, viz. *non-digital EI's* (*NDEI's*), EOT's, and ER's, which are, when written down, also commonly called CEF's of A_1, but which are introduced by certain LFR's (FRS'ta) of A_1. Some of the latter CEF's of A_1 could be introduced by the appropriate EFR's of A_1, but in order to do so the setup of A_1 would have been modified accordingly. The occurrence of the qualifier "euautographic" ("E") in the generic taxonym "euautographic formation rule" ("EFR") or in either of its redundant synonyms ("CEFR" and "EFRC") connotes the fact that the syntactic subject or subjects of an EFR of A_1 are euautographs and that therefore they are used *autonymously*.

4) Analogously, an FR of PLF’s of $\mathbf{A}_1$ is¸ in agreement with Df 6.1(5b′), called a *panlogographic FR* (*PLFR*), [*of PLF’s*] *of* $\mathbf{A}_1$, and also, *explanatorily and redundantly*, a *concrete PLFR* (*CPLFR*), or a *PLFR-concretum* (*PLFRC*, pl. “*PLFRC’ta*”), [*of PLF’s*] *of* $\mathbf{A}_1$, if and only if it *ostensively introduces* one or more *concrete* (*ostended, demonstrated*) *PLF’s* (*CPLF’s*) *of* $\mathbf{A}_1$, which are called the *syntactic subjects of the PLFR*, and which are mentally used *autonymously*, i.e. as *tychautographs* (*accidental autographs*). That is to say, each syntactic subject (syntactic definiendum) of a PLFR (CPLFR, PLFRC) [of PLF’s] of $\mathbf{A}_1$ is a *CPLF of* $\mathbf{A}_1$ *and vice versa*. The occurrence of the qualifier “panlogographic” (“PL”) in the taxonym “PLFR of $\mathbf{A}_1$” or in either of its redundant synonyms connotes the fact that the syntactic subject or subjects of a PLFR of $\mathbf{A}_1$ are panlogographs, which are used *autonymously*, i.e. as *tychautographs*, although they can also be used in the two other mental modes, viz. *xenonymously*, i.e. as *eupanlogographs* (*euxenographs*), and *in the TAEXA-mode*, i.e. as *endosemasiopasigraphs* (*EnSPG’s*).

5) Owing to the occurrence of the postpositive qualifier “of A_1”, the occurrence of the prepositive qualifier “E” for “euautographic” in the taxonym “EFR of A_1” (e.g.) seems at first glance to be redundant. Likewise, owing to the occurrence of the postpositive qualifier “of $\mathbf{A}_1$”, the occurrence of the prepositive qualifier “PL” for “panlogographic” in the taxonym “PLFR of $\mathbf{A}_1$” is also apparently redundant. However, the above prepositive qualifiers become indispensable in some comparative statements. For instance, the DDEI or DDEI’s that is or are introduced by an EFR of A_1 belong to both A_1 and $\mathbf{A}_1$. Hence, it is natural to assert that a*n EFR of* A_1 *is an EFR of* $\mathbf{A}_1$ *and vice versa*. At the same time, the variant of this statement with “FR” in place of “EFR” is false.

6) In the setup of A_1, there are *two and only two sequential PEFR’s* (see the item 2a of this definition), the first one being called the *restricted*, or *former*, *PEFR* (*RPEFR* or *FrPEFR*) and the second one the *extendable*, or *latter*, *PEFR* (*XPEFR* or *LrEFR*), *of* A_1. The RPEFR declares the digits 0 and 1 in this Arial Narrow Font are the only *primary DEI’s* (*PDEI’s*) to be *members of the restricted class of all primary EI’s* (*PEI’s*) *of* A_1, including both these PDEI’s and all *primary non-digital EI’s* (*PNDEI’s*). The XPEFR declares the same digits 0 and 1 are the only *PDEI’s* to be *members of the extendable class of all EI’s including*, *besides the PDEI’s and the PNDEI’s*, all *secondary EI’s* (*SEI’s*), *digital* ones (*SDEI’s*) and *non-digital* ones

(*SNDEI’s*), that can be introduced into A_1 at any current stage of developing A_1. Since 0 and 1 belong to both A_1 and $\mathbf{A}_1$ (see the item 5), therefore the *RPEFR* and *XPEFR of* A_1 are at the same time ones of $\mathbf{A}_1$ and hence ones of A_1.

7) The SEFR’s of A_1 (see the item 2a) form a *single whole compact recursive system of* [*concrete*] *euautographic asymmetric synonymic definitions* (*EASD’s*), which is called the *SEFR-system of* A_1 and which successively defines the infinite set of *secondary decimal digital EI’s* (*SDDEI’s*) 2, 3, …, 9, 10, 11, …19, 20, 21, etc, or, alternatively, the infinite set of *secondary binary digital EI’s* (*SBDEI’s*) 10, 11, 100, 101, 110, 111, 1000, 1001, etc (instead of 2 to 9, etc), – in terms of the two PDEI’s 0 and 1 in both cases. Still, the *decimal system of numeration* (*DSN*) and the *binary system of numeration* (*BSN*) are incompatible if both of them are made of homolographic (photographic) tokens of the same PDEI’s 0 and 1 in this Arial Narrow Font. As a part of A_1, the BSN seems to be more natural than the DSN. Still, I employ the latter because this is more convenient owing to the force of habit and also because I shall have no occasion to use any SDDEI’s larger than 2 both in the EADP’s and in the PLADP’s. Since all DDEI’s, primary ones (PDEI’s) and secondary ones (SDDEI’s), belong to both A_1 and $\mathbf{A}_1$ (see the item 5), therefore the *SEFR-system of* A_1 is at the same time one of $\mathbf{A}_1$ and hence one of A_1.

8) In accordance with the above items 6 and 7, the item 2b can, more precisely, be restated thus. A *PEFR* [*of PDEI’s*] *of* A_1 is an *FKEFR* (*SmnEFR*, *BslEFR*) [*of PDEI’s*] *of* A_1 *and vice versa*, whereas an *SEFR* [*of SDDEI’s*] *of* A_1 is an *SKEFR* (*SntEFR*, *SnmEFR*) [*of SDDEI’s*] *of* A_1 *and vice versa*. This statement applies with ‘$\mathbf{A}_1$’ or ‘A_1’ in place ‘A_1’. Also, a PEFR of A_1 is redundantly called a *PEFR* (*PEFR*) *of* A_1 *of FK* or briefly an *FKPEFR of* A_1, because there are *no* PEFR’s that could be called “*PEFR’s of SK*”. By contrast, an SEFR of A_1 is redundantly called an *SEFR of* A_1 *of SK* or briefly an *SKSEFR* (*SKSEFR*) *of* A_1, because there are *no* SEFR’s that could be called “*SEFR’s of FK*”.

9) In agreement with Df 6.1(6b) and also in agreement with the item 2a of this definition, an FR of EF’s of A_1 is called a *logographic FR* (*LFR*) [*of EF’s*] *of* A_1, and also explanatorily and redundantly a *schematic LFR* (*SchLFR*) or *LFR-schema* (*LFRS*, pl. “*LFRS’ta*”) [*of EF’s*] *of* A_1, if and only if it *semantically determines* a large number (usually an infinite number) of congeneric or conspecific non-concretized EF’s either as members of the range of a *concrete metalogographic formula* (*CMLF*)

of the IML of A_1 or as members of the range of a *concrete panlogographic formula* (*CPLF*) *of* $\boldsymbol{\mathsf{A}}_1$, which is called *a*, or *the*, *syntactic subject* of the respective LFR [of EF's] of A_1 and which is used *xenonymously*, i.e. respectively either as a *metalogographic placeholder* (*MLPH*) or as a *panlogographic placeholder* (*PLPH*), *of EF's of* A_1. An LFR [of EF's] of A_1 is called a *metalogographic FR* (*MLFR*) [*of EF's*] *of* A_1 in the former case and a *panlogographic FR* (*PLFR*) [*of EF's*] *of* A_1 in the latter case. Unlike an MLFR [of EF's] of A_1, a PLFR [of EF's] of A_1 can be *mentally metamorphosed* so that it has three mental *hypostases* (*forms of existence*) with respect to me, which are called:

a) a *PLFR* (*SchPLFR*, *PLFRS*) [*of EF's*] *of* A_1 if I mentally use the PLF's, being its syntactic subjects, *xenonymously*;

b) a *PLFR* (*CPLFR*, *CPLFR*) [*of PLF's*] *of* $\boldsymbol{\mathsf{A}}_1$ if I mentally use the PLF's, being its syntactic subjects, *autonymously*;

c) an *EnSPGFR* [*of EnSPGF's*] *of* A_1, i.e. an *FR of both EF's of* A_1 *and PLF's* $\boldsymbol{\mathsf{A}}_1$ *[as if] simultaneously*, if I mentally use the PLF's, being its syntactic subjects, *in the TAEXA-mode*.•

Df 6.3: ***Schematic logographic placeholders of EF's*** A_1 ***versus abstract atomic ones and descriptive molecular ones.*** 1) The dichotomy of PLPH's (panlogographic placeholders) indicated in Df 4.2(4a–c) applies both to a PLPH, whose range is a certain class of *categorematic*, or *formulary*, *euautographs* (briefly *FE's* or *CtgE's*), i.e. of *euautographic categoremata* or *euautographic formulas* (*ECtg'ta* or *EF's*), *of* A_1 and to a PLPH, whose range is a certain class of *syncategorematic euautographs* (*SctgE's*), i.e. *euautographic syncategoremata* (*ESctg'ta*) , *of* A_1. The former PLPH is called a *categorematic*, or *formulary, PLPH* (*CtgPLPH* or *FPLPH*), and also a *panlogographic categorem* or *panlogographic formula* (*PLCtg* or *PLF*), and the latter is called a *syncategorematic PLPH* (*SctgPLPH*), and also a *panlogographic syncategorem* (*PLSctg*). Consequently, the taxonym "*PLPH of EF's*" and any of the taxonyms "*CtgPLPH*", "*FPLPH*", "*CtgPL*", "*FPL*", and "*PLF*" are synonyms. At the same time, no harm is done by using any of the redundant taxonyms such as "*CtgPLPH of EF's*", "*FPLPH of EF's*", and "*PLF of EF's*", when convenient. The above remark and the above definitions apply with "ML" for "metalogographic" and hence generally with "Lg" or "L" (whenever confusion cannot result) for "logographic" in place of "PL" for "panlogographic", subject to the restrictions, imposed on the MLPH’s useed ad hoc. In order to explicate the *trichotomy* [*of the class*] *of logographic classification rules* (*LCR’s*, *LgClsR*) *of EF’s of* A_1, which has been indicated in the items 11b–11d of Df 6.1, and especially to demarcate the difference between a *schematic logographic formation rule* (*SchLFR*) and a *descriptive logographic sortation rule* (*DLSR*), *of EF’s of* A_1, the items 4a–4c of Df 4.2 are specified below in greater detail so as to relate the above trichotomy to the pertinent trichotomy of the logographic (particularly panlogographic) syntactic subjects of the LCR’s.

2) A StPL (structural panlogograph), i.e. StPLPH (structural panlogographic placeholder) of euautographs of A_1 of its range is, in the general case, either a *formulary* one (*StFPL*), i.e. a *structural panlogographic formula* (*StPLF*), or a *syncaregorematic* one (*StSctgPL*) and simultaneously either an *atomic* one (*StAPL*, i.e. either *StAFPL* or *StASctgPL)* or a *combined* one (*StCbPL*, i.e. either a *StCbFPL* or a *StCbSctgPL*). In any case, a StPL of euautographs of A_1 is *for more clarity* (*explanatorily and redundantly*) called a *schematic StPL* (*SchStPL*), and also a *structural panlogographic schema* (*StPLS*, pl “*StPLS’ta*”), *of the same nomenclature*, i.e. with the same additional qualifiers. In this case, a StAPL, i.e. SchStAPL or StAPLS, of A_1 is also explanatorily called an *abstract StAPL* (*AbStAPL*) or *StAPL-abstractum* (*StAPLA*) of A_1 and vice versa.

a) In accordance with a certain criterion to be explicated in due course, some simplest StCbPL’s of $\mathbf{A}_1$, primary ones (PStCbPL’s) or secondary ones (SStCbPL’s), are called *structural molecular panlogographs* (*StMPL’s*) and also, for more clarity, *StMPL-schemata* (*StMPLS’ta*), *of* $\mathbf{A}_1$. For instance, any one of the *PStCbPLOR’s* ‘$\mathbf{f}^1(\mathbf{x}_1)$’, ‘$\mathbf{f}^2(\mathbf{x}_1,\mathbf{x}_2)$’, etc is a *PStMPLOR of* $\mathbf{A}_1$, whereas any one of the *PStCbPLI’s* ‘$V(\mathbf{p})$’, ‘$V(\mathbf{f}^1(\mathbf{x}_1))$’, ‘$V(\mathbf{f}^2(\mathbf{x}_1,\mathbf{x}_2))$’, etc is a *PStMPLI of* $\mathbf{A}_1$. Parentheses, a comma, and V are elements of B_1, ‘$\mathbf{f}^1$’, ‘$\mathbf{f}^2$’, etc, ‘$\mathbf{x}_1$’, ‘$\mathbf{x}_2$’, etc, and ‘$\mathbf{p}$’ are elements of $\mathbf{B}_{1PSt}$. Therefore, the range range of any one of the above *PStMPLF’s of* $\mathbf{A}_1$ is determined in advance by the ranges of of the pertinent elements of the basis $\mathbf{B}_{1PSt}$.

b) In agreement with the items 1, 3b, 4c, and 6 of Df 5.1 and item 2a of Df 6.1, an element of $\mathbf{B}_{1St}$ is a StAPL that is, for more clarity, called a *basic StAPL* (*BscStAPL*), or a *connotative structural atomic panlogographic taxonym* (*CntStAPLTxm*), *of* $\mathbf{A}_1$, whereas an element of $\mathbf{B}_{1StF}$ ($\mathbf{B}_{1StCtg}$) is a StAFPL (StAPLF) that is, but for more clarity agan, called a *basic StAFPL* (*BscStAFPL*), or *basic*

StAPLF (*BscStAPLF*), *of* $\mathbf{A}_1$, and also a *connotative structural formulary atomic panlogographic taxonym* (*CntStFAPLTxm*).

c) There are *no* StPL's that could be qualified *descriptive* (*not schematic*).

d) I introduce *no* metalogographs (ML's), i.e. *no* metalogographic placeholders (MLPH's), either universal (permanent) ones or ad hoc ones, which could be qualified *structural*. Therefore, the above remarks do not apply with "ML" for "metalogograph" or "metalogographic" in place of "PL" for "panlogograph" or "panlogographic".

3) An AnPL (analytical panlogograph), i.e. AnPLPH (analytical panlogographic placeholder), of euautographs of A_1 of its range is, just as in the previous case, either a *formulary* one (*AnFPL*), i.e. an *analytical panlogographic formula* (*AnPLF*), or a *syncaregorematic* one (*AnSctgPL*) and simultaneously either an *atomic* one (*AnAPL*, i.e. either *AnAFPL* or *AnASctgPL*) or a *combined* one (*AnCbPL*, i.e. either *AnCbFPL* or *AnCbSctgPL*). Like a StAPL, an AnAPL of euautographs of A_1 is explanatorily and redundantly called an *abstract AnAPL* (*AbAnAPL*) or *AnAPL-abstractum* (*AnAPLA*) *of the same nomenclature*, i.e. with the same additional qualifiers. Patticularly, an AnAFPL, or AnAPLF, of EI's or ER's of A_1 is explanatorily and redundantly called an *abstract AnAFPL* (*AbAnAFPL*), or *abstract AnAPLF*, and also an *AnAPL-abstractum* (*AnAPLA*, pl. "*AnAPLAI'ta*"), *of EI's* or *ER's of* A_1. However, unlike a StAPL, a*n AnAPL is not qualified schematic* or *schema*. In agreement with the items 1, 3b, 4c, and 6 of Df 5.1 and item 2a of Df 6.1, an element of $\mathbf{B}_{1An}$ is a AnAPL that is, for more clarity, called a *basic AnAPL* (*BscAnAPL*), or a *connotative analytical atomic panlogographic taxonym* (*CntAnAPLTxm*), *of* $\mathbf{A}_1$, whereas an element of $\mathbf{B}_{1AnF}$ ($\mathbf{B}_{1AnCtg}$) is a AnAFPL (AnAPLF) that is, but for more clarity again, called a *basic AnAFPL* (*BscAnAFPL*), or *basic AnAPLF* (*BscAnAPLF*), *of* $\mathbf{A}_1$, and also a *connotative analytical formulary atomic panlogographic taxonym* (*CntAnFAPLTxm*).

4) A StAFPL (StAPLF, StAFPLS, StAFPLA, BscStAFPL, BscStAPLF) or an AnAFPL (AnAPLF, AnAFPLA, BscAnAFPL, BscAnAPLF), of EF's of A_1 is indiscriminately called an *AFPL* (*APLF, AFPLA, BscAFPL, BscAPLF*) *of EF's of* A_1 and vice versa. In general, a StAPL (StAPLS, StAPLA, BscStAPL) or an AnAPL (AnAPLA, BscAnAPL), of EF's of A_1 is indiscriminately called an *APL* (*APLA, BscAPL*) *of EF's of* A_1 and vice versa.

5) An AnCbFPL (AnCbPLF) of A_1 that is formed by substitution of any appropriate element of B_{1AnF} (B_{1AnCtg}), – such an element, e.g. as '**P**' or '**Q**', whose range is the entire class of ER's of A_1, or as '**I**' or '**J**', whose range is the entire class of EI's of A_1, or as '**i**' or '**j**', whose range is the class of EI's of A_1, which satisfy the idempotent law $\mathbf{i}\hat{\cdot}\mathbf{i} \hat{=} \mathbf{i}$, – for '**Γ**' into any one of the '**Γ**'-based DPAnMML's (PAnMMLD's), such as '$\mathbf{\Gamma}\langle\mathbf{u}\rangle$', '$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v}\rangle$', '$\mathbf{\Gamma}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$', etc, '$\mathbf{\Gamma}_{\langle\mathbf{u}\rangle}\langle\mathbf{u}\rangle$', '$\mathbf{\Gamma}_{\langle\mathbf{u}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$', '$\mathbf{\Gamma}_{\langle\mathbf{u}\rangle}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$', etc, '$\mathbf{\Gamma}_{\langle\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$', '$\mathbf{\Gamma}_{\langle\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$', etc; '$\mathbf{\Gamma}_{\langle\mathbf{u},\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$', '$\mathbf{\Gamma}_{\langle\mathbf{u},\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$', etc, etc, which have been defined in the items 1–7 of Df 5.11, is called a *descriptive analytical molecular formulary panlogograph* (*DAnMFPL*), or *descriptive analytical molecular panlogographic formula* (*DAnMPLF*), *of* A_1, and also an *analytical molecular formulary panlogographic description* (*AnMFPLD*) *of EF's* (either *ER's* or *EI's* depending on the description) *of* A_1 *of its range*.

a) Particularly, the following *semantic definition*, being an instance of Df 5.11(4) with '**P**' in place of '**Γ**', "ER's" for "euautographic relations" in place of "PEA's" for "primary euautographic assemblages", and "PL" for "panlogographic" in place of "ML" for "metalogographic", is an *analytical panlogographic sortation rule* (*AnPLSR*) *of ER's of the range of* '**P**', i.e *of ER's of* A_1: '$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$' is a AnMFPLD (or, more precisely, a PAnMFPLD, – see below) of A_1, whose range is the class of ER's of A_1, any given member $\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$ of which is an ER of A_1 that involves certain *free* occurrences of two given different AEOT's **u** and **v** and perhaps free or bound occurrences of some other AEOT's **w**, **x**, etc, which are not mentioned by using the metalogograph '$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$'. That is, to generalize, the range of '$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$' is a subclass of of only those ER's of the range of '**P**', i.e. of A_1, which have a certain peculiar structural property. Df 6.3(5a)At the same time, in accordance with Dfs 5.1(6b) and 5.11(6), '$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$' *is the panlogographic description of the species of ER's of* A_1 *through the genus* **P**, *denoted by the generic name* '**P**', *and through the differenntia (difference)* $\langle\mathbf{u},\mathbf{v}\rangle$, *denoted by the qualifier* '$\langle\mathbf{u},\mathbf{v}\rangle$'. The *species* $\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$ of ER's of A_1 is alternatively called a *sort*, or *descriptive species*, of ER's of the genus **P**, i.e. the genus *connoted* (or, impartially, *desicnated*) by' **P**', whereas the AnMFPLD '$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$' is called a *sorting*, or *descriptive, analytical molecular panlogographic formula* Df 6.3(5) (*StgAnMPLF* or *DAnMPLF*), or *molecular formulary panlogograph*

(S*tgAnMFPL* or *DAnMFPL*), *of ER's of the genus* **P**. Accordingly, a semantic definition of one or more StgAnMPL's, as that of '$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$' stated above, is called an *analytical panlogographic sortation rule* (briefly *AnPLStnR* or *AnPLSR* whenever confusion cannot result) *of ER's of* A_1. The AnAFPL's '**P**', '**Q**', '**I**', and '**J**' (e.g.) are elements of $\mathbf{B}_{1PAn}$ ($\mathbf{B}_{1PAnF}$), whereas the AnAFPL's '**i**' and '**j**' (e.g.) are elements of $\mathbf{B}_{1SAn}$ ($\mathbf{B}_{1SAnF}$). Therefore, in agreement with the pertinent nomenclature, introduced in Df 5.1(6b), '$\mathbf{P}\langle\mathbf{u}\rangle$', '$\mathbf{Q}\langle\mathbf{u},\mathbf{v}\rangle$', '$\mathbf{I}\langle\mathbf{u}\rangle$', and '$\mathbf{J}\langle\mathbf{u},\mathbf{v}\rangle$', e.g., are elements of $\mathbf{B}'_{1PAn}$ ($\mathbf{B}'_{1PAnF}$), whereas '$\mathbf{i}\langle\mathbf{u}\rangle$' and '$\mathbf{j}\langle\mathbf{u},\mathbf{v}\rangle$', e.g., are elements of $\mathbf{B}'_{1SAn}$ ($\mathbf{B}'_{1SAnF}$). An AnMFPLD [of ER's] of A_1 is called a *primary AnMFPLD* (*PAnMFPLD* or *AnPMFPLD*) [*of ER's*] *of* A_1 if it is an element of $\mathbf{B}'_{1PAnF}$ ($\mathbf{B}'_{1PAn}$) and a *secondary AnMFPLD* (*SAnMFPLD* or *AnSMPLD*) [*of ER's*] *of* A_1 if it is an element of $\mathbf{B}'_{1SAnF}$. Consequently, an AnPLSR [of ER's] of A_1 is called a *primary AnPLSR* (*PAnPLSR* or *AnPPLSR*) [*of ER's*] *of* A_1 if it defines one or more PAnMFPLD's and a *secondary AnPLSR* (*SAnPLSR* or *AnSPLSR*) [*of ER's*] *of* A_1 if it defines one or more SAnMFPLD's, and vice versa.

b) From the above statement, it follows that an AnMFPLD (DAnMPLF, StgAnMFPL) of $\mathbf{A}_1$, index-free (not contracted) or indexed (contracted), stands, *as the definiendum for the definiens*, for a *semi-verbal description of a common euautograph of* A_1 *of a certain class*, being the range of the definiens and hence the range of the definiendum. In the actual fact, the definiens, and hence the definiendum, is a description of the entire range of either of the two terms of the pertinent *semantic definition*, i.e. *AnPLSR*, in the *projective* (*polarized*, *extensional*, *connotative*) *mental mode* (see Df 4.1(4a)), in which I *mentally experience the range as my as if extramental* (*exopsychical*) *object*, which I call a *common* (*general*, *certain*, *concrete but not concretized*) *euautographic denotatum both of the definiens and of the definiendum* and which I call also a *common* (*general*) *element* (*member*) *of the range* that the two terms have in common; the common element *represents the whole range*, thus being *just another hypostasis* (*way of existence*, *aspect*) *of the latter*. An AnMFPLD has *no principal kernel-sign* and *is not therefore an operand*, although it is an An*Cb*FPL. Therefore, any concrete EF (either ER or EI), being a *concretized member* (*element*, *instance*) of the range of the AnMFPLD, is obtained by particularizing (concretizing) the AnMFPLD for the respective concrete (concretized) euautographic denotata of all AnAPL's occurring in it only *as a single whole in accordance with its semantic definition,* i.e. *in accordance with the descriptive* (*interpretative*) *semi-verbal definiens of the pertinent AnPLSR of* $\mathbf{A}_1$*, and not as a schema by interpreting its separate constituent AnAPL's*. This is the very reason for qualifying an AnMFPLD of EF's of A_1 *descriptive* or *description*, and *not schematic* or *schema*, nd also for qualifying it *molecular* (cf. Df 5.11(6)). In contrast to an AnMFPLD, an AnAFPL of $\mathbf{A}_1$ is qualified *abstract* or *abstractum*, and not *descriptive* or *description*. The meaning of any of the above taxonyms (metaterms) that involves the prepositive qualifier "descriptive" or the appositive (postpositive) noun "description" remains unaltered if either of the two words is replaced with the word "mnemonic" either as an adjective (abbreviated as "Mnm") or as a noun (abbreviated as "M") respectively.

c) Either of the synonymous generic taxonyms "*descriptive analytical molecular PLF*" ("*DAnMPLF*") and "*analytical molecular FPL description*" ("*AnMFPLD*") can be abbreviated by omission of the combined qualifier "*analytical molecular* ("*AnM*"), i.e. as "DPLF" and "FPLD" respectively, because there is no *DPLF*, or *FPLD, of* $\mathbf{A}_1$ of any other kind (cf. Df 5.11(9)), particularly no one that could be qualified *structural*.

6) An AnCbFPL, i.e. AnCbPLF, of $\mathbf{A}_1$ is called a *schematic* one (*SchAnCbFPL* or *SchAnCbPLF*), and also *AnCbFPL-schema* (*AnCbFPLS*, pl. "*AnCbFPLS'ta*"), *of EF's* (either *ER's* or *EI's* depending on the schema) *of* A_1 *of its range*, if it it involves at least one elemental (atomic or molecular) endosemasiopasigraphic (euautographic or panlogographic) kernel-sign, namely that serving as its *principal kernel-sign*, and perhaps some others, and if it is patterned (primarily by its constituent syncategorematic endosemasiopasigraphs) in such a way that any EF, being a *member* (*element*, *instance*) *of its range*, is its *detailed euautographic interpretand* in the sense that the EF is obtained by specifying all all *APL's* and all *AnMFPLD's*, occurring in the *AnCbFPLS*, *individually* in accordance with their semantic definitions, and not by specifying the AnCbFPL as a single whole after the manner of an AnMFPLD. Thus, an AnCbFPLS of EF's of A_1, and generally an AnCbPLS of euautographs of A_1, *may contain some AnMFPLD's, and not only AnAPL's* (*AnAPLA'ta*). For instance,

$$‘\left[(\exists\mathbf{x})\mathbf{P}\langle\mathbf{x}\rangle\right]’,\ ‘(\forall\mathbf{z})\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]’,\ ‘(\forall\mathbf{z})\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]’$$

are AnCbFPLS'ta of ER's of A_1 and

$$‘\left[\left(\hat{\cdot}\,\mathbf{x}\right)V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]’,\ ‘1\hat{-}\left(\hat{\cdot}\,\mathbf{z}\right)\left(1\hat{-}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right)’$$

are AnCbFPLS'ta of of EI's (ESpT's) of A_1. At the same time, in accordance with a certain criterion to be explicated in due course, some simplest AnCbPLS'ta of $\mathbf{A}_1$, which involve only APL's of $\mathbf{A}_1$ and no AnMFPLD's of $\mathbf{A}_1$, are called *analytical molecular panlogographic schemata* (*AnMPLS'ta*) *of* $\mathbf{A}_1$ (or *of euautographs of* A_1) the understanding being that these can be *formulary*, or c*ategorematic*, ones (*AnMFPLS'ta* or *AnMCtgPLS'ta*) or *syncategorematic* ones (*AnMSctgPLS'ta*) and and at the same time *primary* ones (*PAnMPLS'ta*) or *secondary* ones (*SAnMPLS'ta*). For instance, '$V(\mathbf{P})$', '$V(\mathbf{Q})$', etc. are *PAnMFPLS'ta of* $\mathbf{A}_1$ or, more specifically, *PSchAnMPLI's of* $\mathbf{A}_1$. Oving to the presense of the qualifiers "Sch" or ""S" and "An", any one of the synonymous taxonyms "SchAnCbFPL", "SchAnCbPLF", and "AnCbFPLS", e.g., can be abbreviated by omission of the qualifier "Cb" without altering the meaning of the taxonym.

7) A StFPLS (StFPL) or AnFPLS of EF's of A_1 is indiscriminately called an *FPLS of EF's of* A_1 and vice versa. In general, a StPLS (StPL) or an AnPLS, of euautographs of A_1 is indiscriminately called a *PLS of euautographs of* A_1 and vice versa.

8) An AnCbFPL, i.e. AnCbPLF, of $\mathbf{A}_1$ is called an *abstract* one (*AbAnCbFPL*, *AbAnCbPLF*), and also *AnCbFPL-abstractum* (*AnCbFPLA*, pl. "*AnCbFPLA'ta*"), *of EF's* (either *ER's* or *EI's* depending on the schema) *of* A_1 *of its range* if it is not a schematic one. For instance, the definienda of the following panlogographic *asymmetric synonymic definitions* (*ASD's*):

$$\begin{gathered}\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\to(\forall\mathbf{z})[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle],\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\to\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle,\\ \mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\to(\forall\mathbf{z})[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle],\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\to\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle;\end{gathered}\tag{6.1}$$

$$\begin{gathered}\mathsf{I}_2\langle\mathbf{u},\mathbf{v}\rangle\to(\exists\mathbf{z})[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle],\mathsf{E}_2\langle\mathbf{u},\mathbf{v}\rangle\to\neg\mathsf{I}_2\langle\mathbf{u},\mathbf{v}\rangle,\\ \mathsf{O}_2\langle\mathbf{u},\mathbf{v}\rangle\to(\exists\mathbf{z})[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle],\mathsf{A}_2\langle\mathbf{u},\mathbf{v}\rangle\to\neg\mathsf{O}_2\langle\mathbf{u},\mathbf{v}\rangle;\end{gathered}\tag{6.2}$$

$$\begin{gathered}\mathsf{I}_3\langle\mathbf{u},\mathbf{v}\rangle\to(\exists\mathbf{z})[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]],\\ \mathsf{O}_3\langle\mathbf{u},\mathbf{v}\rangle\to(\exists\mathbf{z})[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]],\\ \mathsf{A}_3\langle\mathbf{u},\mathbf{v}\rangle\to\neg\mathsf{O}_3\langle\mathbf{u},\mathbf{v}\rangle,\mathsf{E}_3\langle\mathbf{u},\mathbf{v}\rangle\to\neg\mathsf{I}_3\langle\mathbf{u},\mathbf{v}\rangle;\end{gathered}\tag{6.3}$$

$$\begin{gathered}\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\to(\forall\mathbf{z})[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle],\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\to\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle,\\ \mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\to\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle,\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\to\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\end{gathered}\tag{6.4}$$

are AbAnCbPLF's of ER's of A_1, whose the base letters are tokens of the respective prototypal catch vowels "a", "e", "i", and "o" of Aristotelian syllogistics. Consequently, the definienda of the following panlogographic ASD's (e.g) with '1' to '4' in place of '*n*' are some other AbAnCbPLF's of ER's of A_1, whose base words are tokens of the respective prototypical catch words of Aristotelian syllogistics.

$$\begin{gathered}\mathsf{Barbara}_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\rightarrow\left[\mathsf{A}_n\langle\mathbf{w},\mathbf{v}\rangle\wedge\mathsf{A}_n\langle\mathbf{u},\mathbf{w}\rangle\Rightarrow\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle\right],\\ \mathsf{Camestres}_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\rightarrow\left[\mathsf{A}_n\langle\mathbf{v},\mathbf{w}\rangle\wedge\mathsf{E}_n\langle\mathbf{u},\mathbf{w}\rangle\Rightarrow\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle\right],\\ \mathsf{Felapton}_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\rightarrow\left[\mathsf{E}_n\langle\mathbf{w},\mathbf{v}\rangle\wedge\mathsf{A}_n\langle\mathbf{w},\mathbf{u}\rangle\Rightarrow\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle\right].\end{gathered}\tag{6.5}$$

An AbAnAPLF or AbAnCbPLF, i.e. an AnAFPLA or AnCbFPLA, of EF's of A_1 is indiscriminately called an *AbAnPLF*, or *AnFPLA, of EF's of* A_1 and vice versa.

9) I introduce *no* metalogographic placeholders (MLPH's) of PEA's of A_1, either universal (permanent) ones or ad hoc ones, which could be qualified *structural* (*St*) ones. All MLPH's are *analytical* ones (*AnMLPH's*), called also *analytical metalogographs* (*AnML's*). Primary AnAML's (PAnAML's) and primary DAnMML's (DPAnMML's) have been defined in Dfs 5.9 and 5.10. By definition, these are neither categorematic (formulary) nor syncategorematic. Therefore, the above items 2–4 and 7 *do not apply* with "ML" for "metalogograph" or "metalogographic" in place of "PL" for "panlogograph" or "panlogographic". However, if the range of the PAnAML, serving as the generic one of a given DPAnMML, is restricted ad hoc with the help of added words to contain only some ER's or some EI's of A_1 then the DPAnMML turns into an ad hoc StgAnMML's of the ER's or EI's of A_1 in question. Likewise, using ad hoc one or two PAnAML's with a properly restricted range, it is possible to form an *analytical formulary metalogographic schema* (*AnFMLS*) after the manner of an AnFPLS as described above in the item 6. That is to say, the items 5b, 5c and 6 apply with "ML" for "metalogograph" or "metalogographic" in place of "PL" for "panlogograph" or "panlogographic" and with "the IML of A_1 and $\mathbf{A}_1$ in place of '$\mathbf{A}_1$' verbatim, with the understanding that the range of a PAnAML '$\mathbf{\Gamma}$' or $\mathbf{\Delta}$', which occurs in a given AnFMLD or AnFMLS, is properly restricted ad hoc. The unifying AnAMLPH's (UAnAMLPH's) of EF's of A_1 that have been introduced in Df 6.1(2d) are *secondary formulary* AnAMLPH's, but they belong to the IML of A_1 and $\mathbf{A}_1$ and are not systemic ones. Any AnAMLPH, e.g. '$\mathbf{\Phi}$', can in principle be substituted for '$\mathbf{\Gamma}$' into

any one of the ‘**Γ**’-based DPAnMML’s (PAnMMLD’s), which have been defined in the items 1–7 of Df 5.11 (cf. the above item 5). However, just as a UAnAMLPH, any *secondary* DPAnMML thus obtained is usable only in the IML of A_1 and $\mathbf{A}_1$ and not in A_1 and $\mathbf{A}_1$ themselves. Therefore, no such applications of the item 5 are considered.

10) In accordance with the pertinent passage of Df 4.1(2), a *PLPH of EF’s of* A_1 that is used *autonymously* is called a *panlogographic formula* (*PLF*), or *formulary panlogograph* (*FPL*), *of* $\mathbf{A}_1$, and conversely the latter that is used *xenonymously* is the former. Likewise, an *MLPH of EF’s of* A_1 that is used *autonymously* is called a *metalogographic formula* (*MLF*), or *formulary metalogograph* (*FML*), *of the IML of* A_1 (A_1 and $\mathbf{A}_1$), and conversely the latter that is used *xenonymously* is the former. A PLPH of EF’s of A_1 is either a *basic formulary atomic PLPH* (*BscFAPLPH*) *of EF’s of* A_1, i.e. an element of $\mathbf{B}_{1F}$ that is used xenonymously, or a *formulary panlogographic description* (*FPLD*) *of EF’s of* A_1, or else a *formulary panlogographic schema* (*FPLS*) *of EF’s of* A_1. By contrast, an MLPH of EF’s of A_1 is either a *formulary metalogographic description* (*FMLD*) *of EF’s of* A_1 or a *formulary metalogographic schema* (*FMLS*) *of EF’s of* A_1 with the proviso that an BscFAPLPH, FPLS, or FPLD is a *universal* (*permanent*) one, whreas an FMLS or FMLD is an *ad hoc* one subject to the pertinent restriction of the range of every involved AMLPH (see the above item 5). Therefore, it follows from the first sentences of this item that:

a) An *AFPL* (*APLF, BscAFPL, BscAPLF*), or. *AFPLA* (*AFPL-abstractum*), *of EF’s of* A_1 that is used *autonymously* is called an *AFPL* (*APLF, BscAFPL, BscAPLF*), or. *AFPLC* (*AFPL-concretum*), *of* $\mathbf{A}_1$ and conversely the latter that is used *xenonymously* is the former.

b) An *FPLD*, or *FPLS, of EF’s of* A_1 that is used *autonymously* is called an *FPLD*, or *PPLS, of* $\mathbf{A}_1$ and conversely the latter that is used *xenonymously* is the former.

c) An *FMLD*, or *FMLS, of EF’s of* A_1 that is used *autonymously* is called an *FMLD*, or *FMLS, of the XML of* A_1 and conversely the latter that is used *xenonymously* is the former.

In agreement with Df 6.2(9), a PLF of $\mathbf{A}_1$ is, a *concrete PLF* (*CPLF*), i.e. *PLF-concretum* (*PLFC,* pl. “*PLFC’ta*”), *of* $\mathbf{A}_1$ and similarly an MLF of the IML of A_1 is a *concrete MLF* (*CMLF*), i.e. *MLF-concretum* (*MLFC*, pl. “*MLFC’ta*”), of the IML of

A_1. Consequently, an AFPL, FPLD, or FPLS, of $\mathbf{A}_1$ is a *concrete* one, i.e. a *CAFPL*, *CFPLD*, or *CFPLS of* $\mathbf{A}_1$, and similarly an FMLD or FMLS of the IML of A_1 is a *concrete* one, i.e. a *CFMLD* or *CFMLS* of the IML of A_1. By Df 6.1(7), "concrete" ("C") and "concretum" ("C") are antonyms of "abstract" ("Ab") and "abstractum" ("A"), whereas "schematic" ("Sch") and "schema" ("S") are antonyms of "descriptive" ("D") and "description" ("D"). Therefore, the taxonyms "CFMLD", "CFMLS", "CFMLD", and "CFMLS" are *not contradictions in adjecto.*

11) In accordance with the previous definitions, *a PLF* (*FPL*) *of* $\mathbf{A}_1$ *is an FPLA or an FPLD or else an FPLS* subject to the following furcations and synonyms.

a) An FPLA is either an AFPLA or a CbFPLA (as the definiendum of any one of definitions (6.1)–(6.5)). An AFPLA of $\mathbf{A}_1$ is an element of $\mathbf{B}_{1F}$, and hence it is either a StAFPLA of $\mathbf{A}_1$, i.e. an element of $\mathbf{B}_{1FSt}$ (as '**u**', '**p**', or 'i') or an AnAFPLA of $\mathbf{A}_1$, i.e. an element of $\mathbf{B}_{1FAn}$ (as '**P**', '**I**', '**π**', 'p', or '**i**'). AnMFPLD.

b) An FPLD is an AnMFPLD and vice versa.

c) An FPLS is either a StFPLS or an AnFPLS.

i) A StFPLS is either a StAFPLS or a StCbFPLS. A StAFPLS is a StAFPLA and vice versa. A StCbFPLS is either a StMFPLS (as '$\mathbf{f}^2(\mathbf{x}_1,\mathbf{x}_2)$', '$[\mathbf{x} \in \mathbf{y}]$', or '$V(\mathbf{p})$') or a StCxFPLS, "M" being an abbreviation for "molecular", and "Cx" for "complex". A StAFPLS or a StMFPLS is indisctiminately valled a StElFPLS; "El" being an abbreviation for "elemental".

ii) An AnFPLS is an AnCbFPLS and vice versa, so that it is either an AnMFPLS (as '$V(\mathbf{P})$') or an AnCxFPLS. An AnAFPLS or an AnMFPLS is indisctiminately valled an AnElFPLS.

d) By the above items b) and c), an AnMFPLD, StMFPLS, or AnMFPLS is indiscriminately called an *MFPL* or *MPLF* and vice versa.•

Df 6.4: ***PLFR's and MLFR's of EF's of*** A_1 ***versus PLGR's, PLSR's and MLSR's of*** A_1**.** 1) In agreement with Df 5.1(7), a *formulary panlogographic classification rule* (*FPLCR*) *of EF's of* A_1 is called:

a) a *formulary panlogographic generalization rule* (*FPLGR*) *of EOT's*, or *of EI's* (*ESpT*), or *of ER's, of* A_1 if every syntactic subject of it is an *APLF of*

$\mathbf{A}_1$, i.e. an *element of* $\mathbf{B}_{1F}$, *of the respective range*, i.e an *APLOT*, or *APLI*, (*APLSpT*), or *APLR*, *of* $\mathbf{A}_1$ and hence that of $\mathbf{B}_{1F}$, which is used *xenonymously*;

b) a *formulary panlogographic sortation rule* (*FPLSR*) *of EI's* (*ESpT*), or *of ER's*, *of* A_1 if every syntactic subject of it is is an appropriate *FPLD* (*AnMFPLD*) *of* $\mathbf{A}_1$, i.e. an *element of* $\mathbf{B}'_{1F}$ ($\mathbf{B}'_{1AnF}$), *of the respective range*, i.e a *DPLI* (*DPLSpT*) or *DPLR*, or synonymously a *PLID* (*PLSpTD*) or *PLRD*, *of* $\mathbf{A}_1$ and hence that of $\mathbf{B}'_{1F}$, which is used *xenonymously* (the letter "D" stands for "descriptive" if it is prepositive and for "description" if it is postpositive);

c) a *formulary panlogographic formation rule* (*FPLFR*) *of EI's* (*ESpT*), or *of ER's*, *of* A_1 if every syntactic subject of it is an *FPLS of* $\mathbf{A}_1$ *of the respective range*, i.e a *SchtPLI* (*SchtPLSpT*) or *SchPLR*, or synonymously *PLIS* (*PLSpTS*) or *PLRS*, *of* $\mathbf{A}_1$, which is used *xenonymously* (a prepositive "Sch" stands for "schematic" and s postpositive "S" stands for "schema").

By contrast, a *formulary metalogographic classification rule* (*FMLCR*) *of EF's*, i.e.either *of EI's* or *of ER's*, *of* A_1 is called:

b′) a *formulary metalogographic sortation rule* (*FMLSR*) *of EI's* (*ESpT*), or of *ER's*, *of* A_1 if every syntactic subject of it is an appropriate *ad hoc FMLD* (*AnMFMLD*) *of the XML of* A_1 *of the respective range*, i.e an *ad hoc DMLI* (*DMLSpT*) or *DMLR*, or synonymously an *MLID* (*MSpTD*), *of the XML of* A_1, which is used *xenonymously*;

c′) a *formulary metalogographic formation rule* (*FMLFR*) *of EI's* (*ESpT*), or *of ER's*, *of* A_1 if every syntactic subject of it is an *ad hoc FMLS of the XML of* A_1 *of the respective range*, i.e an *ad hoc SchtMLI* (*SchtMLSpT*) or *SchMLR*, or synonymously *MLIS* (*MLSpTS*) or *MLRS*, *of the XML of* A_1, which is used *xenonymously*.

In the process of laying down a system of seven FMLFR's of *primary EI's* (*PEI's*) and *primary ER's* (*PER's*) of A_1 at the very beginning of the setup of A_1, I shall contextually lay down two respective FMLSR's of a constituent PEI of a larger PRI and of a constituent PER of a larger PRR, but I shall have no occasion to lay down any other FMLSR's of EI's or ER's of A_1 as such. I shall not therefore discuss any FMLSR's of EF's of A_1 in what follows.

2) An FMLF of the XML of A_1, i.e. the hypostasis of an FMLF of the IML of A_1 that is used autonymously as a tychautograph, *is not a formula of any organon.* Consequently, an *MLFR* [*of EF's*] *of* A_1, in which each FMLF of the IML of A_1 being its syntactic subject is used autonymously, is nsot an FR of any organon either. At the same time, by the items 10a and 10b of Df 6.3 and in agreement with the item 9b of Df 6.2, the items 1a–1c of this definition imply the following definitions.

a) If every APLF (AFPL, AFPL*A*, AFPLPH) of EF's, i.e. either of EOT's or of EI's (ESpT) or else of ER's, of A_1 being a syntactic subject of the *FPLGR of those EF's of* A_1 is used *autonymously* as the respective *APLF* (*AFPL, AFPLC*), i.e. as the respective *APLOT* or *APLI* (*PLSpT*) or *AER*, *of* $\mathbf{A}_1$ then that FPLGR is mentally turned into the *FPLFR of that APLF* (*AFPL, AFPLC*) *of* $\mathbf{A}_1$.

b) If every DPLF (DFPL, FPLD, AnMFPLD, DFPLPH) of EF's, i.e. either of EI's (ESpT) or of ER's of A_1 being a syntactic subject of the *FPLSR of those EF's of* A_1 is used *autonymously* as the respective *DPLF* (*DFPL, FPLD, AnMFPLD, DFPLC*), i.e. as the respective *DPLI* (*DPLSpT*) or *DPLR*, *of* $\mathbf{A}_1$ then that FPLSR is mentally turned into the *FPLFR of that DPLF* (*DFPL, FPLD, AnMFPLD, DFPLC*) *of* $\mathbf{A}_1$.

c) If every FPLS (SchPLF, SchFPL, SchFPLPH) of EF's, i.e. either of EI's (ESpT) or of ER's of A_1 being a syntactic subject of the *FPLFR of those EF's of* A_1 is used *autonymously* as the respective *FPLS* (*SchPLF, SchFPLC*), i.e. as the respective *CPLI* (*CPLSpT*) or *CPLR*, *of* $\mathbf{A}_1$ then that FPLFR is mentally turned into the *FPLFR of that FPLS* (*SchPLF, SchFPLC*) *of* $\mathbf{A}_1$.

An FPLFR (formulary panlogographic formation rule) of one or more *concrete PLF's* (CAFPL's, CFPLD's, or CFPLS'ta) *of* $\mathbf{A}_1$ is alternatively called a *concrete FPLFR* (*CFPLFR*), or *FPLFR-concretum* (*FPLFRC,* pl. "*FPLFRC'ta*"), and also a [*systemic*] FPLFR, [*of PLF's*] *of* $\mathbf{A}_1$ and vice versa. Consequently, the above items a)–c), can be summarized as follows. Once an *FPLCR of EF's of* A_1, which is either an FPLGR or an FPLSR or else an FPLFR, of EF's of A_1 (see Df 6.1(2c)) is mentally metamorphosed into a definition of the CPLF's of $\mathbf{A}_1$ condensing those EF's in their ranges, that definition is indiscriminately classified as a a *concrete FPLFR* (*CFPLFR*), or simply as *FPLFR*, [*of PLF's*] *of* $\mathbf{A}_1$.

3) In accordance with Df 6.1(2), I shall use the abbreviviations "*PLFGR*", "*PLFSR*", "*PLFFR*", and "*PLFCR*" interchangeably (synonymously) with "*FPLGR*", "*FPLSR*", "*FPLFR*", and "*FPLCR*" respectively. Also, whenever confusion cannot result, particularly as long as I treat of *formulary* classification rules only, I may, in accordance with the convention stated at the end of the item 1 of Df 6.1, abbreviate the above abbreviations as "*PLGR*", "*PLSR*", "*PLFR*", and "*PLCR*" respectively, i.e. by omission of the letter "F" for "formulary".

4) By the previous item, a *PLFR of PLF's of* $\mathbf{A}_1$ is just another mental hypostasis of a certain *PLCR of the EF's of* A_1, which are comprised in the ranges of the PLF's *of* $\mathbf{A}_1$. Therefore, a *PLFR of PLF's of* $\mathbf{A}_1$ is:

a) either a *primary PLFR* (*PPLFR*, *PLPFR*) or a *secondary PLFR* (*SPLFR*, *PLSFR*), *of PLF's of* $\mathbf{A}_1$;

b) a *PLFR of PLF's of one of the following general classes*: *AFPL's*, *FPLD's*, and *FPLS'ta*, *of* $\mathbf{A}_1$.

Consequently, a PLF of $\mathbf{A}_1$ is called a *primary PLF* (*PPLF*) if and only if it is a, or the, syntactic subject (syntactic definiendum) of a PPLFR of $\mathbf{A}_1$ or an *alphabetic variant* of that syntactic subject; and similarly with "*secondary*" ("*S*") in place of "*primary*" ("*P*"). A PPLF is a *concrete* one of the following three kinds: a *primary PLOT* (*PPLOT*), being just a *PLOT* and vice versa, a *primary PLI* (*PPLI*), called also a *primary PLSpT's* (*PPLSpT's*), or a *primary PLR* (*PPER*), – *independent of the presence of any SPLFR's of* $\mathbf{A}_1$. Accordingly, an SPLF is a *concrete* one of the following two kinds: a *secondary PLI* (*SPLI*), called also a *secondary PLSpT's* (*SPLSpT's*), or a *secondary PLR* (*SPLR*).

5) Like the class of PLF's of $\mathbf{A}_1$, the class of EF's of A_1 is divided into two subclasses: the [class of] *primary* (*postulated, undefined, independent*) *EF's* (*PEF's*) and the [class of] *secondary* (*defined, dependent*) *EF's* (*SEF's*). Combination of this dichotomy with the trichotomy of EF's of A_1 into the classes of EOT's, EI's (ESpT's), and ER's, repeatedly indicated previously (particulatly in Df 6.1(1) and in the item 1 of this definition) results in the furcations of the PEF's and SEF's that are similar to those of the PPLF's and SPLF's, indicated in the previous item. Namely, the class of PEF's of A_1 is divided into the following three subclasses, called *minor basic classes*

of EF's of A_1: the *primary EOT's* (*PEOT's*), being just *EOT's* and vice versa, the *primary EI's* (*PEI's*), called also the *primary ESpT's* (*PESpT's*), and the *primary ER's* (*PER's*). Accordingly, the class of SEF's of A_1 is divided into the following two subclasses: the *secondary EI's* (*SEI's*), called also *secondary ESpT's* (*SESpT's*), and the *secondary ER's* (*SER's*).

6) No matter whether or not any SPLFR's (PLSFR's) of $\mathbf{A}_1$ are laid down, when a PPLOT, i.e. PLOT, of $\mathbf{A}_1$, mentioned in the item 1, is used xenonymously, it turns into an PLPH of PEOT's, i.e. EOT's, of A_1. However, given a stage of development of A_1, if some SPLFR's of $\mathbf{A}_1$ have been laid down by that stage and if a certain PPLI (e.g.), of $\mathbf{A}_1$, as mentioned above in the item 4, is used xenonymously, the range of the respective mentally metamorphosed PLPH of EF's of A_1 contains, not only PEI's of A_1, but also some SEI's of A_1; and similarly with "R" for "relation" in place of "I" for "integron". For instance, if the syntactic subject of an SPLFRS is an PLPH of EI's, or of ER's, of A_1, the PKS (principal kernel-sign) of which is a *primary substantival*, or, correspondingly, *primary relational*, *euautographic kernel-sign* (*EKS*), then the range of the PLPH contains every *primary combined EI* (*PCbEI*), or, correspondingly, every *primary combined ER* (*PCbER*), *of the given general form*, whose *euautographic operatum* or *operata* are, depending on the PKS, either *PEI's* or *PER's*, and it also contains every *secondary combined EI* (*SCbEI*), or, correspondingly, every *secondary combined ER* (*SCbER*), *of the same given general form*, whose *euautographic operatum* or *operata* are, but again depending on the PKS, either *SEI's* or *SER's*, which are determined by certain SPLFR's of $\mathbf{A}_1$ that have been laid by the given stage of development of A_1.

7) The CEFR's of A_1 and the CPLFR's of $\mathbf{A}_1$, both *primary* and *secondary*, form a *single whole recursive and coherent*, (*self-adjustable*), i.e. *self-consistent* and *self-updatable*, *system of interrelated formulary classification rules* (*CR's*), or *grammatical rules* (*GmR's*), *of the biune organon* A_1, which is called the *system of endosemasiopasigraphic formulary classification*, or *grammatical*, *rules* (briefly the *EnSPGFCR-system* or *EnSPGFGmR-system*), and also, alternatively, the *system of self-adjustable classification rules* (briefly the *SACR-system*), [*of EnSPGF's*] *of* A_1, i.e. [*of EF's*] *of* A_1 *and* [*of PLF's*] *of* $\mathbf{A}_1$. I recall that, in accordance with Df 4.1(1), the apposition "the biune organon A_1" means *the union and superposition of* A_1 *and*

$\mathbf{A}_1$. Also, by the items 1 and 2 of this definition, *a CPLFR of* $\mathbf{A}_1$ *is at the same time a PLGR or a PLSR or a PLFR, of EF's of* A_1 *and vice versa*. Various rules or groups of rules comprised in the EnSPGFCR-system are scattered throughout the treatise, so that the EnSPGFCR-system is not compact. The *integrity* (*entirety*) and *self-consistency* of the EnSPGFCR-system is achieved through use of the *system of atomic endosemasiopasigraphs of* B_1, i.e. *atomic euautographs of* B_1 and $\Delta\mathsf{B}_1$ and *atomic panlogographs of* $\mathbf{B}_1$, *owing to the properties of meta-autonymy and retroactivity of the latter* (see items 3–5 of subsection 4.4). In this case, the [ranges of the] formulary elements of $\mathbf{B}_1$ are themselves defined in terms of EF's of A_1 by certain PLGR's of EF's of A_1, which are at the same time certain CPLFR's of PLF's of $\mathbf{A}_1$ and which therefore belong to the EnSPGFCR-system. The relations between the PLF's of $\mathbf{A}_1$ on the one hand and the EF's of A_1 of their ranges on the other hand, which have been described above in the item 6, are one of the fundamental properties of that system. The principles of construction of the EnSPGFCR-system [of EnSPGF's] of A_1, i.e. of EF's of A_1 and of PLF's of $\mathbf{A}_1$, this system are explicated in the next subsection.

8) In accordance with Df 6.1(6,13), the previous item applies, *mutatis mutandis*, with 'A_1^0', '$\mathbf{A}_1^0$', and 'A_1^0' or 'A_0', 'A_0', and '$\mathbf{A}_0$' in place of 'A_1', 'A_1', and '$\mathbf{A}_1$' respectively. In this case, the stipulation "mutatis mutandis", being a parasynonym of the expression "*with the corresponding changes*", means that the CPLFR's comprised in the EnSPGFCR-system of A_1 should be restricted properly, both syntactically and semantically, so as to become the *EnSPGFCR-system of* A_1' or the *EnSPGFCR-system of* A_0. However, unless stated otherwise, the name "the EnSPGFCR-system" is thereafter used as an abbreviation of the name "the EnSPGFCR-system of A_1".•

10) Previous items apply with 'A_1^0', '$\mathbf{A}_1^0$', and 'A_1^0' or with 'A_0', 'A_0', and '$\mathbf{A}_0$' in place of 'A_1', 'A_1', and '$\mathbf{A}_1$' respectively. Unless stated otherwise, the name "the EnSPGFCR-system" alone, without any postpositive qualifier, is thereafter used as an abbreviation of the name "the EnSPGFCR-system of A_1", unless stated or obviously understood otherwise. Various rules or groups of rules comprised in the EnSPGFCR-system are scattered throughout the treatise, so that the EnSPGFCR-system is not compact. The entirety of the EnSPGFCR-system is achieved through the

system of atomic endosemasiopasigraphs (*atomic euautographs* and *atomic panlogographs*), which are comprised in B_1 and $\mathbf{B}_1$ and on use of which the EnSPGFCR-system is based. The principles of constructing the EnSPGFCR-system of A_1 are discussed in greater detail in Df 5.13 below in this subsection.•

6.2. An introduction into the EnSPGFCR-system of A_1 and into the GbFCR-system of A_1

Df 6.5: ***The system of restricted primary formation rules (the RPFR-system) of primary euautographic formulas (PEF's) of*** A_1**.** 1) The EnSPGFCR-system of A_1 is based on a certain *temporal single whole compact recursive syntactically complete and semantically close system of conjoined interrelated meta-axioms, twelve in number,* which is called *the system of restricted*, or *former, FR's* (*formation rules*) *of EF's* (*euautographic formulas*) *of* A_1 or, briefly, the *restricted*, or *former*, *primary FR-system* (*RPFR-system* or *FrPFR-system*) *of* A_1. Accordingly, the RPFR-system is a *single whole conjoined meta-axiom*, any separate itemized conjunct of which is called a *restricted primary FR* (*RPFR*) *of* A_1. The RPFR-system effectively (unambiguously) determine *three manor basic classes of PEF's of* A_1, indicated in Df 6.4(5), namely the *PEOT's* (*EOT's),* the *PEI's* (*PESpT's*), and the *PER's*, and it also defines some conspicuous subclasses of these classes, but it does not introduce any SEF's of A_1.

2) The first RPFR is the *RPEFR of* A_1 that has been described in Df 6.2(6). The next three RPFR's are called the first, second, and third *structural panlogographic RPFR* (*StPLRPFR*), and also, explanatorily (for more clarity) and hence redundantly, *schematic StPLRPFR* (*SchStPLRPFR*) or *StPLRPFR-schema* (*StPLRPFRS*, pl. "*StPLRPFRS'ta*"), *[of PEF's] of* A_1 in the order, in which they are laid down. The first four RPFR's are *concursive* (*not recursive*) ones that serve as the *initial conditions* to the next seven RPFR's, which are *recursive* and which are collectively called the *analytical metalogographic RPFR's* (*AnMLRPFR's*), and also explanatorily *schematic AnMLRPFR's* (*SchAnMLRPFR's*) or *AnMLRPFR-schemata* (*AnMLRPFRS'ta*), [*of PEF's*] *of* A_1. The latter are distinguished individually by the prepositive ordinal numerals from "first" to "seventh" in the order, in which they are laid down. In fact, simultaneous occurrences of both qualifiers "structural" ("St") and "panlogographic" ("PL"), or "analytical" ("An") and "metalogographic" ("ML"), to

“RPFR” are redundant, so that either one of the two qualifiers of either pair can be omitted. That is to say, either abbreviation “*StRPFR*” or “*PLRPFR*” can be used interchangeably with “*StPLRPFR*”, whereas either abbreviation “*AnRPFR*” or “*MLRPFR*” can be used interchangeably with “*AnMLRPFR”* without altering the meanings of “StPLRPFR” and “AnMLRPFR”. Besides being one of the four initial conditions to the seven AnRPFR’s (MLRPFR’s, AnMLRPFR’s), the first StRPFR (PLRPFR, StPLRPFR) *predicates* that the *EOT’s* (*PAEOT’s*, *AEOT’s*) *of* A_1, i.e. the *PVOT’s* (*PAPVOT’s*, *APVOT’s*) introduced by Ax 5.1(5) and the *PCOT’s* (*PAPCOT’s*, *APCOT’s*) introduced by Ax 5.1(9), are *the only members of the entire set of primary EOT’s* (*PEOT’s*) *of* A_1. By contrast, the seven recursive AnPFR’s, subject to the first four concursive RPF’R’s, determine the *entire classes of PEI’s* (*PESpT’s*) and *PER’s of* A_1. In agreement with Df 3.1(7), the class of PER’a is divided into two subclasses: the *PEOR’s*) and the *primary euautographic special relations* (*PESpR’s*). The last, twelfth RPFR is a *formal concluding statement of syntactic completeness and semantic closure of the conjunction of the previous eleven RPFR’s with respect to the classes of PEOT’s* (*EOT’s*), *PEI’s* (*PESpT’s*), and *PER’s*, and it is therefore called the *Master*, or *Meta*, *RPFR* (*MrRPFR* or *MtRPFR*) *of PEF’s of* A_1.

3) In spite of the fact that an EOT and a PEI (PESpT) are alien and are treated differently, beyond the RPFR-system, either of the above two euautographs is, for contrasting it with a PER, indiscriminately called a *primary euautographic term* (*PET*), whereas a PET or a PER is indiscriminately called a *primary euautographic formula* (*PEF*), or *primary euautographic categorem* (*PEC*), *of* A_1. In this way, the class of EOT’s and the class of PEI’s are implicitly united into the class of PET’s, whereas the latter is implicitly united with the class of PER’s into the class of PEF’s. Thus, occurrences the nouns “term” (“T”) and “relation” (“R”) in the above taxonyms (taxonomic names, metaterms) should be understood as abbreviations of either compound noun “*term-formula*” or “*formula-term*” and of either compound noun “*relation-formula*” or “*formula-relation*” respectively. A PEF of A_1 is said to be a *primary euautographic ordinary*, or *ordinary euautographic*, *formula* (*PEOF* or *POEF*) if it is a PEOA (POEA) of A_1 and a *primary euautographic special*, or *special euautographic*, *formula* (*PESpF* or *PSpEF*) if it is a PESpA (PSpEA) of A_1, subject to

Df 5.9(2). The StRPFR's (PLRPFR's, StPLRPFR's) and AnRPFR's (MLRPFR's AnMLRPFR's) are described below in greater detail.

4) The EOT''s indiscriminately and the PVOT's and PCOT's separately are represented in the first StRPFR as *common* (*general*, *specific but not specified*) *members* (*elements*) of the three sets, being the ranges of the three *respective* [*primary*] *structural atomic panlogographic schemata* (*PStAPLS'ta* or *StAPLS'ta*) '**x**', '$\mathbf{x}^{\mathbf{pv}}$', and '$\mathbf{x}^{\mathbf{pc}}$', which have been defined previously in Df 5.2(1,6) under the name "*PStAPLOT's*" or "*StAPLOT's*"; '**x**' is the *principal* syntactic subject of the FRS, while '$\mathbf{x}^{\mathbf{pv}}$' and '$\mathbf{x}^{\mathbf{pc}}$' are *explicative* ones. In stating the FRS, I mentally use each of the three syntactic subjects xenonymously, i.e. as **x**, $\mathbf{x}^{\mathbf{pv}}$, or $\mathbf{x}^{\mathbf{pc}}$ respectively, in the *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I mentally experience its range as *my as-if extramental* (*exopsychical*) *object* that represents the whole of the range as its common member. It will be recalled that a common (general) member (element) of a class (particularly, of a set) is just *another hypostasis* (*way of existence*, *aspect*) *of the class*. Accordingly, I say that **x**, $\mathbf{x}^{\mathbf{pv}}$, or $\mathbf{x}^{\mathbf{pc}}$ *is* a [common] EOT, PVOT, or PCOT of A_1 respectively. At the same time, in agreement with Df 6.2(9), the StAPLS '**x**', e.g., has three hypostases with respect to me, which are called:

a) a [*common*] *EOT of* A_1 if I mentally use it xenonymously as **x**;

b) an *EOT-valued concrete StAPLOT* (*CStAPLOT*) *of* $\mathbf{A}_1$ if I mentally use the it autonymously¸ i.e. as the tychautograph '**x**';

c) an *endosemasiopasigraphic ordinary term* (*EnSPGOT*), i.e. *both an EOT of* A_1 *and an EOT-valued StPLOT of* $\mathbf{A}_1$ [*as if*] *simultaneously*, if I mentally use '**x**' in the TAEXA-mode (xenoautonymously, autoxenonymously).

A like statement applies with '$\mathbf{x}^{\mathbf{pv}}$' and 'PVOT' or with '$\mathbf{x}^{\mathbf{pc}}$' and 'PCOT' in place of '**x**' and 'EOT' respectively. Accordingly, the first StRPFR has three respective hypostases with respect to me, which are called:

a′) the first StRPFR of A_1 as such if I mentally use its syntactic subjects, i.e. '**x**', '$\mathbf{x}^{\mathbf{pv}}$', and '$\mathbf{x}^{\mathbf{pc}}$', xenonymously;

b′) the first *concrete StRPFR* (*CStRPFR*), or *StRPFR-concretum* (*StRPFRC*), of $\mathbf{A}_1$ if I mentally its syntactic subjects autonymously;

c′) the first *structural endosemasiopasigraphic RPFR* (*StEnSPGRPFR*) *of* A_1, or, briefly, the first *StRPFR of both* A_1 *and* $\mathbf{A}_1$ *simultaneously*, if I mentally use its syntactic subjects in the TAEXA-mode.

5) The second StRPFR *predicates* that the *atomic ER's* (*AER's*, *AEOR's*, *APVR's*, *APVOR's*, *PAPVOR's*) *of* A_1, introduced by Ax 5.1(6), are *strictly some* (*some but not all*) *members of the class of primary EOR's* (*PEOR's*) and hence *strictly some members of the entire class of primary ER's* (*PER's*).*of* A_1. The second and also the third StRPFR can be analyzed, *mutatis mutandis*, in the same way as the first one. However, in order to elaborate some general principles of terminology and phraseology of the IML of A_1, I shall write down the pertinent statements, although some of them look like repetition of what have already been stated previously regarding the first StRPFR. The AER's (PAER's) are represented in the second StRPFR as a *common* (*general*, *specific but not specified*) *member* (*element*) of the set, being the range of the appropriate *primary structural atomic panlogographic schema* (*PStAPLS*) '**p**', which has been defined previously in Df 5.2(2) under the specific name "*PStAPLOR*", and which is the only syntactic subject of the FRS. In stating the FRS, I mentally use the syntactic subject xenonymously, i.e. as **p**, in the *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I mentally experience its range as *my as-if extramental* (*exopsychical*) *object* that represents the whole of the range in the hypostasis of its common member. Accordingly, I say that **p** *is* a [common] AER (PAER) of A_1. At the same time, just as '**x**', the PStAPLS '**p**' has three hypostases with respect to me, which are called:

a) a [*common*] *AER of* A_1 if I mentally use it xenonymously as **p**;

b) an *AER-valued concrete PStAPLOR* (*CPStAPLOR*) *of* $\mathbf{A}_1$ if I mentally use the it autonymously¸ i.e. as the tychautograph '**p**';

c) an *endosemasiopasigraphic ordinary relation* (*EnSPGOTR*), i.e. *both an AER of* A_1 *and an AER-valued StPLOR of* $\mathbf{A}_1$ [*as if*] *simultaneously*, if I mentally use '**p**' in the TAEXA-mode.

Accordingly, the second StRPFR has three respective hypostases with respect to me, which are called:

a′) the second StRPFR of A_1 as such if I mentally use its syntactic subject, i.e. '**p**', xenonymously;

b′) the second *concrete StRPFR* (*CStRPFR*), or *StRPFR-concretum* (*StRPFRC*) of $\mathbf{A}_1$ if I mentally use its syntactic subject autonymously;

c′) the second *structural endosemasiopasigraphic RPFR* (*StEnSPGRPFR*) *of* A_1, fzor, briefly, the *second StRPFR of both* A_1 *and* $\mathbf{A}_1$ *simultaneously,* if I mentally use its syntactic subject in the TAEXA-mode.

6) The third StRPFR has two interrelated functions. The first one is to *introduce* (*to produce*) an infinite number of infinite classes (sets) of *primary molecular EOR's* (*PMEOR's*) *of* A_1 as the ranges of the following concrete *primary structural molecular panlogographic schemata* (*PStMPLS'ta*) *of* $\mathbf{A}_1$:

the *singulary one* or *one of weight* 1,'$\mathbf{f}^1(\mathbf{x}_1)$' or '$\mathbf{f}^1$'('$\mathbf{x}^1$'); (6.6[1])

the *binary one* or *one of weight* 2, '$\mathbf{f}^2(\mathbf{x}_1,\mathbf{x}_2)$' or '$\mathbf{f}^2$'('$\mathbf{x}^1$','$\mathbf{x}^2$'); (6.6[2])

the *ternary one* or *one of weight* 3,

'$\mathbf{f}^3(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$' or '$\mathbf{f}^3$'('$\mathbf{x}^1$','$\mathbf{x}^2$','$\mathbf{x}^3$'); (6.6[3])

etc, subject to the items 1 and 2 of Ax 5.1 and also subject to the items 1 and 3 of Df 5.2. The above PStMPLS'ta and also ones of all higher weights, which are obviously understood, are the syntactic subjects of the third StRPFR. The second function of this FRS is to *predicate* that, besides the AER's (PAEOR's), the PMEOR's of A_1 are *some more members of the class of PEOR's* and hence *some more members of the entire class of PER's.of* A_1. The PMEOR's are represented in the FRS as *common members* of the classes, being the ranges of the PStMPLS'ta (6.6[1])–(6.6[3]), etc. In stating the FRS, I mentally use the syntactic subjects xenonymously, i.e. as

$$\mathbf{f}^1(\mathbf{x}_1),\ \mathbf{f}^2(\mathbf{x}_1,\mathbf{x}_2),\ \mathbf{f}^3(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3), \text{ etc,} \tag{6.6}$$

in the *projective* (*polarized*, *extensional*, *connotative*) *mental mode,* in which I mentally experience the range of each syntactic subject as *my as-if extramental* (*exopsychical*) *object* that represents the whole of the range in the hypostasis of its common member. Accordingly, I say that $\mathbf{f}^n(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$ *is* an *n*-ary PMEOR of A_1. At the same time, just as '**x**' or '**p**', the *n*-ary PStMPLS of $\mathbf{A}_1$ '$\mathbf{f}^n(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$', which is the appropriate *n-ary primary structural molecular metalogographic schema* (*PStMMLS*) of *n*-ary PMEOR's of A_1, has three hypostases with respect to me, which are called:

a) an *n-ary* [*common*] *PMEOR of* A_1 if I mentally use it xenonymously as $\mathbf{f}^n(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$;

b) a *PMEOR-valued n-ary concrete PStMPLS* (*CPStMPLS*) *of* $\mathbf{A}_1$ if I mentally use the it autonymously¸ i.e. as the tychautograph ‘$\mathbf{f}^{n}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$’;

c) an *n-ary endosemasiopasigraphic ordinary relation* (*EnSPGOR*), i.e. *both an n-ary PMEOR of* A_1 *and* a *PMEOR-valued n-ary PStMPLS of* $\mathbf{A}_1$ [*as if*] *simultaneously*, if I mentally use ‘$\mathbf{f}^{n}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$’ in the TAEXA-mode.

Accordingly, the third StRPFR of A_1 has three respective hypostases with respect to me, which are called:

a′) the third StRPFR of A_1 as such if I mentally use its syntactic subject, i.e. ‘$\mathbf{f}^{n}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$’, xenonymously;

b′) the third *concrete StRPFR* (*CStRPFR*), or *StRPFR-concretum* (*StRPFRC*) of $\mathbf{A}_1$ if I mentally use its syntactic subject autonymously;

c′) the *structural endosemasiopasigraphic RPFR* (*StEnSPGRPFR*) *of* A_1, or, briefly, the third *StRPFR of both* A_1 *and* $\mathbf{A}_1$ *simultaneously*, if I mentally use its syntactic subject, in the TAEXA-mode.

Incidentally, I say that ‘$\mathbf{f}^{n}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$’ *is* an *n*-ary primary structural molecular *panlogographic* schema (PStM*PL*S) in the sense that, upon replacing the metalogographic placeholder ‘*n*’ with any concrete Arabic numeral, the bold-faced single quotation marks should be replaced with light-faced ones. Therefore, I may assert that ‘$\mathbf{f}^{n}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$’ is any one of the PStMPLS’ta) *of* $\mathbf{A}_1$: ‘$\mathbf{f}^{1}(\mathbf{x}_1)$’, ‘$\mathbf{f}^{2}(\mathbf{x}_1,\mathbf{x}_2)$’, ‘$\mathbf{f}^{3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$’, etc. That is to say, ‘‘$\mathbf{f}^{n}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$’’ is a *quasi-homolographic autonymous quotation* (*QHAQ*), i.e. a *metalogographic placeholder* (*MLPH*) of any one of the *homolographic autonymous quotations* (*HAQ’s*) ‘‘$\mathbf{f}^{1}(\mathbf{x}_1)$’’, ‘‘$\mathbf{f}^{2}(\mathbf{x}_1,\mathbf{x}_2)$’’, ‘‘$\mathbf{f}^{3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$’’, etc. In contrast to ‘$\mathbf{f}^{n}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$’, ‘$\mathbf{f}^{n}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$’ is a concrete metalogograph ‘$\mathbf{f}^{n}$’‘(‘‘$\mathbf{x}^{1}$’‘,’‘$\mathbf{x}^{2}$’‘,’‘…’‘,’‘$\mathbf{x}^{n}$’‘)’, – in accordance with the *principle of juxtaposition* of HAQ’s, or ‘$\mathbf{f}^{n}$’(‘$\mathbf{x}^{1}$’,‘$\mathbf{x}^{2}$’,…,‘$\mathbf{x}^{n}$’) if all punctuation marks are used autonymously.

7) Each of the first six AnRPFR’s (MLRPFR’s) is a semi-verbal *hypothetical complex-subordinate statement*, the *consequent* (*principal clause*) of which has exactly one syntactic subject, namely the respective one of the *analytical formulary metalogographic schemata* (*AnFMLS’ta*), called also *analytical metalogographic formulas* (*AnMLF’s*):

$$\text{‘}V(\mathbf{\Gamma})\text{’, ‘}[\hat{-}\mathbf{\Gamma}]\text{’, ‘}[\mathbf{\Gamma}\hat{+}\mathbf{\Delta}]\text{’, ‘}[\mathbf{\Gamma}\hat{\cdot}\mathbf{\Delta}]\text{’, }[\mathbf{\Gamma}\hat{=}\mathbf{\Delta}]\text{’, ‘}[\mathbf{\Gamma}\veebar\mathbf{\Delta}]\text{’} \qquad (6.7)$$

in that order, while the *antecedent* (*subordinate conditional clause*) of which serves to properly restrict *ad hoc* the range of either one of the AnAMLPH's '**Γ**' and '**Δ**', as defined in Df 5.10(1). The seventh AnRPFR has the same character with the only difference that its consequent is a complex-coordinate clause consisting of two simple conjoined clauses having two different syntactic subjects, namely these AnMLF's (AnFMLS'ta):

$$\text{‘}[(\exists\mathbf{x})\mathbf{\Gamma}\langle\mathbf{x}\rangle]\text{’ and ‘}[(\hat{\cdot}\,\mathbf{x})V(\mathbf{\Gamma}\langle\mathbf{x}\rangle)]\text{’.} \qquad (6.8)$$

Therefore, this compound AnRPFR can mentally be developed as two separate simple AnRPFR's, which is supposed to be done in making general statements about an unspecified AnRPFR below. Consequently, I may speak about *eight simple* AnRPFR's rather than to speak about the actual itemized seven AnRPFR's, one of which is compound. It is also noteworthy that, in contrast to the AnFML*S'ta* on the list (6.7), all of which involve exclusively the An*A*ML*'s* '**Γ**' and '**Δ**', the AnFML*S'ta* on the list (6.8) involve the An*M*ML*D* $\mathbf{\Gamma}\langle\mathbf{x}\rangle$.

8) In accordance with the above-said, each of the eight simple AnRPFR's introduces an infinite number of EF's of A_1 as members (elements) of the *current* range of its syntactic subject by *connotatively predicating* that range as the current subclass either of the [class of] PEI's or of the [class of] PER's – depending on the EKS (euautographic kernel-sign) occurring in the AnMLF. In this case, just as a StRPFR, an AnRPFR is an RPFR of A_1 if and only if I mentally use the AnMLF, being its syntactic subject, *xenonymously*. At the same time, the AnMLF is *a formula of the IML of* A_1 *and* $\mathbf{A}_1$, *and not a formula of* $\mathbf{A}_1$, because it is a metalogograph, and not a panlogograph. Therefore, in contrast to a StRPFR, if I mentally use an AnMLF, being the, or a, syntactic subject of a certain AnRPFR of A_1, autonymously then the latter is a statement in the IML, which is not an FR (formation rule) of any organon. Consequently, an AnRPFR is meaningful if and only if its syntactic subject is used xenonymously, so that *the AnRPFR is an FR of* A_1 *and only of* A_1.

9) Owing to the *recursive semantic interrelations* of separate rules of the RPFR-system and also owing to the *Master RPFR*, the RPFR-system unambiguously determines three classes of PEF's: the class of PEOT's, being the same as the class of EOT's, the class of PEI's (PESpT's), and the class of PER's, and it also determines

some subclasses of the class of PER's, – as stated in the item 2 above in this definition.•

Df 6.6: ***Principles of induction of the EnSPGFCR-system of*** A_1 ***from the RPFR-system of*** A_1**.** 1) The RPFR-system of A_1 is intended to create the embryo (priming, entire initial condition) of the EnSPGFCR-system of A_1. However, in accordance with the previous definition, the RPFR-system has the following peculiar properties. Any logographic placeholder (LPH) that is employed in the RPFR-system as a syntactic subject of one of its constituent FR's is either a PStPLF (primary structural panlogographic formula) of $\mathbf{A}_1$ or an AnMLF (analytical metalogographic formula) of the IML of A_1. The range of a PStPLF is a certain class (or particularly a certain set) of PEF's (primary euautographic formulas) of A_1 that is *unambiguously defined and universally* (*permanently*) *fixed in advance beyond the RPFR-system* in accordance with the ranges of the PStAPL's (primary structural atomic panlogographs) and also in accordance with the atomic euautographs, if any, – i.e. in accordance with the endosemasiopasigraphs, which constitute the PStPLF. The range of an AnMLF is also a certain class of PEF's of A_1 that, however, turns out to be *unambiguously defined and fixed only ad hoc*, i.e. *in the result of stating the entire RPFR-system and only in the context of that system*. Therefore, when used beyond RPFR-system, either xenonymously or autonymously, an isotoken of the AnMLF cannot serve as a formula of any *autonomic* (self-contained) calculus. Consequently, the RPFR-system as a single whole introduces (determines, generates) *exclusively certain fixed classes of PEF's of* A_1, i.e. *of primary and only of primary* (*not secondary*) *FR's of* A_1 *and only of* A_1 (and not, say, of $\mathbf{A}_1$) – classes, which cannot be changed either in the framework of the RPFR-system or from the outside of it. That is to say, the RPFR-system as such is a *thing-in-itself* (*noumenon*) in the sense that it cannot immediately serve as an initial condition of the EnSPGFCR-system. In order to perform its predestination, the RPFR-system should be *metamorphosed* as described below in this definition.

2) Immediately upon laying down the RPFR-system of A_1 and hence upon introducing (determining) all conceivable PEF's of A_1, *the PAnFPLAB* (*primary analytical formulary panlogographic atomic basis*) of $\mathbf{A}_1$, $\mathbf{B}_{1PAnF}$, which turns out to be the whole of *the PAnPLAB* of $\mathbf{A}_1$, $\mathbf{B}_{1PAn}$, indicated in Df 5.1(4), is defined by a certain semi-verbal semantic definition, which is properly called the *PLFR of* $\mathbf{B}_{1PAnF}$

or the *primary analytical atomic PLFR* (*PAnAPLFR*, *AnAPPLFR*, *AnAPLPFR*) [*of PAnAPLI's and PAnAPLR's*] *of* $\mathbf{A}_1$ or the *primary analytical PLGR* (*PAnPLGR*, *AnPPLGR*, *AnPLPGR*) [*of EI's and ER's*] *of* A_1; "*F*" is, as before, an abbreviation for "*formation*" and "*G*" for "*generalization*" (cf. items 4, 7, and 8 of Df 5.1). The *primary analytical atomic formulary* (*categorematic*) *panlogographs* (*PAnAFPL's*), i.e. *primary analytical atomic panlogographic formulas* (*PAnAPLF's*), *integrons* (*PAnAPLI's*) or *relations* (*PAnAPLR's*) of $\mathbf{A}_1$, being elements of $\mathbf{B}_{1\mathrm{PAn}}$ (the same as $\mathbf{B}_{1\mathrm{PAnF}}$), are divided into a number of sets in accordance with the specific classes of PEF's, which are determined by the RPFR-system, so that each class is assigned, either as the *initial extendable range* or as the *permanent range*, to every PAnAFPL-member of a certain one of the above sets.

3) After laying down the PLFR of $\mathbf{B}_{1\mathrm{PAnF}}$ (PAnAPLFR of $\mathbf{A}_1$, PAnPLGR of A_1), but *before laying down any SFR* (*secondary FR*) *of* A_1 *or* $\mathbf{A}_1$, *the primary descriptive analytical formulary panlogographic molecular basis* (*PDAnFPLMB*) of $\mathbf{A}_1$, $\mathbf{B}'_{1\mathrm{PAnF}}$, is defined by a certain semi-verbal semantic definition, which is properly called the *PLFR of* $\mathbf{B}'_{1\mathrm{PAnF}}$ and also the *primary descriptive analytical molecular PLFR* (*PDAnMPLFR*, *DAnMPPLFR*, *DAnMPLPFR*), or *primary analytical molecular PLFR-description* (*PAnMPLFRD*, *AnMPPLFRD*, *AnMPLPFRD*), [*of PDAnMFPLI's and PDAnMFPLR's*] *of* $\mathbf{A}_1$, or the *primary analytical PLSR* (*PAnPLSR*, *AnPPLSR*, *AnPLPSR*) [*of EI's and ER's*] *of* A_1 (cf. Dfs 5.1(6b) and 6.3(5a)). The latter definition is a specification of Df 5.11 subject to the RPFR-system and also subject to [the definition of] $\mathbf{B}_{1\mathrm{PAn}}$. The elements of $\mathbf{B}'_{1\mathrm{PAnF}}$ are various *primary descriptive*, or *sorting*, *analytical molecular panlogographic formulas* (*PDAnMPLF's* or *PStgAnMPLF's*), i.e. *primary analytical molecular formulary panlogographic descriptions* (*PAnMFPLD's*), particularly the *primary descriptive analytical molecular panlogographic relations* (*PDAnMPLR's*) such as '$\mathbf{P}\langle\mathbf{u}\rangle$', '$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$', '$\mathbf{P}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$', etc and the *primary descriptive analytical molecular panlogographic integrons* (*PDAnMPLI's*) such as '$\mathbf{I}\langle\mathbf{u}\rangle$', '$\mathbf{I}\langle\mathbf{u},\mathbf{v}\rangle$', '$\mathbf{I}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$', etc (cf. Df 6.3(5)). In this case, '**P**' and '**I**' are elements of $\mathbf{B}_{1\mathrm{PAn}}$, '**u**', '**v**'¸ and '**w**' are elements of $\mathbf{B}_{1\mathrm{PSt}}$ (see the list (5.6)), and ⟨ ⟩ are two elements of $\Delta\mathrm{B}_1$ and hence of B_{1+} (see Df 5.1(1)).

4) After laying down the PLFR of B'_{1PAnF}, which particularly specifies the range of '$\mathbf{P}\langle\mathbf{x}\rangle$', the same as that of '$\mathbf{P}\langle\mathbf{u}\rangle$', but still *before laying down any SFR of* A_1, the RPFR-system of A_1 is restated as a certain *single whole compact recursive syntactically complete and semantically open system of conjoined interrelated twelve meta-theorems,* which is called *the system of extendable*, or *latter*, *FR's* (*formation rules*) *of EF's* (*euautographic formulas*) *of* A_1 or, briefly, the *extendable*, or *latter*, *primary FR-system* (*XPFR-system* or *LrPFR-system*) *of* A_1. Accordingly, the XPFR-system is a *single whole conjoined meta-theorem*, any separate itemized conjunct of which is called an *extendable primary FR* (*XPFR*) *of* A_1. At any given stage of developing A_1, the XPFR-system together with all SFR's of A_1, which have been laid down by that stage, effectively (unambiguously) determine the *three major classes of EF's of* A_1, indicated in Df 6.1(1), namely the *EOT's*, the *EI's* (*ESpT's*), and the *ER's*, and they also determine some conspicuous subclasses of these classes, i.e. some *minor basic classes of EF's of* A_1. the XPFR-system has the following conjuncts.

a) The first XPFR is the *XPEFR of* A_1 that has been described in Df 6.2(6). That is to say, the first RPFR is paraphrased so as the euautographic digits $\mathsf{0}$ and $\mathsf{1}$ being its syntactic subjects are predicated to be members of the class of all EI's of A_1 and not just members of the class of PEI's of A_1 (see Df 6.4(5)). The first StRPFR, as described in Df 6.5(4), remains unaltered and is employed as the first *StXPFR* because the class of EOT's cannot be extended. The second and third StRPFR's of the RPFR-system are paraphrased in analogy with the RPEFR, namely the PER's of A_1, being members of the ranges of the syntactic subjects of these StRPFR's, are now predicated to be members of the entire class of ER's of A_1, while all other aspects of the two StRPFR's, as described in the items 5 and 6 of Df 6.5, remains unaltered. Thus, the first four XPFR's are the *concursive initial conditions* to the *next seven recursive XPFR's*, which are the pertinent extendable modifications of the seven recursive AnRPFR's and which are called briefly the *analytical XPFR's* (*AnXPFR's*) *of* A_1. In analogy with a StRPFR, the StXPFR, being its counterpart or extendable modification, is called in full a *structural panlogographic XPFR* (*StPLRPFR*), and also, explanatorily (for more clarity) and hence redundantly, *schematic StPLXPFR* (*SchStPLXPFR*) or *StPLXPFR-schema* (*StPLXPFRS*, pl. "*StPLXPFRS'ta*"), *[of EF's] of* A_1 and is assigned with the same numeral. Likewise, in analogy with an AnRPFR, the AnXPFR, being its extendable modification, is called in full an *analytical* *panlogographic XPFR* (*AnPLXPFR*), and also explanatorily *schematic AnPLXPFR's* (*SchAnPLXPFR's*) or *AnPLXPFR-schemata* (*AnPLXPFRS'ta*), [*of EF's*] *of* A_1 and is assigned with the same numeral. At the same time, just as in the case of "RPFR", an occurrence of either qualifier "structural" ("St") or "analytical" ("An") to "XPFR" together with an occurrence of the qualifier "panlogographic" ("PL") is redundant, so that either one of the two simultaneous qualifiers can be omitted. Therefore, either abbreviation "*StXPFR*" or "*PLXPFR*" can be used interchangeably with "*StPLXPFR*", whereas either abbreviation "*AnXPFR*" or "*MLXPFR*" can be used interchangeably with "*AnMLXPFR"* without altering the meanings of "StPLXPFR" and "AnMLXPFR".

b) The AnXPFR's of EF's of A_1 are obtained by restating the AnRPFR' of PEF's of A_1 in terms of the appropriate PAnAPL's (PAnAPLF's) of $\mathsf{B}_{1\mathsf{PAnF}}$, namely '**P**' and '**Q**' or '**I**' and '**J**', in place of '**Γ**' and '**Δ**' respectively, while common euautographic members of the ranges of the modified syntactic subjects of the former FR's are [connotatively] predicated to be [members of the entire classes of] EI's or ER's of A_1, as applicable, and not just [members of the classes of] PEI's or PER's of A_1 as predicated in the latter FR's. Accordingly, the former are qualified *extendable* (*X*) in contrast to the latter that are qualified *restricted.* To be specific, under the above-mentioned replacements of '**Γ**' and '**Δ**', the AnFMLS'ta (AnMLF's) on the lists (6.7) and (6.8), being syntactic subjects of the seven sequential AnRPFR's, turn into the following AnFPLS'ta (AnPLF's) as syntactic subjects of the respective seven AnXPFR's:

$$\text{'}V(\mathbf{P})\text{'},\ \text{'}[\hat{-}\,\mathbf{P}]\text{'},\ \text{'}[\mathbf{I}\,\hat{+}\,\mathbf{J}]\text{'},\ \text{'}[\mathbf{I}\,\hat{\cdot}\,\mathbf{J}]\text{'},\ [\mathbf{I}\,\hat{=}\,\mathbf{J}]\text{'},\ \text{'}[\mathbf{P}\veebar\mathbf{Q}]\text{'} \qquad (6.9)$$

$$\text{'}[(\exists\mathbf{x})\mathbf{P}\langle\mathbf{x}\rangle]\text{' and '}[(\hat{\cdot}\,\mathbf{x})V(\mathbf{P}\langle\mathbf{x}\rangle)]\text{'}. \qquad (6.10)$$

Like their metalogographic predecessors on the lists (6.7) and (6.8), the AnFPL*S'ta* on the list (6.9) involve exclusively the An*A*PL's '**P**', '**Q**', '**I**', and '**J**', whereas the AnFPL*S'ta* on the list (6.10) involve the An*M*PL*D* $\mathbf{P}\langle\mathbf{x}\rangle$.

c) In accordance with the peculiarities of the eleven XPFR's of EF's of A_1, indicated in the above items a) and b), the last, twelfth RPFR, which is called the Master, or Meta, RPFR (MrRPFR or MtRPFR) of PEF's of A_1, is restated to become a *statement of syntactic completeness and semantic openness of the preceding eleven*

XPFR's of A_1 – a statement that is called the *Master*, or *Meta*, *XPFR* (*MrXPFRD* or *MtXPFRD*) *of EF's of* A_1.

d) Like the RPFR-system, the XPFR-system is a single whole compact recursive system of conjoined interrelated constituent XPFR's of EF's of A_1. I regard the XPFR-system as a meta-theorem that follows from the RPFR-system as the underlying meta-axiom by replacing, throughout the latter, occurrences of the PAnAML's '**Γ**' and '**Δ**' with occurrences of the appropriate PAnAPL's, as indicated above in the item b), and also by omitting, throughout the RPFR-system, the occurrences of the qualifier "primary" and of the letter "P" as its abbreviation in the *predicatives* in order to allow, in the sequel, automatically extending the initial ranges of the PAnAPL's in the result of making any relevant *SCR* (*secondary classification rules*) of EF's of A_1. The necessity of such order (timing) of the above steps in the setup of A_1 and $\mathbf{A}_1$ and also usefulness of the pertinent terminology introduced above will become clear as I go along.

5) All [systemic] *classification rules* (*CR's*), i.e. all *generalization rules* (*GR's*), *all sortation rules* (*SR's*), and all *formation rules* (*FR's*) [*of EF's*] *of* A_1, which are laid down after laying down the XPFR-system, are called *secondary CR's* (*SCR's*), i.e. discriminately *SGR's*, *SSR's*, and *SFR's*, [*of EF's*] *of* A_1. Except the SEFR's (secondary euautographic formation rules), i.e. SEFR-system, of SDDEI's (secondary decimal digital euautographic integrons) of A_1 and $\mathbf{A}_1$ (see Df 6.1(5a′,7)), any other SCR [of EF's] of A_1 is a *secondary panlogographic* or *CR* (*SPLCR*), i.e. discriminately either an *SPLGR* or an *SPLSR* or else an *SPLFR*, either *of EI's* or *of ER's* (indiscriminately *of EF's*) *of* A_1. By the items 1, 2, and 7 of Df 6.4, an SPLGR, SPLSR, or PLSR of EI's, or of ER's, of A_1 is, at the same time, an *SPLFR of* [*concrete*] *PLI's*, or *of* [*concrete*] *PLR's* (indiscriminately *of* [*concrete*] *PLF's*), *of* $\mathbf{A}_1$ – the *secondary PLI's* (*SPLI's*) or the *secondary PLR's* (*SPLR's*), whose ranges are classes of the respective ones of the above mentioned *secondary EI's* (*SEI's*) or *secondary ER's* (*SER's*), respectively. Conversely, an *SPLFR of PLF's of* $\mathbf{A}_1$ is called:

a) a *generalizing* one (*GgSPLFR*) if all its syntactic subjects are *secondary atomic PLF's* (*SAPLF's*), i.e. either *SAPLI's* or *SAPLR's*, *of* $\mathbf{A}_1$, whose range is a certain *class of secondary EF's* (*SEF's*), i.e. either of *SEI's* or of *SER's*, *of* A_1;

b) a *descriptive*, or *sorting*, one (*DSPLFR* or *StgSPLFR*) if all its syntactic subjects are *secondary descriptive molecular PLF's* (*SDMPLF's*), i.e. either *SDMPLI's* or *SDMPLR's*, *of* $\mathbf{A}_1$, whose range is a certain *sort* of *secondary EF's* (*SEF's*), i.e. either of *SEI's* or of *SER's*, *of* A_1;

c) a *schematic* (*form-giving*) one (*SchSPLFR*) or *SPLFR-schema* (*SPLFRS*) if all its syntactic subjects are *secondary schematic PLF's* (*SSchPLF's*), or *secondary formulary panlogographic schemata* (*SFPLS'ta*), i.e. either *SSchPLI's* or *SSchPLR's*, *of* $\mathbf{A}_1$, whose range is a certain class of *secondary EF's* (*SEF's*), i.e. either of *SEI's* or of *SER's*, *of* A_1, being *synonyms* of some other *EI's* or *ER's of* A_1, primary or secondary.

6) In accordance with Df 6.5 and the previous items of this definition, and also in agrrement with the items 4 and 5 of Df 6.4 of A_1, i.e. of A_1 and $\mathbf{A}_1$, has the following properties (facts).

a) Every EF of A_1 is either an EOT or an EI, called also ESpT, or else an ER, of A_1. And similarly with "PL" and '$\mathbf{A}_1$' in place of "E" and 'A_1' respectively, i.e. every PLF of $\mathbf{A}_1$ is either a PLOT or a PLI, called also PLSpT, or else a PLR, of $\mathbf{A}_1$.

b) Every *EOT of* A_1 is a *primary EOT* (*PEOT*) *of* A_1 and vice versa; i.e .there are *no secondary EOT's of* A_1. By contrast every *EI*, or *ESpT*, *of* A_1 is either a *primary EI* (*PEI*), or *primary ESpT* (*PESpT*), *of* A_1 or a *secondary EI* (*SEI*), or *secondary ESpT* (*SESpT*), *of* A_1, and vice versa. Hence, every PEF of A_1 is either a PEOT, i.e. EOT, or a PEI (PSpT), or else a PER, of A_1, whereas every SEF of A_1 is either an SEI (SESpT) or an SER, of A_1, and vice versa.

c) The above items a) and b) apply verbatim with PL" and '$\mathbf{A}_1$' in place of "E" and 'A_1' respectively.

d) A *primary PLFR* (*PPLFR*, *PLPFR*) *of EI's*, or *of ER's*, *of* A_1 determines both PEI's and SEI's, or correspondingly both PER's and SER's, of A_1. By contrast a *secondary PLFR* (*SPLFR*, *PLSFR*) *of EI's*, or *of ER's*, *of* A_1 determines only SEI's, or correspondingly only SER's, of A_1 (cf. Df 6.4(6)). At the same time, a *PPLFR* (*PLPFR*) *of PLI's*, or *of PLR's*, *of* $\mathbf{A}_1$ determines only PPLI's, or correspondingly only SPLR's, *of* $\mathbf{A}_1$, whereas a *secondary PLFR* (*SPLFR*, *PLSFR*) *of PLI's*, or *of PLR's*, *of* $\mathbf{A}_1$ determines only SPLI's, or correspondingly only SPLR's, of $\mathbf{A}_1$.

e) The above items a)–d) apply with items apply with ‘ $\mathbf{A}_1^0$ ’ and ‘ A_1^0 ’ or with ‘A_0’ and ‘$\mathbf{A}_0$’ in place ‘A_1’ and ‘$\mathbf{A}_1$’ respectively with the proviso that A_0 has no EOT’s and that hence $\mathbf{A}_0$ has no PLOT’s.

7) Until any SFR of EF’s of A_1 is laid down, the XPFR-system determines the same class of PEF’s as that determined by the RPFR-system. That is to say, the *initial* semantic properties of the XPFR-system with respect to EF’s that it determines are the same as the respective *permanent* semantic properties of the RPFR-system. Once, however, a certain SFR of A_1, the first one or any subsequent one – e.g. an SPLFR of A_1 or the SEFR-system of A_1, – is laid down so as to introduce either some SER’s or some SEI’s, the range of every relevant PAnAPL of $\mathbf{B}_{1PAn}$, – such a range, e.g., as the range of ‘**I**’ or ‘**J**’, which is by definition the class of *all* EI’s of A_1, or as the range of ‘**P**’ or ‘**Q**’, which is by definition the class of *all* ER’s of A_1, – is *automatically augmented* (*updated*) by the pertinent new SEF’s. Consequently, the variety of EI’s or ER’s that are introduced by any given AnXPFR is also automatically augmented (updated) to include all new SEI’s or SER’s into the ranges of all pertinent constituent parts of the syntactic subject or subjects of the AnXPFR, – as described in Df 6.4(6). By contrast, *all* EF’s that are introduced by any AnRPFR are either *P*EI’s (*primary* EI’s) or *P*ER’s (*primary* ER’s) and therefore their variety is affected by *no* SFR of A_1. That is to say, the RPFR-system is isolated from any SFR of A_1, while the XPFR-system interrelated with every SFR of A_1. As a result, once at least one SFR of A_1 is laid down, the XPFR-system ceases to be semantically equivalent to the RPFR-system with respect to EF’s that it introduces (determines). Therefore, the RPFR-system is replaced with the XPFR-system before laying down any SFR of A_1. Immediately upon replacing RPFR-system with the XPFR-system, the former *is abandoned* and is not used afterwards any more. In this case, the AMLPH’s ‘**Γ**’ and ‘**Δ**’ are needed exclusively for laying down the seven AnRPFR’s and the Master RPFR. Upon having done this duty, ‘**Γ**’ and ‘**Δ**’ *are abandoned* in the result of replacing the RPFR-system with the RPFR-system.

8) In contrast to an AnRPFR of A_1, which is meaningful if and only if a PAnMLF (PAnMLS) of EF’s of A_1, being its syntactic subject is used xenonymously (see Df 6.5(8)), a PAnPLF (PAnPLS) of EF’s of A_1, being the, or a, syntactic subject of the counterpart AnXPFR of A_1 has, like a PStPLF (PStPLS), three hypostases with respect to me, which are called (cf. the items 4–6 of Df 6.5):

a) a [*common*] *EF of* A_1, i.e. either a [*common*] *EI* (*ESpT*) or a [*common*] *ER* – depending on the PAnPLS, if I mentally use it xenonymously;

b) a *concrete primary analytical panlogographic formula* (*CPAnPLF*) *of* $\mathbf{A}_1$, i.e. either a *concrete primary analytical panlogographic integron* (*CPAnPLI*), called also a *concrete primary analytical panlogographic special term* (*CPAnPLSpT*), or a *concrete primary analytical panlogographic relation* (*CPAnPLR*) – depending on the PAnPLS again, if I mentally use the latter autonymously;

c) an *endosemasiopasigraphic formula* (*EnSPGF*), or, briefly, *formula*, *of* A_1, i.e. *both a common EF of* A_1 *and a CPAnPLF of* $\mathbf{A}_1$ *simultaneously*, if I mentally use the PAnPLS in the TAEXA-mode (xenoautonymously, autxenonymously).

Accordingly, in analogy with the three hypostases of a StRPFR of A_1 indicated in the items 4–6 of Df 6.5, an AnXPFR of A_1 has three hypostases with respect to me, which are called:

a′) an AnXPFR of A_1 as such if I mentally use the PAnPLS, being its syntactic subject, xenonymously;

b′) a *concrete AnXPFR of* $\mathbf{A}_1$ if I mentally use the PAnPLS autonymously;

c′) an *analytical endosemasiopasigraphic XPFR* (*AnEnSPGXPFR*) *of* A_1, or, briefly, an *AnXPFR of both* A_1 *and* $\mathbf{A}_1$ *simultaneously*, if I mentally use the PAnPLS in the TAEXA-mode.

9) The *XPEFR of* A_1, being the first FR of the XPFR-system, and also the *SEFR-system of* A_1 are at the same time ones of $\mathbf{A}_1$ and hence ones of A_1, – in accordance with the items 6 and 7 of Df 6.1. At the same time, any PLF that is employed as a *syntactic subject* in the XPFR-system is a *concrete primary panlogographic formula* (*CPPLF*) *of* $\mathbf{A}_1$, either a *structural* one (*CPStPLF*) or an *analytical* one (*CPAnPLF*), which can intelligibly be used both xenonymously and autonymously. Consequently, in accordance with the items 4–6 of Df 6.5 and item 4 of this definition, the entire *XPFR-system of EF's* (*EOT's, EI's,* and *ER's*) *of* A_1 has three hypostases with respect to me, which are called:

a) the *XPFR-system of EF's of* A_1 as such if all CPPLF's occurring in it are mentally used xenonymously;

b) the *XPFR-system of the CPPLF's of* $\mathbf{A}_1$ if the latter are mentally used autonymously;

c) the *endosemasiopasigraphic XPFR-system* (*EnSPGXPFR-system*) *of EnSPGF's of* A_1, i.e. *of both EF's* A_1 *and the CPPLF's of* $\mathbf{A}_1$ *simultaneously*, if all CPPLF's of $\mathbf{A}_1$ occurring in it are mentally used in the TAEXA-mode.

This epistemological property of the XPFR-system of EF's of A_1 is in contrast to the epistemological property of the RPFR-system of EF's of A_1 to have a single hypostasis as indicated in the item 1 of this definition. In the case a), besides the PEI's and PER's of A_1, which are determined by the AnXPFR's, the latter implicitly introduce (determine, generate) *an infinite number of respectively patterned SEI's* and *SER's of* A_1, whose constituent parts are some SEF's of A_1 that are determined by the relevant SFR's of A_1 once these are laid down. At the same time, in the case b), independently of any SPLFR's of $\mathbf{A}_1$ to be stated in the sequel, the XPFR-system defines the pertinent CPPLF's (CPPLOT's, CPPLEI's, and CPPLER's) of $\mathbf{A}_1$ and also all their alphabetic variants, owing to the properties of meta-autonymy and retroactivity of the elements of $\mathbf{B}_1$, which were described in the items 3–5 of subsection 4.4**.**

10) In the general case, the three hypostases of the XPFR-system of EF's of A_1 indicated in the previous item differ from one another only in the respective mental attitudes that I take towards its constituent XPFR's. In order to illustrate the difference between the *xenonymous hypostasis* and the *autonymous hypostasis* of the XPFR-system, I lay down *the autonymous version of the XPFR-system*, i.e. the XPFR-system in its autonymous hypostasis, explicitly with the help of the pertinent HAQ's and QHAQ's (cf. Df 6.5(6)). I regard the explicit autonymous version of the XPFR-system as corollary from its xenonymous version, i.e. as a statement that does not require any proof.

11) In agreement with Df 6.4(7), the above part of this definition can be summarized as follows. In contrast to the RPFR-system of A_1, which is *semantically restricted only to the PER's of* A_1 and which is therefore *semantically not extendable*, the PLFR of $\mathbf{B}_{1PAnF}$ (PBscAnAPLFR of $\mathbf{A}_1$, PBscAnPLGR of A_1), the PLFR of $\mathbf{B}'_{1PAnF}$ (PBscDAnMPLFR of $\mathbf{A}_1$, PBscAnPLSR of A_1), and the XPFR-system of A_1 form *the embryo* (*priming, entire initial condition*) *and an inseparable part of a single*

whole recursive and coherent (*self-adjustable*), i.e. *self-consistent* and *self-updatable, system of interrelated formulary classification rules* (*CR's*), or *grammatical rules* (*GmR's*), *of the biune organon* A_1, i.e. [*of the union and superposition*] *of* A_1 *and* $\mathbf{A}_1$, – the system that is called the *endosemasiopasigraphic classification*, or *grammatical*, *rules* (briefly the *EnSPGFCR-system* or *EnSPGFGmR-system*), and also, alternatively, the *system of self-adjustable classification rules* (briefly the *SACR-system*), [*of EnSPGF's*] *of* A_1, i.e. [*of EF's*] *of* A_1 *and* [*of PLF's*] *of* $\mathbf{A}_1$. Besides the above initial conditions (embryo, priming), the EnSPGFCR-system includes *all mandatory* (*obligatory, not optional*) *SPLFR's of concrete PLI's and PLR's of* $\mathbf{A}_1$*, which are explicitly stated or obviously understood as ones of the setup of* A_1 *or* $\mathbf{A}_1$ and it also includes *the SEFR-system* [*of SDDEI's*] *of* A_1 *and* $\mathbf{A}_1$. In contrast to the PLFR of $\mathbf{B}_{1PAnF}$, PLFR of $\mathbf{B}'_{1PAnF}$, and XPFR-system of A_1 and $\mathbf{A}_1$, which are complete compact groups of *primary classification rules* (*PCR's*) of A_1 and which are at the same time complete compact groups of *primary PLFR's* (*PPLFR's*) of $\mathbf{A}_1$, various SPLFR's of $\mathbf{A}_1$, any of which *is at the same time a PLGR or a PLSR or a PLFR, of EF's of* A_1, are stated singly or primarily in groups in the appropriate places of the treatise as needed. However, all SPLFR's of $\mathbf{A}_1$ are stated exclusively in terms of homolographic (photographic) tokens of elements of B_1 and $\mathbf{B}_1$, while [the ranges of] all elements of $\mathbf{B}_1$, primary and secondary, are themselves defined by the pertinent *generalizing PLFR's of atomic panlogographs* (*APL's,*) *of* $\mathbf{A}_1$ in terms of some *primary atomic euautographs* (*PAE's*) of A_1. Particularly, all *formulary* elements of $\mathbf{B}_1$, primary and secondary, are defined by the pertinent *generalizing PLFR's of atomic PLF's* (*APLF's,*), *of* $\mathbf{A}_1$, *primary* ones (*PAPLF's, APPLF's,*) or *secondary* ones (*SAPLF's, ASPLF's*) respectively.

12) For more clarity, a SPLFR of $\mathbf{A}_1$ is, when possible, laid down in terms of homolographic (photographic) tokens of the same AnAPLR's and AnAPLI's of $\mathbf{B}_1$ as those employed in the XPFR-system, namely '**P**' and '**Q**', and '**I**' and '**J**' respectively. In any case, the PLFR's of bases $\mathbf{B}_{1PAnF}$ and $\mathbf{B}'_{1PAnF}$ and XPFR-system of A_1 and $\mathbf{A}_1$, and all other mandatory FR's of A_1 and $\mathbf{A}_1$, primary and secondary, are incorporated into the single whole EnSPGFCR-system of A_1 and are thus interrelated owing to the properties of *meta-autonymy* and *retroactivity* of the elements of $\mathbf{B}_1$, which were described in the items 3–5 of subsection 4.4 (cf. the item 10 above in this

definition). Owing to those properties, the range of any *extendable AnAPLR* (as '**P**' or '**Q**'), or *AnAPLI* (as '**I**' or '**J**'), *of* $\mathbf{A}_1$ and the same range of any of its congeners in all *earlier occurrences is automatically updated retroactively after stating any new SFR of* A_1 *or* $\mathbf{A}_1$*, by which some new SEF's of SER's, or SEI's, of* A_1 *are added to that range either directly or obliquely.* Thus, as was stated previously, the XPFR-system determines, not only the PEF's of A_1, but it also determines retroactively all respectively patterned SEF's of A_1. In this case, an EF of A_1 is a PEF if it is postulated to be so by the RPFR-system and an SEF if otherwise. Until any SEF is added to the range of an AnAPLF of $\mathbf{A}_1$, that range is the *initial range* of the AnAPLF, i.e. it is either the class of PER's of A_1 or the class of PEI's of A_1, – depending on the AnAPLF.

13) The PLFR's of $\mathbf{B}_{1\mathrm{PAnF}}$ and $\mathbf{B}'_{1\mathrm{PAnF}}$ serve as *the interface between the RPFR-system of* A_1 *and the EnSPGFCR-system of* $\mathbf{A}_1$– the interface that is a part of the latter system. Although the RPFR-system is replaced with the XPFR-system and then abandoned, it is an inseparable and indispensable part of the entire setup of A_1 and $\mathbf{A}_1$. Accordingly, the totality of the RPFR-system of A_1 and the EnSPGFCR-system of $\mathbf{A}_1$ constitute a *single whole system of classification rules of formulas of* A_1, which is, in accordance with Df 6.1(3), called the *global system of formulary classification rules* (*GbFCR-system*), or more specifically the *global system of formulary formation, generalization, and sortation rules* (*GbFFGSR-system*), *of* A_1, i.e. *of* A_1 *and* $\mathbf{A}_1$. In agreement with Df 6.1(3), a classification rule of formulas of A_1 is called a *systemic* one if it belongs to the GbFCR-system and an *extra systemic* one if otherwise.

14) The logical heading of a single rule or a group of rules of the GbFCR-system of A_1 is be provided with an adherent (prefixing) dagger, †, – just as the logical heading "†Ax 5.1", "†Df 5.1", and "†Df 5.2". In this connection, it will be recalled that Ax 5.1, e.g., is a meta-axiom that postulates which PAE's are admissible in A_1, and it is simultaneously an ostensive definition of names of those characters. At the same time, the GbFCR-system establishes admissible categorematic combinations of PAE's of A_1 and admissible abbreviations of these combinations, and it also establishes admissible PLF's of both. It is therefore natural to relegate Ax 5.1 and all formation rules of A_1 to the same class.•

Df 6.7: ***Postscript on classification rules (CR’s) of euautographic formulas (EF’s) of*** A_1 ***and on panlogographic formation rules (PLFR’s) of panlogographic formulas (PLF’s) of*** $\mathbf{A}_1$. 1) *All* primary CR’s (PCR’s) of EF’s of A_1 and *all* primary PLFR’s (PPLFR’s) of PLF’s, i.e. actually of primary PLF’s (PPLF’s), of $\mathbf{A}_1$, and also *strictly some* (*some but not all*) secondary CR’s (SCR’s) of EF’s, i.e. actually of secondary EF’s (SEFR’s), of A_1 and *strictly some* secondary PLFR’s (SPLFR’s) of PLF’s, i.e. actually of secondary PLF’s (SPLF’s), of $\mathbf{A}_1$ are *semantic definitions*, whereas all other SCR’s of EF’s (SEF’s) of A_1 and all other *S*PLFR’s of PLF’s (SPLF’s) of $\mathbf{A}_1$ are *syntactic definitions*, called also *asymmetric synonymic definitions* (*ASD’s*). Consequently, in agreement with the pertinent terminology introduced in Df 6.2(1c), all PCR’s [of EF’s] of A_1 and all PPLFR’s [of PPLF’s] of $\mathbf{A}_1$ and also strictly some SCR [of SEF’s] of A_1 and strictly some SPLFR’s [of SPLF’s] of $\mathbf{A}_1$ are *semantic* (*Smn*), or *basal* (*Bsl*), or *first kind* (*FK*), ones, i.e. ones *of FK*, whereas all other SCR’s of [of SEF’s] of A_1 and all other SPLFR’s of [SPLF’s] of $\mathbf{A}_1$ are *syntactic* (*Snt*), or *synonymic* (*Snm*), or *of second kind* (*SK*), ones, and also ones of *SK*. Any one of the square-bracketed expressions in the above taxonyms can be omitted whenever there is no danger of confusion. For more clarity, using the prepositive letters “P”, “S”, and “F” as abbreviations for “primary”, “secondary”, and “formulary” respectively, I may also use the following alternative abbreviations: “PFCR” for “PCR of EF’s”, “PFPLGR” for “PPLGR of EF’s”, “PFPLSR” for “PPLSR of EF’s”, “PFEFR” for “PEFR of PDEI’s”, “PFPLFR of A_1” for “PFPLFR of EF’s of A_1”, “PFPLFR of $\mathbf{A}_1$” for “PFPLFR of PPLF’s of $\mathbf{A}_1$”, and the similar abbreviations with “S” for “secondary“ in place of “P” for “primary”, “SEF” for “secondary EF” in place of “EF”, and “SDDEI” for “secondary decimal DEI” in place of “PDEI” for “secondary decimal DEI”.

2) Every PFPLFR or SFPLFR of $\mathbf{A}_1$ is respectively a PFCR or SFCR of A_1, but not vice versa, because the following FCR’s of A_1 are not FPLFR’s of $\mathbf{A}_1$: the RPEFR and XPEFR of A_1 and $\mathbf{A}_1$ (described by Df 6.2(6)) being respectively the first RPFR and the first XPFR of A_1, the AnRPFR’s and MsRPFR of A_1, and the SEFR-system of A_1 and $\mathbf{A}_1$ (described by Df 6.2(7)).

3) A PLFR of an AnCbPLF of $\mathbf{A}_1$ is called an *schematic advanced FPLFR* (*SchAdvPLFR*), or *advanced FPLFR-schema* (*AdvFPLFRS*, pl. “*AdvFPLFRS’ta*”), of both A_1 and $\mathbf{A}_1$ if the AnCbPLF is an *AnCbFPL-schema* (*AnCbFPLS*, pl.

"*AnCbFPLS'ta*") *of EF's of* A_1 and an *abstract advanced FPLFR* (*AbAdvFPLFR*), or *AdvFPLFR-abstractum* (*AdvFPLFRA*, pl. "*AdvFPLFRA'ta*"), of both A_1 and $\mathbf{A}_1$ if the AnCbPLF is an *AnCbFPL-abstractum* (*AnCbFPLA*, pl. "*AnCbFPLA'ta*") *of EF's of* A_1. In contrast to either of the synonyms "SchAdvFPLFR" and "AdvFPLFRS", a PLFR of an StAPLF of $\mathbf{A}_1$, being an element of $\mathbf{B}_{1FSt}$, is called a *schematic basic FPLFR* (*SchBscFPLFR*), or *BscFPLFR-schema* (*BscFPLFRS*, pl. "*BscFPLFRS'ta*"), of both A_1 and $\mathbf{A}_1$. Analogously, in contrast to either of the synonyms "AbAdvFPLFR" and "AdvFPLFRA", a PLFR of an APLF of $\mathbf{A}_1$, being an element of $\mathbf{B}_{1F}$, i.e. of the union $\mathbf{B}_{1FSt}\cup\mathbf{B}_{1FAn}$, is called an *abstract basic FPLFR* (*AbBscFPLFR*), or *BscFPLFR-abstractum* (*BscFPLFRA*, pl. "*BscFPLFRA'ta*"), of both A_1 and $\mathbf{A}_1$. Consequently, a SchBscFPLFR (BscFPLFRS) or SchAdvFPLF (AdvFPLFRS) of A_1 and $\mathbf{A}_1$ is indiscriminately called a SchFPLFR (FPLFRS) of A_1 and $\mathbf{A}_1$. Likewise, an AbBscFPLFR (BscFPLFRA) or AbAdvFPLF (AdvFPLFRA) of A_1 and $\mathbf{A}_1$ is indiscriminately called an AbFPLFR (FPLFRA) of A_1 and $\mathbf{A}_1$. An MLFR of an AnCbMLF of the IML of A_1 and $\mathbf{A}_1$ is alternatively called a *schematic MLFR* (*SchMLFR*), or *MLFR-schema* (*MLFRS*, pl. "*MLFRS'ta*"), of A_1, the understanding being that the AnCbMLF is an *AnCbFML-schema* (*AnCbFMLS*, pl. "*AnCbFMLS'ta*") *of EF's of* A_1. It is also understood that an MLFRS is an AdvMLFRS and vice versa, and that there is *no* MLFR of EF's of A_1 that could be qualified either *abstract* or *secondary*.

4) The set of [systemic] PFCR's of A_1 comprises the following CR's:

a) the PPLFR's of concrete PLOT's (PStAPLOT's) and StAPLOR's (PStAPLOR's) of $\mathbf{A}_1$, being elements of $\mathbf{B}_{1PStF}$, i.e. the PPLGR's of all EOT's (PAEOT's) of A_1, comprised in the range of any one of the PLOT's, and of all AER's (PAEOR's) of A_1, comprised in the range of any one of the StAPLOR's;

b) the separate RPFR's constituting the RPFR-system of A_1, the separate PFFR's constituting the RPFR-system of A_1;

c) the PPLFR of the concrete PAnAPLR's of $\mathbf{A}_1$, *generic* (*comprehensive*, *all-embracing*) ones (*GPAnAPLR's*) and *specific* ones (*ScPAnAPLR's*), and of the concrete [congeneric] primary AnAPLI's (PAnAPLI's) of $\mathbf{A}_1$, all being elements of $\mathbf{B}_{1PAnF}$ (the same as $\mathbf{B}_{1PAn}$), i.e. the PPLGR of the ER's of A_1, comprised in the range of any one of the congeneric or conspecific

PAnAPLR's, and the PPLGR's EI's of A_1, comprised in the range of any one of the [congeneric] PAnAPLI's;

d) the PPLFR's of the concrete PDAnMPLR's and PDAnMPLI's of $\mathbf{A}_1$, being elements of $\mathbf{B}'_{1PAnF}$, i.e. the PPLSR's of the ER's of A_1, comprised in the range of any one of the conspecific PDAnMPLR's, and the PPLSR's of the EI's of A_1, comprised in the range of any one of the conspecific PAnAPLI's;

e) the separate PFFR's constituting the XPFR-system of A_1 and $\mathbf{A}_1$.

Hence, in accordance with the item 3, the set of PFCR's of A_1 includes *BscPPLFRS'ta*, *BscPPLFRA'ta*, *MLFRS'ta* (*AdvPMLFRS'ta*), *DPPLFR*'s or *StgSPLFR's* (*descriptive*, or *sorting*, *PPLFR's*), and *AdvPPLFRS'ta*, all of which are *semantic*, but it *does not include* either any synonymic (syntactic) AdvPPLFRS'ta or any AdvPPLFRA'ta.

5) The set of PFCR's of A_1, is complete by the stage of the setup of A_1 (A_1 and $\mathbf{A}_1$) when the very first SPLFR of A_1 and $\mathbf{A}_1$ is laid down, and it does not change afterwards. By contrast the set of [systemic] SCR's of A_1, and hence the set of [systemic] SFCR's of A_1, is from time to time augmented by various new SCR's of A_1 as needed. First, in contrast to the bases $\mathbf{B}_{1P}$, (i.e. $\mathbf{B}_{1PSt} \cup \mathbf{B}_{1PAnF}$) and $\mathbf{B}'_{1PAnF}$, which are complete, the bases $\mathbf{B}_{1S}$ and $\mathbf{B}'_{1SAnF}$ are, as far as needed, augmented by new elements, which are defined by the appropriate *BscSPLFRA'ta of* $\mathbf{A}_1$ and by the appropriate new *descriptive*, or *sorting*, *SFPLFR's* (*DSFPLFR's* or *StgSFPLFR's*) *of* $\mathbf{A}_1$ respectively. Second, there is need in *abbreviative synonymic SEFR's* and in *synonymic AdvSFPLFRS'ta*, both *abbreviative* and *not*, *of* A_1 and $\mathbf{A}_1$. Third, in the last phase of A_1, dealing with foundations of Aristotelian logic, there is need in the appropriate *abbreviative synonymic AdvSFPLFRA'ta of ER's of* A_1 as the ASD's (6.1)–(6.5). Thus, the class of [systemic] SCR's of A_1 contains abbreviative synonymic SEFR's and synonymic AdvSFPLFRS'ta, both abbreviative and not, of EF's of A_1 and at a certain phase of development of A_1 it also contains abbreviative synonymic AdvSFPLFRA'ta of EF's of A_1. The above SFR's of A_1 have no analogues among the PFR's of A_1. At the same time, in contrast to the set of [systemic] PFCR's of A_1, the class of [systemic] SCR's of A_1 contain *no* MLFR's (MFRS'ta) of EF's of A_1 at any stage of development of A_1.

6) Every separate FR of the SEFR-system of SDDEI's of A_1 and $\mathbf{A}_1$ (see Df 6.4(7)) is an ASD (asymmetric synonymic definition), which is, more specifically, called a *euautographic ASD* (*EASD*). At the same time, in accordance with the item 1 of this definition, an AdvFPLFRS of ER's or EI's of A_1 is one of the following two kinds:

a) a *semantic*, or *basal*, or *first kind*, *AdvFPLFRS* (briefly *SmnAdvFPLFRS*, or *BslAdvFPLFRS*, or *FKAdvFPLFRS*),

b) a *syntactic*, or *synonymic*, or *second kind*, *AdvFPLFRS* (briefly *SntFPLFRS*, or *SnmAdvFPLFRS*, or *SKAdvFPLFRS*).

In accordance with the item 4 of this definition, every *AdvPFPLFRS* is a *SmnAdvPFPLFRS* (*BslAdvPFPLFRS*, *FKAdvPFPLFRS*) and vice versa, whereas in accordance with the item 5 every AdvSFPLFRS is a *SntAdvSFPLFRS* (*SnmAdvSFPLFRS*, *SKAdvSFPLFRS*) and vice versa. At the same time, every AdvFPLFRA of ER's of A_1 is a *SntAdvSFPLFRA* (*SnmAdvSFPLFRA*, *SKAdvSFPLFRA*) *of ER's of* A_1 and vice versa. Both a SnmAdvSFPLFRS and a SnmAdvSFPLFRA are *panlogographic ASD's* (*PLASD's*). Therefore, a SnmAdvSFPLFRS is briefly and monosemantically called an *FPLASD-schema* (*FPLASDS,* pl. "*FPLASDS'ta*"), or *schematic FPLASD* (*SchFPLASD*, pl. "*SchFPLASD's*") [*of SER's*, or *SEI's*] *of* A_1, or [*of SPLR's*, or *SPLI's*], *of* $\mathbf{A}_1$, or of both; whereas a SnmAdvSFPLFRA is likewise called an *FPLASD-abstractum* (*FPLASDA*, pl. "*FPLASDA'ta*"), or *abstract FPLASD* (*AbFPLASD*, pl. "*AbFPLASD's*"), followed by appropriate one of the same postpositive qualifiers.

Incidentally, the generic name "secondary formulary panlogographic formation rule schema" ("SFPLFRS") cannot be used synonymously with "FPLASDS'", because *an SFPLFR of a secondary structural APLR* (*SStAPLR*), or of a *secondary structural APLI* (*SStAPLI*), *of* $\mathbf{A}_1$ can also be called an *SFPLFRS of* $\mathbf{A}_1$. The class of FPLASDS'ta is the widest class of SFR's of A_1 and $\mathbf{A}_1$. Particularly, the set of FPLASDS'ta of SPLF's of $\mathbf{A}_1$, which are based on using B_1 and $\mathbf{B}_{1P}$, is the most immediate supplement to the set of PFCR's of A_1. Therefore, the FPLASDS'ta are discussed in the next definition in greater detail. In this case, the abbreviation "FPLASDS" is hereafter abbreviated by omission of the letter "F" for "formulary", i.e. as "PLASDS", unless stated otherwise.•

Df 6.8: ***PLASDS'ta of*** A_1. 1) A PLASDS is said to be a *binary* one (*BPLASDS*) if it is a *binary ASD* (*BASD*), i.e. an ASD (asymmetric synonymic definition) that has exactly one *concrete panlogographic definiendum* and exactly one *concrete panlogographic definiens*. In the previous definitions, and generally in what follows, by an ASD and hence by an PLASDS, I understand a binary one, unless stated otherwise. In order to introduce several synonymous definienda simultaneously, their BASD's are stated in the *legato style*.

2) The syntactic definiendum of a BPLASDS of $\mathbf{A}_1$ is an *S*FPLS (*secondary* formulary panlogographic schema), i.e. *S*PLF (*secondary* panlogographic formula), of $\mathbf{A}_1$, because in the result of the BPLASDS its syntactic definiendum has, within its scope, *at least one preceding panlogographic synonym*, namely the syntactic definiens of the BPLASDS, called also the *immediate panlogographic definiens* (*IPLD*) *of the BPLASDS*. This definiens is either a *P*PLF (*primary* panlogographic formula) of $\mathbf{A}_1$ or another SPLF of $\mathbf{A}_1$, which is expressed in terms a certain PPLF of $\mathbf{A}_1$ by one or more preceding BPLASDS. In either case, the PPLF in question is called the *primary panlogographic definiens* (*PPLD*) of the ultimate SPLF. Consequently, the IPLD of the SPLF is called the *immediate PPLD* (*IPPLD*) *of the SPLF* if it is the PPLD and the *immediate secondary panlogographic definiens* (*ISPLD*) *of the SPLF* if otherwise.

3) Both the SPLF-definiendum and the PLF-definiens of a given BPLASDS of $\mathbf{A}_1$ contain *homolographic tokens of the same APLF's of* $\mathbf{A}_1$, being formulary (categorematic) elements of $\mathbf{B}_1$. Therefore, the BPLASDS can, alternatively, be construed as a *contextual* (*implicit*) *definition*, whose *effectual definiendum* is *the principal operator of the SPLF-definiendum*, and whose *immediate effectual definiens* is *the entire combined operator of the immediate PLF-definiens*. No matter whether the immediate PLF-definiens is the IPPLD or whether it is the ISPLD of the SPLF-definiendum, *the entire combined operator of the PPLD of the SPLF-definiendum* is called the *primary effectual operator-definiens* of the given *secondary effectual operator-definiendum*. The effectual operator-definiens and hence the effectual operator-definiendum are most often euautographic ones, although the cases when they are panlogographic ones also happen. In contrast to the secondary effectual operator-definiendum and its effectual operator-definiens, primary or immediate (if it is not primary), the SPLF-definiendum containing the former and the PLF-definiens

containing the latter are qualified *apparent*. The act of prescinding the effectual operator-definiendum and the effectual operator-definiens from the apparent SPLF-definiendum and the apparent PLF-definiens respectively can be indicated by replacing all APLF's occurring in the latter two PLF's with 'n'-spaces (e.g.) or with any blank-signs.

4) If both the SPLF-definiendum and the PLF-definiens of a BPLASDS of $\mathbf{A}_1$ are given in the same representation, homogeneous (linear or bilinear) or inhomogeneous (Clairaut-Euler's), and if therefore they contain the respective brackets of the same form, square or round, then the BPLASDS can be construed as a *contextual definition of the principal kernel-sign* of the SPLF-definiendum, rather than that of its entire principal operator.

5) The range of the SPLF-definiendum and the range of the PLF-definiens of a BPLASDS of $\mathbf{A}_1$ stand in a *bijective interrelation* such that to any member of the latter range, which is one of the *semantic euautographic definientia* of the BPLASDS, and which is, depending on the range, either an EI or an ER of A_1, there corresponds exactly one member in the former range, which is the respective one of the *semantic euautographic definienda* of the BPLASDS, and which is respectively either an EI or an ER of A_1, and vice versa. That is to say, *the range of a BPLASDS of* $\mathbf{A}_1$ is a *class of binary euautographic asymmetric synonymic definitions* (*BEASD's*), called also *binary euautographic synonymic formation rules* (*BESnmFR's*), *either of EI's of* A_1 *or of ER's of* A_1, –depending on the BPLASDS.

6) A BPLASDS of $\mathbf{A}_1$, whose IPLD is the PPLD of its SPLF-definiendum, is called an *abbreviative* one if the SPLF-definiendum is designed to stand as an abbreviation for the PPLD. That is to say, such a BPLASDS of $\mathbf{A}_1$ is an *abbreviative* one if the principal operator of its SPLF-definiendum is *shorter* than *the respective combined operator of its PLF-definiens*; both operators are, as a rule, euautographic. If a BPLASDS of $\mathbf{A}_1$ is an abbreviative one then every BESnmFR of A_1 of its range is an abbreviative one as well and vice versa.

7) Most BPLASDS'ta of $\mathbf{A}_1$ and hence most BESnmFR's of A_1 are abbreviative. Therefore, most typically, at least one constituent *atomic syncategorematic euautograph* or *panlogograph* of the SPLF-definiendum of a BPLASDS of $\mathbf{A}_1$ is a *new* one that is not mentioned among the PAE's (primary atomic euautographs) of B_1 or among the PAPL's (primary atomic panlogographs) of

B$_1$ It can, however, happen that some SEF's (secondary euautographic formulas) of the range of the SPLF-definiendum of a BPLASDS of **A**$_1$ are PEA's (primary euautographic assemblages) of A$_1$, which are not, however, defined as PEF's (primary euautographic formulas) by any PFR (primary formation rules) of A$_1$. For instance, in accordance with the fourth XPFR (third StXPFR), '$\mathbf{f}^2(\mathbf{x},\mathbf{y})$' is a *PPLR* (primary panlogographic relation) of **A**$_1$. Therefore, '$\in(\mathbf{x},\mathbf{y})$', being an instance of '$\mathbf{f}^2(\mathbf{x},\mathbf{y})$' with $\in$ in place of '$\mathbf{f}^2$' (see Df 5.2(3)), is a PPLR of **A**$_1$ as well. Consequently, when used xenonymously for mentioning, e.g., $\in(u,v)$ or $\in(x,y)$, $\in(\mathbf{x},\mathbf{y})$ is a PER (primary euautographic relation), while $[\mathbf{x}\in\mathbf{y}]$, as e.g. $[u\in v]$ or $[x\in y]$, is a PEA that is not a PER. In this case, the BPLASDS of **A**$_1$: $[\mathbf{x}\in\mathbf{y}]\to\in(\mathbf{x},\mathbf{y})$ defines '$[\mathbf{x}\in\mathbf{y}]$' or ['**x**'$\in$'**y**'] as a SPLF of **A**$_1$ and hence it defines $[\mathbf{x}\in\mathbf{y}]$ (particularly $[u\in v]$ or $[x\in y]$) as an SER (secondary euautographic relation) of A$_1$. The above BPLASDS of **A**$_1$ is *formative*, .i.e. *form-giving*, but it is not *abbreviative*, and so is any BESnmFR's of A$_1$ of its range.•

Df 6.9. 1) Any *continuous fragment*, without blanks (empty spaces) and without blank-signs, of an EF of A$_1$, *primary* or *secondary*, including the formula itself, is called a *working*, or *purposeful*, *euautographic assemblage* (*WEA*) *of* A$_1$ or simply a *euautograph of* A$_1$. A euautographic assemblage of A$_1$ is called an *idle*, or *purposeless*, *euautographic assemblage* (*IEA*) *of* A$_1$ if it is not a WEA. Hence, a PEA (primary euautographic assemblage) is either a WPEA or an IPEA. Accordingly, a WEA of A$_1$ is called a *primary WEA* (*PWEA*) if it is a PEA and a *secondary WEA* (*SWEA* or *WSEA*) if it is not a PEA. A PWEA is a WPEA and vise versa; i.e. "PWEA" and "WPEA" are synonyms. At the same time, by definition, there is *no SIEA* (*secondary idle euautographic assemblage*) *of* A$_1$. Therefore, a SWEA (WSEA) of A$_1$ is a *secondary euautographic assemblage* (*SEA*) *of* A$_1$ and vise versa; i.e. "SWEA" and "SEA" are synonyms. An assemblage of A$_1$ is said to be *atomic* if it comprises a single atomic euautograph of A$_1$ and *combined*, or *juxtapositional*, if otherwise. A juxtaposition of a PEA and a SEA, of A$_1$ is a SEA of A$_1$. Therefore, any *euautographic assemblage* (*EA*) *of* A$_1$ *is either a PEA or a SEA.*

2) The above item applies with '**A**$_1$', "*PL*" for "*panlogographic*", and "*panlogograph*", or in general with 'A$_1$', "*EnSPG*" for "*endosemasiopasigraphic*",

and “*endosemasiopasigraph*”, in place of ‘A_1’, “*E*” for “*euautographic*”, and “*euautograph*” respectively.•

Cmt 6.1. The class of graphonyms, which is denoted by the count name “primary euautographic assemblage” (“PEA”), is an auxiliary one that has been designed chiefly for conveniently stating the RPFR-system of A_1. After stating the RPFR’-system, all WPEA’s of A_1 are tacitly disregarded. At the same time, any SEF of A_1 is either defined in terms of a certain PEF of A_1 or is derived by combining PEF’s and SEF’s. Therefore, no general notion of SEA’s of A_1 is needed for introducing SEF’s of A_1. Consequently, the class, which is denoted by the count name “secondary euautographic assemblage” (“SEA”) as defined in Df 6.9, is restricted only to the WSEA’s of A_1. The most important of these are *secondary euautographic formulas* (*SEF’s*) and *secondary atomic and molecular euautographic kernel-signs* (*SAEKS’s* and *SMEKS’s*) of A_1, including secondary logical connectives, secondary predicate-signs, and secondary contractors (particularly secondary pseudo-quantifiers).

2) In accordance with the above-said, the general metaterms “panlogographic assemblage” (or “assemblage of $\mathbf{A}_1$”) and “endosemasiopasigraphic assemblage” (or “assemblage of A_1”) are not needed for introducing PLF’s of $\mathbf{A}_1$ and their continuous fragments. From the very beginning, these metaterms can be and are understood as designating the classes of the respective working (purposeful) assemblages and hence as synonyms of the nouns “panlogograph” (or “logograph of $\mathbf{A}_1$”) and “endosemasiopasigraph” (or “pasigraph of A_1”) respectively – just as the noun “euautograph” is understood as a synonym “purposeful euautographic assemblage”

7. Phasing and branching A_1

Df 7.1: ***The root phase and major branches of*** **A_1.** 1) The first phase of A_1, which is, by Df 3.1(19), denoted by ‘A_{1P}’ or ‘A_{1R}’ and is called the *Primordial*, or *Root*, *EAPO* (briefly *PEAPO* or *REAPO*), is an *unbranched* one in the sense that its atomic basis, to be denoted accordingly by ‘B_{1P}’ or ‘B_{1R}’, comprises all atomic euautographs that have been mentioned in Ax 5.1. That is to say, in setting up A_{1P}, the division of the ordinary atomic basis of A_1, B_{1O}, into the mandatory one, B_{1OM}, and the selective one, B_{1OS}, and hence the condition of mutual incompatibility of the BAPCOPS’s =, $\subseteq$, and $\in$ as primary ones and the condition of association of

APCOT's $\emptyset$ and $\emptyset'$ with $\subseteq$, or $\in$, which are imposed on the above atomic euautographs the items 8 and 9 of Ax 5.1, are *ignored* (*not utilized*). In this case, any given BAPCOPS is functionally indistinguishable from another BAPCOPS and from any APVOPS because no *atypical* (*specific*) subject axiom is imposed on any one of the predicate-signs in addition to the *typical* (*general*) subject axioms included in the primordial formation rules of A_1 and in addition to the transformation (inference) rules included in D_1. or by defining at least one *secondary* BAPCOPS in terms of it and by proving some *specific* (*atypical*) *theorems* for the latter with the help of D_1. This fact guarantees that D_1 will remain universal and unaltered in any further development of A_1.

2) Once A_{1P} and D_1 are set up, the division of B_{1O} into B_{1OM} and B_{1OS} and the stipulation of Ax 5.1(8) are recovered, so that various *restrictions* and *branches* of A_1 can be defined. If, for instance, the entire list (5.2) is disregarded, A_1 turns into its restriction, which is called *the Pure Functional EAPO* (*PFEAPO*) and which is accordingly denoted by 'A_1^p'. The first phase of A_1^p is the respective restriction of A_{1P}, which is denoted by 'A_{1P}^p'. If exactly one of the three BAPCOPS's =, $\subseteq$, and $\in$ is selected as the primary one and if in addition some atypical (specific) axioms are imposed on that sign, either *obliquely* by *axiomatically defining* certain *secondary atomic or molecular PCOPS's* in terms of it (in the case of $\subseteq$ or $\in$) or directly, then A_1 turns into its *comprehensive branch*, which is denoted by '$A_{1=}$', '$A_{1\subseteq}$', or '$A_{1\in}$' respectively, whereas A_1^p turns into the respective *restricted branch*, which is denoted by '$A_{1=}^p$', '$A_{1\subseteq}^p$', or '$A_{1\in}^p$' respectively. The above three comprehensive branches are described below in some detail.

3) The first member of the above branch triple will be denoted by '$A_{1=}$' and be called the *Egalitarian EAPO* (*EgEAPO*), because it involves the *ordinary atomic predicate-sign of equality*, =, as the pertinent distinguished optional element of B_{1OS}. In $A_{1=}$, the *conventional laws of reflexivity, symmetry, and transitivity of* =, along with an additional axiom that is called the *incidence law for anti-equalities*, are taken for granted and laid down as *specific* (*atypical*, *individualizing*) *subject* (*intrinsic*) *axioms of* $A_{1=}$. The organon $A_{1=}$ does not involve the PCOT's $\emptyset$ and $\emptyset'$, while the PVOT's of $A_{1=}$ are alternatively called *pseudo nonempty-individuals*, because they are not interrelated by either one of the predicate-signs $\subseteq$ and $\in$, which are not available in

$\mathsf{A}_{1=}$. Accordingly, $\mathsf{A}_{1=}$ is alternatively called the *Pseudo Nonempty-Individual EAPO* (*PNEIEAPO*), i.e. the *EAPO of Pseudo Nonempty Individuals*. The branch of $\mathsf{A}_{1=}$, which results by disregarding the list (5.2), is denoted by '$\mathsf{A}'_{1=}$' and is called the *Pure Functional Egalitarian EAPO* (*PFEgEAPO*) or the *Pseudo Nonempty-Individual EAPO* (*PFPNEIEAPO*).

4) The second member of the branch triple will be denoted by '$\mathsf{A}_{1\subseteq}$' and be called the *Pseudo-Mass EAPO* (*PMsEAPO*), meaning the *EAPO of Pseudo-Masses*, because it involves the *ordinary atomic inclusion* (*part-to-whole*) *predicate-sign* $\subseteq$ as the pertinent distinguished optional element of B_{1OS} and because it can, therefore, serve as the underlying calculus of a full-scale [*one-individual*] *theory of masses* as opposed to a *theory of classes* (see the next item). $\mathsf{A}_{1\subseteq}$ also involves both PCOT's: $\emptyset$, which is in this case alternatively called the *euautographic empty pseudo-mass* (*EEPMs*) or the *euautographic empty pseudo-individual* (*EEPIl*), and $\emptyset'$, which is alternatively called the *subsidiary EEPMs* or the *subsidiary EEPIl*. At the same time, the EOT's, i.e. PCOT's and PVOT's, of $\mathsf{A}_{1\subseteq}$ are alternatively called *pseudo-masses*, because they are interrelated by the predicate-sign $\subseteq$ and are not interrelated by the predicate-sign $\in$, which is not available. In $\mathsf{A}_{1\subseteq}$, the *conventional laws of reflexivity and transitivity of* $\subseteq$ and an additional *incidence law for anti-inclusions* are taken for granted and are laid down as *subject axioms of* $\mathsf{A}_{1\subseteq}$, while the sign = is defined in terms of $\subseteq$. In this case, the conventional laws of reflexivity, symmetry, and transitivity of = and the *incidence law for anti-equalities* turn out to be *theorems of* $\mathsf{A}_{1\subseteq}$. The branch of $\mathsf{A}_{1\subseteq}$, which results by disregarding the list (5.2), is denoted by '$\mathsf{A}'_{1\subseteq}$' and is called the *Pure Functional Pseudo-Mass EAPO* (*PFPMsEAPO*). By way of emphatic comparison with a version of $\mathsf{A}_{1\subseteq}$, which will be described in Cmt 7.6 and which will be denoted by '$\bar{\mathsf{A}}_{1\subseteq}$' and be called the *Pseudo-Confined PMsEAPO*¸ $\mathsf{A}_{1\subseteq}$ can alternatively be called the *Pseudo-Unconfined PMsEAPO*. A certain part of this IML (this treatise), with the help of which and within which $\mathsf{A}_{1\subseteq}$ is developed (set up and executed), is called the *Euautographic Pseudo-Mass Theory* (*EPMsT*) or alternatively and more precisely the *Pseudo-Unconfined EPMsT*.

5) The third member of the branch triple will be denoted by '$\mathsf{A}_{1\in}$' and be called the *Pseudo-Class EAPO* (*PCsEAPO*), because it involves the *ordinary atomic class-membership predicate-sign* $\in$ as the pertinent distinguished optional element of

the atomic basis and because it can therefore serve as the underlying calculus of a full-scale *one-individual theory of classes*. $\mathsf{A}_{1\in}$ also involves both PCOT's: $\emptyset$, which is in this case alternatively called the *euautographic empty pseudo-class* (*EEPCs*) or *euautographic empty pseudo-individual* (*EEPIl*), and $\emptyset'$, which is alternatively called the *subsidiary EEPCs* or *subsidiary EEPIl*. At the same time, the EOT's, i.e. PCOT's and PVOT's, of $\mathsf{A}_{1\in}$ are alternatively called *pseudo-classes* or *pseudo-elements*, because they are interrelated by the predicate-sign $\in$ and also by the predicate-signs $\subseteq$ and =, which are defined in terms of $\in$. In this case, all basic laws of $\subseteq$ and =, which have been mentioned in the previous two items, turn out to be *theorems of* $\mathsf{A}_{1\in}$. The branch of $\mathsf{A}_{1\in}$, which results by disregarding the list (5.2), is denoted by '$\mathsf{A}'_{1\subseteq}$' and is called the *Pure Functional Pseudo-Class EAPO* (*PFPCsEAPO*). By way of emphatic comparison with a version of $\mathsf{A}_{1\in}$, which will be described in Cmt 7.6 and which will be denoted by '$\bar{\mathsf{A}}_{1\in}$' and be called the *Pseudo-Confined PCsEAPO*¸ $\mathsf{A}_{1\in}$ can alternatively be called the *Pseudo-Unconfined PCsEAPO*. A certain part of this IML (this treatise), with the help of which and within which $\mathsf{A}_{1\in}$ is developed (set up and executed), is called the *Euautographic Pseudo-Class Theory* (*EPCsT*) or alternatively and more precisely the *Pseudo-Unconfined EPCsT*.

6) $\mathsf{A}_{1\in}$ has one more phase as its extension, which is called the *Aristotelian*, or *Syllogistic*, *EAPO* (*AEAPO* or *SEAPO*) and is denoted by '$\mathsf{A}_{1\in\mathsf{A}}$ or '$\mathsf{A}_{1\mathsf{A}}$'. This phase results by supplementing $\mathsf{A}_{1\in}$ with axiomatic definitions of various sets of 19 *ordinary ER's* (*OER's*, *EOR's*) in each set – the EOR's, which have the same structure as 19 categorical syllogisms of Aristotelian logic (see, e.g. Hilbert and Ackermann [1950, Chapter II], Łukasiewicz [1951], or Lamontagne and Woo [2008]), which are collectively called the *euautographic syllogistic implications* (*ESI's*). Separate ESI's are distinguished by the same catchwords as those identifying separate categorical syllogisms, e.g. "Barbara", "Bamalip", etc, but these are set in the Roman Arial Narrow Font, and are furnished with various *additional subscripts* distinguishing the different sets of ESI's. Also, for a certain reason, which will be explained in due course and which is, in general outline, relevant to a certain unconventional classification of the ESI's and of the categorical syllogisms, being their so-called *conformal catlogographic* (*CFCL*) *interpretands*, I have replaced the conventional catchword "Darapti" with "Barapti". Together with its subscripts, each modified catchword, is the

euautographic predicate of the pertinent ESI, and it is therefore called a *ternary euautographic syllogistic predicate* (*TESP*). Any ESI comprises three binary EOR's that are called *euautographic syllogistic judgments* (*ESJ's*). There are *four* types of ESJ's in each set of 19 ESI's, which are distinguished from one another by their binary euautographic predicates that are called *binary euautographic syllogistic predicates* (*BESP's*). The latter are denoted by the letters 'A', 'O', 'E', and 'I' furnished with the appropriate subscripts – the letters, which are associated with the conventional *code* (*catch*) *letters* 'A', 'O', 'E', and 'I', or 'a', 'o', 'e', and 'i', serving as logical predicates of *the separate judgments* (*the premises and the conclusion*) *of a categorical syllogism* of Aristotelian logic. These four code letters are derived as the vowels of the two Latin words ***a**ff**i**rmo* and *n**e**g**o***. Some of the BESP's are defined in terms of APVOPS's and are therefore called *binary pseudo-variable syllogistic predicates* (*BPVSP's*). The other BESP's, some of which are interpretable by the syllogistic judgments of Aristotelian logic, are defined in terms of $\in$ or in terms of some secondary *binary pseudo-constant ordinary predicate-signs* (BPPCOPS's) of $A_{1\in}$ and are therefore called *binary pseudo-constant syllogistic predicates* (*BPCSP*'s). Thus, a BESP is either a BPVSP or a BPCSP. The purpose of A_{1A} is to apply D_1 to all defined ESI's and to *calculate* their *validity indices* (*VID's*), which are tantamount to their *validity-values*. In this way, I have proved that 15 categorical syllogisms, other than Bamalip, Barapti (former Darapti), Felapton, and Fesapo, are *universally true* (*tautologous*), because they are the CFCL interpretands of the respective *valid* (*kyrologous*) ESI's, whereas the latter four categorical syllogisms are the CFCL interpretands of the respective *vav-neutral* (*vav-indeterminate, vav-udeterologous*) ESI's, which are *veracious* (*accidentally true*) because they are subjected to a certain additional catlogographic (semantic) axiom. This result is in agreement with the finding of Hilbert and Ackermann [1950, pp. 48–54, 53ff] that all categorical syllogisms in the exclusion of the above four are deducible from Boolean algebra.

7) According to the above items 3–5, all laws (valid relations) for the predicate-sign =, which are postulated (taken for granted) and laid down as *axioms of* $A_{1=}$, turn out to be *theorems of* $A_{1\subseteq}$, whereas all laws for the predicate-sign $\subseteq$, which are laid down as *axioms of* $A_{1\subseteq}$, turn out to be *theorems of* $A_{1\in}$. Consequently, all laws for the predicate-sign =, which are laid down as axioms of $A_{1=}$, are theorems of $A_{1\in}$ as well. That is to say, $A_{1\subseteq}$, *formally* includes $A_{1=}$, whereas $A_{1\in}$ *formally* includes both

$A_{1\subseteq}$ and $A_{1=}$. Since $A_{1=}$ and $A_{1\subseteq}$ *as if converge into* $A_{1\in}$, it seems therefore that the earlier splitting of A_1 into the triple of $A_{1=}$, $A_{1\subseteq}$, and $A_{1\in}$ can be regarded as a *virtual* one or be *disregarded* at all and that hence $A_{1\in}$ can be identified with the entire A_1, including A_{1A} described in the previous item as its extra phase. From this viewpoint, $A_{1\in}$ is the *main branch of* A_1 that may therefore be alternatively called the *Trunk*, or *Stem*, *EAPO*, i.e. the *trunk*, or *stem*, *of* A_1, whereas $A_{1=}$ and $A_{1\subseteq}$ may figuratively be called *boughs*, or *limbs*, *of* A_1. These considerations are however true only as long as A_1 remains semantically uninterpreted. The same PVOT's (APVOT's) of A_1 of the list (5.1), are interpretable in three different ways, when they are employed in $A_{1=}$, $A_{1\subseteq}$, or $A_{1\in}$:

a) a class of *nonempty individuals* if the PVOT is employed in $A_{1=}$;

b) a class comprising both *nonempty masses* and *the empty mass*, i.e. *the empty individual*, if the PVOT is employed in $A_{1\subseteq}$;

c) a class comprising *elements*, i.e. both *nonempty classes* and *the empty class*, i.e. *the empty individual* again, if the PVOT is employed in $A_{1\in}$.

A nonempty individual has neither elements nor parts and it cannot be predicated of any other substance. A mass has no elements, but it has the empty part (empty submass) and it also has nonempty parts (nonempty submasses) if it is nonempty itself, and it can be predicated of other masses. A nonempty class has both *elements* (*members*) and *parts* (*nonempty subclasses* and *the empty subclass*) and it can be predicated of some other classes, nonempty ones and the empty one. The empty class has no elements (no members), but it is a part (subclass) of itself and it can therefore be predicated of another class by stating that the latter is the empty one. Hence, the three classes, indicated in the above items a)–c) are *incomparable* (*not intersecting*). Accordingly, a PVOT or the sign =, which is employed in $A_{1=}$, $A_{1\subseteq}$, and $A_{1\in}$, is *a three-fold homograph*, i.e. *a euautograph that has three different hypostases*, while a PCOT (APOCT) $\emptyset$ or $\emptyset'$, or the sign $\subseteq$, which is employed in $A_{1\subseteq}$ and $A_{1\in}$, is *a two-fold homograph*, i.e. *a euautograph that has two different hypostases*.

8) Thus, $A_{1=}$, $A_{1\subseteq}$, and $A_{1\in}$, and also any other branch of A_1 are different calculi, which have the same AADM, D_1, and which are therefore conveniently treated simultaneously as a single whole calculus, A_1, in terms of common (but homographic) nomenclature. However, such a treatment becomes impossible at the

stage of semantic interpretation $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, and $\mathsf{A}_{1\in}$, because semantic interpretands of the three calculi are *incompatible* and hence *incomparable*. Namely, to be recalled, $\mathsf{A}_{1=}$ is the underlying logical calculus of a *theory of nonempty individuals*; $\mathsf{A}_{1\subseteq}$ is the underlying logical calculus of a *theory of masses*, which unavoidably turns out to be a *one-individual theory* in the sense that it involves *the empty mass as its single empty individual*; $\mathsf{A}_{1\in}$ is the underlying logical calculus of a *theory of classes*, which unavoidably also turns out to be a *one-individual theory* in the sense that it involves *the empty class as its single empty individual*. In this case, a *many-individual class theory* cannot in principle be derived from $\mathsf{A}_{1\in}$ because, in accordance with the formation rules of $\mathsf{A}_{1\in}$, any of its EOT's can stand in ER's to the right of the sign $\in$, and not only to the left of it. Likewise, in accordance with the formation rules of $\mathsf{A}_{1\subseteq}$, any of its EOT's can stand in ER's to the right of the sign $\subseteq$, and not only to the left of it. Therefore, the identity of two nonempty individuals cannot be stated with the help of the sign =, which is defined in terms of the sign $\subseteq$, either a primary (axiomatic) one or secondary one that is defined in terms of $\in$. Incidentally, all existing *many-individual class theories* and particularly *many-individual set theories* are *verbal* (*phonographic*) ones and not *logographic* (see, e.g., Fraenkel et al [1973, pp. 24–25]). This fact can be explained as follows. Since $\emptyset$ and $\emptyset'$ can stand in ER's of $\mathsf{A}_{1\in}$ to the right of the predicate-sign $\in$, therefore the property of emptiness of $\emptyset$ and $\emptyset'$ and hence the property of their individuality (indivisibility) can be expressed by negating the relations $[\mathbf{x} \in \emptyset]$ and that $[\mathbf{y} \in \emptyset']$ for all possible EOT's $\mathbf{x}$ and $\mathbf{y}$ of $\mathsf{A}_{1\in}$, i.e. by asserting '$\neg[\mathbf{x} \in \emptyset]$' and '$\neg[\mathbf{y} \in \emptyset']$' (e.g.) as panlogographic axiom schemata of $\mathsf{A}_{1\in}$, which imply that $\emptyset = \emptyset'$ and that $[\emptyset \subseteq \mathbf{x}]$. By contrast, a name of a nonempty individual is prohibited to stand to the right of the predicate-sign $\in$, while there is no euautographic predicate in $\mathsf{A}_{1\in}$ to be a parasynonym of the verbal predicate "*is not a class*". Therefore, to introduce nonempty individuals into a formal (logographic) *axiomatic class theory* (*AxCsT*), there is no way other than a verbal one.

9) In spite of the fact that $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, and $\mathsf{A}_{1\in}$ are three different organons and that $\mathsf{A}_{1=}$ and $\mathsf{A}_{1\subseteq}$ just *as if converge into* $\mathsf{A}_{1\in}$, I shall, for avoidance of repeatedly stating and proving apparently the same valid relations, set up the above three organons in the reverse order, namely, $\mathsf{A}_{1\in}$ first, $\mathsf{A}_{1\subseteq}$ second, and $\mathsf{A}_{1=}$ last. Consequently, in setting up $\mathsf{A}_{1\subseteq}$ or $\mathsf{A}_{1=}$, I lay down the subject axioms of the

respective organon and indicate the difference between the theorems for $\subseteq$ or = that are stated in terms of $\subseteq$ but proved in terms $\in$ and the theorems for = that are stated and proved in terms of $\subseteq$ in no connections with $\in$.•

Cmt 7.1. A *count name*, i.e. a *count noun* or generally a *count nounal description of the class-species* (*specific class*) *through a class-genus* (*general class*) *and the differences* without any limiting modifier (as a quantifier or as the indefinite or definite article), *denotes* the pertinent class, namely the pertinent class-genus in the former case or the pertinent class-species in the latter case. Therefore, a count name is a *proper class-name* but not necessarily vice versa. By contrast, a *mass-name*, i.e. a *mass noun* or generally a *mass nounal description of the mass-species* (*specific mass*) *through a mass-genus* (*general mass*) *and the differences* without any limiting modifier (as a prepositive qualifier or as the definite article), *denotes* the pertinent mass – homogeneous substance or concept that may have parts but not members, namely the pertinent mass-genus in the former case or the pertinent mass-species in the latter case. Therefore, a mass name is a *proper mass-name* but not necessarily vice versa. Consequently, an EOT of A_1 is replaceable with a *proper individual name* in semantically interpreting $\mathsf{A}_{1=}$, with a *mass-name* in semantically interpreting $\mathsf{A}_{1\subseteq}$, and with a *proper class-name*, particularly with *count name*, in semantically interpreting $\mathsf{A}_{1\in}$.•

Cmt 7.2. 1) As was already mentioned, A_1 comprises an infinite number of branches. Here follows an example of a systematic division of A_1, into an indefinite number of branches. A *singulary EAPO* is by definition a branch of A_1, whose selective atomic basis, b_{1OS}, includes only *singulary* AEOPS's (atomic euautographic ordinary predicate-signs), i.e. the entire list (5.3^1) of singulary APVOPS's. An *exclusive binary EAPO* (*exclusive BEAPO*) is by definition a branch of A_1, whose b_{1OS} includes only the *binary AEOPS's* (*BAEOPS's*), i.e. either the entire list (5.3^2) of binary APVOPS's or, in accordance with items 8 and 9 of Ax 5.1, some one of the binary APCOPS's =, $\subseteq$, and $\in$, or both. By contrast, an *inclusive binary EAPO* is by definition a branch of A_1, which comprises both a singulary EAPO and an exclusive binary EAPO (cf. Church [1956, pp. 173, 174]). *Exclusive* and *inclusive ternary*, *quaternary*, and in general *n-ary EAPO's*, to be denoted by '$\mathsf{A}_1'^n$' and '$\mathsf{A}_1''^n$' respectively, are by definition branches of A_1 that are defined in analogy with exclusive and inclusive binary EAPO's. Accordingly, '$A'^{n}_{1=}$' and '$A''^{n}_{1=}$', '$A'^{n}_{1\subseteq}$' and '$A''^{n}_{1\subseteq}$', and '$A'^{n}_{1\in}$' and '$A''^{n}_{1\in}$' denote the like branches of $A_{1=}$, $A_{1\subseteq}$, and $A_{1\in}$ respectively. The Pure Functional PMsEAPO whose only primary AEOPS is $\subseteq$ and whose specific (atypical) subject axioms are those of $A_{1\subseteq}$ is denoted by '$\hat{A}^{2}_{1\subseteq}$'. Likewise, the Pure Functional PMsEAPO whose only primary AEOPS is $\in$ and whose specific (atypical) subject axioms are those of $A_{1\in}$ is denoted by '$\hat{A}^{2}_{1\in}$'. $\hat{A}^{2}_{1\in}$ can be used as the underlying logical calculus of a full-scale one-individual class, or set, theory, which does not involve any predicate other than $\in$, – just as any *conventional one-individual axiomatic set theory* (*COIAST*) (see, e.g., Fraenkel et al [1973).

2) In accordance with Dfs 3.1(19) and 7.1(1), D_1 is a universal attribute of A_1 as a single whole, which can be applied to any relation of A_1 in no connection with any branch, to which that relation can be relegated. Consequently, any preliminary branching A_1 turn out to be irrelevant to the setup of A_1 and is therefore purposeless. The only kind of partitioning A_1 that turns out to be useful in setting it up and in executing it is its phasing. Once a certain phase (stage) of the setup of A_1 is completed, one may, if he wishes, select any branch of A_1, distinguish it by the appropriate verbal name or logographic constant or both and to treat and particularly to interpret relations of that branch only.•

Df 7.2: *The major phases of the trunk of* A_1. 1) The setup of $A_{1\in}$, e.g., is conveniently phased (staged) as follows. While A_{1R} (A_{1P}) is regarded as the first phase of $A_{1\in}$, which $A_{1\in}$ shares with all other branches of A_1, the next three consecutive phases (stages) of $A_{1\in}$ are called the *Ground PCsEAPO* (*GPCsEAPO*), the *Deficient PCsEAPO* (*DPCsEAPO*), and the *Sufficient PCsEAPO* (*SPCsEAPO*), and are denoted logographically by '$A_{1\in G}$', '$A_{1\in D}$', and '$A_{1\in S}$' in that order, the understanding being that $A_{1\in S}$ is identical with $A_{1\in}$. The above phases of $A_{1\in}$ are characterized as follows.

a) $A_{1\in G}$ is obtained from A_{1R} by the following two successive operations:

i) psychically (mentally) disregarding all formulas, each of which involves = or $\subseteq$ or both as primary predicate-signs;

ii) physically defining, in terms of $\in$ by the appropriate contextual (implicit) formal synonymic definitions, the class of secondary relations, each of which involves any one of the thirteen secondary predicate-signs on the list (5.5) as its principal operator, – in accordance with Cmt 5.3(2).

No individualizing axioms are imposed on $\in$ in $\mathsf{A}_{1\in\mathsf{G}}$, either directly or obliquely via any secondary sign of the list (5.5). However, some important theorems are proved for some of the above signs by means of D_1. The atomic basis of $\mathsf{A}_{1\in\mathsf{G}}$ does not contain $\emptyset$ and $\emptyset'$.

b) $\mathsf{A}_{1\in\mathsf{D}}$ results by supplementing $\mathsf{A}_{1\in\mathsf{G}}$ with two *specific* (*atypical*) subject axioms, which are imposed on $\in$ relative to APVOT's and from which a wide variety of important subject theorems is proved by means of D_1. It is understood that the new specific axioms do not alter the EADM. $\mathsf{A}_{1\in\mathsf{D}}$ is qualified *deficient* because it has the same atomic basis as $\mathsf{A}_{1\in\mathsf{G}}$ and because therefore it does not have $\emptyset$ and $\emptyset'$.

c) $\mathsf{A}_{1\in\mathsf{S}}$, i.e. the ultimate $\mathsf{A}_{1\in}$, results by supplementing $\mathsf{A}_{1\in\mathsf{D}}$ with the APCOT's $\emptyset$ and $\emptyset'$ indicated in Ax 5.1(9) and by simultaneously supplementing it with the additional subject *axiom schemata* '$\neg[\mathbf{x} \in \emptyset]$' and '$\neg[\mathbf{y} \in \emptyset']$', which can be specified by replacing '**x**' and '**y**' either with any APVOT's of the list (5.1) or with either of the two APCOT's $\emptyset$ and $\emptyset'$. Thus, $\neg[x \in \emptyset]$, $\neg[y \in \emptyset']$, $\neg[\emptyset \in \emptyset]$, $\neg[\emptyset' \in \emptyset]$, $\neg[\emptyset \in \emptyset']$, and $\neg[\emptyset' \in \emptyset']$ are some instances of the above axiom schemata of $\mathsf{A}_{1\in}$. In general, the panlogographic axiom schema '$\neg[\mathbf{x} \in \emptyset]$' (e.g.) is concurrent (tantamount) to the euautographic axiom $\neg[x \in \emptyset]$ (e.g.) subject to the rule of substitution, according to which x can be replaced with any APVOT of the list (5.1) or with either of the two APCOT's $\emptyset$ and $\emptyset'$; and similarly with $\emptyset'$ in place of $\emptyset$. With the help of the pertinent AEADP, it is proved from the above two axiom schemata or from their instances $\neg[x \in \emptyset]$ and $\neg[y \in \emptyset']$ that $\emptyset = \emptyset'$. The former two axiom instances and the latter theorem are *valid* ER's. Consequently, the relations '$\neg[x \in \varnothing]$' and '$\neg[y \in \varnothing']$', being the *CFCL* (*conformal catlogographic*) *interpretands* of the above axiom instances, are *tautologies*, i.e. *tautologous* (*universally true*) *relations*, which mean that the *classes* $\varnothing$ and $\varnothing'$ are *empty* (*memberless*), while the relation '$\varnothing = \varnothing'$', being the CFCL interpretand of the theorem $\emptyset = \emptyset'$, is also a tautology, which means that $\varnothing$ and $\varnothing'$ are indistinguishable, i.e. that $\varnothing$ is unique. Accordingly, the axiom schema '$\neg[\mathbf{x} \in \emptyset]$' (or any other equivalent schema as '$\neg[\mathbf{y} \in \emptyset]$', '$\neg[\mathbf{z} \in \emptyset]$', etc) is called an *axiom schema of -emptiness of* $\emptyset$ (or *of the EOZT*) *of* $\mathsf{A}_{1\in}$ and also briefly an *axiom schema of* $\emptyset$, or $\emptyset$-*axiom schema, of* $\mathsf{A}_{1\in}$. At the same time, any given concrete instance of that schema, e.g. $\neg[x \in \emptyset]$, which, along with the pertinent rule of substitutions, is tantamount to that schema, is called an *axiom of emptiness of* $\emptyset$ *of* $\mathsf{A}_{1\in}$ and also briefly an *axiom of* $\emptyset$, or $\emptyset$-*axiom, of* $\mathsf{A}_{1\in}$.

d) $\mathsf{A}_{1\mathsf{A}}$ as described in Df 7.1(6) is the next phase (extension) of $\mathsf{A}_{1\in}$, which involves the appropriate additional definitions and additional theorems, but which does not involve any additional subject axioms.•

Cmt 7.3: *Principles of phasing* A_1**.** If A_{1*} and A_{1*+} are two successive phases of A_1 then A_{1*+} is called:

a) a *structural extension of* A_{1*} if it has at least one additional PAE of $\mathsf{B}_{1\mathsf{OS}}$, at least one composite (secondary) euautograph defined in terms of the former, and at least one theorem involving that secondary euautograph, and if it has no additional subject axioms;

b) a *conceptual extension of* A_{1*} if it has some or no additional PAE's of $\mathsf{B}_{1\mathsf{OS}}$ and at least one additional subject axiom;

c) an *analytical*, or *algebraic*, *extension of* A_{1*} if it has no additional PAE's of $\mathsf{B}_{1\mathsf{OS}}$ and no additional subject axioms, but has at least one new composite euautograph defined in terms of some earlier primary or secondary euautographs and at least one theorem involving a new euautoghaph;

d) an *extension of* A_{1*} if it is one of the previous three, the understanding being that, in any case, all PAE's and all subject axioms of A_{1*} are utilized in A_{1*+}.

By items the above items a)–c), $\mathsf{A}_{1\in\mathsf{G}}$ is a structural extension of $\mathsf{A}_{1\mathsf{R}}$, $\mathsf{A}_{1\in\mathsf{D}}$ is a conceptual extension of $\mathsf{A}_{1\in\mathsf{G}}$, $\mathsf{A}_{1\in\mathsf{S}}$ is a conceptual extension of $\mathsf{A}_{1\in\mathsf{D}}$, and $\mathsf{A}_{1\mathsf{A}}$ is an analytical extension of $\mathsf{A}_{1\in\mathsf{S}}$. By the item d), in any given phase of A_1, following $\mathsf{A}_{1\mathsf{R}}$, all atomic euautographs and all object axioms of the previous phase are utilized, no matter whether or not the latter is mentioned in setting up the former.•

Cmt 7.4. From comparison of the items 4 and 5 of Df 7.1, it follows that $\mathsf{A}_{1\subseteq}$ can be phased, *mutatis mutandis*, in the same way as $\mathsf{A}_{1\in}$ with the most essential difference that $\mathsf{A}_{1\subseteq}$ has no phase (extension) analogous to $\mathsf{A}_{1\mathsf{A}}$. Namely, $\mathsf{A}_{1\mathsf{R}}$ is the first phase of $\mathsf{A}_{1\subseteq}$, which $\mathsf{A}_{1\subseteq}$ shares with all other branches of A_1, while the next three consecutive phases (stages) of $\mathsf{A}_{1\subseteq}$ are called the *Ground PMsEAPO* (*GPMsEAPO*), the *Deficient PMsEAPO* (*DPMsEAPO*), and the *Sufficient PMsEAPO* (*SPMsEAPO*), and are denoted logographically by '$\mathsf{A}_{1\subseteq\mathsf{G}}$', '$\mathsf{A}_{1\subseteq\mathsf{D}}$', and '$\mathsf{A}_{1\subseteq\mathsf{S}}$' in that order, the understanding being that $\mathsf{A}_{1\subseteq\mathsf{S}}$ is identical with $\mathsf{A}_{1\subseteq}$. The above phases of $\mathsf{A}_{1\subseteq}$ are characterized as follows.

a) $\mathsf{A}_{1\mathsf{G}\subseteq}$ is obtained from $\mathsf{A}_{1\mathsf{R}}$ by the following two successive operations:

i) psychically (mentally) disregarding all formulas, each of which involves $=$ or $\in$ or both as primary predicate-signs;

ii) physically defining, in terms of $\subseteq$ by the appropriate contextual (implicit) formal synonymic definitions, the class of secondary relations, each of which involves any one of the nine secondary predicate-signs $=$, $\subset$, $\overline{\subseteq}$, $\equiv$, $\overline{\subset}$, $\supseteq$, $\supset$, $\overline{\supseteq}$, $\overline{\supset}$ as its principal operator, – in accordance with Cmt 5.3(3).

No individualizing axioms are imposed on $\subseteq$ in $\mathsf{A}_{1\subseteq\mathsf{G}}$, either directly or obliquely via any one of the above nine secondary predicate-signs. However, some important theorems can be proved for some of those signs by means of D_1. The atomic basis of $\mathsf{A}_{1\subseteq\mathsf{G}}$ does not contain $\emptyset$ and $\emptyset'$.

b) $\mathsf{A}_{1\subseteq\mathsf{D}}$ results by supplementing $\mathsf{A}_{1\subseteq\mathsf{G}}$ with three *specific* (*atypical*) subject axioms, which are imposed on $\subseteq$ relative to APVOT's and from which a wide variety of important subject theorems can be deduced by means of D_1. It is understood that the new specific axioms do not alter the EADM. $\mathsf{A}_{1\subseteq\mathsf{D}}$ qualified *deficient* because it has the same atomic basis as $\mathsf{A}_{1\subseteq\mathsf{G}}$ and because therefore it does not have $\emptyset$ and $\emptyset'$.

c) $\mathsf{A}_{1\subseteq\mathsf{S}}$, i.e. the ultimate $\mathsf{A}_{1\subseteq}$, results by supplementing $\mathsf{A}_{1\subseteq\mathsf{D}}$ with the APCOT's $\emptyset$ and $\emptyset'$ indicated in Ax 5.1(9) and by simultaneously supplementing it with the additional subject *axiom schemata* '$\emptyset \subseteq \mathbf{x}$' and '$\emptyset' \subseteq \mathbf{y}$' *of* $\mathsf{A}_{1\subseteq\mathsf{D}}$, which can be specified by replacing '$\mathbf{x}$' and '$\mathbf{y}$' either with any APVOT's of the list (5.1) or with either of the two APCOT's $\emptyset$ and $\emptyset'$. In this case, the postpositive qualifier "of $\mathsf{A}_{1\subseteq\mathsf{D}}$" to "axiom schemata" is essential, because the same valid relation schemata are *theorem schemata of* $\mathsf{A}_{1\in}$. Thus, $\emptyset \subseteq x$, $\emptyset' \subseteq y$, $\emptyset \subseteq \emptyset$, $\emptyset \subseteq \emptyset'$, $\emptyset' \subseteq \emptyset$, and $\emptyset' \subseteq \emptyset'$ are some instances of the above axiom schemata of $\mathsf{A}_{1\subseteq}$ and at the same time they are instances of the similar theorem schemata of $\mathsf{A}_{1\in}$. In general, the panlogographic axiom schema '$\emptyset \subseteq \mathbf{x}$' (e.g.) is concurrent (tantamount) to the euautographic axiom $\emptyset \subseteq x$ (e.g.) subject to the rule of substitution, according to which x can be replaced with any APVOT of the list (5.1) or with either of the two APCOT's $\emptyset$ and $\emptyset'$; and similarly with $\emptyset'$ in place of $\emptyset$. With the help of the pertinent AEADP, it is proved from the above two axiom schemata or from their instances $\emptyset \subseteq x$ and $\emptyset' \subseteq y$ that $\emptyset = \emptyset'$. The former two axiom instances and the latter theorem are *valid* ER's. Consequently, the relations '$\varnothing \subseteq x$' and '$\varnothing' \subseteq y$', being the CFCL interpretands of the above axiom instances are *tautologies*, i.e. *tautologous* (*universally true*) *relations*, which mean that the *masses* $\varnothing$ and $\varnothing'$ are *empty*, while the relation '$\varnothing = \varnothing'$', being the CFCL interpretand of the theorem $\emptyset = \emptyset'$, is also a tautology, which means that $\varnothing$ and $\varnothing'$ are indistinguishable, i.e. that $\varnothing$ is unique. Accordingly, the axiom schema '$\emptyset \subseteq \mathbf{x}$' (or any other equivalent schema as '$\emptyset \subseteq \mathbf{y}$', '$\emptyset \subseteq \mathbf{z}$', etc) is called an *axiom schema of emptiness of* $\emptyset$ (or *of the EOZT*) and also briefly an *axiom schema of* $\emptyset$, or $\emptyset$*-axiom schema, of* $\mathsf{A}_{1\subseteq}$. At the same time, any given concrete instance of that schema, e.g. $\emptyset \subseteq x$, which, along with the pertinent rule of substitutions, is tantamount to that schema, is called an *axiom of emptiness of* $\emptyset$ *of* $\mathsf{A}_{1\subseteq}$ and also briefly an *axiom of* $\emptyset$, or $\emptyset$*-axiom, of* $\mathsf{A}_{1\subseteq}$.•

Cmt 7.5. 1) The branching structure and phased development of A_1 is analogous to an extent to the *evolutionary tree* of the kingdom of plants or to that of the kingdom of animals (cf. Campbell [1990, Figs. 27.4, 29.5 – pp. 567, 608]) rather than to a tree as such. This analogy is useful for understanding, not only the structure and development of A_1, but also for explicating the systematic way of verbally naming the phases of A_1 by describing them through the genus and the pertinent differentiae. The following alternative analogy can be driven for the same purposes

2) The noun "English", e.g., can be understood, not only as denoting Modern English, but also as denoting English-in-process, which existed from the ancient times to the present. In order to distinguish among the different historical versions (phases, stages) of English-in-process, the names "Old English" ("OE"), "Middle English" ("ME"), and "Modern English" are used. A like remark applies to the name of any *native language* (*NL*) that has been existent for a long, many-generation, period of time, e.g. "Latin", "Greek", and "Hebrew". In this case, A_1 is analogous to any NL-in-

process, whereas the sequential phases of A_1 are analogous to the different sequential historical versions of the NL with the following essential difference. Gradual changes in the lexicon of an NL occur, as a rule, under the same grammar, while some grammatical rules are changed by grammarians from time to time in no visible connection with accumulated changes in the language's lexicon. Still, not only the lexicon, but also the grammar of an NL changes after all. By contrast, no acts of augmenting A_1 by new characters, new formulas, or new subject axioms alter any rule of its AADM, which is parallel to the grammar of an NL.•

Cmt 7.6. 1) When PVOT's of the list (II.5.1) are employed in $\mathsf{A}_{1\in}$, their CFCL interpretands are *counterparts*, i.e. either *tokens* or *tantamount variants*, of the respective conventional *unrestricted* atomic variables, which are employed in CALC'i, e.g. in Whitehead and Russell [1910; 1962, pp. 4, 5]. Accordingly, the PVOT's are qualified *pseudo-unrestricted* or *pseudo-unconfined*, no matter in which branch, $\mathsf{A}_{1\subseteq}$ or $\mathsf{A}_{1\in}$, they are employed. In this connection, I shall, in accordance with Cmt 5.9, explore the possibility of introducing into both $\mathsf{A}_{1\subseteq}$ and $\mathsf{A}_{1\in}$ a certain *primary atomic pseudo-constant extraordinary term* (*PAPCXOT*), U, which will be called the *atomic pseudo-constant universal term* (*APCUT*). The EAPO's resulted by introducing U, along with the pertinent subject axioms, into $\mathsf{A}_{1\subseteq}$ and $\mathsf{A}_{1\in}$ is denoted logographically by '$\bar{\mathsf{A}}_{1\subseteq}$' and '$\bar{\mathsf{A}}_{1\in}$' and is called verbally (phonographically) the *Pseudo-Restricted*, or *Pseudo-Confined*, *PMsEAPO* and the *Pseudo-Restricted*, or *Pseudo-Confined*, *PCsEAPO* respectively.

2) U does not belong to either of the two EAPO's $\mathsf{A}_{1\subseteq}$ and $\mathsf{A}_{1\in}$. Therefore, U is introduced into each of the two EAPO'a by *separate formation rules*, according to which the new binary euautographic extraordinary relations are formed by placing a token of U on either side of $\subseteq$, or $\in$, and hence on either side of any other pedicate-sign that is defined in terms of $\subseteq$ or $\in$ respectively, while the other side of a predicate-sign is occupied by $\mathbf{x}$ or by another token of U. In order to express the property of universality of U, the new binary ER's $\mathsf{U}\subseteq\mathsf{U}$, $\mathbf{x}\subseteq\mathsf{U}$, and $\neg[\mathsf{U}\subseteq\mathbf{x}]$ are taken for granted as specific (atypical) subject axioms of $\bar{\mathsf{A}}_{1\subseteq}$, whereas the new binary ER's $\neg[\mathsf{U}\in\mathsf{U}]$, $\mathbf{x}\in\mathsf{U}$, and $\neg[\mathsf{U}\in\mathbf{x}]$ are taken for granted as specific (atypical) subject axioms of $\bar{\mathsf{A}}_{1\in}$. Accordingly, U is alternatively called the *universal pseudo-mass* if it is employed in $\bar{\mathsf{A}}_{1\subseteq}$ and the *universal pseudo-class* if it is employed in $\bar{\mathsf{A}}_{1\in}$. Some other new ER's of

academic or practical interest that involve U are proved to be valid or antivalid either with the help of D_1 or with the help of the pertinent new rules of inference and decision. Along with these rules, D_1 is denoted by '$\overline{D}_1$'. The dichotomy of EF's of $\bar{A}_{1\subseteq}$ or $\bar{A}_{1\in}$ into *ordinary* and *special* ones retains with the proviso that an EF is qualified either an *extraordinary* one or an *extraspecial* one if it is a variant respectively of an ordinary or special EF of $\bar{A}_{1\subseteq}$ or $\bar{A}_{1\in}$, in which at least one occurrence of **x** is replaced with an occurrence of U. A certain part of this IML (this treatise), with the help of which and within which $\bar{A}_{1\subseteq}$, or $\bar{A}_{1\in}$, is developed (set up and executed) is called the *Pseudo-Restricted EPMsT*, or the *Pseudo-Restricted EPCsT*, respectively (cf. Df 7.1(4,5)). The CCFCL interpretand of U is denoted by '*U*' and be called the *universal mass* if U is the *universal pseudo-mass* and the *universal class* if U is the *universal pseudo-class.*

The CFCL interpretand of U will be denoted by '*U*' and be called the *universal mass* if U is the *universal pseudo-mass* and the *universal class* if U is the *universal pseudo-class.*

3) In accordance with Df 7.1(4,5), the organons $A_{1\subseteq}$ and $A_{1\in}$ are branches of A_1 and therefore they are treated systematically. By contrast, the organons $\bar{A}_{1\subseteq}$ and $\bar{A}_{1\in}$ are minor digressions from $A_{1\subseteq}$ and $A_{1\in}$, which are neither branches nor phases of A_1, The former include the latter in such a way that all axioms and all theorems of $A_{1\subseteq}$ and $A_{1\in}$ retain in $\bar{A}_{1\subseteq}$ and $\bar{A}_{1\in}$ and do not interfere with any additional axioms and theorems involving U. Therefore, $\bar{A}_{1\subseteq}$ and $\bar{A}_{1\in}$ are treated fragmentarily in order to emphasize only the aspects, by which they differ from $A_{1\subseteq}$ and $A_{1\in}$. $\bar{A}_{1\subseteq}$ and $\bar{A}_{1\in}$ demonstrate that $A_{1\subseteq}$ and $A_{1\in}$ are self-consistent and that the fact that they are treated as pseudo-unrestricted organons does not lead to any loss of generality, while essentially simplifying their structure.

8. An introduction into the conformal catlogographic interpretations of A_1

8.1. The conformal catlogographic (CFCL) pre-interpretations of the atomic euautographic ordinary formulas (AEOF)

Df 8.1. 1) The *pseudo-variable*, or redundantly *atomic pseudo-variable*, *ordinary terms* (briefly *PVOT's* or *APVOT's*) on the list (5.1) and the two *pseudo-constant*, or redundantly *atomic pseudo-constant*, *ordinary terms* (briefly *PCOT's* or *APCOT's*) $\emptyset$ and $\emptyset'$ of Ax 5.1(9), called also the *euautographic ordinary zero-terms* (*EOZT's*), are collectively called the *euautographic ordinary terms* (briefly *EOT's*), or redundantly *atomic ones* (briefly *AEOT's*), *of* A_1. The *atomic euautographic* (or *atomic pseudo-variable*), or redundantly *atomiic euautographic ordinary* (or *atomic pseudo-variable ordinary*), *relations* (briefly *AER's*, *APVR's*, *AEOR's*, or *APVOR's*) on the list (5.2) and the EOT's (AEOT's) are collectively called the *atomic euautographic ordinary formulas* (briefly *AEOF's*), or *like categoremata* (briefly *AEOC'ta*), *of* A_1.

2) A formula, i.e. a term or relation, of A_1 is said to be

a) a *non-degenerate euautographic formula* (briefly *NDgEF*) if and only if it involves at least one AEOF of A_1,

b) a *degenerate euautographic formula* (briefly *DgEF*) if otherwise, i.e. if and only if it involves no AEOF of A_1.

The above terms apply synonymously with "*categorem*" ("*C*") in place of "*formula*" ("*F*").

3) Any given formula of A_1 is one of the four kinds: (a) an EOT, i.e. a PVOT or a PCOT; (b) an EI, i.e. an ESpT; (c) an ESR, i.e. an EOR (EOSR) or an ESpSR; (d) an EMTh (EDTh), i.e. an ESpMR (ESpDR), subject to the standard: "I" for "integron", "R" for "relation", "T" for "term", "Th" for "theorem", "D" for "decision", "M" for "master", "S" for "slave", "E" for "euautographic", "PV" for "pseudo-variable", "PC" for "pseudo-constant", "O" for "ordinary", "Sp" for "special". Whenever there is no danger of misunderstanding, I use the abbreviations "EMT" and "EDT" instead of "EMTh" and "EDTh". Here follow the pertinent instances of the dichotomy defined in the above item 2.

i) An EOT is a kind of an AEOF, so that it is necessarily an NDgEF (ExIEF).

ii) An EOR is necessarily an NDgEF.

iii) Hence, the EMTh (EDTh) for an EOR is necessarily an NDgEMT (NDgEDT).

iv) An EI (ESpT) is either an NDgEI (NDgESpT) or a DgEI (DgESpT).

v) An ESpSR is either an NDgESpSR or a DgESpSR.

vi) Hence, the EMTh (EDTh) for an NDgESpSR is an NDgEMTh (NDgEDTh), whereas the EMTh (EDTh) for a DgESpSR is a DgEMTh (DgEDTh).

4) A formula, i.e. a term or relation, of A_1 is said to be

a) an *externally* (*extrinsically*) *interpretable euautographic formula* (briefly *ExIEF*) if and only if it is one of the following four kinds: (i) a EOT, (ii) an NDgEI (NDgESpT), (iii) an NDgDdESR (non-degenerate decided euautographic slave relation), (iv) the EMTh (EDTh) for an NDgDdESR;

b) an *externally* (*extrinsically*) *non-interpretable euautographic formula* (briefly *ExNIEF*) if and only if it is one of the following four kinds: (i) a DgEI (DgESpT), (ii) a non-decided ESR (euautographic slave relation), (iii) a DgDdESR (degenerate decided euautographic slave relation), (iv) the EMTh (EDTh) for a DgDdESR.

5) All ESpSR's and their EMTh's (EDTh's) are called the *accidental ExIEF's* (*AcExEF's*) or more specifically the *accidental ExIER's* (*AcExER's*) in the sense that they are *accidental* (*by-side*) *products* of the *EAADM* (*euautographic advanced algebraic decision method*), which has been aimed at EOR's, but turned out to be applicable to ESpSR's as well. External (extrinsic) interpretations of any AcESpSR and of its EMTh (EDTh) have no interest, either academic or practical, so that they are not intended to be ever performed. By contrast, a formula of A_1 that is, in principle, intended to be subjected to an external (extrinsic) interpretation is called an *essential ExIEF* (*EsExIEF*), the understanding being that this formula is one of the following four kinds: (i) an EOT, (ii) an NDgEI (NDgESpT), (iii) a DdEOR (decided euautographic ordinary relation), (iv) the EMTh (EDTh) for an DdEOR. Particularly, any AER is a *vav-neutral EOR* and hence it is a DdEOR.

6) At the same time, an external interpretand of an *antivalid EOR* of A_1 is also an *antivalid relation* of the pertinent domain. Also, any EMT (EDT) of A_1 is *a valid ESpR*, but not necessarily vice versa. Hence, an external interpretand of an EMT

(EDT) of A_1 is a *valid special relation* of the pertinent domain. In this case, an external interpretand of the EMT (EDT) for a valid or antivalid EOR cannot be modified further and, when omitted, it can immediately be recovered if the EOR is known. Therefore, from the standpoint of their external interpretands, *antivalid EOR's* and *the EMT's* (*EDT's*) *for valid and antivalid EOR's are of no practical interest*, so that all these ER's can be disregarded. Thus, in accordance with Df 1.7(3), the class $\dot{\mathsf{R}}_1$, which is called *the* [*class of*] *output*, or *sifted decided*, *ER's* (*OptER's* or *SfdDdER's*) *of* A_1, and which is the union of the three classes: the class $\mathsf{R}_{1+}^{\mathrm{O}}$ of valid EOR's, the class $\mathsf{R}_{1\sim}^{\mathrm{O}}$ of the vav-neutral EOR's, and the class $\mathsf{R}_{1\oplus}^{\mathrm{O}\sim}$ of EMTh's (EDTh's) for the vav-neutral EOR's of the class $\mathsf{R}_{1\sim}^{\mathrm{O}}$, is the class of the only ER's of A_1 whose external interpretands may have any academic or practical interest. In this case, the EI's (ESpT's) that occur in EMTh's (EDTh's) of $\mathsf{R}_{1\oplus}^{\mathrm{O}\sim}$ are the only ones whose external interpretands have relevant academic or practical interest. At the same time, $\dot{\mathsf{R}}_1$ is the *least inclusive class of the ER's of* A_1, whose external interpretands are sufficient for recovering such interpretands of *all* DdEOR's of A_1 and of *their* EMTh's (EDTh's). Also, the class $\dot{\mathsf{R}}_1$ of OptER's of A_1 can be restricted to a like class $\dot{\mathsf{R}}_1^0$ of A_1^0 or $\dot{\mathsf{R}}_0$ of A_0, and also to that of a certain preselected branch of A_1 or to that of a certain phase of the branch.

7) Throughout the treatise, the qualifier "*conformal*" to either noun "token" or "isotoken" in general and to any one of the nouns "substiution", "replacement", "interpretation", and "interpretand" in particular has the following meaning. A euautograph or a logograph is indiscriminately called a *pasigraph*. A pasigraph has an indefinite number of isotokens and no phonic paratokens. A pasigraph is called:

a) a *homolograph* or *photograph* if it has only *congruent* or *proportional tokens* (*isotokens*), called *homolographic*, or *photographic*, ones;

b) an *analograph* if, besides homolographic isotokens, it has *recognizably same but stylized* isotokens, qualified *iconographic* or *pictographic*.

An isotoken of a pasigraph is called a *strictly analographic token* (*isotoken*) if it is *not homolographic* (*not photographic*). A pasigraph, i.e. a euautograph or logograph, particularly a panlogograph or a catlogograph, is called a *conformal*, or *analo-homolographic, token* (*isotoken*) *of a given prototypal pasigraph* if it is a *strictly* *analographic token of the latter that has only homolographic* (*photographic*) *tokens* (*isotokens*) *of its own*. Accordingly, a replacement of a *homolograph of one type with a strictly analographic homolograph of the other type* is said to be *conformal* or *analo-homolographic*.

8) The master-to-slave relationships between A_1 and A_0 on the one hand and traditional logic and CALC'i on the other hand are established via two successive *conformal catlogographic* (*CFCL*) *interpretations*, the *conservative* one (*CCFCL* interpretation) and the *progressive*, or *transformative*, one (*PCFCL*, or *TFCFCL*, interpretation), of the OptER's of A_1 including the OptER's of A_0. The CCFCL interpretation, denoted by 'I_1', and the PCFCL interpretation, denoted by '$\mathcal{A}_1$', [*of the OptER's*] *of* A_1 are two *successive principal external* (*extrinsic*) *syntactico-semantic interpretations of* A_1, whose cumulative rules, denoted by 'I_1' and 'D_1' respectively, are based on the following definition.•

Df 8.2: ***The CFCL pre-interpretation of the AEOF's.*** 1) The PVOT's (APVOT's) on the list (5.1), the PCOT's (EOZT's) $\emptyset$ and $\emptyset'$ of Ax 5.1(9), and the AER's (AEOR's, APVOR's) on the list (5.2) are defined *syntactically* (*synonymously*) thus:

$$u\to`u', v\to`v', w\to`w', x\to`x', y\to`y', z\to`z', u_1\to`u_1', v_1\to`v_1', \ldots, \tag{8.1}$$

$$\emptyset\to`\varnothing', \emptyset'\to`\varnothing'', \tag{8.2}$$

$$p\to`p', q\to`q', r\to`r', s\to`s', p_1\to`p_1', q_1\to`q_1', \ldots; \tag{8.3}$$

where *the definientia are the HAQ's* (*homoloautographic quotations*), *and not their interiors, being their denotata*. The interiors of the HAQ's are in turn defined *semantically* as follows.

2) Any one of the following *lexigraphs* (*atomic logographs*), i.e. the interior of any one of the following HAQ's – the interior, which is conventionally mentioned by using its HAQ:

$$`u', `v', `w', `x', `y', `z', `u_1', `v_1', `w_1', `x_1', `y_1', `z_1', `u_2', `v_2', \ldots, \tag{8.4}$$

– is a *logographic variable* that is called a *variable*, or *atomic variable*, *catlogographic ordinary term* (*VCLOT* or *AVCLOT*). Depending on a branch $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, or $\mathsf{A}_{1\in}$, in which the PVOT being the CFE interpretans of a given VCLOT is employed, the range of the VCLOT is the respective one of the following three *classes*:

a) a class of *nonempty individuals* if the VCLOT is utilized in a *domain* (*theory*) *of nonempty individuals*;

b) a class of masses, both *nonempty masses* and *the empty mass*, i.e. *the empty individual*, if the VCLOT is utilized in a *domain* (*theory*) *of masses*;

c) a class of classes, both *nonempty classes* and *the empty class*, i.e. *the empty individual* again, if the VCLOT is utilized in a *domain* (*theory*) *of classes*.

Thus, the VCLOT can assume as its *accidental denotatum* a *nonempty individual* in the case a, *nonempty mass* or *the empty mass*, i.e. *the empty individual*, in the case b, and a *nonempty class* or *the empty class*, i.e. *the empty individual*, in the case c. Accordingly, the VCLOT is called a *nonempty-individual-valued* one in the case a, a *mass-valued* one in the case b, and a *class-valued* one in the case c. A general notion of *domain* is discussed in Appendix 4 (A4). A scientific domain is alternatively called a *theory*. Therefore, in the above context, and generally in what follows, the noun "domain" is used interchangeably with the noun "theory". In Aristotelianism, the terms "*primary substances*" (or "*essential particulars*") and "*secondary substances*" are used for denoting the *entities*, which are respectively called "*nonempty individuals*" and "*classes*" in the presently common terminology (see Cmt 8.1 below for greater detail). Accordingly, a nonempty individual will alternatively be called a *primary substance*. At the same time, A_1 allows distinguishing formally (axiomatically) between classes (including sets) and masses. Therefore, a mass or a class will indiscriminately be called a *secondary substance*.

3) '$\varnothing$' or '$\varnothing'$' is a *logographic constant*, to be called a *constant*, or *atomic constant*, *catlogographic ordinary term* (*CCLOT* or *ACCLOT*) and also, more explicitly, a *catlogographic ordinary zero-term* (*CLOZT*), which denotes either the *empty mass*, if it is utilized in a *domain of masses*, or the *empty class*, if it is utilized in a *domain of classes*. *The empty mass of a domain of masses or the empty class of a domain of classes* is indiscriminately called the *indivisible empty substance* (*entity*, *object*) or briefly the *empty individual*. A general notion of *domain* is discussed in section A4 (Appendix 4). A scientific domain is alternatively called a *theory*. Therefore, in the above context, and generally in what follows, the noun "domain" is used synecdochically interchangeably with the noun "theory". If every class of a domain of classes is a *set*, called also a *regular*, or *small*, *class* (see subsection 9.3 for greater detail), i.e. if the domain is one of sets, then the noun "class" in all pertinent

terms introduced above should be replaced with “set”. Whenever confusion can result, the empty mass will be denoted by ‘$\varnothing_m$’, whereas the empty class will be denoted by ‘$\varnothing_c$’ or, in the domain of sets, by ‘$\varnothing_s$’. Each of the three constants is a CCLOT (CLOZT). A VCLOT or a CCLOT is indiscriminately called a *catlogographic*, or *atomic catlogographic*, *ordinary term* (*CLOT* or *ACLOT*).

4) Any one of the lexigraphs:

$$‘p’, ‘q’, ‘r’, ‘s’, ‘p_1’, ‘q_1’, ‘r_1’, ‘s_1’, ‘p_2’, ‘q_2’, ‘r_2’, ‘s_2’, \ldots \qquad (8.5)$$

is a *logographic variable*, which is called an *atomic catlogographic relation* (*ACLR*) and also, redundantly, an *atomic catlogographic*, or *atomic variable*, *catlogographic ordinary relation* (briefly *ACLOR* or *AVCLOR*), and which can assume as its accidental denotatum *either* (a) any one of the three *formal* (*f*-) *veracity-values*: *f-veracity* (*accidental f-truth*), *f-antiveracity* (*accidental f-antitruth* or *accidental f-falsehood*), and *f-vravr-neutrality* (*f-vravr-indeterminacy*), i.e. *neutrality* (*indeterminacy*) *with respect to f-veracity and f-antiveracity*, or (b) a *simple declarative affirmative sentence*, which can be either *materially* (*m*-) *veracious* (*m-veracious, accidentally m-true*, *conformable to a certain fact*) or *m-antiveracious* (*accidentally m-antitrue*, *accidentally m-false*) or else *m-vravr-neutral* (*m-vravr-indeterminate, neither m-veracious nor m-antiveracious*). The former interpretation of an ACLR is called the *formal veracity-valued*, or *veracity-functional, interpretation* and the latter one the *material veracity-valued*, or *veracity-functional, interpretation.* For the sake of definiteness, I shall adopt the former interpretation, unless stated otherwise.

5) The lexigraphs (atomic logographs) defined in the items 2–4 are collectively called the *catlexigraphs* or *atomic catlogographs*, and also the *atomic catlogographic formulas* (*ACLF’s*) or *atomic catlogographic categoremata* (*ACLC’ta*).

6) The set of definitions (8.1)–(8.3) subject to the items 2–4 is called *the conformal catlogographic pre-interpretation of their euautographic definienda*, i.e. of the respective *AEOF’s of* A_1, and also *the conformal catlogographic pre-interpretation of either organon* A_1 or A_1^0. At the same time, the set of definitions (8.3) subject to the item 4 is called *the conformal catlogographic pre-interpretation of* A_0.•

Cmt 8.1. 1) The cumulative rules I_1 of the CCFCL interpretation I_1 and the cumulative rules D_1 of the PCFCL interpretation of A_1, which are based on Df 8.2, will be made explicit in the next subsection. At the same time, Df 8.2 in general and its constituting definitions (8.1)–(8.3) in particular *are not used* either in the setup of A_1 or in execution of the EADP's (euautographic algebraic decision procedures) for ESR's (euatographic slave relations) of A_1, but they explicate the following three fundamental peculiarities of A_1 as a *euautographic* (*genuinely uninterpreted*, *chess-like*) *calculus*.

a) All AEOF's (AEOC'ta), i.e. the EOT's (PVOT's and PCOT's) and AER's (APVOR's), and hence all *combined euautographic formulas* (*CbEF's*), i.e. *combined euautographic formulas* (*CbEC'ta*), are always used autonymously, so that they do not undergo the psychological phenomenon of *TAEXA* (*tychauto-euxenograph alternation*). In this case, all primary *atomic euautographic ordinary syncategoremata* (*AEOSC'ta*), which unite AEOF's to produce *combined euautographic ordinary formulas* (*CbEOF's*), apply to their operata *contactually*, so that they are also used autonymously.

b) Definitions (8.1)–(8.3) signify that the PVOT's on the list (5.1), AER's on the list (5.2), and PCOT's $\emptyset$ and $\emptyset'$ of Ax 5.1(9) are, in a sense, *constants*. I classify the atomic euautographs on the lists (5.1) and (5.2) as *pseudo*-variables and $\emptyset$ and $\emptyset'$ as *pseudo*-constants in accordance with their future significant (semantic) CFCL interpretations.

c) All branches of A_1 share the same EAADM D_1 independent of any subsequent semantic interpretations of the PVOT's and PCOT's that are involved in the relations of A_1.

In addition, owing to definitions (8.1)–(8.3), *euautographic ordinary relations* (*EOR's*) have various pure syntactic properties, which are *homologues* (*precursors*) of certain semantic properties of their CCFCL interpretands and hence of their counterparts occurring in traditional logic or in CALC'i. In order to emphasize the fundamental character of those syntactic properties of EOR's, I shall make them explicit prior laying down the rules of CCFCL interpretation of the EOR's, which turn out to be trivial after all. One of these syntactic properties is discussed below in the next item of this comment. Some other syntactic properties of EOR's will be explicated in Df 8.3 and Cmt 8.2 to follow.

2) Whitehead and Russell [1910; 1962, p. 4] call their variables, being counterparts of those on the lists (8.4) and (8.5), *unrestricted* in accordance with the following definition

> «We may call a variable *restricted* when its values are confined only to some of those of which it is capable; otherwise, we call it *unrestricted*.»

All variables on either list (8.4) or (8.5) have a certain *broad* (*unlimited*, *unconfined*) *range* (*class*) of values, but the two ranges are obviously distinct, so that any value of any variable of the former list is distinct from any value of any variable of the latter list. Therefore, those variables and hence their counterparts of *Principia Mathematica* are *restricted* in this sense. Consequently, the qualifiers "unrestricted" and "restricted" are confusing *misnomers* in this use. Unfortunately, either pair of antonymous qualifiers "unlimited" and "limited" or "confined" and "unconfined", which can be used instead of the pair of antonyms "unrestricted" and "restricted", has the same ambiguity property. Having no choice, I shall therefore, call the variables on the list (8.4) and (8.5) and also their counterparts employed in *Principia Mathematica* and in other writings on logic and mathematics *unrestricted*, *unlimited*, or *unconfined*, while the variables having any *relatively narrower range* will be called *restricted*, *limited*, or *confined*.

At the same time, the PVOT's as defined by (8.1) and the APVOR's as defined by (8.3) are euautographs, and therefore neither of the qualifiers "unrestricted" ("unconfined") and "restricted" ("confined") is applicable to them. Still, in view of the subsequent CFCL interpretations of the PVOT's and APVOR's by the respective *unrestricted conformal* (*analo-homolographic*) VCLOT's and AVCLOR's, the former atomic *pseudo*-variables will be qualified *pseudo-unrestricted*, *pseudo-unlimited*, or *pseudo-unconfined.* Accordingly, atomic *pseudo*-variable euautographs, which are or are designed to be interpreted by restricted atomic variable catlogographs, will be qualified *pseudo-restricted*, *pseudo-limited*, or *pseudo-confined.*•

2) The most immediate and most common CFCL interpretation of a DdER, called the *conservative CFCL* (*CCFCL*) *interpretation of the DdER*, i*s the act of replacing* the DdER, in accordance with certain rules to be explicated below in this section, with its *significant CFCL token*, which is called the *CCFCL interpretand* (pl. "*interpretands*") *of the DdER* or, less explicitly, a *vavn-decided catlogographic*

relation (*CLR*); the DdER, being the *insignificant graphic template* of its CCFCL interpretand, is called the *conformal euautographic* (*CFE*) *interpretans* (pl. "*interpretantia*") *of the CCFCL interpretand*. The class (set) of the CCFCL interpretations of the OptER's of a certain branch of A_1 or of a certain phase of the branch is called the *CCFCL interpretation of that branch* or *of that phase* respectively. Likewise, the totality of the CCFCL interpretations of the OptER's of A_0 is called the *CCFCL interpretation of* A_0. The CCFCL interpretation of an OptER is based on the following definition, which will, for the obvious reason, be called the *CFCL pre-interpretation of the atomic euautographic ordinary formulas* (*AEOF's*), i.e. *atomic euautographic ordinary categoremata* (*AEOC'ta*), *of* A_1, the understanding being that the AEOF's (AEOC'ta) include the APVOT's (PVOT's) on the list (5.1), the APVOR's (AER's) on the list (5.2), and the APCOT's (PCOT's, EOZT's) $\emptyset$ and $\emptyset'$ of Ax 5.1(9).•

Df 8.3. The organon A_1 allows distinguishing between a *secondary substance* that is called a *class* and a *secondary substance* that is called a *mass* in accordance with the following syntactico-semantic properties of their *proper names*, – such names, e.g., as a *logographic constant* or as an unlimited (particularly not articled) *phonographic* (*verbal*) *count*, or *mass, name* (*substantive, noun equivalent*)

1) A necessary and sufficient condition for a substance to be a class or particularly a set, nonempty or empty, is that a *proper name* of the substance can stand in affirmative or negative statements (relations) *to the right of both the class-membership predicate* $\in$ *and the class-inclusion predicate* $\subseteq$, which is conventionally defined in terms of $\in$ (see, e.g. Appendix 4). A substance whose proper name *cannot stand to the right* of either of the above predicates is called a *nonempty individual*. A class theory, in which the empty class is admitted as the only individual, is called a *one-individual class theory*. A class theory, in which many (usually infinitely many) nonempty individuals are *verbally* postulated to exist besides the empty one, is called a *many-individual class theory*.

2) A *mass is not a class* and hence it is not a set. In a mass theory, the *mass-inclusion predicate* $\subseteq$ is postulated to exit and to have certain axiomatic properties in no connection with the class-membership predicate $\in$, which is not introduced. Therefore, the mass-inclusion predicate $\subseteq$ of a mass theory is a *homograph* of the class-inclusion predicate. $\subseteq$ of a class theory. Accordingly, a necessary and sufficient

condition for a substance to be a *mass*, *nonempty or empty*, is that a proper name of the substance can stand *to the right of the mass-inclusion predicate* $\subseteq$. Just as the empty class, the *empty mass* is alternatively called the *empty individual*, whereas a substance whose proper name cannot stand to the right of $\subseteq$ is as before called a *nonempty individual*. However, any mass theory has the empty mass, i.e. empty individual, as its only individual and is therefore a *one-individual mass theory*.•

Cmt 8.2. 1) In the English translations of Aristotle [350 BCE, *Categories*] by Edghill (referred to as [ACE]) and by Owen (referred to as [ACO]) and in studies of that treatise (e.g., in Studtmann [2008]), the terms "*primary substances*" and "*secondary substances*" are used for denoting the *beings* (τᾶ ὄντα \tá ónta\, singular "τό όν" \tó ón\), which are respectively called "*nonempty individuals*" and "*many-member classes*" in the presently common terminology. In analogy with the conventional term "*singleton*", denoting a *one-member class* (*set*), I use the term "*multipleton*" synonymously (interchangeably) with the term "many-member classes" for denoting any class and particularly any set other than a singleton and other than *the empty*, i.e. *memberless*, *class* (*empty set*), called also *the empty individual.* It is understood that a nonempty individual can be an element, i.e. member, of a class, but it cannot be a subclass, i.e. part, of a class. In Aristotelianism, a multipleton of primary substances (nonempty individuals) is called a *species* (*specific class*), whereas the *whole* (*union*) *of all relevant species* is called a *genus* (*general class, superclass*). For instance, according to [ACE, Part 5], Aristotle says:

> «Substance, in the truest and primary and most definite sense of the word, is that which is neither predicable of a subject nor present in a subject; for instance, the individual man or horse. But in a secondary sense those things are called substances within which, as species, the primary substances are included; also those which, as genera, include the species. For instance, the individual man is included in the species 'man', and the genus to which the species belongs is 'animal'; these, therefore – that is to say, the species 'man' and the genus 'animal', – are termed secondary substances.»

The wording of the corresponding passage of [ACO, Chapter V] is very close to the above. The original Aristotle's parasynonym of "substance" is "ή ουσία" \í usía\ *s.f.* (pl. "ουσίαι" \usíe\). English-writing scholars use expressions "predicable of a subject" or "said-of" and "present in a subject" or "present-in", and also their negations "not predicable of a subject" or "not said-of" and "not present in a subject" or "not present-in" as conventional parasynonyms of the corresponding original technical terms of Aristotle. Unfortunately, Aristotle does not define the term "predicable of a subject" at all, while he defines the term "present in a subject" persuasively and circularly as follows [ACE, Part 2]:

> «By being 'present in a subject' I do not mean present as parts are present in a whole, but being incapable of existence apart from the said subject.»

The property that, in accordance with this obscure definition, some universals are present in primary substances is probably the reason why some contemporary scholars interpret Aristotle's world outlook as *moderate realism* (see, e.g., the section «*Platonic and Aristotelian realism*» of the article «**universal (***logic***)**» in BOE (Britannica.com). But I cannot put my finger on any specific passage of Aristotle's works, where his "present-in"-term is explicitly used as an argument against Plato's doctrine of transcendental *real* Universes, i.e. against Plato's *extreme realism*. Moreover, some other contemporary scholars interpret Aristotle's world outlook as *extreme nominalism* (see, e.g., Durant [1926, pp. 48–49] or *Editorial note* in the article «**ontology**» of Allen [2003]). In this case, it remains unclear how the fact that *multipletons,* i.e. *many-member classes,* such as *animal, man, book, tree,* etc, which are, according to Aristotle's nominalism, seated in the mind (cerebral cortex) of a concrete man (sapient subject), can be reconciled with the property of those same multipletons to *be present in some primary substances*.

3) In order to describe concisely both multipletons as *mental* (*psychical*) *entities,* i.e. as *brain,* or *cortical, symbols, of a particular man* (sapient subject) and the cerebral cortex of the man as *their seat*, a multipleton will alternatively be called a *nominal*, or *conceptual, essential universal*, and also briefly an *essential nom-universal* or *essential couniversal* (in analogy with "*coentity*") or just *eucouniversal* [*of the man*]. By contrast, a singleton or the empty class (empty individual) will indiscriminately be called a *nominal*, or *conceptual, essential particular,* and also briefly *essential nom-particular* or *essential coparticular* or just *eucoparticular* [*of the man*]. In a native language, e.g. in English, a multipleton is or can be *denoted by* an [*unlimited*] *count name* of its members (elements) and therefore it is in this case called the *denotatum* (*denotation value, intended import valu*e, pl. "*denotata*") *of that name*, while the [unlimited] count name, which is used for mentally putting the

multipleton, being one of its import values, forward as its unique denotatum (intended import value), becomes a *proper name of the multipleton* that is less explicitly called a *proper multipleton-name.* The qualifier "*unlimited*" means «*having no limiting modifier such as a quantifier* (*numeral*) *or particularly an article or such as a predicate*». The denotatum of a proper multipleton-name is an *object sui generis* and therefore it automatically, *ipso facto*, produces the *singleton of its own*, which becomes another import value of the proper multipleton-name – the value that is said to be *connoted* by the latter and that is accordingly called the *singleton-connotatum* (*connotation value*, *concomitant import valu*e, pl. "*connotata*"), or impartially *designatum* (*designation value*, pl. "*designata*"), of the proper multipleton-name. In this case, I *use* the proper multipleton-name along with its singleton-connotatum *for mentioning* (*denoting*, *putting forward*) its denotatum, while both the proper multipleton-name and its singleton-connotatum *are used but not mentioned*.

4) Besides multipletons, a properly schooled man (sapient subject) has nominal, or conceptual universals, which are or can be denoted in native English or in any other native language by *mass names* and which will therefore be called *nominal*, or *conceptual*, *masses* and also *concept-masses* or briefly *comasses*. Specifically, a comass will be called:

a) a *universal comass* or briefly *u-comass* and also a *nominal*, or *conceptual*, *accidental universal* or briefly an *accidental nom-universal* or *accidental couniversal* or just *a tychcouniversal* if it is denoted by an *unlimited mass-name*, i.e. by a mass name having no *limiting modifier* (cf. the item 3);

b) a *common particular comass* or briefly *common p-comass* and also a *common nominal*, or *conceptual*, *accidental particular* or briefly a *common accidental nom-particular,* or *coparticular,* or just *common tychcoparticular* if it is denoted by a *limited mass name* having any one of the prepositive *unspecific mass quantifiers* such as: "*some*", "*much*", "*a lot of*", "*a little of*", or "*plenty of*" (e.g. "some water", "much money", "a lot of time", "a little of space", "plenty of trouble", etc).

An unlimited mass name that is used for *denoting* (*mentioning*, *putting forward as its intended import*) a certain *u-comass* (*tychcouniversal*), indicated in the above point a, is called a *proper name of the latter* or, less explicitly, a *proper u-comass-name* or *proper tychcouniversal-name.* A limited mass name that is used for *denoting* (*mentioning*, *putting forward as its intended import*) a *common p-comass* (*tychcoparticular*), indicated in the above point b, is called a *common name of a part of the u-mass* (*tychcouniversal*). A limited name consisting of an unlimited mass name and a prepositive *specific mass quantifier* (*possessive dimensional numeral*) such as "*a barrel of*", "*two bottles of*", "*three cups of*", etc (applied, e.g., to "oil", "water", "juice", "wine", etc) is a *limited proper name of the common member of the respective dimensional number* (e.g., one barrel of oil, two barrels of oil, three barrels of oil, etc).

5) The organon A_1 allows distinguishing formally (axiomatically) between classes (including sets) and comasses. Namely, the *primary predicate of a class theory* is the *class-membership predicate* $\in$, whereas the *lax class-inclusion predicate* $\subseteq$, the *equality-predicate* =, and the *strict class-inclusion predicate* $\subset$ are *secondary predicates* that are defined in terms of $\in$; the *primary predicate of comass theory* is the *lax comass-inclusion predicate* $\subseteq$, homographic with the lax class-inclusion predicate, whereas the *equality-predicate* = and the *strict comass-inclusion predicate* $\subset$ are *secondary predicates* that are defined in terms of $\subseteq$. In this case, there exists *the empty comass, which is identical with the empty class, i.e. with the empty individual*. A comass that is not empty is called a *nonempty comass*. Therefore, I divide the Aristotelian subcategory of *secondary substances* into two distinct narrower subcategories: *multipletons* (*many-member classes*) and *nonempty comasses* (*conceptual masses, concept-masses*).

6) In accordance with the terminology introduced in the above items 3 and 4a, a multipleton or a u-comass is indiscriminately called a *nominal*, or *conceptual*, *universal*, and also briefly a *nom-universal* or *couniversal* (in analogy with "*coentity*") [*of the man*]. In this case, I postulate that some nom-universals have some *real* counterparts, which are called *real universals* or *res-universals* and which are located in nonempty individuals (primary substances), in accordance with the *doctrine of immanence* that is credited to Aristotle. This postulate solves the problem of universals.

7) Names of nonempty individuals, i.e. of primary substances in Aristotelian coinage, are rejected in Aristotelian logic (see, e.g., Łukasiewicz [1951] and Lamontagne and Woo [2008]). Aristotelian logic is often introduced by stating the following argument as a typical example of categorical syllogisms:

«All men are mortal, Socrates is a man, therefore Socrates is mortal.»

This argument is not, however, an Aristotelian syllogism. An appropriate example of categorical syllogisms would be the following:

> «All men are mortal [beings], all Greeks are men, therefore all Greeks are mortal [beings].»

The reason for excluding primary substances, called also *individual subjects* or *singular terms*, in Aristotelian syllogistics is that subjects and predicatives ("predicates" in the Aristotelian terminology) must be exchangeable in the sense that the subject of one proposition (judgment) can be the predicative of another proposition and vice versa. But a primary substance cannot be predicated of ("said-of") any other substance, and therefore it is not admissible in Aristotelian syllogistics.

8) In some writings on logic and mathematics, either term "*null class*" or "*null-class*", or, when applicable, "*null set*" or "*null-set*", is used interchangeably (synonymously) with either of the synonymous terms "*empty class*", or, correspondingly, "*empty set*", and "*empty individual*" (see, e.g., Fraenkel et al [1973, pp. 24, 39, 124]). In this case, in order to simplify the terminology, the term "nonempty individual" is abbreviated by omission of the qualifier "nonempty", so that the noun "*individual*" alone becomes an antonym of either noun "class" or "set". Consequently, the term "empty individual" is disregarded (not used), while the synonymous terms "empty class" and "null class", or, when applicable, "empty set" and "null set", are used instead. I do not adopt the above simplified terminology, i.e. I shall not use the names "null class" ("null-class") and "null set" ("null-set") as synonyms of "empty class" and "empty set" respectively and I shall not use the noun "individual" as an abbreviation of the name "nonempty individual", for the reason that is explained in the item 3 of Cmt A4.1. For convenience in this discussion, the subject matter of that item is summarized below from the pertinent viewpoint.

9) Given a full-scale *axiomatic set theory* (*AST*) to be denoted by 'S', let '$\varnothing$' denote the empty set of S and let '$\mathcal{N}$', '$\mathcal{I}$', '$\mathcal{Q}$', '$\mathcal{R}$', and '$\mathcal{C}$' denote the sets of *natural*, *integer* (*integral*), *rational*, *real*, and *complex numbers* (in that order), which are defined in the framework of S or which can be derived from S. Let the *indexed zeros* '0_N', '0_I', '0_Q', '0_R', and '0_C' denote the *null elements*, i.e. *null numbers*, of the sets $\mathcal{N}$, $\mathcal{I}$, $\mathcal{Q}$, $\mathcal{R}$, and $\mathcal{C}$ respectively. In this case, $0_N=\varnothing$, while 0_I, 0_Q, 0_R, and 0_C turn out to be mutually different *nonempty sets*, and hence they are *not individuals*, either empty or nonempty (see, e.g., Feferman [1964] or Burrill [1967]). The set 0_N, 0_I, 0_Q, 0_R, or 0_C

can be called *the natural, integer, rational, real*, or *complex null-set* respectively and similarly with "*null set*" in place of "*null-set*". Consequently, any one of the sets 0_I, 0_Q, 0_R, and 0_C can indiscriminately be called a *nonempty null-set*, while 0_N, i.e. ∅, should, by way of emphatic comparison with the above term, be called *the empty null-set*. Thus, a null-set can be either empty or nonempty. That is to say, the term "null-set" alone, without the appropriate one of the prepositive qualifiers "empty" and "nonempty", is not a synonym of the terms "empty set" and "empty individual". Also, the impartial term "void" is not applicable to 0_I, 0_Q, 0_R, and 0_C, so that the terms "null" ("nil", nothing") and "void" are not synonyms.

10) An entity (being) and particularly a substance is called a *sensible*, or *real*, or *physical*, *one* and also a *thing* or *concretum* (pl. "*concreta*") if it is known from sensations and an *insensible*, or *ideal*, or *psychical* (*mental*) *one* and also an *abstracttum* (pl. "*abstracta*") if it is known by *prescinding* it from physical entities or from other psychical entities prescinded earlier. In this case, the adjectives "divisible" and "indivisible" are relative and epistemologically relativistic antonyms, whose senses (sense values) depend on the, or a, denotatum of a substantive name (a noun or noun equivalent), to which they apply as qualifiers, and on the mental attitude of the sapient subject towards the denotatum. In any case, criteria of divisibility and indivisibility of concreta and those of abstracta are different. Particularly, a concretum (physical entity) is a necessarily a *nonempty one* that is called a *physical indivisible entity* or a *physical individual* if it preserves its recognizable identity only as a single whole and a *divisible physical entity* if it is regarded as one that consists of a several physical individuals, which are or can be separated from one another physically. For instance, a concrete sensible *higher* (*conscious*) *biont* (*living organism*), i.e. one having a *central nervous system* (*CNS*), especially a concrete man is commonly regarded as a [nonempty] individual. At the same time, in biology or anatomy, for convenience in description and study, a *common* (*general*) *member* of the species *man* (*Homo sapiens*), which is just another, *as if extramental* (*as if exopsychical*), *hypostasis of the species*, is divided into systems of organs, an organ is divided into tissues and cells, while a cell is divided into constituent parts of its own. Analogously, a concrete combined ER (euautographic relation) of A_1 is an individual because it preserves its recognizable identity only as a single whole. Particularly, the ER has its unique property to be valid or antivalid or vav-neutral, but not any two

simultaneously, only as a single whole. On the other hand, the ER can, when applicable and desired, be analyzed (dissected, divided) into the principal operator and its operatum or operata. If an operatum is an ER itself then it has a certain validity value of its own, which is independent of the validity value of its host ER. After all, the initial ER can always be analyzed into atomic euautographs.

11) In accordance with Df 8.2, the adjectives “divisible” and “indivisible” apply to a substance in the following rigorous narrow sense. A substance is said to be *divisible* if and only if it is either a nonempty class or a nonempty mass and it is said to be *indivisible* and also to be an *individual* if and only if it is either the empty individual, i.e. the empty class or the empty mass, or a substance that is neither a class nor a mass. In accordance with the above criterion, a *sensible* (*physical*) substance is neither a class nor a mass and hence it is a nonempty individual. If however the substance in question is *insensible* (*psychical*, *mental*) then it is necessary to decide from some independent considerations whether it is a class or mass, empty or not, or whether it is a nonempty individual, i.e. neither a class nor a mass. Consequently, the above criterion becomes circular. The decision whether a given insensible substance is a class or mass or whether it is a nonempty individual is epistemologically relativistic, i.e. it depends on a *domain* to which the substance belongs (cf. Appendix 4). Some examples are given below.

12) A *vector of an abstract* (*not arithmetical*) *n-dimensional linear* (or *vector*) *Euclidean* (or *inner product*) *space*, $\hat{\boldsymbol{\mathcal{E}}}_{(n)}$, *over the field of real numbers*, $\boldsymbol{\mathcal{R}}$, can be regarded as an insensible nonempty individual if the vector space is treated as an autonomous algebraic system. A *point of an n-dimensional affine Euclidean space*, $\dot{\boldsymbol{\mathcal{E}}}_{(n)}$, *over the field of real numbers*, $\boldsymbol{\mathcal{R}}$, is also an insensible nonempty individual. However, $\dot{\boldsymbol{\mathcal{E}}}_{(n)}$ has $\hat{\boldsymbol{\mathcal{E}}}_{(n)}$ as its *adjont vector space* so that, under the definition that $\dot{\mathcal{E}}_{(n)}$ is *the principal underlying set of points of* $\dot{\boldsymbol{\mathcal{E}}}_{(n)}$ and that $\hat{\mathcal{E}}_{(n)}$ is *the principal underlying set of vectors of* $\hat{\boldsymbol{\mathcal{E}}}_{(n)}$, the following two *affine space axioms* (*ASA’s*) hold.

ASA1: *The vector composition law.* There exists a binary composition function $\hat{V}:\dot{\mathcal{E}}_{(n)}\times\dot{\mathcal{E}}_{(n)}\to\hat{\mathcal{E}}_{(n)}$ such that for each $(\dot{x},\dot{y})\in\hat{\mathcal{E}}_{(n)}\times\hat{\mathcal{E}}_{(n)}$, i.e. for every *ordered pair* of points $\dot{x}$ and $\dot{y}$ in $\dot{\mathcal{E}}_{(n)}$, there is exactly one vector $\hat{z}\in\hat{\mathcal{E}}_{(n)}$ such that

$$\hat{z} = \hat{V}_{\dot{x}}(\dot{y}) \doteqdot \hat{V}(\dot{x}, \dot{y}), \tag{8.6}$$

and conversely, for each $\hat{z} \in \hat{\mathcal{E}}_{(n)}$ there is exactly one $\dot{y} \in \dot{\mathcal{E}}_{(n)}$ such that (8.6) holds, i.e.

$$\dot{y} = \hat{V}_{\dot{x}}^{-1}(\hat{z}). \tag{8.7}$$

Thus, for each $\dot{x} \in \dot{\mathcal{E}}_{(n)}$ the singular functions $\hat{V}_{\dot{x}} : \dot{\mathcal{E}}_{(n)} \to \hat{\mathcal{E}}_{(n)}$ and $\hat{V}_{\dot{x}}^{-1} : \hat{\mathcal{E}}_{(n)} \to \dot{\mathcal{E}}_{(n)}$, defined in terms of the binary function $\hat{V}$ by (8.6) and (8.7), are two mutually inverse *bijections*.

ASA2: *The triangle,* or *Chasle, law.*

$$\hat{V}(\dot{x}, \dot{y}) \hat{+} \hat{V}(\dot{y}, \dot{z}) \hat{+} \hat{V}(\dot{z}, \dot{x}) = \hat{0}_{(n)} \text{ for each } ((\dot{x}, \dot{y}), \dot{z}) \in [\dot{\mathcal{E}}_{(n)} \times \dot{\mathcal{E}}_{(n)}] \times \dot{\mathcal{E}}_{(n)}, \tag{8.8}$$

where $\hat{0}_{(n)} \in \hat{\mathcal{E}}_{(n)}$ is *the null-vector*.

At $\dot{z} = \dot{y} = \dot{x}$ or at $\dot{z} = \dot{y}$, it immediately follows from (8.8) that

$$\hat{V}(\dot{x}, \dot{x}) \hat{=} \hat{0}_{(n)} \text{ for each } \dot{x} \in \dot{\mathcal{E}}_{(n)}, \tag{8.9}$$

$$\hat{V}(\dot{x}, \dot{y}) \hat{+} \hat{V}(\dot{y}, \dot{x}) \hat{=} \hat{0}_{(n)} \text{ for each } (\dot{x}, \dot{y}) \in \dot{\mathcal{E}}_{(n)} \times \dot{\mathcal{E}}_{(n)}, \tag{8.10}$$

so that

$$\hat{V}(\dot{y}, \dot{x}) \hat{=} \hat{-}\hat{V}(\dot{x}, \dot{y}), \tag{8.11}$$

where $\hat{-}\hat{V}(\dot{x}, \dot{y})$ is *the additive inverse of* $\hat{V}(\dot{x}, \dot{y})$.

Under the above axioms, for every $\hat{z} \in \hat{\mathcal{E}}_{(n)}$:

$$\hat{z} = \{(\dot{x}, \dot{y}) | (\dot{x}, \dot{y}) \in \dot{\mathcal{E}}_{(n)} \times \dot{\mathcal{E}}_{(n)} \text{ and } \hat{z} = \hat{V}(\dot{x}, \dot{y})\}, \tag{8.12}$$

the understanding being that (8.12) is a *tautology*. Particularly,

$$\hat{0}_{(n)} = \{(\dot{x}, \dot{x}) | \dot{x} \in \dot{\mathcal{E}}_{(n)} \text{ and } \hat{0}_{(n)} = \hat{V}(\dot{x}, \dot{x})\}. \tag{8.13}$$

Thus, every vector of $\hat{\boldsymbol{\mathcal{E}}}_{(n)}$ is a certain set of ordered pairs of points of $\dot{\boldsymbol{\mathcal{E}}}_{(n)}$, so that it is a class, and not a nonempty individual. A vector is alternatively called a *free vector,* because it can be regarded as a translation of $\dot{\boldsymbol{\mathcal{E}}}_{(n)}$. By contrast, an ordered pair $(\dot{x}, \dot{y}) \in \dot{\mathcal{E}}_{(n)} \times \dot{\mathcal{E}}_{(n)}$ is called *the position vector of the point* $\dot{y}$ *relative to the point* $\dot{x}$.

13) Besides the sets of numbers of five kinds, which have been indicated in the item 9 of this comment and which are called *scalars,* and also besides various algebraic systems, of which $\dot{\boldsymbol{\mathcal{E}}}_{(n)}$ indicated in the previous item is one of the most important algebraic systems, mathematics and physics (especially theoretical physics) deal with *hypernumbers* of various kinds (classes) such as quaternions, tensors of

various valences, and matrices. A hypernumber, called also a *holor* (from the Greek adjective "ὅλος" \ólos\ meaning *all* or *the whole*), consists of several numbers of a certain set, most often of real numbers, which are called the *merates* (from the Greek noun "μέρος" \méros\ meaning *a part*), and also *coordinates* or *components*, of the holor (see, e.g. Moon and Spencer [1965, pp. 1, 14]). Particularly, a complex number is in fact a two-component holor of real numbers. A holor is said to be *univalent*, *bivalent*, *trivalent*, *quadrivalent*, etc if its merates are labeled respectively with one, two, three, four, etc subscripts or superscripts. A scalar is alternatively called a *nilvalent holor*. In any *conventional set theory* (*CST*), an *n*-component univalent holor $\overline{x}_{[1,n]}$ of any elements $x_1, x_2, ..., x_n$ of a given set *X* in that order is called an *ordered n-tuple*, or less explicitly, an *ordered multiple*, and it is defined as a *repeated*, (*n*–1)-*fold ordered pair* such that

$$\overline{x}_{[1,n]} \doteqdot (x_1, x_2, ..., x_n) \doteqdot (((....((x_1, x_2), x_3), ...), x_{n-1}), x_n) \in X^{n\times}, \tag{8.14}$$

where $X^{n\times}$ the pertinent *repeated*, (*n*–1)-*fold*, *direct product of X by itself* so that:

$$X^{n\times} \doteqdot \underbrace{[[...[[X \times X] \times X] \times ...] \times X]}_{n}. \tag{8.15}$$

The pairs of round or square brackets occurring in (8.14) or (8.15) respectively are associated to the left, unless stated otherwise. A single (simple) ordered pair (u, v) of elements *u* and *v* of any given sets *U* and *V* in that order is by definition the set $\{\{u\}, \{u, v\}\}$, i.e.

$$(u, v) \doteqdot \{\{u\}, \{u, v\}\} \tag{8.16}$$

(see, e.g., Halmos [1960, pp. 22–25]). Hence, an ordered *n*-tuple, i.e. an *n*-component univalent holor, is a nonempty set and hence it is not a nonempty individual.

14) An orthonormal coordinate system $c_{(n)}$ in $\dot{\mathcal{E}}_{(n)}$ can be defined as:

$$c_{(n)} \doteqdot \left(\dot{o}, \overline{\hat{e}}_{[1,n]}\right) \in \dot{\mathcal{E}}_{(n)} \times \hat{\mathcal{E}}_{(n)}^{n\times}, \tag{8.17}$$

where the point $\dot{o} \in \dot{\mathcal{E}}_{(n)}$ is the origin of $c_{(n)}$, and $\overline{\hat{e}}_{[1,n]} \doteqdot (\hat{e}_1, \hat{e}_2, ..., \hat{e}_n) \in \hat{\mathcal{E}}_{(n)}^{n\times}$ is the ordered *n*-tuple of orthonormal (normal orthogonal) basis vectors. Given $c_{(n)}$, any point $\dot{x} \in \dot{\mathcal{E}}_{(n)}$ is uniquely determined by the ordered *n*-tuple of its coordinates $\overline{x}_{[1,n]} \doteqdot (x_1, x_2, ..., x_n) \in \mathcal{R}^{n\times}$ relative to $c_{(n)}$, and any vector $\hat{x} \in \hat{\mathcal{E}}_{(n)}$ such that $\hat{x} = \hat{V}(\dot{o}, \dot{x})$ is uniquely determined by the same ordered *n*-tuple of its components relative to that same $c_{(n)}$, i.e. relative to $\bar{\hat{e}}_{[1,n]}$. Therefore, the ordered *n*-tuple $\bar{x}_{[1,n]} \doteqdot (x_1, x_2, ..., x_n) \in \mathcal{R}^{n\times}$ is called an *arithmetical n-point* and also an *arithmetical n-vector*. Accordingly, the *n*-dimensional affine Euclidean space, $\bar{\mathcal{E}}_{(n)}$, over the field of real numbers, $\mathcal{R}$, whose underlying set of points $\bar{E}_{(n)}$ is defined as $\bar{E}_{(n)} \doteqdot \mathcal{R}^{n\times}$ (subject to (8.15) with '$\mathcal{R}$' in place of '*X*') and which is at the same time *the adjoint vector space of itself*, is called *the arithmetical n-dimensional affine Euclidean space* and also *the arithmetical n-dimensional vector* (or *linear*) *Euclidean* (or *inner product*) *space*. An *n*-dimensional affine or linear Euclidean space, $\dot{\mathcal{E}}_{(n)}$ or $\hat{\mathcal{E}}_{(n)}$, over the field of real numbers, $\mathcal{R}$ is called an *abstract* one if it is *not arithmetical*. Thus, a point of $\bar{\mathcal{E}}_{(n)}$, i.e. an arithmetical *n*-point, being at the same time its vector, i.e. an arithmetical *n*-vector, is a nonempty set and hence it is not a nonempty individual.

15) There are in mathematics and theoretical physics a great many of various definitions of the terms "scalar" and "vector" (see, e.g. Moon and Spencer [1965, pp. 317–322]). Depending on a definition, one should decide whether a substance in the range of either term is a class (set) or a nonempty individual.

16) A circumference drawn on paper with the help of a pair of compasses is a physical individual. In the domain of Euclid geometry, an imaginary (abstract) circumference of a given radius is a nonempty individual, whereas in the domain of a set theory the same circumference is regarded as a set of points equidistant from the point called its center, so that it is a divisible substance.

17) In order to compare the nomenclature that has been introduced in Df 8.1 and some nomenclatures that are employed elsewhere in logic and mathematics, I shall assume, unless stated otherwise, that the VCLOT's on the list (8.4) and the CCLOT '∅' introduced in Df 8.1(2) are class-valued, and not mass-valued. In this case, the VCLOT's on the list (8.4) and the ACLR's on the list (8.5) are *counterparts*, i.e. either *tokens* or *tantamount variants*, of the respective *atomic variables* (to use the appropriate term of this treatise), which are employed in all CALC'i, e.g. in Whitehead and Russell [1910; 1962, pp. 4, 5], Hilbert and Ackermann [1950, p. 65], and Church [1956, pp. 69, 169]. At the same time, the VCLOT's on the list (8.4) and the CLOZT '∅' introduced in Df 8.1(2) are counterparts respectively of the *atomic set-valued variables* and of the *empty-set-valued constants*, which are employed in

conventional set theories (*CST's*), *axiomatic ones* (*CAST's*) such as either of the two Zermelo–Fraenkel systems of set theory, namely that without and that with the axiom of choice, denoted by 'ZF' and by 'ZFC' respectively (see e.g. Fraenkel et al [1973, pp. 20–25, 39]), or *naïve* (*quasi-axiomatic*) *ones* (*CNST's*) as that of Halmos [1960, pp. 2, 8]. The empty class (empty set) is most often denoted by '∅' (e.g. in Halmos [1960, p. 8]) or sometimes by 'O' (e.g. in Fraenkel et al [1973, p. 39]). In Whitehead and Russell [1910; 1925; 1962, pp. 216, 217, **∗24.02**], the empty class is denoted by 'Λ'. A proper logographic name of the empty class, as '∅' or 'O', is called a *zero*, but not necessarily vice versa. The VCLOT's on the list (8.4) can also be regarded as counterparts of the variables, which are employed as *terms* of premises and conclusions of Aristotelian categorical syllogisms (see, e.g., Łukasiewicz [1951], Hilbert and Ackermann [1950, chap. II], and Lamontagne and Woo [2008]). The nomenclature of this treatise versus the nomenclatures of CALC'i and CST's will be discussed in section I.9.•

8.2. The conservative conformal catlogographic (CCFCL) interpretation of A_1

†**Ax 8.1:** ***The rules of CCFCL interpretations of ExIEF's of*** A_1. 1) The act of replacing all occurrences of AEOF's, i.e. of EOT's (PVOT's and PCOT's) and AER's (AEOR's), throughout a given *ExIEF* (*externally*, or *extrinsically*, *interpretable euautographic formula*) *of* A_1 with occurrences of *the interiors of the HAQ's* that serve as the definientia of the pertinent ones of definitions (8.1)–(8.3) is called *the conservative conformal catlogographic* (*CCFCL*) *interpretation of the ExIEF*. The formula resulted by the CCFCL interpretation of an ExIEF is called *the CCFCL interpretand of the ExIEF* or, less explicitly, *a catlogographic formula* (*CLF*), whereas *the ExIEF thus interpreted* is called *the conformal*, or *template*, *euautographic* (*CFE* or *CFT*) *interpretans* (pl. "*interpretantia*") *of the CCFCL interpretand.*

2) In accordance with Df 8.1(5), the above item applies particularly with "*EsExIEF*" in place of "*ExIEF*". At the same time, in accordance with Df 8.1(6), the only application of the above item that may have any academic or practical interest is that with "*OptER*", i.e specifically with "*valid EOR*", "*vav-neutral EOR*", or "*EMT* (*EDT*) *for a vav-neutral EOR*", in place of "ExIEF". In this case, the CCFCL interpretand of a *valid* or *vav-neutral EOR of* A_1 is respectively a *valid* or *vav-neutral*

CLR, whereas the CCFCL interpretand of *the EMTh* (*EDTh*) *for a vav-neutral EOR of* A_1 is *the CCFCL EMTh* (*EDTh*) *for the CCFCL interpretand of the vav-neutral EOR of of* A_1. It is understood that the above statement applies particularly in the case, where the *vav-neutral EOR* in question is *any AER's of the list (5.2)*, standing alone.

3) In accordance with the above two items, *the cumulative rule of the CCFCL interpretation of any ExIEF of* A_1 *in general and of any OptER of* A_1 *in particular comprises the pertinent ones of the following substitutions*

$$u \mapsto u,\ v \mapsto v,\ w \mapsto w,\ x \mapsto x,\ y \mapsto y,\ z \mapsto z, \tag{8.18}$$

$$\emptyset \mapsto \varnothing,\ \emptyset' \mapsto \varnothing', \tag{8.19}$$

$$p \mapsto p,\ q \mapsto q,\ r \mapsto r,\ s \mapsto s, \tag{8.20}$$

without any quotation marks, throughout the ExIEF or throughout the OptER respectively. The substitutions (8.18) and (8.20) should be understood both as ones for the index-free PVOT's and AER's (APVOR's) and for the base letters of the indexed PVOT's and AER's. For instance, $u \mapsto u$ means, not only the substitution of the catlexigraph *u* for the PVOT *u*, but it also implies the substitutions: $u_1 \mapsto u_1$, $u_2 \mapsto u_2$, etc. Each of the letters *u* to *z* and *p* to *s* is a homolograph of this *Light-Faced Italic Arial Narrow Type*, while each of the letters *u* to z and *p* to s is a homolograph of this *Light-Faced Italic Times New Roman Type*. In accordance with Df 8.1(7), any one of the former ten letters and the respective one of the latter ten letters are *conformal*, or *analo-homolographic, isotokens* of each other. Like remarks apply, *mutatis mutandis*, to the pasigraphs $\emptyset$ and $\varnothing$. Consequently, the substitutions (8.18)–(8.20), and generally similar substitutions in the sequel, are said to be *conformal* or *analo-homolographic* in the sense that in this case a *homolograph of one type is replaced with an analographic homolograph of the other type*.

4) The rules (8.18)–(8.20) concern only with *the CCFCL interpretations and CCFCL interpretands of EOT's and AER's*, i.e. *of AEOF's* (*atomic euautographic ordinary formulas*), called also *AEOC'ta* (*atomic euautographic ordinary categoremata*). Accordingly, the statement of those and only those rules implies to *tacitly* include the *following additional rule*. Any atomic euautographs other than the AEOF's, which may occur in an ExIEF, – namely any one of the *ten integronic digits from* 0 *to* 9 in this type and any *syncategorematic euautographs*, i.e. any primary or secondary *euautographic kernel-signs* (*EKS's*), including *V*, or any primary *atomic punctuation marks*, *– remain syntactically unaltered under the CCFCL interpretation*

of the ExIEF. At the same time, a syntactically unaltered isotoken (occurrence) of an *euautographic ordinary kernel-sign* (*EOKS's*) in the CCFCL interpretand of the ExIEF, in which at least one of its operata is either a CLOT (catlogographic ordinary term) or an ACLR (atomic catlogographic relation), i.e. either the CCFCL interpretand of a certain EOT or that of a certain AER, is *altered* (*interpreted*) *semantically* (*mentally, psychically*). Indeed, in this case, the isotoken of the EOKS *contactually* applies to every one of its atomic catlogographic operata (CLOT's or ACLR's) and thereby it *slidingly* (*transitively*) applies to the *xenonymous denotata* of those catlogographic operata. By contrast, an occurrence of an EKS in the CCFCL interpretand of the ExIEF is *semantically* (*mentally, psychically*) *uninterpreted* (*unaltered*) if it is an occurrence of an EOKS, in which all its operata are *euautographic special relations* (*ESpR's*), or if it is any occurrence whatever of a *euautographic special kernel-sign* (*ESpKS*), all operata of which are always EI's (ESpT's). Therefore particularly, *the CCFCL interpretand of any EMTh (EDTh) remains semantically uninterpreted*.

5) The conjunction of the *rules of substitution* (8.18)–(8.20) throughout ExIEF's of A_1, along with *the tacit rule of preserving the decimal digital integrons* (*DDI's*) *and the euautographic syncategoremata*, as stated in the previous item, are denoted by 'I_1' and are called *the rules of CCFCL interpretation of the ExIEF's* A_1 or *the CCFCL interpretation of the ExIEF's of* A_1 *in intension*. The above statement applies with 'A_1^0' in place of 'A_1'. The conjunction of the rules (8.20) alone, along with the above-mentioned tacit rule, are denoted by 'I_0' and are called *the rules of CCFCL interpretation of the ExIEF's of* A_0 or *the CCFCL interpretation of the ExIEF's of* A_0 *in intension*. The cumulative rule I_0 is also called the *basic* rule of CCFCL interpretation of the ExIEF's of A_1, whereas the cumulative rule I_1 is in this case qualified *advanced*.

6) In accordance with Df 8.1(6), I shall hereafter assume that the rule I_1 applies only to the OptER's of A_1 and hence to those of A_1^0, so that $I_1^0 = I_1$, and that the rule I_0 applies only to the OptER's of A_0, unless stated otherwise. As commonly done in mathematics, the rules (functions) thus restricted will equivocally (homonymously, homographically) be denoted by the same logographs 'I_1' and 'I_0' respectively. In this case, the class of the CCFCL interpretands of all OptER's of A_1, constituting the class $\dot{\mathsf{R}}_1$, is denoted by '$\dot{R}_1$' and by '$I_1(\dot{\mathsf{R}}_1)$'; the class of the CCFCL interpretands of all

valid, or all *vav-neutral*, *output euautographic ordinary relations* (*OptEOR's*) *of* A_1, constituting the class $\mathsf{R}^{\mathrm{O}}_{1+}$, or correspondingly $\mathsf{R}^{\mathrm{O}}_{1\sim}$, is denoted respectively by 'R^{O}_{1+}' and '$I_1(\mathsf{R}^{\mathrm{O}}_{1+})$', or by '$R^{\mathrm{O}}_{1\sim}$' and '$I_1(\mathsf{R}^{\mathrm{O}}_{1\sim})$'; the class of the CCFCL interpretands of *all EMTh's* (*EDTh's*), i.e. of *all valid output euautographic special relations* (*OptESpR's*) *of* A_1, oconstituting the class $\mathsf{R}^{\mathrm{O}}_{1+}$, is denoted by '$R^{\mathrm{O}\sim}_{1\oplus}$' and '$I_1(\mathsf{R}^{\mathrm{O}\sim}_{1\oplus})$'; hence,

$$\dot{\mathsf{R}}_1 = \left[\mathsf{R}^{\mathrm{O}}_{1+} \cup \left[\mathsf{R}^{\mathrm{O}}_{1\sim} \cup \mathsf{R}^{\mathrm{O}\sim}_{1\oplus}\right]\right],\ \dot{R}_1 = \left[R^{\mathrm{O}}_{1+} \cup \left[R^{\mathrm{O}}_{1\sim} \cup R^{\mathrm{O}\sim}_{1\oplus}\right]\right], \tag{8.21}$$

$$\dot{R}_1 = I_1(\dot{\mathsf{R}}_1),\ R^{\mathrm{O}}_{1+} = I_1(\mathsf{R}^{\mathrm{O}}_{1+}),\ R^{\mathrm{O}}_{1\sim} = I_1(\mathsf{R}^{\mathrm{O}}_{1\sim}),\ R^{\mathrm{O}\sim}_{1\oplus} = I_1(\mathsf{R}^{\mathrm{O}\sim}_{1\oplus}); \tag{8.22}$$

and similarly with either one of the two indices '$_0$' and '${}^{0}_{1}$' in place of '$_1$'. The class of the CCFCL interpretands of all OptER's of a certain preselected branch of A_1 or that of a certain phase of the branch is called *the CCFCL interpretand of that branch or of that phase*.

8) An EOR (euautographic ordinary relation) may involve only some EOKS's, and no ESpKS's, the understanding being that all operata of every isotoken of any EOKS occurring in any given EOR are EOT's or EOR's. Therefore, in accordance with the item 4, the CCFCL interpretand of any valid or vav-neutral OptEOR of A_1 is *semantically* (*mentally, psychically*) *interpreted*, whereas the CCFCL interpretand of the EMTh (EDTh) of any vav-neutral OptEOR of A_1 is *semantically* (*mentally, psychically*) *uninterpreted.*

9) The class (set) of the acts of CCFCL interpretations of the OptER's of A_1, i.e. the class of ordered pairs of an OptER of A_1 in $\dot{\mathsf{R}}_1$ and its CCFCL interpretand in $\dot{R}_1$, is denoted by 'I_1' and is called the C*CFCL interpretation of* A_1 [*in extension*]. An OptER of A_1 is alternatively called an *input ER* (*IptER*) *of* I_1, whereas *its CCFCL interpretand* is alternatively called an *output catlogographic relation* (*OptCLR*) *of* I_1 or less explicitly a *concervative catlogographic relation* (*CCLR*). A CCLR is said to be an *ordinary CCLR* (briefly *CCLOR*), if it is the CCFCL interpretand of a certain *valid* or *vav-neutral OptEOR of* A_1 in $\mathsf{R}^{\mathrm{O}}_{1+}$ or $\mathsf{R}^{\mathrm{O}}_{1\sim}$ respectively, and a *special CCLR* (briefly *CCLSpR*) and also a *conservative catlogographic master theorem* (briefly *CCLMTh*) or a ditto *decision theorem* (briefly *CCLDTh*), if it is the CCFCL interpretand of a certain EMTh (EDTh) in $\mathsf{R}^{\mathrm{O}\sim}_{1\oplus}$. In accordance with the pertinent conventional terminology of the theory of sets of ordered pairs, the CCFCL

interpretation of an OptER of A_1, i.e. the ordered pair of the OptER and its CCFCL interpretand, which belongs to I_1, is called *the cut of* I_1 *at the OptER of* A_1 or, less explicitly, a *cut of* I_1. (cf. Bourbaki [1960, chap. II, §3, 1]). The variants of the above definitions with either one of the two indices '${}^0_1$' and '${}_0$' in place of '${}_1$' in all occurrences applies verbatim, with the understanding that $\mathsf{I}_0 \subset \mathsf{I}_1^0 \subset \mathsf{I}_1$.

10) A CCLR preserves the validity-value of its CFE interpretans and it is therefore alternatively called a *vavn-decided CLR*. At the same time, with allowance for the mental (psychical) *significand* (*signification value*) that a CCLOR assumes, the latter is alternatively said to be *tautologous* or *tautological* (*universally true*) if its CFE interpretans is *valid* (*kyrologous*), and *ttatt-neutral* or *ttatt-indeterminate* (*neutral*, or *indeterminate, with respect to tautologousness and antitautologousness*; *neither tautologous nor antitatologous*), if its CFE interpretans is *vav-neutral* or *vav-indeterminat*, i.e. is *kak-neutral* or *kak-indeterminate* (*neutral*, or *indeterminate, with respect to kyrologousness and antitkyrologousness*, *neither kyrologous nor antikyrologous*) respectively. That is to say, in addition to or instead of its *inherent validity value validity* or *vav-neutrality*, a CCLOR assumes exactly one respective *tautologousness-value tautologousness* (*universal truth*) or *ttatt-neutrality* (*neither tautologousness nor antitatologousness*) – namely that one, which is *inclusive of* and hence *compatible with its validity-value*. It is understood that the negation of a tautologous CCLOR is by definition an *antitautologous* (*universally antitrue*, *universally false*, *contradictory*) *CCLOR*, although this is not defined as one of the OptCLR's of I_1. Accordingly, an antitautologous CCLOR has the *tautologousness-value antitautologousness* (*universal antitruth*, *contradictoriness*). By contrast, a CCLSpR, i.e. the CCFCL interpretand of an EMTh (EDTh), *accepts* (*exports*) *the validity-value validity of its CFE interpretans without any change.*

11) In accordance with the previous item, the CCFCL interpretation of A_1 results in assigning the respective one of the two tautologousness-values tautologousness and ttatt-neutrality to the CCFCL interpretand of an OptEOR and in transferring the validity-value validity of EMTh (EDTh) to its CCFCL interpretand. In reference to this property, I_1 is alternatively called the *Conformal Catlogographic Interpretational Decision Method* (*CFCLIDM*) [*for the OptER's*] *of* A_1, whereas I_0 is alternatively called the *CFCLIDM* [*for the OptER's*] *of* A_0 and also the *Basic CFCLIDM* (*BCFCLIDM*) [*for the OptER's*] *of* A_1. By way of emphatic comparison

with the last name, I_1 is redundantly called the *Advanced CFCLIDM* [*for the OptER's*] *of* A_1. In agreement with the above alternative name of I_1, the CCFCL interpretation of an OptER is alternatively called the *conformal catlogographic interpretational decision procedure* (*CFCLIDP*) *for the OptER* or, less explicitly, *a* CFCLIDP of I_1. A CFCLIDP is called an *advanced* one (*ACFCLIDP*) if it involves the rules (8.18) or (8.19) or both and perhaps the rules (8.20) and a *basic* one (*BCFCLIDP*) if it involves the rules (8.20) only.•

8.3. Progressive conformal catlogographic (PCFCL) interpretations of A_1

Df 8.4. 1) Let '$\mathbf{P}_+$' be a *panlogographic placeholder* (*PLPH*) whose range is the class $\mathsf{R}^{\mathrm{O}}_{1+}$ of *valid* IptEOR's of I_1 (OptEOR's of A_1) and let '$\mathbf{P}_\sim$' be a PLPH whose range is the class $\mathsf{R}^{\mathrm{O}}_{1\sim}$ of *vav-neutral* IptEOR's of I_1 (OptEOR's of A_1), whereas '$\mathsf{T}_{1\sim}(\mathbf{P}_\sim)$' is a PLPH whose range is the class $\mathsf{R}^{\mathrm{O}\sim}_{1\oplus}$ of *EMTh's* (*EDTh's*) *for all* $\mathbf{P}_\sim \in \mathsf{R}^{\mathrm{O}}_{1\sim}$, i.e. *for all vav-neutral IptEOR's of* I_1, the understanding being that such an EMTh is an *IptESpR of* I_1 (*OptESpR of* A_1) and vice versa. Let '$\boldsymbol{P}_+$', '$\boldsymbol{P}_\sim$', and '$\mathsf{T}_{1\sim}(\boldsymbol{P}_\sim)$' be *pancatlogographic placeholders* (*PCLPH's*) for the respective *OptCCLR's of* I_1 in the classes R^{O}_{1+}, $R^{\mathrm{O}}_{1\sim}$, and $R^{\mathrm{O}\sim}_{1\oplus}$ respectively, so that

$$\boldsymbol{P}_+ = I_1(\mathbf{P}_+),\ \boldsymbol{P}_\sim = I_1(\mathbf{P}_\sim),\ \mathsf{T}_{1\sim}(\boldsymbol{P}_\sim) = I_1(\mathsf{T}_{1\sim}(\mathbf{P}_\sim)) = \mathsf{T}_{1\sim}(I_1(\mathbf{P}_\sim)) \tag{8.23}$$

subject to $\mathsf{T}_{1\sim}(\mathbf{P}_\sim) \to [V(\mathbf{P}_\sim) \hat{=} \mathbf{i}|\mathbf{P}_\sim\rangle]$ and $\mathsf{T}_{1\sim}(\boldsymbol{P}_\sim) \to [V(\boldsymbol{P}_\sim) \hat{=} \mathbf{i}|\boldsymbol{P}_\sim\rangle]$.

Let D_1 be the CCFCL interpretand of D_1, so that formally $D_1 = I_1(\mathsf{D}_1)$. The set of rules that is denoted by 'D_1' is called the *catlogographic advanced algebraic decision method* (*CLAADM*). Since $\boldsymbol{P}_+$ is by definition a tautologous and hence valid CCLR, therefore it cannot be modified either syntactically or semantically. By contrast, $\boldsymbol{P}_\sim$ is a ttatt-neutral CCLR and it can therefore either remain *unattended* (*suspended*) or be treated in one of the following two alternative ways.

i) $\boldsymbol{P}_\sim$ can *be postulated to be veracious* (*atautologously*, or *accidentally*, *true*). As a result, $\boldsymbol{P}_\sim$ turns into a *veracious catlogographic slave postulate*, $\boldsymbol{P}^{\mathrm{p}}_{\sim+}$, which satisfies the *progressive*, or *transformative, conformal catlogographic* (*PCFCL* or *TFCFCL*) *master postulate* $V(\boldsymbol{P}^{\mathrm{p}}_{\sim+}) \hat{=} 0$ instead of the *CCFCL MTh* (*DTh*) $\mathsf{T}_{1\sim}(\boldsymbol{P}^{\mathrm{p}}_{\sim+})$, i.e $V(\boldsymbol{P}^{\mathrm{p}}_{\sim+}) \hat{=} \mathbf{i}|\boldsymbol{P}^{\mathrm{p}}_{\sim+}\rangle$. A postulate $\boldsymbol{P}^{\mathrm{p}}_{\sim+}$ is, more specifically, called a *veracious* *catlogographic slave axiom*, $\boldsymbol{P}_{\sim+}^{\mathrm{a}}$, if it is a *permanent* slave postulate and a *veracious catlogographic slave hypothesis*, $\boldsymbol{P}_{\sim+}^{\mathrm{h}}$, if it is an *ad hoc* slave postulate.

ii) If $\mathsf{T}_{1\sim}(\boldsymbol{P}_{\sim})$, being the CCFCL MTh (DTh) for $\boldsymbol{P}_{\sim}$, contains as its constituent parts tokens (isotokens, occurrences) of some catlogographic slave postulates that have been laid down earlier then $\mathsf{T}_{1\sim}(\boldsymbol{P}_{\sim})$ can be developed further with the help of D_1 into a *catlogographic algebraic decision procedure* (*CLADP*) $D_1(\boldsymbol{P}_{\sim})$ *for* $\boldsymbol{P}_{\sim}$ as its *catlogographic slave-relation* (*CLSR*), or *catlogographic relation-slave* (*CLR-slave*). The CLADP $D_1(\boldsymbol{P}_{\sim})$ is similar to an EADP $\mathsf{D}_1(\mathbf{P})$, so that it terminates in a certain *progressive catlogographic master,* or *decision, theorem* (*PCLMTh* or *PCLDTh*) $T_1(\boldsymbol{P}_{\sim})$ *for* $\boldsymbol{P}_{\sim}$ of exactly one of the following three forms:

$$V(\boldsymbol{P}_{\sim}) \hat{=} \boldsymbol{i}_m|\boldsymbol{P}_{\sim}\rangle \hat{=} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \boldsymbol{i}|\boldsymbol{P}_{\sim}\rangle & \text{(c)} \end{cases}, \tag{8.24}$$

which are similar to those given by the euautographic decision-theorem (EDTh) scheme (3.2) (see also (3.2$_1$) and (3.2$_2$)) and which are therefore denoted by '$T_{1+}(\boldsymbol{P}_{\sim})$', '$T_{1-}(\boldsymbol{P}_{\sim})$', or '$T_{1\sim}(\boldsymbol{P}_{\sim})$' respectively, in analogy with '$\mathsf{T}_{1+}(\mathbf{P})$', '$\mathsf{T}_{1-}(\mathbf{P})$', and '$\mathsf{T}_{1\sim}(\mathbf{P})$' introduced in Df 3.1(23). It is understood that the CLADP $D_1(\boldsymbol{P}_{\sim})$ that results in the PCLMTh (PCLDTh) of the respective one of three possible forms a–c is a development of the CCFCL MTh (DTh) $V(\boldsymbol{P}_{\sim}) \hat{=} \mathbf{i}|\boldsymbol{P}_{\sim}\rangle$, subject to $\boldsymbol{i}_n|\boldsymbol{P}_{\sim}\rangle \vec{\hat{=}} \mathbf{i}|\boldsymbol{P}_{\sim}\rangle \vec{\hat{=}} \mathbf{i}_n|\boldsymbol{P}_{\sim}\rangle$ and $m>n$.

2) In order to indicate that $T_{1*}(\boldsymbol{P}_{\sim})$, i.e. $T_{1+}(\boldsymbol{P}_{\sim})$, $T_{1-}(\boldsymbol{P}_{\sim})$, or $T_{1\sim}(\boldsymbol{P}_{\sim})$, is the pertinent catlogographic development of $\mathsf{T}_{1\sim}(\boldsymbol{P}_{\sim})$, all occurrences of the validity-operator V throughout $T_{1*}(\boldsymbol{P}_{\sim})$ can be (but is not recommended to be) replaced with occurrences of the *CFCL veracity-operator V*, which has exactly the same properties, and then the pertinent CLADP $D_1(\boldsymbol{P}_{\sim})$ is performed with V in place of V. Formally, the analo-homolographic substitution $V \mapsto V$ can in this case be included as an additional rule in I_1, which has been used in the item 1.

3) In analogy with the decisional terminology introduced in Df 3.1(24), a *ttatt-neutral CCLR* $\boldsymbol{P}_{\sim}$ is said to be (a) *veracious* (*atautologously*, or *accidentally*, *true*), and it is denoted by '$\boldsymbol{P}_{\sim+}$', if it either is $\boldsymbol{P}_{\sim+}^{\mathrm{p}}$ or if its PCLDTh is $T_{1+}(\boldsymbol{P}_{\sim})$; (b)

antiveracious (*accidentally antitrue*), and it is denoted by '$\boldsymbol{P}_{\sim-}$', if its PCLDTh is $\mathsf{T}_{1-}(\boldsymbol{P}_{\sim})$; (c) *vravr-neutral* (or *vravr-indeterminate*), i.e. *neutral* (or *indeterminate*) *with respect to the veracity-values veracity and antiveracity* or, in other words, *neither veracious nor antiveracious*, and it is denoted by '$\boldsymbol{P}_{\sim\sim}$', if its PCLDTh is $\mathsf{T}_{1\sim}(\boldsymbol{P}_{\sim})$. A ttatt-neutral CCLR $\boldsymbol{P}_{\sim+}$, which is proved to be veracious, is called a *veracious catlogographic slave theorem*.

4) By definition, "veracious" and "antiveracious" mean *accidentally true* and *accidentally antitrue* (*accidentally false*) – in contrast to *universally true* (*tautologously true*, *tautologous*) and *universally antitrue* (*tautologously anitrue*, *tautologously false*, *antitautologous*, *contradictory*), respectively. Accordingly, "veracity" and "antiveracity" mean *accidental truth* and *accidental antitruth* (*accidentally falsehood*) – in contrast to *universal truth* (*tautologous truth*, *tautologousness*) and *universally antitruth* (*tautologous anitruth*, *tautologous falsehood*, *antitautologousness*, *contradictoriness*), respectively. It is understood that the negation of a veracious relation is an antiveracious relation and vice versa, whereas the negation of a vravr-relation is another a vravr-relation.

5) Once a *ttatt-neutral CCLR*, i.e. *ttatt-neutral OptCCLR's of* I_1, $\boldsymbol{P}_{\sim}$ is provided with any one of the above three veracity-values, it turns into its own *homograph* that is called a *progressive CLR* (*PCLR*), while the name "*catlogographic relation*" ("*CLR*") *without either prepositive qualifier* "*conservative*" ("*C*") *or* "*progressive*" ("*P*") *equivocally applies to both homographs*. Thus, a PCLR is syntactically indistinguishable from the *CCLR* being its *predecessor*. A PCLR $\boldsymbol{P}_{\sim}$ is more specifically denoted by '$\boldsymbol{P}_{\sim*}$' and is alternatively called a *vravrn-decided ttatt-neutral CLR.* A totality (set) of compatible (mutually consistent) catlogographic postulates of A_1 along with all PCLMTh's (PCLDTh's) that can be proved from those postulates with the help of D_1 and also along with the *catlogographic slave relations* (*CLSR's*) of the PCLMTh's is called a *progressive CFCL* (*PCFCL*) *interpretation of* A_1 [*in extension*]. The class of PCLR's, being the result of a PCFCL interpretation of A_1, is called a *PCFCL interpretand of* A_1. The division of the PCLR's into three classes as indicated in the item 3, namely veracious, antiveracious, and vravr-neutral (vravr-indeterminate), is called the *primary*, or *basic, decisional trichotomy* (*trisection*, *trifurcation*) *of the PCLR's.* At the same time, a PCLR is said to be:

unveracious if is antiveracious or vravr-neutral, *non-antiveracious* if it is veracious or vravr-neutral, and *vravr-unnutral* or *vravr-determinate* if it is veracious or antiveracious. Consequently, there are three *secondary,* or *subsidiary*, *decisional dichotomies* (*bisections, bifurcations*) *of the PCLR*'s:

a) the *veracious* ones and the *unveracious* ones,

b) the *antiveracious* ones and the *non-antiveracious* ones,

c) the *vravr-neutral* (*vravr-indeterminate*) ones and the *vravr-unneutral* (*vravr-determinate*) ones.

6) All PCFCL interpretations of A_1 have the class of ttatt-neutral OptCCLR's of I_1 as their common source of *vravrn-decided CCLR's* and they also have the CLAADM D_1 as their common ADM. Therefore, the PCFCL interpretations of A_1 form a *single whole interpreted logistic system,* i.e. a *formalized language*, which is denoted by 'A_1' and which is called the *Comprehensive Catlogographic Algebraico-Predicate Organon* (*CCLAPO*) or the *Comprehensive Catlogographic Advanced Algebraico-Logical Organon* (*CCLAALO*). In this case, in accordance with the items 1–5, A_1 has no formation, no transformation (inference), and no decision rules other than those comprised in D_1. Particularly, by the item 1, some selective ttatt-neutral OptCCLR's of I_1 are used as *input CCLR's* (*IptCCLR's*) *of* A_1, whereas the CLAADM D_1 of A_1 is the CCFCL interpretand of the EAADM D_1 of A_1, $D_1 = I_1(\mathsf{D}_1)$. Therefore, D_1 can alternatively be called the *PCFCL interpretation of* A_1 *in intension*, the understanding being that this is unique. At the same time, there is an indefinite number of *PCFCL interpretations of* A_1 *in extension*, each of which is accomplished within A_1. In this case, I_1 plays two interrelated roles: first, it is the *most immediate interpretational supplement to* A_1 and, second, it is the *interpretational interface between* A_1 *and* A_1.

7) It goes without saying that A_1 contains as its *autonomous* (*self-subsistent*) parts two organons, which are denoted by 'A_1^0' and 'A_0' and which stand to A_1^0 and A_0 via I_1^0 and I_0 respectively in the same interpretational relations as that, in which A_1 stands to A_1 via I_1. Accordingly, A_1^0 is called the *Comprehensive Catlogographic Binder-Free*, or *Contractor-Free, Algebraico-Predicate Organon* (*CCLBFAPO* or *CCLCFAPO*) and also the *Comprehensive Catlogographic Rich Basic Algebraico-Logical Organon* (*CCLRBALO*), whereas A_0 is called the *Catlogographic Predicate-*

Free, or *Catlogographic* [*Depleted*] *Basic*, *Algebraico-Logical Organon* (*CLPFALO* or *CLDBALO* or *CLBALO*). A_1^0 and A_0 have the ADM's, which are denoted by 'D_1^0' and 'D_0' and which are related to D_1^0 and D_0 as $D_1^0 = I_1(\mathsf{D}_1^0)$ (because $I_1^0 = I_1$) and as $D_0 = I_0(\mathsf{D}_0)$ respectively. D_1^0 is called the *Catlogographic Rich BADM* (*CLRBADM*) *of* A_1 and D_0 is called the *Catlogographic BADM* (*CLBADM*) *of* A_1, in accordance with the corresponding names of D_1^0 and D_0 suggested in Df 3.1(20). Just as in the case of A_1, a totality (set) of compatible (mutually consistent) catlogographic postulates of A_1^0 or A_0 along with all PCLMTh's (PCLDTh's) that can be proved from those postulates with the help of D_1^0 or D_0 and also along with the *catlogographic slave relations* (*CLSR's*) of the PCLMTh's is called a *progressive CFCL* (*PCFCL*) *interpretation of* A_1^0 or A_0 [*in extension*] respectively. Hence, there is an indefinite number of *PCFCL interpretations of* A_1^0 or A_0, each of which is accomplished within A_1^0 or A_0 respectively (cf. the item 6).•

Cmt 8.3. In spite of the fact that I classify A_1 and its autonomous parts A_1^0' and A_0 as organons and provide them with long pretentious proper names, these are not full-scale effective logistic calculi, but rather they are weak (practically ineffective) semantic supplements to A_1, A_1^0, and A_0, which have with respect to the latter primarily illustrative academic interest. Particularly, A_1 illustrates that the class of *ttatt-neutral OptCCLR's of* I_1 is the source of *mathematical catlogographic postulates* (*veracious catlogographic axioms* and *veracious catlogographic hypotheses*) and of *mathematical catlogographic theorems*. In addition, A_1 illustrates the difference between a *tautologous* (*universally true*) relation and a *veracious* (*atautologously*, or *accidentally*, *true*) relation, which is necessarily a *ttatt-neutral* one. Hence, A_1 also illustrate the difference between a veracious relation and a true relation, which is either a tautologous one or a veracious ttatt-neutral one. The resources of A_1 provide the most general underlying concepts to allow distinguishing with complete rigor between masses and classes but they do not allow distinguishing between irregular (proper) classes and sets (regular, or small, classes). In order to develop a full-scale class, set, or mass theory, A_1 should be augmented by an *additional formation rule*, according to which to any given relation (condition) $\boldsymbol{P}\langle\boldsymbol{x}\rangle$ of a certain class there corresponds a unique class (particularly, a unique set) or a unique mass, which is denoted by **'**$\{\boldsymbol{x}|\boldsymbol{P}\langle\boldsymbol{x}\rangle\}$**'**. This formation rule is in fact a *contextual definition* of the *operator of abstraction* { | }, which allows prescinding a unique class or mass $\{\boldsymbol{x}|\boldsymbol{P}\langle\boldsymbol{x}\rangle\}$ from a given relation (condition) $\boldsymbol{P}\langle\boldsymbol{x}\rangle$ and which is called an *abstract class-builder*, *set-builder*, or *mass-builder* depending on a theory. All other operators that are used in a class or mass theory, – such operators, e.g., as the *binary operators* ∪, ∩, and **-** *of union, intersection*, and *difference of classes* or *masses* or the *operator of aggregation* { , ,…, } *of elements* (*classes* or *sets*), called also a *concrete set-builder,* – can contextually be defined in terms of the respective operator { | }. For instance, in the case of classes or particularly sets,

$$\boldsymbol{y}_1 \cup \boldsymbol{y}_2 \to \{\boldsymbol{x}|\boldsymbol{x} \in \boldsymbol{y}_1 \text{ or } \boldsymbol{x} \in \boldsymbol{y}_2\},\ \boldsymbol{y}_1 \cap \boldsymbol{y}_2 \to \{\boldsymbol{x}|\boldsymbol{x} \in \boldsymbol{y}_1 \text{ and } \boldsymbol{x} \in \boldsymbol{y}_2\},$$

$$\boldsymbol{y}_1 - \boldsymbol{y}_2 \to \{\boldsymbol{x}|\boldsymbol{x} \in \boldsymbol{y}_1 \text{ and } \neg[\boldsymbol{x} \in \boldsymbol{y}_2]\},$$

whereas a *singleton*, i.e. a *one-member set*, and an *unordered pair*, i.e. an *unordered two-member set*, are conventionally defined as

$$\{\boldsymbol{x}_1\} \to \{\boldsymbol{x}|\boldsymbol{x} = \boldsymbol{x}_1\} \text{ and } \{\boldsymbol{x}_1, \boldsymbol{x}_2\} \to \{\boldsymbol{x}|\boldsymbol{x} = \boldsymbol{x}_1 \text{ or } \boldsymbol{x} = \boldsymbol{x}_2\}$$

respectively and then recursively

$$\{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_{n-1}, \boldsymbol{x}_n\} \to \{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_{n-1}\} \cup \{\boldsymbol{x}_n\} \text{ for each natural } n \geq 3.$$

Also, sets (but not irregular classes and not masses) should be allowed to be domains of definitions of various order relations and thus to become ordered. It is understood that all the above operators and all order relations should be subjected to or be introduced by the appropriate *semantic axioms* along with the appropriate definitions. Therefore, a full-scale class or set theory and hence a full-scale mass theory cannot have any decision method after the manner of A_1 and A_1.

Creation of any complete full-scale class, set, or mass theory on the basis of A_1 and I_1 is beyond the scope of this treatise. Nevertheless, in the subsection 9.3 of the next section I shall explicate the difference between an irregular (proper) class and a set (regular, or small, class), and in the section A5 (Appendix 5) I shall lay down the basic axioms of the full-scale nominalistic (intuitionistic) one-individual and many-individual class theories and also the basic axioms of the full-scale mass theory – the theories that are based on $\mathsf{A}_{1\in}$ and $\mathsf{A}_{1\subseteq}$.•

Cmt 8.4. 1) Like validity, antivalidity, or vav-neutrality (vav-indeterminacy) of a DdER, as treated in Df 3.1(28), *tautologousness*, *antitautologousness* (*contradictoriness*), or *ttatt-neutrality* (*ttatt-indeterminacy*) *of a PCLR*, i.e. the *quality* of the PCLR to be tautologous, antitautologous (contradictory), or ttatt-neutral (ttatt-indeterminate), can conveniently be regarded as the *state of membership* of the PCLR in the respective one of the three *decision classes*, which will discriminately be called *the tautologousness-class,* or *tautologousness-value, tautologousness,* ditto *antitautologousness* (*contradictoriness*) respectively, or ditto *ttatt-neutrality* (*ttatt-indeterminacy*), i.e. *neither tautologousness nor antitautologousness,* and hence indiscriminately a *tautologousness-class;* or *tautologousness-value*. Likewise, *veracity*, *antiveracity*, or *vravr-neutrality* (*vravr-indeterminacy*) *of a ttatt-neutral PCLR*, i.e. the *quality* of the ttatt-neutral PCLR to be veracious, antiveracious, or vravr-neutral (vravr-indeterminate) can conveniently be regarded as the *state of membership* of the PCLR in the respective one of the three *decision class*, which will discriminately (respectively) be called *the veracity-class* (*veracity-value*) *veracity, antiveracity*, or *vravr-neutrality* (*vravr-indeterminacy*), i.e. *neither veracious veracity nor antiveracious,* and indiscriminately a *veracity-class,* or *veracity-value*. Logographically, the above three tautologousness-values (tautologousness-classes) are denoted by 'τ_+', 'τ_-', '$\tau_\sim$', and the above three veracity-values (veracity-classes) by '$\tau_{\sim+}$', '$\tau_{\sim-}$', '$\tau_{\sim\sim}$', and also by 'ϕ_+', 'ϕ_-', '$\phi_\sim$' in that order. Consequently, τ_+ or $\tau_{\sim+}$, i.e. ϕ_+, is ambiguously (equivocally) denoted by 'α_+' and is indiscriminately called the *truth-value truth*; τ_- or $\tau_{\sim-}$, i.e. ϕ_-, is ambiguously denoted by 'α_-' and is indiscriminately called the *truth-value antitruth* or *falsity* (*falsehood*); $\tau_{\sim\sim}$, i.e. $\phi_\sim$, is alternatively denoted by '$\alpha_\sim$' and is alternatively called the truth-value *neutrality with respect to the truth-values truth and antitruth* or briefly the *tat-neutrality* (*tat-indeterminacy*). Hence,

$$\phi_+ = \tau_{\sim+},\ \phi_- = \tau_{\sim-},\ \phi_\sim = \tau_{\sim\sim} = \alpha_\sim\,. \tag{8.25}$$

The mnemonic justification for the above notation is that: 'τ´ is the first letter of the Greek noun "ταυτολογία'' \taftolojía\; 'φ' is the first letter of the Greek noun "φιλαλήθεια" \filalíθia\, meaning *veracity*, and of the kindred adjective "φιλαλήθης" \filalíθis\, meaning *veracious*; and 'α' is the first letter of the Greek noun "αλήθεια" \alíθia\, meaning *truth*, and of the kindred adjective "αλήθης" \alíθis\, meaning *true*. In accordance with the above notation, the prepositive abbreviations "ttatt" ("tautologuosness-antitautologuosness"), "vravr" ("veracity-antiveracity"), and "tat" (truth-antitruth") can be used interchangeably with 'τατ', 'φαφ', and 'ααα' respectively; the middle letter ‘α’ in any of the latter three abbreviations stands for the Greek combining form “άντι” \ánti\ denoting *opposition, opposite situation,* or *negation.*

2) In analogy with the *secondary atomic panlogographic relations* (*SAPLR’s*) ‘$\mathbf{P}_*$’, ‘$\mathbf{P}_+$’, ‘$\mathbf{P}_-$’, and ‘$\mathbf{P}_\sim$’, and the *secondary molecular panlogographic integrons* (*SMlPLI’s*) ‘$\mathbf{i}|\mathbf{P}_\sim\rangle$’ and ‘$\bar{\mathbf{i}}|\mathbf{P}_\sim\rangle$’ (see Df 3.1(27)), I introduce the following *pancatlogographs* (*PCL’s*), called also *pancatlogographic placeholders* (*PCLPH’s*).

i) ‘$\boldsymbol{P}_*$’ is an *atomic pancatlogographic relation* (*APCLR*) whose range is the class of *ttattn-decided CCLR’s*; ‘$\boldsymbol{P}_+$’, ‘$\boldsymbol{P}_-$’, or ‘$\boldsymbol{P}_\sim$’ is an APCLR whose range is the class of tautologous, antitautologous, or τατ-neutral CCLR’s respectively. That is to say, in the ordinary *projective phraseology,* $\mathbf{P}_+$ *is* a *valid* OptEOR of A_1 (IptEOR of I_1), $\mathbf{P}_-$` *is* a *vav-neutral* OptEOR of A_1 (IptEOR of I_1), $\mathbf{P}_-$ is the [*antivalid*] *negation of* $\mathbf{P}_+$, and $\mathbf{P}_*$ *is* $\mathbf{P}_+$, $\mathbf{P}_-$, or $\mathbf{P}_\sim$; likewise, $\boldsymbol{P}_+$ *is* a *tautologous* OptCCLOR of I_1, $\boldsymbol{P}_\sim$ *is* a *ttatt-neutral* OptCCLOR of I_1, $\boldsymbol{P}_-$ is the [*antitautologous*] *negation of* $\boldsymbol{P}_+$, and $\boldsymbol{P}_*$ *is* $\boldsymbol{P}_+$, $\boldsymbol{P}_-$, or $\boldsymbol{P}_\sim$

ii) ‘$\mathbf{i}|\boldsymbol{P}_\sim\rangle$’ is a *molecular pancatlogographic integron* (*MlPCLI*) whose range is the class of *catlogographic tautologousness-integrons* (briefly, *CLTtI’s*), called also, more specifically, *catlogographic tautologousness-identifiers* or *catlogographic tautologousness-indices* (briefly, *CLTtID’s* in both cases), *τατ-neutrality.* In other words, the range of ‘$\mathbf{i}|\boldsymbol{P}_\sim\rangle$’ is the class of *CCFCL interpretands of euautographic validity-integrons* (*EVI’s*) *κακ-neutrality* (*vav-neutrality, kak-neutrality*) *of the range of* ‘$\mathbf{i}|\mathbf{P}_\sim\rangle$’. Consequently, in the ordinary *projective phraseology,* $\mathbf{i}|\boldsymbol{P}_\sim\rangle$ *is* thw *catlogographic intrgron* (*CLI*), being a CCFCL interpretand of $\mathbf{i}|\mathbf{P}_\sim\rangle$, so that it *is* a *CLTlI* (*CLTlID*) *τατ-neutrality*.

iii) ‘$\bar{\mathbf{i}}|\boldsymbol{P}_\sim\rangle$’ is an *MlPCLI* whose range is the class of *catlogographic antitautologousness-integrons* (briefly, *CLATtI’s*), called also, more specifically, *catlogographic antitautologousness-identifiers* or *catlogographic antitautologousness-indices* (briefly, *CLATtID’s* in both cases), *τατ-neutrality.* In other words, the range of ‘$\bar{\mathbf{i}}|\boldsymbol{P}_\sim\rangle$’ is the class of *CCFCL interpretands of euautographic antivalidity-integrons* (*EAVI’s*) *κακ-neutrality* (*vav-neutrality, kak-*

neutrality) *of the range of* '$\bar{\mathbf{i}}|\mathbf{P}_{\sim}\rangle$'. Consequently, in the projective phraseology, $\bar{\mathbf{i}}|\boldsymbol{P}_{\sim}\rangle$ *is* a CLI, being a CCFCL interpretand of $\bar{\mathbf{i}}|\mathbf{P}_{\sim}\rangle$, so that it *is* a *CLATlI* (*CLATlID*) *τατ-neutrality*.

iv) '$\boldsymbol{P}_{\sim *}$' is an APCLR whose range is the class of *vravrn-decided τατ–neutral PCLR's*; '$\boldsymbol{P}_{\sim +}$', '$\boldsymbol{P}_{\sim -}$', or '$\boldsymbol{P}_{\sim\sim}$' is an APCLR whose range is the class of *veracious*, *antiveracious*, or *φαφ–neutral* τατ–neutral PCLR's respectively.

v) '$\boldsymbol{i}|\boldsymbol{P}_{\sim\sim}\rangle$' is an *MlPCLI* whose range is the class of *catlogographic veracity-integrons* (briefly, *CLVrI's*), called also, more specifically, *catlogographic veracity-identifiers* or *catlogographic veracity-indices* (briefly, *CLVrID's* in both cases), *φαφ-neutrality* [*of VrVPCLR's of the range of* '$\boldsymbol{P}_{\sim\sim}$']. That is to say, in the projective phraseology, $\boldsymbol{i}|\boldsymbol{P}_{\sim\sim}\rangle$ *is* a CLI or, more specifically, *a CLVrI* (*CLVrID*) *φαφ-neutrality*.

vi) '$\bar{\boldsymbol{i}}|\boldsymbol{P}_{\sim\sim}\rangle$' is an *MlPLI* whose range is the class of *catlogographic antiveracity-integrons* (briefly, *CLAVrI's*), called also, more specifically, *catlogographic antiveracity-identifiers* or *catlogographic antiveracity-indices* (briefly, *CLAVrID's* in both cases), *φαφ-neutrality* [*of AVrVPCLR's of the range of* '$\boldsymbol{P}_{\sim\sim}$']. That is to say, in the projective phraseology, $\bar{\boldsymbol{i}}|\boldsymbol{P}_{\sim\sim}\rangle$ *is* a CLI or, more specifically, *a CLAVrI* (*CLAVrID*) *φαφ-neutrality*.

vii) '$\boldsymbol{P}^{*}$' is a *metalogographic placeholder* (*MLPH*) whose range is the class of *tatn-decided* PCLR's, whereas '$\boldsymbol{P}^{+}$', '$\boldsymbol{P}^{-}$', or '$\boldsymbol{P}^{\sim}$' is an MLPH whose range is the class of *true*, *antitrue* (*false*), or *tat-neutral* (*tat-indeterminate*) PCLR's respectively. That is to say, the range of '$\boldsymbol{P}^{*}$' is the union of the ranges of '$\boldsymbol{P}^{+}$', '$\boldsymbol{P}^{-}$', or '$\boldsymbol{P}^{\sim}$'; the range of '$\boldsymbol{P}^{+}$' is the union of the ranges of '$\boldsymbol{P}_{+}$' and '$\boldsymbol{P}_{\sim +}$' so that "true" means *tautologous* (*universally true*) *or veracious* (*accidentally true*); the range of '$\boldsymbol{P}^{-}$' is the union of the ranges of '$\boldsymbol{P}_{-}$' and '$\boldsymbol{P}_{\sim -}$' so that "antitrue" ("false") means *antitautologous* (*contradictory*, *universally antitrue*) *or antiveracious* (*accidentally antitrue*, *accidentally false*); the range of '$\boldsymbol{P}^{\sim}$' is the same as the range of '$\boldsymbol{P}_{\sim\sim}$' so that "tat-neutral" ("tat-indeterminate") means *vravr-neutral* ("*vravr-indeterminate*") and vice versa.

3) By definitions of the previous item, it follows from (3.15) and (3.16) that

$$V(\boldsymbol{P}_{+}) \hat{=} V(\mathbf{P}_{+}) \hat{=} 0,\ V(\boldsymbol{P}_{-}) \hat{=} V(\mathbf{P}_{-}) \hat{=} 1,\ V(\boldsymbol{P}_{\sim}) \hat{=} V(\mathbf{P}_{\sim}) \hat{=} \mathbf{i}|\mathbf{P}_{\sim}\rangle \hat{=} \mathbf{i}|\boldsymbol{P}_{\sim}\rangle, \quad (8.26)$$

$$\bar{V}(\boldsymbol{P}_+)\hat{=}\bar{V}(\mathbf{P}_+)\hat{=}1,\ \bar{V}(\boldsymbol{P}_-)\hat{=}\bar{V}(\mathbf{P}_-)\hat{=}0,\ \bar{V}(\boldsymbol{P}_\sim)\hat{=}\bar{V}(\mathbf{P}_\sim)\hat{=}\bar{\mathbf{i}}|\mathbf{P}_\sim\rangle\hat{=}\bar{\mathbf{i}}|\boldsymbol{P}_\sim\rangle, \tag{8.27}$$

$$V(\boldsymbol{P}_{\sim+})\hat{=}0,\ V(\boldsymbol{P}_{\sim-})\hat{=}1,\ V(\boldsymbol{P}_{\sim\sim})\hat{=}\boldsymbol{i}|\boldsymbol{P}_{\sim\sim}\rangle, \tag{8.28}$$

$$\bar{V}(\boldsymbol{P}_{\sim+})\hat{=}1,\ \bar{V}(\boldsymbol{P}_{\sim-})\hat{=}0,\ \bar{V}(\boldsymbol{P}_{\sim\sim})\hat{=}\bar{\boldsymbol{i}}|\boldsymbol{P}_{\sim\sim}\rangle, \tag{8.29}$$

$$V(\boldsymbol{P}^+)\hat{=}0,\ V(\boldsymbol{P}^-)\hat{=}1,\ V(\boldsymbol{P}^\sim)\hat{=}V(\boldsymbol{P}_{\sim\sim})\hat{=}\boldsymbol{i}|\boldsymbol{P}_{\sim\sim}\rangle\hat{=}\boldsymbol{i}|\boldsymbol{P}^\sim\rangle, \tag{8.30}$$

$$\bar{V}(\boldsymbol{P}^+)\hat{=}1,\ \bar{V}(\boldsymbol{P}^-)\hat{=}0,\ \bar{V}(\boldsymbol{P}^\sim)\hat{=}\bar{V}(\boldsymbol{P}_{\sim\sim})\hat{=}\bar{\boldsymbol{i}}|\boldsymbol{P}_{\sim\sim}\rangle\hat{=}\bar{\boldsymbol{i}}|\boldsymbol{P}^\sim\rangle. \tag{8.31}$$

Also, in analogy with Df 3.1(28) and Cmt 3.2(4), it follows from the previous two items 1 and 2 of this comment that, besides the pair of mutually dual metalinguistic functions, denoted by 'V' and '$\bar{\mathrm{V}}$' or synonymously by 'K' and '$\bar{\mathrm{K}}$', there are three other pairs of similar functions, to be denoted by 'T' and '$\bar{\mathrm{T}}$', 'Φ' and '$\bar{\Phi}$', and 'A' and '$\bar{\mathrm{A}}$', so that

$$\mathrm{K}(0)=\bar{\mathrm{K}}(1)=\kappa_+,\ \mathrm{K}(1)=\bar{\mathrm{K}}(0)=\kappa_-,\ \mathrm{K}(\mathbf{i}|\mathbf{P}_\sim\rangle)=\bar{\mathrm{K}}(\bar{\mathbf{i}}|\mathbf{P}_\sim\rangle)=\kappa_\sim, \tag{8.32}$$

$$\mathrm{T}(0)=\bar{\mathrm{T}}(1)=\tau_+,\ \mathrm{T}(1)=\bar{\mathrm{T}}(0)=\tau_-,\ \mathrm{T}(\mathbf{i}|\boldsymbol{P}_\sim\rangle)=\bar{\mathrm{T}}(\bar{\mathbf{i}}|\boldsymbol{P}_\sim\rangle)=\tau_\sim, \tag{8.33}$$

$$\Phi(0)=\bar{\Phi}(1)=\phi_+,\ \Phi(1)=\bar{\Phi}(0)=\phi_-,\ \Phi(\boldsymbol{i}|\boldsymbol{P}_{\sim\sim}\rangle)=\bar{\Phi}(\bar{\boldsymbol{i}}|\boldsymbol{P}_{\sim\sim}\rangle)=\phi_\sim, \tag{8.34}$$

$$\mathrm{A}(0)=\bar{\mathrm{A}}(1)=\alpha_+,\ \mathrm{A}(1)=\bar{\mathrm{A}}(0)=\alpha_-,\ \mathrm{A}(\boldsymbol{i}|\boldsymbol{P}^\sim\rangle)=\bar{\mathrm{A}}(\bar{\boldsymbol{i}}|\boldsymbol{P}^\sim\rangle)=\alpha_\sim, \tag{8.35}$$

subject to (3.21) and (8.25), and also subject to the above item 2vii.

4) The intrgrons 0 and 1 belong to A_1, i.e. to both A_1 and $\mathbf{A}_1$. The SAPLR's '$\mathbf{P}_*$', '$\mathbf{P}_+$', '$\mathbf{P}_-$', and '$\mathbf{P}_\sim$' and the SMlPLI's '$\mathbf{i}|\mathbf{P}_\sim\rangle$' and '$\bar{\mathbf{i}}|\mathbf{P}_\sim\rangle$' belong to $\mathbf{A}_1$, whereas their values, $\mathbf{P}_*$, $\mathbf{P}_+$, $\mathbf{P}_-$, $\mathbf{P}_\sim$, $\mathbf{i}|\mathbf{P}_\sim\rangle$, and $\bar{\mathbf{i}}|\mathbf{P}_\sim\rangle$, are EF's of A_1. By the item 2i of this comment, $\mathbf{P}_+$ and $\mathbf{P}_\sim$ are OptEOR's of A_1 and at the same time IptEOR's of I_1, so that these, along with $\boldsymbol{P}_+$ and $\boldsymbol{P}_\sim$, being their CCFCL interprtands and at the same time OptCCLOR's of I_1, belong to I_1. At the same time, PCLR's $\boldsymbol{P}_{\sim+}$, $\boldsymbol{P}_{\sim-}$, and $\boldsymbol{P}_{\sim\sim}$, and CLI's $\boldsymbol{i}|\boldsymbol{P}_{\sim\sim}\rangle$ and $\bar{\boldsymbol{i}}|\boldsymbol{P}_{\sim\sim}\rangle$ belong to A_1.•

9. Nomenclature of the treatise versus nomenclatures of conventional logical calculi and conventional class and set theories

Preliminary Remark 9.1. As stated in Preliminary Remark 8.1, the CFCL interpretation of A_1 and A_0 allows establishing the master-to-slave relationships between these organons on the one hand and traditional logic and CALC'i on the other hand. However, in order to establish these relationships, it is necessary to explicate as far as possible the correspondence between the nomenclature, i.e. verbal terminology and pasigraphic notations, of the master organons A_1 and A_0 and their CFCL interpretands on the one hand and nomenclatures of the slave logistic systems on the other hand. Also, the calculus A_1, along with its CFCL interpretand, is the underlying logical discipline, from which a full-scale class theory and a full-scale set theory can be derived. Therefore, it is also necessary to establish the correspondence between the nomenclature of A_1 and its CFCL interpretand on the one hand and the nomenclatures of *conventional axiomatic class* and *set theories* (*CACT's* and *CAST's*) on the other hand. Unfortunately, there is neither unique conventional nomenclature of CALC'i nor unique conventional nomenclature of CACT's and CAST's. Therefore, in the following discussion, I shall refer only to the most typical or most authoritative nomenclatures of logical calculi and of class and set theories. •

9.1. The taxonomy of endosemasiopasigraphs (euautographs and panlogographs) of A_1 versus the taxonomies of logographs of conventional axiomatic logical calculi (CALC'i)

1. The noun "*term*" is used in the XML (exclusive metalanguage) of A_1, i.e. A_1 and $\mathbf{A}_1$, equivocally in many different senses. In a broad sense, especially without any qualifiers, a *term is an element of the metalinguistic terminology*. Whenever confusion can result, instead of the noun "term" in this sense, I employ the noun "*metaterm*" as an abbreviation of the name "*metalinguistic term*". For instance, the count names "*term of* A_1", "*relation of* A_1", and "*formula of* A_1" are *technical metaterms* that are relevant to the *object* calculus of the IML (inclusive metalanguage) of A_1. Particularly, the metaterm "*term of* A_1" denotes the subclass (specific class, species) of euautographs of A_1, being the complement of the subclass denoted by the count name "*relation* of A_1" in the superclass (generic class, genus) denoted by the count name "*formula* of A_1". In this case, the occurrences of the nouns "term" and "relation" in the names "term of A_1" and "relation of A_1" can, for more clarity, be replaced with occurrences of the compound nouns "formula-term" (or "term-formula") and "formula-relation" (or "relation-formula") respectively. The count name "*term of a syllogistic judgment*" is another *technical metaterm* which applies, e.g., to either of the placeholders '*x*' and '*y*' occurring in any of the four syllogistic sentential forms (judgments) of Aristotelian logic [of categorical syllogisms]: “Every *x* is a *y*”, “Every *x* is not a *y*” (or, equivalently, “No *x* is a *y*”) , “Some *x* is a *y*”, and “Some *x* is not a *y*”.

2. Although the calculi A_0 and A_1 have no homologies among CALC’i, in order to establish master-to-slave relationships between the former and the latter, it is necessary, first of all, to establish an analogy between the taxonyms (taxonomic names) of the euautographs of A_0 and A_1 and the taxonyms of the respective CALC’i. This is done below in this subsection.

3. I use the technical terms “assemblage of A_1”, “formula of A_1”, “term of A_1”, and “relation of A_1” (e.g.), - or, more precisely, “primary assemblage of A_1”, “primary formula of A_1”, “primary term of A_1”, and “primary relation of A_1”, - in *analogy* with the terms “*assemblage of theory* $\mathcal{T}$”, “*formative construction of theory* $\mathcal{T}$”, “*term of theory* $\mathcal{T}$”, and “*relation of theory* $\mathcal{T}$” in this order in Bourbaki [1960, chap. I, §1, subsections 1, 3]. However, the taxa (taxonomic classes) denoted by the former taxonyms (taxonomic names) are classes of euautographs, whereas the taxa denoted by the latter taxonyms are classes of logographs. Consequently, the respective taxa of the two taxonomies are different. That is to say, the above-mentioned analogy in use of the generic taxonyms “assemblage”, “term”, and “relation” of this treatise and of the *homonymous* taxonyms of Bourbaki’s set theory is relevant, not to the taxa denoted by the taxonyms, but largely to the *relative taxonomic ranks* of the respective taxonyms. Likewise, in the following discussion, when I say that a certain taxonym of euautographs of A_1 and a certain taxonym of logographs of a CALC are analogous, I mean that they are analogous in their relative taxonomic ranks and not in their meanings.

4. The treatise of Bourbaki [1960] is not a theory of a CALC, but rather it is an attempt to present a certain system of set theory as an interpretand of a CAPC (conventional axiomatic predicate calculus of first order), which includes the Russell-Bernays sentential calculus as its constituent part (see *ibidem*, chap. I, §3, subsection 1, axiom schemata S1–S4). Typically, the taxonomy of logographs of a CALC does not contains any formal taxonym of the same rank as Bourbaki’s taxonym “*formative construction*” or as the taxonym “*formula*” of this treatise, although the noun “formula” is utilized in all theories (IML’s) of CALC’i. This is at least true, e.g., of the theories of Whitehead and Russell [1910–13; 1925–27; 1962], Hilbert and

Ackermann [1950], Quine [1951], Church [1956], and Suppes [1957]. For instance, in Hilbert and Ackermann [1950, pp. 65, 66], Quine [1951, chap. II, §13], and Suppes [1957, p. 52], the noun "formula" is used as a taxonym analogous to Burbaki's taxonym "relation", and hence analogous to the taxonym "relation" of this treatise. In Church [1956, pp. 49, 70], the taxonyms "formula" and "well-formed formula" (abbreviated as "wff") are used as analogues of Burbaki's taxonyms "assemblage" and "relation" respectively. Terms of Bourbaki's set theory are of two kinds, namely, atomic ones and combined ones; a combined term is obtained from a relation by a certain operation of abstraction denoted by 'τ' (*ibidem*, chap. I, §1, subsection 1). At the same time, Boubaki's taxonyms (metaterms) "term" and "relation" are ones of the same rank, because they are introduced by the dichotomy of the class of formative constructions of his theory into the class of terms and the class of relations. Likewise, the homonymous taxonyms of this treatise are ones of the same analogous rank, because they are introduced by the dichotomy of the class of formulas of any one of the object organons A_n, $\boldsymbol{\mathsf{A}}_n$, and A_n into the class of terms and the class of relations of that organon.

5. In Church [1956, pp. 69, 168], a CALC (conventional axiomatic logical calculus) that I call a CASC (conventional axiomatic sentential calculus) is called a *propositional calculus*, whereas a CALC that I call a CAPC (conventional axiomatic predicate calculus of first order or conventional axiomatic first order predicate calculus) is called a *functional calculus of first order*. Two original propositional calculi that are developed in chapters I and II of Church's book are denoted by 'P_1' and 'P_2'. The most general branching functional calculus of first order that is developed in chapter III of Church's book is denoted by 'F^1'. The simplest way to establish an analogy between the taxonyms (taxonomic names) of logographs of P_1, P_2, or F^1 and some taxonyms of euautographs of A_0 and A_1 is to provide the former with synonyms in which the appropriate ones of the generic nouns "assemblage", "formula", "relation", and "term" of this treatise are used. Here follow the pertinent definitions, in which the angle-quoted definientia (sentential subjects) are Church's original metaterms (*ibidem,* pp. 49, 70, 169), while the definienda (sentential predicatives) are metaterms of this treatise.

a) A «*formula*» of P_1, P_2, or F^1 is called an *assemblage* of P_1, P_2, or F^1 respectively.

b) A «*well-formed formula*» («*wff*») of P_1, P_2, or F^1 is called a *relation* (*relation-formula*, *formula-relation*) of P_1, P_2, or F^1 respectively.

c) A «*propositional variable*» of P_1, P_2, or F^1 is called an *atomic relation-variable* (*atomic variable-relation*) of P_1, P_2, or F^1 respectively.

d) An «*individual variable*», or an «*individual constant*», of F^1 is called an *atomic term-variable* (*atomic variable-term*), or an *atomic term-constant* (*constant-term*), and also an *atomic element-variable* or an *atomic element-constant*, of F^1 respectively.

e) A «*functional variable*», or a «*functional constant*», of F^1 is called a *primary predicate-variable* (*variable-predicate*), or a *primary predicate-constant* (*constant-predicate*), of F^1 respectively.

The items a) and b) are in agreement with the informal remarks which I have made regarding Church's taxonyms "formula" and "well-formed formula" ("wff") in the item **4** above in this subsection. At the same time, the item c)–e) are in agreement with Df 3.1(4) and Cmt 3.1(1), according to which application of either of the words "variable" and "constant" (as a noun or as an adjective) or of any more restricted count name as "propositional variable", "individual variable", "functional variable", "individual constant", "functional constant", etc to a graphonym evidences that the graphonym is regarded as a xenograph or particularly as a logograph and not as a euautograph. Also, Church uses the word "*individual*" in the sense which was attached to it by Russell [1908] in connection with his theory of logical types (see, e.g., Church [1956, p. 174, n. 309]). However, this Russell's term is inappropriate for employing in a CAPC if the latter is used as the underlying discipline of a [system of] set theory (see the item **9** below in this subsection for greater detail). Therefore, Church-Russell's terms "individual variable" and "individual constant" are, by the item d), replaced with the appropriate consistent taxonyms. The latter particularly allow discussing the inconsistency of the former taxonyms and also the inconsistency of the basic Russell's term "individual" conveniently. This will be done in due course below in this section.

6. Hilbert and Ackermann [1950, pp. 65–67] employ the metaterm "individual variable" in the same sense but, instead of Church's metaterms "propositional variable" and "well-formed formula" ("wff"), they employ the metaterms "sentential variable" and "formula" respectively. Consequently, in accordance with the previous item **5(**c–e), the above three metaterms should be replaced with "*atomic term-variable*" ("*atomic variable-term*"), "*atomic relation-variable"* ("*atomic variable-relation*"), and "*relation*" in that order. Editor's Notes to the book of Hilbert and Ackermann (*ibid.*, pp. 165, 166) contain a summary of various systems of nomenclature of CALC'i.

7. The notation and terminology of Whitehead and Russell [1910; 1925; 1962, p. 5] and [1925; 1962, pp. xv–xx, 51] do not stand in straightforward analogy either with those of the above-mentioned books or with those of this treatise; they are confusing and seem to be not self-consistent. First, the criteria, according to which I characterize a graphonym atomic or molecular differs from those used by Whitehead and Russell. Accordingly, the formulas that these writers call *atomic propositions* (the second reference, p. xv) should, in accordance with the criteria of this treatise, be called *molecular relations*. Second, the writers say (*ibidem*, p. xvi): «Let *p*, *q*, *r*, *s*, *t*, denote, to begin with, atomic propositions». Thus defined, these variables are MLPH's (metalinguistic logographic placeholders), whose range is the class of atomic propositions (in the above sense). On the other hand, in the former of the above two references, the writers say: «Of the remaining letters, *p*, *q*, *r* will be called *propositional letters*, and will stand for variable propositions (except that, from **∗40** onward, *p* must not be used for a variable); …». In fact, the letters in question are used in *Principia Mathematica* in the same way, in which Church [1956] uses his homonymous *propositional variables*.

8. *Russell's term* "***individual***". a) Russell [1908] described *individuals* as objects «destitute of complexity», while Whitehead and Russell [1910; 1962, p. 51] described individuals as «objects which are neither propositions nor functions». These two different descriptions cannot of course serve as formal definitions of the meaning of the metaterm "individual". The meaning of the metaterm is determined by its use in the *theory of logical types* (*TLT)* of Russell [1908] and of Whitehead and Russell [1910; 1962, Chapter II], according to which only *relative* logical types are actually relevant in any specific context. In the light of this *epistemologically relativistic property of thinking*, it has become usual to employ the metaterms "individual", "individual variable", and "individual constant" as antonyms of the metaterms "predicate", "predicate variable", and "predicate constant". Particularly, Church uses the three former metaterms in this way. Also, it is presently common to employ those metaterms as antonyms of the metaterms such as "class", "proposition", "function" ("propositional function"), "sentential variable", "sentential constant", "sentence", "functional variable", "functional constant", and the like.

b) From the standpoint of etymological analysis, the word "*individual*", both as a noun and as an adjective, is cognate to the adjective "*indivisible*". At the same time, the adjectives "divisible" and "indivisible" are epistemologically relativistic antonyms, which may assume (take on) many different senses (sense values) depending on a criterion of divisibility or indivisibility of the entities, to which the antonyms are applied. Therefore, as long as a certain uninterpreted [system of] predicate calculus of first order is developed or exercised in isolation from its applications, employing Russell's ambiguous word 'individual" as a technical term is harmless. However, the main predestination of any such calculus is to serve as the underlying discipline of mathematics in general and of class theory or particularly of set theory in the first place. In a class theory, a *divisible substance* is a *nonempty class*, i.e. a *class having members*, and vice versa, whereas an *indivisible substance*, called also an *individual*, is either *the empty*, or *memberless*, *class*, called also *the empty individual*, or, if the class theory is a many-individual one, a *nonempty individual*, i.e. a substance that is *not a class*., and vice versa. In this case, an *atomic* term-variable (or element-variable) that Church and many other logicians call an *individual variable* is allowed to *assume* (*take on*) *nonempty* (*divisible*) *classes* as its *accidental denotata*. Hence, Russell's term "individual" is a *misnomer*, which is inapplicable in class and, particularly, set theories. In modern class and set theories, the term "*element*" is used instead of Russell's term "individual", the understanding being that in a many-individual class theory an element is either a nonempty class or the empty class, i.e. the empty individual, or else a nonempty individual. Thus, in the general case, the term "element" is distinguished from both terms "class" (or "set") and "individual". However, in a *one-individual class* (or *set*) *theory*, an element is a class (or, correspondingly, a set), nonempty or empty, and vice versa (cf. Fraenkel et al [1973, pp. 24–25]).

Cnv 9.1. *Foundations of Set Theory* by Fraenkel et al [1973] will hereafter be often referred to as "*FST*".•

9.2. The endosemasiopasigraphic (euautographic and panlogographic) notation A_1 versus the logographic notations of CALC'i

1. The six logical connectives $\barwedge$ (or $\overline{\wedge}$), $\neg$, $\vee$, $\Rightarrow$, $\wedge$, $\Leftrightarrow$ of the list (5.4) are *functional parasynonyms* of the sentential connectives |, **~**, **v**, ⊃, ., ≡ in Whitehead and Russell [1925; 1962, p. xvi] and of the six logical connectives / (the virgule), $\overline{}$ (the overbar of an adjustable length), **v**, →, **&**, ~ in Hilbert and Ackermann [1950, pp. 3–4, 11] in this order, whereas the connectives $\neg$, $\vee$, $\Rightarrow$, $\wedge$, $\Leftrightarrow$, i.e. the last five of the former six connectives, are also functional parasynonyms of ~, **v**, ⊃, ., ≡ in Quine [1951, pp, 11–15, 20] and of -, **v**, →, **&**, ↔ in Suppes [1957, pp. 3–10] in that order. At the same time, the ten logical connectives $\neg$, $\vee$, $\Rightarrow$, $\Leftarrow$, $\barwedge$ (or $\overline{\wedge}$), $\Leftrightarrow$, $\overline{\vee}$, $\overline{\Rightarrow}$, $\overline{\Leftarrow}$, $\overline{\Leftrightarrow}$ of the list (5.4) are functional parasynonyms of the ten connectives **~**, **v**, ⊃, ⊂, **|**, ≡, $\overline{\text{v}}$, $\not\supset$, $\not\subset$, $\not\equiv$ in that order in Church [1956, pp. 36–37] with the following reservations. The connectives that are depicted in Church's book by crossing each of the signs ⊃, ⊂, ≡ with an upright stroke, **|**, are depicted here as $\not\supset$, $\not\subset$, $\not\equiv$ in consequence of typographical difficulties. Church does not use any sign as a functional parasynonym of the *conjunction connective* $\wedge$ because he expresses a *relation* (in the terminology of this treatise, i.e. a *well-formed formula* or, briefly, *wff* in his terminology) of the conjunction of two relations (wffs) **A** and **B** as [**AB**] (see, e.g., *ibidem*, pp. 37, 78, 258; '**A**' and '**B**' are placeholders whose range is the class of wffs of any given Church's CALC). That is to say, Church adopts a tacit convention of omission of a conjunction sign in analogy with the convention of omission of a multiplication sign in algebra. Accordingly, Church tacitly associates the logical operation of conjunction with the algebraic operation of multiplication. In this connection the following remark should be made.

2. Let us consider a CALC, in which both the *disjunction sign* **v** and the *conjunction sign* **&** (e.g.) are employed, whereas ≡ is the *equivalence sign*. Let '**X**', '**Y**', and '**Z**' be metalinguistc (syntactic) placeholders whose range is the relation-formulas of the CALC. Owing to the object axioms of the CALC, – such axioms, e.g., as the four axioms of the Russell-Bernays system (see, e.g., Hilbert and Ackermann [1950, p. 27]), – and also owing to the rules of inference of the CALC, the signs **v** and **&** satisfy the following laws relative to *the equivalence sign*, ≡ (see *ibidem*, pp. 6–7):

i) *The commutative laws*: $[\mathbf{X}\vee\mathbf{Y}]\equiv[\mathbf{Y}\vee\mathbf{X}], [\mathbf{X}\,\&\,\mathbf{Y}]\equiv[\mathbf{Y}\,\&\,\mathbf{X}]$.

ii) *The associative laws*: $[\mathbf{X}\vee[\mathbf{Y}\vee\mathbf{Z}]]\equiv[[\mathbf{X}\vee\mathbf{Y}]\vee\mathbf{Z}]$,

$$[\mathbf{X}\,\&\,[\mathbf{Y}\,\&\,\mathbf{Z}]]\equiv[[\mathbf{X}\,\&\,\mathbf{Y}]\,\&\,\mathbf{Z}].$$

iii) *The first distributive law*: $[\mathbf{X}\vee[\mathbf{Y}\,\&\,\mathbf{Z}]]\equiv[[\mathbf{X}\vee\mathbf{Y}]\,\&\,[\mathbf{X}\vee\mathbf{Z}]]$.

iv) *The second distributive law*: $[\mathbf{X}\,\&\,[\mathbf{Y}\vee\mathbf{Z}]]\equiv[[\mathbf{X}\,\&\,\mathbf{Y}]\vee[\mathbf{X}\,\&\,\mathbf{Z}]]$.

The laws i and ii are analogous to the like laws in algebra with the multiplication sign · and the addition sign + in place of **∨** and **&** either in this or in the reverse order, and also with the sign of equality for denotata, =, in place of the equivalence sign, **≡**. At the same time, either of the distributive laws iii and iv is analogous to the algebraic distributive law for · over + relative to =, the former with **∨** and **&** and the latter with **&** and **∨** in place of · and + respectively. In accordance with the law iii, a relation $[\mathbf{X}\vee\mathbf{Y}]$ was once called the *logical product* of **X** and **Y** and a relation $[\mathbf{X}\,\&\,\mathbf{Y}]$ the *logical sum* of **X** and **Y**. However, in accordance with the law iv, $[\mathbf{X}\,\&\,\mathbf{Y}]$ might just as well have been called the logical product and $[\mathbf{X}\vee\mathbf{Y}]$ the logical sum. Therefore, for avoidance of confusion, it is presently common to call $[\mathbf{X}\vee\mathbf{Y}]$ "the *disjunction* of **X** and **Y**" or, more precisely, "the *inclusive disjunction* of **X** and **Y**", whereas $[\mathbf{X}\,\&\,\mathbf{Y}]$ is called "the *conjunction* of **X** and **Y**". Nevertheless, in analogy with the product of two numbers in algebra, some writers find it convenient to abbreviate '$[\mathbf{X}\vee\mathbf{Y}]$' by omission of **∨**, while some others abbreviate '$[\mathbf{X}\,\&\,\mathbf{Y}]$' by omission of **&**; Hilbert and Ackermann (*ibidem*, p. 12, n. 1) are among the former and Church is among the latter.

3. Analogy between **∨**, or **&**, and · is not restricted to the laws i-iv of the previous item, but rather it can be extended on the correspondence between the *truth-value* (*truth-class*) of a relation $[\mathbf{X}\vee\mathbf{Y}]$, or $[\mathbf{X}\,\&\,\mathbf{Y}]$, and truth-values of the relations **X** and **Y**. Indeed, in the usual technical phraseology, relations $[\mathbf{X}\vee\mathbf{Y}]$ and $[\mathbf{X}\,\&\,\mathbf{Y}]$ have the following truth-functional properties (see, e.g., Hilbert and Ackermann [1950, p. 14] or Quine [1951, pp. 11–13]):

i) A relation $[\mathbf{X}\vee\mathbf{Y}]$ is *true* if and only if *at least one* of the relations **X** and **Y** is *true*, and $[\mathbf{X}\vee\mathbf{Y}]$ is *false* (*antitrue*) if and only if *both* relations **X** and **Y** are *false*.

ii) A relation $[\mathbf{X}\,\&\,\mathbf{Y}]$ is *false* if and only if and only if *at least one* of the relations **X** and **Y** is *false*, and $[\mathbf{X}\,\&\,\mathbf{Y}]$ is *true* if and only if *both* relations **X** and **Y** are *true*.

The *sentence schemata* (*sentential forms*) i and ii exchange if the signs **∨** and **&** are exchanged and if simultaneously the words "true" and "false" ("antitrue") are exchanged. The predicates "*is true*" and "*is false*" are synonyms of the predicates "*has the truth-value truth*" and "*has the truth-value falsehood*" respectively. Therefore, the sentence schemata i and ii express the correspondence between the truth-value of $[\mathbf{X}\vee\mathbf{Y}]$, or $[\mathbf{X}\,\&\,\mathbf{Y}]$, and truth-values of **X** and **Y**. Let 'T' denote *the truth-value truth* and 'F' *the truth-value falsehood* (*falsity*, *antitruth*). Let also $\dot{\vee}$ and $\dot{\&}$ denote the binary operations on the truth values of **X** and **Y** which are determined by the schemata i and ii respectively. In this case, the schemata i and ii can be rewritten as:

$$[\mathrm{T}\dot{\vee}\mathrm{T}]=[\mathrm{T}\dot{\vee}\mathrm{F}]=[\mathrm{F}\dot{\vee}\mathrm{T}]=\mathrm{T},[\mathrm{F}\dot{\vee}\mathrm{F}]=\mathrm{F}. \tag{9.1}$$

$$[\mathrm{F}\,\dot{\&}\,\mathrm{F}]=[\mathrm{F}\,\dot{\&}\,\mathrm{T}]=[\mathrm{T}\,\dot{\&}\,\mathrm{F}]=\mathrm{F},[\mathrm{T}\,\dot{\&}\,\mathrm{T}]=\mathrm{T}. \tag{9.2}$$

Under either of the two sets of mapping (substitution):

$$\dot{\vee}\mapsto\cdot,\ \mathrm{T}\mapsto 0,\ \mathrm{F}\mapsto 1, \tag{9.3}$$

$$\dot{\&}\mapsto\cdot,\ \mathrm{F}\mapsto 0,\ \mathrm{T}\mapsto 1, \tag{9.4}$$

both trains of identities (9.1) and (9.2) reduce to

$$[0\cdot 0]=[0\cdot 1]=[1\cdot 0]=0,[1\cdot 1]=1. \tag{9.5}$$

It is understood that 'T' and 'F' as introduced above are parasynonyms of 'α_+' and 'α_-' (in that order) as introduced in Cmt 8.3(1).

4. It will be recalled (see Cmt 5.1(4)) that each of the functionally parasynonymous signs |, /, and **|** is called *Sheffer's stroke* after the logician who first recognized that a single logical connective is sufficient for constructing a sentential calculus (see, e.g., Hilbert and Ackermann [1950, pp. 11, 29]). Nicod [1917] was the first logician to set forth an axiomatic sentential calculus on the basis of Sheffer's stroke. Church [1956, p. 37] suggests "*Non-conjunction*" as a synonym of the name "Sheffer's stroke". In this treatise, the connective $\not\wedge$ is a functional parasynonym of Sheffer's stroke and it is called *the former anticonjunction connective* (or *kernel-sign*), as opposed to its functional synonym $\overline{\wedge}$, which is called *the latter* *anticonjunction connective* (or *kernel-sign*). Likewise, the universal logical connective $\not\vee$ of this treatise is called *the former antidisjunction connective* (or *kernel-sign*), as opposed to its functional synonym $\overline{\vee}$, which is called *the latter antidisjunction connective* (or *kernel-sign*). Thus, the relations $[\mathbf{X} \not\wedge \mathbf{Y}]$, $[\mathbf{X} \overline{\wedge} \mathbf{Y}]$, $[\mathbf{X}/\mathbf{Y}]$, $[\mathbf{X} \mid \mathbf{Y}]$, and $[\mathbf{X} \,\|\, \mathbf{Y}]$ are parasynonyms, each of which can be rendered into ordinary English as "*not both* **X** *and* **Y**". At the same time, by the definitions of $\Rightarrow$ and $\not\wedge$, $[\mathbf{X} \not\wedge \mathbf{Y}]$ can be rewritten as $[\mathbf{X} \Rightarrow \neg\mathbf{Y}]$, which can be read as: "**X**, *so that not* **Y**" if both '$[\mathbf{X} \Rightarrow \neg\mathbf{Y}]$' and '**X**' are assumed to be *formally veracious* (*f-veracious*), i.e. *accidentally f-true*. According to Simpson [1968], the connective phrases "*so that not*" and "*by which the less*" are two translations into English of the Latin word "**quōmĭnŭs**" \quominus\, which was particularly employed by Cicero and Livius. Since the signs $\not\wedge$ and $\Rightarrow\neg$ are by definition synonyms ($\not\wedge \leftrightarrow \Rightarrow\neg$), therefore either of the two can alternatively be called the *quominus sign* (*kernel-sign*, *logical connective*), in accordance with Cmt 5.1(5).

5. '$\bigvee_x$' and '$\bigwedge_x$', e.g., are synonyms of the conventional quantifiers '$(\exists \boldsymbol{x})$' and '$(\forall \boldsymbol{x})$', which are read as "*for some* $\boldsymbol{x}$:" or "*for at least one* $\boldsymbol{x}$:" or "*there exists at least one* $\boldsymbol{x}$ *such that*" and as "*for all* $\boldsymbol{x}$:" or "*for every* $\boldsymbol{x}$:" respectively. Still, in Church [1956, p. 171, D13], '$(\boldsymbol{x})$' is used instead of '$(\forall \boldsymbol{x})$'.

6. Most generally, the correspondence between the logical connectives and pseudo-quantifiers that are employed in this treatise and the logical connectives and quantifiers that are employed in the various CALC'i can be established with the help of the verbal expressions that I use for the former when they occur in the CFCL interpretands of OptER's of A_1, namely:

"not" for '$\neg$',

"or" or "ior" for '$\vee$',

"and" or "&" for '$\wedge$',

"if … then –" or "… only if –" for '$\Rightarrow$',

"if" for '$\Leftarrow$',

'if and only if" or "iff" for '$\Leftrightarrow$',

"neither … nor –" for '$\not\vee$' or '$\overline{\vee}$',

"not both … and –" for '$\not\wedge$' or '$\overline{\wedge}$',

"but not" for '$\overline{\Rightarrow}$', “not … but –” for ‘$\overline{\Leftarrow}$’,

“either … or – but not both” or “xor” for ‘$\overline{\Leftrightarrow}$’,

“for some ∗:” or “for at least one ∗:” or “there exists at least one ∗ such that” for ‘$\bigvee_{*}$’,

“for all ∗:” or “for every ∗:” for ‘$\bigwedge_{*}$’,

“for some but not all ∗:” or “for strictly some ∗:” for ‘$\breve{\bigvee}_{*}$’,

“for at most one ∗:” or “there exists at most one ∗ such that” for ‘$\overset{\frown}{\bigvee}{}^{1}_{*}$’,

“for exactly one ∗:” or “there exists exactly one ∗ such that” for ‘$\bigvee^{1}_{*}$’,

the understanding being that in any one of the above definitions alike ellipses should be replaced alike by the appropriate relations or relation-valued variables.

9.3. “Set” versus “class”

9.3.1. General remarks about uses of the metaterms “class” and “set”

Df 9.1. 1) The common name “*a class*”, i.e. the count noun “class” bound (limited) by the indefinite article, is used in this treatise and in Psychologistics in general as an abbreviation of the disjunctive phrase “*a nonempty class or the empty class*”, whereas the constituent common name “a nonempty class” is in turn an abbreviation of the disjunctive phrase “*a singleton, i.e. a one-member class, or a multipleton, i.e. a many-member class*” subject to the following definitions.

a) An object of a sapient subject (as me), – such an object, e.g., as a nonempty individual, a class (see the next item), or a state of affairs, – which the subject regards as a *unique one* and which he provides with a *logographic* or *phonographic* (*verbal*) *proper name*, is *ipso facto* an *object* (*substance*) *sui generis*, i.e. an object *of its own kind* (*class*). That is to say, a properly named *unique* object, i.e. an object having a *unique property that is not shared by any other objects*, generates *ipso facto* a class, in which it is the only member, a class that is accordingly called a *singleton*, i.e. a *one-member class*. A unique distinguishing property of the member of a singleton is called a *concept of the singleton* or less explicitly a *singleton-concept*. Depending on the object in question, its proper name is either a *proper substantive*, i.e. a *proper noun* or *noun equivalent* (e.g. “Aristotle”, “Abraham Lincoln”, “the founder of logic”, or “the 16th president of the USA”), if the object is a substance, or a *proper declarative sentence* (e.g. “Aristotle is the founder of logic” or “Abraham Lincoln is the 16th president of the USA in the years 1861–65”), if the object is a state of affairs (fact).

b) In agreement with Aristotelian doctrine of *nominalism*, if two or more, i.e. finitely or infinitely many, particular (separate, elemental) coentities of a sapient subject (as me), i.e. entities of which the subject is conscious, have a certain single or cumulative *conceptual property in common* with respect to him and if therefore he provides every one of the coentities with the same *common name* [comprising an *unlimited* (and hence *article-free*) *count name and the preceding indefinite article*] then all those particular coentities become *ipso facto members* (*elements*) of a single whole *mental* (*psychical*, *abstract*) *universal coentity* of the sapient subject, which is called *the class*, or *nominal universal*, *of all particular coentities* in question or, more generally (less explicitly), a *multipleton*, i.e. a *many-member class*. Depending on the particular coentities, their common name in question is either an *unlimited* (and hence *article-free*) *count substantive*, i.e. an *unlimited count noun* or *noun equivalent*, in a singular number form (e.g. “animal”, “man”, “biont”, “living organism”, “mortal [being]”, “author of Principia Mathematica”, etc), if the coentities are substances, or a *common declarative sentence* (e.g. “It is raining” or “The sky is blue”, “The sky is dark”, etc), if the coentities are states of affairs (facts). In any case, the above common name turns out to be a *proper name of the class* and at the same time it becomes an *apparent proper name of a common* (*general*, *certain*, *particular but not particularized*, *concrete but not concretized*) *member (element) of the multipleton* – a common member that *represents the whole class*, thus being just *another hypostasis* (*way of existence*, *aspect*) *of the class*. Consequently, the class is alternatively called *the range of the common name of members of the class*, the understanding being that when I (e.g.) mentally metamorphose this name into an apparent proper name of the common member of the class, I use the common name along with its range in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* the range as *my as if extramental* (*exopsychical*) *object* – the very object that I call a common member of the class. I also say that both *the common name and its* [*original*, *unpolarized*] *range are used for mentioning a common element of the range* or that, less explicitly, *they are used but not mentioned*, whereas the range is said to be *connoted by*, or to be *the connotatum* (*connotation value*, pl. “*connotata*”) *of*, *the common name*. At the same time, the common element of the range, which is a metamorphosed (polarized, externalized) hypostasis of the latter, is said to be *denoted by*, or to be *a denotatum* (*denotation value*, pl. “*denotata*”) *of*, the common name, being its apparent proper name. The conceptual property, which all members of a given multipleton have in common and which determines that multipleton, is called a *concept of the multipleton* or less explicitly *a multipleton-concept*.

c) In practice, if the coentities *collectivized* (*classified*) are substances then the common name of the coentities and the count name that is used as an unlimited proper name of their class are related as follows. The former name is the homonym of the latter name if the native language used has no indefinite article. The former name comprises the latter name and the preceding indefinite article if the native language used has such an article. If the common name in question has a logographic synonym, i.e. a synonymous logographic variable, then the whole of the above said applies, *mutatis mutandis*, to that variable in place of the common name. In accordance with the pertinent Aristotelian terminology, a *nonempty individual* is called a *primary substance*, whereas a class of nonempty individuals is called a *secondary substance*.

d) A *singleton-concept* or a *multipleton-concept* is indiscriminately called a *class-concept*, i.e. a *concept of a class*. Consequently, in accordance with the above item 1b, either of the synonymous common names "a multipleton" and "a many-member class" is used in this treatise and in Psychologistics in general as an abbreviation of the descriptive phrase "a *class of equivalence of elemental substances with respect to a certain cumulative conceptual property, which the substances have in common and which is called a concept of the class or less explicitly a class-concept*".

2) The metaterm "class" as defined in the above item 1 is said to be *unrestricted* or to be "*class*" *sensu lato*, while the qualifier "*sensu lato*" means «*in the broadest sense*», although in the general it may mean «*in a broad sense*» (see below). In practice, however, the class-denotatum of the metaterm "class" is often *restricted*, formally or informally (e.g. contextually), so that the metaterm turns into its *homonym* (*homograph* and *homophon*), which is said to be "*class*" *sensu stricto*, while the qualifier "*sensu stricto*" means, generally, «*in a narrow sense*» or, more specifically, «*in the pertinent narrow sense*». In order to distinguish between the two *homographs* "*class*" *sensu lato* and "*class*" *sensu stricto*, they can be replaced with the two different monosemantic graphomyms (graphic names) "*class sensu lato*" and "*class sensu stricto*" respectively. In this case, the former homograph, i.e. discriminately

"*class sensu lato*", assumes (takes on) a *class sensu lato* as its accidental (circumstantial) denotatum and the latter homograph, i.e. discriminately "*class sensu stricto*", assumes a *class sensu stricto* as its accidental denotatum. Thus, either one of the Latin postpositive qualifiers "*sensu lato*" and "*sensu stricto*" is an operator (attributive modifier, function expression), which equivocally applies to the count noun (metaterm) "class", to its class-denotatum, and to a class being its accidental (circumstantial) distributive denotatum. For instance, as was mentioned in Df A4.1(1), in a biological taxonomy of bionts (BTB), the noun "class" is used in a narrow sense, i.e. as "*class*" *sensu stricto*, for denoting a taxon (taxonomic *class sensu lato*) ranking between the *orders* and *divisions* of either kingdom **Plantae** or **Fungi** or between the orders and phyla of the kingdom **Animalia**.

3) Once the class-denotatum of the metaterm "class" *sensu stricto* and hence of its univocal substituend "class *sensu stricto*" is defined, this denotatum can be *extended* to a greater or lesser extent, so that the former metaterm turns into its *homonym* (*homograph* and *homophon*), which is said to be "*class*" *sensu lato* and which can, just as in the previous case, be replaced with the monosemantic graphonym "*class sensu lato*". It is understood that the homograph "*class*" *sensu lato* and its monosemantic subsituend "*class sensu lato*" assume a *class sensu lato* as its accidental denotatum. That is to say, in this case, the qualifier "*sensu lato*" is used in the same way as in the previous items with the only difference that now it means, generally, «*in a broad sense*» or, more specifically, «*in the pertinent broad sense*». Thus, "*sensu stricto*" and "*sensu lato*" are *relative* and. *ad hoc*, i.e. *epistemologically relativistic*, *antonymous qualifiers*, whose absolute senses should be defined specifically in each specific case.•

Preliminary Remark 9.2. 1) "Set" is a technical metaterm (metalinguistic term) of a set theory, which is widely used informally throughout this treatise. After creation of set theory, at first of the *naive* one by Georg Cantor during the years 1878-84[6], and then of the *axiomatic* one by Ernst Zermelo [1908], writers on mathematics, including set theory itself, and on mathematics-related disciplines as symbolic logic and theoretical physics, began using and often *misusing* the technical term "set" of set theory instead of or interchangeably with the word "class". For instance, Hilbert and

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[6] Bibliography of numerous publications of Cantor on set theory in German during those years can be found in Fraenkel et al [1973, p. 354].

Ackermann [1950, p. 46, footnote 1] say: «In mathematics, the term “set” is used rather than “class”». Unfortunately, Hilbert and Ackermann and most users of the nouns “class” and “set” do not explicate the difference between the denotata of the two nouns and do not warn that, owing to that difference, such a use of the noun “set” is often incorrect. When both nouns “set” and “class” are employed in the same writing without explaining the semantic difference between the two, it is often impossible to decide whether this is done just for avoidance of terminological monotony or from some considerations of principle. It is presently common to postulate that *a set is a class but not necessarily vice versa* (see, e.g., Fraenkel et al [1973, Chapter II, §7] or the article «**class**» in Wikipedia). Still, most sets dealt with in axiomatic set theories are unnamed and unnamable in the sense that they have no proper names, so that they are mentioned only as members of the ranges of some variables. Therefore, this postulate implies either that there are objects, which are called “classes” and which have no proper names, but this implication contradicts any *nominalistic* (or *intuitionistic*) *definition* of a class (as Df 9.1), or it means that sets cannot be treated as classes unless they are is freed of the pertinent peculiar properties by properly revising the axiom of power sets and by disregarding the axiom of choice.

2) Letting meanwhile a solution of the above epistemological problem aside, the term “set” can, at first glance, be formally defined thus: *a set* is a class belonging to a *domain*, which is called *a system of axiomatic set theory* or briefly *an axiomatic set theory* and which is determined by the totality of conceptual properties that are collectively called *the axioms of that system* – just as, e.g., the classes *point*, *straight line*, and *circumference* are determined by the totality of conceptual properties collectively called *the axioms of Euclidean geometry*. Still, many writers on axiomatic set theories use the count noun “set”, not only for *verbally* (*phonographically*) *denoting* (*calling*, *mentioning*) certain abstract objects in the ranges of *logographic variables* of an axiomatic set theory, but also for verbally denoting some *classes of concrete* (*not abstract*, and hence *nonempty*) *individuals* with the purpose, as they think, to illustrate abstract set-theoretic relations. Unfortunately, many of such examples are inadequate and are therefore counterproductive and misleading. For instance, either one of the two equivalent statements:

«The set of Irishmen is included in the set of men»,

«The set of Irishmen is a subset of the set of men» of «*Axiomatic set theory*» by Suppes [1960, p. 22], which the author uses for illustrating the set-inclusion (subset-superset) relation, is *meaningless* because the names "the set of Irishmen" and "the set of men" *have no denotata* (*are empty*, *naked*). By contrast, the names "the class of Irishmen" and "the class of men" do have denotata, – just as the count nouns "Irishman" and "man" being their synonyms. Particularly, either of the names: "the class of men" and "man" (without any modifiers) denotes the **species** *Homo sapiens*, i.e. a certain **specific class** *of men*, and **not a specific set**, i.e. **not a fixed collection** *of men*.

3) In general, a *taxon* (*taxonomic class*, pl. "*taxa*" or "*taxons*") of a *biological taxonomy of bionts* (*BTB*) is a class of an *indefinite number of bionts*, i.e. *individual living organisms*, some of which lived, some are living, and some other are expected to live in the future in the *biosphere* (*ecumene*) of the Earth; the taxon is determined by certain conceptual properties, which its any two or more members have in common, and is framed and identified by its *taxonym* (taxonomic name). Here follows, for instance, a biological definition of the *common individual name* "a man" and at the same time of the *count name* (*count noun*) "man" – being an informal *proper class-name* synonymous with the formal *taxonym* (*taxonomic name*) "*Homo sapiens*".

> **Df 9.2.** *A man*, or *human being*, is a member of kingdom Animalia, subkingdom Eumetazoa, phylum Chordata, subphylum Vertebrata, class [sensu stricto] Mammalia, subclass Eutheria, order Primates, family Hominidae, genus *Homo*, species *sapiens*.•

This definition is effective in the framework of the *five-kingdom BTB*, which I call the *Linnaeus-Whittaker-Margulis taxonomy* (*LWMT*), after Whittaker [1969], who revised the classical *two-kingdom Linnaean taxonomy* (*LT*) in the light of some modern concepts of genetic and evolution theories, and after Lynn Margulis, who supplemented Whittaker's revision with some important modifications discussed in Margulis and Schwartz [1987]. The LT is described in general outline, e.g., in Villee [1957, chapter VI] and in Campbell [1990, pp. 484–486]. The LWMT is substantiated and followed closely as a general frame of reference in Campbell [1990, Unit Five, pp. 505–674, 518–520ff]. In the framework of the LT, the occurrence of the taxonym "Eumetazoa" in Df 2.8 should be replaced with an occurrence of "Metazoa". Df 9.2 is in fact a definition of the species, denoted by the count noun "man" (without any modifier), through the kingdom Animalia as the pertinent informal *genus* [sensu lato], denoted by the generic name "Animalia", and the differentia (differences), denoted by the conjunction of the qualifiers of decreasingly narrow ranges: "Eumetazoa", "Chordata", etc to "*sapiens*". In this case, "man" turns out to be an informal synonym of "*Homo sapiens*". In this case, both names can be called denotative taxonyms, In this case, both names "man" and "*Homo sapiens*" can be called *denotative taxonyms*, while the common name "a man" can be called a *connotative taxonym*. Thus, the class *man* is not a set. On the other hand, a group of men that are gathered together in a certain room at a certain time is the pertinent set of men, being a subclass [sensu lato] of species *Homo sapiens*.

4) Likewise, in spite of the fact that the [system of] set theory by Halmos [1960] is called "*Naïve set theory*", it is a *quasi-axiomatic* one. This is likely why Halmos says (*ibid*. pp. 1, 2):

> «By way of examples we might occasionally speak of sets of cabbages, or kings, and the like, but such usage is always to be construed as an illuminating parable only, and not as a part of the theory that is being developed.»

Letting aside the humorous mood of the above quotation, just as the names "set of Irishmen" and "set of men", the *unlimited* names "set of cabbages" and "set of kings" have no denotata and therefore they cannot be used for illustrating any set-theoretic relations. At the same time, like the names "the class of Irishmen" and "the class of men", the names "*the class of cabbages*" and "*the class of kings*" have denotata, – just as the count nouns "*cabbage*" and "*king*" being their synonyms. Particularly, either of the names "the class of cabbages" and "cabbage" (without any modifiers) denotes the genus *Brassica*, which is not certainly a set. The above examples illustrate that, in contrast to what Hilbert and Ackermann state, a class is not necessarily a set. The following definition is designed to demarcate the difference between a set and a class not being a set.•

9.3.2. "Regular class" ("set") versus "irregular class"

Df 9.3: ***A necessary and sufficient condition for a class to be a set.*** 1) Under Df 9.1, a *nonempty class*, which is identifiable by one or more *proper names*, either *direct* (*denotative*, *formal* or *informal*) ones or *oblique* (*connotative* and hence *informal*) ones, is called:

a) a *nonempty regular class* or *nonempty set* if and only if it has *permanent* (*invariable*) *member population*, i.e. if and only if it is either a *singleton* (one-member class) of a distinct *element* or a *multipleton* (many-member class) of a finite or infinite number (two or more) *elements*, which *persistently coexist* (*exist simultaneously*) either in the external (extramental, exopsychical) world or in the mental (psychical) realm of the interpreter (as me) of any one of the proper names of the class;

b) a *nonempty irregular class* if it is not regular, i.e. if it is not a set.

Consequently, in contrast to sets, *neither a bijective function nor a surjective function* can be defined on a nonempty irregular class or from one nonempty class onto another nonempty class if at least one of the two is irregular. Particularly, no *equipollence relation* (as defined for sets, e.g., in Suppes [1960, p. 91]) can be established between such two nonempty classes. Also, *no partial order relation* $\leq$ and *no relation of well ordering* (as defined for sets, e.g., in Halmos [1960, Sections 14 and 17, pp. 54–58 and 66–69]; see also Cmt 9.4 below in this subsection) can be defined on an irregular class.

2) A *unique ideal element* (*abstract entity*) of any given conventional [system of] class or set theory, i.e. of a theory of the class-membership predicate $\in$, – the ideal element, which is postulated to be a *class that has no members* and which turns therefore to be *the empty* (*indivisible*) *subclass* (*part*) *of every nonempty* (*divisible*) *class of the theory and of itself*, is called *the empty class* or *the empty set*, i.e. *the empty regular class*, and also *the empty individual*. The empty class (empty set) is conventionally denoted by '$\varnothing$' (e.g. in Halmos [1960, p. 8]); it is denoted by 'Λ' in Whitehead and Russell [1910; 1925; 1962, pp. 216, 217, **∗24.02**] and by 'O' in Fraenkel et al [1973, p. 39]). A proper logographic name of the empty class, as '$\varnothing$', 'O', or 'Λ', is called a *zero*, but not necessarily vice versa, because any null element is denoted by the appropriate zero (cf. Cmt A4.1(3)). The empty class (empty set) is a *universal substance* independent of a concrete class or set theory, in which it is introduced, – the substance that is called the *universal nothing* or *universal nil*, and also the *void* (cf. Cmt A4.1(3)).

3) A nonempty set or the empty set, i.e. a nonempty regular class or the empty regular class, is indiscriminately called a *set* or *regular class*. A class that is not regular, i.e. that is not a set, is called an *irregular class*. Hence, *an irregular class*

cannot be empty. Therefore, a *nonempty irregular class* is alternatively called an *irregular class* and vice versa. In the contemporary literature on logic and mathematics, an irregular class, i.e. a nonempty class not being a set, is called a *proper class*, whereas a regular class, i.e. a set, is sometimes called a *small class* (see, e.g., Fraenkel et al [1973, pp. 128, 134–135, 167] for the former term or the article «**class**» in Wikipedia for both terms).•

Cmt 9.1. 1) In accordance with Df 9.3, the count noun "class" is a generic substantive [name], whose range includes both regular classes and irregular classes. Hence, a set is a class but not necessarily vice versa, – as usually postulated (see Preliminary Remark 9.2(1)). Formally, this means that any *class-related axiom* (*CRA*) is a *set-related axiom* (*SRA*) *and vice versa*, whereas any *class-related definition* (*CRD*) or any *class-related theorem* (*CRT*) is respectively a *set-related definition* (*SRD*) or a *set-related theorem* (*SRT*) *but not necessarily vice versa*.

2) A consistent full-scale *axiomatic class theory* (*ACT*) that can be developed on the basis of $\mathsf{A}_{1\in}$ and Df 9.1 will be called (denoted phonographically, i.e. verbally) a *nominalistic*, or *intuitionistic*, *ACT* (*NACT* or *IACT*) and be denoted (logographically) by 'C_1'. Likewise, a consistent full-scale *axiomatic set theory* (*AST*) that can be developed on the basis of C_1 and Df 9.3, i.e. on the basis of $\mathsf{A}_{1\in}$ and Dfs 9.1 and 9.3, will be called a *nominalistic*, or *intuitionistic*, *AST (NAST* or *IAST*) and be denoted by 'S_1'. The most immediate and simplest NACT or NAST is a *one-individual one* (*OINACT* or *OINAST*), which will be denoted by '$\mathsf{C}_1^{\mathsf{OI}}$' or '$\mathsf{S}_1^{\mathsf{OI}}$' respectively. In order to develop $\mathsf{C}_1^{\mathsf{OI}}$ (cf. Cmt 8.3), the CFCL interpretands of the pertinent euautographic definitions, axioms, and theorems of $\mathsf{A}_{1\in}$ should be supplemented with a *complete set* (*conjunction*) *of semantic veracious CRA's* and then all basic semantic CRT's should be proved from the the CRA's and from the CFCL interpetands of the pertinent definitions and pertinent valid ER's (euautographic axioms and euautographic theorems) of $\mathsf{A}_{1\in}$, along with making the pertinent CRD's. A full-scale *nominalistic*, or *intuitionistic*, *axiomatic mass theory* (*NAMT* or *IAMT*), which is unavoidably a *one-individual* theory as well and which will denoted by 'M_1', can be developed on the basis of $\mathsf{A}_{1\subseteq}$ likewise.

3) By supplementing $\mathsf{C}_1^{\mathsf{OI}}$ and hence $\mathsf{S}_1^{\mathsf{OI}}$ with appropriate *verbal axioms* that introduce an indefinite number of *indistinguishable nonempty individuals* (cf. Cmt 9.6

in subsection 9.4 below), C_1^{OI} and S_1^{OI} can be turned into a *many-individual NACT* (*MINACT*) and into a *many-individual NAST* (*MINAST*) to be denoted by 'C_1^{MI}' and 'S_1^{MI}' respectively. The basic axioms of each one of the three theories C_1^{OI}, M_1, and C_1^{MI} are given in section A5 (Appendix 5). In this case, in accordance with the above item 1, every CRA of C_1^{OI} or C_1^{MI} is respectively an SRA of S_1^{OI} or S_1^{MI} and vice versa, whereas every CRD and every CRT of C_1^{OI} or C_1^{MI} is respectively an SRD and an SRT of S_1^{OI} or S_1^{MI} but not necessarily vice versa. Therefore, in order to turn C_1^{OI} into S_1^{OI}, or C_1^{MI} into S_1^{MI}, the former should be supplemented by *SRD's* of various relations such as a *bijective or surjective function*, a *partial order relation* ≤, and a *relation of well ordering,* which are known from the conventional set theories, and whose domains of definition and variation can, in accordance with Df 9.3, be only regular classes, i.e. sets, of C_1^{OI} or C_1^{MI} respectively. Accordingly, the classes of C_1^{OI} or C_1^{MI}, for which those relations cannot be defined, are, in accordance with Df 9.3, irregular ones. After disregarding these, SRT's of S_1^{OI} or S_1^{MI} can be proved from both, the SRA's and SRD's of S_1^{OI} or S_1^{MI} respectively.

4) As one pleased, he can supplement S_1^{OI} or S_1^{MI} with any appropriate peculiar axiom of conventional axiomatic set theories as the axiom of power sets or the axiom of choice or both, in order to get a conventional, *non-nominalistic* (*non-intuitionitic, realistic*) *set theory*. Such a theory is an unrestricted intellectual game – a light of fancy, which is irrelevant to Df 1.1.•

Cmt 9.2. In order to illustrate the difference between a *set,* i.e. a *regular class*, and an *irregular class* as explicated in Df 9.3, I shall consider several examples.

1) In accordance with Df 9.3(1a), a class of a finite number of substances, which permanently coexist either in my external (extramental, exopsychical) world or in my mental (psychical) realm, is a set, no matter what the above substances are. Particularly, the following classes are sets.

a) A *singleton*, which is denoted by a certain proper name., no matter what its member is.

b) 21 kings of England from Egbert (A.D. 827) to Harold II (A.D. 1086), i.e. before the Norman Conquest, form the set of Anglo-Saxon, Dane, and Saxon kings of

England, because these coexist in a certain order in my mental realm (for a chart of the above kings, see, e.g., Taylor [1863, p. 509]).

c) The English alphabet, which consists of twenty-six conventionally ordered letter-types (token-classes) and which is represented by any given list of their graphic tokens (materializations), is the set of the above twenty-six token-classes.

2) In accordance with Cmt 8.1(4), in the framework of a certain full-scale AST (axiomatic set theory), $\mathcal{S}$, the *classes* of natural, integer (integral), rational, real, and complex numbers, $\mathcal{N}$, $\mathcal{I}$, $\mathcal{Q}$, $\mathcal{R}$, and $\mathcal{C}$, are sets. Every number of any given one of the above sets, including the pertinent *null-number*, i.e. 0_N, 0_I, 0_Q, 0_R, or 0_C in that order, is also a *set*;. 0_N is the empty set ($0_N=\varnothing$), whereas all other null-numbers are nonempty sets.

3) In accordance with the items 7–9 of Cmt 8.1, the class of vectors, $\hat{\mathcal{E}}_{(n)}$, of an *n*-dimensional linear (vector) Euclidean (inner product) space, $\hat{\boldsymbol{\mathcal{E}}}_{(n)}$, over the field of real numbers, $\boldsymbol{\mathcal{R}}$, is a set, whereas separate vectors of $\hat{\boldsymbol{\mathcal{E}}}_{(n)}$, including *the null-vector* $\hat{0}_{(n)}$, are *nonempty individuals*, unless $\hat{\boldsymbol{\mathcal{E}}}_{(n)}$ is the *arithmetical n*-dimensional linear Euclidean space, $\bar{\boldsymbol{\mathcal{E}}}_{(n)}$, whose vectors are arithmetical ones, i.e. ordered *n*-tuples of real numbers, and are therefore sets.

$$\bar{\mathcal{E}}_{(n)} = \underbrace{[[\ldots[[\mathcal{R}\times\mathcal{R}]\times\mathcal{R}]\times\ldots]\times\mathcal{R}]}_{n}.$$

An *n*-dimensional Euclidean vector space, $\hat{\boldsymbol{\mathcal{E}}}_{(n)}$, over the field of real numbers, $\boldsymbol{\mathcal{R}}$, is called an *abstract* one if it is not *arithmetical*.

4) As I have already pointed out in the item 3 of Preliminary Remark 9.2, the taxa of a BTB, i.e. the class-denotata of the taxonyms of the BTB, are irregular classes [sensu lato], and not sets. However, the taxonyms themselves are finite (although very large) in number and therefore their totality can be regarded as a set..

5) In contrast to the English alphabet (see the item 1c above in this comment), the English lexicon (vocabulary) is an irregular class simply because it is impossible to collect together an indefinite number of English linguistic forms, being in use in any given time, in order to represent this class.

6) My library is an extension of the class of my books. This class is a set until I bye some new books or get rid of some old ones, i.e. until the set turns into another, somewhat modified, updated set. Thus, from an alternative viewpoint, my library is

the class of equivalence of the sets of my books at certain sequential discrete instances, at which a current set of my books is updated.

7) Tychautographs and euxenographs that occur in this treatise form two different complementary classes, but these are not sets because I can, at any moment, mentally turn any tychautograph into a euxenograph and vice versa. Putting it figuratively, the division of graphonyms into tychautographs and euxenographs is similar to two communicating vessels (receptacles): the vessels are enduring (stable), but some elements of their contents can pass from one vessel into the other thus changing their character.•

Cmt 9.3. In order to make explicit the most essential property of an irregular class, by which it is distinguished from a set and which underlies Df 9.3, I shall, by way of example, consider the common name "a man" again. The same applies to any common name that is formed by adhering a count (quantifiable) with the indefinite article "*a*" or "*an*". I construe the class (species) *man* as my entire *conception* (*notion*, *idea*, *cognition*, *recept*, *brain symbol*) of a man, which enables me to *distinguish* a man from *a non-man*, i.e. from any object of mine that is not a man. The conception includes particularly the abstract memory-image of my visual sensations of various men. I may express the fact that I have the conception (cognition) of a man, i.e. the fact that *I have the class man in the realm of my conceptions*, by saying that *I know what a man is*. In order to decide whether or not a certain object that I see at a certain moment of time is a man, I need know nothing about *any set of men*. In particular, when I encounter a man in a street of N, the city where I live, I recognize immediately that I have encountered a man, and not, say, a cat or a stone, without having any knowledge of *the set of all men occurring in N at that moment*, to say nothing of *the set of all men occurring on the Earth at that same moment.* Moreover, I am unable to solve the problem of defining a denotatum of the name "the set of all men occurring in N at that moment" in principle. Indeed, whichever instant of time I may select, some of the men which supposedly occur in N at that instant are perhaps birthing, whereas some others are perhaps dying. Also, some of the supposed inhabitants of N may, at the instant selected, cross the administrative border of N either inward or outward. Fortunately, I do not need to solve any problem of that kind in order to decide whether or not a *concrete* (*particular*) object of mine is entitled to be commonly called "a man". I decide this *immediately*, i.e. *here and now* or *there and* *then*, with the help of the class *man* which I have in the realm of my conceptions. Moreover, the above analysis shows that *the name* “*the set of all men occurring in N at that moment*” *has no denotatum with respect to me*, i.e. *this name is empty*. Loosely speaking, the set of all men occurring in N at any given instant *does not exist*. I may of course *postulate* (*take for granted*) that there is the set (aggregate) of all men, or the set of all trees, or the set of all stones, etc, occurring on the Earth at any given instant of time. However, in the everyday life, I recognize a man or a tree or a stone, and I also distinguish among the three objects, e.g., with the help of the corresponding conceptions (classes of equivalence) which I have in my *mind* (*cerebral cortex*) and which hence exist *within the physical limits of my body*; I do not need to appeal to any obscure hypothetical sets which supposedly exist outside my body. The class man alone does not enable me to recognize (identify) a concrete man which I acquainted previously. In order to do so, I have to utilize the differentia (differences distinguishing properties) that this man has in my mind as compared to any other man.●

Cmt 9.4. 1) A *partial order* [*relation*] (or sometimes simply an *order* [*relation*]) *in a given nonempty set u*, conventionally denoted by ‘≤,’, is a *binary relation in intension*, i.e. a *binary predicate*, which has the following three properties for every three elements x, y, and z in u (see, e.g., Halmos [1960, pp. 54-58]):

i) *Reflexivity* $x \leq x$.

ii) *Asymmetry*: If $x \leq y$ and $y \leq x$ then $x = y$.

iii) *Transitivity*: If $x \leq y$ and $y \leq z$ then $x \leq z$.

By definition, $y \geq x$ if and only if $x \leq y$, so that ≥ is the inverse of the relation ≤, which is therefore called an *inverse partial order* [*relation*] (or simply an *inverse order* [*relation*]). In this case, the partial order [relation] ≤ can for more clarity be qualified *direct.* Also by definition, $x < y$ and, equivalently, $y > x$ if and only if $x \leq y$ and $x \neq y$, or, equivalently, if and only if $y \geq x$ and $x \neq y$. Hence, for no elements x and y, $x < y$ and $y < x$, or, equivalently, $y > x$ and $x > y$, hold simultaneously. That is to say, the relation < does not satisfy the variants of the above items i) and ii) with ‘<’ in place of ‘≤’, but it does satisfy the like variant of the item iii), namely if $x < y$ and $y < z$ then $x < z$. Therefore, if $x \leq y$ is defined so as to satisfy only the latter transitivity relation and to have no properties of reflexivity and symmetry then the relation ≤ that has properties i)–iii) can be defined in terms of < thus: $x \leq y$ if and only if $x<y$ or $x=y$. Thus, the relations < and > are *restrictions* of ≤ and ≥, whereas the latter are extensions of the former. Also, like ≤ and ≥, the relations < and > are *mutually inverse*. Accordingly, < is called a *weak*, or more specifically, *weak direct, partial order* [*relation*], whereas ≤ is, by contrast, called a *strict*, or more specifically, *strict direct*, *partial order* [*relation*]; and similarly with ‘>’, ‘≥’, and “*inverse*” in place ‘<’, ‘≤’, and “*direct*”.

2) In ordinary language, the predicate (relation in intension) ‘≤’ is read as «*is less,* or *smaller, than or equal to*», whereas the predicate ‘≥’ is read as «*is greater,* or *larger*, *than or equal to*». Accordingly, the predicate ‘<’ is read as «*is less,* or *smaller*, *than*» and also as «*is a predecessor of*», whereas the predicate ‘>’ is read as «*is greater,* or *larger, than*» and also as «*is a successor of*». In *alternative* terminology, ‘≤’ is read as «*is less,* or *smaller, than*», whereas ‘≥’ is read as «*is greater,* or *larger, than*». Accordingly, ‘<’ is read as «*is strictly less,* or *strictly smaller, than*» or, as before, «is a predecessor of», whereas the predicate ‘>’ is read as «*is strictly greater,* or *strictly larger, than*» or, as before, «is a successor of». In forming the latter, *alternative,* verbal (phonographic) counterparts of the logographic signs ‘≤’, ‘≥’, ‘<’, and ‘>’, I have been guided by the following general principle (meta-axiom).

> **Ax 9.1:** ***The principle of simplicity of fundamental terms.*** If one of two given comparable classes is more general than, i.e. is a strict superclass of, the other, then the former should be denoted by a simpler sign and be called by a simpler verbal term than the latter.

I have not, however, applied the above principle to the signs ‘≤’, ‘≥’, ‘<’, and ‘>’ themselves. Indeed, in accordance with that principle, the simpler signs ‘<’ and ‘>’ should have been freed from their presently common meanings (modes of use) and be used instead of the conventional compound signs ‘≤’ and ‘≥’. At the same time, the presently common meanings (modes of use) of the signs ‘<’ and ‘>’ should have been assigned to some complex signs, say to ‘<′’ and ‘`>’ respectively. Still, for avoidance of confusion, I have decided to use the signs ‘≤’, ‘≥’, ‘<’, and ‘>’ and to render them into English conventionally. A like remark applies to the signs ‘⊆’, ‘⊇’, ‘⊂’, and ‘⊃’, which are utilized in the treatise.

3) A set is called a *partially ordered set* if it is a *domain of definition* and hence a *domain of variation* of a partial order [relation] ≤. Using the pertinent

conventional phraseology and nomenclature, Halmos [1960, p. 55] states the above definition thus:

> «A *partially ordered set* is a set together with a partial order in it. A precise formulation of this "togetherness" goes as follows: a partially ordered set is an ordered pair $(X, \leq)$, where X is a set and $\leq$ is a partial order in it. This kind of definition is very common in mathematics; a mathematical structure is almost always a set "together" with some specified sets, functions, and relations. The accepted way of making such definitions precise is by reference to ordered pairs, triples, or whatever is appropriate.»

While a statement of togetherness of various aspects of a mathematical structure, such as a partially ordered set or an algebraic system, is an intelligible and consistent informal verbal definition of the structure, a formal and as if precise definition of the structure in the form of certain ordered multiple can be criticized for being inconsistent. For instance, if x is an element of a partially ordered set $\boldsymbol{u}$, which is defined as $\boldsymbol{u} \doteqdot (u, \leq)$, then $x \in u$, but $x \notin \boldsymbol{u}$. In an axiomatic set theory, a binary relation on a set u, – such a relation, e.g., as any singular function f from u to (onto or into) u or such as $\leq$ or $<$ from u to u, – is treated as the respective *set of ordered pairs of elements of u*, i.e. as the respective *subset of the direct product* $u \times u$. Therefore, a partially ordered set $\boldsymbol{u}$ can consistently be defined as $\boldsymbol{u} \doteqdot u \cup \leq$. In this case, if $x \in u$ then $x \in \boldsymbol{u}$, as it must be. Any mathematical structure and particularly any algebraic system can be defined likewise.

4) An ordered set is called a *well ordered* (or *well-ordered*) *set*, while its ordering is called a *well ordering* (or *well-ordering*), if every nonempty subset of it has a smallest member. If x and y are elements of a well ordered set then $\{x, y\}$ is a nonempty subset of the latter and therefore it has either x or y as its first element, so that either $x \leq y$ or $y \leq x$. Hence, *every well ordered set is totally* (*linearly*) *ordered, but not necessarily vice versa*. For instance, the set of natural numbers and the set of nonnegative integers are well ordered and hence they are totally (linearly) ordered. However, a partially ordered set is not necessarily well ordered. In general, a set can have several different compatible but incomparable (mutually independent) orderings.

5) Most axiomatic set theories admit as their last axiom the *axiom of choice*, which is most simply formulated in Suppes [1960, p. 239]) thus:

«For any set *A* there is a function *f* such that for any non-empty subset *B* of *A*, $f(B)\in B$.»

There is a number of propositions equivalent to the axiom of choice (see, e.g., *ibid*, p. 250). The choice axiom or any equivalent proposition allows proving the *well ordering theorem*, which is most simply and most generally formulated in Halmos [1960. p. 68] thus:

«Every set can be well ordered.»

The axiom of choice, the well ordering theorem, and any proposition that is equivalent to either of the former two is a so-called existence proposition, i.e. either an *existence axiom* or an *existence theorem*, – a proposition that asserts the existence of a distinguished psychical (mental) object possessing certain defining properties. In this case, the distinguished object is mentioned by using a logographic variable or a verbal (phonographic) common name, whose range is a class of an indefinite number of objects. However, there is a principal difference between such an as if distinguished object and a distinguished object *sui generis* that is mentioned by its *proper name* consisting of a finite number of intelligible signs, – just as there is a principal difference between the meanings of the predicates «can be done by an unspecified mind» and «is done by the concrete mind». Therefore, it is as a rule impossible to exhibit the conclusion of an existence proposition in concrete finite graphic form that has universal significance. For instance, the set of real numbers has *the power of the continuum*, so that all its members, except those forming certain denumerable subsets of that set, – such subsets, e.g., as the set of rational numbers, or a set of *algebraic real numbers*, i.e. the set of the real roots of an algebraic equation with rational coefficients, or as the singletons of certain distinguished isolated real numbers as *e* (the base of natural, or Napier's, logarithms) or π, – *are unnamable*. At the same time, there is no way to distinguish between any two unnamable numbers and to *express* (*write down*) *the cut* (*instance*, *extension*) *of the relation* $\leq$ *for them*. Not only the set of real numbers, but also the set of rational numbers cannot be well ordered in the same concrete (demonstrable, ostensive) manner as the set of natural numbers or as the set of nonnegative integers. Therefore, many mathematicians, mostly so-called *intuitionists* (see, e.g. Heyting [1966] and FST, pp. 215–216) do not admit the axiom of choice and hence they do not admit a theorem of well ordering either. A discussion of the problem of well ordering sets is not within the scope of this treatise. The interested reader will locate such a discussion elsewhere, e.g. in FST.

6) Any order relation and particularly a well ordering relation that is established in a nonempty set should be *permanent*, which is possible only because a set has permanent member population. Hence, if a nonempty class is ordered or particularly well ordered then it is a set. That is to say, *an irregular class cannot be ordered and particularly it cannot be well ordered*. •

9.3.3. Comparability and compatibility of classes

Df 9.4. 1) Given two classes, if any member of one class is a member of the other class but *not necessarily vice versa* then the former class is called a *subclass*, or *part*, of the latter class, whereas the latter is called a *superclass*, or *whole*, of the former. If any member of one class is a member of the other class *but not vice versa* then the former class is called a *strict subclass*, or a *strict part*, i.e. *a part but not the whole*, *of the latter class*, whereas the latter is called a *strict superclass*, or a *strict whole*, i.e. *a whole but not exactly the whole*, of the former. Accordingly, the terms "part" and "whole" as used in the first case are synonyms of the expressions "*a strict part or the whole*" and "*a strict whole or the whole*" respectively. Also, the terms "part", "whole", "subclass", and "superclass" alone can be understood as synonyms of the redundant terms "*lax part*", "*lax whole*", "*lax subclass*", and "*lax superclass*" respectively, whereas antonymous qualifiers "*strict*" and "*lax*" can be used interchangeably (synonymously) with "*strong*' and "*weak*" respectively.

2) Two classes are said to *intersect* or to be *joint* if they have some members in common and to be *disjoint* if otherwise. The class all members of which belong to both intersecting classes is called the *intersection* of the two.

3) Two classes are said to be *comparable* if and only if one of them is a *subclass* of the other or, equivalently, if and only if one of them is a *superclass* of the other one and *incomparable* or *not comparable* if otherwise. Two classes are said to be *compatible* or *conjoint* if and only if they *intersect*, and *incompatible* or *disjoint* if otherwise. *Comparable classes are compatible but not necessarily vice versa*.

4) A class is said to be *the union of two classes*, joint or disjoint, if and only if every one of its members is a member of at least one of the latter two classes.

5) If *u* and *v* are classes then the *difference* between *u* and *v*, called also the *relative complement of v in u*, is the class *u*–*v* of all members of *u*, which are not members of *v*.

6) A name, usually an ideonym (symbol), that denotes (is used for mentioning) a class is called a *class-name* or *classonym* or *calonym*. If two classes are comparable then the classonym of the subclass is called *the subterm* and the classonym of the superclass is called *the superterm*, *with respect to each other* or *of each other*. Either class-name "subterm" or "superterm" can be prefixed with the same one of the qualifiers "strict" (or "strong") and "lax" (or "weak"), which qualifies the, or a, subclass or superclass denoted by the respective class-name.

7) A class being a unit of taxonomy is called a *taxon* (pl. "*taxa*" or "*taxons*"), whereas its name is called a *taxonym*. If two taxa are comparable then the one being the strict subclass is called *the hypotaxon* and the one being the strict superclass is called *the hypertaxon*, *with respect to each other* or *of each other*. Accordingly, the name (taxonym) of the hypotaxon and the name of the hypertaxon are called *the hypotaxonym* and *the hypertaxonym with respect to each other* or *of each other*.

8) Two classonyms (particularly two taxonyms) are said to be *comparable*, *incomparable*, *compatible*, or *incompatible* (and also *disjoint*) if so are the classes (correspondingly, the taxa) denoted by the classonyms (correspondingly, by the taxonyms).

9) The terms "*species*" and "*genus*" are *mutually relative* and *epistemologically relativistic common names of two classes*, one of which, called a *species* or *specific class*, is a *strict subclass* of the other one, called a *genus* or *generic class* and being therefore a *strict superclass* of the former. Accordingly, a name of a species is called a *specific name* (*SN*) or *idonym*, whereas a name of a genus is called a *generic name* (*GN*) or *genonym*. In general, the adjective "*specific*" means *of, relating to, or constituting a species,* whereas "*conspecific*" means *of the same species,* i.e. *of the same strict subclass,* and analogously the qualifier "*generic*" means *of, relating to, or constituting a genus* whereas "*congeneric*" means *of the same genus* (cf. WTNID), i.e. *of the same strict superclass*. An idonym and a genonym are respectively called an *idograph* (*idographonym*) and a *genograph* (*genographonym*) if they are graphonyms, and an *idophon* (*idophononym*) and a *genophon* (*genophononym*) if they are phononyms.•

9.4. The class-related taxonomy of the metalanguage of this treatise and of semantic interpretations of A_1 versus the class-related and set-related taxonomies of conventional axiomatic class and set theories

Df 9.5. Any *well-known,* i.e. *published and interpersonally verified*, *axiomatic class theory* (*ACT*), which is *not based* on the *theory of logical types* (*TLT)* of Russell [1908] and Whitehead and Russell [1910; 1962, Chapter II], i.e. an axiomatic theory of classes that are *not logically typified* (*not formally ranked into logical types*), will be called a *conventional axiomatic class theory* (*CACT*, pl. "*CACT's*"). A CACT that deals exclusively with sets and their members will be called a *conventional axiomatic set theory* (*CAST*). Among the CACT's, there are some that admit side by side with sets also *irregular* classes. Whenever it is desirable to distinguish such a CACT terminologically, it will be referred to as a *conventional extended axiomatic set theory* (*CXAST*). Any well-known axiomatic class theory, which is based on the TLT, including the TLT itself and also including *Mathematical Logic* by Whitehead and Russell (*ibidem*, Part I), i.e. an axiomatic theory that deals with *logically typified classes and their members,* will be called an *axiomatic theory of logically typified classes* (*ATLTC*) and also an *unconventional axiomatic class theory* (*UCACT*). An ATLTC (UCACT) that deals exclusively with *logically typified sets and their members* will be called an *axiomatic theory of logically typified sets* (*ATLTS*) and also an *unconventional axiomatic set theory* (*UCAST*).

Cmt 9.5. 1) It was reputedly Aristotle who said: «Any well-known thing is known to few». Therefore, unless stated otherwise, I shall, for the sake of being specific, restrict the ranges of the above metaterms as follows. Any one of the numerous *axiomatic set theories* (*AST's*), which are discussed in Chapter II of FST (see Cnv 9.1), is a CACT. The CACT's dealt with in §§1–6 of Chapter II of FST are CAST's, whereas the CACT's dealt with in §§7 of Chapter II are CXAST's. For instance, the set theory of Zermelo [1908] and its modification, which is referred to in the literature as the Zermelo-Fraenkel set theory or, more appropriately, as the Zermelo-Fraenkel-Skolem set theory (see FST¸ p. 22), are CAST's, and so are the theories of Bourbaki [1960, 1963], Suppes [1960], and Halmos [1960], although the last one is entitled "*Naïve set theory*". In order to avoid the terminological conflict, I shall use the substantive "*conventional set theory*" ("*CST*") for both a [system of] set theory, which is included under a title containing the word "axiomatic", and for a set theory, which is, like that by Halmos, included under a title that does not contain the word “axiomatic”, but is axiomatic in its essence. There are two variants of the Zermelo-Fraenkel set theory, which are denoted in FST (p. 22) by ‘ZF’ and ‘ZFC’ in this font. The set (conjubction) of axioms of ZF comprises the axioms of *extensionality, comprehension, pairing, union, power set*, and *infinity*, whereas the set of axioms of ZFC includes in addition the *axiom of choice*. The most conspicuous CXAST is that discussed in FST (pp. 119–135) under the abbreviated name ‘VNB’ (after von Neumann and Bernays). The ACT’s, which are discussed in Chapter III of FST, are UCACT’s. No UCACT is relevant either to $A_{1\in}$ or to its semantic interpretations. Therefore, I do not need to establish any relationship between the terminology of this treatise and the terminology of any UCACT. Nevertheless, a few remarks regarding the latter terminology will be useful.

2) The *theory of classes*, which was formulated by Quine in his book *Mathematical logic* [1951, Chapters 3 and 4] and which he alternatively called the *theory of sets*, the *theory of aggregates*, and *Mengenlehre* (*ibid*., p. 127), is a typical and at the same time the most conspicuous UCACT. This theory is based on Quine’s article *New foundations for mathematics* [1937], conventionally denoted in FST (p. 162) as ‘NF’ in this font, which is, in turn, based on the TLT. *Quine’s class,* or *set, theory* (*QCT* or *QST*) became known by the name of his book, i.e. as Mathematical logic too or briefly ML. Accordingly, the abbreviation ‘ML’ in this font is also used in FST (p. 167, §4) for referring to the FST version of QCT. In order to distinguish between «membership-eligible» classes, i.e. ones that can be members of classes, and classes, which cannot be members of classes, Quine [1951, p. 131] calls the former *elements*. By contrast, in order to compare ML with the other set theories, which are discussed in FST, the authors of the latter «refer to the elements as *sets* and to the classes which are not sets as *proper classes*» (*ibid.*). Since $A_{1\in}$ is irrelevant to the TLT in general and to ML in particular, I shall use the noun “element” for referring to (mentioning) any member-eligible class, regular (a set) or irregular, while a class (particularly a set) that cannot be a member of a class will be called a *universal class*. At the sane time, the metaterm “proper class” of FST is, in accordance with Df 9.3(3), a synonym of “irregular class”, although I shall not use the former.•

Df 9.6. A CST, in which the empty set is admitted as the only individual (in the set-theoretic sense of the word), is called a *one-individual set theory*. A CST, in which many (usually infinitely many) individuals are postulated to exist, is called a *many-individual set theory*. Most CST's are set up as one-individual set theories because such a theory is regarded by some authors as sufficient for mathematical applications (which is however debatable and is likely untrue) and also because a many-individual set theory system has some debatable peculiar properties.

Cmt 9.6. 1) A many-individual set theory is usually developed by supplementing a certain one-individual axiomatic set theory with *verbal* axioms that introduce an indefinite number of *indistinguishable* nonempty individuals. Therefore, a many-individual set theory necessarily has one and only one empty individual (null-individual). All other individuals of the many-individual set theory are certain nonempty individuals, i.e. concrete but not concretized sensible or insensible objects that are not classes and hence not sets, – such sensible objects, e.g., as concrete men, dolphins, books, bicycles, «cabbages, and kings» (Halmos [1960, p. 2]), etc, or such insensible mathematical objects e.g. as elements of algebraic systems (particularly elements of a group, vectors of a linear space, or points of an affine space) or as imaginary figures (circumferences, triangles, quadrangles, etc) of Euclid geometry, provided of course that an insensible mathematical object is not defined as a set (see the items 7–11 of Cmt 8.1). All relevant nonempty individuals are represented in the many-individual set theory by *restricted atomic term-variables*, which may be called *individual variables* or more precisely *nonempty-individual-valued variables* (*NEIVV's*) in the sense that such a variable may assume (take on) a certain nonempty individual of its range as its accidental denotatum. In this case, for avoidance of confusion, '$\varnothing$' should be called an *empty-individual-valued*, or *null-individual-valued*, *constant* (*EIVC* or *NIVC*), or, an *empty-class-valued constant* (*ECVC*), alternatively, and not just an *individual constant*. It will be recalled that the expressions "null-set" and "null-class" are not used in this treatise as synonyms of "empty individual" and "null-individual". *Axiomatic set theory* by Suppes [1960], e.g., is a many-individual set theory, and so is, in fact, *Naïve set theory* by Halmos [1960].

2) No matter whether the underlying one-individual set theory of the given many-individual one is logographic (formal) or phonographic (verbal, informal), all nonempty individuals, along with the sets (regular classes) or irregular classes, to which they belong, are *always* incorporated into the one-individual set-theory by

some *verbal* axioms and hence informally. A usual practicable semi-formal way of setting up a many-individual set theory is described in FST (pp. 24–25) thus:

> «Having decided that we need an individual we now face the question of whether we need *more than one* individual. It turns out that for mathematical purposes there seems to be no real need for individuals other than the null-set [2]). Therefore we shall indeed not admit any such individuals in ZF. Yet, mostly for mathematical purposes, there is also considerable interest in systems of set theory which admit individuals other than the null-set. Therefore we shall formulate the *verbal* 〈italicized by me – Ya. I.〉 versions of the axioms in such a way that they will serve, with possible minor modifications which we shall point out as we go along, also as axioms for the corresponding system of set theory which admits individuals [1]). Thus we shall distinguish between the term 'element', which in such a system refers also to the individuals, and the term 'set'. Also, from now on we shall use the term 'individual' only for individuals other than the null-set; thus every element is a set or an individual, but nothing is both a set and an individual.»

It is noteworthy that the above quotation involves two mutually contradictory remarks about advisability *to have* or *not to have* «*more than one* individual» «for mathematical purposes». Also, it is pointed out in FST (p. 59) that in ZF, which is a one-individual CAST, «*there is no characteristic which distinguishes one individual from another*» (italicized by the authors). Therefore, all nonempty individuals are, as indicated above in the item 1, represented in a many-individual set theory by *atomic term-variables*, and not by term-constant.

3) The fact that nonempty individuals can be introduced into a many-individual set theory only verbally is implied by the formation rules of A_1 and by Df 8.2(1) as explicated in what follows.

a) The property of ∅ to be a class (and not a mass) is formally expressed by allowing '∅' to stand to the right of the class-membership sign ∈, whereas the fact that ∅ is empty can be expressed by denying the relation '$x \in \varnothing$', i.e. by taking the relation '$\neg[x \in \varnothing]$' for granted as a true one. By contrast, the property of a nonempty individual, *a*, not to be a class is formally expressed by disallowing '*a*', being a proper or common name of that individual, to stand to the right of the class-membership sign $\in$, although 'a' may stand to the left of $\in$. In general, the string '$x\in u$', e.g., is admissible as one that expresses a class-membership relation, true or antitrue (false) if and only if 'u' is disallowed to take on any nonempty individual, say, a, a_1, a_2, etc, as its value, although it may take on $\varnothing$ as one of its values. Since the string '$\in a$' is inadmissible, there is no way to express the fact that a is an individual logographically, say by asserting the string '$\neg[x\in a]$' as a true relation after the manner of the true relation '$\neg[x\in\varnothing]$'. This fact can be expressed only verbally, without using the logographic sign '$\in$' or its negation as a predicate.

b) In accordance with Df 5.2(1), '**u**' and '**v**', e.g., are APLPH's, whose range is the set of all APVOT's on the list (5.1) and of the two APCOT's $\emptyset$ and $\emptyset'$ introduced in Ax 5.1(9). The *primary* formation rules of A_1 include one, according to which either primary euautographic assemblage of A_1: $\in(\mathbf{u},\mathbf{v})$ or $\neg\in(\mathbf{u},\mathbf{v})$ is an ER of A_1. Then, by the pertinent *secondary* formation rule of A_1, $[\mathbf{u}\in\mathbf{v}]\to\in(\mathbf{u},\mathbf{v})$ and hence $\neg[\mathbf{u}\in\mathbf{v}]\leftrightarrow\neg\in(\mathbf{u},\mathbf{v})$, so that $[\mathbf{u}\in\mathbf{v}]$ and $\neg[\mathbf{u}\in\mathbf{v}]$ are also ER of A_1. In accordance with the above item a, this rule disallows specifying any variable on the list (8.4) as a nonempty individual. At the same time, it is proved in A_1 that the axioms $\neg[\mathbf{u}\in\emptyset]$ and $\neg[\mathbf{v}\in\emptyset']$ imply that $\emptyset=\emptyset'$. Hence, if $\emptyset'\to$'$\varnothing$'' besides (8.2) then $\varnothing=\varnothing'$, i.e. the *empty class* (*null-individual*) is unique.•

3) To say nothing of a set theory, an attempt to introduce peudo-nonempty individuals into the organon $\mathsf{A}_{1\in}$ seems to be impracticable because formation and transformation rules of such a calculus would unavoidably have been extremely ramified and hence incomprehensible. Incidentally, as was pointed out in Cmt 7.6, the universal term U is introduced into $\bar{\mathsf{A}}_{1\subseteq}$ and $\bar{\mathsf{A}}_{1\in}$ in such a way that it can stand to both sides of either sign $\subseteq$ or $\in$, whereas the property of universality of U is expressed either by the axioms $U\subseteq U$, $\mathbf{x}\subseteq U$, and $\neg[U\subseteq\mathbf{x}]$ of $\bar{\mathsf{A}}_{1\subseteq}$ or by the axioms $\neg[U\in U]$, $\mathbf{x}\in U$, and $\neg[U\in\mathbf{x}]$ of $\bar{\mathsf{A}}_{1\in}$. An attempt to introduce the universal term U alternatively by prohibiting it to stand to *the left of either sign* $\subseteq$ *or* $\in$, i.e. by disallowing the strings $U\subseteq$ and $U\in$, would have made impossible setting up $\bar{\mathsf{A}}_{1\subseteq}$ and $\bar{\mathsf{A}}_{1\in}$ pasigraphically (formally).

Chapter II. The setup of A_1

1. The major formation rules of A_1 and $\mathbf{A}_1$

1.1. The primary formation rules of A_1 and $\mathbf{A}_1$ and the general principles of a single whole recursive system of formation rules of A_1 and $\mathbf{A}_1$

Df 1.1. In agreement with Dfs I.3.1, I.4.2, and I.5.10, I shall use the following abbreviations:

1. a) “EI” for “euautographic integron”, “AEI” for “atomic euautographic integron”, “MEI” for “molecular euautographic integron”, “CbEI” for “combined euautographic integron”, “CxEI” for “complex euautographic integron”, and similarly with “SpT” for “special term” in place of “I” for “integron”, because “special term” and “integron” are synonyms;

b) “EOT” for “euautographic ordinary term”, “PVOT” for “pseudo-variable ordinary term”, “PCOT” for “pseudo-constant ordinary term”;

c) “ER” for “euautographic relation”, “AER” for “atomic euautographic relation”, “MER” for “molecular euautographic relation”, “CbER” for “combined euautographic relation”, “CxER” for “complex euautographic relation”, “EOR” for “euautographic ordinary relation”, “ESpR” for “euautographic special relation”, “ELR” for “euautographic logical relation”, “EAlR” for “euautographic algebraic relation”;

d) “EF” for “euautographic formula”, “AEF” for “atomic euautographic formula”, “CbER” for “combined euautographic formula”;

2. a) “FR” for “formation rule”, “CFR” for “concrete formation rule”, “SchFR” for “schematic formation rule”;

b) “EFR” for “euautographic formation rule”, “CEFR” for “concrete euautographic formation rule”, “SchEFR” for “schematic euautographic formation rule”.

3. The prepositive letter “P” or “S” adhered to any of the above abbreviations stands for the prepositive qualifier “primary” or “secondary” respectively that is adhered to the pertinent full taxonym.

4. The pertinent ones of the above definitions, those stated explicitly in the items 1 and 2b and those obviously understood in the item 3, apply with “PL” for “panlogographic” in place of “E” for “euautographic”.•

†**Ax 1.1:** ***The restricted primary euautographic formation rules system (RPFR-system) of*** A_1**.** All atomic placeholders (APH's), panlogographic ones (PLAPH's) and metalogographic ones (MLAPH's or AMLPH's), that occur in the following statements are used *xenonymously*.

1) Either of the digits 0 and 1 is a PEI, or, more specifically, PAEI, of A_1.

2) $\mathbf{x}$, called an AEOT or, simply, EOT of A_1, i.e. either $\mathbf{x}^{\mathrm{pv}}$, called an APVOT or, simply, PVOT of A_1, or $\mathbf{x}^{\mathrm{pc}}$, called an APCOT or, simply, PCOT of A_1, is a PEOT of A_1 and vice versa, whereas $\mathbf{x}^{\mathrm{pv}}$ is a PPVOT of A_1 and $\mathbf{x}^{\mathrm{pc}}$ is a PPCOT of A_1, and vice versa.

3) $\mathbf{p}$, called an APVOR or AEOR or, simply, AER of A_1 is a PER of A_1, the understanding being that there are neither secondary nor special AER's in A_1.

4) $\mathbf{f}^1(\mathbf{x}_1)$, $\mathbf{f}^2(\mathbf{x}_1,\mathbf{x}_2)$, etc, – generally, $\mathbf{f}^n(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$ or $\mathbf{f}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$, – are PER's or, more specifically, PEOR's or, still more specifically, PMEOR's, of A_1.

5) $V(\mathbf{\Gamma})$ is a PEI of A_1 if $\mathbf{\Gamma}$ is a PER of A_1.

6) $[\hat{\neg}\mathbf{\Gamma}]$ is a PEI of A_1 if $\mathbf{\Gamma}$ is a PEI of A_1.

7) $[\mathbf{\Gamma}\,\hat{+}\,\mathbf{\Delta}]$ is a PEI of A_1 if $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are PEI's of A_1.

8) $[\mathbf{\Gamma}\,\hat{\cdot}\,\mathbf{\Delta}]$ is a PEI of A_1 if $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are PEI's of A_1.

9) $[\mathbf{\Gamma}\,\hat{=}\,\mathbf{\Delta}]$ is a PER, or, more specifically, PESpR, or, still more specifically, PEAlR, of A_1 if $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are PEI's of A_1.

10) $[\mathbf{\Gamma}\,\dot{\vee}\,\mathbf{\Delta}]$ is a PER, or, more specifically, PELR, of A_1, if $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are PER's of A_1.

11) If $\mathbf{x}$ is a PVOT (and not a PCOT) and if $\mathbf{\Gamma}\langle\mathbf{x}\rangle$ is a PER that contains $\mathbf{x}$ and that does not contain either of the assemblages $(\exists\mathbf{x})$ and $(\hat{\cdot}\,\mathbf{x})$ then (a) $[(\exists\mathbf{x})\mathbf{\Gamma}\langle\mathbf{x}\rangle]$ is a PER and (b) $[(\hat{\cdot}\,\mathbf{x})V(\mathbf{\Gamma}\langle\mathbf{x}\rangle)]$ is a PEI. If $\mathbf{x}$ does not occur in $\mathbf{\Gamma}$ then (a′) $[(\exists\mathbf{x})\mathbf{\Gamma}]$ is $\mathbf{\Gamma}$ and (b′) $[(\hat{\cdot}\,\mathbf{x})V(\mathbf{\Gamma})]$ is $V(\mathbf{\Gamma})$.

12) $\mathbf{\Gamma}$ is either a PEOT and at the same time EOT, $\mathbf{x}$, or a PEI (PESpT), or else a PER if and only if its being so follows from the above formation rules.•

Df 1.2. 1) Any one of the twelve interrelated items of Ax 1.1 is indiscriminately called a *restricted primary formation rule of euautographic formulas* *of* A_1 or, briefly, a *restricted primary euautographic formation rule* (*RPFR*) *of* A_1. Accordingly, the single whole coherent recursive system of the above RPFR's is called the *restricted primary euautographic formation rules system* (*RPFR-system*) *of* A_1.

2) The item 1 of Ax 1.1 is called the *concrete RPFR* (*CRPFR*), or *restricted primary CEFR* (*RPCEFR*), *of* A_1.

3) Any one of the items 2–4 of Ax 1.1 is called a *schematic structural RPFR* (*SchStRPFR*), or *structural RPFR-schema* (*StRPFRS*, pl. "*StRPFRS'ta*"), *of* A_1.

4) Any one of the items 5–12 of Ax 1.1 is called a *schematic analytical RPFR*, or *analytical RPFR-schema* (*AnRPFRS*, pl. "*AnRPFRS'ta*"), *of* A_1.

5) All RPFR's of A_1 are FKPPLFR's (SmnPPLFR's, BslPPLFR's) of A_1, – in accordance with Df 6.2(1c).•

†**Df 1.3:** ***The primary analytical atomic panlogographic basis of*** $\mathbf{A}_1$ ***and*** A_1**.**

The entire set of atomic pasigraphs that is denoted by '$\mathbf{B}_{1PAn}$' and is called the *primary analytical panlogographic atomic basis* (*PAnPLAB*) *of* $\mathbf{A}_1$ or *of* A_1 includes the *euautographic angle brackets*, ⟨ ⟩, which are used as a single whole *molecular punctuation sign of demonstration and aggregation*, and which are the only *syncategorematic elements of* $\mathbf{B}_{1PAn}$, and it also includes several *subsets of categorematic* (*formulary*) *panlogographs*. In analogy with the elements of $\mathbf{B}_{1PSt}$ that are defined in Df I.5.2, every categorematic element of $\mathbf{B}_{1PAn}$ is called a *primary analytical panlexigraph* (*PAnPLxg*) and also, synonymously (interchangeably), by a variant of the above name with "*atomic panlogograph*" ("*APL*") or "*atomic panlogographic placeholder*" ("*APLPH*") in place of "*panlexigraph*" ("*PLxg*"). In this case, however, the name "atomic panlogographic schema" ("APLS") is not used interchangeably with any of the last three names because, in contrast to a StPLxg, an AnPLxg (AnAPL, AnAPLPH) is *not a PLS* (panlogographic schema), by Df I.4.2(4c). An occurrence of the qualifier "primary" in the verbal name of $\mathbf{B}_{1Pan}$ and in any of the above taxonyms is descriptive of the fact that all elements of $\mathbf{B}_{1PAn}$ are specified immediately after laying down the RPFR-system (Ax 1.1) and before making any SFR of A_1. Here follow lists of *congeneric* or *conspecific* PAnAPL's (PAnAPLPH's, PAnAPLxg's) of $\mathbf{B}_{1PAn}$.

1) Each one of the logographs:

$$\text{'}\mathbf{P}\text{', '}\mathbf{Q}\text{', '}\mathbf{R}\text{', '}\mathbf{S}\text{', '}\mathbf{P}_1\text{', '}\mathbf{Q}_1\text{', '}\mathbf{R}_1\text{', '}\mathbf{S}_1\text{', '}\mathbf{P}_2\text{', '}\mathbf{Q}_2\text{', '}\mathbf{R}_2\text{', '}\mathbf{S}_2\text{', …} \quad (1.1)$$

is a PAnAPL, the *initial range* of which is the *class of PER's of* A_1 (without any quotation marks) that is determined by the RPFR-system, and which is called a *generic* (*comprehensive*, *all-embracing*) *analytical atomic panlogographic relation* (*GAnAPLR*) *of* $\mathbf{A}_1$.

2) Each one of the logographs:

$$\text{'}\mathbf{I}\text{', '}\mathbf{J}\text{', '}\mathbf{K}\text{', '}\mathbf{L}\text{', '}\mathbf{M}\text{', '}\mathbf{N}\text{', '}\mathbf{I}_1\text{', '}\mathbf{J}_1\text{', '}\mathbf{K}_1\text{', '}\mathbf{L}_1\text{', '}\mathbf{M}_1\text{', '}\mathbf{N}_1\text{', '}\mathbf{I}_2\text{', '}\mathbf{J}_2\text{', } \ldots \quad (1.2)$$

is a PAnAPL, the *initial range* of which is the *class of PEI's of* A_1 (without any quotation marks) that is determined by the RPFR-system, and which is called a *primary analytical atomic panlogographic integron* (*PAnAPLI*), or *primary analytical atomic panlogographic special term* (*PAnAPLSpT*), *of* $\mathbf{A}_1$.

3) Each logograph on any one of the following lists:

$$\text{'}\boldsymbol{\pi}^1\text{', '}\boldsymbol{\rho}^1\text{', '}\boldsymbol{\sigma}^1\text{', '}\boldsymbol{\pi}_1^1\text{', '}\boldsymbol{\rho}_1^1\text{', '}\boldsymbol{\sigma}_1^1\text{', '}\boldsymbol{\pi}_2^1\text{', '}\boldsymbol{\rho}_2^1\text{', '}\boldsymbol{\sigma}_2^1\text{', } \ldots, \quad (1.3^1)$$

$$\text{'}\boldsymbol{\pi}^2\text{', '}\boldsymbol{\rho}^2\text{', '}\boldsymbol{\sigma}^2\text{', '}\boldsymbol{\pi}_1^2\text{', '}\boldsymbol{\rho}_1^2\text{', '}\boldsymbol{\sigma}_1^2\text{', '}\boldsymbol{\pi}_2^2\text{', '}\boldsymbol{\rho}_2^2\text{', '}\boldsymbol{\sigma}_2^2\text{', } \ldots, \quad (1.3^2)$$

$$\text{'}\boldsymbol{\pi}^3\text{', '}\boldsymbol{\rho}^3\text{', '}\boldsymbol{\sigma}^3\text{', '}\boldsymbol{\pi}_1^3\text{', '}\boldsymbol{\rho}_1^3\text{', '}\boldsymbol{\sigma}_1^3\text{', '}\boldsymbol{\pi}_2^3\text{', '}\boldsymbol{\rho}_2^3\text{', '}\boldsymbol{\sigma}_2^3\text{', } \ldots, \quad (1.3^3)$$

etc is a PAnAPL, the *range* of which is *the same set of PER's* (*primary euautographic relations*) *of* A_1 (without any quotation marks) as that of '$\mathbf{f}^1(\mathbf{x}_1)$', '$\mathbf{f}^2(\mathbf{x}_1,\mathbf{x}_2)$', etc respectively and which is called, discriminately, a *specific analytical atomic panlogographic relation* (*ScAnAPLR*) *of weight* 1, 2, etc or, indiscriminately, a *weighed ScAnAPLR* (*WScAnAPLR*), *of* $\mathbf{A}_1$.

4) Each one of the logographs:

$$\text{'}\boldsymbol{\pi}\text{', '}\boldsymbol{\rho}\text{', '}\boldsymbol{\sigma}\text{', '}\boldsymbol{\pi}_1\text{', '}\boldsymbol{\rho}_1\text{', '}\boldsymbol{\sigma}_1\text{', '}\boldsymbol{\pi}_2\text{', '}\boldsymbol{\rho}_2\text{', '}\boldsymbol{\sigma}_2\text{', } \ldots \quad (1.3)$$

is a PAnAPL, the *range* of which is the union of the ranges of (e.g.) $\boldsymbol{\pi}^1$', '$\boldsymbol{\pi}^2$', etc, and which is called an *unweighed ScAnAPLR* (*UWScAnAPLR*) *of* $\mathbf{A}_1$.

5) Each one of the logographs:

$$\text{'}\mathsf{p}\text{', '}\mathsf{q}\text{', '}\mathsf{r}\text{', '}\mathsf{s}\text{', '}\mathsf{p}_1\text{', '}\mathsf{q}_1\text{', '}\mathsf{r}_1\text{', '}\mathsf{s}_1\text{', '}\mathsf{p}_2\text{', '}\mathsf{q}_2\text{', '}\mathsf{r}_2\text{', '}\mathsf{s}_2\text{', } \ldots \quad (1.4)$$

is a PAnAPL, the *range* of which is the union of the ranges of (e.g.) '**p**' and '$\boldsymbol{\pi}$', and which is called a *unified ScAnAPLR* (*UdScAnAPLR*) *of* $\mathbf{A}_1$.

6) The PAnAPL's occurring on any one of the lists (1.1)–(1.4) are called *congeneric ones,* whereas the PAnAPL's occurring on any one of the lists (1.3^1)–(1.3^3), etc are called *conspecific* ones. The order, in which the congeneric or conspecific PAnAPL's are presented on any one of the above lists, is called the *alphabetic order* of those AnAPL's, whereas the list itself is called the *alphabet* of the

AnAPL's listed (cf. Ax I.5.1(v) and Df I.5.2(5)). Any of GAnAPLR's of the list (1.1) and any of the AnAPLI's of the list (1.2) can, when needed, be furnished any number of primes, thus becoming another AnAPL of the same name and hence having the same range.

7) A PAnAPL of any one of the lists (1.1), (1.3^1)–(1.3^3) etc, (1.3), and (1.4) is indiscriminately called an *analytical atomic panlogographic ordinary relation* (*PAnAPLR*), or, redundantly, *primary analytical atomic panlogographic ordinary relation* (*PAnAPLOR*), *of* $\mathbf{A}_1$. A PAnAPLI (PAnAPLSpT) or PAnAPLR (PAnAPLOR) of $\mathbf{A}_1$ is indiscriminately called a *primary analytical atomic panlogographic ordinary formula* (*PAnAPLOF*) *of* $\mathbf{A}_1$.

8) The PAnAPL's that have been introduced in the previous items 2–5 are by definition both *primary* and *ordinary*. Therefore, the generic taxonyms that have been introduced in the items 2 and 3 and that have been abbreviated as "GAnAPLR" and "ScAnAPLR" should have contained the prepositive qualifier "primary" ("P") and midpositive qualifier "ordinary" ("O"), so that the abbreviation "*PLOR*" for "*panlogographic ordinary relation*" should have been used instead of the constituent abbreviation "*PLR*" for "*panlogographic relation*". I have omitted both qualifiers because neither secondary nor special AnAPL's of the same generic taxonyms will be introduced in the sequel.

9) In accordance with the items 1–5, all elements of $\mathbf{B}_{1PAn}$ are *categorematic* (*formulary*) ones, i.e. they are *primary analytical atomic panlogographic formulas* (*PAnAPLF's*) of $\mathbf{A}_1$. The range of every PStAPL (StAPLOR) on the list (I.5.7) is a subset of the range of every PAnAPL on either list (1.1) or (1.4). *All syncategorematic PAPL's* are PStAPL's on the lists $(I.5.8^1)$–$(I.5.8^3)$ etc and (I.5.8). Any StAPL or AnAPL, categorematic or syncategorematic, to be introduced in the sequel will be called a *secondary* one, i.e. briefly, an *SStAPL* or *SAnAPL* respectively. For instance, each of the *idempotent PLI's* '**i**', '**j**', '**k**', and '**l**', whose range is the class of *idempotent EI's,* is an SAnAPL, belonging to of $\mathbf{B}_{1SAn}$, which can be introduced only after laying down some basic transformation rules of D_1 and $\mathbf{D}_1$.

10) Until any SFR of A_1 is laid down, the *current range* of every GAnAPLR on the list (1.1) or the *current range* of every PAnAPLI on the list (1.2) is by definition its *initial range*, i.e. the class of PER's or PEI's respectively that is determined by the RPFR-system. After stating the first or any subsequent SFR of A_1,

the *current range* of every GAnAPLR on the list (1.1) and the *current range* of every PAnAPLI on the list (1.2) and also *the same current ranges of their tokens in all earlier occurrences* are supposed to be automatically, retroactively and recursively, augmented (updated) with all new SER's or SEI's of A_1 introduced by the pertinent SFR.

11) By contrast, the *current range* of any PAnAPLR of the lists (1.3^1)–(1.3^3) etc, (1.3), and (1.4) is affected by no SFR of A_1, so that it is permanently equals the initial range of the PAnAPLR.

12) At any place, the current range of a given token of any one the above PAnAPL's can be restricted by making the appropriate verbal reservation (stipulative definition), while the restriction made can optionally be indicated explicitly by putting the appropriate labels (subscripts, superscripts, or overscripts), alphanumeric or not, on the PAnAPL, thus introducing one or more new PAnAPL's of the restricted range (cf. Df I.5.2). The new PAnAPL's can be included either under the same specific taxonym or be provided with an additional appropriate qualifier to that taxonym.•

Cmt 1.1. By Df I.5.3(1), '*n*' is an AMLPH (atomic metalogographic placeholder). Therefore, the logograph '$\boldsymbol{\pi}^n$', e.g., is a MMLPH (molecular metalogographic placeholder), whose *immediate* range is the set of specific PAnAPL's (ScAnAPLR's): '$\boldsymbol{\pi}^1$', '$\boldsymbol{\pi}^2$', '$\boldsymbol{\pi}^3$', etc (cf. Df I.5.4)). The latter three have, by Df 1.3(3), the same *immediate* ranges as the *structural molecular panlogographic ordinary relations* (*StMPLOR's*) '$\mathbf{f}^1(\mathbf{x}_1)$', '$\mathbf{f}^2(\mathbf{x}_1,\mathbf{x}_2)$', '$\mathbf{f}^3(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$', etc, or, concurrently, '$\mathbf{f}(\mathbf{x}_1)$', '$\mathbf{f}(\mathbf{x}_1,\mathbf{x}_2)$', '$\mathbf{f}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$', etc, in that order, whereas the union of the ranges of all StMPLOR's of either set is the ultimate range of '$\boldsymbol{\pi}^n$'. Unless stated otherwise, I shall assume that the range of '$\boldsymbol{\pi}^n$' is its immediate range.•

Df 1.4. A panlogographic relation (PLR) of $\mathbf{A}_1$ is said to be a *special* one (*SpPLR*), or *panlogographic special relation* (*PLSPR*), of $\mathbf{A}_1$ if it contains at least one [homolographic token of a] PAnAPLI (PAnAPLSpT) or at least one [homolographic token of a] special euautograph or both and an *ordinary* one (*OPLR*), or *panlogographic ordinary relation* (*PLOR*), of $\mathbf{A}_1$ if otherwise.•

Df 1.5: ***A metalogographic unification rule* (*MLUR*) [*of EF's*] *of*** A_1***: Analytical atomic formulary metalogographic placeholders* (*AnAFMLPH's*) *as unifying taxonyms.*** Each of the bold-faced upright capital Greek letters '**Φ**', '**Ψ**', and '**Ω**' is an APH (atomic placeholder) belonging to the XML (exclusive metalanguage) of A_1 and $\boldsymbol{\mathsf{A}}_1$, whose range is the [*class of*] *EF's of* A_1 (without any quotation marks), primary and secondary. Therefore, any of the above APH's is called an *analytical atomic metalogographic* (*metalinguistic logographic*) *placeholder* (*AnAMLPH*) *of euautographic formulas* (*EF's*) or briefly an *EF-valued AnAMLPH*. Any of the above three letters can, if desired, be furnished with any of the upright Arabic numeral subscripts '$_1$', '$_2$', etc in this font or with any number of primes or both thus becoming another AnAMLPH with the same range.•

Df 1.6. 1) An occurrence of an APVOT $\mathbf{x}$ in an EF $\mathbf{\Phi}$, i.e. in an ER $\mathbf{P}$ or in an EI $V(\mathbf{P})$, of A_1 is called a *bound occurrence* of $\mathbf{x}$ in $\mathbf{\Phi}$ if it is an occurrence in an EF of $\mathbf{\Phi}$ either of the form $\left[(\exists\mathbf{x})\mathbf{Q}\langle\mathbf{x}\rangle\right]$ or of the form $\left[(\hat{\cdot}\,\mathbf{x})V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]$, where $\mathbf{Q}\langle\mathbf{x}\rangle$ is an ER of A_1 containing $\mathbf{x}$; otherwise it is called a *free occurrence* of $\mathbf{x}$ in $\mathbf{\Phi}$**.** It is understood that if $\mathbf{\Phi}$ has bound occurrences of $\mathbf{x}$ then it cannot have any free occurrences of $\mathbf{x}$ and, conversely, if $\mathbf{\Phi}$ has free occurrences of $\mathbf{x}$ then it cannot have any bound occurrences of $\mathbf{x}$. The APVOT's that have bound occurrences in $\mathbf{\Phi}$ are called the *bound*, or *dummy*, *APVOT's of* $\mathbf{\Phi}$, and the APVOT's that have free occurrences in $\mathbf{\Phi}$ are called the *free APVOT's of* $\mathbf{\Phi}$ (cf. a like definition in Church [1956, p. 170]).

2) Either of the qualifiers "*contracted*" and "*dummy*" can be used interchangeably with "*bound*". Accordingly, $\left[(\exists\mathbf{x})\mathbf{Q}\langle\mathbf{x}\rangle\right]$ is called a *contracted ER* and $\left[(\hat{\cdot}\,\mathbf{x})V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]$ a *contracted EI*. Either of the two is indiscriminately called a *contracted EF* or, briefly, a *euautographic contraction*. a *kernel-signs* $(\exists\mathbf{x})$ or $(\hat{\cdot}\,\mathbf{x})$ is indiscriminately called a *euautographic contractor*. Discriminately, $(\exists\mathbf{x})$ is commonly called an *inclusive,* or *laxly inclusive* (by way of emphatic comparison with the subsequent qualifier "*strictly inclusive*"), *disjunctive euautographic contractor* (*DjECnt*) or a [*lax*] *existential euautographic pseudo-quantifier* (*EEPQ*) and also, properly, *the one over* $\mathbf{x}$, whereas $(\hat{\cdot}\,\mathbf{x})$ is commonly called a *pseudo-multiplicative euautographic contractor* (*PsdMvECnt*) or a *euautographic transcendental pseudo-multiplier* (*ETPsdMr*) and also, properly, *the* one *over* $\mathbf{x}$. In any of the above terms, the noun "*contractor*" ("*Cnt*") can be used interchangeably with "*binder*" ("*Bnd*").•

Cnv 1.1. In accordance with Df 1.6, all occurrences of a *free* PVOT $\mathbf{x}$ in $\mathbf{P}\langle\mathbf{x}\rangle$ are at the same time *bound* if $\mathbf{P}\langle\mathbf{x}\rangle$ is the operatum of a contractor with respect to $\mathbf{x}$. In order to avoid such *contradictiones in adjecto*, I shall assume that when I say that a PVOT **x** is a free one of a given ER $\mathbf{P}\langle\mathbf{x}\rangle$ or of a given EI $V(\mathbf{P}\langle\mathbf{x}\rangle)$, either of the latter two EF's is prescinded from any contractor with respect to **x**, which may be applied to it.•

Cmt 1.2. 1) The verb "*to contract*", its derivative "*contracted*, and it kindred noun "*contraction*", and also the qualifier "*dummy*" are used here in analogy with use of these words in mathematics in the expressions pertinent to tensors, e.g. in the expression "contraction of a tensor" meaning summation over a pair of coinciding indices of the tensor, which are conventionally called *dummy indices*. Besides "contraction", "contractor" is another kindred noun of the verb "to contract", which I utilize in this treatise.

2) In accordance with Ax 1.1(11) and Df 1.6, a PEA $(\exists\mathbf{x})$ is a single whole *singulary relational* EKS (euautographic kernel-sign), i.e. one that applies to (is united with) a *single ER*, containing at least one occurrence of the pertinent APVOT **x**, to produce another ER. Analogously, a PEA $(\hat{\cdot}\,\mathbf{x})$ is a single whole *singulary termal* (*substantival*) EKS, i.e. one that applies to (is united with) a *single EI*, containing at least one occurrence of the pertinent APVOT **x**, to produce another EI. That is to say, neither $(\exists\ )$ nor $(\hat{\cdot}\ )$ is regarded as an *operator* that applies to **x**, i.e. **x** is not an *operatum* of $\exists$ or $\hat{\cdot}$. Consequently, the occurrence of **x** in $(\exists\mathbf{x})$ is not a constituent term of the ER $[(\exists\mathbf{x})\mathbf{P}\langle\mathbf{x}\rangle]$, being the scope of $(\exists\mathbf{x})$, and the occurrence of **x** in $(\hat{\cdot}\,\mathbf{x})$ is not a constituent term of the EI $[(\hat{\cdot}\,\mathbf{x})V(\mathbf{P}\langle\mathbf{x}\rangle)]$. According to Ax I.5.1, each one of the four euautographs (,), $\exists$, and $\hat{\cdot}$, taken alone, is a *primary atomic euautographic syncategorem*, whereas **x**, i.e. any euautograph of the list (I.5.1), is an APVOT. Therefore, in agreement with Df A3.1(1c), either EKS $(\exists\mathbf{x})$ or $(\hat{\cdot}\,\mathbf{x})$ is called a primary *molecular* euautographic contractor (PMECnt, pl. "PMECnt's"), the first of them being *relational* and the second one *termal* (*substantival*). A *catlogographic* (semantic) interpretand of $(\exists\mathbf{x})$ is an ordinary *existential quantifier*. Therefore, $(\exists\mathbf{x})$ and all other contractors, which will be defined in terms of $(\exists\mathbf{x})$, are alternatively called *pseudo-quantifiers*.

3) In accordance with the above-said, the expression "*a bound occurrence of* **x** *in* $[(\exists\mathbf{x})\mathbf{P}\langle\mathbf{x}\rangle]$", e.g., has two different meanings depending on whether it applies to the

occurrence of **x** in the PMECnt $(\exists \mathbf{x})$, i.e. to the very first occurrence of **x** in the pertinent *operand* (*scope*) *of the operator* $(\exists \mathbf{x})$, or whether it applies to an occurrence of **x** in the *operatum* $\mathbf{P}\langle \mathbf{x} \rangle$ of $(\exists \mathbf{x})$. Namely, the EKS $(\exists \mathbf{x})$ *binds* (*contracts*) *every occurrence of* **x** *in* $\mathbf{P}\langle \mathbf{x} \rangle$ *to the occurrence of* **x** *in* $(\exists \mathbf{x})$ itself and conversely *every occurrence of* **x** *in* $\mathbf{P}\langle \mathbf{x} \rangle$ *is bound* (*contracted*) *by the EKS* $(\exists \mathbf{x})$ *to the occurrence of* **x** *in* $(\exists \mathbf{x})$ *itself*. A like phraseology applies, *mutatis mutandis*, to an EI $[(\hat{\cdot}\,\mathbf{x})V(\mathbf{P}\langle \mathbf{x} \rangle)]$ in place of the ER $[(\exists \mathbf{x})\mathbf{P}\langle \mathbf{x} \rangle]$.

4) There is a certain *homology* (and not just an *analogy*) between the function of the EKS $\vee$ occurring in an ER $[\mathbf{Q} \vee \mathbf{R}]$ and the function of an EKS $(\exists \mathbf{x})$ occurring in an ER $[(\exists \mathbf{x})\mathbf{P}\langle \mathbf{x} \rangle]$. A similar homology exists between the function of the EKS $\hat{\cdot}$ occurring in an EI $[V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})]$ and the function of an EKS $(\hat{\cdot}\,\mathbf{x})$ occurring in an EI $[(\hat{\cdot}\,\mathbf{x})V(\mathbf{P}\langle \mathbf{x} \rangle)]$. In Df 1.6(2), I have emphasized these homologies by qualifying a contractor $(\exists \mathbf{x})$ *disjunctive* and a contractor $(\hat{\cdot}\,\mathbf{x})$ *pseudo-multiplicative*. In the sequel, I shall, also, emphasize these homologies mnemonically by introducing *secondary molecular euautographic contractors* (*SMECnt's*) $\vee_{\mathbf{x}}$ and $\hat{\cdot}_{\mathbf{x}}$ as synonyms of $(\exists \mathbf{x})$ and $(\hat{\cdot}\,\mathbf{x})$ respectively. In essence, the homologies will become evident from the pertinent rules of D_1.•

†**Df 1.7: *The primary panlogographic sortation rule* (*PPLSrtRl, PPLSR*) *of ER's and EI's of*** A_1**.** The items 4–10 of Df I.5.11 apply, *mutatis mutandis* (particularly with the corresponding changes in terminology) with any of the PAnAPL's of either list (1.1) or (1.2) in place of '**Γ**', subject to the item 11 of Ax 1.1. Here follow typical instances

1) '$\mathbf{P}\langle \mathbf{x} \rangle$' is a *primary analytical molecular panlogographic relation* (*PAnMPLR*) *of* $\mathbf{A}_1$, whose *current range* at any given stage of the setup of A_1 and $\mathbf{A}_1$ is the class of ER's of A_1, any given member of which, $\mathbf{P}\langle \mathbf{x} \rangle$, is an ER of A_1 that involves certain *free* occurrences of a given (selected) AEOT **x** and perhaps occurrences of some other AEOT's **u**, **v**, etc, which are not mentioned by using the panlogograph '$\mathbf{P}\langle \mathbf{x} \rangle$'. Under the assumption that $\mathbf{P}\langle \mathbf{x} \rangle$ does not involve, e.g., **y** either as a free AEOT or as a bound APVOT, $\mathbf{P}\langle \mathbf{y} \rangle$ is defined as:

$$\mathbf{P}\langle\mathbf{y}\rangle \to \mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\Big| \tag{1.5}$$

(cf. (I.5.23)), where ‘$\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\ \Big|$’ is the *metalogographic operator of substitution* such that $\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\Big|$ is the ER resulting by substitution of **y** for **x** throughout $\mathbf{P}\langle\mathbf{x}\rangle$. Likewise, ‘$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$’, e.g., is a PAnMPLR of $\mathsf{\mathbf{A}}_1$, whose range is the class of ER’s of A_1, any given member of which, $\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$, is an ER of A_1 that involves occurrences of two different free AEOT’s **u** and **v** and perhaps occurrences of some other AEOT’s **w**, **x**, etc, which are not mentioned by using the panlogograph ‘$\mathbf{P}\langle\mathbf{x}\rangle$’. If **x** and **y** are two different AEOT’s, other than **u** and **v**, which do not occur in $\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$ either as free AEOT’s or as bound ones, then

$$\begin{gathered}\mathbf{P}\langle\mathbf{u},\mathbf{x}\rangle \to \mathrm{S}_{\mathbf{v}}^{\mathbf{x}}\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle\Big|, \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle \to \mathrm{S}_{\mathbf{u}}^{\mathbf{x}}\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle\Big|, \mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle \to \mathrm{S}_{\mathbf{v}}^{\mathbf{y}}\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\Big|,\\ \mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle \to \mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\Big|, \mathbf{P}\langle\mathbf{y},\mathbf{y}\rangle \to \mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\Big|, \text{etc}\end{gathered} \tag{1.6}$$

(cf. (I.5.24)). The above definitions (1.6) are obviously generalized to PAnMPLR’s such as ‘$\mathbf{P}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$’, ‘$\mathbf{P}\langle\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{x}\rangle$’, etc and their appropriate variants.

2) The above item applies with any PAnAPL (AnAPLR) of the list (1.1) in place of ‘**P**’, or, alternatively, with any PAnAPL (PAnAPLI) of the list (1.2) in place of ‘**P**’ and with “integron” (“I”) in place of “relation” (“R”), and, independently, with any mutually different PStAPL’s (StAPLOT’s) of the list (I.5.6), or, alternatively, with any mutually different PStAPL’s (StAPLOR’s) of the list (I.5.7), in place of the AEOT’s used (mentioned) in that item.

3) If it is necessary to indicate that, for instance, occurrences of **x** or of **u** and **v** in **P** or **I** are bound then the respective one of the panlogographs ‘$\mathbf{P}_{\langle\mathbf{x}\rangle}\langle\mathbf{x}\rangle$’ and ‘$\mathbf{P}_{\langle\mathbf{u},\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$’ or ‘$\mathbf{I}_{\langle\mathbf{x}\rangle}\langle\mathbf{u}\rangle$’and ‘$\mathbf{I}_{\langle\mathbf{u},\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$’ can be used. The former two are, as before, called *primary analytical molecular panlogographic relations* (*PAnMPLR’s*) and the latter two *primary analytical molecular panlogographic integrons* (*PAnMPLI’s*). It is understood that when an indexed congeneric PAnAPL, e.g., ‘$\mathbf{P}_1$’ or ‘$\mathbf{I}_1$’, is used in place of the respective base letter ‘**P**’ or ‘**I**’, the angle-bracketed subscript should occur after the numeral subscript, e.g. ‘$\mathbf{P}_{1\langle\mathbf{x}\rangle}\langle\mathbf{x}\rangle$’ and ‘$\mathbf{P}_{1\langle\mathbf{u},\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$’ or ‘$\mathbf{I}_{1\langle\mathbf{x}\rangle}\langle\mathbf{u}\rangle$’and ‘$\mathbf{I}_{1\langle\mathbf{u},\mathbf{v}\rangle}\langle\mathbf{u},\mathbf{v}\rangle$’.

4) The AnMPLR ‘$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$’ (e.g.) can be used either *xenonymously* so as to denote $\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$, i.e. a general ER of A_1 of its range, or *non-xenonymously* in two different mental modes, namely either *autonymously* or *semi-xenonymously* (*semi-autonymously*). When used autonymously, ‘$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$’ denotes the *homoloautographic* (*photoautographic*) *token-class* of the logograph therein depicted between the curly light-faced single quotation marks. In accordance with the *juxtaposition principle of autonymous quotations* (see, e.g., Suppes [1957, pp. 125–127]), the juxtaposition of autonymous quotations: ‘**P**’‘⟨’‘**u**’‘,’**v**’‘⟩’ is a synonym of ‘$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$’. By contrast, when used semi-xenonymously (semi-autonymously), ‘$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$’ denotes **P**⟨‘**u**’,‘**v**’⟩, so that the class of operata of the *atomic panlogographic quasi-predicate* (*AtMLQP*) ‘**P**’ is extended from EOT’s (panlogographic ordinary terms) to PLOT’s (panlogographic ordinary terms). Like remarks apply, *mutatis mutandis,* to ‘$\mathbf{P}\langle\mathbf{x}\rangle$’ (e.g.). The PLR’s **P**⟨‘**x**’⟩ and **P**⟨‘**u**’,‘**v**’⟩ can be defined formally by the following generalizations of definitions (1.5) and (1.6):

$$\mathbf{P}\langle\text{'}\mathbf{x}\text{'}\rangle \to \mathrm{S}^{\mathbf{x}}_{\text{'}\mathbf{x}\text{'}}\mathbf{P}\langle\mathbf{x}\rangle\Big|, \tag{1.5a}$$

$$\mathbf{P}\langle\text{'}\mathbf{u}\text{'},\text{'}\mathbf{v}\text{'}\rangle \to \mathrm{S}^{\mathbf{x}}_{\text{'}\mathbf{u}\text{'}}\mathrm{S}^{\mathbf{y}}_{\text{'}\mathbf{v}\text{'}}\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\Big\|, \tag{1.6a}$$

Therefore, **P**⟨‘**x**’⟩ and **P**⟨‘**u**’,‘**v**’⟩ have the following meanings. If an ER **P**⟨**x**⟩ of A_1 is given then all occurrences of the pertinent EOT **x** of A_1 throughout **P**⟨**x**⟩ are also given. Consequently, **P**⟨‘**x**’⟩ is the variant of **P**⟨**x**⟩, in which all occurrences of the EOT **x** throughout **P**⟨**x**⟩ are replaced with occurrences of its StAtPLPH ‘**x**’ belonging to $\mathbf{A}_1$. Likewise, if an ER **P**⟨**u**,**v**⟩ of A_1 is given then all occurrences of the pertinent EOT’s **u** and **v** of A_1 throughout **P**⟨**u**,**v**⟩ are also given. Consequently, **P**⟨‘**u**’,‘**v**’⟩ is the variant of **P**⟨**u**,**v**⟩, in which all occurrences of the EOT’s **x** throughout **P**⟨**u**,**v**⟩ are replaced with occurrences of their StAtPLPH’s ‘**u**’ and ‘**v**’ belonging to $\mathbf{A}_1$. It is understood that, while **P**⟨**x**⟩ and **P**⟨**u**,**v**⟩ are ER’s of A_1, **P**⟨‘**x**’⟩ and **P**⟨‘**u**’,‘**v**’⟩ are PLR’s of $\mathbf{A}_1$, because ‘**x**’, ‘**u**’, and ‘**v**’ belong to $\mathbf{A}_1$. At the same time, it is clear that ‘**P**⟨‘**x**’⟩’ and ‘**P**⟨‘**u**’,‘**v**’⟩’ are *AnAtPLPH’s* (analytical atomic panlogographic placeholders), to *of* $\mathbf{A}_1$, *whose ranges are certain classes of PLR’s of* $\mathbf{A}_1$. PLR’s such as **P**⟨‘**x**’⟩ and **P**⟨‘**u**’,‘**v**’⟩ will be discussed in a wider context in subsection 2.4.•

†**Th 1.1:** ***The extendable primary euautographic formation rules system (XPFR-system) of*** A_1. All PAPL's (PAPLPH's), structural ones (PStAPL's) and analytical ones (PAnAPL's), that occur in the following statements are used *xenonymously*.

1) Either of the digits $\mathsf{0}$ and $\mathsf{1}$ is an EI or, more specifically, AEI or, still more specifically, PAEI, of A_1.

2) $\mathbf{x}$, i.e. either $\mathbf{x}^{\mathrm{pv}}$ or $\mathbf{x}^{\mathrm{pc}}$, is an EOT of A_1, whereas, more specifically, $\mathbf{x}^{\mathrm{pv}}$ is a PVOT and $\mathbf{x}^{\mathrm{pc}}$ is a PCOT, of A_1.

3) $\mathbf{p}$ is an ER, or, more specifically, PER, or, even more specifically, PAEOR, of A_1, and also, simply, an AER of A_1, the understanding being that there are neither secondary nor special AER's in A_1.

4) $\mathbf{f}^1(\mathbf{x}_1)$, $\mathbf{f}^2(\mathbf{x}_1,\mathbf{x}_2)$, etc, – generally, $\mathbf{f}^n(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$ or $\mathbf{f}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$, – are ER's, or, more specifically, PEOR's, or, still more specifically, PMEOR's, of A_1. $\mathbf{i}\langle\mathbf{x}\rangle$ is a secondary MESpR. $\mathbf{f}^n(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$ is a secondary MEOR.

5) $V(\mathbf{P})$ is an EI of A_1.

6) $[\hat{-}\,\mathbf{I}]$ is an EI of A_1.

7) $[\mathbf{I}\,\hat{+}\,\mathbf{J}]$ is an EI of A_1.

8) $[\mathbf{I}\,\hat{\cdot}\,\mathbf{J}]$ is an EI of A_1.

9) $[\mathbf{I}\,\hat{=}\,\mathbf{J}]$ is an ER, or more specifically, ESpR, and also, still more specifically, EAlR, of A_1.

10) $[\mathbf{P} \forall \mathbf{Q}]$ is an ER or, more specifically, ELR, of A_1.

11) If $\mathbf{x}$ is a PVOT (and not a PCOT) then $[(\exists\mathbf{x})\mathbf{P}\langle\mathbf{x}\rangle]$ is an ER of A_1 and $[(\hat{\cdot}\,\mathbf{x})V(\mathbf{P}\langle\mathbf{x}\rangle)]$ is an EI of A_1.

12) The range of each one of the three count names: "EOT" ("euautographic ordinary term"), "EI" ("euautographic integron") or "ESpT" ("euautographic special term") being its synonym, and "ER" ("euautographic relation"), which is followed by the postpositive possessive qualifier "of A_1", and the same range of a panlogographic synonym of a name as '$\mathbf{x}$', '$\mathbf{I}$', or '$\mathbf{P}$' (e.g.) respectively is determined by the above XPFR's and by all pertinent SEFR's (secondary euautographic formation

rules) of A_1 that will be laid down in the sequel, – in accordance with Df 1.3.

Proof: The theorem is a version of Ax 1.1, in which the following substitutions have been made.

a) In the exclusion of the item 11, all occurrences of the AnAMLPH's of '**Γ**' and '**Δ**' along with the pertinent verbal conditions restricting their ranges have been replaced with semantically adequate occurrences of the PAnAPL's '**P**' and '**Q**' or '**I**' and '**J**' respectively, in accordance with items 1 and 2 of Df 1.3.

b) In the item 11, all occurrences of '**Γ**' along with the pertinent verbal conditions restricting its range to the class of properly structured PER's have been replaced with occurrences of '$\mathbf{P}\langle\mathbf{x}\rangle$' that has the required semantic properties, in accordance with Dfs 1.5–1.7.

c) Most occurrences of the abbreviated taxonyms in the predicatives of RPFR's, which contain the prepositive letter "P" for the qualifier "primary" have been altered by omission of that letter in order to allow automatically extending the initial ranges of the PAnAPL's in the result of making SFR's of A_1 in the sequel.

d) The syntactic (grammatical) panlogographic subjects of the statements 2–11 of Th 1.1 are interrelated semantically via their ranges of EF's.•

Df 1.8. Df 1.2 applies with "Th 1.1", "extendable", and "X" for "extendable" in place of "Ax 1.1", "restricted", and "R" for "restricted" respectively.•

Df 1.9. The following definitions are in agreement with Dfs I.3.1 and 1.3(1).

1) An ER is called an *elemental*, or *primitive*, one (*ElER* or *PvER.*) if and only if it is either an *atomic* one (*AER*) or a *molecular* one (*MER*). Logographically, **P** is an ElER if and only if it is p, i.e. if and only if it is either **p** or **π.**

2) An ER is called is a *complex*, or *compound*, *relation* (*CxER* or *CdER*) if and only if it is not an ElER. Logographically, **P** is a CxER if and only if it is not p, i.e. if and only if it is neither **p** nor **π.**

3) An EOT (euautographic ordinary term), **x**, or PEI (primary euautographic integron), **I**, is indiscriminately called a PET (primary euautographic term), the understanding being that "integron" and "special term" are synonyms.

4) A PET or a PER, **P**, is indiscriminately called PEF (primary euautographic formula).•

Cmt 1.3. 1) The initial range of either one of the assemblage-valued MLPH's '**Γ**' and '**Δ**', which are used in Ax 1.1, is formally determined by Df I.5.10(1). At the same time, the range of every AnAPLPH (AnAPL's) on either list (1.1) or (1.2) is empty (undefined) until Ax 1.1 is stated. Therefore, these placeholders cannot be introduced before stating Ax 1.1. After stating Ax 1.1, the initial ranges of all these placeholders are defined through the class-denotata of the taxonyms (count names) "PEI" ("primary integron") and "PER" ("primary relation") and their hypotaxonyms, which Df 1.3 and Ax 1.1 have in common. Formally, the initial ranges are assigned to the above AnAPL's by Th 1.1, being a version of Ax 1.1 with occurrences of '**P**' and '**Q**' or '**I**' and '**J**' in place of the semantically adequate occurrences of '**Γ**' and '**Δ**'. That version of Ax 1.1 is called:

a) the *extendable primary euautographic formation rules system* (*XPFR-system*) *of* A_1 if all APL's occurring in it are used xenonymously;

b) the *XPFR-system of* $\mathbf{A}_1$ if all APL's occurring in it are used autonymously;

c) the *XPFR-system of* A_1 if all APL's occurring in it are used in the TAEXA-mode.

2) All subsequent *schematic SEFR's* (*SchSEFR's*) of A_1 are also stated primarily in terms of '**P**' and '**Q**' or '**I**' and '**J**', the understanding being that *the class of ER's or EI's of* A_1*, which are determined either by a certain schematic XPFR (SchXPFR) or by a certain SchSEFR* (*schematic secondary euautographic formation rule*) *of* A_1*, is assumed* (*postulated*) *to be independent of the concrete PStAPL's of the pertinent one of the lists (I.5.6)–(I.5.9), including (I.5.8*^{1}*)–(I.5.8*^{3}*) etc,* and *of the concrete PAnAPL's of the pertinent one of the lists (1.1)–(1.4), including (1.3*^{1}*)–(1.3*^{3}*) etc*, which are employed in the respective *schematic formation rule* (*SchFR*). Therefore, once a certain SchRPFR or SchSFR is stated in terms of some PAPL's of a certain list, that rule can be restated in terms of any other congeneric or conspecific AnAPL's of the same list without altering the class of ER's that is determined by the SchFR.

3) In accordance with the items 10 and 11 of Df 1.3, the range of any PStAPL and the range of any PAnAPLR of any given one of the lists (1.3^1)–(1.3^3) etc, (1.3), and (1.4) is affected by no SEFR of A_1, so that it is permanent. By contrast, the range that is attributed to every PAnAPL of a given type (taxon, taxonomic set) and which is called its *initial range*, is the pertinent class of PEF's that is determined by the

RPFR-system. Depending on the type of a PAnAPL, once some SFR's of A_1 are made, the initial range of the PAnAPL's either remains unaltered or is *automatically* augmented by some SEF's that are associated with SEF's introduced by the SFR's. In contrast to the *initial range* of a PAnAPL, which is permanent, *a current range*, or simply *a range*, of the PAnAPL is one that the PAnAPL acquires after making any given SFR or group of SFR's of A_1. Consequently, depending on the type of a PAnAPL, a current range of the PAnAPL either equals its initial range or is a certain superclass of it. In this case, all *euautographic FR's*, i.e. FR's of A_1, – *concrete* ones (*CEFR's*) and *schematic* ones (SchEFR's), which are laid down in terms of PStAPL's introduced in Df I.5.2 or PAnAPL's introduced in Df 1.3 or both, – form a *single whole coherent recursive system of FR's of* A_1 and at the same time *that of* $\mathbf{A}_1$, and hence *that of* A_1, which determines a *single whole system of interrelated classes of EOT's, EI's, and ER's* – classes that are extended automatically as new SFR's are laid down. In this case, in stating any schematic euautographic formation rule (SchEFR) of A_1, I tacitly follow the following convention.•

Cnv 1.2. A *variant* of a SchEFR of A_1, denoted ad hoc by 'K_σ', is another SchEFR of A_1, denoted ad hoc by 'K'_σ', which is obtained by alphabetic changes of the *primary atomic panlogographs* (*PAPL's*) occurring in K_σ, namely PStAPL's (if present), i.e. bound or free StAPLOT's of the list (I.5.6) and StAPLOR's of the list (I.5.7), and also PAnAPL's (if present), i.e. PAnAPLI's of the list (1.2) and GAnAPLR of the list (1.1), in such a way that two occurrences of the same PAPL in K_σ remain occurrences of the same PAPL in K'_σ, and two occurrences of different PAPL's in K_σ remain occurrences of different PAPV's in K'_σ (cf. a like definition of a variant of a *well-formed formula* in Church [1956, p. 86]). The SchEFR's of A_1, K_σ and K'_σ, are at the same time, i.e. in the TAEXA-mode, a CPLFR, denoted ad hoc by '$\mathbf{K}_\iota$' and '$\mathbf{K}'_\iota$' respectively. The above ad hoc notations have the following mnemonic justification. "K" is the first letter of the Greek masculine noun "κανών" \kanón\ meaning *a rule* (*measure*, *principle*) or (eccl. & mus.) *a canon.* The subscript '$_\sigma$' in 'K_σ' is the first letter of the Greek feminine noun "σχήμα" \schima\ meaning *a form, shape, diagram,*or (gram.) *figure*. The subscript '$_\iota$' in '$\mathbf{K}_\iota$' is the first letter of the Greek adjective "ιδιαίτερος" \idiéteros\ meaning *special* or *particular*.•

Crl 1.1. In contrast to the RPFR-system of A_1, which is established by Ax 1.1 and which is designed to be used only *xenonymously*, the XPFR-system of A_1, which is established by Th 1.1, can be used either *xenonymously* or *autonymously* or else *in both modes simultaneously* (*intermittently*), i.e. *in the TAEXA-mode* (see Df I.5.10(1)). When the XPFR-system of A_1 is used autonymously subject to Cnv 1.2, it is mentally turned into a *primary concrete panlogographic formation rules system (PCPLFR-system) of* $\mathbf{A}_1$. With the help of the appropriate HAQ's (homoloautographic quotations) and QHAQ's (quasi-homoloautographic quotations), the PCPLFR-system of $\mathbf{A}_1$ can explicitly be expressed as follows, – in accordance with Dfs I.5.2 and 1.3.

1) Either of the digits $\mathsf{0}$ and $\mathsf{1}$ is an EI, or, more specifically, AEI, or, still more specifically, PAEI, of $\mathbf{A}_1$.

2) '$\mathbf{x}$' is a StAPLOT of $\mathbf{A}_1$, '$\mathbf{x}^{\mathrm{pv}}$' is an APVOT-valued panlogograph of $\mathbf{A}_1$, and '$\mathbf{x}^{\mathrm{pc}}$' is an APCOT-valued panlogograph of $\mathbf{A}_1$.

3) '$\mathbf{p}$' is a StAPLOR of $\mathbf{A}_1$.

4) $'\mathbf{f}^1{}'('\mathbf{x}_1{}')$, $'\mathbf{f}^2{}'('\mathbf{x}_1{}','\mathbf{x}_2{}')$, etc, – generally $'\mathbf{f}^n{}'('\mathbf{x}_1{}','\mathbf{x}_2{}',...,'\mathbf{x}_n{}')$ or $'\mathbf{f}'('\mathbf{x}_1{}','\mathbf{x}_2{}',...,'\mathbf{x}_n{}')$, – are PPLR's, or, more specifically, PPLOR's, or, still more specifically, PMPLOR's, of $\mathbf{A}_1$, and also, simply, PMPLR's of $\mathbf{A}_1$, the understanding being that there are no MPLSpR's in $\mathbf{A}_1$, either primary or secondary, but there are SMPLLR's (SMPLOR's) in $\mathbf{A}_1$.

5) $V('\mathbf{P}')$ is a PPLI, or, more specifically, PMPLI, of $\mathbf{A}_1$.

6) $[\hat{\neg}'\mathbf{I}']$ is a PPLI of $\mathbf{A}_1$.

7) $['\mathbf{I}'\hat{\dotplus}'\mathbf{J}']$ is a PPLI of $\mathbf{A}_1$.

8) $['\mathbf{I}'\hat{\because}'\mathbf{J}']$ is a PPLI of $\mathbf{A}_1$.

9) $['\mathbf{I}'\hat{=}'\mathbf{J}']$ is a PPLR, or more specifically, PPLSpR, and also, still more specifically, PPLAlR, of $\mathbf{A}_1$.

10) $['\mathbf{P}'\hat{\vee}'\mathbf{Q}']$ is a PPLR or, more specifically, PPLOR or PPLLR, of $\mathbf{A}_1$.

11) $[(\exists'\mathbf{x}')\mathbf{P}\langle'\mathbf{x}'\rangle]$ is a PPLR, and also, more specifically, PPLOR or PPLLR, of $\mathbf{A}_1$; and $[(\hat{\because}'\mathbf{x}')V(\mathbf{P}\langle'\mathbf{x}'\rangle)]$ is a PPLI, or PPLSpT, of $\mathbf{A}_1$.

12) The ranges of the taxonyms: "PPLI" ("PPLSpT") and "PPLR", and also the ranges of their hypotaxonyms (as those used above) are determined by

the above RPPLFR’s and by their alphabetic variants, – in accordance with Cnv 1.2.•

Cmt 1.4. In accordance with Df 1.6, Th 1.1, and Clr 1.1, the following terminology is obviously understood.

1) An occurrence of the PLOT (panlogographic ordinary term) ‘**x**’ in a given PLF (panlogographic formula) [of $\mathbf{A}_1$] is called a *bound occurrence* of ‘**x**’ in the given formula if it is an occurrence in a constituent formula of the given formula, which is either a scope of ‘$(\exists\mathbf{x})$’, i.e. of $(\exists'\mathbf{x}')$, or a scope of ‘$(\hat{\cdot}\,\mathbf{x})$’, i.e. of $(\hat{\cdot}'\mathbf{x}')$; otherwise it is called a *free occurrence* of ‘**x**’ in the given formula. It is understood that if a given panlogographic formula has bound occurrences of ‘**x**’ then it cannot have any free occurrences of ‘**x**’ and, conversely, if the given formula has free occurrences of ‘**x**’ then it cannot have any bound occurrences of ‘**x**’. A like definition applies with any other PLOT in place of ‘**x**’. The PLOT’s that have bound occurrences in a panlogographic formula are called the *bound*, or *dummy*, PLOT’s, and the PLOT’s that have free occurrences in the panlogographic formula are called the *free* PLOT’s.

2) A scope of ‘$(\exists\mathbf{x})$’ or a scope of ‘$(\hat{\cdot}\,\mathbf{x})$’ is indiscriminately called a *panlogographic contraction*, whereas either of the *kernel-signs* ‘$(\exists\mathbf{x})$’ and ‘$(\hat{\cdot}\,\mathbf{x})$’ is indiscriminately called a *panlogographic contractor*. Discriminately, ‘$(\exists\mathbf{x})$’ is commonly called *an inclusive*, or *laxly inclusive*, *disjunctive panlogographic contractor* (*DjPLCnt*) or a [*lax*] *existential panlogographic pseudo-quantifier* (*EPLPQ*) and also, properly, *the* one *over* ‘**x**’, and $(\hat{\cdot}\,\mathbf{x})$ is commonly called *a pseudo-multiplicative panlogographic contractor* (*PsdMvPLCnt*) or a *panlogographic pseudo-multiplier* (*PLPsdMr*) and also, properly, *the* one *over* ‘**x’**. In any of the above terms, the noun “*contractor*” (“*Cnt*”) can be used interchangeably with “*binder*” (“*Bnd*”).

3) Every bound PLOT of a panlogographic formula should be understood as an APVOT-valued APLPH. For instance, all occurrences of the bound PLOT ‘**x**’ throughout a PLF should be understood as occurrences ‘$\mathbf{x}^{\mathrm{pv}}$’, i.e. as occurrences of an APLPH that can be replaced with occurrences of any PVOT of A_1. By contrast, if ‘**y**’, e.g., is a free PLOT of a PLF then it can be understood either as the corresponding APVOT-valued APLPH ‘$\mathbf{y}^{\mathrm{pv}}$’ or as the corresponding APCOT-valued APLPH ‘$\mathbf{y}^{\mathrm{pc}}$’.

Consequently, all occurrences of ‘**y**’ throughout the PLF can be replaced either with occurrences of any PVOT of A_1 or with either one of the PCOT’s of A_1 $\emptyset$ and $\emptyset'$.

4) The syntactico-semantic correspondence between Th 1.1 and Crl 1.1 explicates how any secondary *schematic* euautographic formation rule of A_1, which is expressed in terms of panlogographic placeholders and perhaps in terms of some concrete euautographs, can be treated as a secondary *concrete* panlogographic formation rule $\mathbf{A}_1$. In the sequel, I shall not state any corollaries after the manner of Crl 1.1.•

1.2. Synonymic formation rules of the major classes of basic (non-contracted) secondary formulas of A_1 and $\mathbf{A}_1$

†**Df 1.10:** ***A system of basic (non-contracted) secondary (defined) formulas of*** A_1 ***and*** $\mathbf{A}_1$**.**

1) $[\neg\mathbf{P}]\to[\mathbf{P}\veebar\mathbf{P}]$.

2) $[\mathbf{P}\vee\mathbf{Q}]\to[\neg[\mathbf{P}\veebar\mathbf{Q}]]$.

3) $[\mathbf{P}\Rightarrow\mathbf{Q}]\to[[\neg\mathbf{P}]\vee\mathbf{Q}]$.

4) $[\mathbf{P}\Leftarrow\mathbf{Q}]\to[\mathbf{P}\vee[\neg\mathbf{Q}]]$.

5) $[\mathbf{P}\barwedge\mathbf{Q}]\to[[\neg\mathbf{P}]\vee[\neg\mathbf{Q}]]$.

6) $[\mathbf{P}\wedge\mathbf{Q}]\to[\neg[\mathbf{P}\barwedge\mathbf{Q}]]\to[\neg[[\neg\mathbf{P}]\vee[\neg\mathbf{Q}]]]$.

7) $[\mathbf{P}\Leftrightarrow\mathbf{Q}]\to[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{P}\Leftarrow\mathbf{Q}]]\leftrightarrow[\neg[[\neg[[\neg\mathbf{P}]\vee\mathbf{Q}]]\vee[\neg[\mathbf{P}\vee[\neg\mathbf{Q}]]]]]$.

8) $[\mathbf{P}\,\overline{\vee}\,\mathbf{Q}]\to[\neg[\mathbf{P}\vee\mathbf{Q}]]$.

9) $[\mathbf{P}\,\overline{\wedge}\,\mathbf{Q}]\to[\neg[\mathbf{P}\wedge\mathbf{Q}]]\to[\neg[\neg[\mathbf{P}\barwedge\mathbf{Q}]]]$.

10) $[\mathbf{P}\,\overline{\Rightarrow}\,\mathbf{Q}]\to[\neg[\mathbf{P}\Rightarrow\mathbf{Q}]]$.

11) $[\mathbf{P}\,\overline{\Leftarrow}\,\mathbf{Q}]\to[\neg[\mathbf{P}\Leftarrow\mathbf{Q}]]$.

12) $[\mathbf{P}\,\overline{\Leftrightarrow}\,\mathbf{Q}]\to[\neg[\mathbf{P}\Leftrightarrow\mathbf{Q}]]$.

13) $[\mathbf{I}\,\overline{\hat{=}}\,\mathbf{J}]\to[\neg[\mathbf{I}\hat{=}\mathbf{J}]]\leftrightarrow[[\mathbf{I}\hat{=}\mathbf{J}]\veebar[\mathbf{I}\hat{=}\mathbf{J}]]$.

14) $[\mathbf{I}\,\hat{-}\,\mathbf{J}]\to[\mathbf{I}\,\hat{+}\,[\hat{-}\,\mathbf{J}]]$.

15) $\overline{V}(\mathbf{P})\to V(\neg\mathbf{P})\leftrightarrow V(\mathbf{P}\veebar\mathbf{P})$.

No matter whether the relations **P** and **Q** or the integrons **I** and **J** are primary or secondary, the definienda of definitions 1–13 are *SER’s* (*secondary euautographic relations*) *of* A_1, whereas the definienda of definitions 14 and 15 are *SEI’s* (*secondary euautographic integrons*) *of* A_1.•

Cmt 1.5. In Df 1.10, for the sake of universality, I employ an overbar $\bar{\ }$ of an adjustable length as a negation label on a base sign instead of the more common slant upright to the left stroke, /, across the base sign in consequence of typographical difficulties in some cases.•

Cmt 1.6. Every item of Df 1.10 is a partial definition schema, which defines an indefinite number of secondary relations or terms of A_1. For the sake of brevity, the *immediate definientia* of items 3–13 and 15 of Df 1.10 are expressed in terms of the *kernel-signs*, which are defined in some preceding items of that definition. Particularly, the immediate definientia of items 3–9, 13 and 15 are expressed in terms of the signs $\neg$ and $\vee$, which are defined by items 1 and 2 of Df 1.10. By items 3, 4, and 7, the immediate definientia of items 9–12 can also be expressed in terms of $\neg$ and $\vee$ only. Therefore, without loss of generality, the two primitive connectives $\neg$ and $\vee$ could from the very beginning be introduced into A_1 as primary ones instead of the single connective $\forall$. In this case, the following changes should be introduced in the pertinent articles of the setup of A_1.

1). Item 4 of Ax 5.1 should be replaced by this:

"4a) The two *ordinary logical connectives*: $\neg$ and $\vee$."

2). Item 10 of Ax 1.1 should be replaced with the conjunction of these two:

"10a) $[\neg\mathbf{\Gamma}]$ is a relation if $\mathbf{\Gamma}$ is a relation.

10b) $[\mathbf{\Gamma}\vee\mathbf{\Delta}]$ is a relation if $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are relations."

3). Item 10 of Th 1.1 should be replaced with the conjunction of these two:

"10a) $[\neg\mathsf{\mathbf{P}}]$ is a relation.

10b) $[\mathsf{\mathbf{P}}\vee\mathsf{\mathbf{Q}}]$ is a relation."

4) Items 1 and 2 of Df 1.10 should be omitted, while the remaining thirteen items should, upon omission of all definientia involving $\forall$, be renumbered by the successive numerals from '1' through '13'.•

Cmt 1.7. In the current setting, the final definientia of items 3–12 of Df 1.10 can, by items 1 and 2, be expressed in terms of $\forall$ and be thus transformed into the *primary definientia* (*primary synonymous predecessors*) of their definienda. By way of example, here follow the pertinent developments of items 2, 3, 5, and 8 of Df 1.10:

2′) $[\mathbf{P}\vee\mathbf{Q}]\leftrightarrow[\mathbf{P}\,\overline{\forall}\,\mathbf{Q}]\rightarrow[\neg[\mathbf{P}\forall\mathbf{Q}]]\leftrightarrow[[\mathbf{P}\forall\mathbf{Q}]\forall[\mathbf{P}\forall\mathbf{Q}]]$.

$$3')\ [\mathbf{P} \Rightarrow \mathbf{Q}] \rightarrow [[\neg\mathbf{P}] \vee \mathbf{Q}] \leftrightarrow [[[\neg\mathbf{P}] \veebar \mathbf{Q}] \veebar [[\neg\mathbf{P}] \veebar \mathbf{Q}]]$$
$$\leftrightarrow [[[\mathbf{P} \veebar \mathbf{P}] \veebar \mathbf{Q}] \veebar [[\mathbf{P} \veebar \mathbf{P}] \veebar \mathbf{Q}]].$$

$$5')\ [\mathbf{P} \barwedge \mathbf{Q}] \rightarrow [[\neg\mathbf{P}] \vee [\neg\mathbf{Q}]] \leftrightarrow [\neg[[\neg\mathbf{P}] \veebar [\neg\mathbf{Q}]]]$$
$$\leftrightarrow [[[\neg\mathbf{P}] \veebar [\neg\mathbf{Q}]] \veebar [[\neg\mathbf{P}] \veebar [\neg\mathbf{Q}]]]$$
$$\leftrightarrow [[[\mathbf{P} \veebar \mathbf{P}] \veebar [\mathbf{Q} \veebar \mathbf{Q}]] \veebar [[\mathbf{P} \veebar \mathbf{P}] \veebar [\mathbf{Q} \veebar \mathbf{Q}]]].$$

$$8')\ [\mathbf{P}\ \overline{\vee}\ \mathbf{Q}] \rightarrow [\neg[\mathbf{P} \vee \mathbf{Q}]] \leftrightarrow [[\mathbf{P} \vee \mathbf{Q}] \veebar [\mathbf{P} \vee \mathbf{Q}]]$$
$$\leftrightarrow [[[\mathbf{P} \veebar \mathbf{Q}] \veebar [\mathbf{P} \veebar \mathbf{Q}]] \veebar [[\mathbf{P} \veebar \mathbf{Q}] \veebar [\mathbf{P} \veebar \mathbf{Q}]]].$$

In items 13 and 15, the like transformations have already been done. Thus, upon expressing the secondary kernel-signs in all final definientia of Df 1.10 in terms of $\veebar$, if the APH's '**P**', '**Q**', '**I**', and '**J**' occurring in that definition are replaced with *concrete* primary formulas of A_1 of their initial ranges, the immediate definiens schema of any item of Df 1.10 turns into a certain primary formula of A_1, whereas its definiendum schema turns into the respective *concrete* secondary formula of A_1. Consequently, after imaginarily concretizing any item of Df 1.10, the ranges of the tokens of the above four placeholders, which occur in that item and in all other items of that definition, in Th 1.1, and also in all other formation rules to be stated in the sequel, are supposed to be updated in accordance with the items 10 and 11 of Df 1.3 (see also Cmt 1.3(3)). Thus, any item of Df 1.10 is, after all, a schema of an infinite number of concrete *abbreviative definitions* of *secondary formulas of* A_1 in terms of *the respective primary formulas of* A_1.•

Cmt 1.8. [The token of] the definition sign → (e.g) occurring in any item of Df 1.10 can be interpreted in any of the following three ways.

1) According to one interpretation, the sign → applies *contactually* to the definiendum schema and definiens schema so that the two schemata are regarded as *tychautographs*, namely as the *concrete tychaugraphic definiendum* and as the *concrete tychautographic definiens* respectively.

2) The sign → applies *slidingly* (*passingly*, *transitorily*) to any concrete but not concretized object euautographic instance of the definiendum schema and to the respective concrete euautographic instance of the definiens schema, so that the two schemata are regarded as *euxenographs*.

3) The separate items of Df 1.10 are *contextual* (*implicit*) *definitions of the fifteen major secondary kernel-signs*:

$$\neg,\ \vee,\ \Rightarrow,\ \Leftarrow,\ \barwedge,\ \wedge,\ \Leftrightarrow,\ \overline{\vee},\ \overline{\wedge},\ \overline{\Rightarrow},\ \overline{\Leftarrow},\ \overline{\Leftrightarrow},\ \overline{\hat{=}},\ \hat{-},\ \overline{V} \qquad (1.7)$$

in terms of *the six major primary atomic kernel-signs*:

$$\underline{\forall},\ \hat{=},\ \hat{-},\ \hat{+},\ \hat{\cdot},\ V. \qquad (1.8)$$

In this case, the kernel-signs of the list (1.7) are *effectual definienda* of the contextual definitions, whereas the definiendum schemata, which contain those signs, and which have been described above in the previous items 1 and 2, are *apparent definienda* of the contextual definitions.•

1.3. The primary and secondary formation rules of A_0 and $\mathbf{A}_0$

†**Ax 1.2:** ***The restricted primary euautographic formation rules system (RPFR-system) of*** A_0**.** Just as in Ax 1.1, all atomic placeholders (APH's), panlogographic ones (PLAPH's) and metalogographic ones (MLAPH's or AMLPH's), that occur in the following statements are used *xenonymously*.

1) Either of the digits 0 and 1 is a PEI, or, more specifically, PAEI, of A_0.
2) **p**, called an APVOR or AEOR or, simply, AER of A_0 is a PER of A_0, the understanding being that there are neither secondary nor special AER's in A_0.
3) $V(\mathbf{\Gamma})$ is a PEI of A_0 if $\mathbf{\Gamma}$ is a PER of A_0.
4) $[\hat{-}\mathbf{\Gamma}]$ is a PEI of A_0 if $\mathbf{\Gamma}$ is a PEI of A_0.
5) $[\mathbf{\Gamma}\,\hat{+}\,\mathbf{\Delta}]$ is a PEI of A_0 if $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are PEI's of A_0.
6) $[\mathbf{\Gamma}\,\hat{\cdot}\,\mathbf{\Delta}]$ is a PEI of A_0 if $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are PEI's of A_0.
7) $[\mathbf{\Gamma}\,\hat{=}\,\mathbf{\Delta}]$ is a PER, or, more specifically, PESpR, or, still more specifically, PEAlR, of A_0 if $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are PEI's of A_0.
8) $[\mathbf{\Gamma}\,\underline{\forall}\,\mathbf{\Delta}]$ is a PER, or, more specifically, PELR, of A_0, if $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are PER's of A_0.
9) $\mathbf{\Gamma}$ is a PEI (PESpT) of A_0 or a PER of A_0 if and only if its being so follows from the above formation rules.•

Cmt 1.9. All items of Ax 1.2 are selected out of the items of Ax 1.1 and are, when necessary, modified in accordance with the atomic basis B_0 of A_0 as specified in item vii of Ax I.5.1. To be specific, the former items 2, 4, and 11, of Ax 1.1 have been omitted, the former items 1, 3, and 5–10 of Ax 1.1 remain basically unaltered, and the former item 12 of Ax 1.1 have been restated as the latter item 9 of Ax 1.2 with allowance for the fact that A_0 *has no EOT's* (*euautographic ordinary terms*). That is to say, according to Ax 1.2, an *EF* (*euautographic formula*) *of* A_0 is either an *EI* (*euautographic integron*), i.e. or an *ESpT* (*euautographic special term*) or an *ER* (*euautographic relation*). All preserved items of Ax 1.1 are renumbered in Ax 1.2 by successive Arabic numerals from ‘1’ to ‘9’. Thus, Ax 1.2 is in fact a corollary of Ax 1.1 subject to Ax I.5.1(vii). I have classified Ax 1.2 as an *axiom* in order to emphasize its status of being fundamental as groundwork for *autonomous* (*self-contained*) *reasoning*. I shall develop A_0 at large in Chapter III. Meanwhile, it is noteworthy that Df 1.3 and all previous statements based on it apply with ‘A_0’ in place of ‘A_1’ provided that the ranges of all APH’s introduced in that definition are restricted properly. Particularly, the following Th 1.2 theorem of Ax 1.2 is analogous to Th 1.1. of Ax 1.1.•

†**Th 1.2:** ***The extendable primary euautographic formation rules system (XPFR-system) of*** A_0. All PAPL’s (PAPLPH’s), structural ones (PStAPL’s) and analytical ones (PAnAPL’s), that occur in the following statements are used *xenonymously*.

1) Either of the digits $\mathsf{0}$ and $\mathsf{1}$ is an EI or, more specifically, AEI or, still more specifically, PAEI, of A_0.
2) **p** is an ER, or, more specifically, PER, or, even more specifically, PAEOR, of A_1, and also, simply, an AER of A_0, the understanding being that there are neither secondary nor special AER’s in A_0.
3) $V(\mathbf{P})$ is an EI of A_0.
4) $[\hat{-}\mathbf{I}]$ is an EI of A_0.
5) $[\mathbf{I}\,\hat{+}\,\mathbf{J}]$ is an EI of A_0.
6) $[\mathbf{I}\,\hat{\cdot}\,\mathbf{J}]$ is an EI of A_0.
7) $[\mathbf{I}\,\hat{=}\,\mathbf{J}]$ is an ER, or more specifically, ESpR, and also, still more specifically, EAlR, of A_0.
8) $[\mathbf{P}\,\dot{\vee}\,\mathbf{Q}]$ is an ER or, more specifically, ELR, of A_0.
9) The range of either one of the two count names: “EI” (“euautographic integron”), or “ESpT” (“euautographic special term”) being its synonym, and “ER” (“euautographic relation”), which is followed by the postpositive possessive qualifier “of A_0”, and the same range of a panlogographic synonym of a given name as ‘**I**’ or ‘**P**’ (e.g.) respectively is determined by the above XPFR’s and by all pertinent SEFR’s of A_0 (as those constituting

Df 1.10) that can be laid down in any place, – in accordance with the restriction of Df 1.3 from A_1 to A_0.

Proof: The theorem is a restatement of Ax 1.2 under Df 1.3 subject to the pertinent restrictions of the ranges of the PAPLPH's '**P**', '**Q**', '**I**', and '**J**'. That is to say, the theorem is a version of Ax 1.2, which is proved in analogy with Th 1.1.•

Cmt 1.10. It goes without saying that all definitions and all comments that have been stated in connection with Ax 1.1 and Th 1.1 apply, *mutatis mutandis*, to Ax 1.2 and Th 1.2. Also, Th 1.2 has a corollary analogous to Clr 1.1.•

1.4. Proper and common names of basic (not contracted) formulas and basic (not contracting) kernel-signs of A_1

Preliminary Remark 1.1. 1) In order to treat of various basic formulas and kernel-signs of A_1 as objects, they should be provided with the appropriate verbal proper and common names to be descriptive of their distinguishing properties. Particularly, the names of the basic kernel-signs should be descriptive of their functions in forming formulas of the respective classes. *Descriptio per genus et differentiam* or *differentias*, i.e. *a description through the pertinent genus and the pertinent differentia or differentiae*, is a name that satisfies the above requirements. Such a descriptive common name naturally classifies the formulas or kernel-signs, which are comprised in its range (class-connotatum). More generally, the *generic name* (*relative genus name*, *head name*) of a proper or common name of a certain formula or kernel-sign can be used as the proper name of the broader class, to which the formula or kernel-sign belongs.

2) Verbal names of euautographic formulas and kernel-signs have also the following auxiliary function. A euautograph has no phonic paratokens, so that it cannot be read orally. Therefore, its proper verbal name, if the euautograph has one, is the only means, by which it can be mentioned in spoken form. Particularly, once primitive (atomic and molecular) euautographs of A_1 are provided with descriptive proper names, a compound assemblage, or particularly a compound formula, of A_1 can be mentioned orally by sequentially uttering the names of the primitive constituents of the assemblage (to be exemplified in due course). The euautographic kernel-signs of A_1 will acquire phonic token only when they will be used as constituent parts of the catlogographic interpretands of euautographic formulas of A_1.

3) Descriptive names of euautographs of A_1 can be introduced with the help of the pertinent *ostensive*, or *quasi-ostensive*, *nominal definitions*, which will conveniently be stated either with the help of the suitable one of the nominal definition signs ↘ and ↙, which belong to the IML (inclusive metalanguage) and which have been defined in Df I.2.21, or with the help of defining tables of the name-definiendaversus their ostensive definientia of by informal verbal definitions. To be recalled, in accordance with Df I.2.21, both the name-definiendum, which stands at the base of either slant arrow ↘ and ↙, and the ostended (demonstrated) or quasi-ostended definiens, which stands at the head of either slant arrow ↘ and ↙, are used without any special quotation marks. In this case, depending of the *terms*, the *definiendum* and the *definiens,* of a definition and also depending on the mental attitude that I take towards the terms, the arrow used or the entire figure "***↘…" or "…↙***" is rendered into ordinary language in one of the following ways. The arrow ↘ is read as: "*is a name of*" or "*is a nominal definiendum of*"; the arrow ↙ is read as: "*is the ostensive definiens of*", "*is called*", or "*is denoted by*"; either one of the figures "***↘…" or "…↙***" can alternatively be rendered into ordinary language also thus: "I give the name "***" to …", where the alike ellipses "***" or "…" should be replaced alike. When it is desirable to have a synonym or synonyms of a given descriptive name, these will be introduced either in the legato style with the help of a suitable synonymic definition sign →, ←, or ↔ defined in Dfs I.2.17–I.2.19 (see also Cmt I.2.12 and Df I.2.20) or by a separate definition. Some of the names that are introduced below will be justified and become clearer after the pertinent issues of the setup of A_1 are laid down or after the theorems expressing the basic properties of the formulas or kernel-signs named are stated and proved.•

Df 1.11: ***Special (algebraic) terminology.***

A) Integrons (special terms), their elementary (basic, non-contracted) compositions, and their relations

1) $V(\mathbf{P})$↙[the primary, or initial, validity-integron of **P**].

2) $\bar{V}(\mathbf{P})$↙[the primary, or initial, antivalidity-integron of **P**].

3) $[\hat{-}\mathbf{I}]$↙[the additive inverse of **I**]←[hyphen-minus **I**].

4) $[\mathbf{I}\hat{+}\mathbf{J}]$↙[the sum of **I** and **J**] $[\mathbf{I}\hat{+}\mathbf{J}]$←[**I** plus **J**]

↔[the integron obtained by addition of **J** to **I**].

5) $[\mathbf{I} \hat{\cdot} \mathbf{J}]$↙[the product of **I** and **J**]↔[**I** multiplied by **J**]↔[**I** times **J**]

↔[the integron obtained by multiplication of **I** by **J**].

6) $[\mathbf{I} \hat{-} \mathbf{J}]$↙[the difference of **I** and **J**]↔[**I** dash-minus **J**]

↔[the integron obtained by subtraction of **J** from **I**].

7) $[\mathbf{I} \hat{=} \mathbf{J}]$↙[the equality of **I** and **J**]↔[**I** equals **J**],.

8) $[\mathbf{I} \overline{\hat{=}} \mathbf{J}]$↙[the antiequality of **I** and **J**]↔[**I** antiequals **J**].

B) Elementary (basic, non-contracting) special (algebraic) kernel-signs

1) V↙[the special singulary validity-sign].

2) $\overline{V}$ ↙[the special singulary antivalidity-sign].

3) $\hat{-}$↙[the special singulary minus sign]↔[the capped hyphen-minus sign].

4) $\hat{+}$↙[the special plus sign]↔[the capped plus sign].

5) $\hat{\cdot}$↙[the special multiplication sign]↔[the capped dot sign].

6) $\hat{-}$↙[the special binary minus sign]↔[the capped dash-minus sign].

7) $\hat{=}$↙[the special equality sign]↔[the capped equality sign].

8) $\overline{\hat{=}}$ ↙[the special antiequality sign]↔[the capped antiequality sign].

The above *eight* signs are, by Cmt A3.1(4), called the *elementary* (*not advanced, non-contracting*) *special* (*algebraic*) *kernel-signs of* A_1, the understanding being that A_0 has no other special (algebraic) kernel-signs. The *five* signs $\hat{=}$, $\hat{-}$, $\hat{+}$, $\hat{\cdot}$, and V, which have been introduced in Ax I.5.1(10,11), are called the *primary elementary atomic euautographic special* (*algebraic*) *kernel-signs* and the *three* signs $\overline{\hat{=}}$, $\hat{-}$, and $\overline{V}$, which have been defined by Df 1.10(13–15), are called *secondary elementary euautographic special* (*algebraic*) *kernel-signs*; $\hat{-}$ is an *atomic* one, whereas $\overline{\hat{=}}$ and $\overline{V}$ are *molecular* ones. The signs $\hat{-}$, V, and $\overline{V}$ are *singulary*, and the five other elementary special kernel-signs are *binary*. Collectively (indiscriminatorily), the signs $\hat{=}$ and $\overline{\hat{=}}$ are called the *euautographic relational special* (*algebraic*) *kernel-signs* (*predicate-signs*), the signs $\hat{-}$, $\hat{+}$, $\hat{\cdot}$, and $\hat{-}$ are called the *euautographic substantival special* (*algebraic*) *kernel-signs*, and the signs V and $\overline{V}$ are called the *euautographic substantivating* (*substantivizing*, *termizing*, *termization*) *special* (*algebraic*) *kernel-signs*.

Df 1.12: *Ordinary (logical) terminology.*

A) Major elementary (basic) ordinary (logical) relations

i) *Accidental proper names*

1) $[\mathbf{P} \not\vee \mathbf{Q}] \swarrow$[The former antidisjunction of **P** and **Q**].

2) $[\neg \mathbf{P}] \swarrow$[The negation, *or* denial, of **P**].

3) $[\mathbf{P} \vee \mathbf{Q}] \swarrow$[The inclusive disjunction of **P** and **Q**].

4) $[\mathbf{P} \Rightarrow \mathbf{Q}] \swarrow$[The rightward implication of **Q** from **P**].

5) $[\mathbf{P} \Leftarrow \mathbf{Q}] \swarrow$[The leftward implication of **P** from **Q**].

6) $[\mathbf{P} \not\wedge \mathbf{Q}] \swarrow$[The former anticonjunction of **P** and **Q**].

7) $[\mathbf{P} \wedge \mathbf{Q}] \swarrow$[The conjunction of **P** and **Q**].

8) $[\mathbf{P} \Leftrightarrow \mathbf{Q}] \swarrow$[The biimplication, *or* equivalence, of **P** and **Q**].

9) $[\mathbf{P} \overline{\vee} \mathbf{Q}] \swarrow$[The latter antidisjunction of **P** and **Q**].

10) $[\mathbf{P} \overline{\wedge} \mathbf{Q}] \swarrow$[The latter anticonjunction of **P** and **Q**].

11) $[\mathbf{P} \overline{\Rightarrow} \mathbf{Q}] \swarrow$[The rightward antiimplication of **Q** from **P**].

12) $[\mathbf{P} \overline{\Leftarrow} \mathbf{Q}] \swarrow$[The leftward antiimplication of **P** from **Q**].

13) $[\mathbf{P} \overline{\Leftrightarrow} \mathbf{Q}] \swarrow$[The antibiimplication, *or* antiequivalence, *or* exclusive disjunction, of **P** and **Q**].

ii) *Common names*

The expression, which is formed by replacing the prepositive definite article “The” with the indefinite article “A” and by simultaneously omitting the corresponding one of the postpositive adjoined qualifiers “of **P** and **Q**”, “of **Q** from **P**”, and “of **P** from **Q**”, occurring in the definiendum of any given one of the above definitions 1–13, is a *common name* of any euautographic formula of A_1 of the range of the relation schema of A_1 that serves as the definiens schema of the given definition.

B) Major ordinary (logical) connectives

i) *Individual proper names*

1) $\not\vee \swarrow$[The former anticonjunction sign].

2) $\neg \swarrow$[The negation, *or* denial, sign].

3) $\vee \swarrow$[The inclusive disjunction sign].

4) $\Rightarrow \swarrow$[The rightward implication sign].

5) $\Leftarrow\swarrow$[The leftward implication sign].

6) $\not\wedge\swarrow$[The former anticonjunction sign]$\leftrightarrow$[The quominus sign].

7) $\wedge\swarrow$[The conjunction sign].

8) $\Leftrightarrow\swarrow$[The biimplication, *or* equivalence, sign].

9) $\overline{\vee}\swarrow$[The latter antidisjunction sign].

10) $\overline{\wedge}\swarrow$[The latter anticonjunction sign].

11) $\overline{\Rightarrow}\swarrow$[The rightward antiimplication sign].

12) $\overline{\Leftarrow}\swarrow$[The leftward antiimplication sign].

13) $\overline{\Leftrightarrow}\swarrow$[The anti-biimplication, *or* antiequivalence, *or* exclusive disjunction, sign].

ii) *Taxonyms*

The above thirteen ordinary kernel-signs are collectively called *the major elementary* (*basic*) *ordinary logical kernel-signs* and also *the major logical connectives*, *of* A_n, i.e. of both A_1 and A_0, the understanding being that, in the current setup. The very first logical connective, $\not\vee$, is the primary atomic one, the next seven logical connectives are *secondary atomic* ones, and the last five logical connectives are *secondary molecular* ones (to be explicated in Cmt 1.13(4) below). The connective $\neg$ is *singulary*, whereas all other major logical connectives are *binary*.•

Cmt 1.11. In this treatise, the nouns "*form*" and "*matter*" or "*content*" are conventionally used as complementary antonyms and consequently their kindred (derivational) adjectives "*formal*" and "*material*" are used likewise. Since euautographs of A_1 are associated exclusively with form of logical reasoning, and not with its matter, I qualify any euautographic formula of A_1 *formal* – in contrast to an English (e.g.) *true declarative sentence* of the same form, which I qualify *material*.•

Cmt 1.12. 1) The concrete functions, which the separate major logical connectives and special kernel-signs perform in the calculus A_n are determined by the axioms and the rules of inference (transformation) and decision of A_n. Meanwhile, the following preliminary remarks regarding functions of some logical connectives will be in order. The qualifiers "*major*" and "*non-redundant*" are not synonyms. Particularly, either sign of each of the following four pairs: $\not\vee$ and $\overline{\vee}$, $\Rightarrow$ and $\Leftarrow$, $\not\wedge$ and $\overline{\wedge}$, $\overline{\Rightarrow}$ and $\overline{\Leftarrow}$ is redundant. Also, some of the sign mentioned in Df 1.12(B,i) will be used in this treatise rarely, but I have introduced them all for the sake of convenience in discussing relations of these connectives with one another and with some other connectives that will be introduced in the sequel. To be specific, the sings $\veebar$ and $\overline{\vee}$, defined by items 2 and 8 of Df 1.10, are functionally indistinguishable, and so are the signs $\barwedge$ and $\overline{\wedge}$, defined by items 5, 6, and 9 Df 1.10. The sign $\veebar$ is a primary atomic one, whereas the sign $\overline{\vee}$ is a secondary molecular one. At the same time, it is seen from items 5, 6, and 9 Df 1.10 that $\barwedge$ is defined in terms of $\neg$ and $\vee$, and hence in terms of $\veebar$, more ingenuously than $\overline{\wedge}$. Nevertheless, from the practical viewpoint, the signs $\overline{\vee}$ and $\overline{\wedge}$ turn out to be more convenient than $\veebar$ and $\barwedge$ because they are the direct negations of the fundamental affirmative atomic kernel-signs $\vee$ and $\wedge$. Also, in accordance with Cmt 1.6, the sign $\veebar$ is in fact an auxiliary one that serves for introducing the signs $\neg$ and $\vee$, which could from the very beginning be introduced as two primary atomic logical connectives instead of $\veebar$. However, from the standpoint of epistemological analysis of the setup of A_n, the signs $\veebar$ and $\barwedge$ are ones of fundamental importance for establishing some most general *dual* relations of A_n. These relations will be discussed at large in due course. Meanwhile, the following brief preliminary remarks should be made.

2) The sign $\barwedge$ can and will be used for recursively defining some *minor* (*with respect to the current setup*) secondary kernel-signs of A_n, which will visually be distinguished from the remaining twelve *major* logical connectives

$$\veebar, \neg, \vee, \Rightarrow, \Leftarrow, \wedge, \Leftrightarrow, \overline{\vee}, \overline{\Rightarrow}, \overline{\Leftarrow}, \overline{\wedge}, \overline{\Leftrightarrow} \tag{1.9}$$

by an overdot (although any other appropriate label can be used instead), but which will turn out to be functionally indistinguishable from the above signs. Particularly, $\barwedge$ can be used for defining $\dot{\neg}$ and $\dot{\vee}$, while all other overdoted signs: $\dot{\veebar}$, $\dot{\Rightarrow}$, $\dot{\Leftarrow}$, etc to $\dot{\overline{\Leftrightarrow}}$, and hence the sign $\dot{\overline{\;}}$ are defined in terms of the former two. Hence, the sign $\barwedge$ can be used instead of $\veebar$ as a *primary atomic kernel-sign* of an alternative setup of A_n, in which the sign $\veebar$ does not occur, while the functionally equivalent sign $\dot{\veebar}$ is defined in terms of $\barwedge$ as a secondary atomic kernel-sign. In this connection, it should be recalled that, as was already mentioned in Cmt I.5.1(4) and in the item 4 of subsection I.9.2, $\barwedge$ is a *euautographic parasynonym*, or *euautographic interpretans, of*, – i.e. a *euautographic analogue* that can be *interpreted as*, – any of the following logographic signs: '|' that is used in Whitehead and Russell [1962, p. xvi] under the name "*the stroke*"; '/' that is used in Hilbert and Ackermann [1950, pp. 11, 29] under the name "*Sheffer's stroke*"; '**|**' that is used in Church [1956, p. 37] under the names

"*Non-conjunction*" and "*Sheffer's stroke*". The name "Sheffer's stroke" is used in writings on logic after the logician who first recognized that a single logical connective is sufficient for constructing a sentential calculus. Nicod [1917] was the first logician to set forth an axiomatic sentential calculus on the basis of Sheffer's stroke. In thic case, ⩛ is an alternative universal logical connective. Thus, the recursive relations between ⩛ (or $\dot{⩛}$) and ⩚ allow making explicit the most fundamental *dual properties of* A_n. In this case, the *dual character* of the kernel-signs V and $\bar{V}$ will also become evident.

3) It will also be recalled that, as was indicated in Cmt I.5.1(5), either of the two functionally synonymous relations '$[p ⩚ q]$' and '$[p \barwedge q]$' can be rendered into ordinary language either as "*not both p and q*". At the same time, by items 1–3 of Df 1.10, a catlogographic variant o`f item 5 of Df 1.10 can be developed thus:

$$[p ⩚ q] \to [[\neg p] \vee [\neg q]] \to [p \Rightarrow [\neg q]] \to [p \Rightarrow \neg q], \tag{1.10}$$

wheras the relation '$[p \Rightarrow \neg q]$' can be read as: "*p, so that not q*" if both '$[p \Rightarrow \neg q]$' and '*p*' are assumed to be *formally veracious* (*f-veracious*), i.e. *accidentally f-true*. According to Simpson [1968], the phrase "*so that not*" is a translation into English of the Latin word "**quōmĭnŭs**" \quominus\, which was particularly employed in this sense by Cicero and Livius. Since the signs ⩚ and $\Rightarrow\neg$ are by definition synonyms (⩚ $\leftrightarrow \Rightarrow\neg$), therefore either of the two can be called the *quominus sign* (*kernel-sign*, *logical connective*).

4) By Th 1.1(11), I have introduced the following *primary molecular euautographic contractors of* A_1:

$$(\hat{\cdot}\, u), (\hat{\cdot}\, v), (\hat{\cdot}\, w), (\hat{\cdot}\, x), (\hat{\cdot}\, y), (\hat{\cdot}\, z), (\hat{\cdot}\, u_1), (\hat{\cdot}\, v_1), \ldots, \tag{1.11}$$

$$(\exists u), (\exists v), (\exists w), (\exists x), (\exists y), (\exists z), (\exists u_1), (\exists v_1), \ldots, \tag{1.12}$$

as *euautographic interpretands* (*interpretation values*) of the *primary molecular panlgographic contractors primary molecular euautographic contractors* '$(\hat{\cdot}\,\mathbf{x})$' *and* '$(\exists\mathbf{x})$' *of* $\mathbf{A}_1$ respectively. The major secondary molecular euautographic contractors of A_1 will be defined in terms of those occurring on the lists (1.11) and (1.12) in the next section.•

1.5. Major panlogographic kernel-signs of A_1 versus their euautographic interprtands of A_1

Df 1.13. '$\boldsymbol{\zeta}$', '$\boldsymbol{\lambda}$', '$\boldsymbol{\eta}$', '$\boldsymbol{\xi}$', and '$\overline{\boldsymbol{\xi}}$' are *atomic* placeholders whose ranges are defined as follows:

$$\boldsymbol{\zeta}\,\vec{\in}\,\left\{\hat{+},\hat{\cdot},\hat{-},\hat{=},\overline{\hat{=}}\right\}. \tag{1.13}$$

$$\boldsymbol{\lambda}\,\vec{\in}\,\left\{\not\vee,\vee,\Rightarrow,\Leftarrow,\not\wedge,\wedge,\Leftrightarrow,\overline{\vee},\overline{\Rightarrow},\overline{\Leftarrow},\overline{\wedge},\overline{\Leftrightarrow}\right\}, \tag{1.14}$$

$$\boldsymbol{\eta}\,\vec{\in}\,\left\{\not\vee,\vee,\Rightarrow,\Leftarrow,\not\wedge,,\wedge,\Leftrightarrow\right\}, \tag{1.15}$$

$$\boldsymbol{\xi}\,\vec{\in}\,\left\{\vee,\Rightarrow,\Leftarrow,\wedge,\Leftrightarrow\right\},\ \overline{\boldsymbol{\xi}}\,\vec{\in}\,\left\{\overline{\vee},\overline{\Rightarrow},\overline{\Leftarrow},\overline{\wedge},\overline{\Leftrightarrow}\right\}. \tag{1.16}$$

Any of the above placeholders can be furnished either with any of the light-faced numeral subscripts $_1$, $_2$, etc. in the current font or with any number of primes, or else with both labels simultaneously, thus becoming another atomic syntactic placeholder with the same range.•

Df 1.14. The occurrence of a kernel-sign V, $\overline{V}$, $\hat{\div}$, or $\neg$ in the respective one of the formulas $V(\mathbf{P})$, $\overline{V}(\mathbf{P})$, $[\hat{\div}\,\mathbf{I}]$, and $[\neg\mathbf{P}]$ is called the *principal kernel-sign* of that formula. Likewise, $\boldsymbol{\lambda}$ is [called] the principal kernel-sign of the logical relation $[\mathbf{P}\boldsymbol{\lambda}\mathbf{Q}]$, whereas $\boldsymbol{\xi}$ is the principal kernel-sign of the algebraic formula $[\mathbf{I}\boldsymbol{\xi}\mathbf{J}]$.•

Df 1.15. The specific form of any of the formulas occurring in Ax 1.1 or Th 1.1 and in Df 1.10, – particularly, the shape (square or round) of the brackets, which are utilized in forming the formula, and also the position of its kernel-sign either inside of the square brackets or before the round brackets, – is determined exclusively by the pertinent primary and secondary formation rules. After the major formation rules are laid down, one can make any of the following definitions;

$$[V\mathbf{P}]\to V(\mathbf{P}),\ [\overline{V}\mathbf{P}]\to\overline{V}(\mathbf{P}), \tag{1.17}$$

$$[\mathbf{x}\mathbf{f}^{\mathrm{pc}}\mathbf{y}]\to\mathbf{f}^{\mathrm{pc}}(\mathbf{x},\mathbf{y})\ \text{for each}\ \mathbf{f}^{\mathrm{pc}}\,\vec{\in}\,\kappa^{\mathrm{pc}}, \tag{1.18}$$

$$\hat{\div}(\mathbf{I})\to[\hat{\div}\,\mathbf{I}],\ \neg(\mathbf{P})\to[\neg\mathbf{P}], \tag{1.19}$$

$$\boldsymbol{\zeta}(\mathbf{I},\mathbf{J})\to[\mathbf{I}\,\boldsymbol{\zeta}\mathbf{J}],\ \boldsymbol{\lambda}(\mathbf{P},\mathbf{Q})\to[\mathbf{P}\boldsymbol{\lambda}\mathbf{Q}], \tag{1.20}$$

The form of a formula such as that of the definienda of definitions (1.17) and (1.18) or such as that of the definientia of definitions (1.19) and (1.20) is called the *homogeneous form* of the formula. The homogeneous form of a formula is also called a *linear* form if the formula is singulary, and a *bilinear* form if the formula is binary. The form of a formula such as that of the definientia of definitions (1.17) and (1.18) or such as that of definienda of definitions (1.19) and (1.20) is called the *inhomogeneous*, or *Clairaut-Euler*, *form* of the formula. The word "*form*" in any of the above terms can be used interchangeably with either of the words "*notation*" and "*representation*".•

Cmt 1.13. 1) The most appropriate substitute for the qualifier "homogeneous" in the term "homogeneous form" would likely be "*algebraic*". However, I avoid using the latter in this sense because it already is used in this treatise in the completely different sense.

2) It is noteworthy that both types of notation: '$f(x)$' and 'fx', if 'f' is a singulary functional variable, or '$f(x,y)$'and '$[xfy]$', if 'f' is a binary functional variable, are due to A. C. Clairaut and L. Euler.

3) A combined formula can be written in the homogeneous notation if and only if it is either singulary of binary. By contrast, the inhomogeneous (Clairaut-Euler) notation $\mathbf{f}^{n}(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n)$ (e.g.) is applicable in the general case where the principal kernel-sign, $\mathbf{f}^{n}$, of the formula has any given weight *n* from 1 to infinity. For $\mathbf{f}^{2} \rhd \mathbf{f}^{\mathrm{pc}} \vec{\in} \kappa^{\mathrm{pc}}$ (see Df I.5.2), the relation schema '$\mathbf{f}^{2}(\mathbf{x}_1,\mathbf{x}_2)$' becomes '$\mathbf{f}^{\mathrm{pc}}(\mathbf{x}_1,\mathbf{x}_2)$' (e.g. '$\in(\mathbf{x}_1,\mathbf{x}_2)$', '$=(\mathbf{x}_1,\mathbf{x}_2)$', etc). In order to pass to the conventional set-theoretic and algebraic notation (e.g. '$[\mathbf{x}_1 \in \mathbf{x}_2]$', '$[\mathbf{x}_1 = \mathbf{x}_2]$', etc), one should adopt definition (1.18).

4) Definitions (1.20) are opposite in form to definition (1.18). In accordance with (1.13), the first of the former can be particularized, e.g., thus:

$$\hat{=}(\mathbf{I},\mathbf{J}) \to [\mathbf{I} \mathrel{\hat{=}} \mathbf{J}],\ \overline{\hat{=}}(\mathbf{I},\mathbf{J}) \to [\mathbf{I} \mathrel{\overline{\hat{=}}} \mathbf{J}]. \tag{1.21}$$

By item 13 of Df 1.10, it follows from (1.21) that

$$\overline{\hat{=}}(\mathbf{I},\mathbf{J}) \leftrightarrow [\mathbf{I} \mathrel{\overline{\hat{=}}} \mathbf{J}] \leftrightarrow \neg[\mathbf{I} \mathrel{\hat{=}} \mathbf{J}] \leftrightarrow \neg \hat{=}(\mathbf{I},\mathbf{J}), \tag{1.22}$$

whence

$$\overline{\hat{=}} \leftrightarrow \neg \hat{=}, \tag{1.23}$$

Likewise, in accordance with (1.16), items 8–12 of Df 1.10 can be restated as this single definition schema:

$$[\mathbf{P}\bar{\xi}\mathbf{Q}] \to \neg[\mathbf{P}\xi\mathbf{Q}], \tag{1.24}$$

while the second definition (1.20) can be specified thus:

$$\xi(\mathbf{P},\mathbf{Q}) \to [\mathbf{P}\xi\mathbf{Q}],\ \bar{\xi}(\mathbf{P},\mathbf{Q}) \to [\mathbf{P}\bar{\xi}\mathbf{Q}]. \tag{1.25}$$

Therefore,

$$\bar{\xi}(\mathbf{P},\mathbf{Q}) \leftrightarrow [\mathbf{P}\bar{\xi}\mathbf{Q}] \leftrightarrow \neg[\mathbf{P}\xi\mathbf{Q}] \leftrightarrow \neg\xi(\mathbf{P},\mathbf{Q}), \tag{1.26}$$

whence

$$\bar{\xi} \leftrightarrow \neg\xi\,. \tag{1.27}$$

It follows from (1.23) and (1.27) that, as was already indicated in Cmt I.5.1(2), the overbar of an adjustable length, ¯, is an *overscript synonym of the adscript negation sign* ¬ and hence it is a secondary atomic sign of negation of the relational sign over which it is put. Therefore, '$\bar{\xi}$' is a *molecular sign-valued placeholder*, and any of the signs of its range as given in (1.16) is a *molecular logical connective*, i.e. *a molecular ordinary relational kernel-sign*. Although the sign ¯ is not juxtapositional, it can, like ¬, be applied to the same major kernel-sign repeatedly, thus producing some *minor complex relational kernel-signs*. In general, '$\bar{\bar{\xi}}$' is a placeholder defined as $\bar{\bar{\xi}} \rightarrow \neg\neg\xi$ so that, e.g., $\bar{\bar{\hat{=}}} \rightarrow \neg\neg\hat{=}$.•

Cmt 1.14. Concrete instances of the atomic placeholders '**x**', '**y**', '**P**', '**Q**', '**I**', and '**J**' occurring in the definition schemata (1.17)–(1.20) can always be written in terms of primary atomic euautographs as introduced in Ax I.5.1. In this case, the definiendum of any concrete instance of any one of those definition schemata is *a primary assemblage of* A_1, *which is not however a primary formula of* A_1. In the case of A_0, the above-said applies only to the definienda of the definitions (1.19) and (1.20) because the comma, tokens of which occur in the definientia of (1.18) and in the definienda of (1.20), is not a primary atomic euautograph of A_0.•

Cmt 1.15. In accordance with Df 1.15, any combined (not atomic) major formula of A_1 can, in principle, be written either in the homogeneous form (notation, representation) or in the inhomogeneous (Clairaut-Euler) form. In either case, *the pair of square or round brackets is an inseparable supplement to the principal kernel-sign of the formula.* The kernel-sign together with the pertinent pair of brackets forms *the principal operator of the formula*. The kernel-sign alone occurring in a formula is not an operator. However, once an operator is prescinded (mentally disengaged) from its operata, it can be identified with its kernel sign, because the omitted brackets can be restored when desired. Also, some pairs of square brackets will be omitted in accordance with certain conventions to be stated in the sequel.•

Cmt 1.16. 1) There is no generally accepted system of notation for most of the major logical (sentential) connectives being in standard use. Therefore, among the thirteen signs occurring in Df 1.12(B), $\neg$, $\vee$, and $\wedge$ are the only conventional ones, whereas the others are suggestions of my own; they are not in common use, although various one-sided and two-sided arrows are used by different writers. In the sequel, $\neg$, $\vee$, and $\wedge$ will be interpreted by the English words "*not*", "*or*", and "*and*". In this case, as contrasted to English, in which the disjunctive conjunction "or" can equivocally be understood either in the inclusive sense or in the exclusive sense, Latin has two different disjunctive conjunctions, namely, the *inclusive disjunctive conjunction* "**vĕl**" and the *exclusive disjunctive conjunction* "**auf**" (see Simpson [1968]). In this case, the character $\vee$ is a stylized token of the first letter of the Latin word "vĕl", whose meaning is in agreement with the meaning, which will be assigned to the logical connective $\vee$ and which is often expressed by the barbarism "*ior*" ("*inclusive or*"). The meaning of the logical connective $\overline{\Leftrightarrow}$ will be the same as that of the Latin word "aut" – the meaning which is often expressed by the barbarism "*xor*" ("*exclusive or*").

2) Just as the major binary logical (sentential) connectives being in standard use, the major binary euautographic logical connectives of A_1 are not all of the same style. The latter can be unified by setting, e.g.

$$\not\Downarrow \to \not\vee, \Downarrow \to \vee, \not\Uparrow \to \not\wedge, \Uparrow \to \wedge, \overline{\Downarrow} \to \overline{\vee}, \overline{\Uparrow} \to \overline{\wedge}. \tag{1.28}$$

In this case, each major binary logical connective is a properly oriented arrow, either alone or with an overbar or crossbar. Another way to unify the system of binary logical connectives is to set

$$> \to \Rightarrow, < \to \Leftarrow, <> \to \Leftrightarrow, \overline{>} \to \overline{\Rightarrow}, \overline{<} \to \overline{\Leftarrow}, \overline{<>} \to \overline{\Leftrightarrow}. \tag{1.29}$$

Unfortunately, the signs $<$ and $>$ are already reserved in mathematics for their conventional use to denote the corresponding pre-order relations. Therefore, under definitions (1.29), the signs $<$ and $>$ would have been used equivocally in mathematical applications of A_1.

3) In this treatise, the negation of any kernel-sign is denoted by a bar of an adjustable length over the sign, the overbar being a secondary atomic sign (cf. Cmt I.5.1(2) and Cmt 1.13(4)). Alternatively, the synonymous secondary atomic sign / could be used instead of ¯. In this case, $\not\vee$, $\not\wedge$, $\nRightarrow$, $\nLeftarrow$, $\nLeftrightarrow$, $\not\hat{=}$ would have come

instead of $\overline{\vee}$, $\overline{\wedge}$, $\overline{\Rightarrow}$, $\overline{\Leftarrow}$, $\overline{\Leftrightarrow}$, $\hat{=}$, and $\neq$, $\notin$, $\not\ni$, $\not\subseteq$, $\not\supseteq$, $\not\subset$, $\not\supset$ instead of $\overline{\equiv}$, $\overline{\in}$, $\overline{\ni}$, $\overline{\subseteq}$, $\overline{\supseteq}$, $\overline{\subset}$, $\overline{\supset}$, respectively.

2. The major formation rules of A$_1$ and **A**$_1$ (*continued*)

†**Df. 2.1:** ***The major plain contractions of*** A$_1$. The definienda of the following six definitions are *secondary formulas*, and hence *formulas*, *of* A$_1$ which are called *plain contracted formulas*, or briefly *plain contractions*, *of* A$_1$:

1) $\left[\hat{\cdot}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \to \left[(\hat{\cdot}\,\mathbf{x}) V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]$.

2) $\left[\bigvee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle\right] \to \left[(\exists\mathbf{x})\mathbf{P}\langle\mathbf{x}\rangle\right]$.

3) $\left[\bigwedge_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle\right] \to \left[(\forall\mathbf{x})\mathbf{P}\langle\mathbf{x}\rangle\right] \to \left[\neg\bigvee_{\mathbf{x}} \neg\mathbf{P}\langle\mathbf{x}\rangle\right]$.

4) $\left[\breve{\bigvee}_{\mathbf{z}} \mathbf{P}\langle\mathbf{z}\rangle\right] \to \left[\left[\bigvee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle\right] \wedge \left[\bigvee_{\mathbf{y}} \neg\mathbf{P}\langle\mathbf{y}\rangle\right]\right]$.

5) $\left[\widehat{\bigvee}{}^{1}_{\mathbf{z}} \mathbf{P}\langle\mathbf{z}\rangle\right] \to \left[\bigwedge_{\mathbf{x}} \bigwedge_{\mathbf{y}} \left[\left[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{P}\langle\mathbf{y}\rangle\right] \Rightarrow \left[\mathbf{x} = \mathbf{y}\right]\right]\right]$.

6) $\left[\bigvee{}^{1}_{\mathbf{v}} \mathbf{P}\langle\mathbf{v}\rangle\right] \to \left[\left[\widehat{\bigvee}{}^{1}_{\mathbf{z}} \mathbf{P}\langle\mathbf{z}\rangle\right] \wedge \left[\bigvee_{\mathbf{w}} \mathbf{P}\langle\mathbf{w}\rangle\right]\right]$.

The definiendum of item 1) is a plain *contracted validity-integron*, whereas the definienda of items 2–6 are *plain contracted relations*. The primary molecular kernel-signs $(\hat{\cdot}\,\mathbf{x})$ and $(\exists\mathbf{x})$ and the *secondary molecular kernel-signs*

$$\hat{\cdot}_{\mathbf{x}},\ \bigvee_{\mathbf{x}},\ \bigwedge_{\mathbf{x}},\ \breve{\bigvee}_{\mathbf{z}},\ \widehat{\bigvee}{}^{1}_{\mathbf{z}},\ \bigvee{}^{1}_{\mathbf{v}} \tag{2.1}$$

are collectively called *plain contractors*.•

Cmt 2.1. 1) The AEOPS = will be defined in each of the EAPO's A$_{1=}$, A$_{1\subseteq}$, and A$_{1\in}$ (see Df. I.7.1). Once it is done, definitions 5 and 6 of Df 2.1 will be restated and studied at large. Meanwhile, in order to define, from the very beginning, all contractors to be used in A$_1$ and to introduce the pertinent general terminology, it is convenient to assume that = is introduced in accordance with item 8a of Ax I.5.1 and, along with definitions 1–4, to make definitions 5 and 6 subject to

$$\left[\mathbf{x} = \mathbf{y}\right] \to = \left(\mathbf{x}, \mathbf{y}\right), \tag{2.2}$$

in accordance with definition (1.18) (cf. (1.20)).

2) I use the noun "contraction" in analogy with its use in the mathematical term "*contraction of tensor*". The qualifier "contracted" and the generic name (headword) "contraction" are descriptive, not only of the definienda of all the six definitions of Df 2.1, but also of the definientia of definitions 1, 2, and 5. Either of the

two words does not, however, descriptive of the definientia of definitions 3, 4, and 6, because the first one has the form of $[\neg\mathbf{Q}]$, whereas the last two have the form of $[\mathbf{Q}\wedge\mathbf{R}]$.•

Cmt 2.2. 1) In the definienda of the items 2–6 of Df 2.1, the operators $\bigvee_{\mathbf{x}}$ and $\bigwedge_{\mathbf{x}}$ can be used interchangeably with the conventional operators $(\exists\mathbf{x})$ and $(\forall\mathbf{x})$ respectively. However, the rules of *substantivating* (*substantivizing*, *termizing*, *termization of*) relations $\lfloor\bigvee_{\mathbf{x}}\mathbf{P}\rfloor$ and $\lfloor\bigwedge_{\mathbf{x}}\mathbf{P}\rfloor$ will be homological to those of $[\mathbf{P}\vee\mathbf{Q}]$ and $[\mathbf{P}\wedge\mathbf{Q}]$ respectively. Consequently, a *transcendental* relation $\lfloor\bigvee_{\mathbf{x}}\mathbf{P}\rfloor$ can be regarded as a generalization of a *binary inclusive disjunction* $[\mathbf{P}\vee\mathbf{Q}]$, whereas the *transcendental* relation $[\bigwedge_{\mathbf{x}}\mathbf{P}]$ can be regarded as a generalization of a *binary conjunction* $[\mathbf{P}\wedge\mathbf{Q}]$. This is why I give preference to the signs $\bigvee$ and $\bigwedge$ over the conventional signs $\exists$ and $\forall$. For the same reason, in the subsequent taxonomy of the different contractions, I shall use the nouns "disjunction" and "conjunction" as appropriate generic names.

2) It will also be demonstrated in due course that a transcendental relation $[\breve{\bigvee}_{x}\mathbf{P}]$ can be regarded both as another generalization of the binary inclusive disjunction $[\mathbf{P}\vee\mathbf{Q}]$ and as a generalization of a binary exclusive disjunction $[\mathbf{P}\overline{\Leftrightarrow}\mathbf{Q}]$, whereas a transcendental relation $\lfloor\bigvee_{\mathbf{v}}^{1}\mathbf{P}\langle\mathbf{v}\rangle\rfloor$ can be regarded as another generalization of the binary exclusive disjunction $[\mathbf{P}\overline{\Leftrightarrow}\mathbf{Q}]$. Therefore, the signs '$\bigvee$' and '$\bigwedge$' with the appropriate labels are a natural supplement to the thirteen major binary sentential connectives occurring in the item B of Df 1.12.•

†**Df. 2.2:** ***The major pseudo-typical contractions of*** A_1. The definienda of the following six definitions are *secondary formulas*, and hence *formulas, of* A_1 which are called *pseudo-typical contracted formulas*, or briefly *pseudo-typical contractions, of* A_1:

1) $\lfloor\hat{\cdot}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V(\mathbf{P}\langle\mathbf{x}\rangle)\rfloor\to[\hat{\cdot}_{\mathbf{x}}V(\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle)]$.

2) $\lfloor\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\rfloor\to[\bigvee_{\mathbf{x}}[\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle]]$.

3) $\lfloor\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\rfloor\to\lfloor\neg\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\rfloor\leftrightarrow[\neg\bigvee_{\mathbf{x}}[\mathbf{R}\langle\mathbf{x}\rangle\wedge[\neg\mathbf{P}\langle\mathbf{x}\rangle]]]$.

4) $\left\lfloor \breve{\vee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right\rfloor \to \left\lfloor\!\left\lfloor \vee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right\rfloor \wedge \left\lfloor \vee_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}\neg\mathbf{P}\langle\mathbf{y}\rangle\right\rfloor\!\right\rfloor.$

5) $\left\lfloor \hat{\vee}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right\rfloor \to \left\lfloor \hat{\vee}^{1}_{\mathbf{z}}\left[\mathbf{R}\langle\mathbf{z}\rangle \wedge \mathbf{P}\langle\mathbf{z}\rangle\right]\right\rfloor$

$$\leftrightarrow \left\lfloor \wedge_{\mathbf{x}} \wedge_{\mathbf{y}} \left[\!\left[\mathbf{R}\langle\mathbf{x}\rangle \wedge \mathbf{P}\langle\mathbf{x}\rangle\right] \wedge \left[\mathbf{R}\langle\mathbf{y}\rangle \wedge \mathbf{P}\langle\mathbf{y}\rangle\right] \Rightarrow \left[\mathbf{x} = \mathbf{y}\right]\!\right]\right\rfloor.$$

6) $\left\lfloor \vee^{1}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle}\mathbf{P}\langle\mathbf{v}\rangle\right\rfloor \to \left\lfloor\!\left\lfloor \hat{\vee}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right\rfloor \wedge \left\lfloor \vee_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}\mathbf{P}\langle\mathbf{w}\rangle\right\rfloor\!\right\rfloor.$

The definiendum of the item 1 is a *pseudo-typical contracted validity-integron,* whereas the definienda of items 2–6 are *pseudo-typical contracted relations*. The *secondary complex kernel-signs* of the six kinds:

$$\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle},\ \vee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle},\ \wedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle},\ \breve{\vee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle},\ \hat{\vee}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle},\ \vee^{1}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle} \tag{2.3}$$

are collectively called *pseudo-typical contractors*. In any of the above occurrences, the qualifier "*pseudo-typical*" can be used interchangeably with "*pseudo-conditional*".•

Cmt 2.3. All formulas of A_1 are euautographs. Therefore, a relation **R**, occurring in the subscript of any of the *euautographic* contractors on the list (2.3) is *not a condition* on the respective APVOT **x**, **z**, **v**, etc, which **R** in each occurrence supposedly contains in the same place or places (and hence in the same symbolic surrounding). Like any other relation of A_1, **R** is permanently valid or antivalid or else vav-neutral. If, particularly, **R** is vav-neutral and if ***R***, ***x***, ***z***, and ***v***, etc are catlogographic interpretands of **R, x,** z**,** and **v**, etc respectively then ***R*** can serve as a *condition* such that ***R*** is *veracious* (*accidentally true*) for some ***x***, ***z***, or ***v*** and *aniveracious* (*accidentally*, i.e. *not universally*, *antitrue*) for some other. In this case, the contractors

$$\hat{\bullet}_{x|\boldsymbol{R}\langle x\rangle},\ \vee_{x|\boldsymbol{R}\langle x\rangle},\ \wedge_{x|\boldsymbol{R}\langle x\rangle},\ \breve{\vee}_{z|\boldsymbol{R}\langle z\rangle},\ \hat{\vee}^{1}_{z|\boldsymbol{R}\langle z\rangle},\ \vee^{1}_{v|\boldsymbol{R}\langle v\rangle}, \tag{2.4}$$

being *catlogographic interprtands* of the respective euautographic contractors on the list (2.3), can be called *conditional* ones. In Bourbaki [1960, chap. I, §4, sec. 4], the *logographic* (*xenographic*, *interprted*) kernel-signs such as $\vee_{x|\boldsymbol{R}\langle x\rangle}$ and $\wedge_{x|\boldsymbol{R}\langle x\rangle}$ are called *typical quantifiers* (*typiques quantificateurs*). In analogy with the Bourbaki term, I shall call the contractors on the list (2.3) *pseudo-typical*, or *pseudo-conditional*, ones.•

Df 2.3. 1) A plain or typical contracted formula is indiscriminately called a *contracted formula*. A formula as any one defined by D̳f 1.10 is said to be a *non-*

contracted one if it is not contracted. The above definitions apply with “validity-integron” or “relation” in place of “formula”.

2) An atomic or molecular kernel-sign is indiscriminately called a *primitive*, or *elemental, kernel-sign*.•

2.2. Taxonomy of the major advanced (contracted) formulas of A_1 and of their principal operators

Preliminary Remark 2.1. In order to treat of the various major advanced (contracted) formulas (terms or relations) and separate kernel-signs of A_1 as objects, they should be provided with wordy (verbal) or semi-wordy common or proper names, which should be descriptive of the distinguishing properties of their circumstantial or essential denotata. Particularly, the names of the major contractors should be descriptive of their functions in forming formulas of the respective classes. Such a common or proper name is unavoidably an appropriate *description of the species through a genus and the difference* (*differentia*) or *differences* (*differentiae*), i.e., in Latin, *descriptio species per genus et differentiam* or *differentias*, as the pertinent count name, along with the relevant additional *limiting modifier* as the indefinite or definite article, Particularly, such a descriptive *common* name naturally classifies the formulas or kernel-signs, which are comprised in its range (class-connotatum), i.e. in the class that is properly denoted by the pertinent count name. More generally, the *generic name* (*relative genus name, head name*) of a common or proper name of a certain formula or kernel-sign can be used as the proper name of the broader class, to which the formula or kernel-sign belongs. Also, once primitive (atomic and molecular) euautographs of A_1 are provided with descriptive names, a compound assemblage, or particularly a compound formula, of A_1 can be mentioned orally by sequentially uttering the names of the primitive constituents of the assemblage (to be exemplified in due course). Descriptive names of euautographs of A_1 can be introduced with the help of the pertinent *ostensive* or *quasi-ostensive nominal definitions*, which will conveniently be stated with the help of the suitable one of the nominal definition signs $\searrow$ and $\swarrow$ defined in Df I.2.21. When it is desirable to have a synonym or synonyms of a given descriptive name, these will be introduced either in the legato style with the help of a suitable synonymic definition sign $\rightarrow$, $\leftarrow$, or $\leftrightarrow$ defined in Df I.2.17–I.2.19 or by a separate statement. Some of the names that are introduced below will be justified and become clearer after the pertinent issues of the setup of A_1 are laid down or after the theorems expressing the basic properties of the formulas or kernel-signs named are stated and proved.•

Df 2.4.

1) $[\hat{\cdot}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle)] \swarrow$ [the pseudo-product over **x** of $V(\mathbf{P}\langle\mathbf{x}\rangle)$].

2) $[\vee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle] \swarrow$ [the lax inclusive disjunction over **x** of $\mathbf{P}\langle\mathbf{x}\rangle$].

3) $[\wedge_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle] \swarrow$ [the conjunction over **x** of $\mathbf{P}\langle\mathbf{x}\rangle$].

4) $[\breve{\vee}_{\mathbf{z}} \mathbf{P}\langle\mathbf{z}\rangle] \swarrow$ [the strict inclusive disjunction over **z** of $\mathbf{P}\langle\mathbf{z}\rangle$].

5) $[\overset{\frown}{\vee}{}^{1}_{\mathbf{z}} \mathbf{P}\langle\mathbf{z}\rangle] \swarrow$ [the infrafunctional disjunction over **z** of $\mathbf{P}\langle\mathbf{z}\rangle$].

6) $[\vee^{1}_{\mathbf{v}} \mathbf{P}\langle\mathbf{v}\rangle] \swarrow$ [the functional, or exclusive, disjunction over **v** of $\mathbf{P}\langle\mathbf{v}\rangle$].

7) $[\hat{\cdot}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle} V(\mathbf{P}\langle\mathbf{x}\rangle)] \swarrow$ [the pseudo-product over **x** subject to $\mathbf{R}\langle\mathbf{x}\rangle$ of $V(\mathbf{P}\langle\mathbf{x}\rangle)$].

8) $[\vee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle} \mathbf{P}\langle\mathbf{x}\rangle] \swarrow$ [the lax inclusive disjunction over **x** subject to $\mathbf{R}\langle\mathbf{x}\rangle$ of $\mathbf{P}\langle\mathbf{x}\rangle$].

9) $[\wedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle} \mathbf{P}\langle\mathbf{x}\rangle] \swarrow$ [the conjunction over **x** subject to $\mathbf{R}\langle\mathbf{x}\rangle$ of $\mathbf{P}\langle\mathbf{x}\rangle$].

10) $[\breve{\vee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle} \mathbf{P}\langle\mathbf{z}\rangle] \swarrow$ [the strict inclusive disjunction over **z** subject to $\mathbf{R}\langle\mathbf{z}\rangle$ of $\mathbf{P}\langle\mathbf{z}\rangle$].

11) $[\overset{\frown}{\vee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle} \mathbf{P}\langle\mathbf{z}\rangle] \swarrow$ [the infrafunctional disjun**c**tion over **z** subject to $\mathbf{R}\langle\mathbf{z}\rangle$ of $\mathbf{P}\langle\mathbf{z}\rangle$].

12) $[\vee^{1}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle} \mathbf{P}\langle\mathbf{v}\rangle] \swarrow$ [the functional, or exclusive, disjunction over **v** subject to $\mathbf{R}\langle\mathbf{v}\rangle$ of $\mathbf{P}\langle\mathbf{v}\rangle$].•

Df 2.5. A) The expressions, which are formed by omitting all occurrences of the adjoined (postpositive) qualifiers "of $V(\mathbf{P})$", "of $\mathbf{P}\langle\mathbf{x}\rangle$", "of $\mathbf{P}\langle\mathbf{z}\rangle$", and "of $\mathbf{P}\langle\mathbf{v}\rangle$" from the definienda of Df 2.4 and by substituting the generic names "*multiplicative contractor*", "*disjunctive contractor*", and "*conjunctive contractor*" for occurrences of the generic names "*product*", "*disjunction*", and "*conjunction*" respectively throughout the above definienda are proper names of the operators (kernel-signs) occurring in the respective definientia of Df 2.4. Thus, in the result of the above alterations, the items 1–12 of Df 2.4 turn into the following definitions.

1) $\hat{\cdot}_{\mathbf{x}} \swarrow$[The pseudo-multiplicative contractor over **x**].

2) $\vee_{\mathbf{x}} \swarrow$[The lax inclusive disjunctive contractor of **x**].

3) $\wedge_{\mathbf{x}} \swarrow$[The conjunctive contractor over **x**].

4) $\breve{\vee}_{\mathbf{z}} \swarrow$[The strict inclusive disjunctive contractor over **z**].

5) $\overset{\frown}{\vee}{}^{1}_{\mathbf{z}} \swarrow$[The infrafunctional contractor over **z**].

6) $\vee^{1}_{\mathbf{z}} \swarrow$[The functional, or exclusive, disjunctive contractor over **z**].

7) $\hat{\cdot}_{\boldsymbol{x}|\boldsymbol{R}\langle\boldsymbol{x}\rangle} \swarrow$[The pseudo-multiplicative contractor over **x** subject to $\mathbf{R}\langle\mathbf{x}\rangle$].

8) $\vee_{\boldsymbol{x}|\boldsymbol{R}\langle\boldsymbol{x}\rangle} \swarrow$[The lax inclusive disjunctive contractor over **x** subject to $\mathbf{R}\langle\mathbf{x}\rangle$].

9) $\wedge_{\boldsymbol{x}|\boldsymbol{R}\langle\boldsymbol{x}\rangle} \swarrow$[The conjunctive contractor over **x** subject to $\mathbf{R}\langle\mathbf{x}\rangle$].

10) $\breve{\vee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle} \swarrow$[The strict inclusive disjunctive contractor over **z** subject to $\mathbf{R}\langle\mathbf{z}\rangle$].

11) $\overset{\frown}{\vee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle} \swarrow$[The infrafunctional disjunctive contractor over **z** subject to $\mathbf{R}\langle\mathbf{z}\rangle$].

12) $\vee^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle} \swarrow$[The functional, or exclusive, disjunctive contractor over **z** subject to $\mathbf{R}\langle\mathbf{z}\rangle$].•

Df 2.6: ***A supplement to Dfs 2.4 and 2.5.*** 1) The terminology that is introduced in Df 2.4 and in the item B of this definition is formed with allowance for Cmt 2.2. In this case, the following expressions can be used interchangeably:

i) and “pseudo-multiplicative contraction” and “pseudo-product”;

ii) “disjunctive contraction” and “disjunction”;

iii) “conjunctive contraction” and “conjunction”;

iv) “pseudo-multiplier” and “pseudo-multiplicative contractor”;

v) “existential” and “disjunctive”;

vi) “universal” and “conjunctive”;

vii) “functional” and “exclusive”;

viii) “laxly” and “lax”;

ix) “strictly” and “strict”;

x) “weak” and “lax”;

xi) “strong” and “strict”.

2) As opposed to both the contractions occurring as definienda in definitions 5, 6, 11, and 12 of Df 2.4 and the contractors occurring as definientia in definitions 5, 6, 11, and 12 of Df 2.5, which are qualified *infrafunctional* or *functional*, both the contractions occurring as definienda in definitions 2–4 and 8–10 of Df 2.4 and the contractors occurring as definientia in definitions 2–4 and 8–10 of Df 2.5 are qualified *suprafunctional*.

3) The expression “contractor over”, occurring in the names of operators, which are introduced in separate definitions 2–6 and 8–12 of Df 2.5, can be used interchangeably with the noun “*pseudo-quantifier of*” if an operator named is employed in $\mathsf{A}_{1\in}$ and interchangeably with the noun “*pseudo-qualifier of*” if an operator named is employed in $\mathsf{A}_{1\subseteq}$.•

Cmt 2.4. I have mentioned in Cmt 1.16 that for various reasons, the separate sentential connectives are not all of the same style. Also, I have discussed two possible sets of completely unified sentential connectives. One of the sets is based on the definitions (1.28), so that this set of sentential connectives consists exclusively of properly oriented arrows, alone or with an overbar. In this case, in order to preserve the uniform style for all kernel-signs, including sentential connectives and pseudo-quantifiers, definitions (1.28) should be supplemented with these two:

$$\big\Downarrow \to \bigvee, \big\Uparrow \to \bigwedge. \qquad (2.5)$$

Thus, in the framework of this unified system of notation, the figures $\Downarrow$, $\Uparrow$, $\big\Downarrow$, and $\big\Uparrow$ should be used instead of $\vee$, $\wedge$, $\bigvee$ and $\bigwedge$, respectively. The other possible unified system of notation of sentential connectives, which is based on definitions (1.29), incorporates the figures $\bigvee$ and $\bigwedge$ automatically. By Cmt 1.16(2), this system is however inacceptable.•

2.3. Omission of brackets

Preliminary Remark 2.2. Besides informal uses of round and square brackets in the metalanguage, any pair of round or square brackets occurring in a euautographic or panlogographic formula (term or relation) is an integral part of a certain operator. At the same time, all operators, primary and secondary, differ from one another primarily by their kernel-signs while all brackets are auxiliary euautographic marks that are formally used to show the way in which constituent formulary parts of a given host EF (euautographic formula) or PLF (panlogographic formula) are associated to be united (executed) by the pertinent kernel-signs in the desired order. Therefore, for improving the readability of the treatise in general and of EF's and PLF's in particular, I shall, in accordance with the common practice, omit as many *square brackets* as it seems to be save without leading to confusion. In this case, in contrast to what is done, e.g., in Whitehead & Russell [1927] or Church [1956], I shall not introduce any punctuation marks (as heavy dots) instead of the omitted brackets, because some training is required in order to read the additional punctuation automatically. I shall just omit some square brackets from an EF or PLF, provided that no mental effort should be made in order to read the formula as such and hence to recover the omitted square brackets in only one way, when desired. To this end, I shall adjust the habitual rules of omission of brackets, which are familiar from mathematics, to the major operators of A_1 and $\boldsymbol{\mathsf{A}}_1$.The specific rules of omission of square brackets will particularly be based on the conventions, which I shall adopt regarding *execution priorities*, i.e. *an order of precedence*, of some kernel-signs over some others. In order to allow omitting the outermost pair of square brackets from both the definiens and the definiendum, I shall also incorporate the metalinguistic definition signs $\rightarrow$, $\leftarrow$, $\leftrightarrow$, and also their restrictions $\overrightarrow{\hat{=}}$, $\overleftarrow{\hat{=}}$, $\overleftrightarrow{\hat{=}}$, and $\overrightarrow{\Leftrightarrow}$, $\overleftarrow{\Leftrightarrow}$, $\overleftrightarrow{\Leftrightarrow}$ into the hierarchy of the euautographic kernel-signs, primary and secondary. The general initial convention of omission of some square bracket from formulas of A_1 (A_1 or $\boldsymbol{\mathsf{A}}_1$) is stated below. As I go along with the setup of A_1, that convention will be supplemented by some new rules. For instance, some square brackets will be omitted on account of the commutative and associative laws to be established for the EF's and PLF's of certain classes. It should also be emphasized that the rules of omission of square brackets are not the must: some or all square brackets of an EF or PLF, which can be omitted in accordance with a certain rule, may alternatively be retained for more clarity.•

Cnv 2.1. There is the following order of precedence of executions of the constituent parts of a host *endosemasiopasigraphic* (*euautographic or panlogographic*) *formula (EnSPGF*) *of* A_1, which is mentioned as a *formula*.

1) If there is more than one pair of square brackets in a formula, the part of the formula between the innermost pair of square brackets is executed first.

2) Any one of the subject metalinguistic signs: $\rightarrow$, $\leftarrow$, $\leftrightarrow$, $\overrightarrow{\hat{=}}$, $\overleftarrow{\hat{=}}$, $\overleftrightarrow{\hat{=}}$, $\overrightarrow{\Leftrightarrow}$, $\overleftarrow{\Leftrightarrow}$, $\overleftrightarrow{\Leftrightarrow}$ has the least execution priority relative to all object kernel-signs, primary or secondary, which occur in the formulas standing on both sides of the former sign.

3) If a formula [**Φ**] stands in a metalinguistic context apart from any other formula then **Φ**→[**Φ**]; that is, the outermost pair of brackets in [**Φ**] can be omitted.

4) If [**P**] is a relation, primary or secondary, then *V*(**P**)→ *V*([**P**]).

5) The sign $\hat{\cdot}$ has precedence over, i.e. it dominates (is executed prior), $\hat{:}$, $\hat{+}$, and $\hat{-}$, and in turn, each one of the three latter signs has precedence over $\hat{=}$ and $\hat{\equiv}$.

6) The sign $\neg$ has the first execution priority relative to any one of the eleven major binary logical connectives $\vee$, $\Rightarrow$, $\Leftarrow$, $\not\wedge$, $\wedge$, $\Leftrightarrow$, $\overline{\vee}$, $\overline{\Rightarrow}$, $\overline{\Leftarrow}$, $\overline{\wedge}$, $\overline{\Leftrightarrow}$ (see (1.7)), all of which have the same rank of dominance and subordination.

7) The sign $\hat{\cdot}$ has precedence over, i.e. it dominates (is executed prior), any one of the signs $\hat{:}$, $\hat{+}$, and $\hat{-}$, and in turn, each one of the three latter signs has precedence over $\hat{=}$ and $\hat{\equiv}$.

8) The sign $\hat{\cdot}$ has precedence over any one of the signs any one of the signs $\hat{\cdot}$, $\hat{:}$, $\hat{+}$, $\hat{-}$, $\hat{=}$, and $\hat{\equiv}$.

9) Any one of the signs $\vee$, $\wedge$, $\breve{\vee}$, and $\overset{\frown}{\vee}$ has precedence over any one of the signs any one of the twelve logical connectives mentioned in the item 6.•

2.4. Operators of substitution

Df 2.7. 1) In accordance with Dfs I.5.11(3), 1.5, 1.6, and 1.7(1), in order to indicate explicitly the assumption that an EF (euautographic formula) **Φ** contains a certain AEOT (atomic euautographic ordinary term) **x**, i.e. a certain free or bound APVOT (atomic euautographic ordinary term) $\mathbf{x}^{pv}$ or a certain free APCOT $\mathbf{x}^{pc}$, i.e. either $\emptyset$ or $\emptyset'$, I shall suffix '**Φ**' with '$\langle\mathbf{x}\rangle$' thus writing '$\mathbf{\Phi}\langle\mathbf{x}\rangle$' instead of '**Φ**'.

Likewise, I shall write ‘$\mathbf{\Phi}\langle\mathbf{p}\rangle$’ instead of ‘**Φ**’ for explicitly indicating that **Φ** contains a given AER (atomic euautographic relation) **p**, which is always free in accordance with Ax 1.1(3) or Th 1.1(3). Besides **x**, $\mathbf{\Phi}\langle\mathbf{x}\rangle$ may contain some other AEOT’s or AER’s or both. If **y** is a certain one or the only one of the latter AEF’s (atomic euautographic formulas), and if I wish to explicitly indicate the fact that **Φ** contains both **x** and **y**, then I shall write ‘$\mathbf{\Phi}\langle\mathbf{x},\mathbf{y}\rangle$’ instead of both ‘$\mathbf{\Phi}\langle\mathbf{x}\rangle$’ and ‘**Φ**’. Analogously, when appropriate and desirable, I shall write ‘$\mathbf{\Phi}\langle\mathbf{p},\mathbf{q}\rangle$’, ‘$\mathbf{\Phi}\langle\mathbf{x},\mathbf{p}\rangle$’, or in general ‘$\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_m\rangle$’, ‘$\mathbf{\Phi}\langle\mathbf{p}_1,\mathbf{p}_2,...,\mathbf{p}_n\rangle$’, or ‘$\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_m,\mathbf{p}_1,\mathbf{p}_2,...,\mathbf{p}_n\rangle$’ in order to indicate that **Φ** contains occurrences of the corresponding atomic formulas in addition, perhaps, to occurrences of some other atomic formulas that are not indicated explicitly. In all such cases, I use *angle brackets* in order not to confuse an MLPH (metalogographic, i.e metalinguistic logographic, placeholder) such as ‘$\mathbf{\Phi}\langle\mathbf{x}\rangle$’, ‘$\mathbf{\Phi}\langle\mathbf{x},\mathbf{y}\rangle$’, or ‘$\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_m\rangle$’ with a PLPH (panlogographic placeholder) such as ‘$\mathbf{f}^1(\mathbf{x})$’, ‘$\mathbf{f}^2(\mathbf{x},\mathbf{y})$’, or ‘$\mathbf{f}^m(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_m)$’, in which use of parentheses is predetermined by the formation rule Ax 1.1(4).

2) It is understood that the definitions comprised in the above item 1 apply with any AnALPH, e.g. ‘**P**’, ‘**Q**’, ‘**I**’, or ‘**J**’, selected out of those introduced in Df 1.3(1,2), or with any AnAtMLPH, e.g. ‘**Ψ**’ or ‘**Ω**’¸ selected out of those introduced in Df 1.5, instead of ‘**Φ**’, and also with any appropriate mutually different PStAPLOT’s, selected out of the list (I.5.6), instead of ‘**x**’, ‘**y**’, ‘$\mathbf{x}_1$’, ‘$\mathbf{x}_2$’, …, ‘$\mathbf{x}_m$’, or with any appropriate mutually different PStAPLOR’s, selected out of the list (I.5.7), instead of ‘**p**’, ‘**q**’, ‘$\mathbf{p}_1$’, ‘$\mathbf{p}_2$’, …, ‘$\mathbf{p}_n$’.

3) When I use, an MMLPH (molecular MLPH) such as ‘$\mathbf{\Phi}\langle\mathbf{x}\rangle$’, ‘$\mathbf{\Phi}\langle\mathbf{x},\mathbf{y}\rangle$’, or ‘$\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_m\rangle$’ or such as ‘$\mathbf{\Phi}\langle\mathbf{p}\rangle$’, ‘$\mathbf{\Phi}\langle\mathbf{p},\mathbf{q}\rangle$’, or ‘$\mathbf{\Phi}\langle\mathbf{p}_1,\mathbf{p}_2,...,\mathbf{p}_n\rangle$’ instead of the AtMLPH (atomic MLPH) ‘**Φ**’, the occurrence of the letter ‘**Φ**’, preceding an occurrence of the angle bra 〈 is not an AtMLPH anymore, but rather it is an *atomic metalogographic quasi-predicate* (*AtMLQP*), whose range is a certain class of *euautographic operators*, while any of the AtPLPH’s occurring in the angle brackets is called an *operatum* (pl. “*operata*”) of the AtMLQP. Moreover, when ‘**Φ**’ alone is said to be used *non-xenonymously*, it is necessarily used *automymously in the*

homoloautographic (*photoautographic*) *mental mode*, so as to denote the *homolographic* (*photographic*) *token-class* of the logograph therein depicted between the curly light-faced single quotation marks. By contrast, when ‘$\mathbf{\Phi}\left\langle \mathbf{x}_1, \mathbf{x}_2 \right\rangle$’, e.g., is said to be used *non-xenonymously*, it may be used *either autonymously* in the above homoloautographic (photoautographic) mental mode, so as to denote the homolographic (photographic) token-class of the logograph therein depicted between the curly light-faced single quotation marks, or *semi-xenonymously* (*semi-autonymously*), so as to denote the homolographic (photographic) token-class $\mathbf{\Phi}\langle$‘$\mathbf{x}_1$’,‘$\mathbf{x}_2$’$\rangle$ depending on the AtMLQP $\mathbf{\Phi}$. In this case, in accordance with the *juxtaposition principle of autonymous quotations* (see, e.g., Suppes [1957, pp. 125–127]), ‘$\mathbf{\Phi}\left\langle \mathbf{x}_1, \mathbf{x}_2 \right\rangle$’ is a synonym the juxtaposition of quotations: ‘$\mathbf{\Phi}$’‘$\langle$’‘$\mathbf{x}_1$’‘,’$\mathbf{x}_2$’‘$\rangle$’, which essentially differs from $\mathbf{\Phi}\langle$‘$\mathbf{x}_1$’,‘$\mathbf{x}_2$’$\rangle$.

4) More generally, ‘$\mathbf{\Phi}\left\langle \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m \right\rangle$’, e.g., denotes: ‘$\mathbf{\Phi}\left\langle \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m \right\rangle$’, i.e. ‘$\mathbf{\Phi}$’‘$\langle$’‘$\mathbf{x}_1$’‘,’$\mathbf{x}_2$’‘,’‘…’‘,’‘$\mathbf{x}_m$’‘$\rangle$’, when it is used *autonymously*; **‘**$\mathbf{\Phi}\left\langle \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m \right\rangle$**’**, i.e. **‘$\mathbf{\Phi}$’**‘$\langle$’‘$\mathbf{x}_1$’‘,’$\mathbf{x}_2$’‘,’‘…’‘,’**‘**$\mathbf{x}_m$**’**‘$\rangle$’, when it is used *quasi-autonymously*; $\mathbf{\Phi}\langle$‘$\mathbf{x}_1$’,$\mathbf{x}_2$’,…,**‘**$\mathbf{x}_m$**’**$\rangle$ when it is used *semi-xenonymously, semi-autonymously, and semi-quasi-autonymously*. In the last case, the class of operata of the AtMLQP ‘**Φ**’ is extended from EOT’s (panlogographic ordinary terms) to PLOT’s (panlogographic ordinary terms). A like remark applies, e.g., witht ‘**P**’ or ‘**I**’ in place of ‘**Φ**’ or, e.g., with ‘**p**’ in place of ‘**x**’ and “StAtPLR” (“structural atomic panlogographic relation”) in place of “EOT” in any of the above cases.

5) Henceforth, when I say that I use, e.g., a placeholder ‘$\mathbf{\Phi}\left\langle \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m \right\rangle$’ *semi-xenonymously*, or *semi-autonymously*, I mean that, unless stated otherwise, I use it as $\mathbf{\Phi}\langle$‘$\mathbf{x}_1$’,$\mathbf{x}_2$’,…,**‘**$\mathbf{x}_m$**’**$\rangle$, while the latter is equivalent to the sequence: $\mathbf{\Phi}\langle$‘$\mathbf{x}_1$’$\rangle$, $\mathbf{\Phi}\langle$‘$\mathbf{x}_1$’,$\mathbf{x}_2$’$\rangle$, etc. Consequently, when I say that I use ‘$\mathbf{\Phi}\left\langle \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m \right\rangle$’ *xenoautonymously* or *autoxenonymously* or *in the TAEXA-mode,* I mean that I mentally experience ‘$\mathbf{\Phi}\left\langle \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m \right\rangle$’ xenonymously as $\mathbf{\Phi}\left\langle \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m \right\rangle$ and semi-xenonymously (or semi-autonymously) as $\mathbf{\Phi}\langle$‘$\mathbf{x}_1$’,$\mathbf{x}_2$’,…,**‘**$\mathbf{x}_m$**’**$\rangle$. Like definitions apply to every variant and every instance of ‘$\mathbf{\Phi}\left\langle \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m \right\rangle$’ as those mentioned in the previous item.

6) Using '$\mathbf{\Phi}\langle\mathbf{x},\mathbf{y}\rangle$', e.g., instead of '**Φ**' is in fact as inconsistent as using '$\mathbf{f}^2(\mathbf{x},\mathbf{y})$', e.g., instead of '$\mathbf{f}^2$', though use of angle brackets instead of round ones indicates that the former case is somewhat different from the latter. For complete rigor and for avoidance of any confusion, it would be desirable to employ in '$\mathbf{\Phi}\langle\mathbf{x},\mathbf{y}\rangle$' another affine letter instead of '**Φ**', alone or with some label – say, '**ϕ**' or '$\mathbf{\Phi}^2$'. However, the question is, not about a single letter in a given occurrence, but about introducing an entire system of AtMLQP's and their panlogographic instances that should be parallel to the system of AtMLPH's and their panlogographic instances, which has been introduced previously. An attempt of introducing such a system of AtMLQP's seems to be counterproductive, so that using the same letters as '**Φ**', '**P**', or '**I**' in two different hypostases is unavoidable.

7) In connection with the above said, I recall that no matter what a brain symbol is, it is a *dynamic and varying entity*, – in contrast to a graphic (written) symbol, which is *static and invariable* (*unchangeable*). Therefore, any *functional*, i.e. *single-valued*, correspondence (mapping) from brain symbols to graphic symbols is, in the general case, *many-to-one,* i.e. *surjective, and not bijective*. Consequently, the inverse correspondence is a *many-valued*, i.e. *not functional*, one. For instance, an interpreter of a certain graphic symbol (as a word or word group or as a placeholder) in a given occurrence can, depending on his mental attitude towards the symbol, use the symbol either in any of many *autonymous mental modes* or in any of many of *xenonymous mental modes*. The equivocality of exteroceptive symbols is their inherent property that one has to live with.

7) In what follows, I shall generalize the definition of the *metalogographic operator of substitution* '$\mathrm{S}_{-}^{-}-\big|$', introduced in Dfs I.5.11(3) and 1.7(1), it in several ways.•

Df 2.8. 1) Let **x** be a given APVOT that has either free or bound, or else no occurrences in **Φ**, and let **y** be either a given APVOT *other than* **x** or a given APCOT, i.e. a given AEOT, which satisfies the following conditions.

a) If **x** occurs in **Φ** as a free APVOT then **y** may occur in **Φ** if and only if it is either a free APVOT or an APCOT.

b) If **x** occurs in **Φ** as a bound APVOT then **y** is an APVOT, which does not occur in **Φ** at all.

Under the above assumptions, the following notation is used.

i) $\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{\Phi}\Big|$ is the EF, which results by substitution of **y** for each occurrence of **x** throughout **Φ**. If **x** occurs in **Φ** then '$\mathbf{\Phi}\langle\mathbf{x}\rangle$' can be written instead of '**Φ**', so that

$$\mathbf{\Phi}\langle\mathbf{y}\rangle \to \mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{\Phi}\langle\mathbf{x}\rangle\Big|. \tag{2.6}$$

Since **x** is an EF therefore $\mathbf{\Phi}\langle\mathbf{x}\rangle$ can coincide with **x**, so that $\mathbf{\Phi}\langle\mathbf{y}\rangle \to \mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{x}\Big| \to \mathbf{y}$. If **x** does not occur in **Φ** then $\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{\Phi}\Big| \leftrightarrow \mathbf{\Phi}$.

ii) $\dot{\mathrm{S}}_{\mathbf{y}}^{\mathbf{x}}\mathbf{\Phi}\langle\mathbf{x}\rangle\Big|$ is an EF, which results by substitution of **y** for *some* (*strictly some preselected or all*) occurrences of **x** throughout $\mathbf{\Phi}\langle\mathbf{x}\rangle$.

2) Let **p** be a given AER that has either some or no occurrences in **Ψ**; if exist, all the occurrences are *free* in accordance with Axs 1.1(3) and 1.2(2) and Ths 1.1(3) and 1.2(2). Let a given ER **P** and any (each given) APVOT **x** satisfy the following conditions.

a) If **x** has bound occurrences in **Ψ** then it has no occurrences, neither bound nor free, in **P**, and conversely if **x** has bound occurrences in **P** then it has no occurrences, neither bound nor free, in **Ψ**.

b) If **x** has free occurrences in **Ψ** then it has either free or no occurrences in **P**, and conversely if **x** has free occurrences in **P** then it has either free or no occurrences in **Ψ**.

Consequently, if **x** has no occurrences, neither bound nor free, in **Ψ** then it has either bound or free or no occurrences in **P**, and conversely if **x** has no occurrences, neither bound nor free, in **P** then it has either bound or free or no occurrences in **Ψ**. Under the above assumptions, the following notation is used.

i) $\mathrm{S}_{\mathbf{P}}^{\mathbf{p}}\mathbf{\Psi}\Big|$ is the EF, which results by substitution of **P** for each occurrence of **p** throughout **Ψ**. If **p** occurs in **Ψ** then '$\mathbf{\Psi}\langle\mathbf{p}\rangle$' can be written instead of '**Ψ**', so that

$$\mathbf{\Psi}\langle\mathbf{P}\rangle \to \mathrm{S}_{\mathbf{P}}^{\mathbf{p}}\mathbf{\Psi}\langle\mathbf{p}\rangle\Big|. \tag{2.7}$$

It is understood that **P** may contain **p**. It is also understood that $\mathbf{\Psi}\langle\mathbf{p}\rangle$ may coincide with **p**, so that $\mathbf{\Psi}\langle\mathbf{P}\rangle \to \mathrm{S}_{\mathbf{P}}^{\mathbf{p}}\mathbf{p}\Big| \to \mathbf{P}$. If **p** does not occur in **Ψ** then $\mathrm{S}_{\mathbf{P}}^{\mathbf{p}}\mathbf{\Psi}\Big| \leftrightarrow \mathbf{\Psi}$.

ii) $\dot{\mathrm{S}}^{\mathbf{p}}_{\mathbf{P}}\mathbf{\Psi}\langle\mathbf{p}\rangle\big|$ is an EF, which results by substitution of **P** for *some* (*strictly some preselected or all*) occurrences of **p** throughout $\mathbf{\Psi}\langle\mathbf{p}\rangle$.•

Df 2.9. 1) Let $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$ be *m* given mutually (pairwise) different APVOT's, any of which has either free or bound, or else no occurrences in **Φ**. Let $\mathbf{y}_1$, $\mathbf{y}_2$, …, $\mathbf{y}_m$ be *m* given AEOT's, all different or not, each of which can be either an APVOT or an APCOT. It is assumed that for each $i \in \omega_{1,m}$ the following conditions hold.

a) If $\mathbf{x}_i$ has free occurrences in **Φ** then there is no APVOT among $\mathbf{y}_1$, $\mathbf{y}_2$, …, $\mathbf{y}_m$, i.e. no $\mathbf{y}_j$ with $j \in \omega_{1,m}$, which coincides with (i.e. is a homolographic token of) $\mathbf{x}_i$ and which has bound occurrences in **Φ**.

b) If $\mathbf{x}_i$ has bound occurrences in **Φ** then $\mathbf{y}_i$ is an APVOT that differs from any other APVOT $\mathbf{y}_j$ with $j \in \omega_{1,m} - \{i\}$ and that has no occurrences in **Φ** except for the case, where $\mathbf{y}_i$ coincides with $\mathbf{x}_i$ and where hence it has bound occurrences in **Φ**. At the same time, no APVOT $\mathbf{y}_j$ with $j \in \omega_{1,m} - \{i\}$ may coincide with $\mathbf{x}_i$.

Besides having free occurrence of some of the APVOT's $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$, **Φ** may, in the framework of the above conditions, have occurrences of some APCOT's, which are found among $\mathbf{y}_1$, $\mathbf{y}_2$, …, $\mathbf{y}_m$, and free occurrences of some APVOT's, which are also found among $\mathbf{y}_1$, $\mathbf{y}_2$, …, $\mathbf{y}_m$ and which do not occur among $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$. Under the above assumptions, the following notation is used.

i) $\mathrm{S}^{\mathbf{x}_1\mathbf{x}_2\ldots\mathbf{x}_m}_{\mathbf{y}_1\mathbf{y}_2\ldots\mathbf{y}_m}\mathbf{\Phi}\big|$ is the EF, which results by simultaneous substitutions of $\mathbf{y}_1$, $\mathbf{y}_2$, …, $\mathbf{y}_m$ for $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$ throughout **Φ**. If all AEOT's $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$ occur in **Φ** then '$\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle$' can be written instead of '**Φ**', so that

$$\mathbf{\Phi}\langle\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_m\rangle \to \mathrm{S}^{\mathbf{x}_1\mathbf{x}_2\ldots\mathbf{x}_m}_{\mathbf{y}_1\mathbf{y}_2\ldots\mathbf{y}_m}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle\big|. \tag{2.8}$$

ii) Since $m \in \omega_1 = \{1,2,\ldots\}$, therefore definition (2.8) is equivalent to the following sequence of definitions:

$$\mathbf{\Phi}\langle\mathbf{y}_1\rangle \to \mathrm{S}^{\mathbf{x}_1}_{\mathbf{y}_1}\mathbf{\Phi}\langle\mathbf{x}_1\rangle\big|, \mathbf{\Phi}\langle\mathbf{y}_1,\mathbf{y}_2\rangle \to \mathrm{S}^{\mathbf{x}_1\mathbf{x}_2}_{\mathbf{y}_1\mathbf{y}_2}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2\rangle\big|, \ldots \tag{2.8a}$$

(cf. (2.6)). If none of the AEOT's $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$ occurs in $\mathbf{\Phi}$ then $\mathrm{S}^{\mathbf{x}_1\mathbf{x}_2...\mathbf{x}_m}_{\mathbf{y}_1\mathbf{y}_2...\mathbf{y}_m}\mathbf{\Phi}\Big| \leftrightarrow \mathbf{\Phi}$.

iii) $\dot{\mathrm{S}}^{\mathbf{x}_1\mathbf{x}_2...\mathbf{x}_m}_{\mathbf{y}_1\mathbf{y}_2...\mathbf{y}_m}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_m\rangle\Big|$ is an EF, which results by simultaneous substitutions of $\mathbf{y}_1$, $\mathbf{y}_2$, …, $\mathbf{y}_m$ for *some* (*strictly some preselected or all*) occurrences of $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$ throughout $\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_m\rangle$.

2) Let $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$ be *n* given mutually different AER's, any of which has either some [free] occurrences no occurrences in $\mathbf{\Psi}$. Let $\mathbf{P}_1$, $\mathbf{P}_2$, …, $\mathbf{P}_n$ be any *n* given ER's, different or not, each of which may contain some (i.e. strictly some or all) of $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$ or, particularly, coincide with (i.e. be a token of) one of the latter or with any other AER of the list (I.5.2). Let **x** be any (each given) APVOT, such that the following conditions, being generalizations of the conditions a and b of Df 2.8(2), hold.

a) If **x** has bound occurrences in $\mathbf{\Psi}$ then it has no occurrences, neither bound nor free, in $\mathbf{P}_i$ with every $i \in \omega_{1,n}$, and conversely if **x** has bound occurrences in $\mathbf{P}_i$ with some one $i \in \omega_{1,n}$ then it has no occurrences, neither bound nor free, both in $\mathbf{\Psi}$ and in $\mathbf{P}_j$ with every $j \in \omega_{1,n} - \{i\}$.

b) If **x** has free occurrences in $\mathbf{\Psi}$ then it has either free or no occurrences in $\mathbf{P}_i$ with every $i \in \omega_{1,n}$, and conversely if **x** has free occurrences in $\mathbf{P}_i$ with some one $i \in \omega_{1,n}$ then it has either free or no occurrences in both in $\mathbf{\Psi}$ and in $\mathbf{P}_j$ with every $j \in \omega_{1,n} - \{i\}$.

Under the above assumptions, the following notation is used.

i) $\mathrm{S}^{\mathbf{p}_1\mathbf{p}_2...\mathbf{p}_n}_{\mathbf{P}_1\mathbf{P}_2...\mathbf{P}_n}\mathbf{\Psi}\Big|$ is the EF, which results by simultaneous substitutions of $\mathbf{P}_1$, $\mathbf{P}_2$, …, $\mathbf{P}_n$ for $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$ throughout $\mathbf{\Psi}$. If all AER's $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$ occur in $\mathbf{\Psi}$ then '$\mathbf{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,...,\mathbf{p}_n\rangle$' can be written instead of '$\mathbf{\Psi}$', so that

$$\mathbf{\Psi}\langle\mathbf{P}_1,\mathbf{P}_2,...,\mathbf{P}_n\rangle \to \mathrm{S}^{\mathbf{p}_1\mathbf{p}_2...\mathbf{p}_n}_{\mathbf{P}_1\mathbf{P}_2...\mathbf{P}_n}\mathbf{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,...,\mathbf{p}_n\rangle\Big|. \tag{2.9}$$

ii) Since $n \in \omega_1 = \{1,2,...\}$ therefore definition (2.8) is equivalent to the following sequence of definitions:

$$\mathbf{\Psi}\langle\mathbf{P}_1\rangle \to \mathrm{S}^{\mathbf{p}_1}_{\mathbf{P}_1}\mathbf{\Psi}\langle\mathbf{p}_1\rangle\Big|, \mathbf{\Psi}\langle\mathbf{P}_1,\mathbf{P}_2\rangle \to \mathrm{S}^{\mathbf{p}_1\mathbf{p}_2}_{\mathbf{P}_1\mathbf{P}_2}\mathbf{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2\rangle\Big|,... \tag{2.9a}$$

(cf. (2.7)). If none of the AER's $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$ occurs in $\mathbf{\Psi}$ then $\mathrm{S}^{\mathbf{p}_1\mathbf{p}_2\ldots\mathbf{p}_n}_{\mathbf{P}_1\mathbf{P}_2\ldots\mathbf{P}_n}\mathbf{\Psi}\big| \leftrightarrow \mathbf{\Psi}$. This condition is particularly satisfied for any $\mathbf{\Psi}$ if A_1 is restricted to its branch whose atomic basis does not contain the list (I.5.2) of AER's.

iii) $\dot{\mathrm{S}}^{\mathbf{p}_1\mathbf{p}_2\ldots\mathbf{p}_n}_{\mathbf{P}_1\mathbf{P}_2\ldots\mathbf{P}_n}\mathbf{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle\big|$ is an EF, which results by simultaneous substitutions of $\mathbf{P}_1$, $\mathbf{P}_2$, …, $\mathbf{P}_n$ for *some* (*strictly some preselected or all*) occurrences of $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$ throughout $\mathbf{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle$.

3) The EF $\mathrm{S}^{\mathbf{x}_1\mathbf{x}_2\ldots\mathbf{x}_n}_{\mathbf{y}_1\mathbf{y}_2\ldots\mathbf{y}_n}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n\rangle\big|$, or $\mathrm{S}^{\mathbf{p}_1\mathbf{p}_2\ldots\mathbf{p}_n}_{\mathbf{P}_1\mathbf{P}_2\ldots\mathbf{P}_n}\mathbf{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle$, is called an *intrinsic interpretand* (pl. "*interpretands*"), or *interpretandum* (pl. "*interpretanda*"), of the EF $\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n\rangle$, or $\mathbf{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle$, respectively, whereas the latter EF is called the *intrinsic interpretans* (pl. "*interpetantia*") of the former EF.•

Df 2.10. 1) The logographs '$\mathrm{S}^{-}_{-}-\big|$', '$\mathrm{S}^{--}_{--}-\big|$', '$\mathrm{S}^{---}_{---}-\big|$', etc or generally '$\mathrm{S}^{--\cdots-}_{\underbrace{--\cdots-}_{n}}-\Big|$' (not including the indicator '$\underbrace{\quad}_{n}$'), where tokens of the En Dash ('n'-Dash) – are blank-signs, are distributively and discriminately called the *abstract singulary*, *binary*, *ternary*, etc, or generally *n-ary*, *operator* (or *predicate*) *of comprehensive* (i.e. *not selective*) *substitution*, respectively. Indiscriminately, any of the operators with $n\geq 2$ is called an *abstract multiary operator of comprehensive substitution* if $n\geq 2$. The *operations* (*functions*) *in intension*, i.e. the *rules of the pertinent operations in extension*, which are denoted by the above operators, i.e. $\mathrm{S}^{-}_{-}-\big|$, $\mathrm{S}^{--}_{--}-\big|$, $\mathrm{S}^{---}_{---}-\big|$, etc, or generally $\mathrm{S}^{--\cdots-}_{\underbrace{--\cdots-}_{n}}\,|$, are characterized by the same adherent and adjoined qualifiers to the headword "operation" in place of "operator".

2) The above item applies with '$\dot{\mathrm{S}}$' in place of 'S' and "*selective*" in place of "*comprehensive*".

3) In accordance with the pertinent general terminology introduced previously, the EF $\mathbf{\Phi}$, to which a substitution operator applies, is called the *host formula*, or *exclusive scope*, of the operator; a EF that is substituted into $\mathbf{\Phi}$ is called a *substituendum* (pl. "*subtituenda*"), or *substituend* (pl. "*substituends*"), and a constituent EF of $\mathbf{\Phi}$ that is replaced with another EF is called a *substituens* (pl. "*substituentia*").

4) Like '**Φ**' and '**Ψ**' (e.g.), the logographs 'S' and '$\dot{\mathrm{S}}$' and empty spaces and hence blank-signs belong to the XML (exclusive metalanguage) of A_1, i.e. of A_1 and A_1.•

Th 2.1. Each of the formulas EF's $\mathrm{S}^{\mathbf{x}_1\mathbf{x}_2\ldots\mathbf{x}_n}_{\mathbf{y}_1\mathbf{y}_2\ldots\mathbf{y}_n}\boldsymbol{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n\rangle|$ and $\mathrm{S}^{\mathbf{p}_1\mathbf{p}_2\ldots\mathbf{p}_n}_{\mathbf{P}_1\mathbf{P}_2\ldots\mathbf{P}_n}\boldsymbol{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle|$ can always be obtained by at most 2*n* successive single substitution as defined in items 1 and 2 of Df 2.8 respectively (cf. Church [1956, p. 82]).

Proof: 1) In accordance with Df 2.9(1), it has already been assumed that $\boldsymbol{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n\rangle$ contains *n* mutually different APVOT's $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_n$ and perhaps some, i.e. strictly some or all, of the AEOT's $\mathbf{y}_1$, $\mathbf{y}_2$, …, $\mathbf{y}_n$. Let $\mathbf{z}_1$, $\mathbf{z}_2$, ..., $\mathbf{z}_n$ be the first *n* APVOT's in alphabetic order not occurring in the EF $\boldsymbol{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n\rangle$. Such APVOT's always exist because the list (I.5.1) is infinite. In this case, $\mathrm{S}^{\mathbf{x}_1\mathbf{x}_2\ldots\mathbf{x}_n}_{\mathbf{y}_1\mathbf{y}_2\ldots\mathbf{y}_n}\boldsymbol{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n\rangle|$ can be defined as

$$\mathrm{S}^{\mathbf{x}_1\mathbf{x}_2\ldots\mathbf{x}_n}_{\mathbf{y}_1\mathbf{y}_2\ldots\mathbf{y}_n}\boldsymbol{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n\rangle| \to \mathrm{S}^{\mathbf{z}_1}_{\mathbf{y}_1}\mathrm{S}^{\mathbf{z}_2}_{\mathbf{y}_2}\ldots\mathrm{S}^{\mathbf{z}_n}_{\mathbf{y}_n}\boldsymbol{\Phi}\langle\mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_n\rangle\|\ldots| \tag{2.10}$$

subject to

$$\boldsymbol{\Phi}\langle\mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_n\rangle \to \mathrm{S}^{\mathbf{x}_1}_{\mathbf{z}_1}\mathrm{S}^{\mathbf{x}_2}_{\mathbf{z}_2}\ldots\mathrm{S}^{\mathbf{x}_n}_{\mathbf{z}_n}\boldsymbol{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n\rangle\|\ldots|. \tag{2.11}$$

2) Analogously, assuming without loss of generality that a given EF **Ψ** contains *n* mutually different AER's $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$, I write '$\boldsymbol{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle$' instead of '**Ψ**'. Let $\mathbf{q}_1$, $\mathbf{q}_2$, ..., $\mathbf{q}_n$ be the first *n* AER's in alphabetic order not occurring in any of the EF's **Φ**, $\mathbf{P}_1$, $\mathbf{P}_2$, ..., $\mathbf{P}_n$. Such AER's always exist because the list (I.5.2) is infinite. In this case, $\mathrm{S}^{\mathbf{p}_1\mathbf{p}_2\ldots\mathbf{p}_n}_{\mathbf{P}_1\mathbf{P}_2\ldots\mathbf{P}_n}\boldsymbol{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle|$ can be defined as

$$\mathrm{S}^{\mathbf{p}_1\mathbf{p}_2\ldots\mathbf{p}_n}_{\mathbf{P}_1\mathbf{P}_2\ldots\mathbf{P}_n}\boldsymbol{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle| \to \mathrm{S}^{\mathbf{q}_1}_{\mathbf{P}_1}\mathrm{S}^{\mathbf{q}_2}_{\mathbf{P}_2}\ldots\mathrm{S}^{\mathbf{q}_n}_{\mathbf{P}_n}\boldsymbol{\Psi}\langle\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_n\rangle\|\ldots| \tag{2.12}$$

subject to

$$\boldsymbol{\Psi}\langle\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_n\rangle \to \mathrm{S}^{\mathbf{p}_1}_{\mathbf{q}_1}\mathrm{S}^{\mathbf{p}_2}_{\mathbf{q}_2}\ldots\mathrm{S}^{\mathbf{p}_n}_{\mathbf{q}_n}\boldsymbol{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle\|\ldots|. \tag{2.13}$$•

Df 2.11. An *alphabetic variant of an EF* **Φ** *of* A_1 is an *EF* **Φ′** *of* A_1, which is obtained from **Φ** by alphabetic changes of *atomic pseudo-variable ordinary formulas* (*APVOF's*), i.e. of bound or free APVOT's or of AER's or of both, throughout **Φ** in such a way that any two occurrences of the same APVOF in **Φ** remain occurrences of the same A APVOF in **Φ′** and any two occurrences of different APVOF's in **Φ** remain occurrences of different APVOF's in **Φ′** (cf. Church [1956, p. 86]). An alphabetic variant of **Φ** will alternatively be called a *variant interpretand of* **Φ**.•

Cmt 2.5. Here follow three kinds of alphabetic variants of EF's of A_1, which occur most frequently.

1) If $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$ are mutually different APVOT's occurring in an EF **Φ** and if $\mathbf{y}_1$, $\mathbf{y}_2$, …, $\mathbf{y}_m$ are some other mutually different APVOT's such that there is no APVOT among them, which occurs in **Φ** and which does not occur among $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$, then $\mathrm{S}_{\mathbf{y}_1\mathbf{y}_2\ldots\mathbf{y}_m}^{\mathbf{x}_1\mathbf{x}_2\ldots\mathbf{x}_m}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle|$, i.e. $\mathbf{\Phi}\langle\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_m\rangle$, is an alphabetic variant of $\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle$.

2) Likewise, if $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$ are mutually different AER's occurring in an EF **Ψ** and if $\mathbf{q}_1$, $\mathbf{q}_2$, ..., $\mathbf{q}_n$ are some different AER's such that there is no AER among them which occurs in **Ψ** and which does not occurs among $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$ then $\mathrm{S}_{\mathbf{q}_1\mathbf{q}_2\ldots\mathbf{q}_n}^{\mathbf{p}_1\mathbf{p}_2\ldots\mathbf{p}_n}\mathbf{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle|$, i.e $\mathbf{\Psi}\langle\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_n\rangle$, is an alphabetic variant of $\mathbf{\Psi}\langle\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle$.

3) If the above two conditions are satisfied simultaneously with '**Ω**' in place of both '**Φ**' and '**Ψ**' then $\mathbf{\Omega}\langle\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_m,\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_n\rangle$, defined as

$$\mathbf{\Omega}\langle\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_m,\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_n\rangle \to \mathrm{S}_{\mathbf{y}_1\mathbf{y}_2\ldots\mathbf{y}_m}^{\mathbf{x}_1\mathbf{x}_2\ldots\mathbf{x}_m}\mathrm{S}_{\mathbf{q}_1\mathbf{q}_2\ldots\mathbf{q}_n}^{\mathbf{p}_1\mathbf{p}_2\ldots\mathbf{p}_n}\mathbf{\Omega}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m,\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle\|,$$

is an alphabetic variant of $\mathbf{\Omega}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m,\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle$.•

Df 2.12. If it is necessary to indicate explicitly that occurrences of some APVOT's in an EF **Φ** are bound, without indicating specific characters of the bondages, then I shall use the pertinent *indexed* (*contracted*) *metalogograph* after the manner of indexed (contracted) metalogographs introduced in Df I.5.11(5). For instance, '$\mathbf{\Phi}_{\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n\rangle}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle$' with $n \leq m$ is an EF that contains occurrences of $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$, of which occurrences of $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_n$ are bound.•

Df 2.13: ***A generalization of Df 2.9.*** Let **Ψ** and **Ω** be two given different conspecific EF's of A_1, which satisfies the following conditions.

a) If **Ψ** occurs in a given EF **Φ** of A_1, so that the latter can be written as $\mathbf{\Phi}\langle\mathbf{\Psi}\rangle$, but **Ψ** has no bound APVOT's in common with the rest of **Φ**.

b) **Ω** does not occur in $\mathbf{\Phi}\langle\mathbf{\Psi}\rangle$ and it has no bound APVOT's that have bound occurrences in the part of $\mathbf{\Phi}\langle\mathbf{\Psi}\rangle$ complementary of **Ψ**.

Under the above assumptions, the following notation is used.

i) $\mathrm{S}_{\mathbf{\Omega}}^{\mathbf{\Psi}}\mathbf{\Phi}\langle\mathbf{\Psi}\rangle\big|$ is the EF of A_1, which results by substitution of **Ω** for each occurrence of **Ψ** throughout **Φ,** so that

$$\mathbf{\Phi}\langle\mathbf{\Omega}\rangle \to \mathrm{S}_{\mathbf{\Omega}}^{\mathbf{\Psi}}\mathbf{\Phi}\langle\mathbf{\Psi}\rangle\big|. \tag{2.14}$$

Since **Ψ** is an EF therefore $\mathbf{\Phi}\langle\mathbf{\Psi}\rangle$ can coincide with **Ψ**, so that $\mathbf{\Phi}\langle\mathbf{\Omega}\rangle \to \mathrm{S}_{\mathbf{\Omega}}^{\mathbf{\Psi}}\mathbf{\Psi}\big| \to \mathbf{\Omega}$.

ii) Then $\dot{\mathrm{S}}_{\mathbf{\Omega}}^{\mathbf{\Psi}}\mathbf{\Phi}\langle\mathbf{\Psi}\rangle\big|$ is the EF of A_1, which results by substitution of **Ω** for *some* (*strictly some preselected or all*) occurrences of **Ψ** throughout $\mathbf{\Phi}\langle\mathbf{\Psi}\rangle$.•

Df 2.14. The barred arrow $\mapsto$ is a *substitution sign* such that the string **'Ω ↦ Ψ'** indicates the act of substitution of tokens of the EF **Ω** for specified occurrences of the EF **Ψ** in a specified host formula. Formally, the sign $\mapsto$ can be defined in terms of the operator '$\dot{\mathrm{S}} - \big|$' thus:

$$[\mathbf{\Omega} \mapsto \mathbf{\Psi}] \to \dot{\mathrm{S}}_{\mathbf{\Omega}}^{\mathbf{\Psi}} - \big|. \qquad (2.15$$

Depending on the context in which the string **'Ω ↦ Ψ'** occurs, it should be read in accordance with the English grammar and English lexicon. Particularly, the expression **"***with* **Ω ↦ Ψ"** should be read as **"***with* **Ω** *in place of* **Ψ"** or equivalently as **"***with* **Ω** *substituted for* **Ψ".•**

Cmt 2.6. 1) An informal description of substitutions with the help of the substitution sign $\mapsto$ or of its some verbal equivalents as those mentioned in Df 2.14 turns out to be most convenient and often the only possible one if the concrete instance of a certain substitution schema of those introduced above is too cumbersome so that it cannot be printed in consequence of typographical difficulties or if none of those schemata are applicable. For instance, the intended denotatum of the schema '$\dot{\mathrm{S}}_{\mathbf{\Omega}}^{\mathbf{\Psi}}\mathbf{\Phi}\langle\mathbf{\Psi}\rangle\big|$' can be ambiguous in the absence of some immediate contextual verbal explanations accompanying the EF **Φ** and the substitution **Ω ↦ Ψ** when these are stated or mentioned separately.

2) Also, in the sequel, I shall use the sign $\mapsto$ as a most general sign of substitution which can particularly be used when the range of the substituend and the range of the substituens are *incomparable*.

Cmt 2.7. A semi-xenonymous (semi-autonymous) value $\mathbf{\Phi}\langle\text{‘}\mathbf{x}_1\text{’},\mathbf{x}_2\text{’},\ldots,\text{‘}\mathbf{x}_m\text{’}\rangle$ of the MLPH $\text{‘}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle\text{’}$ and its instances or variants, which have informally been introduced in Df 2.7(5), are defined formally by the following generalization of Df 2.9.●

Df 2.15. 1) Just as in Df 2.9(1), let $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$ be *m* given mutually different APVOT's, any of which has either free or bound occurrences in an EF $\mathbf{\Phi}$ of A_1, so that $\mathbf{\Phi}$ can more specifically be written as $\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle$.

i) Then $\mathrm{S}^{\mathbf{x}_1\ \mathbf{x}_2 \ldots\ \mathbf{x}_m}_{\text{'}\mathbf{x}_1\text{''}\mathbf{x}_2\text{'}\ldots\text{'}\mathbf{x}_m\text{'}}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle|$ (e.g) is the PLF (panlogographic formula), which results by simultaneous substitutions of ‘$\mathbf{x}_1$’, ‘$\mathbf{x}_2$’, …, ‘$\mathbf{x}_m$’ for $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$, i.e. by

$$\text{‘}\mathbf{x}_1\text{’}\mapsto\mathbf{x}_1,\ \text{‘}\mathbf{x}_2\text{’}\mapsto\mathbf{x}_2,\ \ldots,\ \text{‘}\mathbf{x}_m\text{’}\mapsto\mathbf{x}_m, \tag{2.16}$$

throughout $\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle$, so that

$$\mathbf{\Phi}\langle\text{'}\mathbf{x}_1\text{'},\text{'}\mathbf{x}_2\text{'},\ldots,\text{'}\mathbf{x}_m\text{'}\rangle\to\mathrm{S}^{\mathbf{x}_1\ \mathbf{x}_2 \ldots\ \mathbf{x}_m}_{\text{'}\mathbf{x}_1\text{''}\mathbf{x}_2\text{'}\ldots\text{'}\mathbf{x}_m\text{'}}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle|. \tag{2.17}$$

Similarly, $\mathrm{S}^{\mathbf{x}_1\ \mathbf{x}_2 \ldots\ \mathbf{x}_m}_{\text{'}\mathbf{y}_1\text{''}\mathbf{y}_2\text{'}\ldots\text{'}\mathbf{y}_m\text{'}}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle|$ (e.g) is the PLF, which results by simultaneous substitutions of ‘$\mathbf{y}_1$’, ‘$\mathbf{y}_2$’, …, ‘$\mathbf{y}_m$’ for $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$, throughout $\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle$, whereas $\mathrm{S}^{\text{'}\mathbf{x}_1\text{''}\mathbf{x}_2\text{'}\ldots\text{'}\mathbf{y}_m\text{'}}_{\text{'}\mathbf{y}_1\text{''}\mathbf{y}_2\text{'}\ldots\text{'}\mathbf{y}_m\text{'}}\mathbf{\Phi}\langle\text{'}\mathbf{x}_1\text{'},\text{'}\mathbf{x}_2\text{'},\ldots,\text{'}\mathbf{x}_m\text{'}\rangle|$ is the PLF, which results by simultaneous substitutions of ‘$\mathbf{y}_1$’, ‘$\mathbf{y}_2$’, …, ‘$\mathbf{y}_m$’ for ‘$\mathbf{x}_1$’, ‘$\mathbf{x}_2$’, …, ‘$\mathbf{x}_m$’ throughout $\mathbf{\Phi}\langle\text{'}\mathbf{x}_1\text{'},\text{'}\mathbf{x}_2\text{'},\ldots,\text{'}\mathbf{x}_m\text{'}\rangle$, so that

$$\begin{aligned}\mathbf{\Phi}\langle\text{'}\mathbf{y}_1\text{'},\text{'}\mathbf{y}_2\text{'},\ldots,\text{'}\mathbf{y}_m\text{'}\rangle&\to\mathrm{S}^{\mathbf{x}_1\ \mathbf{x}_2 \ldots\ \mathbf{x}_m}_{\text{'}\mathbf{y}_1\text{''}\mathbf{y}_2\text{'}\ldots\text{'}\mathbf{y}_m\text{'}}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m\rangle|\\ &\leftrightarrow\mathrm{S}^{\text{'}\mathbf{x}_1\text{''}\mathbf{x}_2\text{'}\ldots\text{'}\mathbf{x}_m\text{'}}_{\text{'}\mathbf{y}_1\text{''}\mathbf{y}_2\text{'}\ldots\text{'}\mathbf{y}_m\text{'}}\mathbf{\Phi}\langle\text{'}\mathbf{x}_1\text{'},\text{'}\mathbf{x}_2\text{'},\ldots,\text{'}\mathbf{x}_m\text{'}\rangle|.\end{aligned} \tag{2.18}$$

ii) Since $m\in\omega_1=\{1,2,\ldots\}$, therefore definitions (2.33) and (2.34) are equivalent to the following sequences of definitions:

$$\mathbf{\Phi}\langle\text{'}\mathbf{x}_1\text{'}\rangle\to\mathrm{S}^{\mathbf{x}_1}_{\text{'}\mathbf{x}_1\text{'}}\mathbf{\Phi}\langle\mathbf{x}_1\rangle|,\ \mathbf{\Phi}\langle\text{'}\mathbf{x}_1\text{'},\text{'}\mathbf{x}_2\text{'}\rangle\to\mathrm{S}^{\mathbf{x}_1\ \mathbf{x}_2}_{\text{'}\mathbf{x}_1\text{''}\mathbf{x}_2\text{'}}\mathbf{\Phi}\langle\mathbf{x}_1,\mathbf{x}_2\rangle|,\ldots, \tag{2.17a}$$

$$\mathbf{\Phi}\langle '\mathbf{y}_1' \rangle \to \mathrm{S}^{\mathbf{x}_1}_{'\mathbf{y}_1'}\mathbf{\Phi}\langle \mathbf{x}_1 \rangle\Big| \leftrightarrow \mathrm{S}^{'\mathbf{y}_1'}_{'\mathbf{y}_1'}\mathbf{\Phi}\langle '\mathbf{x}_1' \rangle\Big|,$$
$$\mathbf{\Phi}\langle '\mathbf{y}_1', '\mathbf{y}_2' \rangle \to \mathrm{S}^{\mathbf{x}_1\ \mathbf{x}_2}_{'\mathbf{y}_1''\mathbf{y}_2'}\mathbf{\Phi}\langle \mathbf{x}_1, \mathbf{x}_2 \rangle\Big| \leftrightarrow \mathrm{S}^{'\mathbf{x}_1''\mathbf{x}_2'}_{'\mathbf{y}_1''\mathbf{y}_2'}\mathbf{\Phi}\langle '\mathbf{x}_1', '\mathbf{x}_2' \rangle\Big|, \ldots \tag{2.18a}$$

(cf. (2.8a)).

iii) $\dot{\mathrm{S}}^{\mathbf{x}_1\ \mathbf{x}_2 \ldots\ \mathbf{x}_m}_{'\mathbf{x}_1''\mathbf{x}_2'\ldots'\mathbf{x}_m'}\mathbf{\Phi}\langle \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m \rangle$ is a PLF, which results by simultaneous substitutions of ‘$\mathbf{x}_1$’, ‘$\mathbf{x}_2$’, …, ‘$\mathbf{x}_m$’ for $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$ for *some* (*strictly some preselected but not all*) occurrences of $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_m$ throughout $\mathbf{\Phi}\langle \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m \rangle$.

2) Just as in Df 2.9(2), let $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$ be *n* given mutually different AER’s, any of which has some [free] occurrences in an EF $\mathbf{\Psi}$ of A_1, so that $\mathbf{\Psi}$ can more specifically be written as $\mathbf{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle$.

i) Then $\mathrm{S}^{\mathbf{p}_1\ \mathbf{p}_2 \ldots\ \mathbf{p}_n}_{'\mathbf{P}_1''\mathbf{P}_2'\ldots'\mathbf{P}_n'}\mathbf{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle\Big|$ is the PLF, which results by simultaneous substitutions of ‘$\mathbf{P}_1$’, ‘$\mathbf{P}_2$’, …, ‘$\mathbf{P}_n$’ for $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$, i.e. by

$$‘\mathbf{P}_1’ \mapsto \mathbf{p}_1,\ ‘\mathbf{P}_2’ \mapsto \mathbf{p}_2,\ \ldots,\ ‘\mathbf{P}_n’ \mapsto \mathbf{p}_n \tag{2.19}$$

throughout $\mathbf{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle$, so that

$$\mathbf{\Psi}\langle '\mathbf{P}_1', '\mathbf{P}_2', \ldots, '\mathbf{P}_n' \rangle \to \mathrm{S}^{\mathbf{p}_1\ \mathbf{p}_2 \ldots\ \mathbf{p}_n}_{'\mathbf{P}_1''\mathbf{P}_2'\ldots'\mathbf{P}_n'}\mathbf{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle\Big|. \tag{2.20}$$

Similarly, $\mathrm{S}^{\mathbf{p}_1\ \mathbf{p}_2 \ldots\ \mathbf{p}_n}_{'\mathbf{Q}_1''\mathbf{Q}_2'\ldots'\mathbf{Q}_n'}\mathbf{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle\Big|$ (e.g) is the PLF, which results by simultaneous substitutions of ‘$\mathbf{Q}_1$’, ‘$\mathbf{Q}_2$’, …, ‘$\mathbf{Q}_n$’ for $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$, throughout $\mathbf{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle$, whereas $\mathrm{S}^{'\mathbf{P}_1''\mathbf{P}_2'\ldots'\mathbf{P}_n'}_{'\mathbf{Q}_1''\mathbf{Q}_2'\ldots'\mathbf{Q}_n'}\mathbf{\Psi}\langle '\mathbf{P}_1', '\mathbf{P}_2', \ldots, '\mathbf{P}_n' \rangle\Big|$ is the PLF, which results by simultaneous substitutions of ‘$\mathbf{Q}_1$’, ‘$\mathbf{Q}_2$’, …, ‘$\mathbf{Q}_n$’ for ‘$\mathbf{P}_1$’, ‘$\mathbf{P}_2$’, …, ‘$\mathbf{P}_n$’ throughout $\mathbf{\Psi}\langle '\mathbf{P}_1', '\mathbf{P}_2', \ldots, '\mathbf{P}_n' \rangle$, so that

$$\mathbf{\Psi}\langle '\mathbf{Q}_1', '\mathbf{Q}_2', \ldots, '\mathbf{Q}_n' \rangle \to \mathrm{S}^{\mathbf{p}_1\ \mathbf{p}_2 \ldots\ \mathbf{p}_n}_{'\mathbf{Q}_1''\mathbf{Q}_2'\ldots'\mathbf{Q}_n'}\mathbf{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle\Big|$$
$$\leftrightarrow \mathrm{S}^{'\mathbf{P}_1''\mathbf{P}_2'\ldots'\mathbf{P}_n'}_{'\mathbf{Q}_1''\mathbf{Q}_2'\ldots'\mathbf{Q}_n'}\mathbf{\Psi}\langle '\mathbf{P}_1', '\mathbf{P}_2', \ldots, '\mathbf{P}_n' \rangle\Big|. \tag{2.21}$$

ii) Since $n \in \omega_1 = \{1,2,\ldots\}$, therefore definitions (2.20) and (2.21) are equivalent to the following sequences of definitions:

$$\mathbf{\Psi}\langle '\mathbf{P}_1' \rangle \to \mathrm{S}^{\mathbf{p}_1}_{'\mathbf{P}_1'}\mathbf{\Psi}\langle \mathbf{p}_1 \rangle\Big|, \mathbf{\Psi}\langle '\mathbf{P}_1', '\mathbf{P}_2' \rangle \to \mathrm{S}^{\mathbf{p}_1\ \mathbf{p}_2}_{'\mathbf{P}_1''\mathbf{P}_2'}\mathbf{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2 \rangle\Big|, \ldots, \tag{2.20a}$$

$$\mathbf{\Psi}\langle '\mathbf{Q}_1' \rangle \to \mathrm{S}^{\mathbf{p}_1}_{'\mathbf{Q}_1'}\mathbf{\Psi}\langle \mathbf{p}_1 \rangle\Big| \leftrightarrow \mathrm{S}^{'\mathbf{P}_1'}_{'\mathbf{Q}_1'}\mathbf{\Psi}\langle '\mathbf{P}_1' \rangle\Big|,$$
$$\mathbf{\Psi}\langle '\mathbf{Q}_1', '\mathbf{Q}_2' \rangle \to \mathrm{S}^{\mathbf{p}_1\ \mathbf{p}_2}_{'\mathbf{Q}_1''\mathbf{Q}_2'}\mathbf{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2 \rangle\Big| \leftrightarrow \mathrm{S}^{'\mathbf{P}_1''\mathbf{P}_2'}_{'\mathbf{Q}_1''\mathbf{Q}_2'}\mathbf{\Psi}\langle '\mathbf{P}_1', '\mathbf{P}_2' \rangle\Big|, \ldots \tag{2.21a}$$

(cf. (2.17a) and (2.18a)).

iii) $\dot{\mathrm{S}}^{\mathbf{p}_1\,\mathbf{p}_2 \ldots\, \mathbf{p}_n}_{'\mathbf{P}_1'\,'\mathbf{P}_2' \ldots '\mathbf{P}_n'}\boldsymbol{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle\big|$ is a PLF, which results by simultaneous substitutions of '$\mathbf{P}_1$', '$\mathbf{P}_2$', …, '$\mathbf{P}_n$' for *some* (*strictly some preselected but not all*) occurrences of $\mathbf{p}_1$, $\mathbf{p}_2$, …, $\mathbf{p}_n$ throughout $\boldsymbol{\Psi}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle$.•

Cmt 2.8. 1) Each one of definitions (2.17), (2.18), (2.20), and (2.21) can be specified by replacing '**Φ**' or '**Ψ**' with occurrences of any one of the AtPLPH's '**P**', '**Q**', '**I**', and '**J**', e.g., without any quotation marks, throughout each definition. Thus, for instance,

$$\mathbf{P}\langle '\mathbf{x}_1', '\mathbf{x}_2', \ldots, '\mathbf{x}_m' \rangle \to \mathrm{S}^{\mathbf{x}_1\,\mathbf{x}_2 \ldots\, \mathbf{x}_m}_{'\mathbf{x}_1'\,'\mathbf{x}_2' \ldots '\mathbf{x}_m'}\mathbf{P}\langle \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m \rangle\big|, \tag{2.17′}$$

$$\mathbf{I}\langle '\mathbf{x}_1', '\mathbf{x}_2', \ldots, '\mathbf{x}_m' \rangle \to \mathrm{S}^{\mathbf{x}_1\,\mathbf{x}_2 \ldots\, \mathbf{x}_m}_{'\mathbf{x}_1'\,'\mathbf{x}_2' \ldots '\mathbf{x}_m'}\mathbf{I}\langle \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m \rangle\big| \tag{2.17″}$$

are some specific instances of (2.17), whereas

$$\mathbf{Q}\langle '\mathbf{P}_1', '\mathbf{P}_2', \ldots, '\mathbf{P}_n' \rangle \to \mathrm{S}^{\mathbf{p}_1\,\mathbf{p}_2 \ldots\, \mathbf{p}_n}_{'\mathbf{P}_1'\,'\mathbf{P}_2' \ldots '\mathbf{P}_n'}\mathbf{Q}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle\big|, \tag{2.20′}$$

$$\mathbf{J}\langle '\mathbf{P}_1', '\mathbf{P}_2', \ldots, '\mathbf{P}_n' \rangle \to \mathrm{S}^{\mathbf{p}_1\,\mathbf{p}_2 \ldots\, \mathbf{p}_n}_{'\mathbf{P}_1'\,'\mathbf{P}_2' \ldots '\mathbf{P}_n'}\mathbf{J}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle\big| \tag{2.20″}$$

are some specific instances of (2.20).

2) Definitions (2.17′), (2.17″), (2.20′), and (2.20″) are equivalent to the following respective sequences of definitions:

$$\mathbf{P}\langle '\mathbf{x}_1' \rangle \to \mathrm{S}^{\mathbf{x}_1}_{'\mathbf{x}_1'}\mathbf{P}\langle \mathbf{x}_1 \rangle\big|,\ \mathbf{P}\langle '\mathbf{x}_1', '\mathbf{x}_2' \rangle \to \mathrm{S}^{\mathbf{x}_1\,\mathbf{x}_2}_{'\mathbf{x}_1'\,'\mathbf{x}_2'}\mathbf{P}\langle \mathbf{x}_1, \mathbf{x}_2 \rangle\big|, \ldots, \tag{2.17′a}$$

$$\mathbf{I}\langle '\mathbf{x}_1' \rangle \to \mathrm{S}^{\mathbf{x}_1}_{'\mathbf{x}_1'}\mathbf{I}\langle \mathbf{x}_1 \rangle\big|,\ \mathbf{I}\langle '\mathbf{x}_1', '\mathbf{x}_2' \rangle \to \mathrm{S}^{\mathbf{x}_1\,\mathbf{x}_2}_{'\mathbf{x}_1'\,'\mathbf{x}_2'}\mathbf{I}\langle \mathbf{x}_1, \mathbf{x}_2 \rangle\big|, \ldots, \tag{2.17″a}$$

$$\mathbf{Q}\langle '\mathbf{P}_1' \rangle \to \mathrm{S}^{\mathbf{p}_1}_{'\mathbf{P}_1'}\mathbf{Q}\langle \mathbf{p}_1 \rangle\big|, \mathbf{Q}\langle '\mathbf{P}_1', '\mathbf{P}_2' \rangle \to \mathrm{S}^{\mathbf{p}_1\,\mathbf{p}_2}_{'\mathbf{P}_1'\,'\mathbf{P}_2'}\mathbf{Q}\langle \mathbf{p}_1, \mathbf{p}_2 \rangle\big|, \ldots, \tag{2.20′a}$$

$$\mathbf{J}\langle '\mathbf{P}_1' \rangle \to \mathrm{S}^{\mathbf{p}_1}_{'\mathbf{P}_1'}\mathbf{J}\langle \mathbf{p}_1 \rangle\big|, \mathbf{J}\langle '\mathbf{P}_1', '\mathbf{P}_2' \rangle \to \mathrm{S}^{\mathbf{p}_1\,\mathbf{p}_2}_{'\mathbf{P}_1'\,'\mathbf{P}_2'}\mathbf{J}\langle \mathbf{p}_1, \mathbf{p}_2 \rangle\big|, \ldots. \tag{2.20″a}$$

Cmt 2.9. From time to time and particularly above in this subsection, I use autonymous and quasi-autonymous quotations (citation forms, quotation nouns, hypotheses) in order to indicate the respective mental attitudes, which the interpreter should take towards the interiors of the quotations and in order to elaborate thus the relevant epistemological notions. In the sequel, however, when I execute the calculus A_1 via executing the calculus $\mathbf{A}_1$, I shall not attempt to systematically maintain the distinction between xenomymous and autonymous uses of PLF's of $\mathbf{A}_1$, because this is impossible, – in accordance with Df 2.7(7). That is to say, I shall use an unquoted PLF for mentioning both a general (common, certain, concrete but not concretized) euautographic value of the PLF and the concrete PLF itself equivocally and intermittently and as if simultaneously by repeatedly switching from one mental attitude to the other – just as it happens in perceiving any of Escher's *Convex and Concave* pictures, e.g. "*Cube with Magic Ribbons*" (see, for instance, Ernst [1985, p. 85f]). *Fortunately* and *amazingly,* such a use of PLF's does not result in confusion and inconsistencies.

3. Preliminaries to the next steps of the setup and to the subsequent execution of A_1 and $\mathbf{A}_1$

3.1. Taxonomy of relations of A_1 and $\mathbf{A}_1$ in general outline

Df 3.1. 1) An ER (euautographic relation) of A_1 that is taken for granted to be *valid* and that is laid down to be so with the help of the appropriate format is called a *subject axiom of* A_1 or briefly an *axiom of* A_1 and also synecdochically a *euautographic axiom* (*EAx*). A [*subject*] *axiom of* A_1 is alternatively called an *object axiom of the IML* (*inclusive metalanguage*) *of* A_1 *that belongs to* A_1.

2) A PLR (panlogographic relation) of $\mathbf{A}_1$ that is taken for granted to be *valid* and that is laid down to be so with the help of the appropriate format is called a *subject axiom of* $\mathbf{A}_1$ or briefly an *axiom of* $\mathbf{A}_1$ and also a *panlogographic axiom* (*PLAx*). A [*subject*] *axiom of* $\mathbf{A}_1$ is alternatively called an *object axiom of the IML* (*inclusive metalanguage*) *of* A_1 *that belongs to* $\mathbf{A}_1$.

3) The fact that a *concrete* ER of A_1 is laid down as an EAx will formally be indicated by writing it, either alone or together with some other *concrete congeneric* or *conspecific* EAxs, under a logical heading, which begins with the string "°**Ax**". A concrete EAx of A_1 involves neither graphonyms of the XLM (exusive metalanguage) of A_1 nor panlogographs of $\mathbf{A}_1$.

4) The fact that a *concrete* PLR of $\mathbf{A}_1$ is laid down as a PLAx will formally be indicated by writing it, either alone or together with some other *concrete congeneric* or *conspecific* PLAxs, under a logical heading, which begins with the string "***Ax**". A concrete PLAx of $\mathbf{A}_1$ involves no graphonyms of the XLM of A_1 (A_1 and $\mathbf{A}_1$), but it can involve some euautographs of A_1.

5) If several concrete congeneric or conspecific EAxs or PLAxs are stated under the same title then each axiom is displayed in its own line, or lines. Also, it is preceded by its own numeral name relative to the common title or relative to the

section, in which it occurs, and is separated from the next concrete axiom by a comma, semicolon, or full stop.

6) When used xenonymously, a concrete PLAx of $\mathbf{A}_1$ is alternatively called a *panlogographic schema* (*PLS*, pl. "*PLS'ta*") *of euautographic axioms* (*EAx's*) *of* A_1 in the sense that it is a *schematic* (*patterned*) *panlogographic placeholder* (*SchPLPH*) condensing a large number (commonly an infinite number) of concrete but not concretized EAxs of A_1 in its range and also condensing an indefinite number of *specific PLPH's*, the ranges of which are *species* (*specific classes*, *strict subclasses*) of the range of the PLS.

7) There are no systematic *metalogographic schemata* (*MLS'ta*), i.e. *schematic* (*patterned*) *metalogographic placeholders* (*SchMLPH's*) to condense a large number of concrete PLAxs of $\mathbf{A}_1$ in a single metalogographic statement, although some such schemata can occasionally be laid down for the sake of clarity.

8) The generic name "*axiom of* A_1" is a synonym of, and the name "*primary valid relation of* A_1", and the generic name "*axiom of* $\mathbf{A}_1$" is a synonym of the name "*primary valid relation of* $\mathbf{A}_1$". Consequently, every axiom of A_1 is a *valid relation of* A_1, but not every valid relation of A_1 is an axiom of A_1. Likewise, every axiom of $\mathbf{A}_1$ is a *valid relation of* $\mathbf{A}_1$, but not every valid relation of $\mathbf{A}_1$ is an axiom of $\mathbf{A}_1$. The ranges of the terms "*valid relation of* A_1" and "*valid relation of* $\mathbf{A}_1$" are defined recursively in the definitions to follow.•

Df 3.2. 1) A statement in the IML of A_1 (A_1 and $\mathbf{A}_1$), according to which a valid relation of A_1 *is immediately inferred as conclusion* from other valid relations of A_1 as *premises*, is called an *inference*, or *transformation, rule of* A_1. In this case, the premises are said to *immediately infer*, or *immediately yield*, the conclusion, while the conclusion is said to *immediately follow* from the premises.

2) I use any grammatical form of the infinitive verb equivalent "*to immediately infer*" and its kindred substantive "*immediate inference*" in accordance with the first part of following definition by Church [1956, p. 49, n. 115] of the latter substantive in modern symbolic logic:

> «…We term the inferences *immediate* in the sense of requiring only one application of a rule of inference – not the traditional sense of (among other things) having only one premiss.»

At the same time, Church's reservation regarding the special *traditional sense of* "*immediate reference*" as «*having only one premiss*» contradicts the fact that any *traditional categorical* (*unconditional*) or *conditional* (*disjunctive* or *hypothetical*) *syllogism* ***immediately infers*** *its conclusion from its* ***two*** *premises* (*premisses*), whereas any *traditional dilemma* (*dilemmatic syllogism*) *immediately infers its conclusion from its* ***three*** *premises.* In this case, *modus ponendo ponens,* being a *traditional hypothetical syllogism,* is conventionally used as *the main primary rule of inference in all axiomatic systems of modern sentential* (*propositional*) *and predicate* (*functional*) *calculi*, so that its applications are *immediate inferences*, in accordance with the first part of Church's definition. Therefore, Church's definition as a single whole is *self-contradictory*. Still, Church's contradictory definition of the traditional sense of "immediate reference" is supported, e.g., by the following definition of WTNID:

> «**immediate inference** *n* **1 :** an inference drown from a single premise **2 :** the operation of drawing an inference from a single premise»

This semantic paradox can be solved by admitting that a syllogistic inference from two or three premises is, by that definition, not immediate. However, if one admits Church's thesis that an inference by *modus ponendo ponens* is immediate in symbolic logic then he must admit that a like inference in traditional logic is immediate as well. Incidentally, *modus ponendo ponens* is a theorem of A_1, and not a primary rule of inference either of A_1 or of $\mathbf{A}_1$.•

3) Besides "to *yield*", the verbs "to *deduce*" and "to *detach*" are two other synonyms of the verb "to *infer*". Therefore, the noun "inference" in either of the term "rule of inference" can be used interchangeably with either of the nouns "*deduction*" and "*detachment*", and also interchangeably with any of the binomials "*inference procedure*", "*deduction procedure*", and "*detachment procedure*".

4) A rule of inference or decision of A_1 is called a *primary one* or a *meta-axiom of inference* or *decision* if it is postulated (taken for granted) and a *secondary one* or a *meta-theorem of inference* or *decision* if it is deduced from some other *rules of inference* with the help of some intuitive rules substitutions, being meta-axioms as well.

5) The logical head, under which one or more *meta-axioms*, or *meta-theorems*, *of inference* or *decision* of A_1 are included, will begin with the string "****Ax**", or "****Th**", respectively.

6) With few exceptions that are done for more clarity, there are *no explicit rules of inference and decision of* $\mathbf{A}_1$. Subject theorems of $\mathbf{A}_1$ are deduced *implicitly* as panlogographic schemata of valid ER's of A_1 with the help of *explicit rules of inference and decision of* A_1.•

Df 3.3. 1) A finite sequence of one or more relations, euautographic, panlogographic, or both, is called a *proof of the last relation* in the sequence if each relation in the sequence either is an axiom or is immediately inferred from preceding relations in the sequence by means of one of the rules of inference (cf. Church [1956, p. 49]). The number of premises required for immediately making a single conclusion depends on the rule of inference used. A proof of a PLR of $\mathbf{A}_1$ is alternatively called a *panlogographic proof schema of ER's* A_1 in the sense that it condenses the proofs of all ER's of A_1 of the range of the PLR. A proof of an ER of A_1 will briefly be called a *proof of* A_1, whereas and a proof of a PLR of $\mathbf{A}_1$ will briefly be called a *proof of* $\mathbf{A}_1$ or a *panlogographic schema of proofs of* A_1 (cf. Df 3.1(6)).When necessary, a proof of A_1 or $\mathbf{A}_1$ can be supplemented by remarks in the IML of A_1 indicating the concrete primary rule of inference and concrete premises, which are used in any given act of inference in the proof.

2) The noun "*identity*" is a synonym of the name "*valid equality*" and therefore "*anti-identity*" is a synonym of "*antivalid equality.* Both identities and anti-identities can particularly be euautographic or panlogographic (and also some other, e.g. catlogographic or generally logographic), whereas a panlogographic identity or anti-identity is a schema of euautographic identities or anti-identities of its range, respectively.

3) A proof that consists exclusively of identities is called an *algebraic proof*.

4) The noun "*argument*" and also, in accordance with Df 3.2(3), any of the terms: "*inference procedure*", "*deduction procedure*", and "*detachment procedure*" can be used synonymously (interchangeably) with "*proof*".•

Df 3.4. 1) A *valid* ER of A_1 that has a proof is called a *subject theorem*, or briefly a *theorem*, *of* A_1 and also synecdochically a *euautographic theorem* (*ETh*). A [*subject*] *theorem of* A_1 is alternatively called an *object theorem of the IML of* A_1 *that belongs to* A_1.

2) A *valid* PLR of $\mathbf{A}_1$ that has a proof is called a *subject theorem*, or briefly a *theorem*, *of* $\mathbf{A}_1$ and also synecdochically a *panlogographic theorem* (*PLTh*). A [*subject*] *theorem of* $\mathbf{A}_1$ is alternatively called an *object axiom of the IML of* A_1 *that belongs to* $\mathbf{A}_1$.

3) The formats of formally laying down axioms of A_1 and $\mathbf{A}_1$, which have been described in the items 3–5 of Df 3.1, will apply also to theorems of A_1 and $\mathbf{A}_1$ with '**Th**', being an abbreviation of '**Theorem**', in place of '**Ax**'. In addition, the statement of a theorem of A_1 or $\mathbf{A}_1$ will as a rule be followed by its proof included under the local head "**Proof:**".

4) A concrete PLTh of $\mathbf{A}_1$ is alternatively called a *panlogographic schema of theorems of* A_1 in the sense that it is a *panlogographic schema* (*schematic placeholder*) condensing a large number (commonly an infinite number) of concrete but not concretized ETh s of A_1 in its range (cf. Df 3.1(6)).

5) There is *no metalogographic schema* (*schematic placeholder*) to condense a large number of concrete PLThs of $\mathbf{A}_1$ in a single metalogographic statement (cf. Df 3.1(7)).

6) The generic name "*theorem of* A_1" is a synonym of, and the name "*secondary valid relation of* A_1", and the generic name "*theorem of* $\mathbf{A}_1$" is a synonym of the name "*secondary valid relation of* $\mathbf{A}_1$". Consequently, every theorem of A_1 is a *valid relation of* A_1, while every valid relation of A_1 is either an axiom of A_1 or a theorem of A_1. Likewise, every axiom of $\mathbf{A}_1$ is a *valid relation of* $\mathbf{A}_1$, while every valid relation of $\mathbf{A}_1$ is either an axiom of $\mathbf{A}_1$ or a theorem of $\mathbf{A}_1$ (cf. Df 3.1(7)).

7) An axiom or theorem of A_1 or $\mathbf{A}_1$ that has the form of an *identity* relative to the sign $\hat{=}$ is called a [*special*] *algebraic*, or *egalitarian*, one. An axiom or theorem of A_1 or $\mathbf{A}_1$, whose principal kernel sign is other than $\hat{=}$ is called a *logical* one.

8) The negation of an axiom or of a theorem of A_1 or $\mathbf{A}_1$ is an *antivalid relation* of A_1 or $\mathbf{A}_1$, which is called an anti-axiom or an anti-theorem of A_1 or $\mathbf{A}_1$, respectively. Consequently, the negation of an anti-axiom or of an anti-theorem of A_1 or $\mathbf{A}_1$ is an axiom or a theorem of A_1 or $\mathbf{A}_1$, respectively. It is understood that, unlike an axiom and a theorem, neither an anti-axiom nor an anti-theorem can be asserted (used assertively).

9) When convenient, the expression "*theorem sensu lato*", i.e. "theorem in a broad sense", can indiscriminately be used as a common name of both an axiom and theorem of A_1 or $\mathbf{A}_1$, the understanding being that an axiom is a theorem whose proof consists of that same axiom. In this case, the term "*theorem*", introduced above in the items 1 and 2 of this definition, should be understood as an abbreviation of the expression "*theorem sensu stricto*", i.e. "theorem in a narrow sense".•

Df 3.5. 1) According to Df I.3.1(2,3), some algebraic theorems *of* A_1 are called *master*, or *decision*, *theorems* (*MT's* or *DT's*) *of* A_1 or synecdochically *euautographic MT's* (*EMT's*) or *euautographic DT's* (*EDT's*). Similarly, according to Df I.4.3(6), some algebraic theorems of $\mathbf{A}_1$ are called *MT's*, or *DT's*, *of* $\mathbf{A}_1$ or synecdochically *panlogographic MT's* (*PLMT's*) or *panlogographic DT's* (*PLDT's*). A PLMT (PLDT) is a *panlogographic schema* (*schematic placeholder*) condensing a large number (commonly an infinite number) of concrete but not concretized EMT's (EDT's). An EMT (EDT) of A_1 is an EMT (EDT) of a certain *euautographic slave-relation* (*ESR*) *of* A_1, so that, in accordance with the syntactic form of the EMT, the ESR is unambiguously classified as a *valid* one or an *antivalid* one, or else as a *vav-neutral* (*vav-indeterminate*) one, i.e. as *neither valid nor antivalid.* Similarly, a PLMT (PLDT) of $\mathbf{A}_1$ is a PLMT (PLDT) of a certain *panlogographic slave-relation* (*PLSR*) *of* $\mathbf{A}_1$, so that, in accordance with the syntactic form of the PLMT, the PLST is unambiguously classified as a *valid* one or an *antivalid* one, or else as a *vav-neutral* (*vav-indeterminate*) one, i.e. as *neither valid nor antivalid.* Consequently, an algebraic proof of an MT (DT) is alternatively called an *algebraic decision procedure* (*ADP*) *for the slave-relation* (*SR*) *of the MT* (*DT*). An ADP is called a *euautographic ADP* (*EADP*) if it applies to an ESR and a *panlogographic ADP* (*PLADP*) if it applies to a PLSR.

2) A PLSR of $\mathbf{A}_1$ is *valid*, or *antivalid*, if and only if *every ESR of* A_1 *of its range*, i.e. *every particular* (*concrete*) *euautographic instance* (*denotatum*) is *valid*, or *antivalid*, respectively.

3) In accordance with Df 3.5(1), a PLSR of $\mathbf{A}_1$ is a *vav-neutral* (*vav-indeterminate*) one if and only if there is an EMT (EMD) of the PLSR, according to which the PLSR is neither valid nor antivalid. A vav-neutral PLSR of $\mathbf{A}_1$ is called

a) a *structural*, or *skeletal*, *one* (*StPLSR* or *SkPLSR*) if and only if *every ESR of* A_1 *of its range is a vav-neutral one*,

b) an *analytical one* (*AnPLSR*) if and only if its range comprises *ESR's of* A_1 *of* all the three classes: valid, antivalid, and vav-neutral.

4) In accordance with Df 3.4(8), the negation of the MST (MDT) of a certain SR is the *master*, or *decision*, *anti-theorem* (*MAT* or *DAT*) *of the same SR* and at the same time it is the *MT* (*DT*) *of the negation of the SR*. Consequently*, the negation of a valid SR is an antivalid SR and vice versa*, whereas *the negation of a vav-neutral* (*vav-indeterminate*) *SR is another vav-neutral* (*vav-indeterminate*) *SR*. A valid SR is alternatively (synonymously) called a *slave-theorem* (*ST* or *STh*), whereas an antivalid SR is alternatively called a *slave-antitheorem* (*SAT* or *SATh*).

5) An algebraic or logical theorem and particularly an MT (DT) or ST, of A_1 or $\mathbf{A}_1$ is indiscriminately called a *theorem* of A_1 or $\mathbf{A}_1$, whereas the negation of a theorem of A_1 or $\mathbf{A}_1$ is indiscriminately called an *antitheorem* of A_1 or $\mathbf{A}_1$, respectively.

6) A relation of A_1 or $\mathbf{A}_1$ is said to be valid if and only if it is either an axiom or a theorem of A_1 or $\mathbf{A}_1$ respectively.

7) *The negation of a valid relation* of A_1 or $\mathbf{A}_1$ *is an antivalid relation and vice versa*. Consequently, a relation of A_1 or of $\mathbf{A}_1$ is antivalid if and only if its negation is valid and vice versa.

8) A relation *of* A_1 is said to be a *vav-neutral* (*vav-indeterminate*) *relation of* A_1 if and only if it is either the vav-neutral (vav-indeterminate) slave-relation (SR) of a concrete EMT (EDT) of A_1 or a euautographic instance of the range of the vav-neutral StPLSR (SkPLSR) of a certain PLMR (PLDR) of $\mathbf{A}_1$.

9) A relation *of* $\mathbf{A}_1$ is said to be a *vav-neutral* (*vav-indeterminate*) one if and only if it is the vav-neutral (vav-indeterminate) slave-relation (SR) of a *concrete* PLMT (PLDT) of $\mathbf{A}_1$.•

Df 3.6. 1) A euautographic or panlogographic *valid*, *antivalid*, or *vav-neutral* (*vav-indeterminate*) *relation* is alternatively (synonymously) called a euautographic or panlogographic *kyrology*, *antikyrology*, or *vav-udeterology* (*vav-anorismenology*) respectively.

2) An ER (particularly an ESR) of A_1 or a PLR (particularly a PLSR) of $\mathbf{A}_1$ is said to be

a) an *invalid one* or an *akyrology* if an only if it is either antivalid or vav-neutral,

b) a *vav-unneutral* (*vav-determinate*) *one* or an *anudeterology* (*orismenology*) if and only if it is either valid or antivalid,

c) a *non-antivalid one* or an *anantikyrology* if and only if it is either valid or *vav-neutral* (*vav-indeterminate*).•

Cmt 3.1. A_1 is a logistic system which is formalized explicitly. Since A_1 is the euautographic interpretand (intrinsic image) and inseparable part of $\mathbf{A}_1$ therefore the latter is also formalized, although implicitly, via A_1. Consequently, as long as I deal explicitly with A_1 and its branches (or with A_0 and A_1^0 being parts but not branches of A_1) and implicitly with $\mathbf{A}_1$ and its respective branches (or with $\mathbf{A}_0$ and $\mathbf{A}_1^0$ being parts but not branches of $\mathbf{A}_1$), the meanings of the words or word groups: "valid", "antivalid", "neutral" (or "indeterminate"), "invalid", "unneutral" (or "determinate"), "non-antivalid", "premise", "conclusion", "infer" ("yield"), "immediately infer", "axiom", "theorem", etc are those bestowed upon them by the pertinent definitions of this section, and also by the actual subject axioms and actual meta-axioms of inference and decision, which will be laid down in the next section. In this case, the criteria for an ER of A_1 or PLR of $\mathbf{A}_1$ to be valid, antivalid, vav-neutral (indeterminate), invalid, unneutral (determinate), or non-antivalid can be formulated syntactically and hence with complete rigor. By contrast, the entire IML (inclusive metalanguage) *of* $\mathcal{A}_1$ (A_1 and $\mathbf{A}_1$) includes a certain part of the English language. Therefore, no rigorous criteria can be elaborated for relations (statements, sentences or sequences of sentences) of the IML of $\mathcal{A}_1$ to be valid or invalid (e.g.) if these relations involve English words or expressions that are subject to informal dictionary definitions. Consequently, the inference meta-axioms and meta-theorems, belonging to the IML, which will be stated in the sequel, are qualified *veracious*, i.e *untautologously*, or *accidentally*, *true*. The mata-axioms are *postulated* (*taken for granted*) to be veracious, while the meta-theorems are inferred (proved) so be so with the help of certain intuitive veracious rules of substitution, being veracious meta-axioms as well.•

3.2. Metalogographic predicates

Df 3.7. The characters '$\vdash$', '$\dashv$', and '$\vdash\!\dashv$' are *metalogographic predicates*, i.e. *logographic predicates belonging to the metalanguage*, which are put before a *vavn-decided ER* (*DdER*) *of* A_1 or before a *vavn-decided PLR* (*DdPLR*) *of* $\mathbf{A}_1$ as the subject and which are, e.g. in the former case, defined thus:

$$\vdash \mathbf{P}_* \to [\mathbf{P}_* \text{ is valid}], \tag{3.1}$$

$$\dashv \mathbf{P}_* \to [\mathbf{P}_* \text{ is antivalid}], \tag{3.2}$$

$$\vdash\!\dashv \mathbf{P}_* \to [\mathbf{P}_* \text{ is vav-neutral}], \tag{3.3}$$

where '$\mathbf{P}_*$' is an AnAtPLR (analytical atomic PLR) of $\mathbf{A}_1$, whose range is the class of DdER's of A_1. "*Indeterminate*" can be used interchangeably with "neutral". The characters '$\vdash$', '$\dashv$', and '$\vdash\!\dashv$' are called the *validity* (or *validness*), *antivalidity* (or *antivalidness*), and *vav-neutrality* (or *vav-indeterminacy*) *predicate-signs* respectively. '$\mathbf{P}_+$', '$\mathbf{P}_-$', or '$\mathbf{P}_\sim$' is an AnAtPCLR whose range is the class of tautologous, antitautologous, or ttatt-neutral CFCLR's respectively, so that $\vdash \mathbf{P}_+$, $\dashv \mathbf{P}_-$, and $\vdash\!\dashv \mathbf{P}_\sim$. Hence, the variants of definitions (3.1)–(3.3) with '$\mathbf{P}_+$', '$\mathbf{P}_-$', and '$\mathbf{P}_\sim$' respectively in place of '$\mathbf{P}_*$' are *tautologies*.•

Cmt 3.2. In accordance with Df 3.7, the signs '$\vdash$', '$\dashv$', and '$\vdash\!\dashv$' are abbreviations of the predicates "is valid", "is antivalid", and "is van-neutral', each of which is put before the appropriate subject to form the corresponding abbreviated simple affirmative declarative sentence or clause belonging to the metalanguage. The sign '$\vdash$' is widely used in writing on symbolic logic in various meanings. Still, all those meanings differ somewhat from the meaning, which this sign has in this treatise. The signs '$\dashv$' and '$\vdash\!\dashv$' are suggestions of my own – they are not in common usage. In accordance with Df 3.5(7),

$$\dashv \mathbf{P}_* \text{ if and only if } \vdash \neg \mathbf{P}_*. \tag{3.4}$$

At the same time, by Dfs I.3.1(23) and 3.5(8,9), to any vav-neutral relation $\mathbf{P}_*$ there is an MT (DT) $\mathsf{T}_{1\sim}(\mathbf{P}_*)$, i.e. a valid relation, proving the vav-neutrality of the former. Hence,

$$\vdash\!\dashv \mathbf{P}_* \text{ if and only if } \vdash \mathsf{T}_{1\sim}(\mathbf{P}_*). \tag{9.5}$$

Therefore, the signs '$\dashv$' and '$\vdash\!\dashv$' are, in fact, redundant. I have introduced them in order to have them available if needed.

Here follow brief remarks regarding the historical origin of the sign '├', and also regarding its acceptation (presently common meaning), which is different from the meaning that the sign has in this treatise.

In order to distinguish between the unassertive and assertive uses of a sentence, Frege [1879] prefixed the sentence with a horizontal line "—" in the former case, and by the character "├—" in the latter case. A version of the Fregean *assertion sign* "├—" with a somewhat shorter horizontal line, "├", was borrowed from Frege by Russell [1903] and by Whitehead and Russell [1927, pp.8, 9ff], but these writers did not used the Fregean *non-assertion sign* "—". Since then the sign "├" has been widely used in writing on logic with various distinctions, but mainly, it was used for prefixing both the axioms of a logical calculus and the theorems, which are derivable from the axioms by the pertinent rules of inference, especially by the rule of *modus ponendo ponens* and by a *rule,* or *rules, of substitution*. In the latter use, the sign "├" can, under a convention analogous to Df 3.4(9), be called *the provability,* or *theoremhood, sign*. In this mode, the sign "├" is particularly used in Quine [1951, pp. 88,161ff], Church [1956, n. 65, § 12], and Kleene [1967, §§3,9-11ff]. In order to indicate that a *formula*, i.e. a *relation* or *relation-formula* in the terminology of this treatise, is *valid*, Kleene prefixes the formula with a token of the sign "⊨", other than "├". Afterwards, he proves his Theorems 12 and 14 (*ibid*, §11) saying: "*If ├E then ⊨E*" and "*If ⊨E then ├E*", respectively; that is, *a formula is provable if and only if it is valid*.

Owing to the effective EAADM (euautographic advanced algebraic decision method), being an integral part of A_1, I assume from the very beginning that, by Df 3.5(6), *a relation of* A_1 *is valid if and only if it is either an axiom or a theorem*, i.e. *if and only if it is either taken for granted to be valid or is proved* (*demonstrated*) *to be valid by the pertinent EADP*. Therefore, there is no need in two different signs '├' and '⊨' in the IML of A_1. A like remark applies, *mutatis mutandis*, with "PLAADM" in place of "EAADM" and 'A_1' in place of '$\mathbf{A}_1$'.

Typical non-trivial uses of the signs '├', '┤', and '┼' can be illustrated as follows. Unless stated otherwise, the range of the AnAtPLR (analytical atomic panlogographic relaion) '**P**' of $\mathbf{A}_1$ is the class of all ER's of A_1. Therefore, when used xenonymously and assertively for mentioning a general (certain, concrete but not concretized) ER of A_1, the sentence (clause) '├**P**', '┤**P**', or '┼**P**' is a condition restricting the range of '**P**' only to valid, antivalid, or vav-neutral ER's respectively. Accordingly, I shall use the sign '$\vdash$', e.g., mainly before *vav-neutral PLR's* occurring as substantivized subjects or objects in conditional sentences of the IML of A_1 in order to indicate that the range of a vav-neutral AnPLR is restricted only to its *valid* euautographic instances. Particularly, such sentences are extensively used in stating rules of inference and decision of A_1. To say nothing of the signs '$\dashv$' and '$+$', the sign '$\vdash$' is an auxiliary one. Therefore, it will be used only if its use is advisable. The special formats of stating valid relations of A_1 and $\mathbf{A}_1$, which have been specified in Dfs 3.1(3,4) and 3.4(3), are themselves effective indications of the validity of the relations stated. In this case, the sign '$\vdash$' would be redundant. In general, the signs '$\vdash$', '$\dashv$', and '$+$' will be used only in cases, in which there might otherwise be either doubt in the meaning or need in a prolix explanatory context.

It should be especially emphasized that the signs '$\vdash$', '$\dashv$', and '$+$' do not belong to the atomic bases of A_1 and $\mathbf{A}_1$. Therefore, in contrast to the expressions such as '$\vdash[V(\mathbf{P})\hat{=}0]$' or '$\vdash[V(\mathbf{P})\hat{=}1]$', which will be used in the sequel for mentioning general ER's that satisfy the respective conditions, the expressions such as '$V(\vdash\mathbf{P})$' or '$V(\dashv\mathbf{P})$' and hence such as '$V(\vdash\mathbf{P})\hat{=}0$' or '$V(\dashv\mathbf{P})\hat{=}0$' are meaningless (unacceptable, unusable).

In what follows, I shall introduce some other metalogographic predicates, which may be useful in various interpretations of *OptER's* (*output ER's*) *of* A_1.•

Df 3.8. The characters '$\Vdash$', '$\dashv\!\dashv$', and '$+\!\!+$' are metalogographic predicates, which are put, e.g., before a *vavn-decided conformal catlogographic relation* (*DdCFCLR*) *of* I_1 as the subject and which are defined thus:

$$\Vdash \boldsymbol{P}_* \rightarrow [\,\boldsymbol{P}_* \text{ is tautologous}], \tag{3.6}$$

$$\dashv\!\dashv \boldsymbol{P}_* \rightarrow [\,\boldsymbol{P}_* \text{ is antitautologous}], \tag{3.7}$$

$$+\!\!+ \boldsymbol{P}_* \rightarrow [\,\boldsymbol{P}_* \text{ is ttatt-neutral}], \tag{3.8}$$

where '$\boldsymbol{P}_*$' is an *AnAtPCtLR* (*analytical atomic pancatlogographic relation*), whose range is the class of DdCFCLR's. "*Tautologic*" and "*universally true*" can be used interchangeably with "tautologous"; "*antitautologic*", "*contradictory*", "*universally antitrue*", and "*universally false*" can be used interchangeably with "antitautologous"; as before, "*indeterminate*" can be used interchangeably with "*neutral*". The characters $\Vdash$', '$\dashv\!\dashv$', and '$+\!\!+$' are called the *tautologousness, antitautologousness* (or "*contradictoriness*)", and *ttatt-neutrality* (or *ttatt-indeterminacy*) *predicate-signs* respectively. '$\boldsymbol{P}_{+}$', '$\boldsymbol{P}_{-}$', or '$\boldsymbol{P}_{\sim}$' is an AnAtPCtLR whose range is the class of tautologous, antitautologous, or ttatt-neutral CFCLR's respectively, so that ╟$\boldsymbol{P}_{+}$, ╢$\boldsymbol{P}_{-}$, and ╫$\boldsymbol{P}_{\sim}$. Hence, the variants of definitions (3.6)–(3.8) with '$\boldsymbol{P}_{+}$', '$\boldsymbol{P}_{-}$', and '$\boldsymbol{P}_{\sim}$' respectively in place of '$\boldsymbol{P}_{*}$' are *tautologies*.•

Df 3.9. The characters '╞', '╡', and '╪' metalogographic predicates, which are put, e.g., before a ttatt-neutral CFCLR $\boldsymbol{P}_{\sim}$ and which are defined thus:

$$╞\boldsymbol{P}_{\sim} \rightarrow [\boldsymbol{P}_{\sim} \text{ is veracious}], \tag{3.9}$$

$$╡\boldsymbol{P}_{\sim} \rightarrow [\boldsymbol{P}_{\sim} \text{ is antiveracious}], \tag{3.10}$$

$$╪\boldsymbol{P}_{\sim} \rightarrow [\boldsymbol{P}_{\sim} \text{ is vravr-neutral}]. \tag{3.11}$$

"*Accidentally true*" and "*untautologously true*" can be used interchangeably with "veracious"; "*accidentally antitrue*", "*non-universally antitrue*", and "*accidentally false*" can be used interchangeably with "antiveracious"; as before, "*indeterminate*" can be used interchangeably with "*neutral*". The characters '╞', '╡', and '╪' are called the *veracity*, *antiveracity*, and *vravr-neutrality* (*vravr-indeterminacy*) predicate-signs respectively. By definition, '$\boldsymbol{P}_{\sim *}$' is an AnAtPCtLR whose range is the class of vravrn-decided ttatt–neutral CFCLR's, whereas '$\boldsymbol{P}_{\sim +}$', '$\boldsymbol{P}_{\sim -}$', or '$\boldsymbol{P}_{\sim\sim}$' is an AnAtPCtLR whose range is the class of veracious, antiveracious, or vravr-neutral CFCLR's respectively, so that ╞$\boldsymbol{P}_{\sim +}$, ╡$\boldsymbol{P}_{\sim -}$, and ╪$\boldsymbol{P}_{\sim\sim}$. Hence, the definitions (3.9)–(3.11) can be restated with '$\boldsymbol{P}_{\sim *}$' in place of '$\boldsymbol{P}_{\sim}$', whereas the variants of those definitions with '$\boldsymbol{P}_{\sim +}$', '$\boldsymbol{P}_{\sim -}$', and '$\boldsymbol{P}_{\sim\sim}$' respectively in place of '$\boldsymbol{P}_{\sim}$' are *tautologies*.•

Df 3.10. Let '$\boldsymbol{P}^{*}$' be a *metalogographic placeholder* (*MLPH*) whose range is the class of *tatn-decided* CFCLR's, whereas '$\boldsymbol{P}^{+}$', '$\boldsymbol{P}^{-}$', or '$\boldsymbol{P}^{\sim}$' is an MLPH whose range is the class of *true*, *antitrue* (*false*), or *tat-neutral* (*tat-indeterminate*) *CFCLR's* respectively. That is to say, the range of '$\boldsymbol{P}^{*}$' is the union of the ranges of '$\boldsymbol{P}^{+}$', '$\boldsymbol{P}^{-}$', or '$\boldsymbol{P}^{\sim}$'; the range of '$\boldsymbol{P}^{+}$' is the union of the ranges of '$\boldsymbol{P}_{+}$' and '$\boldsymbol{P}_{\sim +}$' so that "true" means *tautologous* (*universally true*) *or veracious* (*accidentally true*); the range of '$\boldsymbol{P}^{-}$' is the union of the ranges of '$\boldsymbol{P}_{-}$' and '$\boldsymbol{P}_{\sim -}$' so that "antitrue" ("false") means *antitautologous* (*contradictory*, *universally antitrue*) *or antiveracious* (*accidentally antitrue*, *accidentally false*); the range of '$\boldsymbol{P}^{\sim}$' is the same as the range of '$\boldsymbol{P}_{\sim\sim}$' so that “tat-neutral” (“tat-indeterminate”) means *vravr-neutral* (“*vravr-indeterminate*”) and vice versa. The characters ‘╠’, ‘╣’, and ‘╬’ are metalogographic predicates, which can be defined, e.g., thus:

╠ $\boldsymbol{P}^{*} \to [\,\boldsymbol{P}^{*}$ is true], (3.12)

╣ $\boldsymbol{P}^{*} \to [\,\boldsymbol{P}^{*}$ is antitrue], (3.13)

╬ $\boldsymbol{P}^{*} \to [\,\boldsymbol{P}^{*}$ is tat-neutral], (3.14)

so that ╠ $\boldsymbol{P}^{+}$, ╣ $\boldsymbol{P}^{-}$, and ╬ $\boldsymbol{P}^{\sim}$. Hence, the variants of the definitions (3.12)–(3.14) with ‘ $\boldsymbol{P}^{+}$ ’, ‘ $\boldsymbol{P}^{-}$ ’, and ‘ $\boldsymbol{P}^{\sim}$ ’ respectively in place of ‘ $\boldsymbol{P}^{*}$ ’ are *tautologies*.

4. The subject axioms of A_1 and the meta-axioms (primary rules) of inference and decision of A_1

4.1. Preliminaries

1) In accordance with the relevant nomenclature, which has been introduced in Df I.4.3, especially in its item 2, the above head of this section is a current synonym of the logograph ‘D_1’, whereas D_1 is alternatively called the *Advance Algebraic Decision Method* (*AADM*) *of* A_1 and also the *Endosemasiopasigraphic* (*EnSPG*), or *Biune Euautographic and Panlogographic* (*BUE&PL*), *AADM* (briefly *EnSPGAADM* or *BUE&PLAADM*). D_1 is the union (conjunction) and superposition of D_1 and $\boldsymbol{\mathsf{D}}_1$, which are respectively called the *AADM of* A_1 and the *AADM of* $\boldsymbol{\mathsf{A}}_1$, and also the *Euautographic AADM* (*EAADM*) and the *Panlogographic AADM* (*PLAADM*).

2) D_1 is explicitly set up in the next subsection, while $\boldsymbol{\mathsf{D}}_1$ is implicitly specified as a by-side product of the setup of D_1[, which is, fore more clarity, supplemented by the explicit basic rule of inference and decision of $\boldsymbol{\mathsf{A}}_1$, belonging exclusively to $\boldsymbol{\mathsf{D}}_1$]. Every item of D_1 is included, alone or together with some conjoined conspecific items, under a logical head, which contains the abbreviation “Ax” for “Axiom” and one of the prepositive superscripts °, *, and **, whose meanings have been explained in Dfs 3.1(1–4) and 3.2(5). Particularly, Ax 4.5 is the only *concrete* [subject] axiom of A_1 and therefore it is the only one that is marked with the flag °. This axiom is also a one of $\boldsymbol{\mathsf{A}}_1$.

3) Every *special identity* (*valid special equality*), i.e. every *identity relative to the sign* $\hat{=}$, that is stated below, alone or together with some conjoined conspecific special identities, under a logical head containing the string “*Ax” is a

panlogographic equality, i.e. *egalitarian PLR* (*panlogographic relation*), *of* $\mathbf{A}_1$, which is *taken for granted to be valid*. In setting up A_1, every one of these identities is used *xenonymously* as a *panlogographic schema* (*PLS*, pl. "*PLS'ta*"), i.e. as a *schematic* (*patterned*) *panlogographic placeholder* (*SchPLPH*, pl. "*SchPLPH's*") that condenses in its range an infinite number of similarly patterned *euautographic identities of* A_1 as *subject*, or *intrinsic*, *euautographic axioms* (*EAxs*). Therefore, depending on my mental attitude towards a panlogographic identity, I call it either a *PLS of EAxs of* A_1 or a *concrete subject*, or *intrinsic*, *panlogographic axiom* (*PLAx*) *of* $\mathbf{A}_1$. This biune terminology reflects the mental phenomenon, which I may experience in perceiving a panlogograph and which I have described in Df I.4.1(7) as *TAEXA* (*tychautograph-euxenograph alternation* or briefly *tychauto-euxenograph alternation*). I recall that TAEXA is the mental phenomenon of intermittently using a panlogograph in two alternating ways (mental modes)*: xenonymously*, i.e. as a *eulogograph*, for mentioning a *general* (*common*, *certain*, *concrete but not concretized*) *euautograph of its range*, which is just another *hypostasis* (*way of existence*) *of the range*, and *autonymously*, i.e. as a *tychautograph*, for mentioning either itself or its homolographic (photographic) token-class or else its some concrete homolographic (photographic) token or tokens that are indicated by the pertinent context (added words). In accordance with Df I.4.1(8), when I use a panlogograph (e.g) *autonymously* and *xenonymously intermittently but as if simultaneously* – briefly, *autoxenonymously* or *xenoautonymously*, I say that I use the panlogograph in the *autoxenonymous*, or *xenoautonymous*, *mental mode*, or alternatively in the *TAEXA-mode*. Consequently, in accordance with Df I.4.3(3), when I use a PLAx xenoautonymously (autoxenonymously, in the TAEXA-mode), I call it an *endosemasiopasigraphic* [*subject*] *axiom* (*EnSPGAx*) *of* A_1. In this connection, it will be recalled that, in accordance with Df I.4.1(1), A_1 is called the *Comprehensive Endosemasiopasigraphic*, or *Comprehensive Biune Euautographic and Panlogographic*, *Algebraico-Predicate Organon APO* (briefly *CEnSPGAPO* or *CBUE&PLAPO*).. Every subject axiom of A_1, distinguished by the string "*Ax" in its logical head, is stated exclusively in terms of atomic euautographs of A_1 and atomic panlogographs of $\mathbf{A}_1$. Any words of the XML (exclusive metalanguage) of A_1, which may occur in the statement of such an axiom, do not belong to the body of the axiom; they are auxiliary (explanatory) ones that they can be stated separately. Therefore, the schematic subject axioms of A_1 will also be called the *intrinsic axioms* (or *intrinsic primary premises*) *of* A_1, i.e. of A_1 or $\mathbf{A}_1$ or both.

4) The *subject axioms of* A_1 are divided into two groups, one of which being called the *basic*, or *elementary*, *subject axioms of* A_1 and the other one the *advanced subject axioms of* A_1. The basic axioms are concerned with non-contracted ER's, whereas the advanced axioms are concerned with contracted ER's.

5) In contrast to the [subject] axioms of A_1 that are marked with the flag *, all statements, which are made below under the logical heads containing the string "**Ax", are meta-axioms, i.e. statements in the IML (inclusive metalanguage) of A_1, which are taken for granted to be *veracious* (*accidentally*, or *untautologously*, *true*) and which are collectively called the *meta-axioms,* or *extrinsic primary rules, of inference and decision of* A_1. To be specific, the very last meta-axiom of this group, Ax 4.20, is a *conjoined three-fold meta-axiom of* A_1, the first conjunct of which is *the basic rule of decision with respect to the validity-value validity* and at the same time it is *a rule of algebraic inference,* while the two other conjuncts are *the basic rules of decision with respect to the validity-values antivalidity and vav-neutrality* (*vav-indeterminacy*) respectively. Except Ax 4.20, all other meta-axioms of this group, namely Axs 4.13–4.19, are the *ones of algebraic inference of* A_1.

6) In accordance with Dfs 3.1(6,7) and 3.2(6), there is no need in explicit meta-axioms of inference and decision of $\mathbf{A}_1$, because the appropriate instances (substituends) of the rules of inference of A_1 can be and most often are applied *semantically* (*xenonymously*) to *general* valid ER's of A_1 of the ranges of the pertinent valid PLR's of $\mathbf{A}_1$ as *general euautographic premises or conclusions of* A_1 and at the same time those same inference rule instances are applied *syntactically* (*autonymously*) to those same PLR's as *panlogographic premises or conclusions of* $\mathbf{A}_1$. That is to say, a proof of a theorem is most often done xenoautonymously (autoxenonymously), i.e. in the TAEXA-mode. Particularly, if the proof in question is regarded xenonymously as one of *a panlogographic master*, or *decision*, *theorem-schema* (*PLMTS* or *PLDTS*) for a certain *panlogographic slave-relation schema* (*PLSRS*) *of* A_1, i.e. as a proof of *a panlogographic schema of an infinite number of euautographic master* (*decision*) *theorems of* A_1*, for the respective euautographic slave-relations of* A_1, then that same proof can be regarded autonymously as one of *the pertinent concrete master*, or *decision*, *theorem* (*PLMT* or *PLDT*) *of* $\mathbf{A}_1$ *for the*

pertinent concrete panlogographic slave-relation schema (*PLSRS*) *of* $\boldsymbol{\mathsf{A}}_1$. Under the latter mental attitude, the appropriate instances of the pertinent *panlogographic axiom-schemata* (*PLAxS'ta*) and of the pertinent rules of inference and decision, including Ax 4.20, are used autonymously (to be explained). When D_1 is used autonymously as indicated above, it *is extended from* A_1 *to* $\boldsymbol{\mathsf{A}}_1$. The autonymous version of D_1 is denoted by '$\boldsymbol{\mathsf{D}}_1$' and is called the *Autonymous*, or *Panlogographic*, *Extension of the EAADM*, and also the AADM of $\boldsymbol{\mathsf{A}}_1$ or the Panlogographic AADM, as it was called previously. The union and superposition of D_1 and $\boldsymbol{\mathsf{D}}_1$ is denoted by 'D_1'.

4.2. Subject axioms of A_1

4.2.1. Basic (elementary) subject axioms of A_1

Ax 4.1:** $\hat{\vee}$-Axiom – Algebraization Law of the Universal Logical Connective (ALULC).***

$$V(\mathbf{P}\hat{\vee}\mathbf{Q})\hat{=}1\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q}). \tag{4.1}$$

Ax 4.2:** ***Basic Idempotent Law (BIL).

$$V(\mathbf{P})\hat{\cdot}V(\mathbf{P})\hat{=}V(\mathbf{P}). \tag{4.2}$$

Ax 4.3:** ***Law of Initial Validity-Integrons (IVIL).

$$V(V(\mathbf{P})\hat{=}V(\mathbf{Q}))\hat{=}[V(\mathbf{P})\hat{-}V(\mathbf{Q})]^2\overset{\to}{\hat{=}}[V(\mathbf{P})\hat{-}V(\mathbf{Q})]\hat{\cdot}[V(\mathbf{P})\hat{-}V(\mathbf{Q})]. \tag{4.3}$$

Ax 4.4:** $\hat{=}$-Axioms – Laws of the Special Equality Sign.***

$$\mathbf{I}\hat{=}\mathbf{I}. \qquad (\textit{Reflexive Law}) \tag{4.4}$$

$$V(\mathbf{I}\hat{=}\mathbf{J})\hat{=}V(\mathbf{J}\hat{=}\mathbf{I}). \qquad (\textit{Symmetry Law}) \tag{4.5}$$

$$V([[\mathbf{I}\hat{=}\mathbf{J}]\wedge[\mathbf{J}\hat{=}\mathbf{K}]]\Rightarrow[\mathbf{I}\hat{=}\mathbf{K}])\hat{=}0. \qquad (\textit{Transitivity Law}) \tag{4.6}$$

°**Ax 4.5:** ***Law of Non-Triviality of*** A_1 ***(NTL).***

$$\neg[0\hat{=}1]. \tag{4.7}$$

†**Crl 4.1:** ***Closure Law for*** $\hat{+}$ ***and*** $\hat{\cdot}$ ***in*** A_1**.**

$$[\mathbf{I}\hat{+}\mathbf{J}] \text{ and } [\mathbf{I}\hat{\cdot}\mathbf{J}] \text{ are integrons if } \mathbf{I} \text{ and } \mathbf{J} \text{ are integrons.} \tag{4.8}$$

Proof: The corollary is the conjunction of the items 7 and 8 of Th 1.1. Unlike any subject axiom, the corollary is a *veracious statement of the IML*, in which the informal interpreted metalinguistic sentential connectives "and" and "if" and predicate "are integrons" are used along with the ordinary atomic euautographs [and] and special atomic euautographs $\hat{+}$ and $\hat{\cdot}$, and also and along the AtPLPH's '**I**' and '**J**'. According to its wording, the corollary is a meta-theorem, and therefore it should have been put in subsection 4.3. It has been put here because it underlies the following axiom•

Ax 4.6:** ***Axioms for the Termal (Substantival) Special Algebraic Kernel-Signs Relative to $\hat{=}$**.**

$\mathbf{I} \hat{+} \mathbf{J} \hat{=} \mathbf{J} \hat{+} \mathbf{I}$. (*Commutative Law for Addition*) (4.9)

$\mathbf{I} \hat{+} [\mathbf{J} \hat{+} \mathbf{K}] \hat{=} [\mathbf{I} \hat{+} \mathbf{J}] \hat{+} \mathbf{K}$. (*Associative Law for Addition*) (4.10)

$0 \hat{+} \mathbf{I} \hat{=} \mathbf{I}$. (*Law of the Zero-Integron*) (4.11)

$\mathbf{I} \hat{+} [\hat{-} \mathbf{I}] \hat{=} 0$. (*Law of the Additive Inverse*) (4.12)

$\mathbf{I} \hat{\cdot} \mathbf{J} \hat{=} \mathbf{J} \hat{\cdot} \mathbf{I}$. (*Commutative Law for Multiplication*) (4.13)

$\mathbf{I} \hat{\cdot} [\mathbf{J} \hat{\cdot} \mathbf{K}] \hat{=} [\mathbf{I} \hat{\cdot} \mathbf{J}] \hat{\cdot} \mathbf{K}$. (*Associative Law for Multiplication*) (4.14)

$1 \hat{\cdot} \mathbf{I} \hat{=} \mathbf{I}$. (*Law of the Unity-Integron*) (4.15)

$\mathbf{I} \hat{\cdot} [\mathbf{J} \hat{+} \mathbf{K}] \hat{=} [\mathbf{I} \hat{\cdot} \mathbf{J}] \hat{+} [\mathbf{I} \hat{\cdot} \mathbf{K}]$. (*Distributive Law for* $\hat{\cdot}$ *over* $\hat{+}$) (4.16)

Cmt 4.1. Given any natural number *n*>3 of EI's $\mathbf{I}_1$, $\mathbf{I}_2$, ..., $\mathbf{I}_n$, it can be proved from axioms (4.10) and (4.14) that any two EI's resulted by two different arrangement of *n*–2 pairs of brackets [ ] either in the string $\mathbf{I}_1 \hat{+} \mathbf{I}_2 \hat{+} ... \hat{+} \mathbf{I}_n$ or in the string $\mathbf{I}_1 \hat{\cdot} \mathbf{I}_2 \hat{\cdot} ... \hat{\cdot} \mathbf{I}_n$ are related by the sign $\hat{=}$. The above statement is a *meta-theorem* that is called the *generalized associative laws for the operators* $[\ \hat{+}\ ]$ *and* $[\ \hat{\cdot}\ ]$. For a sufficiently small *concrete* value of '*n*', say 4 or 5, the pertinent instance of either of the two generalized associative laws can be verified straightforwardly. However, in order to prove the generalized associative laws in the general form for *any* unspecified number *n*>3 of integrons, one should utilize the *method of mathematical induction*. This means that the metalanguage (IML) that has been used so far should be extended properly. In this case, it can be proved that, in general, *any binary logical or mathematical operator* (as ∨, ∧, ∩, ∪, +, ·, etc.), *which satisfies the basic associative law for any three operata of the domain of definition of the operator relative to the appropriate equivalence operator* (as ~, ⇔, =, etc), *also satisfy the generalized associative laws for any unspecified number n*>3 *of operata of the same class relative to the same equivalence operator*. It can then be proved that if, side by side with being associative, a given binary operator is *symmetrical*, i.e. if it satisfies the *basic commutative law* for any two operata of its domain of definition, – such a law, e.g., as axiom (4.9) or (4.13), – then that operator also satisfies *the generalized associative and commutative law for any unspecified number n*>3 *of operata of its domain of* *definition relative to the given equivalence operator*. In order to formulate and prove the above two generalized laws rigorously, an auxiliary uninterpreted logistic system after the manner of a semi-group should be formulated. Therefore, I shall discuss the generalized laws for any abstract binary operator relative to any abstract equivalence operator elsewhere.•

4.2.2. The advanced subject axioms of A_1

The subject axioms that are stated below in this sub-subsection are based on the following two definitions.

†**Df 4.1.** 1) Each one of the four bold-faced Roman (upright) small letters '**i**', '**j**', '**k**', '**l**' is an AtPLI (atomic panlogographic integron), whose range is the subclass of euautographic integrons (EI's) of A_1, each of which satisfies the *idempotent law*:

$$\mathbf{i} \,\hat{\cdot}\, \mathbf{i} \;\hat{=}\; \mathbf{i}\,, \tag{4.17}$$

to be called the *Elementary Idempotent Law* (*EIL*); and similarly with '**j**', '**k**', or '**l**' in place of '**i**'. Any of the four letters can, if needed, be furnished either with any of the light-faced numeral subscripts $_1$, $_2$, etc in the current font or with any number of primes or with any other labels, thus becoming another AtPLR of the same name and of the same range.

2) An EI satisfying (4.17) will *impartially* be called an *idempotent euautographic integron* (*IEI*). Under Axs 4.1–4.20, i.e. under the *current setup of* A_1, an IEI is alternatively (synonymously) called a *euautographic validity-integron* (*EVI*). In a certain *dual setup of* A_1, which will briefly be described in due course later on, an IEI will alternatively (synonymously) be called a *euautographic antivalidity-integron* (*EAVI*). It is understood that the only EVI's are the following ones: $\mathsf{0}$, $\mathsf{1}$, and for each ER **P**, $V(\mathbf{P})$ along with, when applicable, all successive EVI's $\mathbf{i}_1|\mathbf{P}\rangle$, $\mathbf{i}_2|\mathbf{P}\rangle$, $\ldots\mathbf{i}_n|\mathbf{P}\rangle$, which are obtained by *reducing* $V(\mathbf{P})$ in the course of a *EADP for* **P** with the help of certain rules (subject axioms and rules of inference) comprised in D_1. The EI $V(\mathbf{P})$ is properly called the *primary*, or *initial*, *validity-integron* (*PVI* or *IVI*) *of* **P** and also commonly a *primary*, or *initial*, *non-digital*, or *non-numeral*, *EVI* (briefly *PNDEVI*, *INDEVI*, *PNNEVI*, or *INNIEVI*) *of* A_1. The purpose of an EADP for **P** is, first, *to eliminate all ordinary* (*logical*) *EKS's* (*euautographic kernel-signs*) *occurring in* **P** *by transducing* $V(\mathbf{P})$ *into a special algebraic form relative to some* (*strictly some or all*) *of the substantival special* (*algebraic*) *EKS's* $\hat{-}$, $\hat{+}$, $\hat{=}$, $\hat{\cdot}$, $\hat{\cdot}_{\mathbf{x}}$ and, second, *to*

decrease as far as possible the total number of occurrences of special EKS's in the algebraic form by performing all performable algebraic operations indicated by those EKS's. In this case, $\mathbf{i}_j|\mathbf{P}\rangle$ with $j \in \omega_{1,n-1}$ is an EVI, which indicates the result of the *j*-th step in the above-mentioned process of reducing $V(\mathbf{P})$ and which is therefore called the *j*-th *intermediate* and hence *reducible non-digital*, or *non-numeral*, *euautographic validity-integron* (briefly *RNDEVI* or *RNNEVI*) *of* **P**. Consequently, the EVI $\mathbf{i}_j|\mathbf{P}\rangle$ with $j \in \omega_{1,n}$ is called the *j*-th *reduced EVI* (*RdEVI*) *of* **P**, whereas $\mathbf{i}_n|\mathbf{P}\rangle$ is supposed to equal 0 or 1, or else a certain *ultimate*, or *irreducible*, *non-digital EVI* (briefly *UNDEVI, IRNDEVI*) $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ of one of the following two kinds: (a) the *molecular*, and hence *irreducible, EVI* (*MIEVI*), $V(\mathsf{p})$, of a certain *elemental* (*atomic* or *molecular*) *ER* (*ElER*), p; (b) an *irreducible algebraic form* in some EIEVI's as 1, $V(\mathsf{p})$, $V(\mathsf{q})$, etc relative to some of the above-mentioned algebraic operators of A_1. It is understood that any of the graphonyms: 'p' to 's', p_1 to s_1, p_2 to s_2, etc is a StAPLR, whose range is the union of the set of all APVOR's of A_1 and the set of all MlEOR's (MlPVOR's and MlPCOR's) of A_1, which are not subject axioms of A_1, i.e. ones that do not satisfy the conditions $V(\mathsf{p}) \hat{=} 0$, $V(\mathsf{q}) \hat{=} 0$, etc.

3) Any panlogographic term, whose range contains IEI's and only IEI's is called a *idempotent panlogographic integron* (*IPLI*). In this case, '**i**' to '**l**', '$\mathbf{i}_1$' to '$\mathbf{l}_1$', '$\mathbf{i}_2$' to '$\mathbf{l}_2$', etc are *atomic IPLI's* (*AIPLI's*), whereas, for instance, '$V(\mathbf{P})$', '$V(\mathsf{p})$', '$\mathbf{i}_{\sim}|\mathbf{P}\rangle$', '$\mathbf{i}_1|\mathbf{P}\rangle$', '$\mathbf{i}_2|\mathbf{P}\rangle$', etc, and also '$\mathbf{i}\langle\mathbf{x}\rangle$', '$\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle$', etc are *molecular IPLI's* (*MIPLI's*). Thus, when necessary, any AIPLI can be suffixed either with a string such as '$|\mathbf{P}\rangle$' in order to indicate that it denotes a certain IEI, which is obtained in the result of transformation of $V(\mathbf{P})$, or with a string such as '$\langle\mathbf{x}\rangle$' or '$\langle\mathbf{x},\mathbf{y}\rangle$' in order to indicate that the AIPLI denotes a certain IEI that involves a certain one or certain two different AEOT's, – in accordance with Dfs 2.6(1), 2.7(1), and 2.8(1). In this case, '$\mathbf{i}\langle\mathbf{x}\rangle$' and '$\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle$' can particularly be specified by replacing them with '$V(\mathbf{P}\langle\mathbf{x}\rangle)$' and '$V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle)$' respectively, which is symbolically indicated as: $\mathbf{i}\langle\mathbf{x}\rangle \rhd V(\mathbf{P}\langle\mathbf{x}\rangle)$ and $\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle \rhd V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle)$ or, alternatively, as $V(\mathbf{P}\langle\mathbf{x}\rangle) \mapsto \mathbf{i}\langle\mathbf{x}\rangle$ and $V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle) \mapsto \mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle$. In

the following definition, the MIPLI's such as '$\mathbf{i}\langle\mathbf{x}\rangle$' and '$\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle$' are defined formally with the help of the pertinent instances of Dfs 2.7(1) and 2.8(1).•

†**Df 4.2.** 1) Let **x**, **y**, and **z** be three mutually different APVOT's. In order to indicate explicitly the assumption that **i** contains **x** as a *free* APVOT, I shall write '$\mathbf{i}\langle\mathbf{x}\rangle$' instead of '**i**'. Likewise, in order to indicate explicitly the assumption that **j** contains **y** as a *free* APVOT, I shall write $\mathbf{j}\langle\mathbf{y}\rangle$ instead of **j**. In this case, in accordance with Df 2.7(1),

$$\mathbf{i}\langle\mathbf{y}\rangle \to \widehat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{y}}\mathbf{i}\langle\mathbf{x}\rangle\Big| \text{ if } \mathbf{y} \text{ has no occurrences in } \mathbf{i}\langle\mathbf{x}\rangle, \tag{4.18}$$

$$\mathbf{i}\langle\mathbf{z}\rangle \to \widehat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{z}}\mathbf{i}\langle\mathbf{x}\rangle\Big| \text{ if } \mathbf{z} \text{ has no occurrences in } \mathbf{i}\langle\mathbf{x}\rangle, \tag{4.19}$$

$$\mathbf{j}\langle\mathbf{x}\rangle \to \widehat{\mathrm{S}}^{\mathbf{y}}_{\mathbf{x}}\mathbf{j}\langle\mathbf{y}\rangle\Big| \text{ if } \mathbf{x} \text{ has no occurrences in } \mathbf{j}\langle\mathbf{y}\rangle, \tag{4.20}$$

$$\mathbf{j}\langle\mathbf{z}\rangle \to \widehat{\mathrm{S}}^{\mathbf{y}}_{\mathbf{z}}\mathbf{j}\langle\mathbf{y}\rangle\Big| \text{ if } \mathbf{z} \text{ has no occurrences in } \mathbf{j}\langle\mathbf{y}\rangle. \tag{4.21}$$

2) Besides an APVOT **x**, $\mathbf{i}\langle\mathbf{x}\rangle$ may contain some other free AEOT's. In no connection with the hypothesis of the previous item, if **y** is one or the only one of such AEOT's, and if I wish to explicitly indicate the assumption that **i** contains both **x** and **y** then I shall write '$\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle$' instead of both '$\mathbf{i}\langle\mathbf{x}\rangle$' and '**i**'. In this case,

$$\mathbf{i}\langle\mathbf{y},\mathbf{y}\rangle \to \widehat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{y}}\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle\Big|. \tag{4.22}$$ •

*__Ax 4.7:__ $\vee_{\mathbf{x}}$***-Axiom – Law of Algebraization of the Existential Pseudo-Quantifier (LAEPQ).**

$$V\left(\vee_{\mathbf{x}}\mathbf{P}\right) \,\hat{=}\, \overset{\frown}{\cdot}_{\mathbf{x}} V(\mathbf{P}). \tag{4.23}$$

*__Ax 4.8:__ ***Idleness*** (***Vacuousness***) ***Law for an Algebraic Contractor*** (***ILAC***).

$$\overset{\frown}{\cdot}_{\mathbf{x}}\mathbf{i} \,\hat{=}\, \mathbf{i} \text{ if } \mathbf{x} \text{ does not occur in } \mathbf{i}. \tag{4.24}$$

*__Ax 4.9:__ ***Law of Dummy Atomic Terms*** (***DATL***).

$$\overset{\frown}{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle \,\hat{=}\, \overset{\frown}{\cdot}_{\mathbf{y}}\mathbf{i}\langle\mathbf{y}\rangle \text{ subject to (4.18).} \tag{4.25}$$

*__Ax 4.10:__ ***Transparency Law for an Algebraic Contractor (TLAC).***

$$\overset{\frown}{\cdot}_{\mathbf{x}}\left[\mathbf{i} \,\hat{\cdot}\, \mathbf{j}\langle\mathbf{x}\rangle\right] \,\hat{=}\, \mathbf{i} \,\hat{\cdot}\, \left[\overset{\frown}{\cdot}_{\mathbf{x}}\mathbf{j}\langle\mathbf{x}\rangle\right] \text{ if } \mathbf{x} \text{ does not occur in } \mathbf{i}. \tag{4.26}$$

*__Ax 4.11:__ ***Two versions of Emission and Absorption Law*** (***EAL***) ***– Advanced Idempotent Law 1*** (***AIL1***).

$$\overset{\frown}{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle \,\hat{=}\, \mathbf{i}\langle\mathbf{y}\rangle \,\hat{\cdot}\, \left[\overset{\frown}{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle\right] \text{ subject to (4.18).} \tag{4.27}$$

$$\overset{\frown}{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle \mathrel{\hat{=}} \mathbf{i}\langle\mathbf{y},\mathbf{y}\rangle \,\hat{}\, \left[\overset{\frown}{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle\right] \text{ subject to (4.22).} \tag{4.28}$$

The class of euautographic or panlogographic identities (valid equalities) (4.27) and (4.28), each of which proceeds from left to right, is called the *Emission Law* (*EL*). The class of the same identities, each of which proceeds from right to left, is called the *Absorption Law* (*AL*). The class of the same identities, independent on the direction in which each of them proceeds, i.e. the union of the above two classes, is called the *Emission and Absorption Law* (*EAL*) or the *Advanced Idempotent Law 1* (*AIL1*).•

Ax 4.12:** ***Fusion and Fission Law **(*FFL*) – *Advanced Idempotent Law 2*** **(*AIL2*)** 1) If **x**, **y**, and **z** are three mutually different APVOT's such that $\mathbf{i}\langle\mathbf{x}\rangle$ contains neither **y** nor **z**, while $\mathbf{j}\langle\mathbf{y}\rangle$ contains neither **x** nor **z**, then

$$\begin{gathered}\left[\overset{\frown}{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle\right]\hat{}\left[\overset{\frown}{\cdot}_{\mathbf{y}}\mathbf{j}\langle\mathbf{y}\rangle\right] \mathrel{\hat{=}} \overset{\frown}{\cdot}_{\mathbf{x}}\overset{\frown}{\cdot}_{\mathbf{y}}\left[\mathbf{i}\langle\mathbf{x}\rangle\,\hat{}\,\mathbf{j}\langle\mathbf{y}\rangle\right] \\ \mathrel{\hat{=}} \overset{\frown}{\cdot}_{\mathbf{x}}\left[\mathbf{i}\langle\mathbf{x}\rangle\,\hat{}\,\mathbf{j}\langle\mathbf{x}\rangle\right] \mathrel{\hat{=}} \overset{\frown}{\cdot}_{\mathbf{y}}\left[\mathbf{i}\langle\mathbf{y}\rangle\,\hat{}\,\mathbf{j}\langle\mathbf{y}\rangle\right] \mathrel{\hat{=}} \overset{\frown}{\cdot}_{\mathbf{z}}\left[\mathbf{i}\langle\mathbf{z}\rangle\,\hat{}\,\mathbf{j}\langle\mathbf{z}\rangle\right] \\ \text{subject to (4.18) – (4.21).}\end{gathered} \tag{4.29}$$

2) The class of trains of euautographic identities (4.29), each of which proceeds from left to right, is called the *Law of Fusion* [*of* $\overset{\frown}{\cdot}_{\mathbf{x}}$ *and* $\overset{\frown}{\cdot}_{\mathbf{y}}$]. The class of the same trains of identities, each of which proceeds from right to left, is called the *Law of Fission* [*of* $\overset{\frown}{\cdot}_{\mathbf{x}}$, $\overset{\frown}{\cdot}_{\mathbf{y}}$, and $\overset{\frown}{\cdot}_{\mathbf{z}}$]. The class of the same trains of identities, independent on the direction in which each of them proceeds, i.e. the union of the above two classes, is called the *Fusion and Fission Law* (*FFL*) or the *Advanced Idempotent Law 2* (*AIL2*) .

3) If as before $\mathbf{i}\langle\mathbf{x}\rangle$ does not contain **y** and $\mathbf{j}\langle\mathbf{y}\rangle$ does not contain **x**, while no assumption is made regarding occurrences of **z** either in $\mathbf{i}\langle\mathbf{x}\rangle$ or in $\mathbf{j}\langle\mathbf{y}\rangle$, then the rightmost term in (4.29) should be omitted.•

4.3. The meta-axioms (primary rules) of inference and decision of A_1

4.3.1. The meta-axioms of algebraic inference of A_1

****Ax 4.13:** ***The First Rule of Realizaton of a Definition (RRD1).***

$$\text{If } [\mathbf{I}' \to \mathbf{I}] \text{ then } \vdash [\mathbf{I}' \mathrel{\hat{=}} \mathbf{I}], \tag{4.30}$$

which can more explicitly be called the *Rule of Realization of the Definition of an Integron* or *Rule of the Implied Identity for Integrons.* While the ASD (asymmetric synonymic definition) $[\mathbf{I}' \to \mathbf{I}]$ belongs to the IML (inclusive metalanguage) of A_1

and is hence extrinsic with respect to A_1, the *identity*, i.e. *valid equality*, $[\mathbf{I}' \hat{=} \mathbf{I}]$ has the logical status of a *subject*, i.e. *intrinsic*, *axiom of* A_1.•

Ax 4.14: *The Second Rule of Realization of a Definition (RRD2).*

$$\text{If } [\mathbf{P}' \to \mathbf{P}] \text{ then } \vdash [V(\mathbf{P}') \hat{=} V(\mathbf{P})], \tag{4.31}$$

which can more explicitly be called the *Rule of Realization of the Definition of a Relation* or *Rule of the Implied Equality for Validity-Integrons.* Just as in the previous axiom, the ASD $[\mathbf{P}' \to \mathbf{P}]$ belongs to the IML of A_1 and is hence extrinsic with respect to A_1, the *identity* $[V(\mathbf{P}') \hat{=} V(\mathbf{P})]$ has the logical status of a *subject*, i.e. *intrinsic, axdiom of* A_1.•

Ax 4.15: *The Primary Rules of Inference for* $\hat{=}$.

If $\vdash [\mathbf{I} \hat{=} \mathbf{J}]$ then $\vdash [\mathbf{J} \hat{=} \mathbf{I}]$. (*Symmetry Law*) (4.32)

If $\vdash [\mathbf{I} \hat{=} \mathbf{J}]$ and $\vdash [\mathbf{J} \hat{=} \mathbf{K}]$ then $\vdash [\mathbf{I} \hat{=} \mathbf{K}]$. (*Transitive Law*) (4.33)

Ax 4.16: *Uniqueness Laws for the Cuts of* $\hat{+}$ ***and*** $\hat{\cdot}$ ***Relative to*** $\hat{=}$.

$$\text{If } \vdash [\mathbf{I} \hat{=} \mathbf{K}] \text{ and } \vdash [\mathbf{J} \hat{=} \mathbf{L}] \text{ then}$$
$$\text{(a) } \vdash [\mathbf{I} \hat{+} \mathbf{J} \hat{=} \mathbf{K} \hat{+} \mathbf{L}] \text{ and (b) } \vdash [\mathbf{I} \hat{\cdot} \mathbf{J} \hat{=} \mathbf{K} \hat{\cdot} \mathbf{L}]. \tag{4.34}$$

Ax 4.17: *Cancellation Law.*

$$\text{If } \vdash [\mathbf{K} \bar{\hat{=}} 0] \text{ and } \vdash [\mathbf{K} \hat{\cdot} \mathbf{I} \hat{=} \mathbf{K} \hat{\cdot} \mathbf{J}] \text{ then } \vdash [\mathbf{I} \hat{=} \mathbf{J}]. \tag{4.35}$$

Ax 4.18: *Rules of Specific Substitutions* (*SSR's*).

1) *SSR1***:** Under the assumptions of Df 2.8(1) with '**P**' in place of '**Φ**',

$$\text{if } \vdash \mathbf{P}\langle \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m \rangle \text{ then } \vdash \mathbf{P}\langle \mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_m \rangle \tag{4.36}$$

subject to

$$\mathbf{P}\langle \mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_m \rangle \to \mathrm{S}^{\mathbf{x}_1 \mathbf{x}_2 ... \mathbf{x}_m}_{\mathbf{y}_1 \mathbf{y}_2 ... \mathbf{y}_m} \mathbf{P}\langle \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_m \rangle \Big| . \tag{4.36+}$$

2) *SSR2***:** Under the assumptions of Df 2.8(2) with '**Q**' in place of '**Ψ**',

$$\text{`if } \vdash \mathbf{Q}\langle \mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_n \rangle \text{ then } \vdash \mathbf{Q}\langle \mathbf{P}_1, \mathbf{P}_2, ..., \mathbf{P}_n \rangle \tag{4.37}$$

subject to

$$\mathbf{Q}\langle \mathbf{P}_1, \mathbf{P}_2, ..., \mathbf{P}_n \rangle \to \mathrm{S}^{\mathbf{p}_1 \mathbf{p}_2 ... \mathbf{p}_n}_{\mathbf{P}_1 \mathbf{P}_2 ... \mathbf{P}_n} \mathbf{Q}\langle \mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_n \rangle \Big| . \tag{4.37+}$$

Ax 4.19: *Rules of General Single Substitutions (GSR's).*

If $\vdash \mathbf{R}\langle \mathbf{I} \rangle$ and $\vdash [\mathbf{I} \hat{=} \mathbf{J}]$ then $\vdash \mathrm{S}^{\mathbf{I}}_{\mathbf{J}} \mathbf{R}\langle \mathbf{I} \rangle$.

(*GSR1*) (4.38)

If $\vdash \mathbf{S}\langle\mathbf{P}\rangle$ and $\vdash[V(\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{Q})]$ then $\vdash \mathrm{S}^{\mathbf{P}}_{\mathbf{Q}}\mathbf{S}\langle\mathbf{P}\rangle|$. (*GSR2*) (4.39)

4.3.2. The meta-axiom of decision of A$_1$

****Ax 4.20:** ***The Conjoined Three-Fold Basic Rule of Decision and Inference (BDIR) of*** **A$_1$.**

$$\vdash \mathbf{P} \text{ if and only if } \vdash[V(\mathbf{P}) \mathrel{\hat{=}} 0], \quad \text{(a)}$$

$$\dashv \mathbf{P} \text{ if and only if } \vdash[V(\mathbf{P}) \mathrel{\hat{=}} 1], \quad \text{(b)} \quad (4.40)$$

$$\not\vdash \mathbf{P} \text{ if and only if } \vdash[V(\mathbf{P}) \mathrel{\hat{=}} \mathbf{i}_{\sim}|\mathbf{P}\rangle], \quad \text{(c)}$$

where $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ is a certain IRNDIEI (irreducible non-digital idempotent euautographic integron) as described in Df 4.1(2). The “if”-parts of the above three meta-axioms (a)–(c) are *the basic rules of decision of* A$_1$ *with respect to the validity-values validity, antivalidity, and vav-neutrality* (*vav-indeterminacy*) respectively, while the “only if”-part of the meta-axiom (a), but not that of (b) or (c), is the *rule of inference of the euautographic special algebraic identity* (*valid equality*) $[V(\mathbf{P}) \mathrel{\hat{=}} 0]$ *of* A$_1$ *from a valid ER* **P**.•

4.4. Immediate implications of Axs 4.1–4.20

Th 4.1. Let ‘$\mathbf{P}^{a}$’ be an AtPLR (atomic panlogographic relation) of **A$_1$**, i.e. an AtPLPH of ER’s (euautographic relations) of A$_1$, whose range is the set of subject axioms of A$_1$, Axs 4.1–4.12. Then

$$\vdash[V(\mathbf{P}^{a}) \mathrel{\hat{=}} 0]. \quad (4.41)$$

Proof: Since $\mathbf{P}^{a}$ is a subject axiom of A$_1$, therefore it is taken for granted to be *valid*, i.e. $\vdash \mathbf{P}^{a}$, and vice versa. Hence, the identity (4.41) immediately follows from (4.40a) with ‘$\mathbf{P}^{a}$’ in place of ‘**P**’.•

Th 4.2.

$$\vdash[V(\mathbf{P}) \mathrel{\hat{=}} 0] \text{ if and only if } \vdash[V(V(\mathbf{P}) \mathrel{\hat{=}} 0) \mathrel{\hat{=}} 0], \quad \text{(a)}$$

$$\vdash[V(\mathbf{P}) \mathrel{\hat{=}} 1] \text{ if and only if } \vdash[V(V(\mathbf{P}) \mathrel{\hat{=}} 1) \mathrel{\hat{=}} 0], \quad \text{(b)} \quad (4.42)$$

$$\vdash \left[V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle\right] \qquad \text{if and only if} \qquad \vdash \left[V(V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle) \hat{=} 0\right].$$

(c)

Proof: The conjuncts (a)–(c) of the theorem immediately follow from the axiom (4.40a) with '$\left[V(\mathbf{P}) \hat{=} 0\right]$', '$\left[V(\mathbf{P}) \hat{=} 1\right]$', or '$\left[V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle\right]$' in place of '**P**' respectively.•

Df 4.3. In accordance with the axiom (4.33),

$$\begin{aligned} &\left[\mathbf{I}_{i_1} \hat{=} \mathbf{I}_{i_2} \hat{=} \mathbf{I}_{i_3} \hat{=} \ldots \hat{=} \mathbf{I}_{i_{n-1}} \hat{=} \mathbf{I}_{i_n}\right] \leftrightarrow \left[\mathbf{I}_1 \hat{=} \mathbf{I}_2 \hat{=} \mathbf{I}_3 \hat{=} \ldots \hat{=} \mathbf{I}_{n-1} \hat{=} \mathbf{I}_n\right] \\ &\rightarrow \left[\mathbf{I}_1 \hat{=} \mathbf{I}_2, \mathbf{I}_2 \hat{=} \mathbf{I}_3, \ldots, \mathbf{I}_{n-1} \hat{=} \mathbf{I}_n\right] \leftrightarrow \left[\mathbf{I}_{i_1} \hat{=} \mathbf{I}_{i_2}, \mathbf{I}_{i_2} \hat{=} \mathbf{I}_{i_3}, \ldots, \mathbf{I}_{i_{n-1}} \hat{=} \mathbf{I}_{i_n}\right], \end{aligned} \tag{4.43}$$

where $i_1, i_2, \ldots, i_{n-1}, i_n$ is any permutation of the numerals 1, 2, ..., n–1, n. A non-redundant sequence of separate valid two-term equalities, which serves as a definiens in the above definition, is said to be written in the *staccato style*. A continuous train of equalities, which serves as a definiendum in the above definition, is said to be written in the *legato style*.•

Cnv 4.1. In the sequel, sequences of interrelated identities, especially those forming algebraic proofs, will most often be written in the legato style for the sake of brevity.•

Cmt 4.2. 1) As was indicated in the items 3 and 6 of subsection 4.1, all PLR's (panlogographic relations) of A_1 that occur in Axs 4.1–4.20 and their *specified panlogographic instances* can be used in the TAEXA-mode, i.e. intermittently in the two alternating ways (mental modes)*: xenonymously*, i.e. as a *eulogograph*, for mentioning a general (common, certain, concrete but not concretized) ER (euautographic relation) of A_1 of its range and either *autonymously*, i.e. as a *tychautograph*, for mentioning either itself or its homolographic (photographic) token-class or, when appropriate, *semi-autonymously* (*semi-xenonymously*) in accordance with Df 2.6(4).

2) In order to illustrate the semi-autonymous mental attitude that an interpreter is supposed to take towards a combined PLR as any of those indicated in Df 2.6(4), Ax 4.18 is restated below as Ax 4.18_1 in such a way that, under the pertinent assumptions of the items 1 and 2 of Df 2.8, all unquoted occurrences of AtPLOT's, AtPLOR's, quasi-AtPLOT's, and quasi-AtPLOR's throughout Ax 4.18_1 are supposed to be used xenonymously, while all quoted occurrences of AtPLOT's and AtPLOR's, particularly of the same ones, are supposed to be used autonymously and all quasi-

quoted occurrences of the same quasi-AtPLOT's and quasi-AtPLOR's are supposed to be used quasi-autonymously. At the same time, one should remember that the HAQ (homoloautographic quotation) and QHAQ (quasi-homoautolographic quotation) marks occurring in Ax 4.18_1 are virtual ones that are not mentioned and are just used for mentioning the respective autonymous mental attitude and distinguishing it from the xenonymous attitude, with which it is intermixed in most cases.•

****Ax 4.18_1:** ***Rules of Specific Autonymous and Quasi-Autonymous Substitutions.***

1) *SASR1***:** Under the assumptions of Df 2.8(1) with '**P**' in place of '**Φ**',

$$\text{if } \vdash \mathbf{P}\langle \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m \rangle \text{ then } \vdash \mathbf{P}\langle \text{'}\mathbf{x}_1\text{'}, \text{'}\mathbf{x}_2\text{'}, \ldots, \text{'}\mathbf{x}_m\text{'} \rangle \tag{4.36$_1$}$$

subject to

$$\mathbf{P}\langle \text{'}\mathbf{x}_1\text{'}, \text{'}\mathbf{x}_2\text{'}, \ldots, \text{'}\mathbf{x}_m\text{'} \rangle \rightarrow \mathrm{S}^{\mathbf{x}_1\ \mathbf{x}_2 \ldots\ \mathbf{x}_m}_{\text{'}\mathbf{x}_1\text{''}\mathbf{x}_2\text{'}\ldots\text{'}\mathbf{x}_m\text{'}} \mathbf{P}\langle \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m \rangle \Big| \tag{4.36$_{1+}$}$$

(cf. (2.17′) and (2.17′)), and vice versa, i.e.

$$\text{if } \vdash \mathbf{P}\langle \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m \rangle \text{ then } \vdash \mathbf{P}\langle \text{'}\mathbf{x}_1\text{'}, \text{'}\mathbf{x}_2\text{'}, \ldots, \text{'}\mathbf{x}_m\text{'} \rangle \tag{4.36$_2$}$$

subject to

$$\mathbf{P}\langle \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m \rangle \rightarrow \mathrm{S}^{\text{'}\mathbf{x}_1\text{''}\mathbf{x}_2\text{'}\ldots\text{'}\mathbf{x}_m\text{'}}_{\mathbf{x}_1\ \mathbf{x}_2 \ldots\ \mathbf{x}_m} \mathbf{P}\langle \text{'}\mathbf{x}_1\text{'}, \text{'}\mathbf{x}_2\text{'}, \ldots, \text{'}\mathbf{x}_m\text{'} \rangle \Big| \tag{4.36$_{2+}$}$$

Also, e.g.,

$$\text{if } \vdash \mathbf{P}\langle \text{'}\mathbf{x}_1\text{'}, \text{'}\mathbf{x}_2\text{'}, \ldots, \text{'}\mathbf{x}_m\text{'} \rangle \text{ then } \vdash \mathbf{P}\langle \text{'}\mathbf{y}_1\text{'}, \text{'}\mathbf{y}_2\text{'}, \ldots, \text{'}\mathbf{y}_m\text{'} \rangle \tag{4.36$_3$}$$

subject to

$$\mathbf{P}\langle \text{'}\mathbf{y}_1\text{'}, \text{'}\mathbf{y}_2\text{'}, \ldots, \text{'}\mathbf{y}_m\text{'} \rangle \rightarrow \mathrm{S}^{\text{'}\mathbf{x}_1\text{''}\mathbf{x}_2\text{'}\ldots\text{'}\mathbf{x}_m\text{'}}_{\text{'}\mathbf{y}_1\text{''}\mathbf{y}_2\text{'}\ldots\text{'}\mathbf{y}_m\text{'}} \mathbf{P}\langle \text{'}\mathbf{x}_1\text{'}, \text{'}\mathbf{x}_2\text{'}, \ldots, \text{'}\mathbf{x}_m\text{'} \rangle \Big| \tag{4.36$_{3+}$}$$

(cf. (2.18)), and vice versa.

2) *SASR2***:** Under the assumptions of Df 2.8(2) with '**Q**' in place of '**Ψ**',

$$\text{if } \vdash \mathbf{Q}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle \text{ then } \vdash \mathbf{Q}\langle \text{'}\mathbf{P}_1\text{'}, \text{'}\mathbf{P}_2\text{'}, \ldots, \text{'}\mathbf{P}_n\text{'} \rangle \tag{4.37$_1$}$$

subject to

$$\mathbf{Q}\langle \text{'}\mathbf{P}_1\text{'}, \text{'}\mathbf{P}_2\text{'}, \ldots, \text{'}\mathbf{P}_n\text{'} \rangle \rightarrow \mathrm{S}^{\mathbf{p}_1\ \mathbf{p}_2 \ldots\ \mathbf{p}_n}_{\text{'}\mathbf{P}_1\text{''}\mathbf{P}_2\text{'}\ldots\text{'}\mathbf{P}_n\text{'}} \mathbf{Q}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle \Big| \tag{4.37$_{1+}$}$$

(cf. (2.20) and (2.20′)), and vice versa, i.e.

$$\text{if } \vdash \mathbf{Q}\langle \text{'}\mathbf{P}_1\text{'}, \text{'}\mathbf{P}_2\text{'}, \ldots, \text{'}\mathbf{P}_n\text{'} \rangle \text{ then } \vdash \mathbf{Q}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle \tag{4.37$_2$}$$

subject to

$$\mathbf{Q}\langle \mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n \rangle \rightarrow \mathrm{S}^{\text{'}\mathbf{P}_1\text{''}\mathbf{P}_2\text{'}\ldots\text{'}\mathbf{P}_n\text{'}}_{\mathbf{p}_1\ \mathbf{p}_2 \ldots\ \mathbf{p}_n} \mathbf{Q}\langle \text{'}\mathbf{P}_1\text{'}, \text{'}\mathbf{P}_2\text{'}, \ldots, \text{'}\mathbf{P}_n\text{'} \rangle \Big| \tag{4.37$_{2+}$}$$

Also, e.g.,

$$\text{if } \vdash \mathbf{Q}\langle '\mathbf{P}_1', '\mathbf{P}_2', \ldots, '\mathbf{P}_n' \rangle \text{ then } \vdash \mathbf{Q}\langle '\mathbf{Q}_1', '\mathbf{Q}_2', \ldots, '\mathbf{Q}_n' \rangle \qquad (4.37_3)$$

subject to

$$\mathbf{Q}\langle '\mathbf{Q}_1', '\mathbf{Q}_2', \ldots, '\mathbf{Q}_n' \rangle \to \mathrm{S}^{'\mathbf{P}_1''\mathbf{P}_2'\ldots'\mathbf{P}_n'}_{'\mathbf{Q}_1''\mathbf{Q}_2'\ldots'\mathbf{Q}_n'} \mathbf{Q}\langle '\mathbf{P}_1', '\mathbf{P}_2', \ldots, '\mathbf{P}_n' \rangle \big| . \qquad (4.37_{3+})\bullet$$

Cmt 4.3. The *primary*, or *initial*, *validity-integron* (*PVI* or *IVI*) of any *AnAtPLR* (*analytical atomic panlogographic relation*) *of* A_1 on the list (1.1), i.e. $V('\mathbf{P}')$, $V('\mathbf{Q}')$, etc, is *irreducible,* so that $\mathbf{i}_{\sim}|'\mathbf{P}'\rangle$, e.g., is the same as $V('\mathbf{P}')$, i.e $\mathbf{i}_{\sim}|'\mathbf{P}'\rangle \leftrightarrow V('\mathbf{P}')$. Therefore, *the predicate* $\vdash$ *is not applicable* either to $[V('\mathbf{P}') \hat{=} 0]$ or to $[V('\mathbf{P}') \hat{=} 1]$, whereas $[V('\mathbf{P}') \hat{=} \mathbf{i}_{\sim}|'\mathbf{P}'\rangle]$ is the same as $[V('\mathbf{P}') \hat{=} V('\mathbf{P}')]$, i.e. $[V('\mathbf{P}') \hat{=} \mathbf{i}_{\sim}|'\mathbf{P}'\rangle] \leftrightarrow [V('\mathbf{P}') \hat{=} V('\mathbf{P}')]$. Hence, the variants of the items (a) and (b) of axiom (4.40) are ineffective (inapplicable), whereas the item (c) of that axiom becomes:

$$\dashv\!\vdash '\mathbf{P}' \text{ if and only if } \vdash [V('\mathbf{P}') \hat{=} V('\mathbf{P}')], \qquad \text{(c)} \qquad (4.40_0)$$

which is effective thus saying that **'P'** *is a vav-neutral* (*vav-indeterminate*) *PLR of* A_1. It is self-evident that this simplest instance of Ax 4.20 cannot be used as a general rule of decision for any given PLR of A_1. In subsection 4.6, I shall make explicit how Axs 4.1–4.20 can be extended formally from A_1 to A_1 in accordance with their informal use for solving the vavn-decision problem, not only for A_1, but also for A_1.•

4.5. Particularization (specification or concretization) of panlogographic schemata of definitions, axioms, and theorems of A_1

4.5.1. Definitions of the pertinent nomenclature

Preliminary Remark 4.1. All *panlogographic identities* (*valid equalities*) which are comprised in Axs 4.1–4.12, are *panlogographic axiom-schemata* (*PLAxS'ta*) *of* A_1, each of which condenses an infinite number of *concrete conspecific euautographic axioms* (*EAx's*) *of* A_1. In order to make explicit any *immediate implications* of these schemata as such, i.e. implications, which are intimately connected with the schemata themselves and which require no application of any rules of inference and decision, I begin with a few general definitions.•

Df 4.4. In contrast to the count name "*genus*" or "*generic class*", which has the same general epistemologically relativistic meaning as "*superclass*", the count name "*species*" or "*specific class*" has the same general epistemologically relativistic meaning as "*subclass*". Consequently, the qualifier "*generic*" means *of, related, or* *constituting a genus*, i.e. a *superclass*, whereas the qualifier "*specific*" means *of, related to, or constituting a species*, i.e. a *subclass* (cf. WTNID). In this case, "*congeneric*" means *of the same genus*, i.e. *of the same superclass*, whereas "*conspecific*" means *of the same species*, i.e. *of the same subclass*. The qualifier "*particular*" to "class" is by definition a synonym of "specific". A specific (particular) class that has *exactly one member* is conventionally called a *singleton.* Accordingly, a *concrete object* and particularly a *concrete member of a many-member class*, called a *multipleton* (in analogy with "singleton"), is a *sui generic object*, i.e. an object forming a class of its own – its singleton.•

Cmt 4.4. When a sapient subject uses a *proper name* (particularly, a proper substantival name or a proper declarative sentence) for mentioning the object, which the name *denotes*, he uses but does not mention the singleton of the object (just as he uses but does not mention the name itself), which is the range of the name and which the name *connotes*, in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which the sapient subject mentally experiences the singleton as *his as if extramental* (*exopsychical*) *object representing the singleton and being its member*. For instance, the singleton of Aristotle, i.e. ${}^{\backslash}$Aristotle${}^{/}$ or {Aristotle}, is used along with using the name "Aristotle" for mentioning Aristotle. Thus, the element (member) of the one-member range of a proper name is just *another hypostasis* (*way of existence*, *aspect*) *of that range*. Likewise, when a sapient subject uses a *common name* (particularly, a common substantival name or a common declarative sentence) for *mentioning* and hence for *denoting a common* (*general*, *certain*, *concrete but not concretized*) *member* (*element*) *of the multipleton*, which is the range of the name and which the name *connotes*, he uses but does not mention the many-member range of the name (just as he uses but does not mention the name itself) in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode,* in which the sapient subject mentally experiences the range as *his as if extramental* (*exopsychical*) *object representing the whole range*. Thus, a common (general) element (member) of the range of a common name is just *another hypostasis* (*way of existence, aspect*) *of that range*. For instance, the multipleton *man*, i.e. ${}^{\backslash}$*a man*${}^{/}$, is used along with using the name "a man" for mentioning a man.

The above mental phenomenon of using xenonyms is analogous to any process of perception, in which a sensation (sense datum) of a sapient subject, being his mental (psychical) coentity – just as his singleton or multipleton, is mentally experienced by the subject as his extramental (exopsychical, physical) object.•

Df 4.5. 1) The act of replacing a *panlogograph* (briefly *PL*) *of* $\mathbf{A}_1$, i.e. of a *panlogographic placeholder* (*PLPH*) [*of euautographs*] *of* A_1, – a *plain* (*non-patterned*) *one* (briefly *PlPL* or *PlPLH*) or a *schematic* (*patterned*) *one* (briefly *SchPL* or *SchPLPH*), called also a *panlogographic schema* (*PLS*, pl. "*PLS'ta*") [*of euautographs*] *of* A_1 (as a definition-schema, PLI-schema, or PLR-schema), – with a *endosemasiopasigraph* (briefly *EnSPG*) *of* A_1, i.e. either with another PL of $\mathbf{A}_1$ (PLPH of A_1) or with a euautograph of A_1, is called:

a) a *specification* of the former PL (PLPH) if the latter EnSPG is a PL (PLPH) whose range is a *species* (*strict subclass*) of the former;

b) a *concretization* of the former PL (PLPH) if the latter EnSPG is a concrete euautograph of A_1, being a member (element) of the range of the former;

c) a *generalization* of the former PL (PLPH) if the latter EnSPG is a PL (PLPH) whose range is a *genus* (*strict superclass*, *strict whole*) of the former.

Specification or concretization of a PL (PLPH) is indiscriminately called *particularization* of the PL (PLPH). The result of specification or concretization, i.e. the pertinent substituend, of a PL (PLPH) is discriminately called a *specific instance* or a *concrete instance* respectively and indiscriminately an *instance*, *of the PL* (*PLPH*). The result of generalization, i.e. again the pertinent substituend, of a PL (PLPH) is a *universal of the PL* (*PLPH*).

2) Particularly, the act of replacing every occurrence of an *AtPL* (*atomic panlogograph*) *of* $\mathbf{A}_1$, i.e. an *AtPLPH* (*atomic panlogographic placeholder*) [*of euautographs*] *of* A_1, throughout a given PLS (SchPLPH) [of euautographs] of A_1 either with another AtPL (AtPLPH) of a less inclusive range or with a suitable less inclusive PLS (SchPLPH) is a specification both of the AtPL (AtPLH) and of the PLS (SchPLPH). A like act of substitution, in which the substituend of the AtPL is a concrete euautograph of A_l, is a concretization both of the AtPL (AtPLH) and of its host PLS (SchPLPH).

3) The plural of any abbreviation which ends with "S" standing for "schema" is formed by suffixing "S" with "*'ta*" in accordance with the plural "schema*ta*". In general, the plural of an abbreviation is normally formed by adjoining to it an

apostrophe and the ending of the plural of the word represented by the last letter of the abbreviation.•

Df 4.6. 1) The more inclusive one of two given comparable *panlogographic* (*PL*) *definition-schemata* (*DfS'ta*), *axiom-schemata* (*AxS'ta*), or *theorem-schemata* (*ThS'ta*) *of* A_1 is called a *generic* (*G*) *one*, i.e. briefly a *GPLDfS*, *GPLAxS*, or *GPLThS of* A_1 respectively and also indiscriminately a *GPLS* (*generic panlogographic schema*), whereas the less inclusive one is called a *specific* (*S*) *one*, i.e. briefly an *SPLDfS*, *SPLAxS*, or *SPLThS of* A_1 respectively and also indiscriminately a *SPLS* (*specifc panlogographic schema*). An AxS or ThS is indiscriminately called a *kyrology-schema* (*KrgS*). Accordingly, a GPLAxS or GPLThS is indiscriminately called a *GPLKrgS*, and an SPLAxS or SPLThS is indiscriminately called an *SPLKrgS*, *of* A_1.

2) An AtPLPH occurring in a given DfS or KrgS is said to be *specifiable* if there is a less inclusive AtPLPH or SchPLPH, which can be substituted for every occurrence of the former AtPLPH throughout the DfS or KrgS. It goes without saying that any AtPLPH in any occurrence is *concretizable*.

3) In accordance with Df 4.5, the result of specification or concretization of a GPLDfS or GPLKrgS with respect to some, i.e. strictly some or all, AtPLPH's occurring in the GPLDfS or GPLKrgS is called an *instance of the respective schema*. The instance of a PLS (SchPLPH) is said to be:

i) a *specific*, or *specified, one* if it results by specification of some (strictly some or all) specifiable AtPLPH's occurring in the schema, i.e. if its range is a species (subclass) of the range of the schema;

ii) a *concrete*, or *concretized, one* if it results by concretization of all AtPLPH's occurring in the schema, i.e. is it is if it is a member (element) of the range of the schema;

iii) a *generic semi-concretized one* if it results by concretization of strictly some AtPLPH's occurring in the schema, while each one the rest of AtPLPH's occurring in the schema either remains unaltered;

iv) a *specific semi-concretized one* if it results by concretization of strictly some AtPLPH's occurring in the schema, while some (strictly some or all) of the rest of AtPLPH's occurring in the schema are specified;

v) a *semi-concretized*, or *semi-concretized*, *one* if it is either a generic semi-concretized one or a schematic semi-concretized one, i.e. if it results by concretization of strictly some AtPLPH's occurring in the schema, while each one the rest of AtPLPH's occurring in the schema either remains unaltered or is specified.•

Cmt 4.5. 1) All terms introduced in Df 4.6 are epistemologically relativistic. Particularly, the fact whether or not a given AtPLPH in a given occurrence is specifiable depends on both the occurrence of the AtPLPH and on the previous definitions of comparable less inclusive AtPLPH's. For instance, a *bound* occurrence of any one of the AtPLOT's '**u**', '**v**', etc introduced in Df I.5.2 is unspecifiable, while a free occurrence of the same AtPLOT is specifiable, because a free occurrence of '**u**', e.g., can be replaced either with an occurrence of '$\mathbf{u}^{pv}$' or with an occurrence of '$\mathbf{u}^{pc}$'. By contrast, any of the AtPLOR's '**p**', '**q**', etc introduced in the same definition is unspecifiable in any occurrence because no less extensive AtPLOR's have been defined.

2) The range of any panlogographic DfS, AxS, or ThS is predetermined by the definitions of and hence by the ranges of the AtPLPH's occurring in a given PLS. There are no rules of specification or concretization of instances of such a PLS. Consequently, any *selected specific*, *concrete*, or *semi-concrete instance* of a given PLS is its *corollary*, which does not have any *formal proof* in the sense of "a proof of a theorem" as defined in Dfs 3.3 and 3.4, and which is defined in the following definition.•

Df 4.7. 1) The *generic* term "*corollary*" will indiscriminately be used for mentioning any instance – specific, concrete, or semi-concrete, which is comprised in the range of a given PLS. Accordingly, the *specific* terms "*definition corollary*", "*axiom corollary*", and "*theorem corollary*" or their hyphenated variants will be used for any instances of a PLDfS, PLAxS, and PLThS respectively. That is to say, any instance of any one of the three PLS'ta is *a corollary, and not a theorem* that requires a formal proof by using some of Axs 4.13–4.20. At the same time, a corollary (instance) of a PLDfS, PLAxS, or PLThS is a definition, axiom, or theorem respectively, but a theorem being a corollary (instance) of the PLThS does not need any proof because it has already been proved earlier in proving the PLThS.

2) A corollary (instance) of a PLS is an *implication* of the PLS, but not necessarily vice versa.•

Cmt 4.6. 1) WTNID defines the acceptation of the noun "corollary" in this manner:

> «a proposition that follows upon one just demonstrated and that requires no additional proof»,

whereas APED defines it thus:

> «in logic, a direct conclusion from a proved proposition».

Df 4.7 agrees with the above two dictionary definitions and it allows avoiding confusion in using either word "axiom" or "theorem" alone. •

Df 4.8. The following three systems of bookmarks of corollaries will be used for the purpose of cross-reference.

1) When a corollary is laid down formally, it will be included under a logical head, which contains the abbreviation '**Crl**' for '**Corollary**', followed by the pertinent double position-numeral and preceded by the appropriate one of the following labels: * if the corollary is specific, º if the corollary is concrete, and º* if the corollary is semi-concrete. In this case, the statement of a corollary is followed by a *comment* under the heading '**Proof:**', in which the GPLS of the corollary is indicated. This heading should be understood as *an hoc* abbreviation of the name '**Corollary Proof:**', being a at the same time an *ad hoc* synonym of '**Comment:**' or "**Explanation:**", while the homonymous heading of *the proof a theorem*, which is based on the rules of inference and decision, Axs 4.13–4.4.20, should be understood as an abbreviation of the term "**Theorem Proof:**".

2) If a GPLS is displayed and is numbered (bookmarked) with the pertinent parenthesized double-position numeral relative to the beginning of the section, in which it is laid down, and if a certain SPLS, being a corollary (instance) of the GPLS, is laid down informally just by displaying it within its current context in the same section then the SPLS is numbered (bookmarked) with the same parenthesized double-position numeral as that numbering the GPLS (generic panlogographic schema), while the *instance kind* of the corollary with respect to its generic source schema is indicated by the respective one of the following adscripts to the double position-numeral:

'γ' if the GPLS is regarded as the result of generalization of its instance, ‘ε’ if the corollary is a specific panlogographic instance of the GPLS,

‘μ’ if the corollary is a concrete euautographic instance of the GPLS,

‘ι’ if the corollary is a semi-concrete instance of the GPLS,

‘γι’ if the corollary is a generic semi-concretized instance of the GPLS,

'ει’ if the corollary is a specific semi-concretized instance of the GPLS.

In this case, mnemonic justification of using the letters ‘γ’, ‘ε, ‘ι’, and ‘μ’ in the above way is that these are the first letters of the following Greek words (Pring [1982]):

«γένος \jénos\ *s. n.* race, tribe; genus; gender…»

«είδος \ídos\ *s. n.* sort, kind; species; article, commodity…»

«ιδιαίτερος \idiéteros\ *a.* special, particular…»

«μέλος \mélos\ *s. n.* member…»

A modified double position-numeral as described above can be used as a bookmark either of a single corollary or of a group of corollaries of the same class. If however separate corollaries or separate groups of corollaries, all of the same class, should be bookmarked (itemized) separately for further references then a bookmark including the pertinent double position-numeral and the pertinent one-letter or two-letter subscript on it can be supplemented with an additional distinguishing label as a numeral subscript ‘$_0$’, ‘$_1$’, ‘$_2$’, etc., a subscript asterisk ‘$_*$’, an adscript letter “a”, “b”, etc, etc. Also, if a PLS has initially been laid down as a GPLS and has therefore bookmarked with the appropriate parenthesized double position-numeral without any additional qualifying adscript and if afterwards it is included into the range of another PLS as its as an instance (corollary) then the latter, more inclusive PLS becomes a new GPLS and therefore its double position-numeral can be attributed with the adscript ‘γ’, while the former GPLS turns into an SPLS (cf. Df 4.5(1c)).

3) The first system of stating and bookmarking corollaries is self-sufficient and effective no matter whether a corollary is laid down occurs in the same section as its GPLS or in a different section. By contrast, the second system is effective only if a corollary of a GPLS is laid down in the same section as the GPLS. Therefore, if a corollary of a GPLS is laid down informally in another section and is supposed to be referred to somewhere in the sequel then it can be displayed and be bookmarked with the regular parenthesized double-position numeral, in which the first constituent numeral denotes the ordinal number of its host section in the treatise and the second

one denotes the ordinal number of the corollary in the section. If needed, the instance kind of corollary can be indicated explicitly in the pertinent context.•

4.5.2. Examples

Crl 4.2: ***Some specifications of PLS'ta.*** 1) The range of any one of the MlPL's (molecular PL's) '$V(\mathbf{P})$', '$V(\mathbf{Q})$', '$V(\mathbf{R})$', and '$V(\mathbf{S})$' is in turn a strict subclass of the range of any one of the AtPL's of any one of the AtPL's '**i**', '**j**', '**k**', and '**l**', whereas the latter range is in turn a strict subclass of the range of any one of the AtPL's '**I**', '**J**', '**K**', and '**L**'. Hence, without loss of generality, I shall assume that

$$\mathbf{I} \triangleright \mathbf{i},\ \mathbf{J} \triangleright \mathbf{j},\ \mathbf{K} \triangleright \mathbf{k},\ \mathbf{L} \triangleright \mathbf{l}, \tag{4.44}$$

i.e.

$$\text{'}\mathbf{i}\text{'} \mapsto \text{'}\mathbf{I}\text{'},\ \text{'}\mathbf{j}\text{'} \mapsto \text{'}\mathbf{J}\text{'},\ \text{'}\mathbf{k}\text{'} \mapsto \text{'}\mathbf{K}\text{'},\ \text{'}\mathbf{l}\text{'} \mapsto \text{'}\mathbf{L}\text{'}, \tag{4.44a}$$

and that

$$\mathbf{i} \triangleright V(\mathbf{P}),\ \mathbf{j} \triangleright V(\mathbf{Q}),\ \mathbf{k} \triangleright V(\mathbf{R}),\ \mathbf{l} \triangleright V(\mathbf{S}), \tag{4.45}$$

i.e.

$$\text{'}V(\mathbf{P})\text{'} \mapsto \text{'}\mathbf{i}\text{'},\ \text{'}V(\mathbf{Q})\text{'} \mapsto \text{'}\mathbf{j}\text{'},\ \text{'}V(\mathbf{R})\text{'} \mapsto \text{'}\mathbf{k}\text{'},\ \text{'}V(\mathbf{S})\text{'} \mapsto \text{'}\mathbf{l}\text{'}, \tag{4.45a}$$

the understanding being that the variants of (4.45) and (4.45a) with '**I**', '**J**', '**K**', and '**L**' in place of '**i**', '**j**', '**k**', and '**l**' also hold by (4.44) and (4.44a).

2) I shall also assume that

$$\mathbf{i} \triangleright V(\mathbf{P}),\ \mathbf{i}\langle\mathbf{x}\rangle \triangleright V(\mathbf{P}\langle\mathbf{x}\rangle),\ \mathbf{j}\langle\mathbf{x}\rangle \triangleright V(\mathbf{Q}\langle\mathbf{x}\rangle),\ \mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle \triangleright V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle)\ , \tag{4.46}$$

and similarly with '**y**' or '**z**' in place of '**x**'. In this case, definitions (4.18)–(4.22) become:

$$V(\mathbf{P}\langle\mathbf{y}\rangle) \mapsto \hat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{y}} V(\mathbf{P}\langle\mathbf{x}\rangle)\big| \leftrightarrow V\big(\hat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{y}}\mathbf{P}\langle\mathbf{x}\rangle\big|\big) \text{ if } \mathbf{y} \text{ has no occurrences in } \mathbf{P}\langle\mathbf{x}\rangle, \tag{4.18ε}$$

$$V(\mathbf{P}\langle\mathbf{z}\rangle) \mapsto \hat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{z}} V(\mathbf{P}\langle\mathbf{x}\rangle)\big| \leftrightarrow V\big(\hat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{z}}\mathbf{P}\langle\mathbf{x}\rangle\big|\big) \text{ if } \mathbf{z} \text{ has no occurrences in } \mathbf{P}\langle\mathbf{x}\rangle, \tag{4.19ε}$$

$$V(\mathbf{Q}\langle\mathbf{x}\rangle) \mapsto \hat{\mathrm{S}}^{\mathbf{y}}_{\mathbf{x}} V(\mathbf{Q}\langle\mathbf{y}\rangle)\big| \leftrightarrow V\big(\hat{\mathrm{S}}^{\mathbf{y}}_{\mathbf{x}}\mathbf{Q}\langle\mathbf{y}\rangle\big|\big) \text{ if } \mathbf{x} \text{ has no occurrences in } \mathbf{Q}\langle\mathbf{y}\rangle, \tag{4.20ε}$$

$$V(\mathbf{Q}\langle\mathbf{z}\rangle) \mapsto \hat{\mathrm{S}}^{\mathbf{y}}_{\mathbf{z}} V(\mathbf{Q}\langle\mathbf{y}\rangle)\big| \leftrightarrow V\big(\hat{\mathrm{S}}^{\mathbf{y}}_{\mathbf{z}}\mathbf{Q}\langle\mathbf{y}\rangle\big|\big) \text{ if } \mathbf{z} \text{ has no occurrences in } \mathbf{Q}\langle\mathbf{y}\rangle, \tag{4.21ε}$$

$$V(\mathbf{P}\langle\mathbf{y},\mathbf{y}\rangle) \mapsto \hat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{y}} V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle)\big| \leftrightarrow V\big(\hat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{y}}\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\big|\big). \tag{4.22ε}$$

3) The instances of axioms (4.4)–(4.6), (4.9)–(4.16), (4.30), (4.32)–(4.35), and (4.38) subject to (4.44) and (4.45) are *specific ones*. For instance,

$$V(\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{P}). \tag{4.4ε}$$

$$\text{If } \vdash[V(\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{Q})] \text{ then } \vdash[V(\mathbf{Q}) \mathrel{\hat{=}} V(\mathbf{P})]. \tag{4.32ε}$$

$$\text{If } \vdash [V(\mathbf{P}) \hat{=} V(\mathbf{Q})] \text{ and } \vdash [V(\mathbf{Q}) \hat{=} V(\mathbf{R})] \text{ then } \vdash [V(\mathbf{P}) \hat{=} V(\mathbf{R})]. \quad (4.33\varepsilon)$$

Consequently, the definition:

$$\begin{aligned} &\left[V\left(\mathbf{P}_{i_1}\right) \hat{=} V\left(\mathbf{P}_{i_2}\right) \hat{=} V\left(\mathbf{P}_{i_3}\right) \hat{=} \ldots \hat{=} V\left(\mathbf{P}_{i_{n-1}}\right) \hat{=} V\left(\mathbf{P}_{i_n}\right)\right] \\ \leftrightarrow &\left[V\left(\mathbf{P}_1\right) \hat{=} V\left(\mathbf{P}_2\right) \hat{=} V\left(\mathbf{P}_3\right) \hat{=} \ldots \hat{=} V\left(\mathbf{P}_{n-1}\right) \hat{=} V\left(\mathbf{P}_n\right)\right] \\ \rightarrow &\left[V\left(\mathbf{P}_1\right) \hat{=} V\left(\mathbf{P}_2\right), V\left(\mathbf{P}_2\right) \hat{=} V\left(\mathbf{P}_3\right), \ldots, V\left(\mathbf{P}_{n-1}\right) \hat{=} V\left(\mathbf{P}_n\right)\right] \\ \leftrightarrow &\left[V\left(\mathbf{P}_{i_1}\right) \hat{=} V\left(\mathbf{P}_{i_2}\right), V\left(\mathbf{P}_{i_2}\right) \hat{=} V\left(\mathbf{P}_{i_3}\right), \ldots, V\left(\mathbf{P}_{i_{n-1}}\right) \hat{=} V\left(\mathbf{P}_{i_n}\right)\right] \end{aligned} \quad (4.43\varepsilon)$$

is a like instance of definition (4.43).

4) The instances of axioms (4.24)–(4.29) subject to (4.46) and (4.18ε)–(4.22ε) are the following *specific ones*.

$$\hat{\forall}_{\mathbf{x}} V(\mathbf{P}) \hat{=} V(\mathbf{P}) \text{ if } \mathbf{x} \text{ does not occur in } \mathbf{P}. \quad (4.24\varepsilon)$$

$$\hat{\forall}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle) \hat{=} \hat{\forall}_{\mathbf{y}} V(\mathbf{P}\langle\mathbf{y}\rangle) \text{ subject to (4.18}\varepsilon). \quad (4.25\varepsilon)$$

$$\hat{\forall}_{\mathbf{x}} [V(\mathbf{P}) \hat{\vee} V(\mathbf{Q}\langle\mathbf{x}\rangle)] \hat{=} V(\mathbf{P}) \hat{\vee} [\hat{\forall}_{\mathbf{x}} V(\mathbf{Q}\langle\mathbf{x}\rangle)] \text{ if } \mathbf{x} \text{ does not occur in } \mathbf{P}. \quad (4.26\varepsilon)$$

$$\hat{\forall}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle) \hat{=} V(\mathbf{P}\langle\mathbf{y}\rangle) \hat{\vee} [\hat{\forall}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle)] \text{ subject to (4.18}\varepsilon). \quad (4.27\varepsilon)$$

$$\hat{\forall}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle) \hat{=} V(\mathbf{P}\langle\mathbf{y},\mathbf{y}\rangle) \hat{\vee} [\hat{\forall}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle)] \text{ subject to (4.22}\varepsilon). \quad (4.28\varepsilon)$$

$$\begin{aligned} &[\hat{\forall}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle)] \hat{\vee} [\hat{\forall}_{\mathbf{y}} V(\mathbf{Q}\langle\mathbf{y}\rangle)] \hat{=} \hat{\forall}_{\mathbf{x}} \hat{\forall}_{\mathbf{y}} [V(\mathbf{P}\langle\mathbf{x}\rangle) \hat{\vee} V(\mathbf{Q}\langle\mathbf{y}\rangle)] \\ &\hat{=} \hat{\forall}_{\mathbf{x}} [V(\mathbf{P}\langle\mathbf{x}\rangle) \hat{\vee} V(\mathbf{Q}\langle\mathbf{x}\rangle)] \hat{=} \hat{\forall}_{\mathbf{y}} [V(\mathbf{P}\langle\mathbf{y}\rangle) \hat{\vee} V(\mathbf{Q}\langle\mathbf{y}\rangle)] \\ &\hat{=} \hat{\forall}_{\mathbf{z}} [V(\mathbf{P}\langle\mathbf{z}\rangle) \hat{\vee} V(\mathbf{Q}\langle\mathbf{z}\rangle)] \text{subject to (4.18}\varepsilon)\text{-(4.21}\varepsilon). \end{aligned} \quad (4.29\varepsilon)$$

5) Each one of the *AnAtPLR's* (*analytical atomic panlogographic relations*) *of* A_1 on the list (1.1) and each one of the *AnAtPLI's* (*analytical atomic* panlogographic *integrons*) *of* A_1 on the list (1.2) are the most inclusive PLPH's of ER's (euautographic relations) of A_1 and the most inclusive PLPH's of EI's (euautographic integrons) of A_1 respectively. Therefore, the act of replacing of any AnAtPLR, – e.g. '**P**', – with any *AnCbPLR* (*analytical combined panlogographic relation*), which involves an occurrence of that AtAnPLR or not, – e.g. with any one of the AnCbPLR's defined in Dfs 1.7(1–3), 1.10(1–13), 2.1, and 2.2 or with '$[V(\mathbf{P}) \hat{=} 0]$', '$[V(\mathbf{P}) \hat{=} 1]$', or '$[V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle]$', – is a *specification* of that AnAtPLR, and hence the result of such a specification is a *specific* instance of that AnAtPLR. A like remark applies, *mutatis mutandis*, to an AnAtPLI as '**I**'.

6) The valid PLR (4.17) is a *universal* of axiom (4.2).•

Crl 4.3: *Some concretizations of PLS'ta.* 1) In order to treat of concrete euautographic instances of PLS'ta conveniently, I proceed from recalling or introducing some most basic pertinent terminology, which is based on Axs 4.1–4.20.

i) An *idempotent euautographic integron* (*IEI*) **i** *of* A_1, i.e. any one of the range of the *idempotent panlogographic integron* (*IPLI*) '**i**' (e.g.) *of* $\mathbf{A}_1$ satisfying the identity (4.17), is alternatively called a *euautographic validity-integron* (*EVI*) *of* A_1, whereas an IPLI is alternatively called a *panlogographic validity-integron* (*PLVI*) *of* $\mathbf{A}_1$.

ii) The digits 0 and 1 in this Light-Faced Roman Arial Narrow Type are collectively called the *atomic* (*At*) or *digital* (*D*) or *numeral* (*N*), *idempotent* (*I*) or *validity* (*V*), *euautographic* (*E*) or more specifically *pseudo-constant* (*PC*), *integrons* (*I's*) or *special terms* (*SpT's*) – briefly *AtIEI's*, *DIEI's*, or *NIEI's* and similarly with "*PC*" in place of "*E*" or "*SpT's*" in place of "*I's*" and also with "*EVI*" (for "*euautographic validity-integron*") in place "*IEI*". Individually, 0 and 1 are called *the validity-integron validity* and *the validity-integron antivalidity* respectively. In the respective *dual setup of* A_1, which will be described in due course later on, 0 and 1 are individually called *the antivalidity-integron antivalidity* and *the antivalidity-integron validity* respectively and therefore they are collectively called or the *euautographic antivalidity-integrons* (*EAVI's*). An IEI (EVI) is called a *non-digital one* (briefly *NDIEI* or *NDEVI*) if it is not digital.

iii) Given a ER **P** of A_1, the EI (ESpT) term V(**P**) is properly called *the primary*, or *initial*, *validity-integron* (briefly *PVI* or *IVI*) *of* **P** and also, commonly, a *primary non-digital idempotent euautographic integron* (*PNDIEI*) or a *primary non-digital euautographic validity-integron* (*PNDEVI*), [*of* A_1]. In the above common descriptive name, the qualifier "*non-digital*" ("*ND*") can be used interchangeably with "*non-numeral*" ("*NN*").

2) The most straightforward concretizations of the StAtPLPH's (structural atomic PLPH's) on the list (I.5.6) and of the AnAtPLPH's on the list (1.1) and of PLS'ta involving some of those and only of those StAtPLPH's or some of those and only of those AnAtPLPH's or both can be performed by the pertinent so-called *analo-homolographic* substitutions, under which an *atomic homolograph of one type is replaced with an analographic homolograph of the another type*, namely

$$\mathbf{u} \triangleright u,\ \mathbf{v} \triangleright v,\ \mathbf{w} \triangleright w,\ \mathbf{x} \triangleright x,\ \mathbf{y} \triangleright y,\ \mathbf{z} \triangleright z, \tag{4.47}$$

$$\mathbf{P} \triangleright p,\ \mathbf{Q} \triangleright q,\ \mathbf{R} \triangleright r,\ \mathbf{S} \triangleright s, \tag{4.48}$$

and similarly for the indexed homolographs (cf. Ax 8.1(2)). By (4.44) and (4.45), substitutions (4.48) imply that

$$\begin{gathered}\mathbf{I} \triangleright \mathbf{i} \triangleright V(\mathbf{P}) \triangleright V(p),\ \mathbf{J} \triangleright \mathbf{j} \triangleright V(\mathbf{Q}) \triangleright V(q),\\ \mathbf{K} \triangleright \mathbf{k} \triangleright V(\mathbf{R}) \triangleright V(r),\ \mathbf{L} \triangleright \mathbf{l} \triangleright V(\mathbf{S}) \triangleright V(s).\end{gathered} \tag{4.49}$$

3) The instances of axioms (4.1)–(4.4), (4.11), (4.12), (4.15), and (4.24) subject to (4.47) and (4.48) (or (4.49)), namely

$$V(p \veebar q) \,\hat{=}\, 1 \,\hat{-}\, V(p) \,\hat{\cdot}\, V(q), \tag{4.1μ}$$

$$V(p) \,\hat{\cdot}\, V(p) \,\hat{=}\, V(p), \tag{4.2μ}$$

$$V(V(p) \,\hat{=}\, V(q)) \,\hat{=}\, [V(p) \,\hat{-}\, V(q)]^2 \,\hat{\bar{=}}\, [V(p) \,\hat{-}\, V(q)] \,\hat{\cdot}\, [V(p) \,\hat{-}\, V(q)], \tag{4.3μ}$$

$$V(p) \,\hat{=}\, V(p), \tag{4.4μ}$$

$$0 \,\hat{+}\, V(p) \,\hat{=}\, V(p), \tag{4.11μ}$$

$$V(p) \,\hat{+}\, [\hat{-} V(p)] \,\hat{=}\, 0, \tag{4.12μ}$$

$$1 \,\hat{\cdot}\, V(p) \,\hat{=}\, V(p), \tag{4.15μ}$$

$$\hat{\cdot}_x V(p) \,\hat{=}\, V(p) \tag{4.24μ}$$

are concrete euautographic ones.

4) Like $V(\text{'}\mathbf{P}\text{'})$, $V(\text{'}\mathbf{Q}\text{'})$, etc (see Cmt 4.3), the PNDEVI’s (PNDIEI’s) $V(p)$, $V(q)$, $V(r)$, and $V(s)$ are irreducible, so that $\mathbf{i}_{\sim}|p\rangle$, e.g., is the same as $V(p)$, i.e . $\mathbf{i}_{\sim}|p\rangle \leftrightarrow V(p)$. Therefore, *the predicate* $\vdash$ *is not applicable* either to $[V(p) \,\hat{=}\, 0]$ or to $[V(p) \,\hat{=}\, 1]$, whereas $[V(p) \,\hat{=}\, \mathbf{i}_{\sim}|p\rangle]$ is the same as $[V(p) \,\hat{=}\, V(p)]$, i.e. $[V(p) \,\hat{=}\, \mathbf{i}_{\sim}|p\rangle]$ $\leftrightarrow [V(p) \,\hat{=}\, V(p)]$. Hence, the variants of the items (a) and (b) of axiom (4.40) are ineffective (inapplicable), whereas the item (c) of that axiom becomes:

$$\dashv\!\vdash p \text{ if and only if } \vdash [V(p) \,\hat{=}\, V(p)], \text{(c)} \tag{4.40μ}$$

which is effective thus saying that *p is a vav-neutral* (*vav-indeterminate*) *ER of* A_1.

5) Besides substitutions (4.49), any one of the AnAtPLPH’s ‘**I**’ to ‘**L**’, ‘**i**’ to ‘**l**’, and '$V(\mathbf{P})$’ to '$V(\mathbf{S})$’ can be replaced with 0 or 1. Therefore, axioms (4.4), (4.11), (4.12), (4.15), and (4.24) can be concretized thus:

$$0 \,\hat{=}\, 0,\ 1 \,\hat{=}\, 1, \tag{4.4μ₁}$$

$$0 \,\hat{+}\, 0 \,\hat{=}\, 0,\ 0 \,\hat{+}\, 1 \,\hat{=}\, 1, \tag{4.11μ₁}$$

$$0 \,\hat{+}\, [\hat{-} 0] \,\hat{=}\, 0,\ 0 \,\hat{+}\, [\hat{-} 1] \,\hat{=}\, 0, \tag{4.12μ₁}$$

$$1 \hat{\cdot} 0 \mathrel{\hat{=}} 0,\ 1 \hat{\cdot} 1 \mathrel{\hat{=}} 1, \tag{4.15μ1}$$

$$\hat{\cdot}_x 0 \mathrel{\hat{=}} 0, \hat{\cdot}_x 1 \mathrel{\hat{=}} 1, \tag{4.24μ1}$$

whereas the identity $0 \hat{\cdot} 0 \mathrel{\hat{=}} 0$ is a concrete instance of the theorem $0 \hat{\cdot} \mathbf{I} \mathrel{\hat{=}} 0$ of A_1, which will be proved from certain items of Ax 4.6 in the next section.

6) By (4.48), any one of the PLS'ta of PNDEVI's '*V*(**P**)' to '*V*(**S**)' can be replaced with any so-called *basic*, or *predicate-free*, *molecular*, *euautographic validity-integron* (briefly *BMlEVI* or *PFrMlEVI*) *of* A_1 of the following infinite list:

$$\begin{gathered} V(p), V(q), V(r), V(s), V(p_1), V(q_1), V(r_1), V(s_1), \\ V(p_2), V(q_2), V(r_2), V(s_2), \ldots, \end{gathered} \tag{4.50}$$

which represents the range of the StMlPLPH '*V*(**p**)'.

7) Besides the substitutions (4.48), any one of the AnAtPLR's '**P**' to '**S**' can be replaced with any one of the *MlEOR's* (*molecular euautographic ordinary relations*) such as:

$$\begin{gathered} f^1(u), g^2(v,w), h^3(x,y,z), = (x,y) \text{ or } [x = y], \\ \subseteq (x,y) \text{ or } [x \subseteq y], \in (x,y) \text{ or } [x \in y], \text{ etc,} \end{gathered} \tag{4.51}$$

being some concrete euautographic instances of the range of the *MlMLPH* (*molecular metalogographic placeholder*) '$\mathbf{f}^n(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$' with all $n \mathrel{\vec{\in}} \omega_1$ or, concurrently, ones of the range of the AnAtPLPH '$\boldsymbol{\pi}$' defined in Df 1.3(4). Consequently, besides (4.50), any one of the PLS'ta of PNDEVI's '*V*(**P**)' to '*V*(**S**)' can be replaced with any one of the so-called *rich basic*, or *predicate-containing*, *molecular euautographic validity-integrons* (*RBMlEVI's* or *PCtMlEVI's*) such as:

$$\begin{gathered} V(f^1(u)), V(g^2(v,w)), V(h^3(x,y,z)), V(= (x,y)) \text{ or } V(x = y), \\ V(\subseteq (x,u)) \text{ or } V(x \subseteq u), V(\in (x,u)) \text{ or } V(x \in u), \text{ etc,} \end{gathered} \tag{4.52}$$

which are some concrete euautographic instances of the range of the *rich basic molecular metalogographic placeholder* (*RBMlMLPH*) '$V(\mathbf{f}^n(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n))$' with all $n \mathrel{\vec{\in}} \omega_1$ or, concurrently, ones of the range of AnMlPLPH '*V*($\boldsymbol{\pi}$)'. Therefore, axiom (4.23) can be concretized, e.g., as any one of the following identities:

$$\begin{gathered} V(\vee_u f^1(u)) \mathrel{\hat{=}} \hat{\cdot}_u V(f^1(u)), V(\vee_w g^2(v,w)) \mathrel{\hat{=}} \hat{\cdot}_w V(g^2(v,w)), \\ V(\vee_x h^3(x,y,z)) \mathrel{\hat{=}} \hat{\cdot}_z V(h^3(x,y,z)), \\ V(\vee_x [x = y]) \mathrel{\hat{=}} \hat{\cdot}_x V(x = y), V(\vee_x [x \subseteq y]) \mathrel{\hat{=}} \hat{\cdot}_x V(x \subseteq y), \\ V(\vee_y [x \in y]) \mathrel{\hat{=}} \hat{\cdot}_y V(x \in y). \end{gathered} \tag{4.23μ}$$

8) In accordance with Df 1.3(4,5), a BMlEVI $V(\mathbf{p})$ or an RBMlEVI $V(\boldsymbol{\pi})$ is an MlEVI $V(\mathsf{p})$ and vice versa. Every BMlEVI $V(\mathbf{p})$ is a basic molecular *pseudo-variable* validity-integron (BMl*PV*VI) and vice versa. An RBMlEVI $V\left(\mathbf{f}^{n}\left(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_n\right)\right)$ is a rich basic molecular *pseudo-variable* validity-integron (RBMl*PV*VI) if at least one of the atomic euautographs $\mathbf{f}^{n}$, $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_n$ is a pseudo-variable and a rich basic molecular *pseudo-constant* validity-integron (RBMl*PC*VI) if all the above-mentioned atomic euautographs are pseudo-constants. A *B*MlPVVI or an *RB*MlPVVI is an MlPVVI and vice versa. An *RB*MlPCVI is an MlPCVI and vice versa. An Ml*PV*VI or an Ml*PC*VI is an Ml*E*VI (molecular *euautographic* validity-integron), i.e. $V(\mathsf{p})$, and vice versa. An EVI (IEI) is called an *elemental*, or *primitive*, *one* (briefly *ElEVI* or *ElIEI*) if and only if it is either an AtEVI (AtIEI, DEVI, DIEI) (see Crl 4.3(1)**)** or an MlEVI (MlIEI), while an MlEVI (MlIEI) is alternatively called a *non-digital one* (*NDMlEVI*, *NDMlIEI*, *MlNDEVI*, *MlNDIEI*). Consequently, an NDEVI (NDIEI) is called a *complex one* (*CxNDEVI*, *CxNDIEI*, *NDCxEVI*, *NDCxIEI*) if and only if it is not a molecular one, i.e. not $V(\mathsf{p})$, and hence if and only if it has either at least one occurrence of a euautographic ordinary kernel-sign other than a predicate-sign or at least one occurrence of a special euautographic kernel-sign other than V. It is understood that an NDEVI (NDIEI) is either an MlNDEVI (MlNDIEI) or a CxNDEVI (CxNDIEI).

9) In general, any one of the AnAtPLR's '**P**' to '**Q**' can be replaced with any ER satisfying any pertinent formation rule of A_1, whereas any one of the AnAtPLI's '**I**' to '**L**' can be replaced with any EI satisfying any pertinent formation rule.•

Crl 4.4: ***Some semi-concretizations of PLS'ta.*** 1) Under the substitutions $\mathbf{x} \triangleright x$, $\mathbf{y} \triangleright y$, and $\mathbf{z} \triangleright z$ (see (4.47)), definitions (4.18)–(4.22) become:

$$\mathbf{i}\langle y\rangle \to \widehat{\mathrm{S}}_y^x\mathbf{i}\langle x\rangle\Big| \text{ if } y \text{ has no occurrences in } \mathbf{i}\langle x\rangle, \tag{4.18γι}$$

$$\mathbf{i}\langle z\rangle \to \widehat{\mathrm{S}}_z^x\mathbf{i}\langle x\rangle\Big| \text{ if } z \text{ has no occurrences in } \mathbf{i}\langle x\rangle, \tag{4.19γι}$$

$$\mathbf{j}\langle x\rangle \to \widehat{\mathrm{S}}_x^y\mathbf{j}\langle y\rangle\Big| \text{ if } x \text{ has no occurrences in } \mathbf{j}\langle y\rangle, \tag{4.20γι}$$

$$\mathbf{j}\langle z\rangle \to \widehat{\mathrm{S}}_z^y\mathbf{j}\langle y\rangle\Big| \text{ if } z \text{ has no occurrences in } \mathbf{j}\langle y\rangle, \tag{4.21γι}$$

$$\mathbf{i}\langle y,y\rangle \to \widehat{\mathrm{S}}_y^x\mathbf{i}\langle x,y\rangle\Big|. \tag{4.22γι}$$

Consequently, definitions (4.18ε)–(4.22ε) and axioms (4.23)–(4.29) and (4.24ε)–(4.29ε) are, *mutatis mutandis*, valid with x, y, z in place of '**x**', '**y**', '**z**' respectively.

2) On the other hand, axiom (4.24) can be particularized (semi-concretized) thus:

$$\hat{\cdot}_{\mathbf{x}} V(p) \hat{=} V(p), \tag{4.24γι}$$

$$\hat{\cdot}_{\mathbf{x}} 0 \hat{=} 0, \hat{\cdot}_{\mathbf{x}} 1 \hat{=} 1, \tag{4.24γι$_1$}$$

instead of (4.24μ) and (4.24μ_1) respectively.•

4.6. Formal aspects of extension of Axs 4.1–4.20 from A_1 to $\mathbf{A}_1$

Every PLAx, i.e. *panlogographic axiomatic identity*, *of* $\mathbf{A}_1$ and every meta-axiom of inference or decision of A_1 necessarily involves one or more occurrences (tokens) of one or more *analytical atomic PLR's* (*AnAtPLR's*) as '**P**' to '**J**', *analytical atomic panlogographic integrons* (*AnAtPLI's*) as '**I**' to '**L**' or as '**i**' to '**l**', or *stractural atomic panlogographic ordinary terms* (*StAtPLOT's*) as '**u**' to '**z**'. When a PLAx is used *xenonymously* as a PLS of EAx's of A_1 of its range, every *atomic panlogograph* (*AtPL*), *analytical one* (*AnAtPL*) or *structural one* (*StAtPL*), occurring in the PLS is also used *xenonymously* as the corresponding *atomic PLPH* (*AtPLPH*), *analytical one* (*AnAtPLPH*) or *structural one* (*StAtPLPH*) respectively, *of euautographs of* A_1 *of its range*. Every AtPL, in a meta-axiom of inference or decision of A_1 is used xenonymously as an AtPLPH in the same way. In any xenonymous occurrence, an AtPLPH has the following *psychologistic* (*psychological and logical*) *peculiarity* (*peculiar property*).

i) An AtPLPH can be used for mentioning a *general* (*common, certain, concrete but not concretized*) *euautograph of its range*, which is just another mental (psychical) hypostasis of the range.

ii) The AtPLPH can be replaced with any *concrete* (*concretized*) *euautograph of its range.*

iii) The AtPLPH can be replaced with any concrete *specified PLPH* (*SPLPH*) *of its range*, i.e. with any concrete PLPH, whose range is a *species* (*strict subclass*) *of the range of the AtPLPH*. For instance, '**P**' can be replaced with any one of the *analytical combined PLR's* (*AnCbPLR's*) defined in Dfs 1.7(1–3), 1.10(1–13), 2.1, and 2.2 or with '$\left[V(\mathbf{P}) \hat{=} 0\right]$', '$\left[V(\mathbf{P}) \hat{=} 1\right]$', or '$\left[V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim} \middle| \mathbf{P} \right\rangle\right]$', and '**u**' can be replaced with either one of the StAtPLOT's '$\mathbf{u}^{pv}$' and '$\mathbf{u}^{pc}$'.

iv) An SPLPH can then be used xenonymously in analogy with AtPLPH as indicated in the above items i–iii.

In accordance with the above items iii and iv, any concrete AtPLPH that is initially defined as one of any concrete euautograph of A_1 of its range turns also into an AtPLPH of any concrete SPLPH, i.e. a PLPH whose range is a *species* (*specific class*, *strict subclass*) of the range of the AtPLPH. Therefore, enclosing the AtPLPH in *light-faced* curly or straight single quotation marks, ‘ ’ or ' ', called *homolographic*, or *photographic*, *quotation* (*HAQ*) *marks*, is not appropriate for indicating formally that the AtPLPH is used for denoting a *concrete endosemasiopasigraph* (*EnSPG*), i.e. a *concrete euautograph or panlogograph*, being a *concrete instance* of the AtPLPH. This can be done in either one of the two ways as stated below in the respective two meta-axioms, Axs 4.21 and 4.22. Ax 4.22 is based on Df 4.9 preceding it.

****Ax 4.21.** In order to extend the set (conjunction) of Axs 4.1–4.20 from of A_1 to $\mathbf{A}_1$ formally, each occurrence of each AtPLPH such as ‘**P**’ to ‘**S**’, ‘**I**’ to ‘**L**’, or ‘**i**’ to ‘**l**’¸ which is not followed either by an occurrence of ⟨ ⟩ or by an occurrence of | ⟩, or such as or ‘**u**’ to ‘**z**’ throughout Axs 4.1–4.20 should be enclosed in *bold-faced* curly or straight single quotation marks, **‘ ’** or **' '**, called *quasi-homolographic*, or *quasi-photographic*, *quotation* (*QHAQ*) *marks*, the understanding being that upon replacing the AtPLPH with a concrete EnSPG, being its instance, the QHAQ marks should either be omitted if the EnSPG is a euautograph or they should be replaced with HAQ marks if the EnSPG is a panlogograph.•

Df 4.9. The following logographs in the **Bold-Faced Roman Comic Sans MS Type** are *analytical atomic metalogographic placeholders* (*AnAtMLPH*’s) *of panlogographic formulas of specific classes* as specified.

1) Each one of the four letters ‘**P**’ to ‘**S**’ is an AnAtMLPH, whose range is the class of all PLR’s of $\mathbf{A}_1$, unless it is restricted somehow, e.g. by attributing the qualifier “*of academic or practical interest*” to the taxonym (count name) “*PLR of* $\mathbf{A}_1$”.

2) Each one of the four letters ‘**I**’ to ‘**L**’ is an AnAtMLPH, whose range is the class of all PLI’s of $\mathbf{A}_1$, unless stated otherwise.

3) Each one of the four letters ‘**i**’ to ‘**l**’ is an AnAtMLPH, whose range is the class of all idempotent PLI’s of $\mathbf{A}_1$, so that

$$\mathsf{i}\,\hat{\cap}\,\mathsf{i} \mathrel{\hat{=}} \mathsf{i} \tag{4.53}$$

in analogy with (4.17), and similarly with '**j**', '**k**', or '**l**' in place of '**i**'.

4) Each one of the six letters '**u**' to '**z**' is StAtMLPH, whose range is the class of all PLOT's of $\mathbf{A}_1$.

5) Any of the above letters can, if desired, be furnished with any of the upright Arabic numeral subscripts '$_1$', '$_2$', etc in this font or with any number of primes or both thus becoming another AtMLPH with the same range.•

Ax 4.22. In order to extend the set (conjunction) of Axs 4.1–4.20 from of A_1 to $\mathbf{A}_1$ formally, each occurrence of each one of the AtPLPH's 'P**' to '**S**', '**I**' to '**L**', and '**i**' to '**l**'¸ which is not followed either by an occurrence of ⟨ ⟩ or by an occurrence of | ⟩, and each occurrence of each one of the letters '**u**' to '**z**', throughout Axs 4.1–4.20 should be replaced with an occurrence of the respective *analo-homolographic AtMLPH* '**P**' to '**S**', '**I**' to '**L**', '**i**' to '**l**'¸ or '**u**' to '**z**', i.e.

$$\mathsf{P} \mapsto \mathbf{P},\ \mathsf{Q} \mapsto \mathbf{Q},\ \mathsf{R} \mapsto \mathbf{R},\ \mathsf{S} \mapsto \mathbf{S}, \tag{4.54}$$

$$\mathsf{I} \mapsto \mathbf{I},\ \mathsf{J} \mapsto \mathbf{J},\ \mathsf{K} \mapsto \mathbf{K},\ \mathsf{L} \mapsto \mathbf{L}, \tag{4.55}$$

$$\mathsf{i} \mapsto \mathbf{i},\ \mathsf{j} \mapsto \mathbf{j},\ \mathsf{k} \mapsto \mathbf{k},\ \mathsf{l} \mapsto \mathbf{l}, \tag{4.56}$$

$$\mathsf{u} \mapsto \mathbf{u},\ \mathsf{v} \mapsto \mathbf{v},\ \mathsf{w} \mapsto \mathbf{w},\ \mathsf{x} \mapsto \mathbf{x},\ \mathsf{y} \mapsto \mathbf{y},\ \mathsf{z} \mapsto \mathbf{z}, \tag{4.57}$$

without any quotation marks.•

For instance, the following two meta-axioms, Ax 4.20_1 and Ax 4.20_2, are variants of Ax 4.20 subject to Ax 4.21 and subject to Ax 4.22 respectively; Ax 4.20_1 is called the *quasi-homoloautographic variant of Ax 4.20*, and Ax 4.20_1 is called the *analo-homolographic variant* of Ax 4.20.

Ax 4.20_1: *The quasi-homoloautographic variant of Ax 4.20 – The Conjoined Three-Fold Basic Rule of Decision and Inference (BDIR) of* A_1 *and* $\mathbf{A}_1$.

$$\vdash \text{'}\mathbf{P}\text{'} \text{ if and only if } \vdash\left[V\left(\text{'}\mathbf{P}\text{'}\right) \mathrel{\hat{=}} 0\right], \tag{a}$$

$$\dashv \text{'}\mathbf{P}\text{'} \text{ if and only if } \vdash\left[V\left(\text{'}\mathbf{P}\text{'}\right) \mathrel{\hat{=}} 1\right], \qquad \text{(b)} \tag{4.40$_1$}$$

$$\dashv\vdash \text{'}\mathbf{P}\text{'} \text{ if and only if } \vdash\left[V\left(\text{'}\mathbf{P}\text{'}\right) \mathrel{\hat{=}} \mathbf{i}_{\sim}\middle|\text{'}\mathbf{P}\text{'}\right\rangle\right], \tag{c}$$

the understanding being that upon replacing the AtPLR ‘**P**’ with a concrete EnSPG, being its instance, the QHAQ marks should either be omitted if the EnSPG is a euautograph or they should be replaced with HAQ marks if the EnSPG is a panlogograph, – in accordance with Ax 4.21.•

Ax 4.20$_2$: *The analo-homolographic variant of Ax 4.20 – The Conjoined Three-Fold Basic Rule of Decision and Inference (BDIR) of* $\mathbf{A}_1$

$$\vdash \mathbf{P} \text{ if and only if } \vdash [V(\mathbf{P}) \hat{=} 0], \tag{a}$$

$$\dashv \mathbf{P} \text{ if and only if } \vdash [V(\mathbf{P}) \hat{=} 1], \tag{b}$$
(4.40$_2$)

$$+\mathbf{P} \text{ if and only if } \vdash [V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle], \tag{c}$$

where $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ is a certain *IRNDIPLI* (*irreducible non-digital idempotent panlogographic integron*) that is analogous to an IRNDIEI $\mathbf{i}_{\sim}|\mathbf{P}\rangle$. Just as in Ax 4.20, the “if”-parts of the above three meta-axioms (a)–(c) are *the basic rules of decision of* $\mathbf{A}_1$ *with respect to the validity-values validity, antivalidity, and vav-neutrality* (*vav-indeterminacy*) respectively, while the “only if”-part of the meta-axiom (a), but not that of (b) or (c), is the *rule of inference of the panlogographic special algebraic identity* (*valid equality*) $[V(\mathbf{P}) \hat{=} 0]$ *of* $\mathbf{A}_1$ *from a valid PLR* $\mathbf{P}$.•

4.7. Taxonomy of ADM’s A_1 and $\mathbf{A}_1$

Df 4.10. 1) The set (conjunction) of Axs 4.1–4.20 and Crl 4.1 is denoted by:

a) ‘D_1’, when the pertinent items of it apply xenonymously either to a concrete ER of A_1 or to a concrete PLPH of ER’s of A_1 as to such;

b) ‘$\mathbf{D}_1$’, when the pertinent items of it apply to a concrete PLPH of ER’s of A_1 autonymously as to a concrete PLR of $\mathbf{A}_1$;

c) ‘D_1’, when the pertinent items of it apply to a concrete PLPH of ER’s of A_1 as if simultaneously but intermittently in two different mental modes: xenonymously as to a such and autonymously as to PLR’s of $\mathbf{A}_1$, i.e. in the TAEXA-mode (xenoautonymously, autoxenonymously) as to an EnSPG (endosemasiopasigraph) of A_1.

2) The set (conjunction) of 4.1–4.6, Crl 4.1, and Axs 4.13–4.20 is denoted by:

a) 'D_1^0', when the pertinent items of it apply xenonymously either to a concrete ER of A_1^0 or to a concrete PLPH of ER's of A_1^0 as to such;

b) '$\mathbf{D}_1^0$', when the pertinent items of it apply to a concrete PLPH of ER's of A_1^0 autonymously as to a concrete PLR of $\mathbf{A}_1^0$;

c) 'D_1^0', when the pertinent items of it apply to a concrete PLPH of ER's of A_1^0 as if simultaneously but intermittently in two different mental modes: xenonymously as to a such and autonymously as to PLR's of $\mathbf{A}_1^0$, i.e. in the TAEXA-mode (xenoautonymously, autoxenonymously) as to an EnSPG (endosemasiopasigraph) of A_1^0.

3) The set (conjunction) of 4.1–4.6, Crl 4.1, and Axs 4.13–4.17, 4.19, and 4.20 (i.e. Axs 4.13–4.20 in the exclusion of Ax 4.18) is denoted by:

a) 'D_0', when the pertinent items of it apply xenonymously either to a concrete ER of A_0 or to a concrete PLPH of ER's of A_0 as such;

b) '$\mathbf{D}_0$', when the pertinent items of it apply to a concrete PLPH of ER's of $\mathbf{A}_0$ autonymously as to a concrete PLR of $\mathbf{A}_0$;

c) 'D_0', when the pertinent items of it apply to a concrete PLPH of ER's of A_0 as if simultaneously but intermittently in two different mental modes: xenonymously as to a such and autonymously as to PLR's of $\mathbf{A}_0$, i.e. in the TAEXA-mode (xenoautonymously, autoxenonymously) as to an EnSPG (endosemasiopasigraph) of A_0.

xenonymously as to such and autonymously as to PLR's of $\mathbf{A}_0$, i.e. intermittently (xenoautonymously, autoxenonymously, in the TAEXA-mode) as to EnSPG (endosemasiopasigraph) of A_0.

4) The following phonoxenographic (wordy, verbal) terminology is in agreement with the pertinent one, which has been introduced earlier in Dfs I.3.1(1,20).

i) D_1 and is called the *Advanced Algebraic Decision Method* (*AADM*) *of* A_1 and also the *Euautographic AADM* (*EAADM*). D_1^0 is called the *Rich Basic Algebraic Decision Method* (*RBADM*) *of* A_1^0 *and* A_1 and also the *Euautographic RBADM* (*ERBADM*). D_0 is called in that order the *Basic*, or *Depleted Basic*, *Algebraic*

Decision Method (*BADM* or *DBADM*) *of* A_0, A_1^0, *and* A_1 and also the *Euautographic BADM* (*EBADM*) or *Euautographic DBADM* (*EDBADM*).

ii) $\mathbf{D}_1$ is called the *AADM of* $\mathbf{A}_1$ and also the *Panlogographic AADM* (*PLAADM*) or the *Autonymous*, or *Panlogographic*, *Extension of the EAADM*, in accordance with item 6 of subsection 4.1. $\mathbf{D}_1^0$' and is called the *RBADM of* $\mathbf{A}_1^0$ *and* $\mathbf{A}_1$ and also the *Panlogographic RBADM* (*PLRBADM*) or the *Autonymous*, or *Panlogographic*, *Extension of the ERBADM*. D_0 is called in that order the *Basic*, or *Depleted Basic*, *Algebraic Decision Method* (*BADM* or *DBADM*) *of* A_0, A_1^0, *and* A_1 and also the *Panlogographic BADM* or *DBADM* (*PLBADM* or *PLDBADM*), or the *Autonymous*, or *Panlogographic*, *Extension of the EBADM*.

iii) D_1 is the union and at the same time a superposition of D_1 and $\mathbf{D}_1$, which is called the *AADM of* A_1 and also the *Endosemasiopasigraphic* (*EnSPG*), or *Biune Euautographic and Panlogographic* (*BUE&PL*), *AADM* (*EnSPGAADM* or *BUE&PLAADM*). D_1^0 is the union and superposition of D_1^0 and $\mathbf{D}_1^0$, which is called the *RBADM of* A_1^0 *and* A_1 and also the *Endosemasiopasigraphic* (*EnSPG*), or *Biune Euautographic and Panlogographic* (*BUE&PL*), *RBADM* (*EnSPGRBADM* or *BUE&PLRBADM*). D_0 is the union and superposition of D_0 and $\mathbf{D}_0$, which is called the *BADM*, or *DBADM*, *of* A_0, A_1^0, *and* A_1 and also the *Endosemasiopasigraphic* (*EnSPG*), or *Biune Euautographic and Panlogographic* (*BUE&PL*), *BADM* (*EnSPGAADM* or *BUE&PLAADM*).

5) It is understood that all primary rules of inference and decision, Axs 4.13–4.20, are recursive in the sense that their *premises* can be either any euautographic or panlogographic instances of Axs 4.1–4.12 or any subject euautographic or panlogographic theorems, which can be proved from some of the subject axioms by those same rules.

5. The integronic domain of A_1

5.1. The integronic domains of A_1, A_1^0, and A_0 defined

Df 5.1. 1) Let R_1 be the class of ER's (euautographic relations) of A_1 and I_1 be the class of EI's (euautographic integrons) of A_1. In other words, R_1 and I_1 are the classes, which are verbally denoted by the count names "relation of A_1" or "ER of A_1"

and "integron of A_1" or "EI of A_1" respectively. Both classes are determined by the primary and secondary formation rules of A_1. Thus, R_1 is the range of the AnAtPLR '**P**' (e.g.), whereas I_1 is the range of the AnAtPLI '**I**' (e.g.). The class I_1 contains the *digital idempotent euautographic integrons* (*DIEI's*) 0 and 1 and, for each ER **P**, it also contains the *non-digital idempotent euautographic integron* (*NDIEI*) $V(\mathbf{P})$ along with, when applicable, all its successive algebraic forms $\mathbf{i}_1|\mathbf{P}\rangle$, $\mathbf{i}_2|\mathbf{P}\rangle$, $\ldots\mathbf{i}_n|\mathbf{P}\rangle$, which are obtained by reduction of $V(\mathbf{P})$ in the EADP of **P**. This reduction is made with the help of the respective subject axioms and rules of inference comprised in D_1, i.e. by successively eliminating the logical operators occurring in **P** and by reducing the results properly. In this case, $\mathbf{i}_1|\mathbf{P}\rangle$ to $\mathbf{i}_{n-1}|\mathbf{P}\rangle$ are supposed to be *intermediate* and hence *reducible algebraic forms* relative to some of the *substantival algebraic operators* (*kernel-signs*) *of* A_1: $\hat{-}$, $\hat{+}$, $\hat{\cdot}$, $\hat{\cdot}_{\mathbf{x}}$, whereas $\mathbf{i}_n|\mathbf{P}\rangle$ is supposed to equal 0 or 1 or $V(\mathsf{p})$, or else a certain *ultimate*, and hence *irreducible*, *algebraic form*, $\mathbf{i}|\mathbf{P}\rangle$ of the above kind in terms of IEI's such as 1, $V(\mathsf{p})$, $V(\mathsf{q})$, etc. Thus, algebraic forms of the above kind arise every time when the *primary validity-integron* (*PVI*) $V(\mathbf{P})$ of a complex ER **P** is computed with the purpose to reduce $V(\mathbf{P})$ to a certain *irreducible* (*ultimate*) *form* that is either 0 or 1 or else an irreducible IEI other than 0 or 1. Besides the above IEI's, I_1 also contains all non-idempotent integrons of the range of '**I**', particularly digital ones, which are determined by the pertinent secondary formation rules of A_1 and which will rigorously be defined in subsection 5.3. Thus, *to every relation* **P** *there is its PVEI* along with all is transforms (if exist), while to any two integrons **I** and **J** there is the relation $\mathbf{I}\hat{=}\mathbf{J}$, valid or not. Therefore, the member populations of the classes R_1 and I_1 are not fixed and are interrelated like the contents of communicating vessels. Hence, the two classes *are not sets*.

2) The class I_1, whose members are united by the operators $\hat{-}$, $\hat{+}$, $\hat{\cdot}$, $\hat{\cdot}_{\mathbf{x}}$ subject to the axioms (4.2), (4.7)–(4.17), (4.24)–(4.29), (4.34), and (4.35), is denoted by '$Z_1(I_1,\hat{-},\hat{+},\hat{\cdot},\hat{\cdot}_{\mathbf{x}})$' or briefly by '$Z_{11}$' and is called the *Advanced Integronic Domain* (*AID*) *of* A_1.

3) Let I_1^0 be the restriction of I_1, and hence the restriction of the range of '**I**' (e.g.), to the class of *predicate-free* relations of A_1. The class I_1^0, whose members are

united by the operators $\hat{-}$, $\hat{+}$, and $\hat{\cdot}$, subject to the axioms (4.2), (4.7)–(4.17), (4.34), and (4.35), is denoted by ‘$\mathsf{Z}_0\left(\mathsf{I}_1^0,\hat{-},\hat{+},\hat{\cdot}\right)$’ or briefly by ‘$\mathsf{Z}_{01}^0$’ and is called the *Rich Basic Integronic Domain* (*RBID*) *of* A_1..

4) The item 1 applies word for word with the subscript $_0$ in place of $_1$, the understanding being that the ranges of ‘**P**’ and ‘**I**’ are restricted to the class of ER’s of A_0 and to the class of EI’s of A_0 respectively. It will be recalled that, in contrast to A_1, *all* terms of A_0 are *special* ones, i.e. *integrons*. The class I_0, whose members are united by the operators $\hat{-}$, $\hat{+}$, and $\hat{\cdot}$, subject to the axioms (4.2), (4.7)–(4.17), (4.34), and (4.35), is denoted by ‘$\mathsf{Z}_0\left(\mathsf{I}_0,\hat{-},\hat{+},\hat{\cdot}\right)$’ or briefly by ‘$\mathsf{Z}_{00}$’ and is called the *Integronic Domain* (*ID*) *of* A_0 and also the *Basic*, or *Depleted Basic*, *Integronic Domain* (*BID* or *DBID*). Z_{00} and Z_{01}^0 differs only in their underlying classes I_1^0 and I_0.•

Cmt 5.1. 1) In algebra, a system of abstract or concrete substantive objects and of operations on the objects, which satisfy the conjunction of axioms such as (4.7)–(4.16), (4.34), and (4.35), subject to the three fundamental laws of logic for equality such as (4.4), 4.32), and (4.33), i.e. a system that is based primarily on the same axioms as Z_{01}^0 or Z_{00}, is called an *integral domain* (see, e.g., Birkhoff and Mac Lane [1965, pp. 1–3] or Mac Lane & Birkhoff [1967, pp. 132–134]). An algebraic integral domain is a *system*, or from a somewhat different viewpoint a *predicate* (as ‘Z_0’ in ‘$\mathsf{Z}_0\left(\mathsf{I}_1^0,\hat{-},\hat{+},\hat{\cdot}\right)$’ or ‘$\mathsf{Z}_0\left(\mathsf{I}_0,\hat{-},\hat{+},\hat{\cdot}\right)$’) of its underlying set, comprising *abstract* (*mental*) *objects* as functions or numbers, and of the *operations* (*functions*) of additive inversion (-), addition (+), and multiplication (·), which are defined on that set. Both the elements of the underlying set and the functions defined on the set are mentioned by using the appropriate logographic variables and constants.

2) The predicates (systems, domains) Z_{01}^0 and Z_{00} are qualified *integronic*, and not *integral*, while their objects are accordingly called *integrons*, and not, say, *integers*, in order to emphasize the fact that either of the two systems is governed by the same set of axioms as an integral domain and that at the same time the former essentially differs from the latter. Since Z_{11} is an extension of Z_{01}^0, therefore the same nomenclature is applied to it by analogy. The difference between Z_{01}^0 or Z_{00} and an integral domain is explicated below.

a) An integral domain is a *self-contained system* because its *underlying class* is a *set*, i.e. a class, which has a permanent member population and which *can* therefore *be ordered* in the sense that it can be used as a *domain of definition of the linear order relation* ≤. I call a class a *regular class* if it is a set and an *irregular class* if otherwise (see subsection I.9.3). In the contemporary literature on logic and mathematics, an irregular class, i.e. a nonempty class not being a set, is called a *proper class*, whereas a regular class, i.e. a set, is sometimes called a *small class* (see, e.g., Fraenkel et al [1973, pp. 128, 134–135, 167] for the former term or the article «**class**» in Wikipedia for both terms).

In contrast to an integral domain, which is a *closed* (*self-contained*) *genuine algebraic system*, Z_{01}^{0} is an *integral built-in part of* D_{1}^{0}, which is *inseparably associated with* A_{1}^{0} via D_{1}^{0}. The underlying class I_{1}^{0} of Z_{01}^{0} is therefore an *irregular* one (cf. Df 5.1(1)), so that Z_{01}^{0} is a *quasi-algebraic system*. Z_{00} is a further, *autonomous, restriction* o*f* Z_{01}^{0}, which is inseparably associated with A_{0} via D_{0} in the same way as Z_{01}^{0} is associated with A_{1}^{0} via D_{1}^{0}.

b) All integrons and all operators of Z_{01}^{0} or Z_{00} are *euautographs*, i.e. *uninterpreted graphic* and hence *visible* objects – like chessmen. In this case, the sign $\hat{-}$ is put before an integron to produce another integron, which is conventionally called the *additive inverse of the former*, whereas either sign $\hat{+}$, $\hat{\cdot}$, or $\hat{-}$ (defined by Df 1.10(14) as $\hat{-} \rightarrow \hat{+}\hat{-}$) is put between two integrons to produce another integron, which is conventionally called the *sum*, *product*, or *difference of the former two* respectively. Putting $\hat{=}$ between two integrons produces a *special relation*, conventionally called a *special equality*. A valid special equality is called a *special identity* or simply an identity whenever there is no danger of confusing it with an *ordinary identity* that is stated with the sign =. When treated in general, integrons are represented by the appropriate *panlogographic placeholders* (*PLPH's*), i.e. *place-holding variables*, and not "*abstract metalogographic variables*" ("*AbMLV*"); "abstract" means *not place-holding* or *non-place-holding*,

c) Besides the two AtIEI's (AtEVI's) $\mathsf{0}$ and $\mathsf{1}$, I_{1}^{0} contains an indefinite number of *non-digital euautographic integrons* (*NDEI's*), idempotent or not. Among the idempotent NDEI's of I_{1}^{0}, are MlEVI's (molecular euautographic validity-intrgrons) as $V(\mathbf{p})$ and $V(\boldsymbol{\pi})$ (see Df's I.5.2 and 1.3(3)). In this case, it will be proved from Df 1.10(2) by axiom (4.1) that the relation $\left[V(p) \hat{=} 0\right] \vee \left[V(p) \hat{=} 1\right]$ is *valid*. However, I *may not assume*, even for a while, either that $\vdash\left[V(p) \hat{=} 0\right]$ or that $\vdash\left[V(p) \hat{=} 1\right]$, i.e. that p either is valid or is antivalid (see Df 3.7)), because $V(p)$ is *irreducible* so that the atomic relation p is *vav-neutral* (*vav-indeterminate*). Likewise, I may not, e.g., assume either that $\vdash\left[V(x \in y) \hat{=} 0\right]$ or that $\vdash\left[V(x \in y) \hat{=} 1\right]$, because the first assumption would have meant that the relation $x \in y$ is a subject axiom of A_1, whereas the second assumption would have meant that the relation $x \in y$ is a subject anti-axiom of A_1. Therefore, the condition '$\vdash\left[\mathbf{K} \,\bar{\hat{=}}\, 0\right]$', which occurs in axiom (4.35), is *not* satisfied for an infinite number of values of the panlogographic placeholder '**K**'. By contrast, in the case of an ordinary integral domain as that of natural integers, the above condition is not satisfied only for $\mathbf{K} \rhd 0$, i.e. for '0' $\mapsto$ '**K**'

3) In Z_{01}^{0}, axioms (4.23), and (4.24)–(4.29) remain ineffective. By contrast, in Z_{11}, $\hat{\cdot}_{\mathbf{x}}$ is an additional singulary kernel-sign (operator), which is subjected to the above-mentioned axioms. If an idempotent integron **i** contains an APVOT **x** then putting the sign $\hat{\cdot}_{\mathbf{x}}$ before **i** produces another idempotent integron, $\hat{\cdot}_{\mathbf{x}}\mathbf{i}$, which is called the *pseudo-product*, or *pseudo-multiplicative contraction*, *of* **i** *over* **x**. A wide variety of theorems, which are not provable by means of Z_{01}^{0}, can be proved by means of Z_{11}.

4) In the next two subsections, I shall state and prove *major* (most fundamental) *theorems of* Z_{01}^{0} and Z_{11} and make the pertinent definitions when appropriate. These theorems are called *euautographic plain theorems* (*EPT's*) *of* A_1^0 *and* A_1 – in contrast to *euautographic master, or decision, theorems* (*EMT's* or *EDT's*). It is understood that all theorems of Z_{01}^{0} are at the same time theorems of Z_{11}. All these theorems are called *rich basic theorems* of Z_{11}, whereas all theorems of Z_{11}, which are not theorems of Z_{01}^{0}, are qualified *advanced*. It is understood that all theorems of Z_{01}^{0} that are stated in schematic form apply also to Z_{00} provided that the range of each AtPLPH involved in a theorem-schema is restricted from I_1^0 to I_0. All theorems of Z_{00} are called *basic theorems of* Z_{01}^{0} *and* Z_{11}. In the subsection 5.4, I

shall define the complete sets of decimal and binary digital integrons, which belong to each of the three integronic domains Z_{11}, Z_{01}^{0}, and Z_{00}.•

5.2. Major theorems of Z_{01}^{0}

Cnv 5.1: ***A supplement to Cnv 2.1.*** Owing to (4.10) and (4.14),

$$[[\mathbf{I} \hat{+} \mathbf{J}] \hat{+} \mathbf{K}] \leftarrow [\mathbf{I} \hat{+} \mathbf{J} \hat{+} \mathbf{K}] \rightarrow [\mathbf{I} \hat{+} [\mathbf{J} \hat{+} \mathbf{K}]], \tag{5.1}$$

$$[\mathbf{I} \hat{\cdot} [\mathbf{J} \hat{\cdot} \mathbf{K}]] \leftarrow [\mathbf{I} \hat{\cdot} \mathbf{J} \hat{\cdot} \mathbf{K}] \rightarrow [[\mathbf{I} \hat{\cdot} \mathbf{J}] \hat{\cdot} \mathbf{K}]; \tag{5.2}$$

that is, the inner pair of square brackets in any one of the expressions $[[\mathbf{I} \hat{+} \mathbf{J}] \hat{+} \mathbf{K}]$, $[\mathbf{I} \hat{+} [\mathbf{J} \hat{+} \mathbf{K}]]$, $[[\mathbf{I} \hat{\cdot} \mathbf{J}] \hat{\cdot} \mathbf{K}]$, and $[\mathbf{I} \hat{\cdot} [\mathbf{J} \hat{\cdot} \mathbf{K}]]$ can be omitted, while the omission of the outer pair of square brackets is subjugated to Cnv 2.1.•

The simplest and most immediate metalinguistic theorem of Z_{01}^{0} is the following one.

†**Th 5.1.** Any two items or multipliers in (4.10)–(4.12) and (4.14)–(4.16) can be exchanged.

Proof: The theorem follows from the pertinent variants or instances of (4.9) and (4.13) by the pertinent variants or instances of (4.33).•

Cnv 5.2. In proving further theorems, Th 5.1 will, as a rule, be used without mentioning it.•

†**Th 5.2.**

$$\vdash[\mathbf{I} \hat{+} \mathbf{M} \hat{=} \mathbf{K}] \text{ if and only if } \vdash[\mathbf{I} \hat{=} \mathbf{K} \hat{+} [\hat{-}\mathbf{M}]], \tag{5.3}$$

the understanding being that

$$\mathbf{K} \hat{+} [\hat{-}\mathbf{M}] \hat{=} \mathbf{K} \hat{-} \mathbf{M}, \tag{5.4}$$

by Df 1.10(14).•

Proof: Replacement of '**I**' with '$\mathbf{I} \hat{+} \mathbf{M}$' and of both '**J**' and '**L**' with '$\hat{-}\mathbf{M}$' in (4.34a) yields the following veracious relation:

$$\text{If } \vdash[\mathbf{I} \hat{+} \mathbf{M} \hat{=} \mathbf{K}] \text{ and } \vdash[\hat{-}\mathbf{M} \hat{=} \hat{-}\mathbf{M}] \text{ then } \vdash[[\mathbf{I} \hat{+} \mathbf{M}] \hat{+} [\hat{-}\mathbf{M}] \hat{=} \mathbf{K} \hat{+} [\hat{-}\mathbf{M}]]. \tag{5.3$_1$}$$

The equality $[\hat{-}\mathbf{M} \hat{=} \hat{-}\mathbf{M}]$ is valid by (4.4), and therefore it can be omitted from (5.3$_1$). At the same time,

$$[\mathbf{I} \hat{+} \mathbf{M}] \hat{+} [\hat{-}\mathbf{M}] \hat{=} \mathbf{I} \hat{+} [\mathbf{M} \hat{+} [\hat{-}\mathbf{M}]] \hat{=} \mathbf{I} \hat{+} 0 \hat{=} 0 \hat{+} \mathbf{I} \hat{=} \mathbf{I}, \tag{5.3$_2$}$$

where use of the following identities has been made in that order: the pertinent variants (4.10) and (4.12), the instance of (4.9) at $\mathbf{J} \rhd 0$, and (4.11). QED.•

***Th 5.3.**

$$\hat{-}0 \mathrel{\hat{=}} 0. \tag{5.5}$$

Proof: From (4.11) at $\mathbf{I} \triangleright \hat{-}0$ (i.e. with '$\hat{-}0$' $\mapsto$ '**I**'), which is written in the reverse direction, and from (4.12) at $\mathbf{I} \triangleright 0$ (i.e. with '0' $\mapsto$ '**I**'), it follows that $\hat{-}0 \mathrel{\hat{=}} 0 \mathbin{\hat{+}} [\hat{-}0] \mathrel{\hat{=}} 0$. QED.•

***Th 5.4.**

$$\mathbf{I} \mathbin{\hat{\cdot}} 0 \mathrel{\hat{=}} 0 \mathbin{\hat{\cdot}} \mathbf{I} \mathrel{\hat{=}} 0. \tag{5.6}$$

Proof: By (4.9) at $\mathbf{J} \triangleright 0$ (or by Th 5.1)), it follows from (4.11) that $\mathbf{I} \mathbin{\hat{+}} 0 \mathrel{\hat{=}} \mathbf{I}$. Multiplication of both sides of this identity by **I** from the left yields: $\mathbf{I} \mathbin{\hat{\cdot}} [\mathbf{I} \mathbin{\hat{+}} 0] \mathrel{\hat{=}} \mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I}$ by (4.34b), and alternatively that multiplication yields also: $\mathbf{I} \mathbin{\hat{\cdot}} [\mathbf{I} \mathbin{\hat{+}} 0] \mathrel{\hat{=}} \mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I} \mathbin{\hat{+}} \mathbf{I} \mathbin{\hat{\cdot}} 0$ by (4.16). Hence, $\mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I} \mathrel{\hat{=}} \mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I} \mathbin{\hat{+}} \mathbf{I} \mathbin{\hat{\cdot}} 0$, by (4.33). At the same time, $\mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I} \mathrel{\hat{=}} \mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I} \mathbin{\hat{+}} 0$, by the version of '$\mathbf{I} \mathbin{\hat{+}} 0 \mathrel{\hat{=}} \mathbf{I}$' with '$\mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I}$' in place of '**I**'. Therefore, $\mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I} \mathbin{\hat{+}} \mathbf{I} \mathbin{\hat{\cdot}} 0 \mathrel{\hat{=}} \mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I} \mathbin{\hat{+}} 0$, by (4.33) again. Addition of $\hat{-}[\mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I}]$ to (i.e. subtraction of $[\mathbf{I} \mathbin{\hat{\cdot}} \mathbf{I}]$ from) both sides of the above identity yields: $\mathbf{I} \mathbin{\hat{\cdot}} 0 \mathrel{\hat{=}} 0$, by the pertinent versions of (4.10), (4.9), (4.12), and (4.11) in this order. Making use of (4.13) at $\mathbf{J} \triangleright 0$ (i.e. with '0' $\mapsto$ '**J**') completes the proof. QED.•

Cmt 5.2. If $0 \mathrel{\hat{=}} 1$ then $\mathbf{I} \mathrel{\hat{=}} \mathbf{I} \mathbin{\hat{\cdot}} 1 \mathrel{\hat{=}} \mathbf{I} \mathbin{\hat{\cdot}} 0 \mathrel{\hat{=}} 0$, by (4.15) and (5.6). This result explains the necessity in axiom (4.7).•

***Th 5.5.**

$$\hat{-}[\hat{-}\mathbf{I}] \mathrel{\hat{=}} \mathbf{I}. \tag{5.7}$$

Proof:

$$\begin{aligned}\hat{-}[\hat{-}\mathbf{I}] &\mathrel{\hat{=}} 0 \mathbin{\hat{+}} [\hat{-}[\hat{-}\mathbf{I}]] \mathrel{\hat{=}} [\mathbf{I} \mathbin{\hat{+}} [\hat{-}\mathbf{I}]] \mathbin{\hat{+}} [\hat{-}[\hat{-}\mathbf{I}]] \mathrel{\hat{=}} \mathbf{I} \mathbin{\hat{+}} [\hat{-}\mathbf{I} \mathbin{\hat{+}} [\hat{-}[\hat{-}\mathbf{I}]]] \\ &\mathrel{\hat{=}} \mathbf{I} \mathbin{\hat{+}} 0 \mathrel{\hat{=}} 0 \mathbin{\hat{+}} \mathbf{I} \mathrel{\hat{=}} \mathbf{I},\end{aligned} \tag{5.7$_1$}$$

where use of the following equations has been made in that order: (i) the variant of (4.11), written in the reverse direction, with $\hat{-}[\hat{-}\mathbf{I}]$ in place of **I** (or with '$\hat{-}[\hat{-}\mathbf{I}]$' in place of '**I**'), (ii) (4.12), (iii) the pertinent variant of (4.10), (iv) the variant of (4.12) with '$\hat{-}\mathbf{I}$' $\mapsto$ '**I**', (vi) (5.3_2). QED.•

***Th 5.6.**

$$[\hat{-}1] \mathbin{\hat{\cdot}} \mathbf{I} \mathrel{\hat{=}} \hat{-}\mathbf{I}. \tag{5.8}$$

Proof:

$$[\hat{-}1]\mathbin{\hat{\cdot}}\mathbf{I}\mathrel{\hat{=}}[\hat{-}1]\mathbin{\hat{\cdot}}\mathbf{I}\mathbin{\hat{+}}0\mathrel{\hat{=}}[\hat{-}1]\mathbin{\hat{\cdot}}\mathbf{I}\mathbin{\hat{+}}[\mathbf{I}\mathbin{\hat{+}}[\hat{-}\mathbf{I}]]\mathrel{\hat{=}}[[\hat{-}1]\mathbin{\hat{\cdot}}\mathbf{I}\mathbin{\hat{+}}\mathbf{I}]\mathbin{\hat{+}}[\hat{-}\mathbf{I}]$$
$$\mathrel{\hat{=}}[[\hat{-}1]\mathbin{\hat{\cdot}}\mathbf{I}\mathbin{\hat{+}}1\mathbin{\hat{\cdot}}\mathbf{I}]\mathbin{\hat{+}}[\hat{-}\mathbf{I}]\mathrel{\hat{=}}[[\hat{-}1]\mathbin{\hat{+}}1]\mathbin{\hat{\cdot}}\mathbf{I}\mathbin{\hat{+}}[\hat{-}\mathbf{I}] \qquad (5.8_1)$$
$$\mathrel{\hat{=}}0\mathbin{\hat{\cdot}}\mathbf{I}\mathbin{\hat{+}}[\hat{-}\mathbf{I}]\mathrel{\hat{=}}0\mathbin{\hat{+}}[\hat{-}\mathbf{I}]\mathrel{\hat{=}}\hat{-}\mathbf{I},$$

where use of the following equations has been made in that order: (i) the variant of (4.11), written in the reverse direction, with ‘$[\hat{-}1]\mathbin{\hat{\cdot}}\mathbf{I}$’ $\mapsto$ ‘**I**’, (ii) (4.12), (iii) the pertinent variant of (4.10), (iv) (4.15), (v) the instance of (4.16) with ‘$\hat{-}1$’ $\mapsto$ ‘**J**’ and ‘1’ $\mapsto$ ‘**K**’, (vi) the instance of (4.12) with ‘1’ $\mapsto$ ‘**I**’, (vii) (5.6), (viii) the variant of (4.11) (or (5.3_2)) with ‘$[\hat{-}\mathbf{I}]$’ $\mapsto$ ‘**I**’.

***Th 5.7.**

$$[\hat{-}\mathbf{I}]\mathbin{\hat{\cdot}}[\hat{-}\mathbf{J}]\mathrel{\hat{=}}\mathbf{I}\mathbin{\hat{\cdot}}\mathbf{J}. \qquad (5.9)$$

Proof:

$$[\hat{-}\mathbf{I}]\mathbin{\hat{\cdot}}[\hat{-}\mathbf{J}]\mathrel{\hat{=}}[[\hat{-}1]\mathbin{\hat{\cdot}}\mathbf{I}]\mathbin{\hat{\cdot}}[\hat{-}\mathbf{J}]\mathrel{\hat{=}}\mathbf{I}\mathbin{\hat{\cdot}}[[\hat{-}1]\mathbin{\hat{\cdot}}[\hat{-}\mathbf{J}]]\mathrel{\hat{=}}\mathbf{I}\mathbin{\hat{\cdot}}[\hat{-}[\hat{-}\mathbf{J}]]\mathrel{\hat{=}}\mathbf{I}\mathbin{\hat{\cdot}}\mathbf{J}, \qquad (5.9_1)$$

where use of the following equations has been made in that order: (i) (5.8), (ii) the pertinent variants of (4.13) and (4.14), (iii) the variant of (5.8) with ‘$[\hat{-}\mathbf{J}]$’ in place of ‘**I**.•

***Th 5.8.**

$$[\mathbf{I}\mathbin{\hat{+}}\mathbf{J}\mathbin{\hat{\cdot}}\mathbf{i}]\mathbin{\hat{\cdot}}\mathbf{i}\mathrel{\hat{=}}[\mathbf{I}\mathbin{\hat{+}}\mathbf{J}]\mathbin{\hat{\cdot}}\mathbf{i}, \qquad (5.10)$$

$$[\mathbf{I}\mathbin{\hat{+}}\mathbf{J}\mathbin{\hat{\cdot}}V(\mathbf{P})]\mathbin{\hat{\cdot}}V(\mathbf{P})\mathrel{\hat{=}}[\mathbf{I}\mathbin{\hat{+}}\mathbf{J}]\mathbin{\hat{\cdot}}V(\mathbf{P}). \qquad (5.10\varepsilon)$$

Proof: The expression on the left-hand side of the identity (5.10) can be developed thus:

$$[\mathbf{I}\mathbin{\hat{+}}\mathbf{J}\mathbin{\hat{\cdot}}\mathbf{i}]\mathbin{\hat{\cdot}}\mathbf{i}\mathrel{\hat{=}}\mathbf{I}\mathbin{\hat{\cdot}}\mathbf{i}\mathbin{\hat{+}}[\mathbf{J}\mathbin{\hat{\cdot}}\mathbf{i}]\mathbin{\hat{\cdot}}\mathbf{i}\mathrel{\hat{=}}\mathbf{I}\mathbin{\hat{\cdot}}\mathbf{i}\mathbin{\hat{+}}\mathbf{J}\mathbin{\hat{\cdot}}[\mathbf{i}\mathbin{\hat{\cdot}}\mathbf{i}]\mathrel{\hat{=}}\mathbf{I}\mathbin{\hat{\cdot}}\mathbf{i}\mathbin{\hat{+}}\mathbf{J}\mathbin{\hat{\cdot}}\mathbf{i}$$
$$\mathrel{\hat{=}}[\mathbf{I}\mathbin{\hat{+}}\mathbf{J}]\mathbin{\hat{\cdot}}\mathbf{i}, \qquad (5.10_1)$$

where use of the pertinent versions of (4.16) and (4.14) and also use of (4.17) have been made. The identity (5.10ε) is proved likewise by using ‘$V(\mathbf{P})$’ in place oi ‘**i**’ and (4.2) instead of (4.17). Alternatively, since (4.2) is a specific instance of (4.17), the identity (5.10ε) is the specific instance of (5.10) with ‘$V(\mathbf{P})$’ in place of ‘**i**’.•

Cmt 5.3. Every separate two-term identity, which is written either singly, in the staccato style, or together with other identities, in the legato style, as a link of a train of identities, is a valid relation (kyrology). In this case, in accordance with (4.40a) and (4.42a), identities (5.5) and (5.7), e.g., imply that

$$V(\hat{-}0\mathrel{\hat{=}}0)\mathrel{\hat{=}}0,\ V(V(\hat{-}0\mathrel{\hat{=}}0)\mathrel{\hat{=}}0)\mathrel{\hat{=}}0,\text{ etc,} \qquad (5.5a)$$

$$V(\hat{-}[\hat{-}\mathbf{I}]\hat{=}\mathbf{I})\hat{=}0,\ V(V(\hat{-}[\hat{-}\mathbf{I}]\hat{=}\mathbf{I})\hat{=}0)\hat{=}0\text{, etc,} \tag{5.7a}$$

respectively, whereas the train (5.7$_1$), e.g., implies the following sequence of isolated identities:

$$\begin{gathered} V(\hat{-}[\hat{-}\mathbf{I}]\hat{=}0\hat{+}[\hat{-}[\hat{-}\mathbf{I}]])\hat{=}0, V(0\hat{+}[\hat{-}[\hat{-}\mathbf{I}]]\hat{=}[\mathbf{I}\hat{+}[\hat{-}\mathbf{I}]]\hat{+}[\hat{-}[\hat{-}\mathbf{I}]])\hat{=}0, \\ V([\mathbf{I}\hat{+}[\hat{-}\mathbf{I}]]\hat{+}[\hat{-}[\hat{-}\mathbf{I}]]\hat{=}\mathbf{I}\hat{+}0)\hat{=}0, V(\mathbf{I}\hat{+}0\hat{=}0\hat{+}\mathbf{I})\hat{=}0, \\ V(0\hat{+}\mathbf{I}\hat{=}\mathbf{I})\hat{=}0, \end{gathered} \tag{5.7b}$$

each of which implies, in turn, an infinite number of recursive identities analogous to (5.5a) and (5.7a).•

Th 5.9.

$$\mathbf{i}\,\hat{\cdot}\,[1\hat{-}\mathbf{i}]\hat{=}0. \tag{5.11}$$

Proof: The expression on the left-hand side of the identity (5.11) can be developed thus:

$$\begin{gathered} \mathbf{i}\,\hat{\cdot}\,[1\hat{-}\mathbf{i}]\hat{=}\mathbf{i}\,\hat{\cdot}\,[1\hat{+}[\hat{-}\mathbf{i}]]\hat{=}\mathbf{i}\hat{+}\mathbf{i}\,\hat{\cdot}\,[\hat{-}\mathbf{i}]\hat{=}\mathbf{i}\hat{+}\mathbf{i}\,\hat{\cdot}\,[[\hat{-}1]\,\hat{\cdot}\,\mathbf{i}] \\ \hat{=}\mathbf{i}\hat{+}[\hat{-}1]\,\hat{\cdot}\,[\mathbf{i}\,\hat{\cdot}\,\mathbf{i}]\hat{=}\mathbf{i}\hat{+}[\hat{-}1]\,\hat{\cdot}\,\mathbf{i}\hat{=}\mathbf{i}\hat{+}[\hat{-}\mathbf{i}]\hat{=}0, \end{gathered} \tag{5.11$_1$}$$

where the rules, according to which all immediate inferences are made, can be identified easily.•

3. Major advanced theorems of Z_{11}

Th 5.10.

$$[\hat{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle]\,\hat{\cdot}\,[\hat{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle]\hat{=}\hat{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle. \tag{5.12}$$

i.e. $\hat{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle$ is an idempotent integron.

Proof: The train of identities (4.29) with '**i**' in place of '**j**' becomes:

$$\begin{gathered} [\hat{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle]\,\hat{\cdot}\,[\hat{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle]\hat{=}[\hat{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle]\,\hat{\cdot}\,[\hat{\cdot}_{\mathbf{y}}\mathbf{i}\langle\mathbf{y}\rangle]\hat{=}\hat{\cdot}_{\mathbf{x}}\hat{\cdot}_{\mathbf{y}}[\mathbf{i}\langle\mathbf{x}\rangle\,\hat{\cdot}\,\mathbf{i}\langle\mathbf{y}\rangle] \\ \hat{=}\hat{\cdot}_{\mathbf{x}}[\mathbf{i}\langle\mathbf{x}\rangle\,\hat{\cdot}\,\mathbf{i}\langle\mathbf{x}\rangle]\hat{=}\hat{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle, \end{gathered} \tag{5.12$_1$}$$

where use of the idempotent law (4.17) with '$\mathbf{i}\langle\mathbf{x}\rangle$' in place of '**i**' has been made in developing the final expression.•

Th 5.11.

$$\begin{gathered} [\hat{\cdot}_{\mathbf{x}}V(\mathbf{P}\langle\mathbf{x}\rangle)]\,\hat{\cdot}\,[\hat{\cdot}_{\mathbf{x}}V(\mathbf{P}\langle\mathbf{x}\rangle)]\hat{=}V(\vee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle)\,\hat{\cdot}\,V(\vee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle) \\ \hat{=}V(\vee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}\hat{\cdot}_{\mathbf{x}}V(\mathbf{P}\langle\mathbf{x}\rangle). \end{gathered} \tag{5.13}$$

Proof: (5.13) is a development of the instance of (4.2) with '$\vee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle$' in place of '**P**' with the help of (4.23). Alternatively, (5.13) is concurrent to the train of identities

$$\left[\widehat{\bullet}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \hat{\cdot} \left[\widehat{\bullet}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \hat{=} \left[\widehat{\bullet}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \hat{\cdot} \left[\widehat{\bullet}_{\mathbf{y}} V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\right]$$
$$\hat{=} \widehat{\bullet}_{\mathbf{x}} \widehat{\bullet}_{\mathbf{y}} \left[V\left(\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\right] \hat{=} \widehat{\bullet}_{\mathbf{x}} \left[V\left(\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \hat{=} \widehat{\bullet}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right), \tag{5.13$_1$}$$

which is an instance of (5.12$_1$) with ‘$V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)$’ in place of ‘$\mathbf{i}\langle\mathbf{x}\rangle$’.•

Cnv 5.3. In proving further algebraic theorems, Ths 5.1–5.11 will be used without mentioning them (for Th 5.1, see also Cnv 5.2 in the previous subsection).•

5.3. Digital integrons

†**Df 5.2:** ***Formation rules of the positive secondary decimal digital integrons of*** **A$_1$.**

0) 0←00←000←00…0;

1) $2 \to [1 \hat{+} 1]$, $3 \to [2 \hat{+} 1]$, …, $9 \to [8 \hat{+} 1]$;

2) $10 \to [9 \hat{+} 1]$, $11 \to [10 \hat{+} 1]$, $12 \to [11 \hat{+} 1]$…, $19 \to [18 \hat{+} 1]$;

3) $20 \to [19 \hat{+} 1]$, $21 \to [20 \hat{+} 1]$, $22 \to [21 \hat{+} 1]$, …, $29 \to [28 \hat{+} 1]$;

etc;

10) $90 \to [89 \hat{+} 1]$, $91 \to [90 \hat{+} 1]$, $92 \to [91 \hat{+} 1]$, …, $99 \to [98 \hat{+} 1]$;

11) $100 \to [99 \hat{+} 1]$, $101 \to [100 \hat{+} 1]$, $102 \to [102 \hat{+} 1]$, …, $109 \to [108 \hat{+} 1]$;

etc ad infinitum.•

Df 5.3. 1) In accordance with Ax 1.1(1,7) or Th 1.1(1,7) and in agreement with Df I.3.1(26), the Arabic numerals

$$0, 1, 2, \ldots, 9, 10, 11, 12, \text{etc ad infinitum}, \tag{5.14}$$

and also the *redundant* euautographs

$$00, 000, 0000, \text{etc ad infinitum}, \tag{5.15}$$

all in this Light-Faced Roman Arial Narrow Type, are *integrons*, or *special terms*, *of* A$_1$. More specifically, they are called the *positive*, or *nonnegative*, *decimal digital integrons* (briefly *PsDDI's* or *NNDDI's*) *of* A$_1$; 0 and 1 are the *primary* one, and the rest are the *secondary* one.

2) In accordance with Ax 1.1(6) or Th 1.1(6), to each PsDDN 1, 2, etc ad infinitum there is the respective unique integron, $\hat{-}1$, $\hat{-}2$, etc ad infinitum, subject to (5.5), which is called *negative decimal digital integron* (briefly, *NgDDI*) *of* A$_1$. A PsDDI or a NgDDI is indiscriminately called a DDI. The qualifier “digital” to “integron” can be used interchangeably with the qualifier “*numeral*”, so that the abbreviations “*DDI*” and “*DNI*” are synonyms. An integron is called a *non-digital*, or *non-numeral*, *integron* (briefly, *NDI* or *NNI*) if it is not digital (not numeral).

3) The ten Arabic digits

$$0, 1, 2, 3, \ldots, 9 \tag{5.16}$$

in this font are called the *atomic decimal digital integrons* (*AtDDI's*), and also *decimal integron-digits*, *of* A_1; 0 and 1 are the primary AtDDI's and the others the secondary AtDDI's. The secondary DDI's

$$10, 20, \ldots, 90, 100, 200, \ldots, 900, 1000, 2000, \text{etc and infinitum,} \tag{5.17}$$

and also the redundant integrons (5.15) are called the *molecular DDI's* (*MlDDI's*) *of* A_1. The atomic and molecular decimal digital integrons altogether are called the *elemental*, or *primitive*, *DDI's* (*ElDDI's*) *of* A_1. The rest of the *DDI's* are called the *compound DDI's* (*CpDDI's*) *of* A_1. In this case, in accordance with Cmt A3.1(3), I avoid using the qualifier "complex", which is by Df A3.1(1e) a synonym of "compound", for avoidance of undesirable associations with the meaning that the latter has in the name "complex number". The set of CpDDI's is the union of the set of *positive* CpDDI's (briefly *PsCpDD's* or *CpPsDDI's*) and of the set of *negative* CpDDI's (briefly *NgCpDD's* or *CpNgDDI's*). The MlDDI's and the CpDDI's form the set of *combined DDI's* (*CbDDI's*).•

Df 5.4. 1) The minuscule letter 'i' in this Bold-Faced Roman Arial Narrow type Type is an AtPLPH (atomic panlogographic placeholder) whose range is the [set of] *nine* non-zero digits 1, 2, …, 9.

2) The bold-faced letter 'j' of the same font and size is an AtPLPH whose range is the [set of] *ten* digits 0, 1, 2, …, 9.

3) Either of the four letters 'k','l', 'm', and 'n' is an AtPLPH whose range is the [set of] *strictly positive* DDI's 1, 2, etc ad infinitum.

4) Either of the six letters 'I', 'J', 'K', 'L', 'M', and 'N' is an AtPLPH whose range is the [set of] all PsDDI's 0, 1, 2, etc ad infinitum.

5) Any of the above-mentioned letters can be furnished either with any number of primes or with any of the lightfaced roman Arabic numeral subscripts $_1$, $_2$, etc in the current font, or else with both labels simultaneously, thus becoming another AtPLPH with the same range.•

Cmt 5.4. It is understood that the syntactic relations such as $\mathbf{I} \triangleright \mathsf{i}$, $\mathbf{I} \triangleright \mathsf{j}$, $\mathbf{I} \triangleright \mathsf{k}$, $\mathbf{I} \triangleright \mathsf{l}$, $\mathbf{I} \triangleright \mathsf{m}$, $\mathbf{I} \triangleright \mathsf{n}$, or the concurrent relations such as 'i' $\mapsto$ '**I**', 'j' $\mapsto$ '**I**', 'k' $\mapsto$ '**I**', 'l' $\mapsto$ '**I**', 'm' $\mapsto$ '**I**', 'n' $\mapsto$ '**I**', and also the similar relations with '**J**' to '**N**' in place of '**I**' determine the corresponding restrictions [of the range] of the latter AtPLPH's.•

Cmt 5.5. 1) The PsDDI's (5.14) form the conventional *decimal system of numeration*. With the help of the syntactic placeholders with the base letters 'i' and 'j', the variety of all DDI's, (5.14), can be represented as

$$\mathsf{j}_1,\ \mathsf{i}_2\mathsf{j}_1,\ \mathsf{i}_3\mathsf{j}_2\mathsf{j}_1,\ \mathsf{i}_4\mathsf{j}_3\mathsf{j}_2\mathsf{j}_1,\ \text{etc ad infinitum,} \tag{5.18}$$

whereas the variety of all MlDDI's, (5.15) and (5.17), as

$$\mathsf{j}_2 0,\ \mathsf{j}_3 00,\ \mathsf{j}_4 000,\ \text{etc ad infinitum.} \tag{5.19}$$

The laws of forming the non-zero CpPsDDI's and of the MlDDI's can be made explicit with the help of the following relations:

$$\begin{gathered}\mathsf{i}_2\mathsf{j}_1 \hat{=} [\mathsf{i}_2 0 \hat{+} \mathsf{j}_1],\ \mathsf{i}_3\mathsf{j}_2\mathsf{j}_1 \hat{=} [\mathsf{i}_3\mathsf{j}_2 0 \hat{+} \mathsf{j}_1] \hat{=} [[\mathsf{i}_3 00 \hat{+} \mathsf{j}_2 0] \hat{+} \mathsf{j}_1],\\ \mathsf{i}_4\mathsf{j}_3\mathsf{j}_2\mathsf{j}_1 \hat{=} [\mathsf{i}_4\mathsf{j}_3\mathsf{j}_2 0 \hat{+} \mathsf{j}_1] \hat{=} [[\mathsf{i}_4\mathsf{j}_3 00 \hat{+} \mathsf{j}_2 0] \hat{+} \mathsf{j}_1]\\ \hat{=} [[[\mathsf{i}_4 000 \hat{+} \mathsf{j}_3 00] \hat{+} \mathsf{j}_2 0] \hat{+} \mathsf{j}_1],\ \text{etc,}\end{gathered} \tag{5.20}$$

$$\mathsf{j}_2 0 \hat{=} \mathsf{j}_2 \hat{\cdot} 10,\ \mathsf{j}_3 00 \hat{=} \mathsf{j}_3 \hat{\cdot} 100,\ \mathsf{j}_4 000 \hat{=} \mathsf{j}_4 \hat{\cdot} 1000,\ \text{etc,} \tag{5.21}$$

which are *theorems*, i.e. *secondary valid relations, of* A_1. These relations can also be restated with the metalinguistic synonymity sign ↔ in place of the subject equality sign $\hat{=}$ of A_1 and $\mathbf{A}_1$ and be thus treated as *meta-theorems*. In any case, it is seen from (5.20) and (5.21) that the AtDDI's (5.16) and MlDDI's (5.15) and (5.17) can be regarded as *elemental* (*primitive*) members of the decimal system of numeration. However, in contrast to the AtDDI's, which contain no shorter constituent euautographs, the MlDDI's are made up of the AtDDI's. Therefore, in this case, the qualifiers "atomic", "molecular', and "compound" are used in agreement with Df A3.1.

2) Thus, like the class of EVI's, the class (set) of DDI's is divided into three subclasses (subsets): the [set of] AtDDI's, the MlDDI's, and the CpDDI's. However, the criteria for the two trichotomies are completely different. The trichotomy of the VI's is relevant to the *ADM* (*algebraic decision method*), whereas the trichotomy of the DDI's is relevant exclusively to the recursive formative properties of successive DDI's. Therefore, the two trichotomies are *analogous but not homologous.* Consequently, it would be counterproductive to consider the union of the classes of AtEVI's and AtDDI's as a single whole class of atomic EI's, the union of the classes of MlEVI's and MlDDI's as a single whole class of molecular EI's, and the union of the classes of CxNDEVI's and CpDDI's as a single whole class of complex EI's. It is noteworthy that the two-member set of *primary AtEVI's* (*PAtEVI's*), i.e. {0,1}, is a subset of the ten-member set of AtDDI's of the list (5.16). That is, the PAtEVI's

belong both to the class of EVI's and to the set of DDI's. It is psychologically difficult to avoid associating PsDDI's with the respective natural numbers. Under this association, a PsDDI is a *xenographic constant-term* or *xenographic term-constant*, because it denotes the corresponding unique natural number. In the actual fact, this involuntary (but conscious) mental interpretation of PsDDI's is harmless. Still, in order to avoid any confusion between a euautographic calculus and its interpretands, I regard all integrons of A_1 as *euautographs*, and therefore I qualify all DDI's as *pseudo-constant special terms* (*PCSpT's*) (cf. Df 1.6(2)). In this connection the following preliminary remark should be made.

3) Construction of the set of DDI's is not the end in itself of this study. The main subclass of the class of integrons of A_1 is the subclass of EVI's (IEI's), which has been defined and partitioned in Dfs 4.1 and 4.2. The set of DDI's is just a by-side and auxiliary product of the pertinent recursive primary formation rule, Ax 1.1(7) or Th 1.1(7). Use of 2 and, perhaps, of 3 or 4 will essentially simplify the calculations constituting some EADP's, although use of any of these DDI's can in principle be avoided. It is unlikely that the PsDDI's strictly larger than 2 and the NgDDI's strictly smaller than $\hat{-}2$ will ever be used in the EADP's of any ER's of academic or practical interest. The whole infinite set of DDI's has been defined simply because it is recursive, so that it is impossible to define any restricted part of it along with the binary operators $\hat{+}$, $\hat{\cdot}$, and $\hat{-}$, which are unavoidably defined on the whole set of DDI's.●

†**Df 5.2a (***An alternative to Df 5.2***):** ***Formation rules of the positive secondary binary digital integrons of*** A_1**.**

0) 0←00←000←00…0;

1) $10 \to [1 \hat{+} 1]$, $11 \to [10 \hat{+} 1]$, $100 \to [11 \hat{+} 1]$, $101 \to [100 \hat{+} 1]$, $110 \to [101 \hat{+} 1]$, $111 \to [110 \hat{+} 1]$, $1000 \to [111 \hat{+} 1]$, $1001 \to [1000 \hat{+} 1]$, $1010 \to [1001 \hat{+} 1]$, etc ad infinitum.●

Cmt 5.6. The integrons defined by Df 5.2a are *secondary positive binary digital integrons* (*PsBDI's*), which, along with the primary atomic integrons 0 and 1, form the *binary system of numeration*. In this case, the secondary PsBDI's:

10, 100, 1000, 10000, etc, (5.22)

are *molecular ones* (*MlBDI's*), while the secondary PsBDI's

11, 101, 110, 111, 1001, 1010, 1011, etc, (5.23) and all *negative BDI's* (*NgBDI's*) are *compound ones* (*CpBDI's*).

It is noteworthy that all secondary BDI's are made up of tokens of the primary atomic integrons 0 and 1 so that they are primary assemblages of A_1, which are not, however, primary formulas of A_1 (cf. Cmt and 1.14). In the framework of the decimal system of numeration, the homonymous DDI's have the same property.

When used xenonymously, the MlBDI's of the list (5.22) denote the same numbers as the DDI's 2, 4, 8, 16, etc (in this order), whereas the CpBDI's of the list (5.23) denote the same numbers as the DDI's 3, 5, 6, 7, 9, 10, etc (in this order). Since the meanings of the BDI's (5.22) and (5.23) are completely different from the meanings of the homonymous DDI's, therefore Df 5.2a is incompatible with Df 5.2. In order to make the two systems of numeration compatible, the two AtBDI's 0 and 1 should be set in a different font. For instance, these can be replaced with *0* and *1*respectively.

The binary system of numeration seems to be more natural as a part of A_1 than the decimal one. Still, I have decided to employ the latter because it is more convenient owing to the force of habit and also because any secondary DDI's larger than 2 and smaller than $\hat{-}2$ will hardly be ever used. I have made Df 5.2a and briefly discussed the BDI's as an instructive example. As I have already notice in Cmt 5.5, any ADP of practical interest can, in principle, be performed with the help of 0 and 1 only, so that use of secondary DDI's or BDI's can be avoided.

In what follows, I shall state and prove some additional theorems (identities), which are based on the DDI's. Some of the theorems will extensively be used in subsequent ADP's, whereas the others are given in order to have them available if needed.•

Th 5.12:** ***A basic table of multiplication of a DDI by the integron.

$$
\begin{gathered}
2\,\hat{\cdot}\,\mathbf{I} \triangleq [1\,\hat{+}\,1]\,\hat{\cdot}\,\mathbf{I} \triangleq 1\,\hat{\cdot}\,\mathbf{I}\,\hat{+}\,1\,\hat{\cdot}\,\mathbf{I} \triangleq \mathbf{I}\,\hat{+}\,\mathbf{I},\\
3\,\hat{\cdot}\,\mathbf{I} \triangleq [2\,\hat{+}\,1]\,\hat{\cdot}\,\mathbf{I} \triangleq 2\,\hat{\cdot}\,\mathbf{I}\,\hat{+}\,1\,\hat{\cdot}\,\mathbf{I} \triangleq 2\,\hat{\cdot}\,\mathbf{I}\,\hat{+}\,\mathbf{I} \triangleq [\mathbf{I}\,\hat{+}\,\mathbf{I}]\,\hat{+}\,\mathbf{I},\\
4\,\hat{\cdot}\,\mathbf{I} \triangleq [3\,\hat{+}\,1]\,\hat{\cdot}\,\mathbf{I} \triangleq 3\,\hat{\cdot}\,\mathbf{I}\,\hat{+}\,1\,\hat{\cdot}\,\mathbf{I} \triangleq 3\,\hat{\cdot}\,\mathbf{I}\,\hat{+}\,\mathbf{I} \triangleq [[\mathbf{I}\,\hat{+}\,\mathbf{I}]\,\hat{+}\,\mathbf{I}]\,\hat{+}\,\mathbf{I},\\
\text{etcetera.}
\end{gathered}
\tag{5.24}
$$

Proof: The trains of equalities (5.24) successively follow from each other by Df. 5.2 and by aioms (4.15), and (4.16).•

Th 5.13:** ***A basic multiplication table for DDI’s.

$$
\begin{gathered}
2 \hat{\cdot} 1 \hat{=} 1 \hat{+} 1 \hat{=} 2, \\
2 \hat{\cdot} 2 \hat{=} 2 \hat{+} 2 \hat{=} 2 \hat{+} [1 \hat{+} 1] \hat{=} [2 \hat{+} 1] \hat{+} 1 \hat{=} 3 \hat{+} 1 \hat{=} 4, \\
2 \hat{\cdot} 3 \hat{=} 2 \hat{\cdot} [2 \hat{+} 1] \hat{=} 2 \hat{\cdot} 2 \hat{+} 2 \hat{\cdot} 1 \hat{=} 4 \hat{+} 2 \hat{=} 4 \hat{+} [1 \hat{+} 1] \hat{=} [4 \hat{+} 1] \hat{+} 1 \hat{=} 5 \hat{+} 1 \hat{=} 6, \\
\text{etcetera.}
\end{gathered}
\tag{5.25}
$$

Proof: The first train of identities in (5.25) follows from the first train of identities in (5.24) at $\mathbf{I} \triangleright 1$ by Df 5.2. The second train of identities in (5.25) follows from the first train of identities in (5.24) at $\mathbf{I} \triangleright 2$ by Df 5.2 and (4.10). The third train of identities in (5.25) is developed with the help of Df 5.2, (4.16), and the previous train. Etc.•

†Df 5.5: ***A power of an integron.***

$$\mathbf{I}^{0} \rightarrow 1, \tag{5.26}$$

$$\mathbf{I}^{\mathrm{n}} \rightarrow \mathbf{I}^{\mathrm{n} \hat{-} 1} \hat{\cdot} \mathbf{I}. \tag{5.27}$$

***Th 5.14.**

$$\mathbf{I}^{0} \hat{=} 1, \tag{5.28}$$

$$\mathbf{I}^{\mathrm{n}} \hat{=} \mathbf{I}^{\mathrm{n} \hat{-} 1} \hat{\cdot} \mathbf{I}. \tag{5.29}$$

Proof: The theorem schemata immediately follow from Df. 5.5 by Ax 4.13.•

***Th 5.15.**

$$\mathbf{I}^{1} \hat{=} \mathbf{I}. \tag{5.30}$$

Proof:

$$\mathbf{I}^{1} \hat{=} \mathbf{I}^{1 \hat{-} 1} \hat{\cdot} \mathbf{I} \hat{=} \mathbf{I}^{1 \hat{+} [\hat{-} 1]} \hat{\cdot} \mathbf{I} \hat{=} \mathbf{I}^{0} \hat{\cdot} \mathbf{I} \hat{=} 1 \hat{\cdot} \mathbf{I} \hat{=} \mathbf{I}, \tag{5.30$_1$}$$

where use of the following identities has been made in that order: the instance of (5.28) at $\mathrm{n} \triangleright 1$, the instances of Df 1.10(14)) and of (4.12) at $\mathbf{I} \triangleright 1$, (5.28), and (4.15).•

***Th 5.16.**

$$\mathbf{i}^{\mathrm{n}} \hat{=} \mathbf{i}, \tag{5.31}$$

$$[V(\mathbf{P})]^{\mathrm{n}} \hat{=} V(\mathbf{P}). \tag{5.31ε}$$

Proof: The instance of (5.29) at $\mathbf{I} \triangleright \mathbf{i}$, i.e. with ‘**i**’ $\mapsto$ ‘**I**’, can be developed thus:

$$\mathbf{i}^{\mathrm{n}} \hat{=} \mathbf{i}^{\mathrm{n} \hat{-} 1} \hat{\cdot} \mathbf{i} \hat{=} \left[\mathbf{i}^{[\mathrm{n} \hat{-} 1] \hat{-} 1} \hat{\cdot} \mathbf{i}\right] \hat{\cdot} \mathbf{i} \hat{=} \mathbf{i}^{\mathrm{n} \hat{-} [1 \hat{+} 1]} \hat{\cdot} [\mathbf{i} \hat{\cdot} \mathbf{i}] \hat{=} \mathbf{i}^{\mathrm{n} \hat{-} 2} \hat{\cdot} \mathbf{i} = \mathbf{i}^{[\mathrm{n} \hat{-} 2] \hat{+} 1} \hat{=} \mathbf{i}^{\mathrm{n} \hat{-} 1}, \tag{5.31$_1$}$$

where use of the following articles has been made: (i) the instance of (5.29) with ‘**i**’ $\mapsto$ ‘**I**’ and ‘$\mathrm{n} \hat{-} 1$’ $\mapsto$ ‘n’, (ii) the pertinent instance of (4.10), (iii) (4.17), (iv). the instance of (5.29) with ‘**i**’ $\mapsto$ ‘**I**’ and ‘$\mathrm{n} \hat{-} 1$’ $\mapsto$ ‘n’, Thus, at each $\mathrm{n} \bar{\in} \{2,3,4,...\}$ which may occur in any concrete ADP of A_1, (5.31) is valid. In the general case, if a value of the placeholder ‘n’ remains unspecified, (5.31) can be proved by induction on n. However, such a proof is beyond the scope of this theory, and it is not required for performing any concrete EADP of A_1. The identity (5.31ε) is the instance of (5.31) at $\mathbf{i} \triangleright V(\mathbf{P})$.•

Cnv 5.4. In proving further algebraic theorems, Ths 5.12–5.16 will, as a rule, be used without mentioning them (cf. Cnv 5.3).•

Cmt 5.7. Neither D_1 nor $\mathbf{D}_1$, being its extension, is designed for proving general theorems of number theory. Particularly, the item a of axiom (4.40) applies with $\mathbf{I} \hat{=} \mathbf{J}$ in place of **P** thus becoming

$$\vdash[\mathbf{I} \hat{=} \mathbf{J}] \text{ if and only if } \vdash[V(\mathbf{I} \hat{=} \mathbf{J}) \hat{=} 0]. \tag{5.32}$$

However, in accordance with Cmt 4.3, (4.40a) and (4.40b) are not satisfiable with ‘**P**’ in place of **P**, i.e. in the case, where ‘**P**’ is used autonymously. The item c of (4.40) is the only one, which is satisfiable in this case thus meaning that ‘**P**’ is a *vav-neutral PLR*, i.e. one whose range contains ER’s of all the three kinds: valid, antivalid and vav-neutral. A like remark applies with ‘(5.32)’ in place of ‘(4.40)’ and with ‘$\mathbf{I} \hat{=} \mathbf{J}$’ in place of ‘**P**’, and it also applies with any further euautographic concretizations of ‘**I**’ and ‘**J**’ by DDI’s and with any further specifications of ‘**I**’ and ‘**J**’ by PLS’ta (panlogographic schemata), whose ranges are classes of DDI’s, because any *concrete digital ER* of the range of ‘$\mathbf{I} \hat{=} \mathbf{J}$’ is either valid or antivalid, but it is never vav-neutral. For instance, under definition 5.4, with ‘$L^n \hat{+} M^n$’ in place of ‘**I**’ and ‘N^n’ in place of ‘**J**’, the pertinent PLR ‘$L^n \hat{+} M^n \hat{=} N^n$’ is a *vav-neutral* one, whose every concrete digital euautographic instance such as $1^2 \hat{+} 2^2 \hat{=} 3^2$, $1^3 \hat{+} 2^3 \hat{=} 3^3$, or $3^2 \hat{+} 4^2 \hat{=} 5^2$ is either valid or antivalid. Specifically, the above three relations reduce to $5 \hat{=} 9$, $9 \hat{=} 27$, and $25 \hat{=} 25$ respectively, so that the first two of them are antivalid, whereas the last one is valid. These trivial remarks are in principle the only ones that can be made on the operatum of *Fermat’s Last Theorem* in the framework of D_1 or $\mathbf{D}_1$. A proof or disproof of this theorem is beyond the scope of D_1 and $\mathbf{D}_1$.•

Cmt 5.8. 1) In accordance with Crl 4.3(1), besides the set of DDI’s, the class of EI’s of A_1 includes the subclass of *non-digital idempotent euautographic integrons* (*PNDIEI’s*), called also or *non-digital euautographic validity-integrons* (*PNDEVI’s*) This class comprises *primary* NDEVI’s (*PNDEVI’s*) of the form of V(**P**) and their idempotent transforms other than 0 and 1. If $\mathbf{P}^a$ is an axiom of A_1 then it is taken for granted that $\vdash\mathbf{P}^a$, i.e. that $\mathbf{P}^a$ is valid. Hence, $V(\mathbf{P}^a)\hat{=}0$ by axiom (4.40a) (see Th 4.1). In the general case, the PNDEVI $V(\mathbf{P})$ of any given ER $\mathbf{P}$ of A_1 of academic or practical interest can be reduced with the help of some subject axioms and some rules of inference to either to 0 or to 1 or else to a certain irreducible NDEVI (NDIEI) $\mathbf{i}|\mathbf{P}\rangle$ – an idempotent algebraic form in some *irreducible molecular* NDEVI's and, perhaps, in 1 or 2 with respect to some of the signs $\hat{\cdot}$, $\hat{+}$, $\hat{\cdot}$, $\hat{-}$, $\hat{\cdot}_{\mathbf{x}}$. Consequently, the initial identity $V(\mathbf{P})\hat{=}V(\mathbf{P})$, being a specific instance of axiom (4.4) turns into a valid ER of exactly one of the three forms: $V(\mathbf{P})\hat{=}0$, $V(\mathbf{P})\hat{=}1$, and $[V(\mathbf{P})\hat{=}\mathbf{i}|\mathbf{P}\rangle]$, which is called a euautographic *master*, or *decision*, *theorem* (*EMT* or *EDT*) *for* $\mathbf{P}$¸ which is in turn called the *euautographic slave relation* (*ESR*) or *ER-slave*. In accordance with Ax 4.20, $\mathbf{P}$ is said to be *valid* if $V(\mathbf{P})\hat{=}0$, *antivalid* if $V(\mathbf{P})\hat{=}1$, and *vav-neutral (*or *vav-indeterminate*) if $[V(\mathbf{P})\hat{=}\mathbf{i}|\mathbf{P}\rangle]$. Therefore, a proof of the EMT (EDT) for $\mathbf{P}$ is called a *euautographic algebraic decision procedure* (*EADP*) *for* $\mathbf{P}$. EADP's are discussed in the next section.•

6. Euautographic and panlogographic algebraic decision procedures

6.1. Euautographic decision procedures of A_1

Df 6.1. 1) Given an ER (primarily an EOR) $\mathbf{P}$ of A_1 of *academic or practical interest* (see Dfs I.3.1(22) and I.4.3(4)), an *algebraic proof*, is denoted by **'**$\mathsf{D}_1(\mathbf{P})$**'**, which begins with application of the appropriate rule of D_1 to the *euautographic algebraic identity* (*EAlI*)

$$V(\mathbf{P})\hat{=}V(\mathbf{P}) \tag{6.1}$$

(see (4.4ε)) as the *initial premise* and which ends with the pertinent *ultimate concluding identity* of one of the following three forms:

$$V(\mathbf{P})\hat{=}\begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \mathbf{i}_{\sim}|\mathbf{P}\rangle & \text{(c)} \end{cases} \tag{6.2}$$

as the *pertinent theorem thus proved*. The EI (euautographic integron) $V(\mathbf{P})$ satisfies the idempotent law (4.2) and, by Df 4.1(2) or Crl 4.3(1), it is properly called the *primary*, or *initial*, *validity-integron* (briefly *PVI* or *IVI*) *of* $\mathbf{P}$ and also, commonly (less explicitly), a *primary*, or *initial*, *non-digital euautographic validity-integron* (briefly *PNDEVI* or *INDEVI*) *of* A_1, without the qualifier "*of* **P**". The EI $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ is a certain *irreducible*, or *ultimate*, *validity-integron* (*IRVI* or *UVI*) *of* **P** other than $\mathsf{0}$ or $\mathsf{1}$, which is commonly (less explicitly) called an *irreducible*, or *ultimate*, *non-digital euautographic validity-integron* (briefly *IRNDEVI* or *UNDEVI*), without the qualifier "*of* **P**", and which satisfies the idempotent law:

$$\mathbf{i}_{\sim}|\mathbf{P}\rangle \hat{\cdot} \mathbf{i}_{\sim}|\mathbf{P}\rangle \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle, \tag{6.3}$$

– just as $\mathsf{0}$ and $\mathsf{1}$ do. To be more specific, $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ is either a certain MlEVI (molecular EVI) $V(\mathsf{p})$ subject to Df 1.3(6) or a certain irreducible idempotent algebraic form in some ElEVI's (elemental EVI's) as $\mathsf{0}$, $\mathsf{1}$, $V(\mathsf{p})$, $V(\mathsf{q})$, etc relative to some (strictly some or all) of the special (algebraic) EKS's (euautographic kernel-signs) of A_1:

$$\hat{-},\ \hat{+},\ \hat{=},\ \hat{\cdot},\ \hat{\bullet}_{\mathbf{x}}. \tag{6.4}$$

2) The theorem (a), (b), or (c) of (6.2) that is proved by $\mathsf{D}_1(\mathbf{P})$ is denoted by '$\mathsf{T}_{1+}(\mathbf{P})$', '$\mathsf{T}_{1-}(\mathbf{P})$', or '$\mathsf{T}_{1\sim}(\mathbf{P})$' respectively or indiscriminately by '$\mathsf{T}_1(\mathbf{P})$' and, in accordance with Ax 4.20, it is called the *euautographic master-theorem* (*EMT*), or *euautographic decision theorem* (*EDT*), *for* **P** and also more generally an *EMT* (*EDT*), or *MT* (*DT*), *of* A_1. Accordingly, the ER **P** is called the *euautographic slave-relation* (*ESR*), or *euautographic relation-slave* (*ER-slave*), or *object ER*, *of the algebraic proof* $\mathsf{D}_1(\mathbf{P})$ and *of the EMT* (*EDT*) $\mathsf{T}_1(\mathbf{P})$, whereas the proof $\mathsf{D}_1(\mathbf{P})$ of $\mathsf{T}_1(\mathbf{P})$ is alternatively called a *euautographic algebraic decision procedure* (*EADP*) *for* **P** or less explicitly an *EADP*, or *ADP*, *of* A_1. An EADP is called a *basic one* (*BEADP*) if it is performed by means of D_0, a *rich basic one* (*RBEADP*) if it is performed by means of D_1^0, and an *advanced* one (*AEADP*) if it involves applications of at least one rule of D_1 that does not belong either to D_0 or to D_1^0. A BEADP of **P** is denoted by '$\mathsf{D}_0(\mathbf{P})$', whereas the pertinent EDT $\mathsf{T}_{1+}(\mathbf{P})$, $\mathsf{T}_{1-}(\mathbf{P})$, or $\mathsf{T}_{1\sim}(\mathbf{P})$ will, when desired, be denoted more specifically by '$\mathsf{T}_{0+}(\mathbf{P})$', '$\mathsf{T}_{0-}(\mathbf{P})$', or '$\mathsf{T}_{0\sim}(\mathbf{P})$' respectively or indiscriminately by '$\mathsf{T}_0(\mathbf{P})$' instead of '$\mathsf{T}_1(\mathbf{P})$'. An RBEADP of **P** is denoted by '$\mathsf{D}_1^0(\mathbf{P})$', whereas the pertinent EDT $\mathsf{T}_{1+}(\mathbf{P})$, $\mathsf{T}_{1-}(\mathbf{P})$, or $\mathsf{T}_{1\sim}(\mathbf{P})$ will, when desired, be denoted more specifically by '$\mathsf{T}_{1+}^0(\mathbf{P})$', '$\mathsf{T}_{1-}^0(\mathbf{P})$', or '$\mathsf{T}_{1\sim}^0(\mathbf{P})$' respectively or indiscriminately by '$\mathsf{T}_1^0(\mathbf{P})$' instead of '$\mathsf{T}_1(\mathbf{P})$'.

3) In accordance with Ax 4.20, an ER **P** of A_1 is said to be *valid* if its EDT has the form (6.2a), *antivalid* if its EDT has the form (6.2b), and *vav-neutral* (or *vav-indeterminate*), i.e. *neutral* (or *indeterminate*) *with respect to validity and antivalidity* or, in other words, *neither valid nor antivalid*, if its EDT has the form (6.2c) subject to (6.3). Thus, the *form* of an EDT $T_1(\mathbf{P})$ allows unambiguously classifying (qualifying) its ESR **P** as *valid*, *antivalid*, or *vav-neutral*, thus relegating the ESR to exactly one of the three *decision*, or *validity*, *classes*: *validity*, *antivalidity*, and *vav-neutrality*. Alternatively, these classes are called, indiscriminately (collectively), the *validity-values* and also, discriminately, the *validity-value validity* (or *validness*), *validity-value antivalidity* (or *antivalidness*), and *validity-value vav-neutrality* (or *vav-indeterminacy*) in that order. I call the validity (decision) classes "the validity-*values*" in analogy with the *number-classes* 1, 2, etc, which are conventionally called "*numeric values*". The schema (6.2) of three possible forms of the EDT for its ESR, **P**, is called the *EDT* (*euautographic decision theorem*) *schema,* or *pattern*. An ER of A_1 that is subjected to an EADP, in the result of which it is relegated to one of the three decision (validity) classes or, in other words, is attributed with one of the three validity-values, is called a *decided ER* (briefly, *DdER*) or, more precisely, a *vavn-decided ER* (*vavn-DdER*), i.e. *decided with respect to validity*, *anivalidity*, or *vav-neutrality* (*vav-indeterminacy*). Accordingly, in reference to an ER of A_1, the noun "*decision*", kindred of the adjective "*decided*", is briefly called *vavn-decision*. Particularly, the abbreviations "EADP", "EDT", and "DT", introduced above, should, more precisely, be understood as "*vavn-EADP*", "*vavn-EDT*", and "*vavn-DT*" respectively. The division of the vavn-decided ER's of A_1 into valid ones, antivalid ones, and vav-neutral (vav-indeterminate) ons is called the *basic decisional trichotomy* (*trisection*, *trifurcation*) *of the vavn-decided ER's*. A vavn-decided ER of A_1 is said to be: *invalid* if it is either antivalid or vav-neutral, *non-antivalid* if it is either valid or vav-neutral, and *vav-unneutral* if it is either valid or antivalid. In all above-mentioned terms, the words "*neutral*", "*unneutral*", "*neutrality*", and "*unneutrality*" can be used interchangeably with "*indeterminate*", "*determinate*", "*indeterminacy*", and "*determinacy*" respectively. The latter three divisions of the vavn-decided ER's into two complementary classes each, namely: (a) *valid* and *invalid*, (b) *antivalid* and *non-antivalid*, (c) *vav-neutral* (*vav-indeterminate*) and *vav-unneutral* (*vav-determinate*) are called the *subsidiary decisional dichotomies*

(*bisections*, *bifurcations*) *of the vavn-decided ER's*. *Orismological* (term-formation) aspects of these dichotomies are made explicit in Appendix 2 (A2).

4) The token of the EVI $\mathsf{0}$, $\mathsf{1}$, or $\mathbf{i}_{\sim}\left|\mathbf{P}\right\rangle$ occurring in the respective EDT $\mathsf{T}_{1+}(\mathbf{P})$, $\mathsf{T}_{1-}(\mathbf{P})$, or $\mathsf{T}_{1\sim}(\mathbf{P})$ is indiscriminately called the *validity-identifier* or *validity-index* (briefly *VID* in both cases) *of* **P** or, discriminately, *the VID validity*, *the VID antivalidity*, or *a VID neutrality* (or *indeterminacy*) respectively, the understanding being that a vavn-decided **P** has *exactly one VID*. A VID $\mathsf{0}$ or $\mathsf{1}$ is called a *digital one* (*DVID*), while a VID $\mathbf{i}_{\sim}\left|\mathbf{P}\right\rangle$ is called a *non-digital one* (*NDVID*). At the same time, independently of their associations with certain ER's, $\mathsf{0}$ is called *the validity-integron validity*, $\mathsf{1}$ is called *the validity-integron antivalidity*, and $\mathbf{i}_{\sim}$ is called *a validity-integron neutrality* or *indeterminacy* subject to a tacit self-evident definition, which can, for convenience in further uses (when necessary), be generalized as follows.

5) Each one of the logographs '$\mathbf{i}_{\sim}$', '$\mathbf{j}_{\sim}$', '$\mathbf{k}_{\sim}$', and '$\mathbf{l}_{\sim}$', alone or with any of the numeral subscripts $_1$, $_2$, etc preceding $_{\sim}$, is an *analytical atomic idempotent panlogographic integron* (*AnAtIPLI*), which is called an *analytical atomic panlogographic validity-integron* (*AnAtPLVI*) without any postpositive qualifier, because *its range is the class of IRNDEVI's* $\mathbf{i}_{\sim}\left|\mathbf{P}\right\rangle$ *for all* **P** *of* A_1. Accordingly, $\mathbf{i}_{\sim}$, $\mathbf{j}_{\sim}$, $\mathbf{k}_{\sim}$, or $\mathbf{l}_{\sim}$, without quotation marks, *is* [*said to be*] an *IRNDEVI*, or *UNDEVI*), *of* A_1. By contrast, $\mathsf{0}$ and $\mathsf{1}$ are, in agreement with Ax I.5.1(12), collectively called the *digital* EVI's (*D*EVI's), the understanding being that these are also *irreducible* (*IR*) or *ultimate* (*U*).

6) The fact that the decision procedure $\mathsf{D}_1(\mathbf{P})$ is qualified *algebraic* means that it is *analytical* (*computational*, *transformative*) – as opposed to both a *tabular* decision procedure, i.e. one based on truth-tables, and a *conformal interpretational* (*substitutional*) decision procedure, i.e. one based on analo-homologrphic replacements of uninterpreted vavn-decided ER's with interpreted (e.g. catlogographic) relations. Schematically, $\mathsf{D}_1(\mathbf{P})$ can be written in the *staccato style* as a sequence of *intermediate theorems*, i.e. *proved* (*deduced*) *valid ER's*, of the following form:

$$V(\mathbf{P}) \hat{=} \mathbf{i}_0|\mathbf{P}\rangle, \mathbf{i}_0|\mathbf{P}\rangle \hat{=} \mathbf{i}_1|\mathbf{P}\rangle, \mathbf{i}_1|\mathbf{P}\rangle \hat{=} \mathbf{i}_2|\mathbf{P}\rangle, ..., \mathbf{i}_{n-1}|\mathbf{P}\rangle \hat{=} \mathbf{i}_n|\mathbf{P}\rangle,$$
$$\mathbf{i}_n|\mathbf{P}\rangle \hat{=} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)}, \\ \mathbf{i}_{\sim}|\mathbf{P}\rangle & \text{(c)} \end{cases} \tag{6.5}$$

where $\mathbf{i}_0|\mathbf{P}\rangle \vec{\hat{=}} V(\mathbf{P})$. For the sake of being specific, I shall assume that passage from the identity $V(\mathbf{P}) \hat{=} \mathbf{i}_0|\mathbf{P}\rangle$ to the identity $\mathbf{i}_0|\mathbf{P}\rangle \hat{=} \mathbf{i}_1|\mathbf{P}\rangle$ as the pertinent conclusion or passage from the identity $\mathbf{i}_{j-1}|\mathbf{P}\rangle \hat{=} \mathbf{i}_j|\mathbf{P}\rangle$ to the identity $\mathbf{i}_j|\mathbf{P}\rangle \hat{=} \mathbf{i}_{j+1}|\mathbf{P}\rangle$ as the pertinent conclusion for each $j \in \omega_{1,n-1}$ is an *immediate inference*, i.e. a conclusion is the result of application of only one rule of inference to some identities preceding the conclusion as the pertinent premises. In practice, however, several successive immediate transformations of any given identity can be performed mentally, so that each identity in the proof (6.5) can be regarded as one that just fixes the result of several successive immediate transformations of the preceding identity. In any case, by the pertinent instance of (4.43), the proof (6.5) can be restated in the *legato style* as:

$$V(\mathbf{P}) \hat{=} \mathbf{i}_1|\mathbf{P}\rangle \hat{=} \mathbf{i}_2|\mathbf{P}\rangle \hat{=} ... \hat{=} \mathbf{i}_{n-1}|\mathbf{P}\rangle \hat{=} \mathbf{i}_n|\mathbf{P}\rangle \vec{\hat{=}} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \mathbf{i}_{\sim}|\mathbf{P}\rangle & \text{(c)} \end{cases}. \tag{6.5$_1$}$$

In contrast to $\mathbf{i}_{\sim}|\mathbf{P}\rangle$, which is an *irreducible non-digital EVI* (*IRNDEVI*), any intermediate EVI $\mathbf{i}_j|\mathbf{P}\rangle$ for $j \in \omega_{1,n-1}$ is a *reducible non-digital EVI* (*RNDEVI*) *of* **P**, so that it is either a certain *reducible monomial EVI* of the form $V(\mathbf{P}_j)$ or a certain *reducible idempotent algebraic form* in some reducible or irreducible monomial EVI's such as $V(\mathbf{P}_j^1)$, $V(\mathbf{P}_j^2)$, etc and, perhaps, in 0 or 1 or both relative to some (strictly some or all) of the special (algebraic) EKS's of the list (6.4). By (4.33), it follows from the proof (6.5) or (6.5$_1$) that its terminal theorem: $\mathbf{i}_n|\mathbf{P}\rangle \hat{=} 0$ or $\mathbf{i}_n|\mathbf{P}\rangle \hat{=} 1$ or $\mathbf{i}_n|\mathbf{P}\rangle \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle$, depending on **P**, implies the theorem (6.2) of the respective form.

7) An EVI $\mathbf{i}_j|\mathbf{P}\rangle$ with $j \in \omega_{0,n-1}$, subject to $\mathbf{i}_0|\mathbf{P}\rangle \vec{\hat{=}} V(\mathbf{P})$, is said to be *reducible* if and only if there is an inference rule that allows deducing the EAlI as the conclusion from some identities preceding it in $\mathsf{D}_1(\mathbf{P})$ as premises, whereas the EVI's

$\mathbf{i}_j|\mathbf{P}\rangle$ and $\mathbf{i}_{j+1}|\mathbf{P}\rangle$ satisfies the following two *criteria* (*conditions*), called the *criteria*, or *conditions*, *of decisional reducibility*:

i) The number of ordinary (logical) kernel-signs occurring in $\mathbf{i}_{j+1}|\mathbf{P}\rangle$ is smaller at least by one than the like number for $\mathbf{i}_j|\mathbf{P}\rangle$, while the number of special (algebraic) kernel-signs occurring in $\mathbf{i}_{j+1}|\mathbf{P}\rangle$ is greater at least by one than the like number for $\mathbf{i}_j|\mathbf{P}\rangle$.

b) The number of special (algebraic) kernel-signs occurring in $\mathbf{i}_{j+1}|\mathbf{P}\rangle$ is smaller at least by one than the like number for $\mathbf{i}_j|\mathbf{P}\rangle$, while the number of ordinary (logical) kernel-signs occurring in $\mathbf{i}_{j+1}|\mathbf{P}\rangle$ (if any) either is the same as or is smaller at least by one than the like number for $\mathbf{i}_j|\mathbf{P}\rangle$. Particularly, $\mathbf{i}_{j+1}|\mathbf{P}\rangle$ can have the form of a reducible monomen (monomial) $V(\mathbf{P}_{j+1})$, where $\mathbf{P}_{j+1}$ is an ER that involves less relational ordinary (logical) kernel-signs than **P**.

An act of reducing $\mathbf{i}_j|\mathbf{P}\rangle$ in either of the above two ways is called an *act of decisional reduction of* $\mathbf{i}_j|\mathbf{P}\rangle$. The EVI $\mathbf{i}_{j+1}|\mathbf{P}\rangle$ is called the (j+1)-th *reduced EVI* (*RdEVI*) *of* **P**.

8) A CxNDEVI is called an *irreducible one* (briefly *IRCxNDEVI* or *CxIRNDEVI*) if and only if it is not reducible, i.e. if and only if there is no inference rule that could be used for transforming the CxNDEVI in either of the two ways a and b indicated in the previous item. As opposed to a CxIRNDEVI, an ElEVI is alternatively called an *elemental* (*primitive*) *irreducible validity-integron* (*ElIRVI*). An ElIRVI or a CxIRNDEVI is indiscriminately called an *irreducible EVI* (*IREVI*) of A_1. An IREVI, elemental or complex, cannot be simplified by means of any rules of inference of A_1. Particularly, an MlEVI, $V(\mathsf{p})$, *cannot be reduced either to* 0 *or to* 1. Therefore, a relation p is said to be *vav-neutral* (or *vav-indefinite*), i.e. *neutral* (or *indeterminate*) *with respect to validity and antivalidity* or, in other words, *neither valid nor antivalid*.

9) The train of identities:

$$V(\neg\mathbf{P}) \hat{=} V(\mathbf{P} \veebar \mathbf{P}) \hat{=} 1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{P}) \hat{=} 1 \hat{-} V(\mathbf{P}) \quad (6.6)$$

follows from Df 1.10(1) by Axs 4.1 and 4.2 and is therefore a *theorem*, i.e. a *proved valid relation*, *of* A_1. Therefore,

$$V(\mathbf{P}) \hat{=} 1 \text{ if and only if } V(\neg\mathbf{P}) \hat{=} 0, \tag{6.7}$$

$$V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle \text{ if and only if } V(\neg\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\neg\mathbf{P}\rangle, \tag{6.8}$$

the understanding being that, in accordance with (6.6),

$$\mathbf{i}_{\sim}|\neg\mathbf{P}\rangle \hat{=} 1 \hat{-} \mathbf{i}_{\sim}|\mathbf{P}\rangle. \tag{6.9}$$

In this case, it follows from (6.2) and (6.3) by (6.6) and (6.9) that

$$V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{P}) \hat{=} V(\neg\mathbf{P}), \tag{6.10}$$

$$\mathbf{i}_{\sim}|\neg\mathbf{P}\rangle \hat{\cdot} \mathbf{i}_{\sim}|\neg\mathbf{P}\rangle \hat{=} \mathbf{i}_{\sim}|\neg\mathbf{P}\rangle. \tag{6.11}$$

Therefore, any one of the three identity schemata (6.2) subject to (6.3) holds if and only if the respective one of the following three identity schemata:

$$V(\neg\mathbf{P}) \hat{=} \begin{cases} 1 & \text{(a)} \\ 0 & \text{(b)} \\ \mathbf{i}_{\sim}|\neg\mathbf{P}\rangle & \text{(c)} \end{cases} \tag{6.12}$$

subject to (6.11) holds. In accordance with (6.2) and (6.12), *the negation of a valid ER*, **P**, *is an antivalid ER*, ¬**P**, *and vice versa*, whereas *the negation of a vav-neutral* (*vav-indeterminate*) *ER*, **P**, *is another vav-neutral* (*vav-indeterminate*) *ER*, ¬**P**.

10) Under the definitions

$$\bar{V}(\mathbf{P}) \hat{\overset{\rightarrow}{=}} V(\neg\mathbf{P}) \hat{=} 1 \hat{-} V(\mathbf{P}), \tag{6.13}$$

$$\bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle \hat{\overset{\rightarrow}{=}} \mathbf{i}_{\sim}|\neg\mathbf{P}\rangle \hat{=} 1 \hat{-} \mathbf{i}_{\sim}|\mathbf{P}\rangle, \tag{6.14}$$

which are based on (6.6) and (6.9), the decision theorem pattern (schema) (6.12) for ¬**P** becomes the *dual euautographic decision theorem* (*DEDT*) *pattern* (*schema*) *for* **P**:

$$\bar{V}(\mathbf{P}) \hat{=} \begin{cases} 1 & \text{(a)} \\ 0 & \text{(b)} \\ \bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle & \text{(c)} \end{cases}, \tag{6.15}$$

which is *dual* of (6.2), while the identities (6.10) and (6.11) turn into

$$\bar{V}(\mathbf{P}) \hat{\cdot} \bar{V}(\mathbf{P}) \hat{=} \bar{V}(\mathbf{P}), \tag{6.16}$$

$$\bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle \hat{\cdot} \bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle \hat{=} \bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle, \tag{6.17}$$

which are *dual* of (4.2) and (6.3) respectively. In analogy with $V(\mathbf{P})$ (see the item 1 of this definition), $V(\neg\mathbf{P})$ is [properly called] the *PVI* (*IVI*) *of* $\neg\mathbf{P}$. By contrast, in accordance with definition (6.13) and in analogy with $V(\mathbf{P})$ again, $\bar{V}(\mathbf{P})$ is properly called *the primary*, or *initial*, *antivalidity-integron* (briefly *PAVI* or *IAVI*) *of* **P** and also commonly (less explicitly), a *primary*, or *initial*, *non-digital euautographic antivalidity-integron* (briefly *PNDEAVI* or *INDEAVI*) *of* A_1, without the qualifier "*of* **P**", which is the variant of the variety of synonymous common names of $V(\mathbf{P})$ with "*EAVI*" ("*euautographic antivalidity-integron*") in place of "*EVI*" ("*euautographic validity-integron*"). Hence, using the abbreviations of two pertinent *accidental* (*relative*) *proper names* of $\bar{V}(\mathbf{P})$ and $V(\neg\mathbf{P})$, definition (6.13) can be read (interpreted) as: «*The PAVI of an ER* (as **P**) *is the PVI of the negation of that ER* (as $\neg$**P**) *and vice versa*.» In accordance with the above terminology, the EI $\bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle$ is an *irreducible*, or *ultimate*, *antivalidity-integron* (*IRAVI* or *UAVI*) *of* **P** other than $\mathsf{0}$ or $\mathsf{1}$, which is commonly (less explicitly) called an *irreducible*, or *ultimate*, *non-digital euautographic antivalidity-integron* (briefly *IRNDEAVI* or *UNDEAVI*), without the qualifier "*of* **P**". To be more specific, $\bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle$ is either a certain MlEVI (molecular EVI) $\bar{V}(\mathsf{p})$ subject to Df 1.3(6) or a certain irreducible idempotent algebraic form in some ElEVI's (elemental EVI's) as $\mathsf{0}$, $\mathsf{1}$, $\bar{V}(\mathsf{p})$, $\bar{V}(\mathsf{q})$, etc relative to some special (algebraic) EKS's of the list (6.4). It is noteworthy that, for instance,

$$\bar{V}(\mathsf{p}) \,\hat{\vec{=}}\, V(\neg\mathsf{p}) \,\hat{=}\, \mathsf{1} \,\hat{-}\, V(\mathsf{p}), \tag{6.13ε}$$

which is a specific instance of (6.13) with 'p' in place of '**P**'. Hence, $\bar{V}(\mathsf{p})$ is reducible in terms of *validity-integrons*, but it is *relatively irreducible as an antivalidity-intrgton*. In any case, the algebraic form $\mathsf{1} \,\hat{-}\, V(\mathsf{p})$ is *absolutely irreducible.* It goes without saying that the identities (6.14) and (6.17) apply with any of the logographs '$\bar{\mathbf{i}}_1$' to '$\bar{\mathbf{i}}_n$' in place of '$\bar{\mathbf{i}}_{\sim}$'. The theorem (a), (b), or (c) of (6.15) is denoted by '$\bar{\mathsf{T}}_{1+}(\mathbf{P})$', '$\bar{\mathsf{T}}_{1-}(\mathbf{P})$', or '$\bar{\mathsf{T}}_{1\sim}(\mathbf{P})$' respectively or indiscriminately by '$\bar{\mathsf{T}}_1(\mathbf{P})$', so that $\bar{\mathsf{T}}_{1+}(\mathbf{P})$, $\bar{\mathsf{T}}_{1-}(\mathbf{P})$, or $\bar{\mathsf{T}}_{1\sim}(\mathbf{P})$ is the EDT for **P**, which is *dual* of $\mathsf{T}_{1+}(\mathbf{P})$, $\mathsf{T}_{1-}(\mathbf{P})$, or $\mathsf{T}_{1\sim}(\mathbf{P})$ respectively.

11) The *secondary* (*defined*) *singulary special EKS* (euautographic kernel-sign) $\bar{V}$, which is *implicitly* (*contextually*) defined by definition (6.13) in terms of V

and $\neg$, is *dual* of the *primary* (*postulated*, *undefined*) *singulary special EKS*, which is introduced by Ax I.5.1(11). Consequently, in contrast to and in analogy with the proper name of V that is also introduced thereby, $\bar{V}$ is called *the antivalidity-sign* (or *antivalidity-operator*, when regarded as an abbreviation of $\bar{V}(\ )$) *of termizing* (*substantivating*, *substantivizing*) *an ER*, because its function is, like that of V, *converting an ER into a computable special term* (*substantive*), which is called the PAVI (IAVI) of that ER. In this case, the whole of the terminology that is associated with V should be changed so as to be adjusted to the wordy name of $\bar{V}$. Particularly, the terminology that has been introduced in the item 4 of this definition should be changed as follows.

12) The token of the EVI 1, 0, or $\bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle$ occurring in the respective EDT $\bar{\mathsf{T}}_{1+}(\mathbf{P})$, $\bar{\mathsf{T}}_{1-}(\mathbf{P})$, or $\bar{\mathsf{T}}_{1\sim}(\mathbf{P})$ is indiscriminately called the *antivalidity-identifier* or *antivalidity-index* (briefly *AVID* in both cases) *of* **P** or, discriminately, *the AVID validity*, *the AVID antivalidity*, or *a AVID neutrality* (or *indeterminacy*) respectively, the understanding being that a vavn-decided **P** has *exactly one AVID*. An AVID 1 or 0 is called a *digital one* (*DAVID*), while an AVID $\bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle$ is called a *non-digital one* (*NDAVID*). At the same time, independently of their associations with certain ER's, 1 is called *the antivalidity-integron validity*, 0 is called *the antivalidity-integron antivalidity*, and $\bar{\mathbf{i}}_{\sim}$ is called *an antivalidity-integron neutrality* subject to the following general definition.

13) Each one of the logographs '$\bar{\mathbf{i}}_{\sim}$', '$\bar{\mathbf{j}}_{\sim}$', '$\bar{\mathbf{k}}_{\sim}$', and '$\bar{\mathbf{l}}_{\sim}$', alone or with any of the numeral subscripts $_1$, $_2$, etc preceding $_\sim$, is an *analytical atomic idempotent panlogographic integron* (*AnAtIPLI*), which is called an *analytical atomic panlogographic antivalidity-integron* (*AnAtPLVI*), without any postpositive qualifier, because *its range is the class of IRNDEVI's* $\bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle$ *for all* **P** *of* A_1 and also because it is understood (assumed) that

$$'\bar{\mathbf{i}}_{\sim}' \to 1 \hat{-}\ '\mathbf{i}_{\sim}',\ '\bar{\mathbf{j}}_{\sim}' \to 1 \hat{-}\ '\mathbf{j}_{\sim}',\ '\bar{\mathbf{k}}_{\sim}' \to 1 \hat{-}\ '\mathbf{k}_{\sim}',\ '\bar{\mathbf{l}}_{\sim}' \to 1 \hat{-}\ '\mathbf{l}_{\sim}', \qquad (6.14_1)$$

subject to the definition of '$\mathbf{i}_{\sim}$', '$\mathbf{j}_{\sim}$', '$\mathbf{k}_{\sim}$', and '$\mathbf{l}_{\sim}$' given in the item 4. Accordingly, $\bar{\mathbf{i}}_{\sim}$, $\bar{\mathbf{j}}_{\sim}$, $\bar{\mathbf{k}}_{\sim}$, or $\bar{\mathbf{l}}_{\sim}$, without quotation marks, *is* [*said to be*] an *irreducible*, or *ultimate*, non-digital *IRNDEAVI*, or *UNDEAVI*, *of* A_1, the understanding being that

$$\bar{\mathbf{i}}_{\sim} \to 1 \hat{-}\ \mathbf{i}_{\sim},\ \bar{\mathbf{j}}_{\sim} \to 1 \hat{-}\ \mathbf{j}_{\sim},\ \bar{\mathbf{k}}_{\sim} \to 1 \hat{-}\ \mathbf{k}_{\sim},\ \bar{\mathbf{l}}_{\sim} \to 1 \hat{-}\ \mathbf{l}_{\sim}, \qquad (6.14_2)$$

By contrast, 1 and 0 are collectively called the *digital* EAVI's (*D*EAVI's).

14) In the item 10 of this definition, definition (6.13) has been read (interpreted) in terms of the abbreviations of two pertinent *accidental* (*relative*) *proper names* of $\bar{V}(\mathbf{P})$ and $V(\neg\mathbf{P})$. At the same time, the two synonymous proper names of $\bar{V}(\mathbf{P})$, which have been introduced and abbreviated as "PNDEAVI" and "INDEAVI" in the item 11, have the same range as the two synonymous proper names of $V(\mathbf{P})$, which have been introduced and abbreviated as "PNDEVI" and "INDEVI" in the item 1 and which are also ones of $V(\neg\mathbf{P})$. Consequently, in terms of these names, definition (6.13) can be read (interpreted), e.g., by stating that *a PNDEAVI is a PNDEVI and vice versa*. The *generic* common names "*euautographic validity-integron*" ("*EVI*") and "*euautographic antivalidity-integron*" ("*EAVI*") also have the same range. Therefore, it is possible to state that generally *an EAVI is an EVI and vice versa*. Both above statements are apparently paradoxical or counterproductive at the best.

15) The reason for appearance of such apparent paradoxes is that A_1 can be set up self-consistently either on the basis of V or on the basis of $\bar{V}$, but not on the basis of both *mutually dual EKS's* V and $\bar{V}$ simultaneously. This setup of A_1 is based on V. The alternative setup of A_1 can be made on the basis of the *dual homonymous EKS* V, i.e. on the basis of $\bar{V}$, which can, in the absence of the current validity-sign V, be denoted by 'V' and be called the *validity-sign*, and in relation to which the roles of the AtEI's 0 and 1 exchange so that 1 becomes the *validity-integron validity* and 0 *becomes the validity-integron antivalidity*. It is understood that the dual homonymous V should be introduced axiomatically like its current homonym. In the presence of the current V, which has already been called the *validity-sign*, $\bar{V}$ is called the *antivalidity-sign* and accordingly 0 and 1, which have respectively been called the *validity-integron validity* and the *validity-integron antivalidity* in relation to V, are alternatively (synonymously) and univocally called, in that order, the *antivalidity-integron antivalidity* and the *antivalidity-integron validity* in relation to $\bar{V}$. The like remarks apply with "*index*" or "*identifier*" in place of "*integron*". Also, like (6.13), definition (6.14) can be read by using the pertinent *accidental* (*relative*) *proper names* of $\bar{\mathbf{i}}_{\sim}|\mathbf{P}\rangle$ and $\mathbf{i}_{\sim}|\neg\mathbf{P}\rangle$ as: «*The AVID neutrality of an ER* (as $\mathbf{P}$) *is the VID neutrality of the negation of that ER* (as $\neg\mathbf{P}$) *and vice versa*.» By contrast, both $\mathbf{i}_{\sim}$ and $1\hat{-}\mathbf{i}_{\sim}$ are

validity-integrons neutrality, whereas both $\bar{\mathbf{i}}_{\sim}$ and $1 \hat{-} \bar{\mathbf{i}}_{\sim}$ are *antivalidity-integrons neutrality*. Hence, by (6.14), each of the four *IEI's* (*idempotent euautographic integrons*): $\mathbf{i}_{\sim}$, $1 \hat{-} \mathbf{i}_{\sim}$, $\bar{\mathbf{i}}_{\sim}$, and $1 \hat{-} \bar{\mathbf{i}}_{\sim}$ *is a validity-integron neutrality and an antivalidity-integron neutrality simultaneously*.

16) I shall not use the EKS $\bar{V}$ and relevant terminology in the main stream of this theory. As long as the kernel-sign $\bar{V}$ and the term "antivalidity-integron" are not used, the names "idempotent integron" and "validity-integron" can be used interchangeably. Just as in this particular discussion, I shall use $\bar{V}$ only in discussing the pertinent *dual properties* of A_1 and also of A_1^0 and A_0 (cf. Cmt 1.12(1,2)). In this case, the names "validity-integron" and "antivalidity-integron" alone or together with the same *prepositive* (adherent, prefixal) qualifiers (as "euautographic", "panlogographic", "digital" "non-digital", etc) are respectively two *generic* taxonyms or two descriptive *specific* taxonyms that have the same range, i.e. they are they are two *synonyms*. At the same time, two descriptive *specific* taxonyms that are formed by the above two generic taxonyms together with the same *postpositive* (adjoined, suffixal) or particularly *appositive* qualifier can, depending on the qualifier, either be two *synonyms* or two *antonyms*. For instance, "the primary validity-integron *of* **P**" ("the PVI *of* **P**") and "the primary antivalidity-integron *of* **P**" ("the PAVI *of* **P**"), "the validity-integron *validity*" (i.e. 0) and "the antivalidity-integron *validity*" (i.e. 1), and "the validity-integron *antivalidity*" (i.e. 1) and "the antivalidity-integron *antivalidity*" (i.e. 0) are three pairs of antonyms, whereas "the validity-integron *neutrality*" (i.e. $\mathbf{i}_{\sim}$, $1 \hat{-} \mathbf{i}_{\sim}$, $\bar{\mathbf{i}}_{\sim}$, or $1 \hat{-} \bar{\mathbf{i}}_{\sim}$) and "the antivalidity-integron *neutrality*" (i.e. again $\mathbf{i}_{\sim}$, $1 \hat{-} \mathbf{i}_{\sim}$, $\bar{\mathbf{i}}_{\sim}$, or $1 \hat{-} \bar{\mathbf{i}}_{\sim}$) is a pair of synonyms. Synonyms as described above are *misnomers*. Therefore, in discussing dual properties of $\bar{V}$ with respect to V, *use* of such synonyms, especially in the same context, should be avoided, although *mention* of them can, paradoxically, illustrate some of those properties.•

Cmt 6.1. The axiom (4.2) is by definition a *valid relation*, i.e. $\vdash\left[V(\mathbf{P}) \hat{\cdot} V(\mathbf{P}) \hat{=} V(\mathbf{P})\right]$ (cf. the proof of Th 4.1). At the same time, by (6.2), either (a) $\vdash\left[V(\mathbf{P}) \hat{=} 0\right]$ or (b) $\vdash\left[V(\mathbf{P}) \hat{=} 1\right]$ or else (c) $\vdash\left[V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle\right]$, – depending on **P**. Consequently, application of (4.38) with identity (4.2) as $\mathbf{R}(V(\mathbf{P}))$, $V(\mathbf{P})$ as **I**, and 0, 1, or $\mathbf{i}|\mathbf{P}\rangle$ in turn as **J** yields the following three theorems of A_1:

$$0 \dot{\times} 0 \hat{=} 0,\ 1 \dot{\times} 1 \hat{=} 1,\ \mathbf{i}_{\sim}|\mathbf{P}\rangle \dot{\times} \mathbf{i}_{\sim}|\mathbf{P}\rangle \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle. \tag{6.18}$$

The last identity can be rewritten as (4.17), the understanding being that (4.17) is satisfied at $\mathbf{i} \rhd 0$, by (7.6) at $\mathbf{I} \rhd 0$, and at $\mathbf{i} \rhd 1$, by (4.15) at $\mathbf{I} \rhd 1$ (cf. ($4.15\mu_1$)). The following two conjoined theorems of A_1 are proved successively from axiom (4.3) likewise. These theorems are indispensable in EADP's for euautographic special equalities and for ER's involving such equalities.•

Th 6.1: ***Reduced Laws of Initial Validity-Integrons 1 (IVIL1).***

$$V(V(\mathbf{P}) \hat{=} 0) \hat{=} V(\mathbf{P}), \tag{6.19}$$

$$V(V(\mathbf{P}) \hat{=} 1) \hat{=} 1 \hat{-} V(\mathbf{P}), \tag{6.20}$$

$$V(V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{Q}\rangle) \hat{=} [V(\mathbf{P}) \hat{-} \mathbf{i}_{\sim}|\mathbf{Q}\rangle]^2 \hat{=} V(\mathbf{P}) \hat{+} \mathbf{i}_{\sim}|\mathbf{Q}\rangle \hat{-} 2 \dot{\times} V(\mathbf{P}) \dot{\times} \mathbf{i}_{\sim}|\mathbf{Q}\rangle, \tag{6.21}$$

where $\mathbf{i}_{\sim}|\mathbf{Q}\rangle$ is the *IRNDIEI* (*irreducible, non-digital idempotent euautographic integron*) of a certain vav-neutral ER **Q**, so that

$$\mathbf{i}_{\sim}|\mathbf{Q}\rangle \dot{\times} \mathbf{i}_{\sim}|\mathbf{Q}\rangle \hat{=} \mathbf{i}_{\sim}|\mathbf{Q}\rangle. \tag{6.22}$$

Proof: By the pertinent instances of the distributive law (4.16) and of the idempotent law (4.2), the law of initial validity integrons (4.3) can be developed thus:

$$\begin{aligned} V(V(\mathbf{P}) \hat{=} V(\mathbf{Q})) &\hat{=} [V(\mathbf{P}) \hat{-} V(\mathbf{Q})]^2 \\ &\hat{=} [V(\mathbf{P})]^2 \hat{+} [V(\mathbf{Q})]^2 \hat{-} 2 \dot{\times} V(\mathbf{P}) \dot{\times} V(\mathbf{Q}) \\ &\hat{=} V(\mathbf{P}) \hat{+} V(\mathbf{Q}) \hat{-} 2 \dot{\times} V(\mathbf{P}) \dot{\times} V(\mathbf{Q}). \end{aligned} \tag{6.23}$$

Let the vavn-decision problem for **Q** be solved, so that $V(\mathbf{Q})$ satisfies one of the three identities:

$$V(\mathbf{Q}) \hat{=} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \mathbf{i}_{\sim}|\mathbf{Q}\rangle & \text{(c)} \end{cases} \tag{6.24}$$

subject to (6.22). Application of (4.38) with identity (6.23) as $\mathbf{R}(V(\mathbf{Q}))$, $V(\mathbf{Q})$ as **I**, and 0, 1, or $\mathbf{i}_{\sim}|\mathbf{Q}\rangle$ in turn as **J** yields (6.19)–(6.21) respectively.•

Th 6.2: ***Reduced Laws of Initial Validity-Integrons 2 (IVIL2).***

$$V(0 \hat{=} 0) \hat{=} 0, V(1 \hat{=} 0) \hat{=} 1, V(0 \hat{=} 1) \hat{=} 1, V(1 \hat{=} 1) \hat{=} 1; \tag{6.25}$$

$$V(\mathbf{i}_{\sim}|\mathbf{P}\rangle \hat{=} 0) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle, V(\mathbf{i}_{\sim}|\mathbf{P}\rangle \hat{=} 1) \hat{=} 1 \hat{-} \mathbf{i}_{\sim}|\mathbf{P}\rangle; \tag{6.26}$$

$$\begin{gathered} V(\mathbf{i}_{\sim}|\mathbf{Q}\rangle \hat{=} 0) \hat{=} \mathbf{i}_{\sim}|\mathbf{Q}\rangle, V(\mathbf{i}_{\sim}|\mathbf{Q}\rangle \hat{=} 1) \hat{=} 1 \hat{-} \mathbf{i}_{\sim}|\mathbf{Q}\rangle, \\ V(\mathbf{i}_{\sim}|\mathbf{P}\rangle \hat{=} \mathbf{i}_{\sim}|\mathbf{Q}\rangle) \hat{=} [\mathbf{i}_{\sim}|\mathbf{P}\rangle \hat{-} \mathbf{i}_{\sim}|\mathbf{Q}\rangle]^2 \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle \hat{+} \mathbf{i}_{\sim}|\mathbf{Q}\rangle \hat{-} 2 \dot{\times} \mathbf{i}_{\sim}|\mathbf{P}\rangle \dot{\times} \mathbf{i}_{\sim}|\mathbf{Q}\rangle. \end{gathered} \tag{6.27}$$

The first two identities in (6.27) are variants of identities (6.26) with '$\mathbf{i}_{\sim}|\mathbf{Q}\rangle$' in place of '$\mathbf{i}_{\sim}|\mathbf{P}\rangle$', while identities (6.26) are the same as (6.19) and (6.20). If $\mathbf{i}_{\sim}|\mathbf{P}\rangle \hat{=} \mathbf{i}_{\sim}|\mathbf{Q}\rangle$ then the third identity in (6.27) turns into the first identity in (6.25).

Proof: Let the vavn-decision problem for the ER **P** be solved, so that $V(\mathbf{P})$ satisfies one of the three identities (6.2) subject to (6.3). Application of (4.38) with each one of the identities (6.19)–(6.21) in turn as $\mathbf{R}(V(\mathbf{P}))$, with $V(\mathbf{P})$ as **I**, and with 0, 1, or $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ in turn as **J** yields (6.25)–(6.27) respectively.•

Cmt 6.2. The last identity (6.27) can be generalized as:

$$V(\mathbf{i} \hat{=} \mathbf{j}) \hat{=} [\mathbf{i} \hat{-} \mathbf{j}]^2 \hat{=} \mathbf{i} \hat{+} \mathbf{j} \hat{-} 2 \hat{\cdot} \mathbf{i} \hat{\cdot} \mathbf{j}, \tag{6.28}$$

where **i** and **j** are two arbitrary IEI's (idempotent euautographic integrons), i.e.

$$\mathbf{i} \hat{\cdot} \mathbf{i} \hat{=} \mathbf{i}, \ \mathbf{j} \hat{\cdot} \mathbf{j} \hat{=} \mathbf{j}, \tag{6.29}$$

the understanding being that both (6.28) and (6.29) remain valid for either value 0 or 1 of '**i**' or '**j**' or of both, – in accordance with (6.25) and (6.26).•

6.2. Panlogographic decision procedures of $\mathbf{A}_1$

Df 6.2. 1) In accordance with Df 4.9, let '**P**' be an AnAtMLPH (metalinguistic placeholder), whose range is the class of all PLR's of $\mathbf{A}_1$, and let '**i**' be an AnAtMLPH, whose range is the class of idempotent integrons of $\mathbf{A}_1$ as 0, 1, $V(\mathbf{P})$, '**i**', '**j**', '**k**', '**l**', '$\mathbf{i}_1$', '$\mathbf{k}_1$', etc. Therefore, **i** (but not '**i**') is itself an *idempotent panlogographic integron* (*IPLI*), called also a *panlogographic validity-integron* (*PLVI*), i.e. a *panlogographic special term PLSpT*), which satisfies the idempotent law (4.53). Let **P** (but not '**P**' and not **'P'**) be a given patterned (template) *panlogographic slave-relation* (*PLSR*) *of* $\mathbf{A}_1$, when it is used autonymously, and a *panlogographic schema* (*PLS*) [of a large number (usually an infinite number)] *of euautographic slave-relations* (*ESR's*) *of* A_1, when it is used xenonymously. In the latter case, the class of ESR's of A_1 is the range of **P**. In either case, $V(\mathbf{P})$ *is* the *primary*, or *initial*, *validity-integron* (*PVI* or *IVI*) *of* **P**, which satisfies the pertinent variant of the idempotent law (4.2):

$$V(\mathbf{P}) \hat{\cdot} V(\mathbf{P}) \hat{=} V(\mathbf{P}). \tag{6.30}$$

At the same time, $\mathbf{i}_{\sim}|\mathsf{P}\rangle$ is a certain *irreducible*, or *ultimate*, *validity-integron* (*IRVI* or *UVI*) *of* P other than 0 or 1, which is *commonly* (less explicitly) called a *non-digital* or *pseudo-variable*, *irreducible* or *ultimate, panlogographic validity-integron* (briefly, *NDIRPLVI*, *PVIRPLVI*, *NDUPLVI*, or *PVUPLVI)*, without the qualifier "*of* P", and which satisfies the pertinent variant of the idempotent law (6.3):

$$\mathbf{i}_{\sim}|\mathsf{P}\rangle \hat{\cdot}\, \mathbf{i}_{\sim}|\mathsf{P}\rangle \hat{=} \mathbf{i}_{\sim}|\mathsf{P}\rangle; \tag{6.31}$$

$\mathbf{i}_{\sim}|\mathsf{P}\rangle$ belongs to the range of 'i'. Consequently, *a PLS of EADP's for the PLS* P *of ESR's* is at the same time *a panlogographic algebraic decision procedure* (*PLADP*) *for the homonymous panlogographic slave-relation* (*PLSR*), which is just *another hypostasis* (*way of existence*) *of the PLS* P, in which it is used in the appropriate *autonymous* mental mode. Therefore, the *PLS of the EMT's* (*EDT's*), $\mathsf{T}_1(\mathsf{P})$, which is proved by the PLS of EADP's in question, is at the same time the pertinent *panlogographic master*, or *decision, theorem* (*PLMT* or *PLDT*) *for* P *as the PLSR*. In the case when P is treated autonymously as the PLSR, and not as the PLS of ESR's, $\mathsf{T}_1(\mathsf{P})$ is denoted by '$\mathbf{T}_1(\mathsf{P})$'. That is to say, $\mathbf{T}_1(\mathsf{P})$ is the extension (continuation) of $\mathsf{T}_1(\mathsf{P})$ from the case when the range of P is the class of ESR's of A_1 to the case when P is a PLSR of $\mathbf{A}_1$, i.e. the range of P is the singleton of the PLSR. Thus, in agreement with Df 4.10, Df 6.1 applies, *mutatis mutandis*, with 'P', '$\mathbf{A}$', '$\mathbf{D}$', '$\mathbf{T}$', "*panlogographic*" ("*PL*"), "*PLADP*", "*PLMT*", "*PLDT*", "*PLR*" ,"*PLSRI*", and "*PLR-slave*" in place of '**P**', 'A', 'D', 'T', "*euautographic*" ("*E*"), "*EADP*", "*EMT*", "*EDT*", "*ER*", "*ESR*", and "*ER-slave*" respectively. Still, in order to discuss conveniently relationship between the *basic decisional trichotomy of vavn-decided PLR's of* $\mathbf{A}_1$ *into the classes of valid, antivalid*, and *vav-neutral* (*vav-indeterminate*) *PLR's* and the like trichotomy of *vavn-decided ER's of* A_1, I shall begin from making explicit some most fundamental aspects of the former trichotomy, which are implied by items 1–3, 8, and 9 of Df 6.1 subject to the above substitutions.

2) In analogy with (6.1) subject to (6.2), the PLADP (panlogographic algebraic decision procedure) for a given PLR (primarily a PLOR) P of $\mathbf{A}_1$, of *academic or practical interest* (see Dfs I.3.1(22) and I.4.3(4)), is an *algebraic proof*,

denoted by ‘$\mathbf{D}_1(\mathbf{P})$’, which begins with application of the appropriate rule of $\mathbf{D}_1$ to the *panlogographic algebraic identity* (*PLAlI*):

$$V(\mathbf{P}) \hat{=} V(\mathbf{P}) \tag{6.32}$$

as the *initial premise* and which ends with the pertinent *ultimate concluding identity* of one of the following three forms:

$$V(\mathbf{P}) \hat{=} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \mathbf{i}_{\sim}|\mathbf{P}\rangle & \text{(c)} \end{cases} \tag{6.33}$$

as the *pertinent theorem thus proved*.

3) In analogy with the pertinent nomenclature introduced in Df 6.1(2), the theorem (a), (b), or (c) of (6.2) that is proved by $\mathbf{D}_1(\mathbf{P})$ is denoted by ‘$\mathbf{T}_{1+}(\mathbf{P})$’, ‘$\mathbf{T}_{1-}(\mathbf{P})$’, or ‘$\mathbf{T}_{1\sim}(\mathbf{P})$’ respectively or indiscriminately by ‘$\mathbf{T}_1(\mathbf{P})$’ and, in accordance with Ax 4.20_2, it is called the *panlogographic master-theorem* (*PLMT*), or *panlogographic decision theorem* (*PLDT*), *for* $\mathbf{P}$ and also more generally a *PLMT* (*PLDT*), or *MT* (*DT*), *of* $\mathbf{A}_1$. Accordingly, the PLR $\mathbf{P}$ is called the *panlogographic slave-relation* (*PLSR*), or *panlogographic relation-slave* (*PLR-slave*), or *object PLR, of the algebraic proof* $\mathbf{D}_1(\mathbf{P})$ and *of the PLMT* (*PLDT*) $\mathbf{T}_1(\mathbf{P})$, whereas the proof $\mathbf{D}_1(\mathbf{P})$ of ` is alternatively called a *panlogographic algebraic decision procedure* (*PLADP*) *for* $\mathbf{P}$ or less explicitly an *PLADP*, or *ADP*, *of* $\mathbf{A}_1$. A PLADP is called a *basic one* (*BPLADP*) if it is performed by means of $\mathbf{D}_0$, a *rich basic one* (*RBPLADP*) if it is performed by means of $\mathbf{D}_1^0$, and an *advanced one* (*APLADP*) if it involves applications of at least one rule of $\mathbf{D}_1$ that does not belong either to $\mathbf{D}_0$ or to $\mathbf{D}_1^0$. A BPLADP of $\mathbf{P}$ is denoted by ‘$\mathbf{D}_0(\mathbf{P})$’, whereas the pertinent PLDT $\mathbf{T}_{1+}(\mathbf{P})$, $\mathbf{T}_{1-}(\mathbf{P})$, or ‘$\mathbf{T}_{1\sim}(\mathbf{P})$ will, when desired, be denoted more specifically by ‘$\mathbf{T}_{0+}(\mathbf{P})$’, ‘$\mathbf{T}_{0-}(\mathbf{P})$’, or ‘$\mathbf{T}_{0\sim}(\mathbf{P})$’ respectively or indiscriminately by ‘$\mathbf{T}_0(\mathbf{P})$’ instead of ‘$\mathbf{T}_1(\mathbf{P})$’. An RBPLADP of $\mathbf{P}$ is denoted by ‘$\mathbf{D}_1^0(\mathbf{P})$’, whereas the pertinent PLDT $\mathbf{T}_{1+}(\mathbf{P})$, $\mathbf{T}_{1-}(\mathbf{P})$, or $\mathbf{T}_{1\sim}(\mathbf{P})$ will, when desired, be denoted more specifically by ‘$\mathbf{T}_{1+}^0(\mathbf{P})$’, ‘$\mathbf{T}_{1-}^0(\mathbf{P})$’, or ‘$\mathbf{T}_{1\sim}^0(\mathbf{P})$’ respectively or indiscriminately by ‘$\mathbf{T}_1^0(\mathbf{P})$’ instead of ‘$\mathbf{T}_1(\mathbf{P})$’.

4) In accordance with the previous item, the subject matter of item 3 of Df 6.1 can be immediately be extended from ER’s of A_1 to PLR’s of $\mathbf{A}_1$ by paraphrasing the

latter item as follows. In accordance with Ax 4.20_2, a PLR **P** of $\mathbf{A}_1$ is said to be *valid* if its PLDT has the form (6.31a), *antivalid* if its PLDT has the form (6.31b), and *vav-neutral* (or *vav-indeterminate*), i.e. *neutral* (or *indeterminate*) *with respect to validity and antivalidity* or, in other words, *neither valid nor antivalid*, if its PLDT has the form (6.31c) subject to (6.32). Thus, the *form* of a PLDT $\mathbf{T}_1(\mathbf{P})$ allows unambiguously attributing its slave PLR **P** to one of the following three kinds: *valid*, *antivalid*, or *vav-neutral*. Therefore, the *schema* (6.31) of three possible forms of the PLDT for its slave PLR, **P**, is called the *PLDT* (*panlogographic decision theorem*) *schema*, or *pattern*, *for* **P**. A PLR of $\mathbf{A}_1$ that has been subjected to a successful PLADP, in the result of which it is relegated to one of the above three *decision*, or *validity*, *classes*, is called a *decided PLR* (briefly, *DdPLR*) or, more precisely, a *vavn-decided PLR*, i.e. decided with respect to validity, anivalidity, or vav-neutrality (vav-indeterminacy). Accordingly, in reference to a relation of A_1, the noun "*decision*", kindred of the adjective "*decided*", should be understood as *decision with respect to validity, antivalidity, and vav-neutrality* or briefly as *vavn-decision*. Particularly, the abbreviations "PLADP", "PLDT", and "DT", introduced above", should, more precisely, be replaced with the abbreviations "*vavn-PLADP*", "*vavn-PLDT*", and "*vavn-DT*" respectively. The division of the vavn-decided PLR's of $\mathbf{A}_1$ into the three classes: valid, antivalid, and vav-neutral (vav-indeterminate) is called the *basic decisional trichotomy* (*trisection*, *trifurcation*) *of the vavn-decided PLR's*. A vavn-decided PLR of $\mathbf{A}_1$ is said to be: *invalid* if it is either antivalid or vav-neutral, *non-antivalid* if it is either valid or vav-neutral, and *vav-unneutral* if it is either valid or antivalid. In all above-mentioned terms, the words "*neutral*", "*unneutral*", "*neutrality*", and "*unneutrality*" can be used interchangeably with "*indeterminate*", "*determinate*", "*indeterminacy*", and "*determinacy*" respectively. The latter three divisions of the vavn-decided PLR's into two complementary classes each, namely: (a) *valid* and *invalid*, (b) *antivalid* and *non-antivalid*, (c) *vav-neutral* (*vav-indeterminate*) and *vav-unneutral* (*vav-determinate*) are called the *subsidiary decisional dichotomies* (*bisections*, *bifurcations*) *of the vavn-decided PLR's*.

5) With '**P**' in place of '**P**', identity schema (6.12) becomes:

$$V(\neg\mathsf{P}) \hat{=} \begin{cases} 1 & \text{(a)} \\ 0 & \text{(b)} \\ \mathbf{i}_{\sim}|\neg\mathsf{P}\rangle & \text{(c)} \end{cases} \tag{6.34}$$

that is equivalent to (6.33), because

$$V(\neg\mathsf{P}) \hat{=} 1 \hat{-} V(\mathsf{P}), \tag{6.35}$$

$$\mathbf{i}_{\sim}|\neg\mathsf{P}\rangle \hat{=} 1 \hat{-} \mathbf{i}_{\sim}|\mathsf{P}\rangle, \tag{6.36}$$

which are the pertinent variants of (6.6) and (6.9). At the same time, the identities

$$V(\neg\mathsf{P}) \hat{\cdot} V(\neg\mathsf{P}) \hat{=} V(\neg\mathsf{P}), \tag{6.37}$$

$$\mathbf{i}_{\sim}|\neg\mathsf{P}\rangle \hat{\cdot} \mathbf{i}_{\sim}|\neg\mathsf{P}\rangle \hat{=} \mathbf{i}_{\sim}|\neg\mathsf{P}\rangle \tag{6.38}$$

are the pertinent variants of the identities (6.10) and (6.11). In accordance with (6.33) and (6.34), *the negation of a valid PlR*, P, *is an antivalid PLR*, $\neg\mathsf{P}$, *and vice versa*, whereas *the negation of a vav-neutral* (*vav-indeterminate*) *PLR*, P, *is another vav-neutral* (*vav-indeterminate*) *PLR*, $\neg\mathsf{P}$.

6) Under the definitions

$$\bar{V}(\mathsf{P}) \vec{\hat{=}} V(\neg\mathsf{P}) \hat{=} 1 \hat{-} V(\mathsf{P}), \tag{6.39}$$

$$\bar{\mathbf{i}}_{\sim}|\mathsf{P}\rangle \vec{\hat{=}} \mathbf{i}_{\sim}|\neg\mathsf{P}\rangle \hat{=} 1 \hat{-} \mathbf{i}_{\sim}|\mathsf{P}\rangle, \tag{6.40}$$

which are based on (6.35) and (6.36) and which are variants of definitions (6.13) and (6.14) with 'P' in place of '**P**', the decision theorem pattern (schema) (6.34) for $\neg\mathsf{P}$ becomes the *dual panlogographic decision theorem* (*DPLDT*) *pattern* (*schema*) *for* P:

$$\bar{V}(\mathsf{P}) \hat{=} \begin{cases} 1 & \text{(a)} \\ 0 & \text{(b)} \\ \bar{\mathbf{i}}_{\sim}|\mathsf{P}\rangle & \text{(c)} \end{cases}, \tag{6.41}$$

which is *dual* of (6.31), while the identities (6.36) and (6.37) turn into

$$\bar{V}(\mathsf{P}) \hat{\cdot} \bar{V}(\mathsf{P}) \hat{=} \bar{V}(\mathsf{P}), \tag{6.42}$$

$$\bar{\mathbf{i}}_{\sim}|\mathsf{P}\rangle \hat{\cdot} \bar{\mathbf{i}}_{\sim}|\mathsf{P}\rangle \hat{=} \bar{\mathbf{i}}_{\sim}|\mathsf{P}\rangle, \tag{6.43}$$

which are *dual* of (6.30) and (6.31) respectively. The theorem (a), (b), or (c) of (6.41) is denoted by '$\bar{\mathsf{T}}_{1+}(\mathsf{P})$', '$\bar{\mathsf{T}}_{1-}(\mathsf{P})$', or '$\bar{\mathsf{T}}_{1\sim}(\mathsf{P})$' respectively or indiscriminately by '$\bar{\mathsf{T}}_{1}(\mathsf{P})$', so that $\bar{\mathsf{T}}_{1+}(\mathsf{P})$, $\bar{\mathsf{T}}_{1-}(\mathsf{P})$, or $\bar{\mathsf{T}}_{1\sim}(\mathsf{P})$ is the PLDT for P, which is *dual* of $\mathsf{T}_{1+}(\mathsf{P})$, $\mathsf{T}_{1-}(\mathsf{P})$, or $\mathsf{T}_{1\sim}(\mathsf{P})$ respectively.

7) The three *decision* (*validity*) *classes of ER's*: *validity*, *antivalidity*, and *vav-neutrality* (*vav-indeterminacy)* and the similar three *decision* (*validity*) *classes of PLR's* are interrelated as follows.

a) A PLR **P** of $\mathbf{A}_1$ is valid, or antivalid, if and only if every ER **P** of A_1 in its range is a valid, or antivalid, respectively.

b) If **P** is a vav-neutral (vav-indeterminate) *analytical* (*not structural*) *PLR* (*AnPLR*) *of* $\mathbf{A}_1$ then its range contains ER's of A_1 of all the three kinds: valid, antivalid, and vav-neutral. In other words, a *vav-neutral AnPLR is not vavn-decided with respect to all ER's of its range.* Therefore, given a concrete ER **P** of academic or practical interest, of the range of a vav-neutral AnPLR **P**, the appropriate EADP D_1(**P**) should be applied to **P** in order to decide, which one of the three validity-classes **P** belongs to, provided of course that this has not been done earlier.

c) If **P** is a vav-neutral *structural PLR* (*StPLR*) *of* $\mathbf{A}_1$ then its range contains only vav-neutral ER's of A_1.

In accordance with the above relations a–c, in order to solve the vavn-decision problem for a given ER, it seems preferable to solve the vavn-decision problem for an adequately patterned PLR, an analytical one or at least a structural one, because the PLR condenses an infinite number of other ER's, for which the vavn-decision problem will be solved simultaneously with that for the given ER by the same work input.

8) There is the following simple bilateral relation between a vav-neutral AnPLR and a certain distinguished euautographic instance of its range. Let $\mathbf{P}_{\sim}$ be a *vav-neutral PLR of* $\mathbf{A}_1$, which involves some AtPLPH's with the base letters '**P**' to '**S**' and '**u**' to '**z**' with the proviso that none of the former AtPLPH's is followed by $\langle\ \rangle$. The ER $\mathbf{P}_{\sim}$, which results by the *analo-homolographic substitutions*:

$$p \mapsto \mathbf{P},\ q \mapsto \mathbf{Q},\ r \mapsto \mathbf{R},\ s \mapsto \mathbf{S}, \tag{6.44}$$

$$u \mapsto \mathbf{u},\ v \mapsto \mathbf{v},\ w \mapsto \mathbf{w},\ x \mapsto \mathbf{x},\ y \mapsto \mathbf{y},\ z \mapsto \mathbf{z} \tag{6.45}$$

(without any quotation marks) throughout $\mathbf{P}_{\sim}$, is a *vav-neutral ER of* A_1, Conversely, if $\mathbf{P}_{\sim}$ is a *vav-neutral ER of* A_1, which involves some of the primary atomic formulas with the base letters p to s and u to z, then the PLR $\mathbf{P}_{\sim}$, which results by the *analo-homolographic substitutions*:

$$\mathbf{P} \mapsto p,\ \mathbf{Q} \mapsto q,\ \mathbf{R} \mapsto r,\ \mathbf{S} \mapsto s, \tag{6.46}$$

$$\mathbf{u}\mapsto u,\ \mathbf{v}\mapsto v,\ \mathbf{w}\mapsto w,\ \mathbf{x}\mapsto x,\ \mathbf{y}\mapsto x,\ \mathbf{z}\mapsto z \qquad (6.47)$$

(without any quotation marks) throughout $\mathbf{P}_{\sim}$, is a *vav-neutral PLR of* $\mathbf{A}_1$. It is understood that the substitutions (6.44), e.g., imply that $p_1\mapsto\mathbf{P}_1$, $q_1\mapsto\mathbf{Q}_1$, etc, and similarly for (6.45)–(6.47).•

6.3. Summary

Df 6.3. Logographically, the three basic validity-values (validity-classes): *validity*, *antivalidity*, and *vav-neutrality* (*vav-indetterminacy*) will be denoted by 'v_+', 'v_-', and '$\mathsf{v}_\sim$' in that order, the understanding being that these three logographic constants belong to the XML (exclusive metalanguage) of A_1, i.e. of both A_1 and $\mathbf{A}_1$. This definition implies that there are two *mutually dual metalinguistic*, which will be denoted by 'V' and '$\bar{\mathsf{V}}$', such that

$$\mathsf{V}(0)=\mathsf{v}_+,\ \mathsf{V}(1)=\mathsf{v}_-,\ \mathsf{V}(\mathbf{i}_\sim)=\mathsf{V}(1\hat{-}\mathbf{i}_\sim)=\mathsf{V}(\mathsf{i}_\sim)=\mathsf{V}(1\hat{-}\bar{\mathsf{i}}_\sim)=\mathsf{v}_\sim, \qquad (6.49)$$

$$\bar{\mathsf{V}}(1)=\mathsf{v}_+,\ \bar{\mathsf{V}}(0)=\mathsf{v}_-,\ \bar{\mathsf{V}}(\bar{\mathbf{i}}_\sim)=\bar{\mathsf{V}}(1\hat{-}\bar{\mathbf{i}})=\bar{\mathsf{V}}(\bar{\mathsf{i}}_\sim)=\bar{\mathsf{V}}(1\hat{-}\bar{\mathsf{i}}_\sim)=\mathsf{v}_\sim, \qquad (6.50)$$

subject to Dfs 6.1(5,13) and 6.2(1,6).•

Cmt 6.3. a) "*The validity-integron validity*", "*the antivalidity-integron antivalidity*", "*the validity-identifier* (or *index*) *validity of* **P**", and "*the antivalidity-identifier* (or *index*) *antivalidity of* **P**" are synonymous names of 0.

b) "*The validity-integron antivalidity*", "*the antivalidity-integron validity*", "*the validity-identifier* (or *index*) *antivalidity of* **P**", and "*the antivalidity-identifier* (or *index*) *validity of* **P**" are synonymous names of 1;

c) "*Euautographic validity-integron neutrality*" and "*euautographic antiavalidity-integron neutrality*" are synonymous common names of any one of *idempotent euautographic validity-integrons* $\mathbf{i}_\sim$, $1\hat{-}\mathbf{i}_\sim$, $\bar{\mathbf{i}}_\sim$, and $1\hat{-}\bar{\mathbf{i}}_\sim$; "*the validity-identifier* (or *validity-index*) *neutrality of* **P**" is an accidental proper name of $\mathbf{i}|\mathbf{P}\rangle$; "*the antivalidity-identifier* (or *antivalidity-index*) *neutrality of* **P**" is an accidental proper name of $\bar{\mathbf{i}}|\mathbf{P}\rangle$.

d) "*Panlogographic validity-integron neutrality*" and "*panlogographic antiavalidity-integron neutrality*" are synonymous common names of any one of *idempotent panlogographic validity-integrons* $\mathsf{i}_\sim$, $1\hat{-}\mathsf{i}_\sim$, $\bar{\mathsf{i}}_\sim$, and $1\hat{-}\bar{\mathsf{i}}_\sim$; "*the panlogographic validity-identifier* (or *validity-index*) *neutrality of* P" is an accidental proper name of $\mathbf{i}_{\sim}|\mathsf{P}\rangle$; "*the panlogographic antivalidity-identifier* (or *antivalidity-index*) *neutrality of* P" is an accidental proper name of $\bar{\mathbf{i}}_{\sim}|\mathsf{P}\rangle$.•

Cmt 6.4. The main properties of the AEADM, D_1, and of the APLADM, $\mathbf{D}_1$, can be recapitulated as follows.

1) To any given euautographic slave-relation (ESR) of A_1 that I regard as one having academic or practical interest, there is an EADP, whose final relation is the pertinent euautographic master-theorem (EMT), or decision theorem (EDT), of A_1, according to the form of which the processed ESR is unambiguously classified either as a valid one (kyrology) or as an antivalid one (antikyrology) or else as a vav-neutral (vav-indeterminate) one (kak-udeterology, kak-anorismenology). Although I use the qualifier "indeterminate" as a synonym of "neutral', *there is no indeterminacy* (*uncertainty*) *in attributing the ESR to the class of vav-neutral* (*vav-indeterminate*) *ER's of* A_1 *if it is the case*. A vav-neutral ESR of A_1 is *not an improvable relation of the Gödelian type*, because it *is proved* to be vav-neutral – just as a valid ESR, other than a subject euautographic axiom of A_1, is proved to be valid and just as an antivalid ESR is proved to be antivalid. *Th 4.2, i.e. the three-fold metatheorem-schema (4.42), expresses this fact explicitly*. For instance, since $V(p)$ and $V(q)$ are irreducible, the atomic euautographic relations p and q, are vav-neutral. At the same time, by the pertinent instance of Ax 4.1 and by the pertinent instances items 1, 2, and 6 of Df 1.10, it is elementarily *proved* (see section 7 for greater detail) that $p\vee\neg p$ and $\neg[p\wedge\neg p]$ are theorems, $\neg[p\vee\neg p]$ and $p\wedge\neg p$ are antitheorems, whereas $\neg p$, $\neg q$, $p\vee q$, and $p\wedge q$, $\neg[p\vee q]$, and $\neg[p\wedge q]$ are vav-neutral relations, of A_1.

2) An ESR of A_1 may have several EADP's, which differ in orders of the elementary operations involved in the EADP's. All the procedures result in the same EDT and hence in the same decision regarding the ESR. However, one of the procedures may turn out to be shorter and simpler than another one. Therefore, in spite of the fact that any EADP is mechanical, choice of the optimal EADP for a complex ESR or for a complex PLS (panlogographic schema) of ESR's of its range is a kind of art that is acquired by experience – just as in the case of mental arithmetical calculations with natural integers.

3) In accordance with Df 6.2, the same remarks apply to PLADP's and PLDT's of $\mathbf{A}_1$ and to their PLSR's in place of EADP's and EDT's of A_1 and to their ESR's respectively. Particularly, the following three-fold metatheorem-schema:

$$\vdash[V(\mathbf{P}) \hat{=} 0] \quad \text{if and only if} \vdash[V(V(\mathbf{P}) \hat{=} 0) \hat{=} 0], \qquad \text{(a)}$$

$$\vdash[V(\mathbf{P}) \hat{=} 1] \quad \text{if and only if} \vdash[V(V(\mathbf{P}) \hat{=} 1) \hat{=} 0], \qquad \text{(b)} \qquad (6.48)$$

$$\vdash[V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle] \quad \text{if and only if} \vdash[V(V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle) \hat{=} 0]. \qquad \text{(c)}$$

is the variant of the three-fold metatheorem-schema (4.42) with '**P**' in place of '**P**' – the variant, which expresses explicitly the fact that the property of a PLSR to be valid, antivalid, or vav-neutral (vav-indeterminate) is provable.•

Cmt 4.5. An EADP has the following general properties.

i) All algebraic calculations that are involved in any EADP are performed *at the syntactic level* with the help of the pertinent primary or secondary rules of inference.

ii) Every EADP is a sequence of interrelated *identities*, i.e. *valid equalities*, *of* A_1, each of which is either a subject axiom or a subject theorem of A_1, Particularly, the ultimate identity of an EADP is a theorem of A_1 that is called a euautographic master, or decision, theorem (EMT or EDT) of A_1.

iii) An EADP does not, as a rule, involve any DDI's larger than 2 or smaller than $\hat{-}2$. As was already pointed out in Cmt 5.5, the whole infinite set of DDI's has been defined and classified simply because it is recursive, so that it is impossible to define any restricted part of it along with the binary operators $\hat{+}$, $\hat{\cdot}$, and $\hat{-}$, which are unavoidably defined on the whole set of DDI's.

iv) The sequence of identities forming an EADP is as rule written in *the legato style*, i.e. in the form of one or more trains of identities, which are supplied with the appropriate comments in the metalanguage.

v) If an EADP is written in terms of AtPLPH's and if the latter are mentally used autonymously then the EADP turns into a PLADP.•

Cmt 4.6. Besides the various impartial descriptive taxonyms which are based on employing the qualifier "idempotent" to "integron" in place of either of the qualifiers "validity-" and "antivalidity-", the impartial Latin names "*terminus a quo*", "*terminus ad quem*", and "*terminus per quem*" can be used synonymously (interchangeably) with "*primary idempotent integron*" ("PII"), "*irreducible* *idempotent integron*” (“III”), and “*reducible idempotent integron*” (“RII”), respectively. In Latin, “*terminus a quo*” means «*term*, *point*, or *limit from which*», “*terminus ad quem*” means «*term, point,* or *limit to which*», and “*terminus per quem*” means “«*term* or *point though which*». The first two expressions are established barbarisms of the English language (see, e.g., WTNID), whereas the third one is an analogous suggestion of my own – it is not in common usage.•

Cmt 4.7. The class of EVI’s of A_1^0 or A_0 is partitioned in the same way as that of A_1 with the following two essential differences.

a) EVI’s of A_1^0 and A_0 do not have any occurrences of the EKS $\hat{\cdot}_*$.

b) The set of MlEVI’s of A_0 coincides with the set of BMlEVI (PFrMlEVI) occurring or obviously understood as occurring on the list (4.50).

7. The BEADP’s and BPLADP’s for the major predicate-free relations of A_1

7.1. The EADP’s for the major predicate-free relations of A_1

°**Th 7.1.**

$$V(p \veebar q) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p) \mathbin{\hat{\cdot}} V(q). \tag{7.0}$$

$$V(\neg p) \mathrel{\hat{=}} V(p \veebar p) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p) \mathbin{\hat{\cdot}} V(p) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p). \tag{7.1}$$

$$V(p \vee q) \mathrel{\hat{=}} V(\neg[p \veebar q]) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p \veebar q) \mathrel{\hat{=}} V(p) \mathbin{\hat{\cdot}} V(q). \tag{7.2}$$

$$V(p \Rightarrow q) \mathrel{\hat{=}} V([\neg p] \vee q) \mathrel{\hat{=}} V(\neg p) \mathbin{\hat{\cdot}} V(q) \mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(p)] \mathbin{\hat{\cdot}} V(q). \tag{7.3}$$

$$V(p \Leftarrow q) \mathrel{\hat{=}} V(p) \vee [\neg q] \mathrel{\hat{=}} V(p) \mathbin{\hat{\cdot}} V(\neg q) \mathrel{\hat{=}} V(p) \mathbin{\hat{\cdot}} [1 \mathbin{\hat{-}} V(q)] \mathrel{\hat{=}} V(q \Rightarrow p). \tag{7.4}$$

$$\begin{aligned} V(p \barwedge q) &\mathrel{\hat{=}} V([\neg p] \vee [\neg q]) \mathrel{\hat{=}} V(\neg p) \mathbin{\hat{\cdot}} V(\neg q) \\ &\mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(p)] \mathbin{\hat{\cdot}} [1 \mathbin{\hat{-}} V(q)] \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p) \mathbin{\hat{-}} V(q) \mathbin{\hat{+}} V(p) \mathbin{\hat{\cdot}} V(q). \end{aligned} \tag{7.5}$$

$$\begin{aligned} V(p \wedge q) &\mathrel{\hat{=}} V(\neg[p \barwedge q]) \mathrel{\hat{=}} V([\neg p] \veebar [\neg q]) \mathrel{\hat{=}} V(\neg[[\neg p] \vee [\neg q]]) \\ &\mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\neg p) \mathbin{\hat{\cdot}} V(\neg q) \mathrel{\hat{=}} V(p) \mathbin{\hat{-}} V(q) \mathbin{\hat{+}} V(p) \mathbin{\hat{\cdot}} V(q). \end{aligned} \tag{7.6}$$

$$\begin{aligned} V(p \Leftrightarrow q) &\mathrel{\hat{=}} V([p \Rightarrow q] \wedge [p \Leftarrow q]) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\neg[p \Rightarrow q]) \mathbin{\hat{\cdot}} V(\neg[p \Leftarrow q]) \\ &\mathrel{\hat{=}} V(p \Rightarrow q) \mathbin{\hat{+}} V(p \Leftarrow q) \mathrel{\hat{=}} V(\neg p) \mathbin{\hat{\cdot}} V(q) \mathbin{\hat{+}} V(p) \mathbin{\hat{\cdot}} V(\neg q) \\ &\mathrel{\hat{=}} V(p) \mathbin{\hat{+}} V(q) \mathbin{\hat{-}} 2 \mathbin{\hat{\cdot}} V(p) \mathbin{\hat{\cdot}} V(q) \mathrel{\hat{=}} [V(p) \mathbin{\hat{-}} V(q)]^2. \end{aligned} \tag{7.7}$$

$$V(p \mathbin{\overline{\vee}} q) \mathrel{\hat{=}} V(\neg[p \vee q]) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p \vee q) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p) \mathbin{\hat{\cdot}} V(q) \mathrel{\hat{=}} V(p \veebar q). \tag{7.8}$$

$$\begin{aligned} V(p \mathbin{\overline{\wedge}} q) &\mathrel{\hat{=}} V(\neg[p \wedge q]) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p \wedge q) \\ &\mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p) \mathbin{\hat{-}} V(q) \mathbin{\hat{+}} V(p) \mathbin{\hat{\cdot}} V(q). \mathrel{\hat{=}} V(p \barwedge q). \end{aligned} \tag{7.9}$$

$$V(p \overline{\Rightarrow} q) \hat{=} V(\neg[p \Rightarrow q]) \hat{=} 1 \hat{-} V(p \Rightarrow q). \tag{7.10}$$

$$\begin{gathered} V(p \overline{\Leftarrow} q) \hat{=} V(\neg[p \Leftarrow q]) \hat{=} 1 \hat{-} V(p \Leftarrow q) \\ \hat{=} 1 \hat{-} V(q \Rightarrow p) \hat{=} V(\neg[q \Rightarrow p]) \hat{=} V(q \overline{\Rightarrow} p). \end{gathered} \tag{7.11}$$

$$\begin{gathered} V(p \overline{\Leftrightarrow} q) \hat{=} V(\neg[p \Leftrightarrow q]) \hat{=} 1 \hat{-} V(p \Leftrightarrow q) \\ \hat{=} 1 \hat{-} [V(p) \hat{-} V(q)]^2 \hat{=} [1 \hat{-} V(p) \hat{+} V(q)] \hat{\cdot} [1 \hat{+} V(p) \hat{-} V(q)] \\ \hat{=} [V(\neg p) \hat{+} V(q)] \hat{\cdot} [V(p) \hat{+} V(\neg q)]. \end{gathered} \tag{7.12}$$

Proof: The identity (7.1) is a concrete euautographic instance (corollary, axiom-corollary) of the PLS (panlogographic schema) of EAxs (euautographic axioms) (4.1) subject to $\mathbf{P} \triangleright p$ and $\mathbf{Q} \triangleright q$, i.e. subject to the *analo-homolographic substitutions*

$$p \mapsto \mathbf{P} \text{ and } q \mapsto \mathbf{Q} \tag{7.13}$$

without quotation marks (cf. (4.48)). The trains of identities (7.1)–(7.12) follow from the instances of the respective items 1–12 of Df. 1.10) subject to $\mathbf{P} \triangleright p$ and $\mathbf{Q} \triangleright q$, i.e. under the substitutions (7.13), by the pertinent instances of the meta-axiom (4.31) and by (7.0). All the calculations (inferences) are self-explanatory: they are performed with the help of the pertinent elementary rules of the BID Z_{00}, primary (axiomatic) ones and secondary ones, which have been derived in subsection 5.2. These rules are also ones of the RBID Z_{01}^{0} and ones of the AID Z_{11} up to the difference between the underlying classes I_0, I_1^0, and I_1 of Z_{00}, Z_{01}^{0}, and Z_{11} respectively. Among the primary rules used, there is the idempotent law

$$V(p) \hat{\cdot} V(p) \hat{=} V(p) \tag{7.14}$$

and its variant with q in place of p, being concrete euautographic instances (corollaries) of the PLS (4.2). Among the secondary (derived) rules used, there is the *algebraic law of excluded middle* (*ALEM*):

$$V(p \vee \neg p) \hat{=} V(p) \hat{\cdot} V(\neg p) \hat{=} V(p) \hat{\cdot} [1 \hat{-} V(p)] \hat{=} 0 \tag{7.15}$$

and its variant with q in place of p, which follow from Df 1.10(2) by (7.1) and (7.14) Also, in developing any one of the trains (7.2)–(7.12), use of some identities preceding it or of their pertinent variants is made. For instance, (7.1) implies the following *algebraic law of double negation* (*ALDN*):

$$V(\neg\neg p) \hat{=} 1 \hat{-} V(\neg p) \hat{=} 1 \hat{-} [1 \hat{-} V(p)] \hat{=} V(p), \tag{7.1$_1$}$$

and its variant with q in place of p, whereas making use of (7.3) and (7.4) along with (7.15) yields:

$$
\begin{aligned}
V([p \Rightarrow q] \vee [p \Leftarrow q]) &\mathrel{\hat{=}} V(p \Rightarrow q) \mathbin{\hat{\cdot}} V(p \Leftarrow q) \\
&\mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(p)] \mathbin{\hat{\cdot}} V(q) \mathbin{\hat{\cdot}} V(p) \mathbin{\hat{\cdot}} [1 \mathbin{\hat{-}} V(q)] \mathrel{\hat{=}} 0.
\end{aligned}
\tag{7.16}
$$

Use of the variant of (7.6) with $[p \Rightarrow q]$ in place of p and $[p \Leftarrow q]$ in place of q and also use of (7.16) have been made in developing (7.7).•

Cmt 7.1. 1) Any one of the trains of identities (7.0)–(7.12) is a BEADP (basic EADP) for the ESR (euautographic slave-relation) occurring in the PVI (primary validity-integron) of the train. According to the final expressions of the BEADP's, which are VID's (validity-indices) of the respective EMT's (euautographic master-theorems), i.e. EDT's pertinent (euautographic decision theorems), all ESR's processed are *vav-neutral*, like p and q themselves. By contrast, the ESR's $p \vee \neg p$ and $[p \Rightarrow q] \vee [p \Leftarrow q]$ have been *vavn-decided* by the BEADP's (7.15) and (7.16) to be *valid.*

2) Calculating the VID's of the pertinent EDT's has been the main object of Th 7.1. Still, the ultimate or intermediate results occurring in some of the trains (7.2)–(7.12) can be modified for convenience in utilizing them in further computations. For instance, making use of (7.1), the variant of (7.1) with q in place of p, and (7.2), the trains (7.6) and (7.7) can be supplemented by the following ones:

$$
\begin{aligned}
V(p \wedge q) &\mathrel{\hat{=}} V(p) \mathbin{\hat{+}} V(\neg p) \mathbin{\hat{\cdot}} V(q) \mathrel{\hat{=}} V(q) \mathbin{\hat{+}} V(\neg q) \mathbin{\hat{\cdot}} V(p) \\
&\mathrel{\hat{=}} V(p) \mathbin{\hat{+}} V(q) \mathbin{\hat{-}} V(p \vee q),
\end{aligned}
\tag{7.6$_1$}
$$

$$
V(p \Leftrightarrow q) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p) \mathbin{\hat{\cdot}} V(q) \mathbin{\hat{-}} V(\neg p) \mathbin{\hat{\cdot}} V(\neg q) \mathrel{\hat{=}} V(p \wedge q) \mathbin{\hat{-}} V(p \vee q). \tag{7.7$_1$}
$$

By (7.1),

$$
V(p) \mathbin{\hat{+}} V(\neg p) \mathrel{\hat{=}} 1 \tag{7.17}
$$

and similarly with q in place of p. Therefore, the next to last expression in (7.7$_1$) can be developed from (7.7), e.g., thus:

$$
\begin{aligned}
V(p \Leftrightarrow q) &\mathrel{\hat{=}} V(\neg p) \mathbin{\hat{\cdot}} V(q) \mathbin{\hat{+}} V(p) \mathbin{\hat{\cdot}} V(\neg q) \\
&\mathrel{\hat{=}} V(\neg p) \mathbin{\hat{\cdot}} [1 \mathbin{\hat{-}} V(\neg q)] \mathbin{\hat{+}} V(p) \mathbin{\hat{\cdot}} [1 \mathbin{\hat{-}} V(q)] \\
&\mathrel{\hat{=}} V(p) \mathbin{\hat{+}} V(\neg p) \mathbin{\hat{-}} V(p) \mathbin{\hat{\cdot}} V(q) \mathbin{\hat{-}} V(\neg p) \mathbin{\hat{\cdot}} V(\neg q) \\
&\mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(p) \mathbin{\hat{\cdot}} V(q) \mathbin{\hat{-}} V(\neg p) \mathbin{\hat{\cdot}} V(\neg q).
\end{aligned}
\tag{7.7$_2$}
$$

7.2. The BPLADP’s for the major predicate-free relations of A_1: The panlogographic interpretantia of the major predicate-free ER’s

***Th 7.2.**

$$V(\mathbf{P} \not\vee \mathbf{Q}) \hat{=} 1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}). \tag{7.0γ}$$

$$V(\neg\mathbf{P}) \hat{=} V(\mathbf{P} \not\vee \mathbf{P}) \hat{=} 1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{P}) \hat{=} 1 \hat{-} V(\mathbf{P}). \tag{7.1γ}$$

$$V(\mathbf{P} \vee \mathbf{Q}) \hat{=} V(\neg[\mathbf{P} \not\vee \mathbf{Q}]) \hat{=} 1 \hat{-} V(\mathbf{P} \not\vee \mathbf{Q}) \hat{=} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}). \tag{7.2γ}$$

$$V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{=} V([\neg\mathbf{P}] \vee \mathbf{Q}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{=} [1 \hat{-} V(\mathbf{P})] \hat{\cdot} V(\mathbf{Q}). \tag{7.3γ}$$

$$\begin{gathered} V(\mathbf{P} \Leftarrow \mathbf{Q}) \hat{=} V(\mathbf{P} \vee [\neg\mathbf{Q}]) \hat{=} V(\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \hat{=} V(\mathbf{P}) \hat{\cdot} [1 \hat{-} V(\mathbf{Q})] \\ \hat{=} V(\mathbf{Q} \Rightarrow \mathbf{P}). \end{gathered} \tag{7.4γ}$$

$$\begin{gathered} V(\mathbf{P} \not\wedge \mathbf{Q}) \hat{=} V([\neg\mathbf{P}] \vee [\neg\mathbf{Q}]) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \\ \hat{=} [1 \hat{-} V(\mathbf{P})] \hat{\cdot} [1 \hat{-} V(\mathbf{Q})] \hat{=} 1 \hat{-} V(\mathbf{P}) \hat{-} V(\mathbf{Q}) \hat{+} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}). \end{gathered} \tag{7.5γ}$$

$$\begin{gathered} V(\mathbf{P} \wedge \mathbf{Q}) \hat{=} V(\neg[\mathbf{P} \not\wedge \mathbf{Q}]) \hat{=} V([\neg\mathbf{P}] \not\vee [\neg\mathbf{Q}]) \hat{=} V(\neg[[\neg\mathbf{P}] \vee [\neg\mathbf{Q}]]) \\ \hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \hat{=} V(\mathbf{P}) \hat{+} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{=} V(\mathbf{Q}) \hat{+} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{P}) \\ \hat{=} V(\mathbf{P}) \hat{+} V(\mathbf{Q}) \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}). \end{gathered} \tag{7.6γ}$$

$$\begin{gathered} V(\mathbf{P} \Leftrightarrow \mathbf{Q}) \hat{=} V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Leftarrow \mathbf{Q}]) \hat{=} 1 \hat{-} V(\neg[\mathbf{P} \Rightarrow \mathbf{Q}]) \hat{\cdot} V(\neg[\mathbf{P} \Leftarrow \mathbf{Q}]) \\ \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\mathbf{P} \Leftarrow \mathbf{Q}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{+} V(\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \\ \hat{=} 1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \\ \hat{=} V(\mathbf{P}) \hat{+} V(\mathbf{Q}) \hat{-} 2 \hat{\cdot} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{=} [V(\mathbf{P}) \hat{-} V(\mathbf{Q})]^2. \end{gathered} \tag{7.7γ}$$

$$V(\mathbf{P} \mathbin{\overline{\triangledown}} \mathbf{Q}) \hat{=} V(\mathbf{P} \not\vee \mathbf{Q}) \hat{=} V(\neg[\mathbf{P} \vee \mathbf{Q}]) \hat{=} 1 \hat{-} V(\mathbf{P} \vee \mathbf{Q}) \hat{=} 1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}). \tag{7.8γ}$$

$$\begin{gathered} V(\mathbf{P} \mathbin{\overline{\wedge}} \mathbf{Q}) \hat{=} V(\mathbf{P} \not\wedge \mathbf{Q}) \hat{=} V(\neg[\mathbf{P} \wedge \mathbf{Q}]) \hat{=} 1 \hat{-} V(\mathbf{P} \wedge \mathbf{Q}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \\ \hat{=} V(\neg\mathbf{P} \vee \neg\mathbf{Q}) \hat{=} 1 \hat{-} V(\mathbf{P}) \hat{-} V(\mathbf{Q}) \hat{+} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}). \end{gathered} \tag{7.9γ}$$

$$V(\mathbf{P} \mathbin{\overline{\Rightarrow}} \mathbf{Q}) \hat{=} V(\neg[\mathbf{P} \Rightarrow \mathbf{Q}]) \hat{=} 1 \hat{-} V(\mathbf{P} \Rightarrow \mathbf{Q}). \tag{7.10γ}$$

$$\begin{gathered} V(\mathbf{P} \mathbin{\overline{\Leftarrow}} \mathbf{Q}) \hat{=} V(\neg[\mathbf{P} \Leftarrow \mathbf{Q}]) \hat{=} 1 \hat{-} V(\mathbf{P} \Leftarrow \mathbf{Q}) \\ \hat{=} 1 \hat{-} V(\mathbf{Q} \Rightarrow \mathbf{P}) \hat{=} V(\neg[\mathbf{Q} \Rightarrow \mathbf{P}]) \hat{=} V(\mathbf{Q} \mathbin{\overline{\Rightarrow}} \mathbf{P}). \end{gathered} \tag{7.11γ}$$

$$\begin{gathered} V(\mathbf{P} \mathbin{\overline{\Leftrightarrow}} \mathbf{Q}) \hat{=} V(\neg[\mathbf{P} \Leftrightarrow \mathbf{Q}]) \hat{=} 1 \hat{-} V(\mathbf{P} \Leftrightarrow \mathbf{Q}) \\ \hat{=} 1 \hat{-} [V(\mathbf{P}) \hat{-} V(\mathbf{Q})]^2 \hat{=} [1 \hat{-} V(\mathbf{P}) \hat{+} V(\mathbf{Q})] \hat{\cdot} [1 \hat{+} V(\mathbf{P}) \hat{-} V(\mathbf{Q})] \\ \hat{=} [V(\neg\mathbf{P}) \hat{+} V(\mathbf{Q})] \hat{\cdot} [V(\mathbf{P}) \hat{+} V(\neg\mathbf{Q})] \\ \hat{=} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{+} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}). \end{gathered} \tag{7.12γ}$$

Proof: The trains of identities (7.0γ)–(7.12γ) are variants of (7.0)–(7.12) under the *analo-homolographic substitutions*

$$\mathbf{P} \mapsto p \text{ and } \mathbf{Q} \mapsto q \tag{7.18}$$

without any quotation marks, which are opposite of (7.13). That is to say, the trains (7.0γ)–(7.12γ) follow from (7.0)–(7.12) by the pertinent autonymous version (4.37_1) subject to (4.37_{1+}) of the inference rule (rule of substitutions) (4.37) subject to (4.37_+). The identity (7.0γ) is the PLS of ER's (4.1), which is rewritten here for convenience in further references. The trains of identities (7.1γ)–(7.12γ) can alternatively be deduced straightforwardly from the items 1–12 of Df 1.10 in the same way as (7.1)–(7.12) by making use of the axioms (4.1) (identity (7.0γ)) and (4.2) instead of (7.1) and (7.14), being concrete euautographic instances of the former, and also by making use of all pertinent elementary rules of Z_{00} in the appropriate form. Particularly, the axiom (4.2) implies the *algebraic law of excluded middle* (*ALEM*) in the form:

$$V(\mathbf{P}\vee\neg\mathbf{P})\,\hat{=}\,V(\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{=}\,V(\mathbf{P})\,\hat{\cdot}\,[1\,\hat{-}\,V(\mathbf{P})]\,\hat{=}\,0 \qquad (7.15\gamma)$$

instead of (7.15), being its concrete instance, and similarly with '**Q**' in place of '**P**'. The BEADP (7.1γ) obviously implies the following *algebraic law of double negation* (*ALDN*):

$$V(\neg\neg\mathbf{P})\,\hat{=}\,1\,\hat{-}\,V(\neg\mathbf{P})\,\hat{=}\,1\,\hat{-}\,[1\,\hat{-}\,V(\mathbf{P})]\,\hat{=}\,V(\mathbf{P}), \qquad (7.1\gamma_1)$$

instead (7.1_1), being its concrete instance, and similarly with '**Q**' in place of '**P**', whereas making use of (7.3γ) and (7.4γ) along with (7.15γ) yields:

$$\begin{aligned} V([\mathbf{P}\Rightarrow\mathbf{Q}]\vee[\mathbf{P}\Leftarrow\mathbf{Q}])\,&\hat{=}\,V(\mathbf{P}\Rightarrow\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{P}\Leftarrow\mathbf{Q})\\ &\hat{=}\,[1\,\hat{-}\,V(\mathbf{P})]\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,[1\,\hat{-}\,V(\mathbf{Q})]\,\hat{\cdot}\,V(\mathbf{P})\,\hat{=}\,0\end{aligned} \qquad (7.16\gamma)$$

instead of (7.16), being its concrete instance.•

Cmt 7.2. 1) Any one of the trains of identities (7.0γ)–(7.12γ) is the BPLADP for the PLSR (panlogographic slave-relation) occurring as argument in the PVI (primary validity-integron) of the train and at the same time it is a PLS (panlogographic schema) of the BEADP's for all ESR's comprised in its ranges. Like AnAtPLR's '**P**' and '**Q**', all PLSR's processed are *vav-neutral*, in accordance with their VID's. Particularly, the BPLADP (7.1γ) implies that

$$V(\neg\mathbf{P})\,\hat{=}\,V(\mathbf{P}\veebar\mathbf{P})\,\hat{=}\,\mathbf{i}_{\sim}|\neg\mathbf{P}\rangle\,\hat{=}\,\mathbf{i}_{\sim}|\mathbf{P}\veebar\mathbf{P}\rangle\,\hat{=}\,1\,\hat{-}\,V(\mathbf{P}). \qquad (7.19)$$

At the same time, all binary ESR's processed in the BPLADP's (7.0γ) and (7.2γ)–(7.12γ) can be condensed into the relation-schema'**Pλ****Q**' subject to definition (1.14) of '**λ**'. Consequently, any one of these BPLADP's has the form:

$$V(\mathbf{P}\boldsymbol{\lambda}\mathbf{Q})\,\hat{=}\,\mathbf{i}_{\sim}|\mathbf{P}\boldsymbol{\lambda}\mathbf{Q}\rangle \qquad (7.20)$$

subject to a certain instance (value) of ‘**λ**’. By contrast, the relations **P**∨¬**P** and [**P**⇒**Q**]∨[**P**⇐**Q**] have been *vavn-decided* by the BPLADP’s (7.15γ) and (7.16γ) to be *valid.*

2) In accordance with the axiom (6.37), a BPLADP is a variant of the respective BEADP subject to (7.18). Therefore, the following PLR’s (panlogographic relations), which are variants with $\mathbf{P}\mapsto p$ and $\mathbf{Q}\mapsto q$ of the ER’s (7.6_1), (7.7_1), (7.7_2), and (7.17) occurring in Cmt 7.1(2), are also valid:

$$\begin{gathered} V(\mathbf{P}\wedge\mathbf{Q})\hat{=}V(\mathbf{P})\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{=}V(\mathbf{Q})\hat{+}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{P}) \\ \hat{=}V(\mathbf{P})\hat{+}V(\mathbf{Q})\hat{-}V(\mathbf{P}\vee\mathbf{Q}), \end{gathered} \tag{7.6γ$_1$}$$

$$V(\mathbf{P}\Leftrightarrow\mathbf{Q})\hat{=}1\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{P})\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{=}V(\mathbf{P}\wedge\mathbf{Q})\hat{-}V(\mathbf{P}\vee\mathbf{Q}). \tag{7.7γ$_1$}$$

$$V(\mathbf{P})\hat{+}V(\neg\mathbf{P})\hat{=}1. \tag{7.16γ}$$

$$\begin{gathered} V(\mathbf{P}\Leftrightarrow\mathbf{Q})\hat{=}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{+}V(\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q}) \\ \hat{=}V(\neg\mathbf{P})\hat{\cdot}[1\hat{-}V(\neg\mathbf{Q})]\hat{+}V(\mathbf{P})\hat{\cdot}[1\hat{-}V(\mathbf{Q})] \\ \hat{=}V(\mathbf{P})\hat{+}V(\neg\mathbf{P})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q}) \\ \hat{=}1\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q}). \end{gathered} \tag{7.7γ$_2$}$$

Cmt 7.3: ***The major predicate-free panlogographic relations (PLR’s) versus euautographic relations (ER’s) of their ranges.*** 1) Comparison of the euautographic identities established in subsection 7.1 and their panlogographic counterparts established above in this subsection allows illustrating the thesis that I have already repeatedly posited earlier, particularly in Df 6.2(7). Namely, a PLR (panlogographic relation) **P** of A_1 is valid, or antivalid, if and only if every ER (euautographic relation) **P** of A_1 in its range is valid, or antivalid, respectively. If however **P** is a *vav-neutral* (*vav-indeterminate*) *AnPLR* (*analytical panlogographic relation*) *of* A_1, – as contrasted to a *vav-neutral StPLR* (*structural panlogographic relation*) *of* A_1, – then its range contains ER’s of A_1 of all the three kinds: valid, antivalid, and vav-neutral. In this case, all vav-neutral PLR’s, e.g. ‘**P**⊻**Q**’, ‘**P**∨**Q**’, ‘**P**⇒**Q**’, ‘**P**⇐**Q**’, etc, and also the valid PLR’s **P**∨¬**P** and [**P**⇒**Q**]∨[**P**⇐**Q**] belong to the range of ‘**P**’. In turn, for instance, the valid ER (euautographic relation) *p*∨¬*q* is in the range of the valid PLR ‘**P**∨¬**P**’, whereas the valid ER [*p*⇒*q*]∨[*p*⇐*q*] is in the range of the valid PLR ‘[**P**⇒**Q**]∨[**P**⇐**Q**]’. At the same time, both above valid ER’s are in the range of the vav-neutral PLR ‘**P**∨**Q**’. The latter range contains also the ER *p*∨*q*, which is vav-neutral by (7.2), and it also contains, e.g., the ER $[p\wedge\neg p]\vee[q\wedge\neg q]$, which is antivalid by the following BEADP:

$$V([p\wedge\neg p]\vee[q\wedge\neg q])\hat{=}V(p\wedge\neg p)\hat{\cdot}V(q\wedge\neg q)\hat{=}1\hat{\cdot}1\hat{=}1, \tag{7.21}$$

being the variant of (7.2) with $[p\wedge\neg p]$ in place of p and with $[q\wedge\neg q]$ in place of q. The final result in (7.21) is determined, first, by the BEADP:

$$V(p\wedge\neg p)\hat{=}V(p)\hat{+}V(\neg p)\hat{-}V(p)\hat{\cdot}V(\neg p)\hat{=}1, \tag{7.22}$$

which is the variant of (7.6) with $\neg p$ in place of q, subject to (7.14) and (7.16), and, second, by the variant of (7.22) with q n place of p.

2) In accordance with Df 6.2(7), the analo-homolographic substitutions (interpretations, particularizations) (7.13) throughout any of the vav-neutral PLR's processed in (7.1γ)–(7.12γ) result in the respective one of the vav-neutral ER's processed in (7.1)–(7.12).

3) The variant of any ER of subsection 7.1 with $\mathbf{p}\mapsto p$ and $\mathbf{q}\mapsto q$ without any quotation marks or the identical variant of the conformal PLR of the above portion of this subsection with $\mathbf{p}\mapsto\mathbf{P}$ and $\mathbf{q}\mapsto\mathbf{Q}$, i.e. at $\mathbf{P}\triangleright\mathbf{p}$ and $\mathbf{Q}\triangleright\mathbf{q}$), is a StCbPLR (structural combined panlogographic relation), provided that the euautographic instances of StAtPLR's '**p**' and '**q**' occurring in the StCbPLR are supposed to be different. In contrast to a vav-neutral AnCbPLR, all ER's in the range of a vav-neutral StCbPLR are vav-neutral.

4) I have already repeatedly pointed out that p, e.g., is a vav-neutral AtER (atomic euautographic relation), because its PVI, $V(p)$, is irreducible. Consequently, I may not assume either that $\vdash[V(p)\hat{=}0]$ or that $\vdash[V(p)\hat{=}1]$ subject to Df 3.7 (see, e.g., Cmt 5.1(2c)). A like remark applies with any other AtER (atomic euautographic relation) **p**, e.g. q, in place of p. Also, neither p nor q can assume any xenovalues including truth-values. Therefore, *the EDT's that the BEADP's (7.0)–(7.12) prove and include as their final identities cannot be used for establishing any validity-tables, any truth-tables, and any veracity-tables for the pertinent major logical connectives.*

5) When the AnAtPLPH '**P**', e.g., is used autonymously, it has the like properties, although its autonymous use is not necessarily indicated by its HAQ (homolographic autonymous quotation). Using nevertheless the appropriate HAQ's for indicating my pertinent mental attitudes, I may not, for instance, assume either that $\vdash[V(\text{'}\mathbf{P}\text{'})\hat{=}0]$ or that $\vdash[V(\text{'}\mathbf{P}\text{'})\hat{=}1]$ subject to Df 3.7, because $V(\text{'}\mathbf{P}\text{'})$ is, like $V(p)$,

irreducible, so that $\vdash\left[V(\text{'}\mathbf{P}\text{'})\hat{=}\mathbf{i}_{\sim}\left|\text{'}\mathbf{P}\text{'}\right\rangle\right]$, which should be understood as the statement that V(‘**P**’) is an *IRNDPLVI* (*irreducible non-digital panlogographic validity-integron*) (cf. Cmt 4.3). When however ‘**P**’ is used xenonymously, it may assume (take on) any ER, – valid, antivalid, or vav-neutral, – as its accidental denotatum (denotation value). Also, an ER in the range of ‘**P**’ can be either a predicate-free one or predicate-containing one, while the latter can be either a non-contracted (contractor-free) one or a contracted (contractor-containing) one. In reference to potential denotata of ‘**P**’ in its range, I may therefore assume either that $\vdash[V(\mathbf{P})\hat{=}0]$ or that $\vdash[V(\mathbf{P})\hat{=}1]$, or else that $\vdash[V(\mathbf{P})\hat{=}\mathbf{i}_{\sim}|\mathbf{P}\rangle]$, where $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ is an IRNDEVI (irreducible non-digital euautographic validity-integron). Each one of the above three assumptions is a *condition* (*hypothesis*) that is imposed on **P**, i.e. on the ER’s that are comprised in the respective restricted range of ‘**P**’ as its allowable concrete instances (accidental denotata). The first condition is satisfied by valid ER’s, $\mathbf{P}_{+}$, the second one by antivalid ER’s, $\mathbf{P}_{-}$ or $\neg\mathbf{P}_{+}$, and the last one is satisfied by vav-neutral ER’s, $\mathbf{P}_{\sim}$. These three conditions can be expressed by the EDT schema (6.2). In this case, in accordance with Ax 4.20,

$$\vdash\mathbf{P} \text{ if and only if } \vdash[V(\mathbf{P})\hat{=}0], \quad \text{(a)}$$
$$\dashv\mathbf{P} \text{ if and only if } \vdash[V(\mathbf{P})\hat{=}1], \quad \text{(b)} \qquad (7.23)$$
$$+\!\!\!\vdash\mathbf{P} \text{ if and only if } \vdash[V(\mathbf{P})\hat{=}\mathbf{i}_{\sim}|\mathbf{P}\rangle], \quad \text{(c)}$$

the understanding being that $\dashv\mathbf{P}$ is equivalent to $\vdash\neg\mathbf{P}$.

6) ‘**P**’ is a vav-neutral AnAtPLR. Therefore, when any of the three equalities: ‘$V(\mathbf{P})\hat{=}0$’, $V(\mathbf{P})\hat{=}1$’, and ‘$V(\mathbf{P})\hat{=}\mathbf{i}_{\sim}|\mathbf{P}\rangle$’ is asserted, that equality is obviously understood as the respective one of the conditions: $\vdash[V(\mathbf{P})\hat{=}0]$, $\vdash[V(\mathbf{P})\hat{=}1]$, or $\vdash[V(\mathbf{P})\hat{=}\mathbf{i}_{\sim}|\mathbf{P}\rangle]$. That is to say, any of the latter three MLR’s (metalogographic relations) can be abbreviated by omission of the metalinguistic predicate ‘$\vdash$’ without any danger of confusion. The previous item and the above remarks of this item apply, *mutatis mutandis*, with any AnAtPLR, particularly with ‘**Q**’ in place of ‘**P**’.

7) Once any one of the three conditions (hypotheses, ad hoc axioms):

$$\vdash[V(\mathbf{P})\hat{=}0] \text{ (a)},\ \vdash[V(\mathbf{P})\hat{=}1] \text{ (b)},\ \vdash[V(\mathbf{P})\hat{=}\mathbf{i}_{\sim}|\mathbf{P}\rangle] \text{ (c)}, \qquad (7.24)$$

or any one of the three similar conditions

$$\vdash[V(\mathbf{Q})\hat{=}0] \text{ (a)},\ \vdash[V(\mathbf{Q})\hat{=}1] \text{ (b)},\ \vdash[V(\mathbf{Q})\hat{=}\mathbf{i}_{\sim}|\mathbf{Q}\rangle] \text{ (c)}, \qquad (7.25)$$

or both are imposed on ESR's processed in BPLADP's (7.0γ)–(7.12γ), the latter can be developed further in accordance with the same rules of inference.•

***Crl 7.1.**

$$V\left(\mathbf{I}\,\overline{\hat{=}}\,\mathbf{J}\right)\hat{=}V\left(\neg\left[\mathbf{I}\,\hat{=}\,\mathbf{J}\right]\right)\hat{=}1\hat{-}V\left(\mathbf{I}\,\hat{=}\,\mathbf{J}\right). \tag{7.1γε}$$

Proof: In accordance with Df 1.10(13), the above train of identities is the instance of (7.1γ) with '$\left[\mathbf{I}\,\hat{=}\,\mathbf{J}\right]$' in place of '**P**'.•

7.3. The conformal catlogographic interpretands of the major predicate-free euautographic relations

Preliminary Remark 7.1. In accordance with the definitions (I.8.3),

$$p \to \text{'}p\text{'} \text{ and } q \to \text{'}q\text{'}. \tag{7.26}$$

Therefore, all *vavn-decided predicate-free euautographic relations* (*vavn-DdPFER's*), which occur in subsection 7.1 and which involve p and q, can be rewritten in the *concurrent form* by making substitutions

$$\text{'}p\text{'} \mapsto p \text{ and } \text{'}q\text{'} \mapsto q \tag{7.26a}$$

together with the quotation marks throughout that subsection. Upon omission of all single quotation marks from the vavn-DdPFER variants thus obtained, the later turn into the *conformal catlogographic* (*CFCL*), or more precisely *conservative CFCL* (*CCFCL*), *interpretands* of the respective initial vavn-DdPFER's, being *their conformal euautographic interpretantia*, – in accordance with Ax I.8.1 and Df I.8.4(2). The CFCL interpretand of a vavn-DdPFER's is less explicitly called a *vavn-decided predicate-free catlogographic relation* (*vavn-DdPFCLR*).

The above CCFCL interpretands of the vavn-DdPFER's can alternatively be obtained straightforwardly by the *analo-homolographic substitutions*

$$p \mapsto p \text{ and } q \mapsto q, \tag{8.27}$$

without any quotation marks, throughout their euautographic interpretantia. Particularly, these substitutions throughout the trains of euautographic identities comprised in Th 7.1 result in the following *catlogographic Th 7.3*, which is analogous and conformal to Th 7.2; the latter results by the similar analo-homolographic substitutions (7.18) throughout Th 7.1.•

+Th 7.3.

$$V\left(p \veebar q\right)\hat{=}1\hat{-}V\left(p\right)\hat{\cdot}V\left(q\right). \tag{7.0κ}$$

$$V\left(\neg p\right)\hat{=}V\left(p \veebar p\right)\hat{=}1\hat{-}V\left(p\right)\hat{\cdot}V\left(p\right)\hat{=}1\hat{-}V\left(p\right). \tag{7.1κ}$$

$$V(p\vee q)\hat{=}V(\neg[p\veebar q])\hat{=}1\hat{-}V(p\veebar q)\hat{=}V(p)\hat{\cdot}V(q). \tag{7.2κ}$$

$$V(p\Rightarrow q)\hat{=}V([\neg p]\vee q)\hat{=}V(\neg p)\hat{\cdot}V(q)\hat{=}[1\hat{-}V(p)]\hat{\cdot}V(q). \tag{7.3κ}$$

$$V(p\Leftarrow q)\hat{=}V(p)\vee[\neg q]\hat{=}V(p)\hat{\cdot}V(\neg q)\hat{=}V(p)\hat{\cdot}[1\hat{-}V(q)]\hat{=}V(q\Rightarrow p). \tag{7.4κ}$$

$$\begin{gathered}V(p\barwedge q)\hat{=}V([\neg p]\vee[\neg q])\hat{=}V(\neg p)\hat{\cdot}V(\neg q)\\ \hat{=}[1\hat{-}V(p)]\hat{\cdot}[1\hat{-}V(q)]\hat{=}1\hat{-}V(p)\hat{-}V(q)\hat{+}V(p)\hat{\cdot}V(q).\end{gathered} \tag{7.5κ}$$

$$\begin{gathered}V(p\wedge q)\hat{=}V(\neg[p\barwedge q])\hat{=}V([\neg p]\veebar[\neg q])\hat{=}V(\neg[[\neg p]\vee[\neg q]])\\ \hat{=}1\hat{-}V(\neg p)\hat{\cdot}V(\neg q)\hat{=}V(p)\hat{-}V(q)\hat{+}V(p)\hat{\cdot}V(q).\end{gathered} \tag{7.6κ}$$

$$\begin{gathered}V(p\Leftrightarrow q)\hat{=}V([p\Rightarrow q]\wedge[p\Leftarrow q])\hat{=}1\hat{-}V(\neg[p\Rightarrow q])\hat{\cdot}V(\neg[p\Leftarrow q])\\ \hat{=}V(p\Rightarrow q)\hat{+}V(p\Leftarrow q)\hat{=}V(\neg p)\hat{\cdot}V(q)\hat{+}V(p)\hat{\cdot}V(\neg q)\\ \hat{=}V(p)\hat{+}V(q)\hat{-}2\hat{\cdot}V(p)\hat{\cdot}V(q)\hat{=}[V(p)\hat{-}V(q)]^2.\end{gathered} \tag{7.7κ}$$

$$V(p\mathbin{\overline{\vee}}q)\hat{=}V(\neg[p\vee q])\hat{=}1\hat{-}V(p\vee q)\hat{=}1\hat{-}V(p)\hat{\cdot}V(q)\hat{=}V(p\veebar q). \tag{7.8κ}$$

$$\begin{gathered}V(p\mathbin{\overline{\wedge}}q)\hat{=}V(\neg[p\wedge q])\hat{=}1\hat{-}V(p\wedge q)\\ \hat{=}1\hat{-}V(p)\hat{-}V(q)\hat{+}V(p)\hat{\cdot}V(q)\hat{=}V(p\barwedge q).\end{gathered} \tag{7.9κ}$$

$$V(p\mathrel{\overline{\Rightarrow}}q)\hat{=}V(\neg[p\Rightarrow q])\hat{=}1\hat{-}V(p\Rightarrow q). \tag{7.10κ}$$

$$\begin{gathered}V(p\mathrel{\overline{\Leftarrow}}q)\hat{=}V(\neg[p\Leftarrow q])\hat{=}1\hat{-}V(p\Leftarrow q)\\ \hat{=}1\hat{-}V(q\Rightarrow p)\hat{=}V(\neg[q\Rightarrow p])\hat{=}V(q\mathrel{\overline{\Rightarrow}}p).\end{gathered} \tag{7.11κ}$$

$$\begin{gathered}V(p\mathrel{\overline{\Leftrightarrow}}q)\hat{=}V(\neg[p\Leftrightarrow q])\hat{=}1\hat{-}V(p\Leftrightarrow q)\\ \hat{=}1\hat{-}[V(p)\hat{-}V(q)]^2\hat{=}[1\hat{-}V(p)\hat{+}V(q)]\hat{\cdot}[1\hat{+}V(p)\hat{-}V(q)]\\ \hat{=}[V(\neg p)\hat{+}V(q)]\hat{\cdot}[V(p)\hat{+}V(\neg q)].\end{gathered} \tag{7.12κ}$$ •

Proof: The trains of identities (7.0κ)–(7.12κ) are variants of (7.0)–(7.12) under the analo-homolographic substitutions (7.27). That is to say, the trains (7.0κ)–(7.12κ) follow from (7.0)–(7.12) by the rule of substitutions (I.8.20), which is one of the rules of CFCL interpretations of DdER's A_1 that are comprised in Ax I.8.1.•

Cmt 7.4. 1) Th 7.3 is analogous and conformal to Th 7.2. Particularly, the analo-homolographic substitutions (7.27) throughout Th 7.1, which result in Th 7.3, are analogous to the analo-homolographic substitutions (7.18) throughout Th 7.1, which result in Th 7.2. In order to indicate that the trains of identities (7.0κ)–(7.12κ) are *cat*logographic, I have attributed their double position-numerals with tokens of the adscript 'κ', which is the first letter of the Greek combining form "κατα"- \kata\ denoting *down* and also the first letter of the Greek adverb "κάτω" \káto\ meaning *down*, *below*, *beneath*, *under* (see (Pring [1982]). Also, Th 7.3 and all subsequent theorems that deal with CFCL interpretands of DdER's (decided euautographic relations) are provided with the flag $^{+}$.

2) Any one of the trains of identities (7.0κ)–(7.12κ) is a *basic CCFCL* (*BCCFCL*) *ADP* for the *CCFCL slave-relation* (*CCFCLSR*), which occurs as argument in the PVI (primary validity-integron) of the train and which is the *CCFCL interpretand* of the pertinent vavn-DdESR. The train contains as its final result the *CCFCL MT* (*master-theorem*), or *DT* (*decision theorem*), which is the *CCFCL interpretand* of the pertinent EMT (EDT). Therefore, the CCFCLSR preserves the *validity-value* of the ESR, being its *conformal*, or *template*, *euautographic* (*CFE*) *interpretans* (pl. "*interpretantia*"). All CCFCLSR's processed are *vav-neutral*, in accordance with their VID's. Particularly, the BCCFCLADP (7.1κ) implies that

$$V(\neg p) \hat{=} V(p \not\vee p) \hat{=} \mathbf{i}_{\sim}\left|\neg p\right\rangle \hat{=} \mathbf{i}_{\sim}\left|p \not\vee p\right\rangle \hat{=} 1 \hat{-} V(p) \tag{7.28}$$

(cf. (7.19)). At the same time, all binary CCFCLSR's processed in the BCCFCLADP's (7.0κ) and (7.2κ)–(7.12κ) can be condensed into the relation-schema'$p\boldsymbol{\lambda}q$' subject to definition (1.14) of '$\boldsymbol{\lambda}$'. Consequently, any one of these BCCFCLADP's has the form:

$$V(p\boldsymbol{\lambda}q) \hat{=} \mathbf{i}_{\sim}\left|p\boldsymbol{\lambda}q\right\rangle \tag{7.29}$$

(cf. (7.19)) subject to a certain instance (value) of '$\boldsymbol{\lambda}$'.

1) All other EADP's (EDT's) occurring in subsection 7.1 can be interpreted catlogographically likewise. For instance, here follow the CCFCL interpretands of the EADP's (EDT's) (7.15) and (7.16):

$$V(p \vee \neg p) \hat{=} V(p) \hat{\cdot} V(\neg p) \hat{=} V(p) \hat{\cdot} [1 \hat{-} V(p)] \hat{=} 0, \tag{7.15κ}$$

$$\begin{gathered} V([p \Rightarrow q] \vee [p \Leftarrow q]) \hat{=} V(p \Rightarrow q) \hat{\cdot} V(p \Leftarrow q) \\ \hat{=} [1 \hat{-} V(p)] \hat{\cdot} V(q) \hat{\cdot} V(p) \hat{\cdot} [1 \hat{-} V(q)] \hat{=} 0. \end{gathered} \tag{7.16κ}$$ •

Cmt 7.5. 1) The AnAtCLPH's (analytical atomic catlogographic placeholders) 'p' and 'q' occurring in the BCFCLADP's (7.0κ)–(7.12κ), (7.15κ), and (7.16κ) are analogous to the AnAtPLPH's '**P**' and '**Q**' occurring, e.g., in the conformal BPLADP's (7.0γ)–(7.12γ), (7.15γ), and (7.16γ). Still, semantic properties of 'p', e.g., essentially differ from those of '**P**' in no connection with their ranges, which are of course distinct. First, all autonymous properties of 'p' have already been studied and utilized by using 'p' in the hypostasis of the AtER p. Therefore, if 'p' is used autonymously then it has the same properties as p, no matter whether or not such use is indicated by HAQ (homoloautographic, single quotation) marks. That is to say, under Df 3.7, I may not assume either that $\vdash$‘*p*’, i.e. that ‘*p*’ is valid, or that $\dashv$ ‘*p*’, i.e. that ‘*p*’ is antivalid, because $\dashv\vdash$‘*p*’, i.e. because ‘*p*’ is vav-neutral. Since V(‘*p*’) is, by (7.26), the same as $V(p)$, therefore V(‘*p*’) is an IRNDEVI (irreducible non-digital euautographic validity-integron) (cf. Cmts 4.3 and 7.3(5)). That is to say, I may assert that $\vdash\left[V('p')\hat{=}\mathbf{i}_{\sim}\left|'p'\right\rangle\right]$, which should be understood as a statement that V(‘*p*’) is an IRNDEVI, and hence I may not assume either that $\vdash[V(‘p’)\hat{=}0]$ or that $\vdash[V(‘p’)\hat{=}1]$. Even if ‘*p*’ is used xenonymously then it still preserves the *validity-value* of p being its CFE interpretans. That is to say, I may not assume either that $\vdash p$, i.e. that *p* is valid, or that $\dashv p$, i.e. that *p* is antivalid, because $\dashv\vdash p$, i.e. because ‘*p*’ is vav-neutral just as p being its synonym. Consequently, I may assert that $\vdash\left[V(p)\hat{=}\mathbf{i}_{\sim}\left|p\right\rangle\right]$, which should be understood as a statement that $V(p)$ is an *irreducible non-digital CCFCL validity-intrgron* (*IRNDCCFCLVI*), and hence I may not assume either that $\vdash[V(p)\hat{=}0]$ or that $\vdash[V(p)\hat{=}1]$.

2) In general, in accordance with Ax I.8.1(6), a *CCFCL relation* (*CCFCLR*) that is used xenonymously preserves the validity-value of its CFE interpretans and it is therefore alternatively called a *vavn-decided CCFCLR* (*vavn-DdCCFCLR*). At the same time, with allowance for an additional mental (psychical) *significand* (*signification value*) that the CCFCLR assumes, it is alternatively said to be *tautologous* (*universally true*) or *antitautologous* (*universally antitrue*, *universally false*, *contradictory*) or else *ttatt-neutral* (*neutral with respect to tautologousness and antitautologousness*, *neither tautologous nor antitatologous*) if and only if its vavn-decided CFE interpretans is *valid* (*kyrologous*) or *antivalid* (*antikyrologous*) or *vav-neutral* (*kak-neutral, neutral with respect to kyrologousness and antitkyrologousness*, *neither kyrologous nor antikyrologous*) respectively. That is to say, in addition to or instead of its *inherent validity-value validity* or *antivalidity* or *vav-neutrality*, which have been denoted by ‘v_{+}’, ‘v_{-}’, and ‘$\mathrm{v}_{\sim}$’ in that order by Dfs I.1.3(28) and 6.3, a CCFCLR assumes exactly one respective *tautologousness-value*: *tautologousness* (*universal truth*), *antitautologousness* (*universal antitruth*, *contradictoriness*), or *ttatt-neutrality* (*neither tautologousness nor antitatologousness*) – namely that one, which is *inclusive of* and is, hence, *compatible with its validity-value*. In Cmt I.8(1), the above three tautologousness-values have been denoted by ‘τ_{+}’, ‘τ_{-}’, and ‘$\tau_{\sim}$’ in that order.

3) Therefore, if ‘*p*’, e.g., is used xenonymously as a CCFCLR, it is a *ttatt-neutral CCFCLR*. That is to say, under definition Df 3.8, ╫*p* and hence I may not assume either that ╟*p*, i.e. that *p* is tautologous (universally true), or that ╢*p*, i.e. that *p* is antitautologous (universally untitrue, contradictory). Consequently, I may assert that $\Vdash\left[V(p)\hat{=}\mathbf{i}_{\sim}\middle|p\rangle\right]$, which should be understood as a statement that *V*(*p*) is an *irreducible non-digital CCFCL tautologousness-intrgron* (*IRNDCCFCLTTI*) and which is equivalent to the statement that $\vdash\left[V(p)\hat{=}\mathbf{i}_{\sim}\middle|p\rangle\right]$, but I may not assume either that $\Vdash[V(p)\hat{=}0]$ or that $\Vdash[V(p)\hat{=}1]$, which would be equivalent to stating either that $\vdash[V(p)\hat{=}0]$ or that $\vdash[V(p)\hat{=}1]$ respectively.

4) Apart from the case, where ‘*p*’ is used xenonymously as a *conservative* CFCL (*C*CFCL) interpretand of its CFE interpretans, it can be used xenonymously as a *transformative CFCL* (*TCFCL*) interpretand of its CFE interpretans in the sense that it can assume (take on), as its *accidental (circumstantial) material denotatum* (*denotation value*), an affirmative (positive) declarative sentence of any of the three kinds: (a) *materially-veracious* (*conformed to facts*), i.e. *accidentally* (*circumstantially, untautologically, non-universally*) *materially-true*; (b) *materially-antiveracious*, i.e. *accidentally materially-antitrue*, (c) *accidentally neutral* (*indeterminate*) *with respect to material veracity* (*accidental material truth*) *and material antiveracity* (*accidental material antitruth*) – briefly *materially-vravr-neutral*, i.e. *accidentally materially-tat-neutral*. Within the scope of *formal* logic, the above semantic property of ‘*p*’ can consistently be treated by allowing ‘*p*’ itself to be one of the three kinds: (a) *formally-veracious*, symbolized as ⊨*p*; (b) *formally-antiveracious*, symbolized as ⫤*p*; (c) *formally-vravr-neutral* (*vravr-indeterminate*), symbolized as ╪*p*, – in accordance with Df 3.9. That is to say, ‘*p*’ is allowed assuming (taking on) any one of the three *formal veracity-values*: (a) *formal veracity*, (b) *formal antiveracity*, (c) *formal vravr-neutrality* (*formal vravr-indterminacy*).

5) In Cmt I.8(1), the three *veracity-values*: *veracity* (*accidental truth*), *antiveracity* (*accidental antitruth, accidental falsity*), and *vravr-neutrality* (*vravr-indeterminacy*) are denoted by ‘$\tau_{\sim+}$’, ‘$\tau_{\sim-}$’, and ‘$\tau_{\sim\sim}$’, and also by ‘ϕ_{+}’, ‘ϕ_{-}’, and ‘$\phi_{\sim}$’ in that order. In this case, τ_{+} or $\tau_{\sim+}$, i.e. ϕ_{+}, is ambiguously (equivocally) denoted by ‘α_{+}’ and is indiscriminately called the *truth-value truth*; τ_{-} or $\tau_{\sim-}$, i.e. ϕ_{-}, is ambiguously denoted by ‘α_{-}’ and is indiscriminately called the *truth-value*

antitruth or *falsity* (*falsehood*); $\tau_{\sim\sim}$, i.e. $\phi_{\sim}$, is *alternatively* denoted by '$\alpha_{\sim}$' and is alternatively called the *truth-value neutrality with respect to the truth-values truth and antitruth* or briefly the *tat-neutrality* (*tat-indeterminacy*). Hence,

$$\phi_{+} = \tau_{\sim+},\ \phi_{-} = \tau_{\sim-},\ \phi_{\sim} = \tau_{\sim\sim} = \alpha_{\sim}\,. \qquad (7.30)$$

6) It is understood that '*p*' takes on materially-veracious, materially-antiveracious, or materially-vravr-neutral sentences if it is a formally veracious, formally antiveracious, or formally vravr-neutral AtCLR respectively. Therefore, if '*p*', e.g., is used xenonymously as a *TCFCL relation* (*TCFCLR*) then, under Df 3.9, I may, in reference to potential material denotata of '*p*' in its range, assume either that $\models[V(p) \mathrel{\hat{=}} 0]$ or that $\models[V(p) \mathrel{\hat{=}} 1]$, or else that $\models\left[V(p) \mathrel{\hat{=}} \mathbf{i}_{\sim} \middle| p\right\rangle]$, which means that $V(p)$ an *irreducible non-digital TCFCL veracity-integron* (*IRNDTCFCLVrI*) (cf. Cmt 7.3(5)). Each one of these three assumptions is a condition that imposed on *p*, i.e. on sentences as potential material denotata of '*p*'. The first condition is satisfied by veracious sentences, p_{+}, the second one by antiveracious sentences, p_{-}, i.e. by $\neg p_{+}$, and the last one is satisfied by vravr-neutral sentences, $p_{\sim}$. These three conditions can be expressed by the EDT schema (6.2). In this case, in analogy with the schema (7.23),

$$\models p \text{ if and only if } \models[V(p) \mathrel{\hat{=}} 0], \qquad (a)$$

$$\mathrel{=\!\!\mid} p \text{ if and only if } \models[V(p) \mathrel{\hat{=}} 1], \qquad (b) \qquad (7.31)$$

$$\mathrel{=\!\!\mid\!\!=} p \text{ if and only if } \models\left[V(p) \mathrel{\hat{=}} \mathbf{i}_{\sim} \middle| p\right\rangle], \qquad (c)$$

the understanding being that $\mathrel{=\!\!\mid} p$ is equivalent to $\models\neg p$. The following remark is analogous to Cmt 7.3(6).

6) If '*p*' is used xenonymously as a CCFCLR then, in accordance with items 1 and 3 of this comment, none of the three conditions $\vdash[V(p) \mathrel{\hat{=}} 0]$¸ $\vdash[V(p) \mathrel{\hat{=}} 1]$, and $\vdash\left[V(p) \mathrel{\hat{=}} \mathbf{i}_{\sim} \middle| p\right\rangle]$, and none of the three equivalent conditions with $\models$ in place of $\vdash$ can be imposed on *p*. Therefore, when any of the three equalities: '$V(p) \mathrel{\hat{=}} 0$'¸ '$V(p) \mathrel{\hat{=}} 1$', and '$V(p) \mathrel{\hat{=}} \mathbf{i}_{\sim} | p\rangle$' is asserted, it is obviously understood as the respective one of the conditions $\models[V(p) \mathrel{\hat{=}} 0]$, $\models[V(p) \mathrel{\hat{=}} 1]$, and $\models\left[V(p) \mathrel{\hat{=}} \mathbf{i}_{\sim} \middle| p\right\rangle]$. That is to say, any of the latter three metalogographic relations (MLR's) can be abbreviated by omission of the metalinguistic predicate '$\models$' without any danger of confusion. The previous item and the above remarks of this item apply, *mutatis mutandis*, with any AnAtCCFCLR, particularly with '*q*', in place of '*p*'.

7) Comparison of the item 6 of Cmt 7.3 and the previous item of this comment shows that sign 'V' occurring in '$V(\mathbf{P})$' (`e.g.) and the same kernel-sign occurring in '$V(p)$' (e.g.) denote two distinct functions. Therefore, rigorously, in accordance with the definitions (I.8.1)–(I.8.3) in general and (7.26) in particular, instead of the occurrence of 'V' in '$V(p)$', an occurrence of another appropriate symbol, say of 'V', should be used subject to the general definition:

$$V(\ )\to V(`\ '), \tag{7.32}$$

where alike spaces should be replaced with congruent tokens of any one of the AnAtCCFCLR's p, q, etc without quotation marks, e.g.

$$V(p)\to V(`p'),\ V(q)\to V(`q'), \text{ etc.} \tag{7.32a}$$

Hence, V is an extension of V from the ER's and PLR's onto the CCFCLR's. In mathematics, the extension of a function is, as a rule, denoted by the same functional operator (functional constant). For instance, 'sin' denotes (represents) the respective homonymous functions both on the set of real numbers and on the set of complex number. Since V satisfies the same rules of inference and decision as V, it is convenient to denote both functions by the same operator 'V'. The following item is analogous to the item 7 of Cmt 7.3.

8) Once any one of the three conditions (hypotheses, ad hoc axioms):

$$\vdash[V(p)\hat{=}0] \text{ (a)},\ \vdash[V(p)\hat{=}1] \text{ (b)},\ \vdash[V(p)\hat{=}\mathbf{i}_{\sim}|p\rangle] \text{ (c)}, \tag{7.33}$$

or any one of the three similar conditions

$$\vdash[V(q)\hat{=}0] \text{ (a)},\ \vdash[V(q)\hat{=}1] \text{ (b)},\ \vdash[V(q)\hat{=}\mathbf{i}_{\sim}|q\rangle] \text{ (c)}, \tag{7.34}$$

or both are imposed on CCFCLSR's processed in BCCFCLADP's (7.0κ)–(7.12κ), the latter can be developed further in accordance with the same rules of inference.•

7.4. A cumulative table of the unneutral (determinate) validity-indices of the major PLR's and of the unneutral veracity-indices of the major CLR's

Df 7.1. 1) If the PVI $V(\mathbf{P})$ is specified as 0 or 1 by imposing the respective one of the conditions $V(\mathbf{P})\hat{=}0$ and $V(\mathbf{P})\hat{=}1$ on **P** then $V(\neg\mathbf{P})$ will be specified as 1 or 0 respectively, in accordance with (7.1γ). Analogously, given **λ** subject to (1.14), if the ordered pair of PVI's $(V(\mathbf{P}),V(\mathbf{Q}))$ is specified as one of the four: (0,0), (0,1), (1,0), (1,1) by imposing on **P** and **Q** the respective conditions selected by one out of each of the ordered pairs: $(V(\mathbf{P})\hat{=}0,V(\mathbf{P})\hat{=}1)$ and of $(V(\mathbf{Q})\hat{=}0,V(\mathbf{Q})\hat{=}1)$ then $V(\mathbf{P\lambda Q})$ will assume exactly one of the DVI's 0 and 1, in accordance with the pertinent one of the

BPLADP's (7.0γ) and (7.2γ)–(7.12γ). A table presenting $V(\neg\mathbf{P})$ versus $V(\mathbf{P})$ and also presenting $V(\mathbf{P}\boldsymbol{\lambda}\mathbf{Q})$ at each $\boldsymbol{\lambda}$ versus $(V(\mathbf{P}),V(\mathbf{Q}))$, subject to $V(\mathbf{P})\hat{=}0$ or $V(\mathbf{P})\hat{=}1$ and $V(\mathbf{Q})\hat{=}0$ or $V(\mathbf{Q})\hat{=}1$, is called the *cumulative table of the unneutral* (*determinate*) *validity-indices of the major PLSR's* and also the *cumulative validity-antivalidity table* (*CVAVT*) *for the major logical connectives.*

2) Under the convention of using the operator '*V*' homonymously as stated in Cmt 7.5(7), the above remarks can be restated, *mutatis mutandis*, with '*p*' and '*q*' in place of '**P**' and '**Q**' as follows. If the *PVrI* (*primary veracity-integron*) $V(p)$ is specified as 0 or 1 by imposing the respective one of the conditions $V(p)\hat{=}0$ and $V(p)\hat{=}1$ on p then $V(\neg p)$ will be specified as 1 or 0 respectively, in accordance with (7.1κ). Analogously, given $\boldsymbol{\lambda}$ subject to (1.14)), if the ordered pair of PVrI's $(V(p),V(q))$ is specified as one of the four: (0,0), (0,1), (1,0), (1,1) by imposing on p and q the respective conditions selected by one out of each of the pairs: $(V(p)\hat{=}0,V(p)\hat{=}1)$ and of $(V(q)\hat{=}0,V(q)\hat{=}1)$ then $V(p\boldsymbol{\lambda}q)$ will assume exactly one of the DVI's 0 and 1, in accordance with the pertinent one of the BCCFCLADP's (identity trains) (7.0κ) and (7.2κ)–(7.12κ). A table presenting $V(\neg p)$ versus $V(p)$ and also presenting $V(p\boldsymbol{\lambda}q)$ at each $\boldsymbol{\lambda}$ versus $(V(p),V(q))$, subject to $V(p)\hat{=}0$ or $V(p)\hat{=}1$ and $V(q)\hat{=}0$ or $V(q)\hat{=}1$, is called the *cumulative table of the unneutral* (*determinate*) *veracity-indices of the major CCFCLSR's* and also the *cumulative veracity-antiveracity table* (*CVtAVrT*) *for the major logical connectives.*

3) I avoid using the loose names "*validity table*" and "*veracity table*" (in analogy with the dictionary loose name "*truth table*") instead of the rigorous names "*validity-antivalidity table*" ("*VAVT*") and "*veracity-antiveracity table*" ("*VrAVrT*"), because the tables denoted by the latter names are not complete as indicated by those names. Indeed, the validity-antivalidity table, e.g., does not include either the input validity-value neutrality $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ for the PVI $V(\mathbf{P})$ occurring in the (7.1γ) or any of the five possible ordered pairs of the validity-values: $(0,\ \mathbf{i}_{\sim}|\mathbf{Q}\rangle)$, $(\mathbf{i}_{\sim}|\mathbf{P}\rangle,0)$, $(1,\ \mathbf{i}_{\sim}|\mathbf{Q}\rangle)$, $(\mathbf{i}_{\sim}|\mathbf{P}\rangle,1)$, $(0,\ \mathbf{i}_{\sim}|\mathbf{Q}\rangle)$, $(\mathbf{i}_{\sim}|\mathbf{P}\rangle,\mathbf{i}_{\sim}|\mathbf{Q}\rangle)$ of the ordered pair $(V(\mathbf{P}),V(\mathbf{Q}))$ occurring in any of the identity trains (7.0γ) and (7.2γ)–(7.12γ). Similarly, the veracity-antiveracity table does not include either the input veracity-value neutrality $\mathbf{i}_{\sim}|p\rangle$ for the PVrI $V(p)$ occurring in the (7.1κ) or any of the five possible ordered pairs of the validity-

values: (0, $\mathbf{i}_{\sim}|p\rangle$), ($\mathbf{i}_{\sim}|p\rangle$,0), (1, $\mathbf{i}_{\sim}|q\rangle$), ($\mathbf{i}_{\sim}|p\rangle$,1), (0, $\mathbf{i}_{\sim}|q\rangle$), ($\mathbf{i}_{\sim}|p\rangle,\mathbf{i}_{\sim}|q\rangle$) of the ordered pair ($V(p)$,$V(q)$) occurring in any of the identity trains (7.0κ) and (7.2κ)–(7.12κ).

4) The CVAVT and the CVrAVrT as defined in the above items 1 and 2 are *same* in the sense that they exchange when '**P**' and '**Q**' are exchanged with '*p*' and '*q*' respectively. Therefore, the two tables can be laid down as a single table, which is given below and which is called logically *Table 7.1* and descriptively the *cumulative table of the unneutral* (*determinate*) *validity-indices of the major PLSR's and of the unneutral* (*determinate*) *veracity-indices of the major CCFCLSR's*, and also the *cumulative validity-antivalidity and veracity-antiveracity table* (*CVAV&VtAVrT*) *for the major logical connectives.*

5) Four possible ordered pairs of validity-values of $V(\mathbf{P})$ and $V(\mathbf{Q})$ and the formally same ordered pairs of veracity-values of $V(p)$ and $V(q)$, are given in the rows 1 and 2 of Table 7.1. Rows 3-13 of Table 7.1 give the respective validity-values of $V(\neg\mathbf{P})$ and $V(\mathbf{P}\boldsymbol{\lambda}\mathbf{Q})$ and the same veracity-values of $V(\neg p)$ and $V(p\boldsymbol{\lambda}q)$ – the values, which is determined by the identity trains (7.0γ)–(7.12γ) and (7.0κ)–(7.12κ). •

Table 7.1: ***The cumulative validity-antivalidity and veracity-antiveracity table for the major logical connectives***

1	$V(\mathbf{P})$	$V(p)$	0	0	1	1
2	$V(\mathbf{Q})$	$V(q)$	0	1	0	1
3	$V(\mathbf{P} \not\vee \mathbf{Q})$ $V(\mathbf{P} \overline{\vee} \mathbf{Q})$	$V(p \not\vee q)$ $V(p \overline{\vee} q)$	1	1	1	0
4	$V(\neg\mathbf{P})$	$V(\neg p)$	1	1	0	0
5	$V(\mathbf{P} \vee \mathbf{Q})$	$V(p \vee q)$	0	0	0	1
6	$V(\mathbf{P} \Rightarrow \mathbf{Q})$ $V(\mathbf{Q} \Leftarrow \mathbf{P})$	$V(p \Rightarrow q)$ $V(q \Leftarrow p)$	0	1	0	0
7	$V(\mathbf{P} \Leftarrow \mathbf{Q})$ $V(\mathbf{Q} \Rightarrow \mathbf{P})$	$V(p \Leftarrow q)$ $V(q \Rightarrow p)$	0	0	1	0
8	$V(\mathbf{P} \not\wedge \mathbf{Q})$ $V(\mathbf{P} \overline{\wedge} \mathbf{Q})$	$V(p \not\wedge q)$ $V(p \overline{\wedge} q)$	1	0	0	0
9	$V(\mathbf{P} \wedge \mathbf{Q})$	$V(p \wedge q)$	0	1	1	1
10	$V(\mathbf{P} \Leftrightarrow \mathbf{Q})$	$V(p \Leftrightarrow q)$	0	1	1	0
11	$V(\mathbf{P} \overline{\Rightarrow} \mathbf{Q})$ $V(\mathbf{Q} \overline{\Leftarrow} \mathbf{P})$	$V(p \overline{\Rightarrow} q)$ $V(q \overline{\Leftarrow} p)$	1	0	1	1
12	$V(\mathbf{P} \overline{\Leftarrow} \mathbf{Q})$ $V(\mathbf{Q} \overline{\Rightarrow} \mathbf{P})$	$V(p \overline{\Leftarrow} q)$ $V(q \overline{\Rightarrow} p)$	1	1	0	1
13	$V(\mathbf{P} \overline{\Leftrightarrow} \mathbf{Q})$	$V(p \overline{\Leftrightarrow} q)$	1	0	0	1

7.5. Major secondary rules of inference and decision

****Crl 7.2.** Any inference or decision rule of A_1 that follows from (7.0γ) and (7.2γ)–(7.12γ) under the condition '$V(\mathbf{P}\boldsymbol{\lambda}\mathbf{Q})\hat{=}0$' subject to (I.1.14) is indiscriminately called a *major secondary inference or decision rule of* A_1. All these rules are self-evident. The most conspicuous of them are given below for convenience in further references.

1) By (7.0γ),

$$V(\mathbf{P}\veebar\mathbf{Q})\hat{=}1\dot{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{=}0 \text{ if and only if } V(\mathbf{P})\hat{=}V(\mathbf{Q})\hat{=}1. \tag{7.35}$$

Hence,

$$\vdash[\mathbf{P}\veebar\mathbf{Q}] \text{ if and only if } \vdash\neg\mathbf{P} \text{ and } \vdash\neg\mathbf{Q}. \tag{7.36}$$

Here, and generally in all other items of this corollary, either function word "Hence" or "whence" should be understood as an abbreviation of the phrase "Hence, by the pertinent variant of the axiom (4.40)". By (7.8γ), the rules (7.35) and (7.36) hold with $\bar{\triangledown}$ in place of $\veebar$.

2) By (7.2),

$$\text{if } V(\mathbf{P}\vee\mathbf{Q})\hat{=}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{=}0 \text{ and } V(\mathbf{P})\hat{=}1, \text{ or } V(\mathbf{Q})\hat{=}1,$$

$$\text{then } V(\mathbf{Q})\hat{=}0, \text{ or correspondingly } V(\mathbf{P})\hat{=}0. \tag{7.37}$$

Hence,

$$\text{if } \vdash[\mathbf{P}\vee\mathbf{Q}] \text{ and } \vdash\neg\mathbf{P} \text{ then } \vdash\mathbf{Q}, \quad \text{(a)}$$

$$\text{if } \vdash[\mathbf{P}\vee\mathbf{Q}] \text{ and } \vdash\neg\mathbf{Q} \text{ then } \vdash\mathbf{P}. \quad \text{(b)} \tag{7.38}$$

3) By (7.3),

$$\text{if } V(\mathbf{P}\Rightarrow\mathbf{Q})\hat{=}[1\dot{-}V(\mathbf{P})]\hat{\cdot}V(\mathbf{Q})\hat{=}0 \text{ and } V(\mathbf{P})\hat{=}0 \text{ then } V(\mathbf{Q})\hat{=}0. \tag{7.39}$$

Hence,

$$\text{if } \vdash[\mathbf{P}\Rightarrow\mathbf{Q}] \text{ and } \vdash\mathbf{P} \text{ then } \vdash\mathbf{Q}. \tag{7.40}$$

4) By (7.4γ),

$$\text{if } V(\mathbf{P}\Leftarrow\mathbf{Q})\hat{=}V(\mathbf{P})\hat{\cdot}[1\dot{-}V(\mathbf{Q})]\hat{=}0 \text{ and } V(\mathbf{Q})\hat{=}0 \text{ then } V(\mathbf{P})\hat{=}0. \tag{7.41}$$

Hence,

$$\text{if } \vdash[\mathbf{P}\Leftarrow\mathbf{Q}] \text{ and } \vdash\mathbf{Q} \text{ then } \vdash\mathbf{P}. \tag{7.42}$$

5) By (7.5γ),

$$\text{if} V(\mathbf{P}\barwedge\mathbf{Q})\hat{=}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{=}0 \text{ and } V(\mathbf{P})\hat{=}0, \text{ or } V(\mathbf{Q})\hat{=}0,$$

(7.43)

$$\text{then } V(\neg\mathbf{Q}) \mathrel{\hat{=}} 0, \text{ or correspondingly } V(\neg\mathbf{P}) \mathrel{\hat{=}} 0.$$

Hence,

$$\begin{aligned} &\text{if } \vdash[\mathbf{P} \mathbin{\not\wedge} \mathbf{Q}] \text{ and } \vdash\mathbf{P} \text{ then } \vdash\neg\mathbf{Q}, \quad \text{(a)} \\ &\text{if } \vdash[\mathbf{P} \mathbin{\not\wedge} \mathbf{Q}] \text{ and } \vdash\mathbf{Q} \text{ then } \vdash\neg\mathbf{P}. \quad \text{(b)} \end{aligned} \tag{7.44}$$

By (7.9γ), the rules (7.43) and (7.44) hold with $\overline{\wedge}$ in place of $\not\wedge$.

6) By (7.6γ),

$$V(\mathbf{P} \wedge \mathbf{Q}) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\neg\mathbf{Q}) \mathrel{\hat{=}} 0 \text{ if and only if } V(\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{Q}) \mathrel{\hat{=}} 0. \tag{7.45}$$

Hence,

$$\vdash[\mathbf{P}\wedge\mathbf{Q}] \text{ if and only if } \vdash\mathbf{P} \text{ and } \vdash\mathbf{Q}. \tag{7.46}$$

7) It follows from (7.7γ) that

$$V(\mathbf{P} \Leftrightarrow \mathbf{Q}) \mathrel{\hat{=}} 0 \text{ if and only if } V(\mathbf{P} \Rightarrow \mathbf{Q}) \mathrel{\hat{=}} V(\mathbf{P} \Leftarrow \mathbf{Q}) \mathrel{\hat{=}} 0, \tag{7.47}$$

whence

$$\vdash[\mathbf{P}\Leftrightarrow\mathbf{Q}] \text{ if and only if } \vdash[\mathbf{P}\Rightarrow\mathbf{Q}] \text{ and } \vdash[\mathbf{P}\Leftarrow\mathbf{Q}], \tag{7.48}$$

and also that

$$V(\mathbf{P} \Leftrightarrow \mathbf{Q}) \mathrel{\hat{=}} 0 \text{ if and only if } V(\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{Q}), \tag{7.49}$$

whence

$$[\mathbf{P}\Leftrightarrow\mathbf{Q}] \text{ if and only if } [V(\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{Q})]. \tag{7.50}$$

From (6.2) and from the variant of (6.2) with ‘**Q**’ in place of ‘**P**’, it follows that

$$V(\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{Q}) \text{ if and only if } V(\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{Q}) \mathrel{\hat{=}} \begin{cases} 0 & \text{(a)} \\ 1 & \text{(b)} \\ \mathbf{i}_{\sim}|\mathbf{P}\rangle \mathrel{\hat{=}} \mathbf{i}_{\sim}|\mathbf{Q}\rangle & \text{(c)} \end{cases}. \tag{7.49$_+$}$$

Relation (7.50) implies that

$$\text{if } \vdash[\mathbf{P}\Leftrightarrow\mathbf{Q}] \text{ and } \vdash\mathbf{P} \text{ then } \vdash\mathbf{Q}, \tag{7.50$_1$}$$

$$\text{if } \vdash[\mathbf{P}\Leftrightarrow\mathbf{Q}] \text{ and } \vdash\mathbf{Q} \text{ then } \vdash\mathbf{P}. \tag{7.50$_2$}$$

8) It follows from (7.12γ) that

$$V(\mathbf{P} \mathbin{\overline{\Leftrightarrow}} \mathbf{Q}) \mathrel{\hat{=}} 0 \text{ if and only if } V(\neg\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{Q}) \mathrel{\hat{=}} 0 \text{ or } V(\mathbf{P}) \mathrel{\hat{=}} V(\neg\mathbf{Q}) \mathrel{\hat{=}} 0, \tag{7.51}$$

whence

$$\vdash[\mathbf{P} \mathbin{\overline{\Leftrightarrow}} \mathbf{Q}] \text{ if and only if } \vdash\neg\mathbf{P} \text{ and } \vdash\mathbf{Q} \text{ or } \vdash\mathbf{P} \text{ and } \vdash\neg\mathbf{Q}. \tag{7.52}$$

Cmt 7.6. If **P** is antivalid, i.e. if $V(\mathbf{P}) \mathrel{\hat{=}} 1$, then (7.3γ) and (7.4γ) reduce to

$$V(\mathbf{P} \Rightarrow \mathbf{Q}) \mathrel{\hat{=}} V(\mathbf{Q} \Leftarrow \mathbf{P}) \mathrel{\hat{=}} [1 \mathbin{\hat{-}} 1] \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} 0 \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} 0$$

independent of **Q**. That is to say, **Q** remains vav-neutral, so that neither of the two equivalent assumptions that $V(\mathbf{P}\Rightarrow\mathbf{Q})\hat{=}0$ and $V(\mathbf{Q}\Leftarrow\mathbf{P})\hat{=}0$ can be used along with the assumption that $V(\mathbf{P})\hat{=}1$ in order to state any secondary decision rule for ER's condensed in the range of **Q**. This is the well-known property of the logical connectives ⇒ and ⇐, no matter how they are depicted.•

Cmt 7.7. By (7.50), it follows from relations (4.4ε), (4.32ε), and (4.33ε) of Cmt 6.4 that

$$\mathbf{P}\Leftrightarrow\mathbf{P}, \tag{7.53}$$

$$\text{If } \vdash[\mathbf{P}\Leftrightarrow\mathbf{Q}] \text{ then } \vdash[\mathbf{Q}\Leftrightarrow\mathbf{P}], \tag{7.54}$$

$$\text{If } \vdash[\mathbf{P}\Leftrightarrow\mathbf{Q}] \text{ and } \vdash[\mathbf{Q}\Leftrightarrow\mathbf{R}] \text{ then } \vdash[\mathbf{P}\Leftrightarrow\mathbf{R}], \tag{7.55}$$

whereas definition (6.43ε) of the same comment implies that

$$\begin{gathered}\left[\mathbf{P}_{i_1}\Leftrightarrow\mathbf{P}_{i_2}\Leftrightarrow\mathbf{P}_{i_3}\Leftrightarrow\ldots\Leftrightarrow\mathbf{P}_{i_{n-1}}\Leftrightarrow\mathbf{P}_{i_n}\right]\\ \leftrightarrow\left[\mathbf{P}_1\Leftrightarrow\mathbf{P}_2\Leftrightarrow\mathbf{P}_3\Leftrightarrow\ldots\Leftrightarrow\mathbf{P}_{n-1}\Leftrightarrow\mathbf{P}_n\right]\\ \rightarrow\left[\mathbf{P}_1\Leftrightarrow\mathbf{P}_2,\mathbf{P}_2\Leftrightarrow\mathbf{P}_3,\ldots,\mathbf{P}_{n-1}\Leftrightarrow\mathbf{P}_n\right]\\ \leftrightarrow\left[\mathbf{P}_{i_1}\Leftrightarrow\mathbf{P}_{i_2},\mathbf{P}_{i_2}\Leftrightarrow\mathbf{P}_{i_3},\ldots,\mathbf{P}_{i_{n-1}}\Leftrightarrow\mathbf{P}_{i_n}\right]\end{gathered} \tag{7.56}$$

This definition defines a continuous train of equivalences, which are written in the *legato style*, in terms of a non-redundant sequence of separate two-term equivalences, which are written in the *staccato style*.•

***Th 7.4.**

$$V([\mathbf{P}\Leftrightarrow\mathbf{Q}]\Leftrightarrow[V(\mathbf{P})\hat{=}V(\mathbf{Q})])\hat{=}0\,. \tag{7.57}$$

Proof: By (4.3) and (7.7γ), the variant of (7.7γ) with '[**P**⇔**Q**]' and '$[V(\mathbf{P})\hat{=}V(\mathbf{Q})]$' in place of '**P**' and '**Q**' respectively can be developed thus:

$$\begin{gathered}V([\mathbf{P}\Leftrightarrow\mathbf{Q}]\Leftrightarrow[V(\mathbf{P})\hat{=}V(\mathbf{Q})])\hat{=}[V(\mathbf{P}\Leftrightarrow\mathbf{Q})\hat{-}V(V(\mathbf{P})\hat{=}V(\mathbf{Q}))]^2\\ \hat{=}\left[[V(\mathbf{P})\hat{-}V(\mathbf{Q})]^2\hat{-}[V(\mathbf{P})\hat{-}V(\mathbf{Q})]^2\right]^2\hat{=}0,\end{gathered} \tag{7.57$_1$}$$

which is the BEADP for an ER's $[\mathbf{P}\Leftrightarrow\mathbf{Q}]\Leftrightarrow[V(\mathbf{P})\hat{=}V(\mathbf{Q})]$ and at the same time the BPLADP for the PLR $['\mathbf{P}'\Leftrightarrow'\mathbf{Q}']\Leftrightarrow[V('\mathbf{P}')\hat{=}V('\mathbf{Q}')]$. QED.•

Cmt 7.8. If a given valid ER, say, **P** is laid down and *asserted plainly*, i.e. by putting it in a certain *assertive layout*, and *not as the slave-relation of its EMT* (*euautographic master-theorem*) $V(\mathbf{P})\hat{=}0$, then it is said to be *asserted*, or *stated,* in the *subjective*, or *plain*, *form*. The *validity-value validity* is assigned to this ER *implicitly* by its assertive layout, i.e. in the *subjective form* as well. For instance, the trains of relations (7.0)–(7.12), (7.14)–(7.16), (7.0γ)–(7.12γ), (7.15γ), and (7.16γ) are valid ones, which are stated in the subjective form and whose validity-value validity is not mentioned. By contrast, if a given ER of A_1, say, **P** is stated to be valid or antivalid or vav-neutral by stating its EMT (EDT) in *the subjective form*, i.e. by stating the pertinent valid ER $V(\mathbf{P}) \hat{=} 0$ or $V(\mathbf{P}) \hat{=} 1$ or $V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle$, then the respective validity-value validity or antivalidity or vav-neutrality is said to be assigned to **P** *explicitly*, i.e. the pertinent *objective form*. In this case, if **P** is valid then by stating its EMT $V(\mathbf{P}) \hat{=} 0$ in the subject form, **P** itself is *ipso facto* stated in the *objective form*. Thus, a valid relation can be stated (asserted) in two forms, subjective (plain) and objective, and the validity-value validity can be assigned to that relation in the same two forms. For instance, either one of the valid relations (7.15) and (7.15γ) is *the law of excluded middle in the respective objective form*, whereas *the same law in the respective subjective form* is conventionally *asserted* as $p \vee \neg p$ or as $\mathbf{P} \vee \neg \mathbf{P}$ respectively. However, if **P** is antivalid or vav-neutral then it cannot be asserted, and therefore the respective validity-value, antivalidity or vav-neutrality, can be assigned to **P** only explicitly, i.e. only in the objective form. At the same time, since an EMT (EDT) is by definition a valid ER, therefore it can be stated in objective form *recursively* (*repeatedly*); that is,

$$\text{if } \vdash [V(\mathbf{P}) \hat{=} 0] \text{ then } \vdash [V(V(\mathbf{P}) \hat{=} 0) \hat{=} 0],\ \vdash [V(V(V(\mathbf{P}) \hat{=} 0) \hat{=} 0) \hat{=} 0], \ldots; \quad (7.58a)$$

$$\text{if } \vdash [V(\mathbf{P}) \hat{=} 1] \text{ then } \vdash [V(V(\mathbf{P}) \hat{=} 1) \hat{=} 0],\ \vdash [V(V(V(\mathbf{P}) \hat{=} 1) \hat{=} 0) \hat{=} 0], \ldots; \quad (7.58b)$$

$$\text{if } \vdash [V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle] \text{ then } \vdash [V(V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle) \hat{=} 0],$$

$$\vdash [V(V(V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle) \hat{=} 0) \hat{=} 0]. \ldots \quad (7.58c)$$

****Th 7.5:** ***The Third Rule of Realization of a Definition (RRD3).***

$$\text{If } \mathbf{P}' \to \mathbf{P} \text{ then } \vdash [\mathbf{P}' \Leftrightarrow \mathbf{P}]. \quad (7.59)$$

Proof: (7.59) immediately follows from (4.31) by (7.50).•

Cmt 7.9. Th 7.5 justifies use of the specific definition signs $\vec{\Leftrightarrow}$, $\overleftarrow{\Leftrightarrow}$, and $\overleftrightarrow{\Leftrightarrow}$ instead of or interchangeably with the respective general definition signs →, ←, and ↔ in the case, where the terms of an asymmetric or symmetric synonymic definition are relations, – as suggested in Df I.2.20.•

7.6. Duality, symmetry, and anti-symmetry properties of the major logical connectives

*Th 7.6: ***Dual properties of** $\veebar$ **and** $\barwedge$**, and of** $\vee$**and** $\wedge$ **with respect to** $\hat{=}$ **and** $\Leftrightarrow$*.

$$V(\mathbf{P}\barwedge\mathbf{P})\hat{=}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{P})\hat{=}V(\neg\mathbf{P})\hat{=}1\hat{-}V(\mathbf{P}). \tag{7.60}$$

$$V(\mathbf{P}\vee\mathbf{Q})\hat{=}V(\neg[\mathbf{P}\veebar\mathbf{Q}])\hat{=}V([\neg\mathbf{P}]\barwedge[\neg\mathbf{Q}])\hat{=}V(\neg[[\neg\mathbf{P}]\wedge[\neg\mathbf{Q}]]). \tag{7.61}$$

$$V(\mathbf{P}\veebar\mathbf{Q})\hat{=}V([\neg\mathbf{P}]\wedge[\neg\mathbf{Q}]). \tag{7.62}$$

By the pertinent variants of (7.50), the trains of identities (7.60)–(7.62) are concurrent (equivalent) to the following trains of equivalences:

$$[\neg\mathbf{P}]\Leftrightarrow[\mathbf{P}\barwedge\mathbf{P}], \tag{7.60'}$$

$$[\mathbf{P}\vee\mathbf{Q}]\Leftrightarrow[\neg[\mathbf{P}\veebar\mathbf{Q}]]\Leftrightarrow[[\neg\mathbf{P}]\barwedge[\neg\mathbf{Q}]]\Leftrightarrow[\neg[[\neg\mathbf{P}]\wedge[\neg\mathbf{Q}]]], \tag{7.61'}$$

$$[\mathbf{P}\veebar\mathbf{Q}]\Leftrightarrow[[\neg\mathbf{P}]\wedge[\neg\mathbf{Q}]] \tag{7.62'}$$

in that order, whereas the respective fragments of the trains of identities (7.1γ), (7.6γ), and (7.5γ), are concurrent to the following trains of equivalences:

$$[\neg\mathbf{P}]\Leftrightarrow[\mathbf{P}\veebar\mathbf{P}], \tag{7.1γ'}$$

$$[\mathbf{P}\wedge\mathbf{Q}]\Leftrightarrow[\neg[\mathbf{P}\barwedge\mathbf{Q}]]\Leftrightarrow[[\neg\mathbf{P}]\veebar[\neg\mathbf{Q}]]\Leftrightarrow[\neg[[\neg\mathbf{P}]\vee[\neg\mathbf{Q}]]], \tag{7.6γ'}$$

$$[\mathbf{P}\barwedge\mathbf{Q}]\Leftrightarrow[[\neg\mathbf{P}]\vee[\neg\mathbf{Q}]] \tag{7.5γ'}$$

in that order. The identities (7.60)–(7.62) are said to be *dual* of (7.1γ), (7.6γ), and (7.5γ), while the equivalences (7.60')–(7.62') are said to be *dual* of (7.1γ'), (7.6γ'), and (7.5γ'), respectively.

Proof: By ($7.1\gamma_1$), the variant of (7.5γ) with '¬**P**' and '¬**Q**' in place of '**P** 'and '**Q**' respectively becomes:

$$\begin{aligned}V([\neg\mathbf{P}]\barwedge[\neg\mathbf{Q}])&\hat{=}V([\neg\neg\mathbf{P}]\vee[\neg\neg\mathbf{Q}])\hat{=}V(\neg\neg\mathbf{P})\hat{\cdot}V(\neg\neg\mathbf{Q})\\&\hat{=}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{=}V(\mathbf{P}\vee\mathbf{Q}),\end{aligned} \tag{7.61_1}$$

The conjunction of (7.2γ) and (7.61_1) proves (7.61). At the same time, by (7.61), it follows from (7.2γ) that

$$\begin{aligned}V(\mathbf{P}\veebar\mathbf{Q})&\hat{=}1\hat{-}V(\mathbf{P}\vee\mathbf{Q})\hat{=}V(\neg[\mathbf{P}\vee\mathbf{Q}])\\&\hat{=}V(\neg\neg[[\neg\mathbf{P}]\wedge[\neg\mathbf{Q}]])\hat{=}V([\neg\mathbf{P}]\wedge[\neg\mathbf{Q}]),\end{aligned} \tag{7.62_1}$$

which proves (7.62).•

*Th 7.7: ***The commutative and anti-commutative panlogographic schemata for major binary logical connectives with respect to** $\hat{=}$ **and** $\Leftrightarrow$**.** Let

$$\boldsymbol{\theta} \vec{\in} \left\{\not\vee, \vee, \not\wedge, \wedge, \Leftrightarrow, \overline{\vee}, \overline{\wedge}, \overline{\Leftrightarrow}\right\}. \tag{7.63}$$

Then

$$V(\mathbf{P}\theta\mathbf{Q}) \hat{=} V(\mathbf{Q}\theta\mathbf{P}), \tag{7.64}$$

$$V(\mathbf{P} \Leftarrow \mathbf{Q}) \hat{=} V(\mathbf{Q} \Rightarrow \mathbf{P}), \tag{7.65}$$

$$V(\mathbf{P} \overline{\Leftarrow} \mathbf{Q}) \hat{=} V(\mathbf{Q} \overline{\Rightarrow} \mathbf{P}), \tag{7.66}$$

whence, by (7.50),

$$[\mathbf{P}\theta\mathbf{Q}] \Leftrightarrow [\mathbf{Q}\theta\mathbf{P}], \tag{7.64'}$$

$$[\mathbf{P} \Leftarrow \mathbf{Q}] \Leftrightarrow [\mathbf{Q} \Rightarrow \mathbf{P}], \tag{7.65'}$$

$$[\mathbf{P} \overline{\Leftarrow} \mathbf{Q}] \Leftrightarrow [\mathbf{Q} \overline{\Rightarrow} \mathbf{P}]. \tag{7.66'}$$

Proof: (7.64) follows from comparison of (7.0γ), (7.2γ), (7.5γ)–(7.9γ), and (7.12γ) and of their variants with ‘**P**’ and ‘**Q**’ exchanged. (7.65) and (7.66) have already been demonstrated in (7.4γ) and (7.11γ) respectively.•

Cnv 7.1. In accordance with Th 7.7, **PθQ** and **QθP**, or $\mathbf{P} \Leftarrow \mathbf{Q}$ and $\mathbf{Q} \Rightarrow \mathbf{P}$, or $\mathbf{P} \overline{\Leftarrow} \mathbf{Q}$ and $\mathbf{Q} \overline{\Rightarrow} \mathbf{P}$ will, as a rule, be used interchangeably without any comments.•

Cmt 7.9. 1) (7.64) and (7.64') are called the *commutative laws for* **θ** *with respect to* $\hat{=}$ *and* $\Leftrightarrow$ respectively. (7.65) and (7.65'), or (7.66) and (7.66'), are called the *anti-commutative laws for* $\Leftarrow$ *and* $\Rightarrow$*,* or *for* $\overline{\Leftarrow}$ *and* $\overline{\Rightarrow}$*, with respect to* $\hat{=}$ *and* $\Leftrightarrow$ respectively. The qualifier “commutative” to “law” can be used interchangeably with either of the qualifiers “*symmetric*” and “*bilateral*”, whereas “anti-commutative” interchangeably with either of the qualifiers “*anti-symmetric*” and “*unilateral*”.

2) In the presence of $\Rightarrow$ and $\overline{\Rightarrow}$, the connectives $\Leftarrow$ and $\overline{\Leftarrow}$ are redundant. However, the former and the latter are rendered into English differently. For instance, $\Rightarrow$ is rendered by the active predicate “implies”, while $\Leftarrow$ is rendered by the passive predicate “is implied by”. Therefore, it is useful to have both pair of connective in order to make the pertinent English expressions univocal.

3) The terms “rightward implication”, “leftward implication”, “rightward antiimplication”, and “rightward antiimplication”, as defined by items 4, 5, 11, and 12 of Df 1.12A, are syntactic ones and therefore they are monosemantic. For instance, both implications **P**$\Rightarrow$**Q** and **Q**$\Rightarrow$**P** are rightward, whereas both implications **P**$\Leftarrow$**Q** and **Q**$\Leftarrow$**P** are leftward. At the same time, in a process of reasoning, a certain

implication, either a rightward one, e.g. **P⇒Q**, or its leftward equivalent, **Q⇐P**, can be qualified *direct* in the sense that it is an *initial* one. In this case, the variant of either implication with **P** and **Q** exchanged, i.e. either the rightward implication **Q⇒P** or the equivalent leftward implication **P⇐Q**, is said to be *converse of,* or *with respect to, either of the two equivalent direct* (*initial*) *implications* **P⇒Q** and **Q⇐P**. Thus, in contrast to the qualifiers “rightward” and “leftward”, which are absolute, the qualifiers “direct” and “converse” are epistemologically relativistic. The difference between the above two pairs of qualifiers can be illustrated by the fact that a formal equivalence (biimplication) of two relations, **P** and **Q**, can be described in words as the conjunction of two *mutually converse* implications, no matter whether either of them is rightward or leftward.•

7.7. Associative and distributive laws for ∨ and ∧ relative to = or ⇔

Th 7.8:** ***Basic associative laws for* ∨ *and* ∧ *relative to $\hat{=}$.

$$V([\mathbf{P}\vee\mathbf{Q}]\vee\mathbf{R})\,\hat{=}\,V(\mathbf{P}\vee[\mathbf{Q}\vee\mathbf{R}])\,\hat{=}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R}), \tag{7.67}$$

$$\begin{aligned}&V([\mathbf{P}\wedge\mathbf{Q}]\wedge\mathbf{R})\,\hat{=}\,V(\mathbf{P}\wedge[\mathbf{Q}\wedge\mathbf{R}])\,\hat{=}\,V(\mathbf{P})\,\hat{+}\,V(\mathbf{Q})\,\hat{+}\,V(\mathbf{R})\\ &\hat{-}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{-}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{-}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{+}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R}),\end{aligned} \tag{7.68}$$

subject to the pertinent instance of convention (5.2).

Proof: By the pertinent instances of (7.2γ) and (7.6γ), it follows that

$$\begin{aligned}&V([\mathbf{P}\vee\mathbf{Q}]\vee\mathbf{R})\,\hat{=}\,V(\mathbf{P}\vee\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{=}\,[V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})]\,\hat{\cdot}\,V(\mathbf{R})\\ &\hat{=}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\\ &\hat{=}\,V(\mathbf{P})\,\hat{\cdot}\,[V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})]\,\hat{=}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q}\vee\mathbf{R})\,\hat{=}\,V(\mathbf{P}\vee[\mathbf{Q}\vee\mathbf{R}]),\end{aligned} \tag{7.67$_1$}$$

$$\begin{aligned}&V([\mathbf{P}\wedge\mathbf{Q}]\wedge\mathbf{R})\,\hat{=}\,V(\mathbf{P}\wedge\mathbf{Q})\,\hat{+}\,V(\mathbf{R})\,\hat{-}\,V(\mathbf{P}\wedge\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\\ &\hat{=}\,V(\mathbf{P})\,\hat{+}\,V(\mathbf{Q})\,\hat{-}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{-}\,[V(\mathbf{P})\,\hat{+}\,V(\mathbf{Q})\,\hat{-}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})]\,\hat{\cdot}\,V(\mathbf{R}),\\ &V(\mathbf{P}\wedge[\mathbf{Q}\wedge\mathbf{R}])\,\hat{=}\,V(\mathbf{P})\,\hat{+}\,V(\mathbf{Q}\wedge\mathbf{R})\,\hat{-}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q}\wedge\mathbf{R})\\ &\hat{=}\,V(\mathbf{P})\,\hat{+}\,V(\mathbf{Q})\,\hat{+}\,V(\mathbf{R})\,\hat{-}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{-}\,V(\mathbf{P})\,\hat{\cdot}\,[V(\mathbf{Q})\,\hat{+}\,V(\mathbf{R})\,\hat{-}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})],\end{aligned} \tag{7.68$_1$)}$$

subject to the pertinent instance of convention (5.2). QED.•

***Th 7.9:** ***Basic associative laws for* ∨ *and* ∧ *relative to* ⇔.**

$$[[\mathbf{P}\vee\mathbf{Q}]\vee\mathbf{R}]\Leftrightarrow[\mathbf{P}\vee[\mathbf{Q}\vee\mathbf{R}]], \tag{7.67a}$$

$$[[\mathbf{P}\wedge\mathbf{Q}]\wedge\mathbf{R}]\Leftrightarrow[\mathbf{P}\wedge[\mathbf{Q}\wedge\mathbf{R}]]. \tag{7.68a}$$

Proof: Equivalences (7.67a) and (7.68a) immediately follow from identities (7.67) and (7.68) respectively by the pertinent instances of theorem (7.50).•

Cnv 7.2: ***A supplement to Cnv. 2.1.***

$$[[\mathbf{P} \vee \mathbf{Q}] \vee \mathbf{R}] \overset{\leftarrow}{\Leftrightarrow} [\mathbf{P} \vee \mathbf{Q} \vee \mathbf{R}] \overset{\rightarrow}{\Leftrightarrow} [\mathbf{P} \vee [\mathbf{Q} \vee \mathbf{R}]], \tag{7.67b}$$

$$[[\mathbf{P} \wedge \mathbf{Q}] \wedge \mathbf{R}] \overset{\leftarrow}{\Leftrightarrow} [\mathbf{P} \wedge \mathbf{Q} \wedge \mathbf{R}] \overset{\rightarrow}{\Leftrightarrow} [\mathbf{P} \wedge [\mathbf{Q} \wedge \mathbf{R}]]; \tag{7.68b}$$

that is, the inner pair of square brackets in any one of the expressions $[[\mathbf{P} \vee \mathbf{Q}] \vee \mathbf{R}]$, $[\mathbf{P} \vee [\mathbf{Q} \vee \mathbf{R}]]$, $[[\mathbf{P} \wedge \mathbf{Q}] \wedge \mathbf{R}]$, and $[\mathbf{P} \wedge [\mathbf{Q} \wedge \mathbf{R}]]$ can be omitted, while the omission of the outer pair of square brackets is subjugated to Cnv. 2.1 (cf. Cnv 5.1).•

Cmt. 7.10. Cmt 4.1 applies, *mutatis mutandis*, to the basic associative laws (7.67a) and (7.68a) in place of (4.10) and (4.14). To be specific, given any natural number n>3 of ER's $\mathbf{P}_1$, $\mathbf{P}_2$, ..., $\mathbf{P}_n$ of A_1 it can be proved from theorems (7.67a) and (7.68a) that any two ER resulted by two different arrangement of n–2 pairs of brackets [ ] either in the string $\mathbf{P}_1 \vee \mathbf{P}_2 \vee ... \vee \mathbf{P}_n$ or in the string $\mathbf{P}_1 \wedge \mathbf{P}_2 \wedge ... \wedge \mathbf{P}_n$ are related by the sign ⇔. The above statement is a *meta-theorem* that is called the *generalized associative laws for the operators* $[\vee]$ *and* $[\wedge]$. For a sufficiently small *concrete* value of '*n*', say 4 or 5, the pertinent instance of either of the two generalized associative laws can be verified straightforwardly. However, in order to prove the generalized associative laws in the general form for *any* unspecified number n>3 of integrons, one should utilize the *method of mathematical induction*. Thus, the proof of each of the two generalized associative laws in question is just an instance of the abstract generalized associative law that has been mentioned in Cmt 4.1. In this case, by Th 7.7, either operator $[\vee]$ or $[\wedge]$ satisfies the basic *commutative law* (7.64), i.e., loosely speaking, it is *symmetrical*, relative to the equivalence operator ⇔. Therefore, in accordance with Cmt 4.1, each one of the operators $[\vee]$ or $[\wedge]$ satisfies *the generalized associative and commutative law for any unspecified number n*>3 *of ER's relative to* ⇔.•

7.8. Distributive laws for ∨ over ∧ and for ∧ over ∨ relative to = or ⇔.

Th. 7.10:** ***The distributive laws for ∨ ***over*** ∧ ***and for*** ∧ ***over*** ∨***, relative to*** $\hat{=}$.

$$\begin{aligned} &V(\mathbf{P} \vee [\mathbf{Q} \wedge \mathbf{R}]) \,\hat{=}\, V([\mathbf{P} \vee \mathbf{Q}] \wedge [\mathbf{P} \vee \mathbf{R}]) \\ &\hat{=}\, V(\mathbf{P}) \,\hat{\cdot}\, [V(\mathbf{Q}) \,\hat{+}\, V(\mathbf{R}) \,\hat{-}\, V(\mathbf{Q}) \,\hat{\cdot}\, V(\mathbf{R})], \end{aligned} \tag{7.69}$$

$$\begin{aligned} &V(\mathbf{P} \wedge [\mathbf{Q} \vee \mathbf{R}]) \,\hat{=}\, V([\mathbf{P} \wedge \mathbf{Q}] \vee [\mathbf{P} \wedge \mathbf{R}]) \\ &\hat{=}\, V(\mathbf{P}) \,\hat{+}\, V(\mathbf{Q}) \,\hat{\cdot}\, V(\mathbf{R}) \,\hat{-}\, V(\mathbf{P}) \,\hat{\cdot}\, V(\mathbf{Q}) \,\hat{\cdot}\, V(\mathbf{R}), \end{aligned} \tag{7.70}$$

subject to the pertinent instance of convention (5.2).

Proof: By the pertinent instances of (7.2γ) and (7.6γ), it follows that

$$
\begin{aligned}
V(\mathbf{P}\vee[\mathbf{Q}\wedge\mathbf{R}]) &\hat{=} V(\mathbf{P})\hat{\cdot}V(\mathbf{Q}\wedge\mathbf{R}) \\
&\hat{=} V(\mathbf{P})\hat{\cdot}[V(\mathbf{Q})\hat{+}V(\mathbf{R})\hat{-}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R})],
\end{aligned}
\tag{7.69$_1$}
$$

$$
\begin{aligned}
V([\mathbf{P}\vee\mathbf{Q}]\wedge[\mathbf{P}\vee\mathbf{R}]) &\hat{=} V(\mathbf{P}\vee\mathbf{Q})\hat{+}V(\mathbf{P}\vee\mathbf{R})\hat{-}V(\mathbf{P}\vee\mathbf{Q})\hat{\cdot}V(\mathbf{P}\vee\mathbf{R}) \\
&\hat{=} V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{+}V(\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\mathbf{P})\hat{\cdot}V(\mathbf{R}) \\
&\hat{=} V(\mathbf{P})\hat{\cdot}[V(\mathbf{Q})\hat{+}V(\mathbf{R})\hat{-}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R})];
\end{aligned}
\tag{7.69$_2$}
$$

$$
\begin{aligned}
V(\mathbf{P}\wedge[\mathbf{Q}\vee\mathbf{R}]) &\hat{=} V(\mathbf{P})\hat{+}V(\mathbf{Q}\vee\mathbf{R})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q}\vee\mathbf{R}) \\
&\hat{=} V(\mathbf{P})\hat{+}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R}),
\end{aligned}
\tag{7.70$_1$}
$$

$$
\begin{aligned}
V([\mathbf{P}\wedge\mathbf{Q}]\vee[\mathbf{P}\wedge\mathbf{R}]) &\hat{=} V(\mathbf{P}\wedge\mathbf{Q})\hat{\cdot}V(\mathbf{P}\wedge\mathbf{R}) \\
&\hat{=} [V(\mathbf{P})\hat{+}V(\mathbf{Q})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})]\hat{\cdot}[V(\mathbf{P})\hat{+}V(\mathbf{R})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{R})] \\
&\hat{=} V(\mathbf{P})\hat{+}V(\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{R}) \\
&\quad\hat{+}V(\mathbf{Q})\hat{\cdot}V(\mathbf{P})\hat{+}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R})\hat{-}V(\mathbf{Q})\hat{\cdot}V(\mathbf{P})\hat{\cdot}V(\mathbf{R}) \\
&\quad\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R})\hat{+}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R}) \\
&\hat{=} V(\mathbf{P})\hat{+}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R}),
\end{aligned}
\tag{7.70$_1$}
$$

subject to the pertinent instance of convention (5.2). In the legato style, the trains (7.69$_1$) and (7.69$_2$) reduce to (7.69), whereas (7.70$_1$) and (7.70$_2$) reduce to (7.70). QED•

Th. 7.11:** ***The distributive laws for $\vee$ ***over*** $\wedge$ ***and for*** $\wedge$ ***over*** $\vee$ ***relative to*** $\Leftrightarrow$.

$$[\mathbf{P}\vee[\mathbf{Q}\wedge\mathbf{R}]]\Leftrightarrow[[\mathbf{P}\vee\mathbf{Q}]\wedge[\mathbf{P}\vee\mathbf{R}]], \tag{7.69a}$$

$$[\mathbf{P}\wedge[\mathbf{Q}\vee\mathbf{R}]]\Leftrightarrow[[\mathbf{P}\wedge\mathbf{Q}]\vee[\mathbf{P}\wedge\mathbf{R}]]. \tag{7.70a}$$

Proof: Equivalences (7.69a) and (7.70a) immediately follow from identities (7.69) and (7.70) respectively by the pertinent instances of theorem (7.50).•

Cmt 7.11. Any distributive law occurring in algebra is of exactly one kind, namely that for a binary multiplication operator, as $\hat{\cdot}$, over the corresponding binary addition operator, as $\hat{+}$, relative to an equality operator, as $\hat{=}$ (cf. *Ax. 19.4(8)). By contrast, there are in A_0 (and in any system of sentential calculus) two similar distributive laws relative to the material equivalence operator $\Leftrightarrow$: the first is one for $\vee$ over $\wedge$, and the second is one for $\wedge$ over $\vee$.•

Cmt 7.12. The following identities are some *simplifications* (*simplest instances*) of (7.69) and (7.70), which are inferred from the latter by the pertinent instances of the idempotent law (4.17):

$$
\begin{aligned}
V(\mathbf{P}\vee[\mathbf{P}\wedge\mathbf{R}]) &\hat{=} V([\mathbf{P}\vee\mathbf{P}]\wedge[\mathbf{P}\vee\mathbf{R}]) \\
&\hat{=} V(\mathbf{P})\hat{\cdot}[V(\mathbf{P})\hat{+}V(\mathbf{R})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{R})]\hat{=}V(\mathbf{P}),
\end{aligned}
\tag{7.69'}
$$

$$V(\mathbf{P}\wedge[\mathbf{P}\vee\mathbf{R}])\hat{=}V([\mathbf{P}\wedge\mathbf{P}]\vee[\mathbf{P}\wedge\mathbf{R}])$$
$$\hat{=}V(\mathbf{P})\hat{+}V(\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{=}V(\mathbf{P}), \tag{7.70'}$$

$$V(\mathbf{P}\vee[\mathbf{Q}\wedge\mathbf{Q}])\hat{=}V([\mathbf{P}\vee\mathbf{Q}]\wedge[\mathbf{P}\vee\mathbf{Q}])$$
$$\hat{=}V(\mathbf{P})\hat{\cdot}[V(\mathbf{Q})\hat{+}V(\mathbf{Q})\hat{-}V(\mathbf{Q})\hat{\cdot}V(\mathbf{Q})]\hat{=}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{=}V(\mathbf{P}\vee\mathbf{Q}), \tag{7.69''}$$

$$V(\mathbf{P}\wedge[\mathbf{Q}\vee\mathbf{Q}])\hat{=}V([\mathbf{P}\wedge\mathbf{Q}]\vee[\mathbf{P}\wedge\mathbf{Q}])$$
$$\hat{=}V(\mathbf{P})\hat{+}V(\mathbf{Q})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\mathbf{Q}) \tag{7.70''}$$
$$\hat{=}V(\mathbf{P})\hat{+}V(\mathbf{Q})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{=}V(\mathbf{P}\wedge\mathbf{Q}).$$

Hence, by the pertinent instances of (7.50), it follows from (7.69')–(7.70'') that

$$\mathbf{P}\vee[\mathbf{P}\wedge\mathbf{R}]\Leftrightarrow\mathbf{P}\wedge[\mathbf{P}\vee\mathbf{R}]\Leftrightarrow\mathbf{P}, \tag{7.69'a}$$

$$\mathbf{P}\vee[\mathbf{Q}\wedge\mathbf{Q}]\Leftrightarrow\mathbf{P}\vee\mathbf{Q}, \tag{7.69''a}$$

$$\mathbf{P}\wedge[\mathbf{Q}\vee\mathbf{Q}]\Leftrightarrow\mathbf{P}\wedge\mathbf{Q}. \tag{7.70''a}$$ •

7.9. Transitive laws

Th 7.12:** ***Transitive laws for ⇔ and ⇒ relative to ⇒ in the objective form.

$$V([[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Leftrightarrow\mathbf{R}])\hat{=}0. \tag{7.71}$$

$$V([[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]]\Rightarrow[\mathbf{Q}\Leftrightarrow[\mathbf{P}\wedge\mathbf{R}]])\hat{=}0. \tag{7.72}$$

$$V([[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{R}])\hat{=}0. \tag{7.73}$$

$$V([[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{R}])\hat{=}0. \tag{7.74}$$

$$V([[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{R}])\hat{=}0. \tag{7.75}$$

Proof: 1) By the pertinent instances of (7.1γ), (7.3γ), (7.6γ), (7.7γ), (7.12γ), and (7.15γ), it follows that

$$V(\neg[[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]])\hat{=}V(\neg[\mathbf{P}\Leftrightarrow\mathbf{Q}])\hat{\cdot}V(\neg[\mathbf{Q}\Leftrightarrow\mathbf{R}])$$
$$\hat{=}V(\mathbf{P}\,\overline{\Leftrightarrow}\,\mathbf{Q})\hat{\cdot}V(\mathbf{Q}\,\overline{\Leftrightarrow}\,\mathbf{R})$$
$$\hat{=}[V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})]\hat{\cdot}[V(\mathbf{Q})\hat{\cdot}V(\mathbf{R})\hat{+}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{R})] \tag{7.71$_1$}$$
$$\hat{=}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R})\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{R})$$
$$\hat{=}V(\mathbf{P})\hat{+}V(\mathbf{Q})\hat{+}V(\mathbf{R})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{-}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R}),$$

$$V(\mathbf{Q}\Leftrightarrow[\mathbf{P}\wedge\mathbf{R}])\hat{=}V(\mathbf{Q})\hat{\cdot}V(\neg[\mathbf{P}\wedge\mathbf{R}])\hat{+}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{P}\wedge\mathbf{R})$$
$$\hat{=}V(\mathbf{Q})\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{R})\hat{+}V(\neg\mathbf{Q})\hat{\cdot}[1\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{R})] \tag{7.72$_1$}$$
$$\hat{=}[V(\mathbf{Q})\hat{-}V(\neg\mathbf{Q})]\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{R})\hat{+}V(\neg\mathbf{Q}).$$

$$V(\neg[[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]])\hat{=}V(\neg[\mathbf{P}\Leftrightarrow\mathbf{Q}])\hat{\cdot}V(\neg[\mathbf{Q}\Rightarrow\mathbf{R}])$$
$$\hat{=}[V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})]\hat{\cdot}[1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R})]$$
$$\hat{=}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R}) \tag{7.73$_1$}$$
$$\hat{=}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{R}),$$

$$\begin{aligned} &V(\neg[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]])\,\hat{=}\,V(\neg[\mathbf{P}\Rightarrow\mathbf{Q}])\,\hat{\cdot}\,V(\neg[\mathbf{Q}\Leftrightarrow\mathbf{R}])\\ &\hat{=}\,[1\,\hat{-}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})]\,\hat{\cdot}\,[V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{+}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})]\\ &\hat{=}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{+}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})\,\hat{-}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\\ &\hat{=}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})\,\hat{+}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R}), \end{aligned} \qquad (7.74_1)$$

$$\begin{aligned} &V(\neg[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]])\,\hat{=}\,V(\neg[\mathbf{P}\Rightarrow\mathbf{Q}])\,\hat{\cdot}\,V(\neg[\mathbf{Q}\Rightarrow\mathbf{R}])\\ &\hat{=}\,[1\,\hat{-}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})]\,\hat{\cdot}\,[1\,\hat{-}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})]\\ &\hat{=}\,1\,\hat{-}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{-}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R}), \end{aligned} \qquad (7.75_1)$$

$$V(\mathbf{P}\Rightarrow\mathbf{R})\,\hat{=}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{R}), \qquad (7.3\gamma_1)$$

$$\begin{aligned} &V(\mathbf{P}\Leftrightarrow\mathbf{R})\,\hat{=}\,V([\mathbf{P}\Rightarrow\mathbf{R}]\wedge[\mathbf{P}\Leftarrow\mathbf{R}])\\ &\hat{=}\,1\,\hat{-}\,V(\neg[\mathbf{P}\Rightarrow\mathbf{Q}])\,\hat{\cdot}\,V(\neg[\mathbf{P}\Leftarrow\mathbf{Q}])\\ &\hat{=}\,V(\mathbf{P}\Rightarrow\mathbf{R})\,\hat{+}\,V(\mathbf{P}\Leftarrow\mathbf{R})\,\hat{=}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{+}\,V(\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{R}). \end{aligned} \qquad (7.7\gamma_1)$$

2) From the pertinent instance of (7.3γ), it follows that

$$\begin{aligned} &V([[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Leftrightarrow\mathbf{R}])\\ &\hat{=}\,V(\neg[[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]])\,\hat{\cdot}\,V(\mathbf{P}\Leftrightarrow\mathbf{R})\\ &\hat{=}\,V(\neg[[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]])\,\hat{\cdot}\,[V(\mathbf{P}\Rightarrow\mathbf{R})\,\hat{+}\,V(\mathbf{R}\Rightarrow\mathbf{P})]\\ &\hat{=}\,[V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{+}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})]\\ &\hat{\cdot}\,[V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{+}\,V(\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{R})]\,\hat{=}\,0, \end{aligned} \qquad (7.71_2)$$

by (7.71_1) and $(7.7\gamma_1)$;

$$\begin{aligned} &V([[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]]\Rightarrow[\mathbf{Q}\Leftrightarrow[\mathbf{P}\wedge\mathbf{R}]])\\ &\hat{=}\,V(\neg[[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]])\,\hat{\cdot}\,V(\mathbf{Q}\Leftrightarrow[\mathbf{P}\wedge\mathbf{R}])\\ &\hat{=}\,[V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{+}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})]\\ &\hat{\cdot}\,[[V(\mathbf{Q})\,\hat{-}\,V(\neg\mathbf{Q})]\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{R})\,\hat{+}\,V(\neg\mathbf{Q})]\\ &\hat{=}\,[V(\mathbf{Q})\,\hat{-}\,V(\neg\mathbf{Q})]\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})\,\hat{+}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})\\ &\hat{=}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})\,\hat{-}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})\,\hat{=}\,0, \end{aligned} \qquad (7.72_2)$$

by (7.71_1) and (7.72_1);

$$\begin{aligned} &V([[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{R}])\\ &\hat{=}\,V(\neg[[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]])\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{R})\\ &\hat{=}\,[V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{+}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})]\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{=}\,0, \end{aligned} \qquad (7.73_2)$$

by (7.73_1) and $(7.3\gamma_1)$;

$$\begin{aligned} &V([[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{R}])\\ &\hat{=}\,V(\neg[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]])\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{R})\\ &\hat{=}\,[V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{R})\,\hat{+}\,V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{R})]\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{R})\,\hat{=}\,0, \end{aligned} \qquad (7.74_2)$$

by (7.74_1) and $(7.3\gamma_1)$;

$$
\begin{aligned}
&V([[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{R}])\\
&\hat{=}V(\neg[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]])\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{R})\\
&\hat{=}[1\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R})]\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{R})\\
&\hat{=}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\mathbf{R})\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R})\\
&\hat{=}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{\cdot}[V(\mathbf{Q})\hat{+}V(\neg\mathbf{Q})]\\
&\hat{=}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{\cdot}1\hat{=}0,
\end{aligned}
\tag{7.75$_2$}
$$

by (7.75$_1$) and (7.3γ_1). QED.•

*Th 7.13: ***Transitive laws for ⇔ and ⇒ relative to ⇒ in the subjective form.**

$$[[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Leftrightarrow\mathbf{R}]. \tag{7.71a}$$

$$[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]\Rightarrow[\mathbf{Q}\Leftrightarrow[\mathbf{P}\wedge\mathbf{R}]]. \tag{7.72a}$$

$$[[\mathbf{P}\Leftrightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{R}]. \tag{7.73a}$$

$$[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{R}]. \tag{7.74a}$$

$$[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{R}]. \tag{7.75a}$$

Proof: (7.71a)–(7.75a) immediately follow from (7.71)–(7.75) respectively by the pertinent instances of (4.40a).•

Cmt 7.13. Identities (7.71)–(7.75) or equivalences (7.71a)–(7.75a) can be called, in that order, the *first transitive law for ⇔ relative to ⇒*, the *second transitive law for ⇔ relative to ⇒*, the *first transitive law for ⇒ and ⇔ relative to ⇒*, the *second transitive law for ⇒ and ⇔ relative to ⇒*, and the *transitive law for ⇒relative to ⇒*, in the respective form.•

*Th 7.14: ***Transitive laws for conjunctions relative to ⇒.**

$$V([[\mathbf{P}\wedge\mathbf{Q}]\wedge[\mathbf{Q}\wedge\mathbf{R}]]\Rightarrow[\mathbf{P}\wedge\mathbf{R}])\hat{=}0, \tag{7.76}$$

whence, by the pertinent instance of (4.40a),

$$[[\mathbf{P}\wedge\mathbf{Q}]\wedge[\mathbf{Q}\wedge\mathbf{R}]]\Rightarrow[\mathbf{P}\wedge\mathbf{R}]. \tag{7.76a}$$

Proof: By the pertinent instances of (7.3γ), (7.6γ), and (7.68), it follows that

$$
\begin{aligned}
&V([[\mathbf{P}\wedge\mathbf{Q}]\wedge[\mathbf{Q}\wedge\mathbf{R}]]\Rightarrow[\mathbf{P}\wedge\mathbf{R}])\\
&\hat{=}[1\hat{-}V([[\mathbf{P}\wedge\mathbf{Q}]\wedge\mathbf{Q}]\wedge\mathbf{R})]\hat{\cdot}V(\mathbf{P}\wedge\mathbf{R})\\
&\hat{=}[1\hat{-}V(\mathbf{R})\hat{-}V(\mathbf{P}\wedge\mathbf{Q})\hat{\cdot}V(\neg\mathbf{R})]\hat{\cdot}V(\mathbf{P}\wedge\mathbf{R})\\
&\hat{=}[1\hat{-}V(\mathbf{P}\wedge\mathbf{Q})]\hat{\cdot}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{P}\wedge\mathbf{R})\\
&\hat{=}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{R})\hat{\cdot}[V(\mathbf{P})\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{R})]\hat{=}0.
\end{aligned}
\tag{7.76$_1$}
$$

8. The AEADP’s and APLADP’s for plain contracted relations

8.1. The AEADP’s for the major plain contracted relations of A_1

***Th 8.1.**

$$V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\overset{\frown}{\cdot}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{x}\rangle\right), \tag{8.1}$$

whence, by (7.50),

$$\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\Leftrightarrow\left[\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y}\rangle\right]. \tag{8.1'}$$

Proof: (8.1) follows from the instance of (4.25) with $V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)$ in place of $\mathbf{i}\langle\mathbf{x}\rangle$ and $V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)$ in place of $\mathbf{i}\langle\mathbf{y}\rangle$, by (4.23) and by the variant of (4.23) with ‘**y**’ in place of ‘**x**’.•

***Th 8.2.**

$$V\left(\bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\bigvee_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]. \tag{8.2}$$

Proof: The self-explanatory train of identities (8.2) follow from Df 2.1(3) by the pertinent instances of (4.31), (4.23), (7.1γ), and (7.6γ).•

***Th 8.3.**

$$V\left(\neg\bigwedge_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right). \tag{8.3}$$

$$V\left(\neg\bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\overset{\frown}{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]. \tag{8.4}$$

$$V\left(\bigwedge_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right). \tag{8.5}$$

Proof: By (7.1γ_1), the trains (8.3)–(8.5) follow from (8.2) respectively thus:

$$\begin{aligned}&V\left(\neg\bigwedge_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\neg\bigvee_{\mathbf{x}}\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\\&\hat{=}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right),\end{aligned} \tag{8.3$_1$}$$

$$\begin{aligned}&V\left(\neg\bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\neg\bigvee_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\\&\hat{=}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\overset{\frown}{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right],\end{aligned} \tag{8.45}$$

$$\begin{aligned}&V\left(\bigwedge_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\bigvee_{\mathbf{x}}\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}V\left(\bigvee_{\mathbf{x}}\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\\&\hat{=}1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right).\end{aligned} \tag{8.5$_1$}$$ •

Cmt 8.1. By (7.50), the trains of identities (8.2)–(8.5) are tantamount to the following trains of equivalences:

$$\left[\bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\Leftrightarrow\left[\neg\bigvee_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right], \tag{8.2'}$$

$$\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\Leftrightarrow\left[\neg\bigwedge_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right], \tag{8.3'}$$

$$\left[\neg\bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\Leftrightarrow\left[\bigvee_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right], \tag{8.4'}$$

$$\left[\bigwedge_{\mathbf{x}}\neg\mathbf{P}\langle\mathbf{x}\rangle\right]\Leftrightarrow\left[\neg\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right], \tag{8.5'}$$

The identities (8.2) and (8.4) are said to be *dual* of (8.3) and (8.5), while the equivalences (8.2') and (8.4') are said to be *dual* of (8.3') and (8.5'), respectively.•

Th 8.4.

$$V\left(\bigvee_{\mathbf{x}}\bigvee_{\mathbf{y}}\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V\left(\bigvee_{\mathbf{y}}\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\hat{\boldsymbol{\cdot}}_{\mathbf{y}}V\left(\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right). \tag{8.6}$$

$$\begin{gathered}V\left(\bigvee_{\mathbf{x}}\bigwedge_{\mathbf{y}}\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{=}\hat{\boldsymbol{\cdot}}_{x}V\left(\bigwedge_{\mathbf{y}}\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\\ \hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{y}}V\left(\neg\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\right]\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{y}}\left[1\hat{-}V\left(\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\right]\right]\end{gathered} \tag{8.7}$$

$$\begin{gathered}V\left(\bigwedge_{\mathbf{x}}\bigvee_{\mathbf{y}}\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}V\left(\bigvee_{\mathbf{y}}\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\right]\\ \hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{y}}V\left(\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\right]\end{gathered} \tag{8.8}$$

$$\begin{gathered}V\left(\bigwedge_{\mathbf{x}}\bigwedge_{\mathbf{y}}\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}V\left(\bigwedge_{\mathbf{y}}\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\right]\\ \hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}\left[1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{y}}V\left(\neg\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\right]\right]\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\hat{\boldsymbol{\cdot}}_{\mathbf{y}}V\left(\neg\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\\ \hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\hat{\boldsymbol{\cdot}}_{\mathbf{y}}\left[1\hat{-}V\left(\mathbf{Q}\langle\mathbf{x},\mathbf{y}\rangle\right)\right]\end{gathered} \tag{8.9}$$

Proof: Each of the trains of identities (8.6)-(8.9) is proved by the two-fold application of the pertinent variants of (4.23) or (8.2).•

***Th 8.5.**

$$\begin{gathered}V\left(\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\vee\left[\bigvee_{\mathbf{y}}\neg\mathbf{P}\langle\mathbf{y}\rangle\right]\right)\hat{=}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{\cdot}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{y}}V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\right]\\ \hat{=}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{\cdot}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{y}}V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\right]\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}0\hat{=}0,\end{gathered} \tag{8.10}$$

whence, by (7.50),

$$\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\vee\left[\bigvee_{\mathbf{y}}\neg\mathbf{P}\langle\mathbf{y}\rangle\right]. \tag{8.10'}$$

Proof: (8.10) follows from the pertinent instance of (7.2γ), by the pertinent instance of (4.29) (Fusion Law) and by (7.15γ).•

***Th 8.6.**

$$\begin{gathered}V\left(\breve{\bigvee}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{=}V\left(\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\wedge\left[\bigvee_{\mathbf{y}}\neg\mathbf{P}\langle\mathbf{y}\rangle\right]\right)\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{+}\hat{\boldsymbol{\cdot}}_{\mathbf{y}}V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\\ \hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{+}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right),\end{gathered} \tag{8.11}$$

$$\begin{gathered}V\left(\overset{\frown 1}{\bigvee}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{=}V\left(\bigwedge_{\mathbf{x}}\bigwedge_{\mathbf{y}}\left[\left[\mathbf{P}(\mathbf{x})\wedge\mathbf{P}\langle\mathbf{y}\rangle\right]\Rightarrow\left[\mathbf{x}=\mathbf{y}\right]\right]\right)\\ \hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\hat{\boldsymbol{\cdot}}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{\cdot}V\left(\mathbf{x}=\mathbf{y}\right)\right],\end{gathered} \tag{8.12}$$

provided that $\mathbf{P}\langle\mathbf{x}\rangle$ contains **x** and does not contain **y** and **z**.

Proof: The AEADP's, or, from a somewhat different viewpoint, the APLADP's, underlying the DT (8.11), follows from Df 2.1(4) by the pertinent instances of (4.31), (4.23), (7.1γ), and (7.6γ) thus:

$$\begin{aligned} V\left(\breve{\bigvee}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right) &\hat{=} V\left(\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\wedge\left[\bigvee_{\mathbf{y}}\neg\mathbf{P}\langle\mathbf{y}\rangle\right]\right) \\ &\hat{=} 1 \hat{-} V\left(\neg\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\neg\bigvee_{\mathbf{y}}\neg\mathbf{P}\langle\mathbf{y}\rangle\right) \\ &\hat{=} 1 \hat{-} \left[1 \hat{-} \hat{:}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle)\right] \hat{\cdot} \left[1 \hat{-} \hat{:}_{\mathbf{y}} V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\right] \\ &\hat{=} 1 \hat{-} \left[1 \hat{-} \hat{:}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle) \hat{-} \hat{:}_{\mathbf{y}} V(\neg\mathbf{P}\langle\mathbf{y}\rangle) \hat{+} \left[\hat{:}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle)\right] \hat{\cdot} \left[\hat{:}_{\mathbf{y}} V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\right]\right] \\ &\hat{=} \hat{:}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle) \hat{+} \hat{:}_{\mathbf{y}} V(\neg\mathbf{P}\langle\mathbf{y}\rangle) \hat{=} \hat{:}_{\mathbf{z}} V(\mathbf{P}\langle\mathbf{z}\rangle) \hat{+} \hat{:}_{\mathbf{z}} V(\neg\mathbf{P}\langle\mathbf{z}\rangle), \end{aligned} \tag{8.11$_1$}$$

where use of (8.10) and of the pertinent instances of (4.25) has been made. At the same time, (8.12) is proved from Df. 2.1(5) by the pertinent instances of (4.31), (8.11), (7.6γ), and (7.3γ) as follows:

$$\begin{aligned} V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right) &\hat{=} V\left(\bigwedge_{x}\bigwedge_{y}\left[\left[\mathbf{P}(\mathbf{x})\wedge\mathbf{P}\langle\mathbf{y}\rangle\right]\Rightarrow[\mathbf{x}=\mathbf{y}]\right]\right) \\ &\hat{=} 1 \hat{-} \hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}\left[1 \hat{-} V\left(\left[\mathbf{P}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{y}\rangle\right]\Rightarrow[\mathbf{x}=\mathbf{y}]\right)\right] \\ &\hat{=} 1 \hat{-} \hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}\left[1 \hat{-} V\left(\neg\left[\mathbf{P}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{y}\rangle\right]\right) \hat{\cdot} V(\mathbf{x}=\mathbf{y})\right] \\ &\hat{=} 1 \hat{-} \hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}\left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{y}\rangle) \hat{\cdot} V(\mathbf{x}=\mathbf{y})\right]. \end{aligned} \tag{8.12$_1$}\bullet$$

Cmt 8.2. 1) By (8.10), it follows from (8.11) that

$$\begin{aligned} \left[V\left(\breve{\bigvee}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\right]^2 &\hat{=} \left[\hat{:}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle) \hat{+} \hat{:}_{\mathbf{y}} V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\right]^2 \\ \hat{=} \left[\hat{:}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle)\right]^2 &\hat{+} \left[\hat{:}_{\mathbf{y}} V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\right]^2 \hat{+} 2 \hat{\cdot} \left[\hat{:}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle)\right] \hat{\cdot} \left[\hat{:}_{\mathbf{y}} V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\right] \\ &\hat{=} \hat{:}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle) \hat{+} \hat{:}_{\mathbf{y}} V(\neg\mathbf{P}\langle\mathbf{y}\rangle) \hat{=} V\left(\breve{\bigvee}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right), \end{aligned} \tag{8.11$_5$}$$

– in agreement with the fact that $V\left(\breve{\bigvee}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)$ must satisfy the pertinent version of the idempotent law(4.2).

2) With allowance for (8.4) or (8.4'), the train (8.11) is tantamount to

$$\breve{\bigvee}_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle \Leftrightarrow \left[\left[\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y}\rangle\right]\wedge\left[\bigvee_{\mathbf{z}}\neg\mathbf{P}\langle\mathbf{z}\rangle\right]\right] \Leftrightarrow \left[\left[\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y}\rangle\right]\wedge\left[\neg\bigwedge_{\mathbf{z}}\mathbf{P}\langle\mathbf{y}\rangle\right]\right]. \tag{8.11'}\bullet$$

Th 8.7.

$$V\left(\left[\overset{\frown}{\bigvee}{}^{1}_{\mathbf{x}}\mathbf{P}\langle\mathbf{z}\rangle\right]\vee\left[\bigvee_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right]\right) \hat{=} V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{x}}\mathbf{P}\langle\mathbf{z}\rangle\right) \hat{\cdot} V\left(\bigvee_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right) \hat{=} 0 \tag{8.13}$$

(cf. (8.2)), whence, by (7.50),

$$\left[\overset{\frown}{\bigvee}{}^{1}_{\mathbf{x}}\mathbf{P}\langle\mathbf{z}\rangle\right]\vee\left[\bigvee_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right]. \tag{8.13'}$$

Proof: By (4.23) and (8.12), it follows that

$$\begin{aligned}
&V\left(\left[\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right]\vee\left[\bigvee{}_{\mathbf{u}}\mathbf{P}\langle\mathbf{u}\rangle\right]\right)\hat{=}V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{\cdot}V\left(\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\\
&\hat{=}\left[1\hat{-}\hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right]\right]\hat{\cdot}\hat{:}_{\mathbf{w}}V(\mathbf{P}\langle\mathbf{w}\rangle)\\
&\hat{=}\hat{:}_{\mathbf{w}}V(\mathbf{P}\langle\mathbf{w}\rangle)\hat{-}\hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}\left[\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right]\hat{\cdot}V(\mathbf{P}\langle\mathbf{x}\rangle)\right]\\
&\hat{=}\hat{:}_{\mathbf{w}}V(\mathbf{P}\langle\mathbf{w}\rangle)\hat{-}\hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}\hat{:}_{\mathbf{x}}V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{-}\hat{:}_{\mathbf{x}}V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}0,
\end{aligned}\qquad(8.13_1)$$

where in developing the final result use of the pertinent instance of (4.29) (Fusion Law) and also use of the identity

$$V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}0\qquad(8.13_2)$$

(see (7.15γ)) have been made.•

Th 8.8.

$$\begin{aligned}
&V\left(\bigvee{}^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right)\hat{=}V\left(\left[\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right]\wedge\left[\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right]\right)\hat{=}V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{+}V\left(\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\\
&\hat{=}1\hat{-}\hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right]\hat{+}\hat{:}_{\mathbf{w}}V(\mathbf{P}\langle\mathbf{w}\rangle).
\end{aligned}\qquad(8.14)$$

Proof: (8.14) follows from Df. 2.1(6) by the pertinent instances of (4.31) and (7.6γ), and also by (8.12) and (8.13) thus:

$$\begin{aligned}
&V\left(\bigvee{}^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right)\hat{=}V\left(\left[\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right]\wedge\left[\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right]\right)\\
&\hat{=}V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{+}V\left(\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\hat{-}V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{\cdot}V\left(\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\\
&\hat{=}V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{+}V\left(\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\\
&\hat{=}1\hat{-}\hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right]\hat{+}\hat{:}_{\mathbf{w}}V(\mathbf{P}\langle\mathbf{w}\rangle).
\end{aligned}\qquad(8.14_1)\bullet$$

Cmt 8.3. Any validity integron **i** satisfies the idempotent law (4.17). As regards $V\left(\bigvee{}^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right)$, this fact can readily be demonstrated from (8.14) with the help of (8.13) as follows:

$$\begin{aligned}
&\left[V\left(\bigvee{}^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right)\right]^2\hat{=}\left[V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{+}V\left(\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\right]^2\\
&\hat{=}\left[V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\right]^2\hat{+}\left[V\left(\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\right]^2\hat{+}2\hat{\cdot}V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{\cdot}V\left(\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\\
&\hat{=}V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{+}V\left(\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\hat{=}V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right).
\end{aligned}\qquad(8.14_2)\bullet$$

Cmt 8.4. Comparison of (8.12) and (8.14) shows that

$$\begin{aligned}
&V\left(\bigvee{}^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right)\hat{=}V\left(\overset{\frown}{\bigtriangledown}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\\
&\hat{=}1\hat{-}\hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right]\\
&\text{if } V\left(\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\hat{=}\hat{:}_{\mathbf{w}}V(\mathbf{P}\langle\mathbf{w}\rangle)\hat{=}0,\text{ i.e. if }\bigvee{}_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle.
\end{aligned}\qquad(8.14_3)\bullet$$

It is understood that (8.14_3) is a semantic statement about relations of A_1, which belongs to its IML (inclusive metalanguage).•

Cmt 8.5. In accordance with the items 1–3 of Df 2.1, occurrences of '$\hat{\cdot}_{\mathbf{x}}$', '$\vee_{\mathbf{x}}$', and '$\wedge_{\mathbf{x}}$' in all pertinent relations can be replaced with occurrences of '$(\hat{\cdot}\,\mathbf{x})$', '$(\exists\mathbf{x})$', and '$(\forall\mathbf{x})$' respectively without altering the identity of the relations; and similarly with '**y**' or '**z**' in place of '**x**'.•

Cmt 8.6. 1) (4.23) is the *panlogographic schema* (*PLS*) of an infinite number of *euautographic axioms* (*EAxs*) *for contracted* (*pseudo-quantified*) *euautographic slave-relations* (*CdESR's*) *condensed in its range* and at the same time it is the *panlogographic axiom* (*PLAx*) *for the contracted* (*pseudo-quantified*) *panlogographic slave-relation* (*CdPLSR*) '$\vee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle$' or, more precisely, $\vee_{\text{'}\mathbf{x}\text{'}}\mathbf{P}\langle\text{'}\mathbf{x}\text{'}\rangle$. According to (4.23), the latter is *vav-neutral*. Hence, any *concrete* CdESR $\vee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle$ of the range of '$\vee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle$' is either valid or antivalid or vav-veutral.

2) Consequently, any one of the trains of identities (8.2)–(8.14) *is a PLS of the AEADP* (*advanced euautographic decision procedure*) *for a common* (*general*) *CdESR* $\wedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle$, $\breve{\vee}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$, $\overset{\frown}{\vee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$, or $\vee^{1}_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle$ and, *at the same time* or *in other words*, *an APLADP* (*advanced panlogographic decision procedure*) for the CdPLSR processed, as '$\wedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle$', '$\breve{\vee}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$', '$\overset{\frown}{\vee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$', or '$\vee^{1}_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle$' respectively. According to the *PLDT* (*panlogographic decision theorem*), resulted by and contained in an APLADP, each of the above-mentioned CdPLSR's is *vav-neutral* and therefore a concrete CdESR in its range can be either valid or antivalid or else vav-veutral – just as a CdESR in the range of '$\vee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle$'. Therefore, given a concrete ER *adjusted* (*fitted*) to the common (general) ER $\mathbf{P}\langle\mathbf{x}\rangle$, which is another hypostasis (way of existence) of the range of '$\mathbf{P}\langle\mathbf{x}\rangle$', in order to establish the validity-value of the respective *concrete CdESR's* adjusted to any one of the *common* (*general*) *CdESR's:*

$$\vee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle,\ \wedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle,\ \breve{\vee}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle,\ \overset{\frown}{\vee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle,\ \vee^{1}_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle, \tag{8.15}$$

one should subject the concrete CdESR to the appropriate *AEADP* (*advanced euautographic decision procedure*) so as to deduce the *EDT* (*euautographic decision theorem*) for that concrete CdESR. At the same time, if '$\mathbf{P}\langle\mathbf{x}\rangle$' is replaced with a *patterned PLSR*, denoted ad hoc by '$\boldsymbol{\mathsf{P}}\langle\boldsymbol{\mathsf{x}}\rangle$', such that a certain one of the pertinent common (general) PLSR's:

$$\vee_{\mathbf{x}}\mathsf{P}\langle\mathbf{x}\rangle,\ \wedge_{\mathbf{x}}\mathsf{P}\langle\mathbf{x}\rangle,\ \breve{\vee}_{\mathbf{z}}\mathsf{P}\langle\mathbf{z}\rangle,\ \overset{\frown}{\vee}{}^{1}_{\mathbf{z}}\mathsf{P}\langle\mathbf{z}\rangle,\ \vee^{1}_{\mathbf{v}}\mathsf{P}\langle\mathbf{v}\rangle \tag{8.16}$$

turns out to be vav-unneutral, i.e. either valid or antivalid, then every concrete CdESR in the range of that PLSR will be valid or antivalid respectively. In this case, no additional AEADP is needed in order to establish the validity-value of any given CdESR in the range of such a pattern PLSR. This is the most typical and most important case, which will occur in practice. Still, a concrete CdER, the validity-value of which is established in either way, individually or via the range of the appropriate valid or antivalid CdPLSR collectivizing it, is, like a chess position, *insignificant*, and therefore it can be intelligibly rendered into ordinary language only via its *CFCL* (*conformal catlogographic*) *interpretand*.

3) The common CdESR's (8.15) can be *specified* (*restricted*, *semi-concretized*) by substituting occurrences of any five different APVOT's of the list (I.5.1), say of x, y, z, v, and w, for occurrences of **x**, **y**, **z**, **v**, and **w** throughout the items 1–6 of Df 2.1, subject to the assumptions made in Df 1.7(1), so that (8.15) become the respective *specific*, or *semi-concretized*, *common CdESR's*:

$$\vee_{x}\mathbf{P}\langle x\rangle,\ \wedge_{x}\mathbf{P}\langle x\rangle,\ \breve{\vee}_{\mathbf{z}}\mathbf{P}\langle z\rangle,\ \overset{\frown}{\vee}{}^{1}_{z}\mathbf{P}\langle z\rangle,\ \vee^{1}_{v}\mathbf{P}\langle v\rangle. \tag{8.15ι}$$

Any specific CdESR thus obtained can be *concretized* by selecting a *concrete* CdESR out of it, i.e. by concretizing **P**. In order to indicate that this is [*as if*] done, I shall replace '**P**' with '***P***', so that (8.15ι) become the respective [*as if*] *concrete CdESR's*:

$$\vee_{x}\boldsymbol{P}\langle x\rangle,\ \wedge_{x}\boldsymbol{P}\langle x\rangle,\ \breve{\vee}_{\mathbf{z}}\boldsymbol{P}\langle z\rangle,\ \overset{\frown}{\vee}{}^{1}_{z}\boldsymbol{P}\langle z\rangle,\ \vee^{1}_{v}\boldsymbol{P}\langle v\rangle. \tag{8.15μ}$$

Then the occurrences of x, y, z, v, and w throughout the [*as if*] CdESR's (8.15μ) and throughout their *eautographic definientia* should be replaced with occurrences of the analo-homolographic AVCLOT's '*x*', '*y*', '*z*', '*v*', and '*w*' respectively, thus obtaining:

$$\vee_{x}\boldsymbol{P}\langle x\rangle,\ \wedge_{x}\boldsymbol{P}\langle x\rangle,\ \breve{\vee}_{z}\boldsymbol{P}\langle z\rangle,\ \overset{\frown}{\vee}{}^{1}_{z}\boldsymbol{P}\langle z\rangle,\ \vee^{1}_{v}\boldsymbol{P}\langle v\rangle, \tag{8.15κ}$$

which are the [*as if*] *CFCL interpretands of the* [*as if*] *concrete CdESR's* (8.15μ). It goes without saying that these interpretands are *significant* (*interpreted*) *vav-neutral contracted CFCL relations* (briefly *CdCFCLR's*) and therefore, together with their definientia, they can be supplemented by the appropriate *wordy* (*verbal*) *denotative definienda*, which are at the same time *connotative definientia* that explicate the meanings of the CdCFCLR's.

4) The *common* (*general*) *contracted euautographic validity-integron* (*CdEVI*) $\hat{\cdot}_{\mathbf{x}}V(\mathbf{P}\langle\mathbf{x}\rangle)$ and its definiens $(\hat{\cdot}\,\mathbf{x})V(\mathbf{P}\langle\mathbf{x}\rangle)$ can be *specified* (*restricted, semi-concretized* likewise as $\hat{\cdot}_{x}V(\mathbf{P}\langle x\rangle)$ and as $(\hat{\cdot}\,x)V(\mathbf{P}\langle x\rangle)$ respectively, which can, in turn, be *as if concretized* as $\hat{\cdot}_{x}V(\boldsymbol{P}\langle x\rangle)$ and $(\hat{\cdot}\,x)V(\boldsymbol{P}\langle x\rangle)$, while $\hat{\cdot}_{x}V(\boldsymbol{P}\langle x\rangle)$ and $(\hat{\cdot}\,x)V(\boldsymbol{P}\langle x\rangle)$) are the [*as if*] CFCL interpretands of the [*as if*] *concrete* CdEVI's.

5) Here follows [*as if*] CFCL interpretands of the items 1–6 of Df 2.1, which are augmented by the appropriate wordy definienda, which render the pertinent euautographic contractors into ordinary language when they apply to CFCLR's.•

Df 8.1.

1) $\left[\hat{\cdot}_{x}V(\boldsymbol{P}\langle x\rangle)\right]\rightarrow\left[(\hat{\cdot}\,x)V(\boldsymbol{P}\langle x\rangle)\right]\leftarrow$[the contraction over x of $V(\boldsymbol{P}\langle x\rangle)$]

$\leftarrow$[the product over x of $V(\boldsymbol{P}\langle x\rangle)$].

2) $\left[\bigvee_{x}\boldsymbol{P}\langle x\rangle\right]\rightarrow\left[(\exists x)\boldsymbol{P}\langle x\rangle\right]\leftarrow$[there exists at least one x such that $\boldsymbol{P}\langle x\rangle$]

$\leftarrow$[for at least one x: $\boldsymbol{P}\langle x\rangle$]$\leftarrow$[for some x: $\boldsymbol{P}\langle x\rangle$].

3) $\left[\bigwedge_{x}\boldsymbol{P}\langle x\rangle\right]\rightarrow\left[(\forall x)\boldsymbol{P}\langle x\rangle\right]\rightarrow\left[\neg\bigvee_{x}\neg\boldsymbol{P}\langle \boldsymbol{x}\rangle\right]\leftarrow$[for all x: $\boldsymbol{P}\langle x\rangle$]

$\leftarrow$[for every x: $\boldsymbol{P}\langle x\rangle$].

4) $\left[\breve{\bigvee}_{z}\boldsymbol{P}\langle z\rangle\right]\rightarrow\left[\left[\bigvee_{x}\boldsymbol{P}\langle x\rangle\right]\wedge\left[\bigvee_{y}\neg\boldsymbol{P}\langle y\rangle\right]\right]\leftarrow$[for some but not all z: $\boldsymbol{P}\langle z\rangle$]

$\leftarrow$[for strictly some z: $\boldsymbol{P}\langle z\rangle$].

5) $\left[\hat{\bigvee}_{z}^{1}\boldsymbol{P}\langle z\rangle\right]\rightarrow\left[\bigwedge_{x}\bigwedge_{y}\left[\left[\boldsymbol{P}\langle x\rangle\wedge\boldsymbol{P}\langle y\rangle\right]\Rightarrow[x=y]\right]\right]$

$\leftarrow$[there exists at most one z such that $\boldsymbol{P}\langle z\rangle$]

$\leftarrow$[for at most one z: $\boldsymbol{P}\langle z\rangle$].

6) $\left[\bigvee_{v}^{1}\boldsymbol{P}\langle v\rangle\right]\rightarrow\left[\left[\hat{\bigvee}_{z}^{1}\boldsymbol{P}\langle z\rangle\right]\wedge\left[\bigvee_{w}\boldsymbol{P}\langle w\rangle\right]\right]$

$\leftarrow$[there exists exactly one v such that $\boldsymbol{P}\langle v\rangle$]

$\leftarrow$[for exactly one v: $\boldsymbol{P}\langle v\rangle$].

In the above occurrences, "*exists*" can be used interchangeably with "*is*".•

Cmt 8.7. 1) According to (8.12) and (8.14), the common CdESR's $\hat{\bigvee}_{\mathbf{z}}^{1}\mathbf{P}\langle\mathbf{z}\rangle$ and $\bigvee_{\mathbf{z}}^{1}\mathbf{P}\langle\mathbf{z}\rangle$, and the concurrent concrete CdPLSR's '$\hat{\bigvee}_{\mathbf{z}}^{1}\mathbf{P}\langle\mathbf{z}\rangle$' and '$\bigvee_{\mathbf{z}}^{1}\mathbf{P}\langle\mathbf{z}\rangle$' are vav-neutral, – like all other common CdESR's and their concurrent concrete

CdPLPSR's that have been processed above in this section. Therefore, either common CdESR $\overset{\frown}{\vee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$ or $\vee^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$ *is adjustable to*, i.e. the range of either concrete CdPLSR '$\overset{\frown}{\vee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$' or '$\vee^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$' *contains*, concrete CdESR's of the pertinent general pattern, of all the three kinds: valid antivalid, and vav-neutral. As long as no additional subject axioms other than Axs 4.1–4.12 (which, along with meta Axs 6.13–6.20, constitute the initial D_1, i.e. D_1 and $\boldsymbol{\mathsf{D}}_1$) are imposed on the sign =, either directly or obliquely (via $\subseteq$ or $\in$), for *individuating* it, *no specific subject theorem* can be stated and proved for any one of the common contracted relations $\overset{\frown}{\vee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$, $\vee^{1}_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle$, $\overset{\frown}{\vee}{}^{1}_{\boldsymbol{\mathsf{z}}}\boldsymbol{\mathsf{P}}\langle\boldsymbol{\mathsf{z}}\rangle$, and $\vee^{1}_{\boldsymbol{\mathsf{v}}}\boldsymbol{\mathsf{P}}\langle\boldsymbol{\mathsf{v}}\rangle$, so as to *individuate* the signs $\overset{\frown}{\vee}{}^{1}_{*}$ and $\vee^{1}_{*}$. Nevertheless, certain *conditional meta-theorems* can be stated for the above relations at any stage of the setup of A_1, i.e. A_1 and $\boldsymbol{\mathsf{A}}_1$, including this one, because the *antecedent* (*hypothesis*) of a conditional statement serves as a certain *ad hoc* axiom. An instructive conditional meta-theorem, which is independent of any special axiom or theorem of =, is stated and proved below this comment.

2) Once the sign = is defined axiomatically in one way or another, either equality

$$V\left(\overset{\frown}{\vee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{=}0 \text{ (a) or } V\left(\vee^{1}_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right)\hat{=}0 \text{ (b)} \tag{8.17}$$

is a condition that is imposed on a ER **P**, whereas either equality

$$V\left(\overset{\frown}{\vee}{}^{1}_{\boldsymbol{\mathsf{z}}}\boldsymbol{\mathsf{P}}\langle\boldsymbol{\mathsf{z}}\rangle\right)\hat{=}0 \text{ (a) or } V\left(\vee^{1}_{\boldsymbol{\mathsf{v}}}\boldsymbol{\mathsf{P}}\langle\boldsymbol{\mathsf{v}}\rangle\right)\hat{=}0 \text{ (b)} \tag{8.18}$$

is a similar condition that is imposed on a PLR $\boldsymbol{\mathsf{P}}$.•

****Th 8.9.** If

$$V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1, \tag{8.19}$$

i.e. if $\mathbf{P}\langle\mathbf{x}\rangle$ is an *antivalid common ER* (*common euautographic antikyrology*), then

$$V\left(\overset{\frown}{\vee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{=}0, \tag{8.20}$$

$$V\left(\vee^{1}_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right)\hat{=}1. \tag{8.21}$$

Proof: From (7.1γ), it follows that

$$V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1 \text{ if and only if } V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}0 \tag{8.19$_1$}$$

and similarly with '**y**' or '**w**' (e.g.) in place of '**x**'. Therefore, from the instance of (4.36) with '¬**P**', '1', '**x**', and '**y**' or '**w**' in place of '**P**', '*m*', '$\mathbf{x}_1$', and '$\mathbf{y}_1$'

respectively, it follows that (8.19) holds with ‘**y**’ or ‘**w**’ in place of ‘**x**’. Hence, by the identity (8.19) and its variant with ‘**y**’ in place of ‘**x**’, the common EDT, concluding the common AEADP (8.12), can be developed thus:

$$V\left(\widehat{\bigvee}_{\mathbf{z}}^{1}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{=}1\hat{-}\hat{\wedge}_{\mathbf{x}}\hat{\wedge}_{\mathbf{y}}\left[1\hat{-}0\hat{\cdot}0\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right]\hat{=}1\hat{-}\hat{\wedge}_{\mathbf{x}}\hat{\wedge}_{\mathbf{y}}1\hat{=}1\hat{-}1\hat{=}0, \qquad (8.20_1)$$

where use of the second identity ($4.24\gamma\iota_1$), being the pertinent instance of (4.24), has been made. Thus, (8.20) is established. Making use of (8.20_1) and of the variants of the identity (8.19) and of the second identity ($4.24\gamma\iota_1$), the common EDT (8.14) can be developed thus:

$$V\left(\bigvee_{\mathbf{v}}^{1}\mathbf{P}\langle\mathbf{v}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\hat{=}\hat{\wedge}_{\mathbf{w}}V(\mathbf{P}\langle\mathbf{w}\rangle)\hat{=}\hat{\wedge}_{\mathbf{w}}1\hat{=}1, \qquad (8.21_1)$$

which proves (8.21).•

Cmt 8.8. 1) Just as Cmt 8.4, Th 8.9 is a semantic statement about relations of A_1, which belongs to its IML.

2) In accordance with Th 8.9, $\widehat{\bigvee}_{\mathbf{z}}^{1}\mathbf{P}\langle\mathbf{z}\rangle$ is a valid common ER and $\bigvee_{\mathbf{v}}^{1}\mathbf{P}\langle\mathbf{v}\rangle$ is an antivalid common ER, independent of the common ER $[\mathbf{x}=\mathbf{y}]$ and hence independent on any additional axioms or theorems will be imposed on = in the sequel. The fact that (8.12) subject to (8.19) implies (8.20), i.e. (8.17a), is just a manifestation of the well-known peculiar property of the logical connective ⇒, which has been explicated in Cmt 7.6 and which occurs in (8.12). Namely, in accordance with Cmt 7.6, if $[\mathbf{P}(\mathbf{x})\wedge\mathbf{P}\langle\mathbf{y}\rangle]$ is antivalid, as assumed by (8.19), then $[\mathbf{P}(\mathbf{x})\wedge\mathbf{P}\langle\mathbf{y}\rangle]\Rightarrow[\mathbf{x}=\mathbf{y}]$ is valid independent of $[\mathbf{x}=\mathbf{y}]$. Consequently, by (8.21), the condition (8.17b) cannot be satisfied by any ER **P** in the class of antivalid ER’s.•

8.2. The General Law of Denial of Russell’s Paradox

***Th 8.10.** In accordance with Df 1.7, let $\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$ be a relation of $\mathsf{A}_{1\mathsf{P}}$ ($\mathsf{A}_{1\mathsf{R}}$) that contains two different free APVOT’s **x** and **y** and perhaps some other APVOT’s, free or bound. Let also

$$\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\rightarrow\widehat{\mathrm{S}}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\Big|, \qquad (8.22)$$

i.e. $\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle$ is the ER that results by substitution of **x** for each occurrence of **y** throughout $\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$. Then

$$V\left(\neg\bigwedge_{\mathbf{x}}[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle]\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\neg[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle]\right)\hat{=}0. \qquad (8.23)$$

Proof: The variant of (8.4) with ‘$\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]$’ in place of ‘$\mathbf{P}\langle\mathbf{x}\rangle$’ becomes:can be developed thus:

$$\begin{aligned}&V\left(\neg\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\\ &\hat{=}\hat{\cdot}_{\mathbf{x}}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}\hat{\cdot}_{\mathbf{x}}\left[1\hat{-}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right)\right]\right]\\ &\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{\cdot}V\left(\neg\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right)\right]\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right)\right].\end{aligned}\tag{8.23$_1$}$$

a) If the ER $\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$ is *reflexive*, i.e. if

$$V\left(\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right)\hat{=}0,\tag{8.23$_2$}$$

then the final expression in (8.23$_1$) reduces straightforwardly developed thus:

$$\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right)\right]\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{\cdot}0\right]\hat{=}\hat{\cdot}_{\mathbf{x}}0\hat{=}0,\tag{8.23$_3$}$$

by (4.24$\gamma\iota_1$).

b) If an ER $\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$ is *not reflexive*, i.e. if

$$V\left(\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right)\hat{=}1\text{ or }V\left(\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right)\hat{=}\mathbf{i}_{\sim}\left|\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right\rangle,\tag{8.23$_4$}$$

then making use of the instance of the Emission Law (4.28) with ‘$\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right)\right]$’ in place of ‘$\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle$’ followed by making use of the identity:

$$V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{y}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{y}\rangle\right)\hat{=}0,\tag{8.23$_5$}$$

being an instance of (7.15γ), reduces the final expression in (8.23$_1$) straightforwardly thus:

$$\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right)\right]\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{\cdot}0\right]\hat{=}\hat{\cdot}_{\mathbf{x}}0\hat{=}0.\tag{8.23$_6$}$$

QED.•

Th 8.11: ***Major objective implications of Th 8.10.***

$$\begin{aligned}&V\left(\neg\bigvee_{\mathbf{y}}\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}V\left(\bigwedge_{\mathbf{y}}\neg\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\\ &\hat{=}V\left(\neg\bigwedge_{\mathbf{y}}\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}0.\end{aligned}\tag{8.24}$$

Proof: By the pertinent variants of (7.1γ), (7.1γ_1), and (8.2), each separate validity-integron of the train (8.24) is expressed in terms of $V\left(\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)$ or $V\left(\neg\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)$, so that it is elementarily computed with the help of (8.23) as follows:

$$\begin{aligned}&V\left(\neg\bigvee_{\mathbf{y}}\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\\ &\hat{=}1\hat{-}\hat{\cdot}_{\mathbf{y}}V\left(\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}1\hat{-}\hat{\cdot}_{\mathbf{y}}1\hat{=}1\hat{-}1\hat{=}0,\end{aligned}\tag{8.24$_1$}$$

$$\begin{gathered} \mathcal{V}\left(\bigwedge_{\mathbf{y}}\neg\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right) \\ \hat{=}\,1\,\hat{-}\,\overset{\frown}{\cdot}_{\mathbf{y}}\mathcal{V}\left(\neg\neg\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right) \\ \hat{=}\,1\,\hat{-}\,\overset{\frown}{\cdot}_{\mathbf{y}}\mathcal{V}\left(\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}\,1\,\hat{-}\,\overset{\frown}{\cdot}_{\mathbf{y}}1\,\hat{=}\,1\,\hat{-}\,1\,\hat{=}\,0, \end{gathered} \tag{8.24$_2$}$$

$$\begin{gathered} \mathcal{V}\left(\neg\bigwedge_{\mathbf{y}}\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right) \\ \hat{=}\,\mathcal{V}\left(\neg\neg\bigvee_{\mathbf{y}}\neg\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right) \\ \hat{=}\,\overset{\frown}{\cdot}_{\mathbf{y}}\mathcal{V}\left(\neg\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}\,\overset{\frown}{\cdot}_{\mathbf{y}}0\,\hat{=}\,0. \end{gathered} \tag{8.24$_3$}$$ •

Cmt 8.9. By (4.40a), the common euautographic identities (valid equalities, algebraic kyrologies) (8.23) and (8.24) are equivalent to the conjunction of the following respective common valid euautographic logical relations (common euautographic logical kyrologies):

$$\bigvee_{\mathbf{x}}\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]. \tag{8.23'}$$

$$\neg\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right], \tag{8.23''}$$

$$\neg\bigvee_{\mathbf{y}}\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right], \tag{8.24'}$$

$$\bigwedge_{\mathbf{y}}\neg\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right], \tag{8.24''}$$

$$\neg\bigwedge_{\mathbf{y}}\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right], \tag{8.24'''}$$

The above statement is a subject theorem of both A_1 and $\boldsymbol{\mathsf{A}}_1$ together with its immediate proof. At the same time, the negation of any of the kyrologies (8.23')–(8.24''') is an antikyrology, which can be asserted only objectively as the train of the identities:

$$\begin{gathered} \mathcal{V}\left(\neg\bigvee_{\mathbf{x}}\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}\,\mathcal{V}\left(\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right) \\ \hat{=}\,\mathcal{V}\left(\bigvee_{\mathbf{y}}\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}\,\mathcal{V}\left(\neg\bigwedge_{\mathbf{y}}\neg\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right) \\ \hat{=}\,\mathcal{V}\left(\bigwedge_{\mathbf{y}}\bigwedge_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}\,1, \end{gathered} \tag{8.25}$$

which follows straightforwardly from (8.23) and (8.24).•

Df 8.2. The PLDT (panlogographic decision theorem) (8.23) will be called the *Weak*, or *Unbound*, *General Law of Denial of Russell's Paradox* (briefly *Weak GLDRP* or *WGLDRP*) *in* A_1, whereas the PLDT (8.24) will be called the *Strong*, or *Bound*, *GLDRP* (*SGLDRP*) *in* A_1, *in the objective form both.* The two identities with 0 on the right-hand side of each of them, which are present in (8.23), are called *versions of the WGLDP in the objective* (or *algebraic*) *form*. The three identities, which are present in (8.24), are called *versions of the SGLDP in the objective form*.

Consequently, the two valid PLR’s (8.23') and (8.23'') are *two versions of the WGLDP in the subjective* (or *logical*) *form*, whereas the three valid PLR’s (8.24')–(8.24''') are *three versions of the SGLDP in the subjective form*.•

Cmt 8.10. 1) In stating and proving Th 8.10, the order of ‘**x**’ and ‘**y**’ is fixed. However, this theorem and hence Th 8.10 and Cmt 8.9 remain valid with ‘$\mathbf{P}\langle\mathbf{y},\mathbf{x}\rangle$’ in place of ‘$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$’, while all occurrences of the pertinent contractors (pseudo-quantifiers) remain unaltered.

2) The AnPLS’ta (analytical panlogographic schemata) (8.23), (8.24), and (8.23')–(8.24''') can be *specified* by replacing ‘$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$’ with any StPLS’ta (structural panlogographic schemata) that contains ‘**x**’ and ‘**y**’ as free APLOT’s (atomic panlogographic ordinary terms) and by replacing ‘$\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle$’ accordingly, – for instance, as follows:

$$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle \triangleright \mathbf{f}^{2}(\mathbf{x},\mathbf{y}) \text{ and } \mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle \triangleright \mathbf{f}^{2}(\mathbf{x},\mathbf{x}) \text{ subject to } \mathbf{f}^{2} \vec{\in} \left[\kappa^{2\mathrm{pv}} \cup \kappa^{\mathrm{pc}}\right], \quad (8.26)$$

$$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle \triangleright \mathbf{f}^{3}(\mathbf{x},\mathbf{y},\mathbf{x}_{1}) \text{ and } \mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle \triangleright \mathbf{f}^{3}(\mathbf{x},\mathbf{x},\mathbf{x}_{1}) \text{ subject to } \mathbf{f}^{3} \vec{\in} \kappa^{3\mathrm{pv}}, \quad (8.27)$$

$$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle \triangleright \mathbf{f}^{4}(\mathbf{x},\mathbf{y},\mathbf{x}_{1},\mathbf{x}_{2}) \text{and } \mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle \triangleright \mathbf{f}^{4}(\mathbf{x},\mathbf{x},\mathbf{x}_{1},\mathbf{x}_{2}) \text{ subject to } \mathbf{f}^{4} \vec{\in} \kappa^{4\mathrm{pv}}, \quad (8.28)$$

etc (see Df I.5.2(3,4)). The *specific* instances of (8.23)–(8.24'''), which are resulted by the substitutions (8.26), e.g., can be specified further by the substitution $\mathbf{f}^{2} \triangleright \mathbf{f}^{2\mathrm{pv}} \vec{\in} \kappa^{2\mathrm{pv}}$ or by the substitution $\mathbf{f}^{2} \triangleright \mathbf{f}^{\mathrm{pc}} \vec{\in} \kappa^{\mathrm{pc}}$ subject to $\mathbf{f}^{\mathrm{pc}} \stackrel{\rightarrow}{=} \mathbf{f}^{2\mathrm{pc}}$ if κ^{pc} subject to $\kappa^{\mathrm{pc}} \stackrel{\rightarrow}{=} \kappa^{2\mathrm{pc}}$ is present (see Df I.5.2(7)). The latter *specific* instances can then be *concretized* by substituting any of the APVOPS’s (atomic pseudovariable ordinary predicate-signs) of the set $\kappa^{2\mathrm{pv}}$ (*ibid*; e.g. f^{2}, g^{2}, etc) for ‘$\mathbf{f}^{2\mathrm{pv}}$’ and at most one of the APCOPS’s (atomic pseudoconstant ordinary predicate-signs) $\in$, $\subseteq$, and = as specified for ‘$\mathbf{f}^{2\mathrm{pc}}$’, and simultaneously substituting any two different APVOT’s (ordinary atomic pseudovariable predicate-signs) of the set τ^{pv} (*ibid*; e.g. x and y or u and v) for ‘**x**’ and ‘**y**’. Once ‘$\mathbf{f}^{2}$’ is replaced by ‘$\mathbf{f}^{\mathrm{pc}}$’, the relation-schema ‘$\mathbf{f}^{\mathrm{pc}}(\mathbf{x},\mathbf{y})$’, which is written in this nonlinear (Clairaut-Euler) form in accordance with the pertinent formation rule of A_1, can be rewritten in the conventional bilinear form ‘$\left[\mathbf{x}\mathbf{f}^{\mathrm{pc}}\mathbf{y}\right]$’ in accordance with definition (I.1.18). Thus, the Russell’s paradox and its solution are not concerned with any specified or unspecified predicate of any weight, and not specifically with the binary set-theoretic predicate $\in$. Th 8.10 guaranties that any formalized language that is based on sound interpretation of the calculus A_1 will be free of any paradox of Russell's type.

3) It is understood that Th 8.10 holds with any MlPLR (molecular panlogographic relation) $\mathsf{P}\langle \mathsf{x}, \mathsf{y}\rangle$ (as '$\mathbf{Q}\langle \mathbf{u}, \mathbf{v}\rangle$' or '$\mathbf{R}\langle \mathbf{v}, \mathbf{w}\rangle$') in place of '$\mathbf{P}\langle \mathbf{x}, \mathbf{y}\rangle$' and with any two different APLOT's (atomic panlogographic terms) x and y (as '**u**' and '**v**' or '**v**' and '**w**') in placed of '**x**' and '**y**'.•

8.3. Miscellaneous plain contracted relations

Preliminary Remark 8.1. In contrast to Th 8.10 and its implications, which have been stated and proved above and which are of fundamental importance, the theorems that will deduced belowin this subsection, are primarily simplest examples of AEADP's or, depending on a viewpoint, of APLADP's.•

*__Th 8.12:__ ***Laws of particularization and generalization.***

$$V\left(\left[\bigwedge_{\mathbf{x}} \mathbf{P}\langle \mathbf{x}\rangle\right] \Rightarrow \mathbf{P}\langle \mathbf{y}\rangle\right) \hat{=} V\left(\neg \bigwedge_{\mathbf{x}} \mathbf{P}\langle \mathbf{x}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle \mathbf{y}\rangle\right) \hat{=} 0\,. \tag{8.29}$$

$$V\left(\mathbf{P}\langle \mathbf{y}\rangle \Rightarrow \bigvee_{\mathbf{x}} \mathbf{P}\langle \mathbf{x}\rangle\right) \hat{=} 0\,. \tag{8.30}$$

$$V\left(\bigvee_{\mathbf{y}} \left[\left[\bigvee_{\mathbf{x}} \mathbf{P}\langle \mathbf{x}\rangle\right] \Rightarrow \mathbf{P}\langle \mathbf{y}\rangle\right]\right) \hat{=} 0. \tag{8.31}$$

$$V\left(\bigvee_{\mathbf{y}} \left[\mathbf{P}\langle \mathbf{y}\rangle \Rightarrow \bigwedge_{\mathbf{x}} \mathbf{P}\langle \mathbf{x}\rangle\right]\right) \hat{=} 0. \tag{8.32}$$

The four identities (8.29)–(8.32) are called the *Strong Law of Particularization*, the *Strong Law of Generalization*, the *Weak Law of Particularization*, and the *Weak Law of Generalization* in that order.

Proof: By the pertinent rules of D_1, it follows that

$$\begin{gathered} V\left(\left[\bigwedge_{\mathbf{x}} \mathbf{P}\langle \mathbf{x}\rangle\right] \Rightarrow \mathbf{P}\langle \mathbf{y}\rangle\right) \hat{=} V\left(\neg \bigwedge_{\mathbf{x}} \mathbf{P}\langle \mathbf{x}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle \mathbf{y}\rangle\right) \\ \hat{=} \left[\hat{\cdot}_{\mathbf{x}} V\left(\neg \mathbf{P}\langle \mathbf{x}\rangle\right)\right] \hat{\cdot} V\left(\mathbf{P}\langle \mathbf{y}\rangle\right) \\ \hat{=} \left[\hat{\cdot}_{\mathbf{x}} V\left(\neg \mathbf{P}\langle \mathbf{x}\rangle\right)\right] \hat{\cdot} \left[V\left(\neg \mathbf{P}\langle \mathbf{y}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle \mathbf{y}\rangle\right)\right] \hat{=} \left[\hat{\cdot}_{\mathbf{x}} V\left(\neg \mathbf{P}\langle \mathbf{x}\rangle\right)\right] \hat{\cdot}\, 0 \hat{=} 0, \end{gathered} \tag{8.29$_1$}$$

$$\begin{gathered} V\left(\mathbf{P}\langle \mathbf{y}\rangle \Rightarrow \bigvee_{\mathbf{x}} \mathbf{P}\langle \mathbf{x}\rangle\right) \hat{=} V\left(\neg \mathbf{P}\langle \mathbf{y}\rangle\right) \hat{\cdot} \left[\hat{\cdot}_{\mathbf{x}} V\left(\neg \mathbf{P}\langle \mathbf{x}\rangle\right)\right] \\ \hat{=} \left[V\left(\neg \mathbf{P}\langle \mathbf{y}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle \mathbf{y}\rangle\right)\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{x}} V\left(\neg \mathbf{P}\langle \mathbf{x}\rangle\right)\right] \hat{=} 0 \hat{\cdot} \left[\hat{\cdot}_{\mathbf{x}} V\left(\neg \mathbf{P}\langle \mathbf{x}\rangle\right)\right] \hat{=} 0, \end{gathered} \tag{8.30$_1$}$$

$$\begin{gathered} V\left(\bigvee_{\mathbf{y}} \left[\left[\bigvee_{\mathbf{x}} \mathbf{P}\langle \mathbf{x}\rangle\right] \Rightarrow \mathbf{P}\langle \mathbf{y}\rangle\right]\right) \hat{=} \hat{\cdot}_{\mathbf{y}} V\left(\neg \bigvee_{\mathbf{x}} \mathbf{P}\langle \mathbf{x}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle \mathbf{y}\rangle\right) \\ \hat{=} \hat{\cdot}_{\mathbf{y}} \left[\left[1 \hat{-} \hat{\cdot}_{\mathbf{x}} V\left(\mathbf{P}\langle \mathbf{x}\rangle\right)\right] \hat{\cdot} V\left(\mathbf{P}\langle \mathbf{y}\rangle\right)\right] \hat{=} \left[1 \hat{-} \hat{\cdot}_{\mathbf{x}} V\left(\mathbf{P}\langle \mathbf{x}\rangle\right)\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{y}} V\left(\mathbf{P}\langle \mathbf{y}\rangle\right)\right] \\ \hat{=} \left[1 \hat{-} \hat{\cdot}_{\mathbf{x}} V\left(\mathbf{P}\langle \mathbf{x}\rangle\right)\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{x}} V\left(\mathbf{P}\langle \mathbf{x}\rangle\right)\right] \hat{=} 0, \end{gathered} \tag{8.31$_1$}$$

$$\begin{aligned} V\left(\bigvee_{\mathbf{y}}\left[\mathbf{P}\langle\mathbf{y}\rangle \Rightarrow \bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\right) &\hat{=} \overset{\frown}{\cdot}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{\cdot}V\left(\bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \\ &\hat{=} \overset{\frown}{\cdot}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{\cdot}\left[1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\right] \\ &\hat{=} \left[\overset{\frown}{\cdot}_{\mathbf{y}}V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \\ &\hat{=} \left[\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{=}0, \end{aligned} \tag{8.32$_1$}$$

which prove (8.29)–(8.32). In developing the final result in (8.29_1) and (8.30_1), use of the pertinent instances of the EL (Emission Law) (4.27) has been made. In developing the final results in (8.31_1) and (8.32_1), use of the appropriate instances of the variant of TLAC (4.26) with '**y**' in place '**x**', i.e. of this one

$$\overset{\frown}{\cdot}_{\mathbf{y}}\left[\mathbf{i}\,\hat{\cdot}\,\mathbf{j}\langle\mathbf{y}\rangle\right]\hat{=}\mathbf{i}\,\hat{\cdot}\left[\overset{\frown}{\cdot}_{\mathbf{y}}\mathbf{j}\langle\mathbf{y}\rangle\right] \text{ if } \mathbf{y} \text{ does not occur in } \mathbf{i}, \tag{8.33}$$

has been made at first. Namely, in (8.31_1) the law (8.33) applies with $\left[1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]$ as **i** and with $V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)$ as $\mathbf{j}\langle\mathbf{y}\rangle$, whereas in ($8.32_1$) the law (8.33) applies with $\left[1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]$ as **i** and with $V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)$ as $\mathbf{j}\langle\mathbf{y}\rangle$. The two expressions thus obtained reduce to 0 by making use of the following two laws in sequence: first, (8.1) or its variant with '¬**P**' in place of '**P**' and, second, the instance of the general identity (5.11) with '$\left[\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]$' or '$\left[\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]$' in place of '**i**'.•

Cmt 8.11. The following two simple APLADP's (trains of identities) underlie (8.31_1) and (8.32_1), and hence (8.31) and (8.32), respectively:

$$\begin{aligned} V\left(\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\Rightarrow\mathbf{P}\langle\mathbf{y}\rangle\right) &\hat{=} V\left(\neg\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y}\rangle\right) \\ &\hat{=}\left[1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y}\rangle\right), \end{aligned} \tag{8.33}$$

$$\begin{aligned} V\left(\mathbf{P}\langle\mathbf{y}\rangle\Rightarrow\bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right) &\hat{=} V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{\cdot}V\left(\bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right) \\ &\hat{=} V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{\cdot}\left[1\hat{-}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]. \end{aligned} \tag{8.34}$$

It follows from (8.33) and (8.34) that

$$\begin{gathered} V\left(\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right]\Rightarrow\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{=}0 \text{ if and only if} \\ V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{=}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}0 \text{ (a) or } V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{=}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1 \text{ (b)}, \end{gathered} \tag{8.33$_1$}$$

$$\begin{gathered} V\left(\mathbf{P}\langle\mathbf{y}\rangle\Rightarrow\bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}0 \text{ if and only if} \\ V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{=}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}0 \text{ (a) or } V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{=}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1 \text{ (b)}, \end{gathered} \tag{8.34$_1$}$$

because, under either of the two conditions (a) and (b) in (8.33_1) or (8.34_1), it follows by the instance of (4.24) with '$V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)$' or '$V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)$' in place of '**i**' that

$$\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\overset{\frown}{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{=}V\left(\mathbf{P}\langle\mathbf{y}\rangle\right), \tag{8.33$_2$}$$

$$\dot{\wedge}_{\mathbf{x}} V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \hat{=} \dot{\wedge}_{\mathbf{x}} V(\neg\mathbf{P}\langle\mathbf{y}\rangle) \hat{=} V(\neg\mathbf{P}\langle\mathbf{y}\rangle), \qquad (8.34_2)$$

At the same time, by (4.40a), it follows from (8.33_1) or (8.34_1) that

$$[\vee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle] \Rightarrow \mathbf{P}\langle\mathbf{y}\rangle, \qquad (8.33_3)$$

$$\mathbf{P}\langle\mathbf{y}\rangle \Rightarrow \wedge_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle \qquad (8.34_3)$$

respectively. However, owing to (8.33_2), relation (8.33_3) reduces to the kyrology $\mathbf{P}\langle\mathbf{y}\rangle \Rightarrow \mathbf{P}\langle\mathbf{y}\rangle$ and is therefore trivial. At the same time, (8.34_2) is tantamount to

$$V(\mathbf{P}\langle\mathbf{y}\rangle) \hat{=} 1 \hat{-} \dot{\wedge}_{\mathbf{x}} V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \hat{=} \wedge_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x}\rangle), \qquad (8.34_4)$$

so that (8.34_3) reduces to the trivial kyrology $\mathbf{P}\langle\mathbf{y}\rangle \Rightarrow \mathbf{P}\langle\mathbf{y}\rangle$ as well. Therefore, relations (8.33_1) and (8.34_1) themselves are trivial. However, if $V(\mathbf{P}\langle\mathbf{x}\rangle) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\langle\mathbf{x}\rangle\rangle$ and if, hence, $V(\mathbf{P}\langle\mathbf{y}\rangle) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\langle\mathbf{y}\rangle\rangle$, then identities (8.33) and (8.34) become:

$$V([\vee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle] \Rightarrow \mathbf{P}\langle\mathbf{y}\rangle) \hat{=} [1 \hat{-} \dot{\wedge}_{\mathbf{x}} \mathbf{i}_{\sim}|\mathbf{P}\langle\mathbf{x}\rangle\rangle] \dot{\wedge} \mathbf{i}_{\sim}|\mathbf{P}\langle\mathbf{y}\rangle\rangle, \qquad (8.33a)$$

$$V(\mathbf{P}\langle\mathbf{y}\rangle \Rightarrow \wedge_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle) \hat{=} [1 \hat{-} \mathbf{i}_{\sim}|\mathbf{P}\langle\mathbf{y}\rangle\rangle] \dot{\wedge} [1 \hat{-} \dot{\wedge}_{\mathbf{x}} [1 \hat{-} \mathbf{i}_{\sim}|\mathbf{P}\langle\mathbf{x}\rangle\rangle]]. \qquad (8.34a)$$

That is to say, ‘$[\vee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle] \Rightarrow \mathbf{P}\langle\mathbf{y}\rangle$’ and ‘$\mathbf{P}\langle\mathbf{y}\rangle \Rightarrow \wedge_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle$’ are analytical panlogographic udeterologies (vav-neutral PLR’s), so that each of them can assume euautographic kyrologies, antikyrologies, and udeterologies as its concrete accidental denotata (concrete instances).•

Th 8.13.

$$\begin{aligned} & V(\vee_{\mathbf{x}} [\mathbf{P}\langle\mathbf{x}\rangle \Rightarrow \mathbf{P}\langle\mathbf{y}\rangle]) \\ & \hat{=} \dot{\wedge}_{\mathbf{x}} [V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \dot{\wedge} V(\mathbf{P}\langle\mathbf{y}\rangle)] \hat{=} V(\mathbf{P}\langle\mathbf{y}\rangle) \dot{\wedge} \dot{\wedge}_{\mathbf{x}} V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \\ & \hat{=} [V(\mathbf{P}\langle\mathbf{y}\rangle) \dot{\wedge} V(\neg\mathbf{P}\langle\mathbf{y}\rangle)] \dot{\wedge} \dot{\wedge}_{\mathbf{x}} V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \hat{=} 0 \dot{\wedge} \dot{\wedge}_{\mathbf{x}} V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \hat{=} 0. \end{aligned} \qquad (8.34)$$

$$\begin{aligned} & V(\vee_{\mathbf{y}} [\mathbf{P}\langle\mathbf{x}\rangle \Rightarrow \mathbf{P}\langle\mathbf{y}\rangle]) \\ & \hat{=} \dot{\wedge}_{\mathbf{y}} [V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \dot{\wedge} V(\mathbf{P}\langle\mathbf{y}\rangle)] \hat{=} V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \dot{\wedge} \dot{\wedge}_{\mathbf{y}} V(\mathbf{P}\langle\mathbf{y}\rangle) \\ & \hat{=} [V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \dot{\wedge} V(\mathbf{P}\langle\mathbf{x}\rangle)] \dot{\wedge} \dot{\wedge}_{\mathbf{y}} V(\mathbf{P}\langle\mathbf{y}\rangle) \hat{=} 0 \dot{\wedge} \dot{\wedge}_{\mathbf{y}} V(\mathbf{P}\langle\mathbf{y}\rangle) \hat{=} 0. \end{aligned} \qquad (8.35)$$

$$\begin{aligned} & V([\wedge_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle] \Rightarrow \mathbf{P}\langle\mathbf{y}\rangle) \\ & \hat{=} V(\neg \wedge_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle) \dot{\wedge} V(\mathbf{P}\langle\mathbf{y}\rangle) \hat{=} V(\mathbf{P}\langle\mathbf{y}\rangle) \dot{\wedge} \dot{\wedge}_{\mathbf{x}} V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \\ & \hat{=} [V(\mathbf{P}\langle\mathbf{y}\rangle) \dot{\wedge} V(\neg\mathbf{P}\langle\mathbf{y}\rangle)] \dot{\wedge} \dot{\wedge}_{\mathbf{x}} V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \hat{=} 0 \dot{\wedge} \dot{\wedge}_{\mathbf{x}} V(\neg\mathbf{P}\langle\mathbf{x}\rangle) \hat{=} 0. \end{aligned} \qquad (8.36)$$

$$
\begin{aligned}
&V\left(\mathbf{P}\langle\mathbf{y}\rangle \Rightarrow \vee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle\right) \\
&\hat{=} V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right) \hat{\cdot} V\left(\vee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x}\rangle\right) \hat{=} V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right) \hat{\cdot} \hat{\prod}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right) \\
&\hat{=} \left[V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\right] \hat{\cdot} \hat{\prod}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{=} 0 \hat{\cdot} \hat{\prod}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{=} 0.
\end{aligned}
\tag{8.37}
$$

Proof: In developing (8.34)–(8.37), use of the pertinent instances of the EL, (4.27), has been made.•

Th 8.14: ***Laws of Simplification.***

$$V\left(\vee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right] \Rightarrow \vee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y}\rangle\right) \hat{=} 0\,. \tag{8.38}$$

$$V\left(\vee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right] \Rightarrow \vee_{\mathbf{y}} \mathbf{Q}\langle\mathbf{y}\rangle\right) \hat{=} 0\,. \tag{8.39}$$

$$V\left(\vee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right] \Rightarrow \left[\left[\vee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y}\rangle\right] \wedge \left[\vee_{\mathbf{z}} \mathbf{Q}\langle\mathbf{z}\rangle\right]\right]\right) \hat{=} 0. \tag{8.40}$$

Proof: Identity (8.38) is proved as follows:

$$
\begin{aligned}
&V\left(\vee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right] \Rightarrow \vee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y}\rangle\right) \\
&\hat{=} V\left(\neg\vee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right]\right) \hat{\cdot} V\left(\vee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y}\rangle\right) \\
&\hat{=} \left[1 \hat{-} \hat{\prod}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right)\right] \hat{\cdot} \hat{\prod}_{\mathbf{y}} V\left(\mathbf{P}\langle\mathbf{y}\rangle\right) \\
&\hat{=} \left[1 \hat{-} \hat{\prod}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]\right] \hat{\cdot} \hat{\prod}_{\mathbf{y}} V\left(\mathbf{P}\langle\mathbf{y}\rangle\right) \\
&\hat{=} \hat{\prod}_{\mathbf{y}} V\left(\mathbf{P}\langle\mathbf{y}\rangle\right) \hat{-} \hat{\prod}_{\mathbf{x}}\left[\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\right] \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \\
&\hat{=} \hat{\prod}_{\mathbf{y}} V\left(\mathbf{P}\langle\mathbf{y}\rangle\right) \hat{-} \hat{\prod}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{=} 0,
\end{aligned}
\tag{8.38$_1$}
$$

where use of the appropriate instance of the FL (Fusion Law) (4.29), and also use of identity (8.1) have been made in developing the final result. Identity (8.39) is proved in the same way:

$$
\begin{aligned}
&V\left(\vee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right] \Rightarrow \vee_{\mathbf{y}} \mathbf{Q}\langle\mathbf{y}\rangle\right) \\
&\hat{=} V\left(\neg\vee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right]\right) \hat{\cdot} V\left(\vee_{\mathbf{y}} \mathbf{Q}\langle\mathbf{y}\rangle\right) \\
&\hat{=} \left[1 \hat{-} \hat{\prod}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right)\right] \hat{\cdot} \hat{\prod}_{\mathbf{y}} V\left(\mathbf{Q}\langle\mathbf{y}\rangle\right) \\
&\hat{=} \left[1 \hat{-} \hat{\prod}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]\right] \hat{\cdot} \hat{\prod}_{\mathbf{y}} V\left(\mathbf{Q}\langle\mathbf{y}\rangle\right) \\
&\hat{=} \hat{\prod}_{\mathbf{y}} V\left(\mathbf{Q}\langle\mathbf{y}\rangle\right) \hat{-} \hat{\prod}_{\mathbf{x}}\left[\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\right] \hat{\cdot} V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\right] \\
&\hat{=} \hat{\prod}_{\mathbf{y}} V\left(\mathbf{Q}\langle\mathbf{y}\rangle\right) \hat{-} \hat{\prod}_{\mathbf{x}} V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right) \hat{=} 0.
\end{aligned}
\tag{8.39$_1$}
$$

Identity (8.40) can be rewritten as:

$$
\begin{aligned}
&V\left(\vee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right] \Rightarrow \left[\left[\vee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y}\rangle\right] \wedge \left[\vee_{\mathbf{z}} \mathbf{Q}\langle\mathbf{z}\rangle\right]\right]\right) \\
&\hat{=} V\left(\neg\vee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{Q}\langle\mathbf{x}\rangle\right]\right) \hat{\cdot} V\left(\left[\vee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y}\rangle\right] \wedge \left[\vee_{\mathbf{z}} \mathbf{Q}\langle\mathbf{z}\rangle\right]\right),
\end{aligned}
\tag{8.40$_1$}
$$

where the first multiplier can be developed in the same way as that in (8.38$_1$) or (8.39$_1$), whereas the second multiplier can be developed thus:

$$V\left(\left[\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y}\rangle\right]\wedge\left[\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\right)\hat{=}1\hat{-}V\left(\neg\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{\cdot}V\left(\neg\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right)$$
$$\hat{=}1\hat{-}\left[1\hat{-}\hat{\cdot}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}\hat{\cdot}_{\mathbf{z}}V\left(\mathbf{Q}\langle\mathbf{z}\rangle\right)\right] \qquad (8.40_2)$$
$$\hat{=}\hat{\cdot}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{+}\hat{\cdot}_{\mathbf{z}}V\left(\mathbf{Q}\langle\mathbf{z}\rangle\right)\hat{-}\left[\hat{\cdot}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\right]\hat{\cdot}\left[\hat{\cdot}_{\mathbf{z}}V\left(\mathbf{Q}\langle\mathbf{z}\rangle\right)\right].$$

Hence, (8.40_1) can be developed further thus:

$$V\left(\bigvee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x}\rangle\wedge\mathbf{Q}\langle\mathbf{x}\rangle\right]\Rightarrow\left[\left[\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y}\rangle\right]\wedge\left[\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\right]\right)$$
$$\hat{=}\left[1\hat{-}\hat{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]\right] \qquad (8.40_3)$$
$$\hat{\cdot}\left[\hat{\cdot}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{+}\hat{\cdot}_{\mathbf{z}}V\left(\mathbf{Q}\langle\mathbf{z}\rangle\right)\hat{-}\left[\hat{\cdot}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\right]\hat{\cdot}\left[\hat{\cdot}_{\mathbf{z}}V\left(\mathbf{Q}\langle\mathbf{z}\rangle\right)\right]\right]\hat{=}0,$$

where the final result is obtain by combining (8.38_1) and (8.39_1).•

Th 8.15: ***Law of Separation.***

$$V\left(\bigvee_{\mathbf{y}}\left[\mathbf{P}\wedge\mathbf{Q}\langle\mathbf{x}\rangle\right]\Rightarrow\mathbf{Q}\langle\mathbf{y}\rangle\right)\hat{=}V\left(\left[\mathbf{P}\wedge\mathbf{Q}\langle\mathbf{x}\rangle\right]\Rightarrow\bigvee_{\mathbf{y}}\mathbf{Q}\langle\mathbf{y}\rangle\right)\hat{=}0. \qquad (8.41)$$

Proof: The following argument proves the first identity in (8.41):

$$V\left(\bigvee_{\mathbf{y}}\left[\mathbf{P}\wedge\mathbf{Q}\langle\mathbf{x}\rangle\right]\Rightarrow\mathbf{Q}\langle\mathbf{y}\rangle\right)\hat{=}\hat{\cdot}_{\mathbf{y}}\left[V\left(\neg\left[\mathbf{P}\wedge\mathbf{Q}\langle\mathbf{x}\rangle\right]\right)\hat{\cdot}V\left(\mathbf{Q}\langle\mathbf{y}\rangle\right)\right]$$
$$\hat{=}V\left(\neg\left[\mathbf{P}\wedge\mathbf{Q}\langle\mathbf{x}\rangle\right]\right)\hat{\cdot}\hat{\cdot}_{\mathbf{y}}V\left(\mathbf{Q}\langle\mathbf{y}\rangle\right)\hat{=}V\left(\left[\mathbf{P}\wedge\mathbf{Q}\langle\mathbf{y}\rangle\right]\Rightarrow\bigvee_{\mathbf{y}}\mathbf{Q}\langle\mathbf{y}\rangle\right). \qquad (8.41_1)$$

At the same time,

$$V\left(\neg\left[\mathbf{P}\wedge\mathbf{Q}\langle\mathbf{x}\rangle\right]\right)\hat{\cdot}\hat{\cdot}_{\mathbf{y}}V\left(\mathbf{Q}\langle\mathbf{y}\rangle\right)\hat{=}\left[V\left(\neg\mathbf{P}\right)\hat{\cdot}V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]\hat{\cdot}\hat{\cdot}_{\mathbf{y}}V\left(\mathbf{Q}\langle\mathbf{y}\rangle\right)$$
$$\hat{=}\left[V\left(\neg\mathbf{P}\right)\hat{\cdot}V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\hat{\cdot}V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]\hat{\cdot}\hat{\cdot}_{\mathbf{y}}V\left(\mathbf{Q}\langle\mathbf{y}\rangle\right)\hat{=}0\hat{\cdot}\hat{\cdot}_{\mathbf{y}}V\left(\mathbf{Q}\langle\mathbf{y}\rangle\right)\hat{=}0, \qquad (8.41_2)$$

where use of the appropriate instance of the EL (4.27) has been made. QED.

9. Pseudo-typical logical and algebraic contractors

***Th 9.1.**

$$V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\left[\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right]\right)\hat{=}\hat{\cdot}_{\mathbf{x}}V\left(\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right)$$
$$\hat{=}\hat{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{=}\hat{\cdot}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right). \qquad (9.1)$$

$$V\left(\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\bigvee_{\mathbf{x}}\left[\mathbf{R}\langle\mathbf{x}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x}\rangle\right]\right)$$
$$\hat{=}1\hat{-}\hat{\cdot}_{\mathbf{x}}V\left(\mathbf{R}\langle\mathbf{x}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}\hat{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{\cdot}V\left(\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \qquad (9.2)$$
$$\hat{=}1\hat{-}\hat{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{=}1\hat{-}\hat{\cdot}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right).$$

Proof: The trains of identities (9.1) and (9.2) follow from items 2 and 3 of Df 2.2 by the pertinent instances of (4.31), (4.23), and (7.6γ) in that order, while in writing the very last result in either train, use of the item 1 of Df 2.2 or of the instance of that item with '$\neg\mathbf{P}\langle\mathbf{x}\rangle$' in place of '$\mathbf{P}\langle\mathbf{x}\rangle$', along with the pertinent instance of (4.31), has been made.•

***Th 9.2.**

$$V\left(\neg\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right). \tag{9.3}$$

$$\begin{gathered}V\left(\neg\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\\ \hat{=}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right].\end{gathered} \tag{9.4}$$

$$\begin{gathered}V\left(\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\\ \hat{=}1\hat{-}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right).\end{gathered} \tag{9.5}$$

Proof: By ($7.1\gamma_1$), it follows from (9.2) that

$$\begin{gathered}V\left(\neg\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\\ V\left(\neg\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\neg\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\\ \hat{=}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right),\end{gathered} \tag{9.3$_1$}$$

$$\begin{gathered}V\left(\neg\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\neg\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\\ \hat{=}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right],\end{gathered} \tag{9.4$_1$}$$

$$\begin{gathered}V\left(\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\\ \hat{=}1\hat{-}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\neg\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right).\end{gathered} \tag{9.5$_1$}$$

QED.•

Cmt 9.1. 1) By (7.50), the trains of identities (9.2)–(9.5) are tantamount to the following trains of equivalences:

$$\left\lfloor\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right\rfloor\Leftrightarrow\left\lfloor\neg\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right\rfloor, \tag{9.2'}$$

$$\left\lfloor\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right\rfloor\Leftrightarrow\left\lfloor\neg\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right\rfloor, \tag{9.3'}$$

$$\left\lfloor\neg\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right\rfloor\Leftrightarrow\left\lfloor\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right\rfloor, \tag{9.4'}$$

$$\left\lfloor\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\mathbf{P}\langle\mathbf{x}\rangle\right\rfloor\Leftrightarrow\left\lfloor\neg\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right\rfloor, \tag{9.5'}$$

The identities (9.2) and (9.4) are said to be *dual* of (9.3) and (9.5), while the equivalences (9.2') and (9.4') are said to be *dual* of (9.3') and (9.5'), respectively.

2) Relations (9.2)–(9.5) and (9.2')–(9.5') are variants of (8.2)–(8.5) and (8.2')–(8.5') with ‘$_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}$’ in place of ‘$_{\mathrm{x}}$’.•

***Th 9.3.**

$$V\left(\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{=}V\left(\bigwedge_{\mathbf{x}}\left[\mathbf{R}\langle\mathbf{x}\rangle\Rightarrow\mathbf{P}\langle\mathbf{x}\rangle\right]\right), \tag{9.6}$$

whence, by (7.50),

$$\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle \Leftrightarrow \bigwedge_{\mathbf{x}}\left[\mathbf{R}\langle\mathbf{x}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x}\rangle\right]. \tag{9.6'}$$

Proof: By (7.3γ), the variant of (9.2) with ‘$\left[\mathbf{R}\langle\mathbf{x}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x}\rangle\right]$’ in place of ‘$\mathbf{P}\langle\mathbf{x}\rangle$’ can be developed thus:

$$\begin{aligned} &V\left(\bigwedge_{\mathbf{x}}\left[\mathbf{R}\langle\mathbf{x}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x}\rangle\right]\right) \hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{x}} V\left(\neg\left[\mathbf{R}\langle\mathbf{x}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x}\rangle\right]\right) \\ &\hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\mathbf{R}\langle\mathbf{x}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x}\rangle\right)\right] \hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]. \end{aligned} \tag{9.6$_1$}$$

Comparison of (9.2) and (9.6$_1$) yields (9.6).•

***Th 9.4.**

$$V\left(\left[\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right] \vee \left[\bigvee_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}\neg\mathbf{P}\langle\mathbf{y}\rangle\right]\right) \hat{=} \hat{\cdot}_{\mathbf{x}} V\left(\mathbf{R}\langle\mathbf{x}\rangle\right) \hat{=} V\left(\bigvee_{\mathbf{x}}\mathbf{R}\langle\mathbf{x}\rangle\right), \tag{9.7}$$

$$V\left(\left[\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right] \vee \left[\bigvee_{\mathbf{y}|\neg\mathbf{R}\langle\mathbf{y}\rangle}\mathbf{P}\langle\mathbf{y}\rangle\right]\right) \hat{=} \hat{\cdot}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{=} V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right), \tag{9.8}$$

whence, by (7.50),

$$\left[\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right] \vee \left[\bigvee_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}\neg\mathbf{P}\langle\mathbf{y}\rangle\right] \Leftrightarrow \bigvee_{\mathbf{x}}\mathbf{R}\langle\mathbf{x}\rangle, \tag{9.7'}$$

$$\left[\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right] \vee \left[\bigvee_{\mathbf{y}|\neg\mathbf{R}\langle\mathbf{y}\rangle}\mathbf{P}\langle\mathbf{y}\rangle\right] \Leftrightarrow \bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle. \tag{9.8'}$$

Proof: (9.7) and (9.8) are deduced from the pertinent variants of (7.2γ) by the pertinent variants of (9.1) as follows:

$$\begin{aligned} &V\left(\left[\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right] \vee \left[\bigvee_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}\neg\mathbf{P}\langle\mathbf{y}\rangle\right]\right) \hat{=} V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\bigvee_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}\neg\mathbf{P}\langle\mathbf{y}\rangle\right) \\ &\hat{=} \left[\hat{\cdot}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle} V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\right] \\ &\hat{=} \left[\hat{\cdot}_{\mathbf{x}} V\left(\mathbf{R}\langle\mathbf{x}\rangle \wedge \mathbf{P}\langle\mathbf{x}\rangle\right)\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{y}} V\left(\mathbf{R}\langle\mathbf{y}\rangle \wedge \neg\mathbf{P}\langle\mathbf{y}\rangle\right)\right] \\ &\hat{=} \left[\hat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{y}}\left[1 \hat{-} V\left(\neg\mathbf{R}\langle\mathbf{y}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{y}\rangle\right)\right]\right] \\ &\hat{=} \hat{\cdot}_{\mathbf{x}}\left[\left[1 \hat{-} V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \hat{\cdot} \left[1 \hat{-} V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\right] \\ &\hat{=} \hat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{-} V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right] \\ &\hat{=} \hat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right) \hat{\cdot} \left[V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{+} \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\right] \hat{=} \hat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right) \hat{\cdot} 1\right] \\ &\hat{=} \hat{\cdot}_{\mathbf{x}} V\left(\mathbf{R}\langle\mathbf{x}\rangle\right) \hat{=} V\left(\bigvee_{\mathbf{x}}\mathbf{R}\langle\mathbf{x}\rangle\right), \end{aligned} \tag{9.7$_1$}$$

$$
\begin{aligned}
V\left(\left[\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right]\vee\left[\bigvee_{\mathbf{y}|\neg\mathbf{R}\langle\mathbf{y}\rangle}\mathbf{P}\langle\mathbf{y}\rangle\right]\right) &\hat{=} \left[V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{\cdot}\left[V\left(\bigvee_{\mathbf{y}|\neg\mathbf{R}\langle\mathbf{y}\rangle}\mathbf{P}\langle\mathbf{y}\rangle\right)\right]\\
&\hat{=} \left[\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V(\mathbf{P}\langle\mathbf{x}\rangle)\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}|\neg\mathbf{R}\langle\mathbf{y}\rangle}V(\mathbf{P}\langle\mathbf{y}\rangle)\right]\\
&\hat{=} \left[\hat{\bullet}_{\mathbf{x}}V(\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle)\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}V(\neg\mathbf{R}\langle\mathbf{y}\rangle\wedge\mathbf{P}\langle\mathbf{y}\rangle)\right]\\
&\hat{=} \left[\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\right]\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V(\neg\neg\mathbf{R}\langle\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\right]\right]\\
&\hat{=} \left[\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\right]\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V(\mathbf{R}\langle\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\right]\right]\\
&\hat{=} \hat{\bullet}_{\mathbf{x}}\left[\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\right]\hat{\cdot}\left[1\hat{-}V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\right]\right]\\
&\hat{=} \hat{\bullet}_{\mathbf{x}}\left[\left[1\hat{-}\left[V(\neg\mathbf{R}\langle\mathbf{x}\rangle)\hat{+}V(\mathbf{R}\langle\mathbf{x}\rangle)\right]\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\right]\right]\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\left[1\hat{-}1\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\right]\right]\\
&\hat{=} \hat{\bullet}_{\mathbf{x}}V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right),
\end{aligned}
\qquad (9.8_1)
$$

where use of the pertinent instance of the Fusion Law (4.29), has been made.•

***Th 9.5.**

$$
\begin{aligned}
V\left(\breve{\bigvee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)&\hat{=}V\left(\left[\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right]\wedge\left[\bigvee_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}\neg\mathbf{P}\langle\mathbf{y}\rangle\right]\right)\\
&\hat{=}V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{+}V\left(\bigvee_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\hat{-}V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}\neg\mathbf{P}\langle\mathbf{y}\rangle\right)\\
&\hat{=}\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{+}\hat{\bullet}_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\hat{-}\hat{\bullet}_{\mathbf{x}}V(\mathbf{R}\langle\mathbf{x}\rangle),
\end{aligned}
\qquad (9.9)
$$

$$
\begin{aligned}
V\left(\overset{1}{\widehat{\bigvee}}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)&\hat{=}V\left(\bigwedge_{\mathbf{x}}\bigwedge_{\mathbf{y}}\left[\left[\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right]\wedge\left[\mathbf{R}\langle\mathbf{y}\rangle\wedge\mathbf{P}\langle\mathbf{y}\rangle\right]\Rightarrow[\mathbf{x}=\mathbf{y}]\right]\right)\\
&\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{x}}\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V(\neg[\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle])\hat{\cdot}V(\neg[\mathbf{R}\langle\mathbf{y}\rangle\wedge\mathbf{P}\langle\mathbf{y}\rangle])\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right]\\
&\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{x}}\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{R}\langle\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right].
\end{aligned}
\qquad (9.10)
$$

Proof: The trains of identities (9.9) and (9.10) follow from items 4 and 5 of Df 2.2 respectively by (4.31) and by the pertinent instances of (7.6γ). In addition, use of (9.1) and (9.7) has been made in developing (9.9) and of (8.12) and (7.3γ) in developing (9.10) (cf. the deduction of (8.11) and (8.12)).•

***Th 9.6.**

$$
V\left(\left[\overset{1}{\widehat{\bigvee}}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right]\vee\left[\bigvee_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}\mathbf{P}\langle\mathbf{w}\rangle\right]\right)\hat{=}0, \qquad (9.11)
$$

whence, by (7.50),

$$
\left[\overset{1}{\widehat{\bigvee}}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right]\vee\left[\bigvee_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}\mathbf{P}\langle\mathbf{w}\rangle\right]. \qquad (9.11')
$$

Proof: By (9.1) and (9.10), it follows that

$$
\begin{aligned}
&V\left(\left[\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right]\vee\left[\bigvee_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}\mathbf{P}\langle\mathbf{w}\rangle\right]\right)\\
&\hat{=}V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}\mathbf{P}\langle\mathbf{w}\rangle\right)\hat{=}V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{\cdot}\hat{\bullet}_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}V\left(\mathbf{P}\langle\mathbf{w}\rangle\right)\\
&\hat{=}\left[1\hat{-}\hat{\bullet}_{\mathbf{x}}\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\left[\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right]\right)\hat{\cdot}V\left(\neg\left[\mathbf{R}\langle\mathbf{y}\rangle\wedge\mathbf{P}\langle\mathbf{y}\rangle\right]\right)\hat{\cdot}V\left(\mathbf{x}=\mathbf{y}\right)\right]\right]\\
&\quad\hat{\cdot}\hat{\bullet}_{\mathbf{w}}\left[V\left(\mathbf{R}\langle\mathbf{w}\rangle\wedge\mathbf{P}\langle\mathbf{w}\rangle\right)\right]\hat{=}\hat{\bullet}_{\mathbf{w}}\left[V\left(\mathbf{R}\langle\mathbf{w}\rangle\wedge\mathbf{P}\langle\mathbf{w}\rangle\right)\right]\\
&\hat{-}\hat{\bullet}_{\mathbf{x}}\hat{\bullet}_{\mathbf{y}}\left[\left[1\hat{-}V\left(\neg\left[\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right]\right)\hat{\cdot}V\left(\neg\left[\mathbf{R}\langle\mathbf{y}\rangle\wedge\mathbf{P}\langle\mathbf{y}\rangle\right]\right)\hat{\cdot}V\left(\mathbf{x}=\mathbf{y}\right)\right]\right.\\
&\left.\hat{\cdot}V\left(\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{=}\hat{\bullet}_{\mathbf{w}}\left[V\left(\mathbf{R}\langle\mathbf{w}\rangle\wedge\mathbf{P}\langle\mathbf{w}\rangle\right)\right]\hat{-}\hat{\bullet}_{\mathbf{x}}\hat{\bullet}_{\mathbf{y}}V\left(\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right)\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}V\left(\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{-}\hat{\bullet}_{\mathbf{x}}V\left(\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}0,
\end{aligned}
\tag{9.11$_1$}
$$

where in developing the final result use of the pertinent instance of the Fusion Law (4.29) and also use of the identity

$$
\left[1\hat{-}V\left(\neg\left[\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right]\right)\right]\hat{\cdot}V\left(\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}0 \tag{9.11$_2$}
$$

have been made (cf. the proof of Th 8.7).●

***Th 9.7.**

$$
\begin{aligned}
&V\left(\bigvee{}^{1}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle}\mathbf{P}\langle\mathbf{v}\rangle\right)\hat{=}V\left(\left[\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right]\wedge\left[\bigvee_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}\mathbf{P}\langle\mathbf{w}\rangle\right]\right)\\
&\hat{=}V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{+}V\left(\bigvee_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}\mathbf{P}\langle\mathbf{w}\rangle\right)\hat{=}V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{+}\hat{\bullet}_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}V\left(\mathbf{P}\langle\mathbf{w}\rangle\right)
\end{aligned}
\tag{9.12}
$$

subject to (9.10).

Proof: (9.12) follows from Df 2.2(6) by the pertinent instances of (4.31), (7.6γ), and (9.11) thus:

$$
\begin{aligned}
&V\left(\bigvee{}^{1}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle}\mathbf{P}\langle\mathbf{v}\rangle\right)\hat{=}V\left(\left[\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right]\wedge\left[\bigvee_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}\mathbf{P}\langle\mathbf{w}\rangle\right]\right)\\
&\hat{=}V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{+}V\left(\bigvee_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}\mathbf{P}\langle\mathbf{w}\rangle\right)\\
&-V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}\mathbf{P}\langle\mathbf{w}\rangle\right)\\
&\hat{=}V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{+}\hat{\bullet}_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}V\left(\mathbf{P}\langle\mathbf{w}\rangle\right).
\end{aligned}
\tag{9.12$_1$}
$$

●

Cmt 9.2. Comparison of the above theorems of this subsection and of the respective theorems of subsection 8.1 shows that, in the exclusion of (9.7) and (9.8), the former are variants of the latter wih ‘ $_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}$ ’, ‘ $_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}$ ’, ‘ $_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}$ ’, and ‘ $_{\mathbf{w}|\mathbf{R}\langle\mathbf{w}\rangle}$ ’ in place of ‘$_{\mathrm{x}}$’, ‘$_{\mathrm{y}}$’, ‘$_{\mathrm{z}}$’, and ‘$_{\mathrm{w}}$’ respectively.●

Crl 9.1: ***The pseudo-typical GLDRP.*** The EDT (8.23), i.e. the pseudo-typical WGLDRP in the objective form, applies with ‘ $_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}$ ’ in place of ‘$_{\mathrm{x}}$’, namely:

$$V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)$$
$$\hat{=}V\left(\neg\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\left[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle\right]\right)\hat{=}0. \qquad (9.13)$$

All other versions of the GLDRP, which have been deduced from (9.26) in subsection 9.2, and also all identities, which have been deduced from in subsection 9.3, apply with ‘ $_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}$ ’ and ‘ $_{\mathbf{y}|\mathbf{R}\langle\mathbf{y}\rangle}$ ’ in place of ‘$_{\mathrm{x}}$’ and ‘$_{\mathrm{y}}$’ respectively.

Proof: The corollary follows from Cmt 9.2.•

***Th 9.8.**

$$\hat{\cdot}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\hat{\cdot}_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right). \qquad (9.14)$$

$$V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle\right). \qquad (9.15)$$

$$V\left(\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigwedge_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle\right). \qquad (9.16)$$

$$V\left(\breve{\bigvee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{=}V\left(\breve{\bigvee}_{\mathbf{z}|V(\mathbf{R}\langle\mathbf{z}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{z}\rangle\right). \qquad (9.17)$$

$$V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{=}V\left(\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|V(\mathbf{R}\langle\mathbf{z}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{z}\rangle\right). \qquad (9.18)$$

$$V\left(\bigvee^{1}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle}\mathbf{P}\langle\mathbf{v}\rangle\right)\hat{=}V\left(\bigvee^{1}_{\mathbf{v}|V(\mathbf{R}\langle\mathbf{v}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{v}\rangle\right). \qquad (9.19)$$

Hence, by (7.50):

$$\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\Leftrightarrow\bigvee_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle. \qquad (9.15')$$

$$\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\Leftrightarrow\bigwedge_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle. \qquad (9.16')$$

$$\breve{\bigvee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\Leftrightarrow\breve{\bigvee}_{\mathbf{z}|V(\mathbf{R}\langle\mathbf{z}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{z}\rangle. \qquad (9.17')$$

$$\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\Leftrightarrow\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}|V(\mathbf{R}\langle\mathbf{z}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{z}\rangle. \qquad (9.18')$$

$$\bigvee^{1}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle}\mathbf{P}\langle\mathbf{v}\rangle\Leftrightarrow\bigvee^{1}_{\mathbf{v}|V(\mathbf{R}\langle\mathbf{v}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{v}\rangle. \qquad (9.19')$$

Proof: In accordance with (9.1), (9.2), (9.9), (9.10), and (9.12), the ultimate irreducible expressions for the validity-integrons on the left-hand sides of the identities of (9.13)–(9.18) involve ‘**R**’ exclusively in the form of ‘$V(\mathbf{R})$’. At the same time,

$$V\left(V(\mathbf{R})\hat{=}0\right)\hat{=}V(\mathbf{R}), \qquad (9.20)$$

by (6.19). QED.•

***Th 9.9.**

$$V\left(\neg\left[V(\mathbf{R})\hat{=}0\right]\right)\hat{=}V(\neg\mathbf{R})\hat{=}1\hat{-}V(\mathbf{R})\hat{=}V\left(V(\mathbf{R})\hat{=}1\right), \qquad (9.21)$$

whence, by (7.50),

$$\neg[V(\mathbf{R})\hat{=}0]\Leftrightarrow\neg\mathbf{R}\Leftrightarrow[V(\mathbf{R})\hat{=}1]. \tag{9.21'}$$

Proof: From the variant of (7.1γ) with '$V(\mathbf{R})\hat{=}0$' in place of '**P**', it follows that

$$V(\neg[V(\mathbf{R})\hat{=}0])\hat{=}1\hat{-}V(V(\mathbf{R})\hat{=}0)\hat{=}1\hat{-}V(\mathbf{R})\hat{=}V(\neg\mathbf{R}). \tag{9.21$_1$}$$

At the same time, the variant of (9.20) with '¬**R**' in place of '**R**' can be developed thus:

$$V(\neg\mathbf{R})\hat{=}V(V(\neg\mathbf{R})\hat{=}0)\hat{=}V(1\hat{-}V(\mathbf{R})\hat{=}0)\hat{=}V(V(\mathbf{R})\hat{=}1). \tag{9.21$_2$}$$

Combination of (9.21$_1$) and (9.21$_2$) yields (9.21).•

Cmt 9.3. R, which occurs in the subscript of any contractor of the following six kinds:

$$\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle},\ \bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle},\ \bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle},\ \breve{\bigvee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle},\ \overset{\frown 1}{\bigvee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle},\ \overset{1}{\bigvee}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle} \tag{9.22}$$

or, in accordance with Th 9.8, in the respective concurrent contractor of the following six kinds:

$$\hat{\bullet}_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0},\ \bigvee_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0},\ \bigwedge_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0},\ \breve{\bigvee}_{\mathbf{z}|V(\mathbf{R}\langle\mathbf{z}\rangle)\hat{=}0},\ \overset{\frown 1}{\bigvee}_{\mathbf{z}|V(\mathbf{R}\langle\mathbf{z}\rangle)\hat{=}0},\ \overset{1}{\bigvee}_{\mathbf{v}|V(\mathbf{R}\langle\mathbf{v}\rangle)\hat{=}0}, \tag{9.23}$$

is a certain (concrete but not concretized) relation of A_1, which can be a kyrology or an antikyrology, or else a vav-udeterology, *and not a semantic condition on the APVOT* (ordinary atomic pseudovariable term) **x**, **z**, or **v**, occurring at a certain fixed place or places, and hence in a certain fixed symbolic surrounding, in **R**. An APVOT is a euautograph of A_1 and hence it is incapable of assuming any denotata. That is to say, the relation-subscript $V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0$, $V(\mathbf{R}\langle\mathbf{z}\rangle)\hat{=}0$, or $V(\mathbf{R}\langle\mathbf{v}\rangle)\hat{=}0$ of a pseudo-typical euautographic contractor on the list (9.23) is neither a condition on the respective APVOT **x**, **z**, or **v** indicated thereby nor an instruction for making the pertinent substitution: $0\mapsto V(\mathbf{R}\langle\mathbf{x}\rangle)$, $0\mapsto V(\mathbf{R}\langle\mathbf{y}\rangle)$, or $0\mapsto V(\mathbf{R}\langle\mathbf{z}\rangle)$ in the train of identities obtained by developing the respective one of the validity-integrons

$$\hat{\bullet}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V(\mathbf{P}\langle\mathbf{x}\rangle),\ V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right),\ V\left(\bigwedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right),\ V\left(\overset{\frown 1}{\bigvee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle\right),\ V\left(\overset{1}{\bigvee}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle}\mathbf{P}\langle\mathbf{v}\rangle\right) \tag{9.24}$$

Use of the pseudo-typical contractors just allows presenting the appropriate ER's formulas or their PLS'ta in an alternative way. Here follow some examples illustrating the above-said.

1) It follows from the variants of (9.1) and (9.14) with '**P**' and '**R**' exchanged and from (9.1) and (9.14) themselves that

$$\begin{gathered}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}0}V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|\mathbf{P}\langle\mathbf{x}\rangle}V(\mathbf{R}\langle\mathbf{x}\rangle)\\ \hat{=}V\left(\bigvee\nolimits_{\mathbf{x}|\mathbf{P}\langle\mathbf{x}\rangle}\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee\nolimits_{\mathbf{x}}[\mathbf{P}\langle\mathbf{x}\rangle\wedge\mathbf{R}\langle\mathbf{x}\rangle]\right)\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V(\mathbf{P}\langle\mathbf{x}\rangle\wedge\mathbf{R}\langle\mathbf{x}\rangle)\\ \hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{R}\langle\mathbf{x}\rangle)]\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)]\\ \hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V(\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}V\left(\bigvee\nolimits_{\mathbf{x}}[\mathbf{R}\langle\mathbf{x}\rangle\wedge\mathbf{P}\langle\mathbf{x}\rangle]\right)\hat{=}V\left(\bigvee\nolimits_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle\right)\\ \hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}V(\mathbf{P}\langle\mathbf{x}\rangle).\end{gathered}\tag{9.25}$$

2) (9.1) is a train of identities (valid equalities). Therefore, the variant of (9.1) with '$\mathbf{P}\langle\mathbf{x}\rangle\wedge\mathbf{Q}\langle\mathbf{x}\rangle$', e.g., in place of '$\mathbf{P}\langle\mathbf{x}\rangle$' is also valid. Omitting '$\langle\mathbf{x}\rangle$' for the sake of brevity, the above variant can be developed recursively thus:

$$\begin{gathered}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|\mathbf{R}}V(\mathbf{P}\wedge\mathbf{Q})\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|V(\mathbf{R})\hat{=}0}V(\mathbf{P}\wedge\mathbf{Q})\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V(\mathbf{R}\wedge[\mathbf{P}\wedge\mathbf{Q}])\\ \hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}[1\hat{-}V(\neg\mathbf{R})\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})],\end{gathered}\tag{9.26}$$

because it follows by the pertinent variants of (7.6γ) that

$$\begin{gathered}V(\mathbf{R}\wedge[\mathbf{P}\wedge\mathbf{Q}])\hat{=}1\hat{-}V(\neg\mathbf{R})\hat{\cdot}V(\neg[\mathbf{P}\wedge\mathbf{Q}])\\ \hat{=}1\hat{-}V(\neg\mathbf{R})\hat{\cdot}[V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})]\hat{=}1\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{R}).\end{gathered}\tag{9.26$_1$}$$

The fact that final expression in (9.26) is invariant under any permutation of '**P**', '**Q**', and '**R**' means that

$$\begin{gathered}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|\mathbf{R}}V(\mathbf{P}\wedge\mathbf{Q})\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|\mathbf{P}}V(\mathbf{Q}\wedge\mathbf{R})\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|\mathbf{Q}}V(\mathbf{P}\wedge\mathbf{R})\\ \hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|\mathbf{P}\wedge\mathbf{Q}}V(\mathbf{R})\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|\mathbf{P}\wedge\mathbf{R}}V(\mathbf{Q})\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}|\mathbf{Q}\wedge\mathbf{R}}V(\mathbf{P}),\end{gathered}\tag{9.27}$$

whence, by (9.1) and (7.50),

$$\begin{gathered}\bigvee\nolimits_{\mathbf{x}|\mathbf{R}}[\mathbf{P}\wedge\mathbf{Q}]\Leftrightarrow\bigvee\nolimits_{\mathbf{x}|\mathbf{P}}[\mathbf{Q}\wedge\mathbf{R}]\Leftrightarrow\bigvee\nolimits_{\mathbf{x}|\mathbf{Q}}[\mathbf{P}\wedge\mathbf{R}]\\ \Leftrightarrow\bigvee\nolimits_{\mathbf{x}|\mathbf{P}\wedge\mathbf{Q}}\mathbf{R}\Leftrightarrow\bigvee\nolimits_{\mathbf{x}|\mathbf{P}\wedge\mathbf{R}}\mathbf{Q}\Leftrightarrow\bigvee\nolimits_{\mathbf{x}|\mathbf{Q}\wedge\mathbf{R}}\mathbf{P}.\end{gathered}\tag{9.27'}$$

The trains (9.27) and (9.27') can be generalized further to the case of any number of conjuncts.

3) By (9.14) and (9.15), it follows from (9.1) and (9.2) that

$$V\left(\bigvee\nolimits_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x}\rangle)],\tag{9.28}$$

$$V\left(\bigwedge\nolimits_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{x}\rangle)],\tag{9.29}$$

which can be specified in two ways as follows.

a) If $\vdash\left[V\left(\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{=}0\right]$, i.e. if $\mathbf{R}\langle\mathbf{x}\rangle$ is a kyrology, then (9.28) and (9.29) become

$$V\left(\bigvee_{\mathbf{x}|0\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{x}}\left[1\,\hat{-}\,1\,\hat{\cdot}\,V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{=}\hat{\bullet}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right), \qquad (9.28_1)$$

$$V\left(\bigwedge_{\mathbf{x}|0\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\,\hat{-}\,\hat{\bullet}_{\mathbf{x}}\left[1\,\hat{-}\,1\,\hat{\cdot}\,V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{=}1\,\hat{-}\,\hat{\bullet}_{\mathbf{x}}V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\bigwedge_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\right), \qquad (9.29_1)$$

where the final results follow from (4.23) and (8.2) respectively.

b) If $\vdash\left[V\left(\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{=}1\right]$, i.e. if $\mathbf{R}\langle\mathbf{x}\rangle$ is an antikyrogy, then (9.28) and (9.29) become

$$V\left(\bigvee_{\mathbf{x}|1\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{x}}\left[1\,\hat{-}\,0\,\hat{\cdot}\,V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{=}\hat{\bullet}_{\mathbf{x}}1\hat{=}1, \qquad (9.28_2)$$

$$V\left(\bigwedge_{\mathbf{x}|1\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\,\hat{-}\,\hat{\bullet}_{\mathbf{x}}\left[1\,\hat{-}\,0\,\hat{\cdot}\,V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right]\hat{=}1\,\hat{-}\,\hat{\bullet}_{\mathbf{x}}1\hat{=}0\,. \qquad (9.29_2)$$

c) If $\vdash\left[V\left(\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{=}\mathbf{i}_{\sim}\left|\mathbf{R}\langle\mathbf{x}\rangle\right\rangle\right]$, i.e. if $\mathbf{R}\langle\mathbf{x}\rangle$ is an udeterology, then (9.28) and (9.29) become

$$V\left(\bigvee_{\mathbf{x}|\mathbf{i}_{\sim}|\mathbf{R}\langle\mathbf{x}\rangle\rangle\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{x}}\left[1\,\hat{-}\left[1\,\hat{-}\,\mathbf{i}_{\sim}\left|\mathbf{R}\langle\mathbf{x}\rangle\right\rangle\right]\hat{\cdot}\,V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right)\right], \qquad (9.28_3)$$

$$V\left(\bigwedge_{\mathbf{x}|\mathbf{i}_{\sim}|\mathbf{R}\langle\mathbf{x}\rangle\rangle\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle\right)\hat{=}1\,\hat{-}\,\hat{\bullet}_{\mathbf{x}}\left[1\,\hat{-}\left[1\,\hat{-}\,\mathbf{i}_{\sim}\left|\mathbf{R}\langle\mathbf{x}\rangle\right\rangle\right]\hat{\cdot}\,V\left(\mathbf{P}\langle\mathbf{x}\rangle\right)\right], \qquad (9.29_3)$$

because

$$V\left(\neg\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{=}\mathbf{i}_{\sim}\left|\neg\mathbf{R}\langle\mathbf{x}\rangle\right\rangle\hat{=}1\,\hat{-}\,\mathbf{i}_{\sim}\left|\mathbf{R}\langle\mathbf{x}\rangle\right\rangle. \qquad (9.29_4)$$

It is noteworthy, that any substitution $0\mapsto V\left(\mathbf{R}\langle\mathbf{x}\rangle\right)$ or $1\mapsto V\left(\mathbf{R}\langle\mathbf{x}\rangle\right)$ or $\mathbf{i}_{\sim}\left|\mathbf{R}\langle\mathbf{x}\rangle\right\rangle\mapsto V\left(\mathbf{R}\langle\mathbf{x}\rangle\right)$ that is made in (9.28) and (9.29) for obtaining (9.28_1)–(9.28_3) and (9.29_1)–(9.29_3) is relevant to the respective extrinsic assumption stated in the metalanguage, and that it is irrelevant to the relation $V\left(\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{=}0$ occurring in the subscript of the pertinent pseudo-typical contractor $\bigvee_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}$ and $\bigwedge_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}$. Also, any of the above three substitutions does not affect the operatum $\mathbf{P}\langle\mathbf{x}\rangle$. That is to say, the condition that is imposed by a pseudo-typical euautographic operator on its operatum is a pure syntactic *conjoined condition*, and not a semantic condition on denotata (denotation values) of any term-variables. Accordingly, if $V\left(\mathbf{R}\langle\mathbf{x}\rangle\right)\hat{=}\mathbf{i}_{\sim}\left|\mathbf{R}\langle\mathbf{x}\rangle\right\rangle$ then the identities (9.28) and (9.29) are not simplified.•

Cmt 9.4. 1) Cmt 8.6 applies, *mutatis mutandis*, with

$$`{}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\text{'},\ `{}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\text{'},\ `{}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle}\text{'},\ `{}_{x|\boldsymbol{R}\langle x\rangle}\text{'},\ `{}_{z|\boldsymbol{R}\langle z\rangle}\text{'},\ `{}_{v|\boldsymbol{R}\langle v\rangle}\text{'} \qquad (9.30)$$

in place of the respective subscripts

$$`\mathbf{x}\text{'}, `\mathbf{z}\text{'}, `\mathbf{v}\text{'}, `x\text{'}, `z\text{'}, `v\text{'} \quad (9.31)$$

in the contractor-signs. Also, by Th 9.8, the subscripts (9.30) can be used interchangeably with the respective subscripts

$$`\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0\text{'}, `\mathbf{z}|V(\mathbf{R}\langle\mathbf{z}\rangle)\hat{=}0\text{'}, `\mathbf{v}|V(\mathbf{R}\langle\mathbf{v}\rangle)\hat{=}0\text{'}, `x|V(\boldsymbol{R}\langle x\rangle)\hat{=}0\text{'}, `z|V(\boldsymbol{R}\langle z\rangle)\hat{=}0\text{'}, `v|V(\boldsymbol{R}\langle v\rangle)\hat{=}0\text{'}. \quad (9.32)$$

2) Consequently,

$$\vee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle,\ \wedge_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle,\ \breve{\vee}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle,\ \hat{\vee}^{1}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle}\mathbf{P}\langle\mathbf{z}\rangle,\ \vee^{1}_{\mathbf{v}|\mathbf{R}\langle\mathbf{v}\rangle}\mathbf{P}\langle\mathbf{v}\rangle \quad (9.33)$$

are *common CdESR's*, in analogy with (8.15);

$$\vee_{x|\mathbf{R}\langle x\rangle}\mathbf{P}\langle x\rangle,\ \wedge_{x|\mathbf{R}\langle x\rangle}\mathbf{P}\langle x\rangle,\ \breve{\vee}_{z|\mathbf{R}\langle z\rangle}\mathbf{P}\langle z\rangle,\ \hat{\vee}^{1}_{z|\mathbf{R}\langle z\rangle}\mathbf{P}\langle z\rangle,\ \vee^{1}_{v|\mathbf{R}\langle v\rangle}\mathbf{P}\langle v\rangle \quad (9.33\iota)$$

are *specified* (*restricted*, *semi-concretized*) *CdESR's*, in analogy with (8.15ι);

$$\vee_{x|\boldsymbol{R}\langle x\rangle}\boldsymbol{P}\langle x\rangle,\ \wedge_{x|\boldsymbol{R}\langle x\rangle}\boldsymbol{P}\langle x\rangle,\ \breve{\vee}_{z|\boldsymbol{R}\langle z\rangle}\boldsymbol{P}\langle z\rangle,\ \hat{\vee}^{1}_{z|\boldsymbol{R}\langle z\rangle}\boldsymbol{P}\langle z\rangle,\ \vee^{1}_{v|\boldsymbol{R}\langle v\rangle}\boldsymbol{P}\langle v\rangle \quad (9.37\mu)$$

are [*as if*] *concrete CdESR's*, in analogy with (8.15μ);

$$\vee_{x|\boldsymbol{R}\langle x\rangle}\boldsymbol{P}\langle x\rangle,\ \wedge_{x|\boldsymbol{R}\langle x\rangle}\boldsymbol{P}\langle x\rangle,\ \breve{\vee}_{z|\boldsymbol{R}\langle z\rangle}\boldsymbol{P}\langle z\rangle,\ \hat{\vee}^{1}_{z|\boldsymbol{R}\langle z\rangle}\boldsymbol{P}\langle z\rangle,\ \vee^{1}_{v|\boldsymbol{R}\langle v\rangle}\boldsymbol{P}\langle v\rangle \quad (9.37\kappa)$$

are the [*as if*] *CFCL interpretands of the* [*as if*] *CdESR's* (8.15μ), in analogy with (8.15κ). These interpretands are *significant* (*interpreted*) *vav-neutral contracted CFCL relations* (*CdCFCLR's*) and therefore, together with their definientia, they can be supplemented by the appropriate *wordy* (*verbal*) *denotative definienda*, which are at the same time *connotative definientia* that explicate the meanings of the CdCFCLR's.

3) The *common* (*general*) *pseudo-typically contracted euautographic validity-integron* (*CdEVI*) $\hat{\cdot}_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}V(\mathbf{P}\langle\mathbf{x}\rangle)$ along with its definiens can be specified (restricted, semi-concretized) likewise as $\hat{\cdot}_{x|\mathbf{R}\langle x\rangle}V(\mathbf{P}\langle x\rangle)$, which can, in turn, be [as if] concretized as $\hat{\cdot}_{x|\boldsymbol{R}\langle x\rangle}V(\boldsymbol{P}\langle x\rangle)$, while $\hat{\cdot}_{x|\boldsymbol{R}\langle x\rangle}V(\boldsymbol{P}\langle x\rangle)$ is the [as if] CFCL interpretand of the [as if] concrete CdEVI $\hat{\cdot}_{x|\boldsymbol{R}\langle x\rangle}V(\boldsymbol{P}\langle x\rangle)$.

4) It has been pointed out in Cmt 9.3 that, for instance, the additional relation-subscript '$\mathbf{R}\langle\mathbf{x}\rangle$' in the expression '$\vee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}\mathbf{P}\langle\mathbf{x}\rangle$' or '$V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0$' in the equivalent expression '$\vee_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}\mathbf{P}\langle\mathbf{x}\rangle$' is neither a condition on the APVOT **x** nor an instruction for making the substitution $0\mapsto V(\mathbf{R}\langle\mathbf{x}\rangle)$ in the ER $\mathbf{P}\langle\mathbf{x}\rangle$, being the operatum of either

one of the two concurrent pseudo-typical contractors '$\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x}\rangle}$' and '$\bigvee_{\mathbf{x}|V(\mathbf{R}\langle\mathbf{x}\rangle)\hat{=}0}$'. A like remark applies, e.g., with x as $\mathbf{x}$. The CFCL interpretand of a DdER (decided ER) preserves the validity-value of the latter. For instance, if the [as if] concrete ER $\boldsymbol{R}\langle x\rangle$ is valid or antivalid or vav-neutral then $\boldsymbol{R}\langle x\rangle$, being its [as if] CFCL interpretand, is also valid or antivalid or vav-neutral respectively. If the ER $\boldsymbol{R}\langle x\rangle$ is valid or antivalid, i.e. if it is a tautology or an anitautology (contradiction), then the equality $V(\boldsymbol{R}\langle x\rangle)\hat{=}0$ is also a tautology or an anitautology (contradiction), respectively, and vice versa. In either case, the above equality cannot serve as a condition on x. If, however, the ER $\boldsymbol{R}\langle x\rangle$ is vav-neutral, i.e. ttatt-neutral (neither tautologous nor antitautologous), then the equality $V(\boldsymbol{R}\langle x\rangle)\hat{=}0$ can be *veracious* (*accidentally true*) for some x, *antiveracious* (*accidentally antitrue*) for some other x, and it can be *vravr-neutral* (*neither veracious nor antiveracious*) for the rest of x. That is to say, in this case, the statement that the equality $V(\boldsymbol{R}\langle x\rangle)\hat{=}0$ *is veracious*, i.e. that $\models[V(\boldsymbol{R}\langle x\rangle)\hat{=}0]$ by Df 3.9, is a condition on x. Consequently, any contractor that has the subscript '$_{x|\boldsymbol{R}\langle x\rangle}$' or '$_{x|V(\boldsymbol{R}\langle x\rangle)\hat{=}0}$' or its variant with any AVCLOT as 'y', 'z', 'u', 'v', or 'w' in place of 'x' is a meaningful *typical contractor*.

4) Here follows [*as if*] CFCL interpretands of the items 1–6 of Df 2.2, which are augmented by the appropriate wordy definienda, which render the pertinent euautographic contractors into ordinary language when they apply to *vav-neutral* (*ttatt-neutral*) *typically contrated CFCLR's*.•

Df 9.1.

1) $\left[\hat{\cdot}_{x|V(\boldsymbol{R}\langle x\rangle)\hat{=}0}V(\boldsymbol{P}\langle x\rangle)\right]\hat{=}\left[\hat{\cdot}_{x|\boldsymbol{R}\langle x\rangle}V(\boldsymbol{P}\langle x\rangle)\right]\to\left[\hat{\cdot}_{x}V(\boldsymbol{R}\langle x\rangle\wedge\boldsymbol{P}\langle x\rangle)\right]$

←[the contraction over x such that $\boldsymbol{R}\langle x\rangle$, of $V(\boldsymbol{P}\langle x\rangle)$]

←[the product over x such that $\boldsymbol{R}\langle x\rangle$, of $V(\boldsymbol{P}\langle x\rangle)$].

2) $\left[\bigvee_{x|V(\boldsymbol{R}\langle x\rangle)\hat{=}0}\boldsymbol{P}\langle x\rangle\right]\Leftrightarrow\left[\bigvee_{x|\boldsymbol{R}\langle x\rangle}\boldsymbol{P}\langle x\rangle\right]\to\left[\bigvee_{x}\left[\boldsymbol{R}\langle x\rangle\wedge\boldsymbol{P}\langle x\rangle\right]\right]$

←[for at least one x such that $\boldsymbol{R}\langle x\rangle$: $\boldsymbol{P}\langle x\rangle$]

←[for some x such that $\boldsymbol{R}\langle x\rangle$: $\boldsymbol{P}\langle x\rangle$].

3) $\left\lfloor \bigwedge_{x|V(\boldsymbol{R}\langle x\rangle)\hat{=}0} \boldsymbol{P}\langle x\rangle \right\rfloor \Leftrightarrow \left\lfloor \bigwedge_{x\{\boldsymbol{R}\langle x\rangle} \boldsymbol{P}\langle x\rangle \right\rfloor \rightarrow \left\lfloor \neg \bigvee_{x|\boldsymbol{R}\langle x\rangle} \neg \boldsymbol{P}\langle x\rangle \right\rfloor$

$\rightarrow \left\lfloor \neg \bigvee_x \left[\boldsymbol{R}\langle x\rangle \wedge \neg \boldsymbol{P}\langle x\rangle\right]\right\rfloor$

$\leftarrow$[for all x such that $\boldsymbol{R}\langle x\rangle$: $\boldsymbol{P}\langle x\rangle$]

$\leftarrow$[for every x such that $\boldsymbol{R}\langle x\rangle$: $\boldsymbol{P}\langle x\rangle$].

4) $\left\lfloor \breve{\bigvee}_{z|V(\boldsymbol{R}\langle z\rangle)\hat{=}0} \boldsymbol{P}\langle z\rangle \right\rfloor \Leftrightarrow \left\lfloor \breve{\bigvee}_{z|\boldsymbol{R}\langle z\rangle} \boldsymbol{P}\langle z\rangle \right\rfloor \rightarrow \left\lfloor\!\left\lfloor \bigvee_{x|\boldsymbol{R}\langle x\rangle} \boldsymbol{P}\langle x\rangle \right\rfloor \wedge \left\lfloor \bigvee_{y|\boldsymbol{R}\langle y\rangle} \neg \boldsymbol{P}\langle y\rangle \right\rfloor\!\right\rfloor$

$\leftarrow$[for strictly some z such that $\boldsymbol{R}\langle z\rangle$: $\boldsymbol{P}\langle z\rangle$].s

5) $\left\lfloor \overset{\frown}{\bigvee}{}^{1}_{z|V(\boldsymbol{R}\langle z\rangle)\hat{=}0} \boldsymbol{P}\langle z\rangle \right\rfloor \Leftrightarrow \left\lfloor \overset{\frown}{\bigvee}{}^{1}_{z|\boldsymbol{R}\langle z\rangle} \boldsymbol{P}\langle z\rangle \right\rfloor \rightarrow \left\lfloor \overset{\frown}{\bigvee}{}^{1}_{z} \left[\boldsymbol{R}\langle z\rangle \wedge \boldsymbol{P}\langle z\rangle\right]\right\rfloor$

$\leftrightarrow \left\lfloor \bigwedge_x \bigwedge_y \left[\left[\boldsymbol{R}\langle x\rangle \wedge \boldsymbol{P}\langle x\rangle\right] \wedge \left[\boldsymbol{R}\langle y\rangle \wedge \boldsymbol{P}\langle y\rangle\right] \Rightarrow [x = y]\right]\right\rfloor$

$\leftarrow$[for at most one z such that $\boldsymbol{R}\langle z\rangle$: $\boldsymbol{P}\langle z\rangle$].

6) $\left\lfloor \bigvee^{1}_{v|V(\boldsymbol{R}\langle v\rangle)\hat{=}0} \boldsymbol{P}\langle v\rangle \right\rfloor \Leftrightarrow \left\lfloor \bigvee^{1}_{v|\boldsymbol{R}\langle v\rangle} \boldsymbol{P}\langle v\rangle \right\rfloor \rightarrow \left\lfloor\!\left\lfloor \overset{\frown}{\bigvee}{}^{1}_{z|\boldsymbol{R}\langle z\rangle} \boldsymbol{P}\langle z\rangle \right\rfloor \wedge \left\lfloor \bigvee_{w|\boldsymbol{R}\langle w\rangle} \boldsymbol{P}\langle w\rangle \right\rfloor\!\right\rfloor$

$\leftarrow$[for exactly one v such that $\boldsymbol{R}\langle v\rangle$: $\boldsymbol{P}\langle v\rangle$].•

Chapter III. The organon A_0: selected valid predicate-free panlogographic ordinary relations (PLOR's) of A_1

1. Introduction to basic panlogographic algebraic decision procedures (BPLADP's) of A_1

1.1. Preliminaries

1.1.1. Classifying definitions

For convenience in description and study, I shall group the PLOR's (panlogographic ordinary relations) of $\mathbf{A}_1$ and EOR's (euautogographic ordinary relations) of A_1, which I deal with in this chapter, in accordance with the following two definitions. In the first of them, I recall and summarize some pertinent terms that have been introduced in different places earlier. The second definition is a new one, in which I introduce a certain classifying characteristic of complexity of a PLOR – its rank.

Df 1.1. 1) The operator (kernel-sign) of a combined PLR (panlogographic relation) that is executed last is called the *principal operator of the PLR*, the understanding being that if the PLR has a single operator then the latter is the principal operator of the PLR. Hence, a CbPLR is *the scope*, or *operand*, *of its principal operator*.

2) The above definition applies with "ER" ("euautographic relation") in place of "PLR" ("panlogographic relation"). Particularly, it applied to any ER in the range of the PLR, the understanding being that if the principal operator (kernel-sign) of a PLR is a euautographic one then every ER in the range of the PLR has the same principal operator.

3) A combined PLR or ER is called a formal *former antidisjunction*, *negation*, *inclusive disjunction*, *rightward implication*, *leftward implication*, *former anticonjunction* (or *quominus*), *conjunction*, *equivalence* (or *biimplication*), *latter antidisjunction*, *latter anticonjunction*, *rightward antiimplication*, *leftward antiimplication*, *exclusive disjunction* (and also *antibiimplication*, *antibihypothetical*, or *antiequivalence*) if its principal kernel-sign is the respective one of the following list (in that order):

$$\not\vee,\ \neg,\ \vee,\ \Rightarrow,\ \Leftarrow,\ \not\wedge,\ \wedge,\ \Leftrightarrow,\ \overline{\vee},\ \overline{\wedge},\ \overline{\Rightarrow},\ \overline{\Leftarrow},\ \overline{\Leftrightarrow}.\bullet$$

Df 1.2. The number of different *atomic euautographic relations* (*AER's*) from p to s, p_1 to s_1, p_2 to s_2, etc, which occur in a given *combined euautographic ordinary relation* (*CbEOR*) *of* A_0, is called *the rank of that CbEOR*. Analogously, the total number of different *atomic panlogographic relations* (*APLR's*), e.g. **P** to **S**, $\mathbf{P}_1$ to $\mathbf{S}_1$, $\mathbf{P}_2$ to $\mathbf{P}_2$, etc or **p** to **s**, $\mathbf{p}_1$ to $\mathbf{s}_1$, $\mathbf{p}_2$ to $\mathbf{s}_2$, etc, and of different *AER's*, which occur in a given *combined panlogographic ordinary relation* (*CbPLOR*) *of* $\mathbf{A}_0$, is called *the rank of that CbPLOR*. An EOR or PLOR is said to be *one of a higher rank* if its rank is equal to or strictly greater than 2.•

1.1.2. A summary of underlying facts

1. In accordance with the AEADM D_1 of A_1 and APLADM $\mathbf{D}_1$ of $\mathbf{A}_1$, a *valid*, or *antivalid*, ER (euautographic relation) or PLR (panlogographic relation) is necessarily a *combined* ER (*Cb*ER) or *combined* PLR (*Cb*PLR) respectively, whereas a *vav-neutral* (*vav-indeterminate*) ER or PLR is either an *atomic* one (*AER* or *A*PLR) or a *combined* one (*Cb*ER or *Cb*PLR). I also recall that a PLR is valid, or antivalid, if and only if every ER of its range is valid, or antivalid, respectively, whereas in the general case the range of a vav-neutral (vav-indeterminate) PLR comprises ER's of all the three classes: valid, antivalid, and vav-neutral.

2. Under the above chapter head I include and put forward the most conspicuous *valid predicate-free* PLOR's (panlogographic ordinary relations) of academic or practical interest, whose *validity* (*validness*) are established by the appropriate *basic panlogographic algebraic decision procedures of* $\mathbf{A}_1$ (*BPLADP's*) from the pertinent *basic panlogographic master*, or *decision*, *theorems* (*BPLMT's* or *BPLDT's*) *of* $\mathbf{A}_1$, which have been proved in section II.7. Accordingly, section II.7 could, or perhaps should, have been included as the first section of this chapter. In any case, the subject matter of section II.7 and of this chapter is associated with the organon A_0 as follows. Every EMT (EDT) of A_0 is a BEMT (BEDT) of A_1, whereas a BEMT of A_1 is either an EMT of A_0 or a version (syntactic interpretand) of a certain EMT of A_0. At the same time, the range a BPLMT of $\mathbf{A}_1$ comprises both all properly patterned EMT's of A_0 and all properly patterned BEMT's of A_1 being versions (syntactic interpretands) of certain EMT's of A_0 of that range.

3. The general way to establish the *validity-value* of a given *panlogographic slave relation* (*PLSR*) $\mathbf{P}$ of $\mathbf{A}_0$ is to compute its validity-integron $V(\mathbf{P})$ by the appropriate PLADP (panlogographic decision procedure) and thus to arrive at the

PLMT of $\mathbf{P}$, $\mathbf{T}_1(\mathbf{P})$, of one of the three forms (a) $V(\mathbf{P}) \hat{=} 0$, (b) $V(\mathbf{P}) \hat{=} 1$, or (c) $V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle$, in accordance with the schema (II.6.33). Consequently, by the pertinent instance of meta-axiom (II.4.40), $\mathbf{P}$ is valid, antivalid, or vav-neutral (vav-indeterminate if $\mathbf{T}_1(\mathbf{P})$ has the form (a), (b), or (c) respectively. Most of the PLSR's, which are selected in this chapter to be processed, turn out to be valid.

4. If $\mathbf{P}$ is has the form of an equivalence $\mathbf{Q}\Leftrightarrow\mathbf{R}$ then its PLADP is as rule essentially simplified by computing first $V(\mathbf{Q})$ and $V(\mathbf{R})$ separately, especially in the case where it is turned out that $V(\mathbf{Q}) \hat{=} V(\mathbf{R})$ as expected. In this case, by the latter identity, the pertinent version of (II.7.50) immediately infers that $\vdash[\mathbf{Q} \Leftrightarrow \mathbf{R}]$, i.e. that $\mathbf{Q}\Leftrightarrow\mathbf{R}$ is valid. By that identity, it also follows from the pertinent instance of (II.7.7γ) that

$$V(\mathbf{Q} \Leftrightarrow \mathbf{R}) \hat{=} [V(\mathbf{Q}) \hat{-} V(\mathbf{R})]^2 \hat{=} 0,$$

by which the fact of validity of $\mathbf{Q}\Leftrightarrow\mathbf{R}$ is alternatively inferred by the pertinent instance of (II.4.40a).

5. In accordance with (II.7.7γ), a slave theorem that is stated in the form of a *valid equivalence* $\mathbf{Q}\Leftrightarrow\mathbf{R}$ *immediately implies two implicative slave theorems* $\mathbf{Q}\Rightarrow\mathbf{R}$ and $\mathbf{Q}\Leftarrow\mathbf{R}$ (or $\mathbf{R}\Rightarrow\mathbf{Q}$). In this case, the theorem $\mathbf{Q}\Leftrightarrow\mathbf{R}$ is called the pertinent *bilateral law*, while the slave theorems $\mathbf{Q}\Rightarrow\mathbf{R}$ and $\mathbf{Q}\Leftarrow\mathbf{R}$ (or $\mathbf{R}\Rightarrow\mathbf{Q}$) are called the pertinent *direct law* and the pertinent *converse law* respectively. Most often, the two latter laws are not written down.•

6. Just as everywhere throughout the treatise, in this chapter, every *assertive valid* PLOR (panlogographic ordinary relation) is use in the natural

projective (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience the class of ER's, being the desigtatum* (*range*) *of the PLOR*, as *my as if extramental* (*exopsychical*) *object* that I call a *common* (*general*, *certain*, *particular but not particularized*) *euautographic element* (*member*), i.e. a *common ER*, *of the designatum* and also a *common denotatum of the PLOR*. The common element of the designatum *represents the whole designatum*, thus being just *another hypostasis* (*way of existence*, *aspect*) *of the latter*. In this case, I also say that both *the PLOR and its* [*original*, *unpolarized*] *designatum are used for mentioning* its common denotatum, i.e. the ER (common element) of the designatum of the PLOR, or that, less explicitly, *they are used but not mentioned*, whereas the designatum is said to be *connoted by*, or to be *the connotatum* (*connotation value*, pl. "*connotata*") *of*, *the PLOR*. Consequently, a *concrete* (*and concretized*) *valid ER* of the designatum (range) of a valid PLOR is a *concrete euautographic corollary* (*instance*, *interpretand*) *of the PLOR*, which can be just written down and which does not require any proof.

7. In general, *panlogographic slave kyrologies*, i.e. *panlogographic slave relations* (*PLSR's*) and their PLMT's (PLDT's) will be referred to by their double position-numerals, serving as their *logical names* (*bookmarks*). However, for purposes of brief explanations, most of the specific PLSR's and their PLMT's that are established in this chapter will be provided with English names – either the same ones as the standard names of the counterparts (interpretands) of the PLSR's, which known as *laws of sentential traditional logic* (*STL*), or some suggested names in the cases, where there are no appropriate standard names in use.

1.2. Valid PLOR's of rank 1 and their PLMT's

Preliminary Remark 1.1. In this section, I shall make explicit most conspicuous valid PLOR's and also some valid EOR's of rank 1 of their ranges as their concrete *euautographic interpretands* (*instances*, *corollaries*), which serve in turn as *interpretantia* of the respective CFCLR's (conformal catlogographic relations), which are some simplest laws of *traditional sentential logic* (*TrSL*). •

*__Th 1.1:__ ***The law of double negation.***

$$[\neg\neg\mathbf{P}] \Leftrightarrow \mathbf{P}. \tag{1.1}$$

Proof: Making use of the variant of (II.7.1γ) with $\neg\mathbf{P}$ and then (II.7.1γ) itself yields:

$$V(\neg\neg\mathbf{P}) \hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{=} 1 \hat{-} [1 \hat{-} V(\mathbf{P})] \hat{=} V(\mathbf{P}). \tag{1.1$_1$}$$

The master-theorem (1.1$_1$) is the *algebraic law*, i.e. *law in the algebraic* (*objective*) *form, of double negation* (*ALODN*), which immediately infers (1.1) by the pertinent instance of (II.7.50). Alternatively, by (1.1$_1$), the instance of (II.7.7γ) with $\neg\neg\mathbf{P}$ as $\mathbf{Q}$ yields

$$V(\mathbf{P} \Leftrightarrow \neg\neg\mathbf{P}) \hat{=} [V(\mathbf{P}) \hat{-} V(\neg\neg\mathbf{P})]^2 \hat{=} [V(\mathbf{P}) \hat{-} V(\mathbf{P})]^2 \hat{=} 0. \tag{1.1$_2$}$$

whence (1.1) immediately follows by the variant of (II.4.40a) with the relation (1.1) in place of **P**.•

Th 1.2:** ***Two trial laws of excluded middle **(*tertium non datur*).**

$\mathbf{P}\vee\neg\mathbf{P}$. (*The weak trial law of excluded middle*) (1.2)

$\mathbf{P}\mathrel{\overline{\Leftrightarrow}}\neg\mathbf{P}$. (*The strong trial law of excluded middle*) (1.3)

Proof: By (II.4.2) and (II.7.1γ), it follows from the pertinent instances of (II.7.2γ) and (II.7.12γ) that

$$V(\mathbf{P}\vee\neg\mathbf{P})\mathrel{\hat{=}}V(\mathbf{P})\mathbin{\hat{\cdot}}V(\neg\mathbf{P})\mathrel{\hat{=}}V(\mathbf{P})\mathbin{\hat{\cdot}}[1\mathbin{\hat{-}}V(\mathbf{P})]\mathrel{\hat{=}}0, \tag{1.2$_1$}$$

$$\begin{aligned} &V(\mathbf{P}\mathrel{\overline{\Leftrightarrow}}\neg\mathbf{P})\mathrel{\hat{=}}1\mathbin{\hat{-}}[V(\mathbf{P})\mathbin{\hat{-}}V(\neg\mathbf{P})]^2\\ &\mathrel{\hat{=}}1\mathbin{\hat{-}}[V(\mathbf{P})\mathbin{\hat{+}}V(\neg\mathbf{P})\mathbin{\hat{-}}2\mathbin{\hat{\cdot}}V(\mathbf{P})\mathbin{\hat{\cdot}}V(\neg\mathbf{P})]\mathrel{\hat{=}}1\mathbin{\hat{-}}1\mathrel{\hat{=}}0, \end{aligned} \tag{1.3$_1$}$$

(1.2_1) being the same as (II.7.15γ). The master theorems (1.2_1) and (1.3_1) immediately infer their slave theorems (1.2) and (1.3) respectively by the instances of (II.4.40a) with each one of the relations (1.2) and (1.3) in turn in place of **P**. The master theorems (1.2_1) and (1.3_1) are respectively called the *weak* and *strong trial laws of excluded middle* (briefly *WTLOXM* and *STLOXM*) *in algebraic* (*objective*) *form* or the *algebraic ones* (*AlgWTLOXM* and *AlgSTLOXM*). By contrast, slave theorems (1.2) and (1.3) are respectively called the *WTLOXM* and *STLOXM in logical* (*subjective*) *form* or the *logical ones* (*LogWTLOXM* and *LogSTLOXM*). The qualifiers "*lax*" and "*strict*" can be used interchangeably with "*weak*" and "*strong*" respectively.•

Cmt 1.1. 1) Besides (1.3_1), it is instructive to make use of the pertinent variants of (II.7.1γ)–(II.7.7γ) and (II.7.12γ) for developing the following train of identities

$$\begin{aligned} &V(\mathbf{P}\mathrel{\overline{\Leftrightarrow}}\neg\mathbf{P})\mathrel{\hat{=}}V(\neg[\mathbf{P}\Leftrightarrow\neg\mathbf{P}])\mathrel{\hat{=}}1\mathbin{\hat{-}}V(\mathbf{P}\Leftrightarrow\neg\mathbf{P})\\ &\mathrel{\hat{=}}1\mathbin{\hat{-}}V([\mathbf{P}\Rightarrow\neg\mathbf{P}]\wedge[\mathbf{P}\Leftarrow\neg\mathbf{P}])\mathrel{\hat{=}}1\mathbin{\hat{-}}V(\neg[\neg[\mathbf{P}\Rightarrow\neg\mathbf{P}]\vee\neg[\mathbf{P}\Leftarrow\neg\mathbf{P}]])\\ &\mathrel{\hat{=}}V(\neg[\neg\mathbf{P}\vee\neg\mathbf{P}]\vee\neg[\mathbf{P}\vee\neg\neg\mathbf{P}])\mathrel{\hat{=}}V(\neg[\neg\mathbf{P}\vee\neg\mathbf{P}])\mathbin{\hat{\cdot}}V(\neg[\mathbf{P}\vee\neg\neg\mathbf{P}])\\ &\mathrel{\hat{=}}[1\mathbin{\hat{-}}V(\neg\mathbf{P}\vee\neg\mathbf{P})]\mathbin{\hat{\cdot}}[1\mathbin{\hat{-}}V(\mathbf{P}\vee\neg\neg\mathbf{P})]\\ &\mathrel{\hat{=}}[1\mathbin{\hat{-}}V(\neg\mathbf{P})\mathbin{\hat{\cdot}}V(\neg\mathbf{P})]\mathbin{\hat{\cdot}}[1\mathbin{\hat{-}}V(\mathbf{P})\mathbin{\hat{\cdot}}V(\neg\neg\mathbf{P})]\mathrel{\hat{=}}[1\mathbin{\hat{-}}V(\neg\mathbf{P})]\mathbin{\hat{\cdot}}[1\mathbin{\hat{-}}V(\mathbf{P})]\\ &\mathrel{\hat{=}}V(\mathbf{P})\mathbin{\hat{\cdot}}V(\neg\mathbf{P})\mathrel{\hat{=}}V(\mathbf{P}\vee\neg\mathbf{P})\mathrel{\hat{=}}0, \end{aligned} \tag{1.3$_2$}$$

which evidences that (1.2) and (1.3) are *formally equivalent* owing to (1.1) or (1.1_1), i.e. owing to the PLMT (panlogographic master theorem) (II.7.1γ) after all. However, *on their own right*, i.e. as PLR's that are *as if postulated to be valid in no connection* *with (II.7.1γ)* (in agreement with the ancient wordy precursor of both (1.2) and (1.3), which was talen for granted to be true), the PLR's (1.2) and (1.3) *are not equivalent materially*. Indeed, in this case, the next to last term in (1.2_1), which is implied by (II.7.1γ), should be omitted, so that (1.2_1) becomes:

$$V(\mathbf{P} \vee \neg\mathbf{P}) \hat{=} V(\mathbf{P}) \hat{\cdot} V(\neg\mathbf{P}) \hat{=} 0, \qquad (1.2_2)$$

whereas (1.3_1) remains unaltered. At the same time, the APLOR (atomic panlogographic ordinary relation) '**P**' can assume, as its accidental denotatum, any valid, antivalid, and vav-neutral (vav-indeterminate) ER's. Consequently, (1.2_2) and (1.3_1) have the following implication.

i) It follows from (1.2_2) that

a) if $V(\mathbf{P}) \hat{=} 0$, i.e. if an ER **P** is valid, then either $V(\neg\mathbf{P}) \hat{=} 0$ or $V(\neg\mathbf{P}) \hat{=} 1$, or else $V(\neg\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\neg\mathbf{P}\rangle$, i.e. the ER $\neg$**P** is either valid or antivalid, or else vav-neutral;

b) if $V(\mathbf{P}) \hat{=} 1$, i.e. if an ER **P** is antivalid, then $V(\neg\mathbf{P}) \hat{=} 0$, i.e. the ER $\neg$**P** is valid;

c) if $V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle$, i.e. if an ER **P** is vav-neutral, then either $V(\neg\mathbf{P}) \hat{=} 0$ or $V(\neg\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\neg\mathbf{P}\rangle$, i.e. is the ER $\neg$**P** is either valid or vav-neutral;

d) the points a–c apply with '**P**' and '$\neg$**P**' exchanged.

ii) It follows from (1.3_1) that

a) if $V(\mathbf{P}) \hat{=} 0$, i.e. if an ER **P** is valid, then $V(\neg\mathbf{P}) \hat{=} 1$, i.e. the ER $\neg$**P** is antivalid;

b) if $V(\mathbf{P}) \hat{=} 1$, i.e. if an ER **P** is antivalid, then $V(\neg\mathbf{P}) \hat{=} 0$, i.e. the ER $\neg$**P** is valid;

c) if $V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle$, i.e. if an ER **P** is vav-neutral, then $[\mathbf{i}_{\sim}|\mathbf{P}\rangle \hat{-} V(\neg\mathbf{P})]^2 \hat{=} 1$, which has the unique solution $V(\neg\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\neg\mathbf{P}\rangle \hat{=} 1 \hat{-} \mathbf{i}_{\sim}|\mathbf{P}\rangle$ subject to $V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{P}) \hat{=} V(\neg\mathbf{P})$, i.e. the ER $\neg$**P** is vav-neutral;

d) the points a–c apply with '**P**' and '$\neg$**P**' exchanged.

As compared to the points a and c of the item ii, the respective points of the item i have some additional solutions for $V(\neg\mathbf{P})$, which are eliminated owing to (II.7.1γ).

2) In analogy with the items 1.i and 1.ii, the PLMT (II.7.1γ), i.e. *the algebraic law of negation* (*AlgLON*), implies the following two versions of the *metatheorem* (*metalinguistic theorem*, *theorem belonging to the IML of* A_1) that can be called the *biune trial law of excluded middle and dual contradiction* (briefly *BUTLOXM&DC1* and *BUTLOXM&DC2*).

> ***BUTLOXM&DC1.*** The negation of a valid ER **P**, i.e. of any one satisfying the PLR $V(\mathbf{P}) \hat{=} 0$, is the antivalid ER ¬**P**, i.e. the one satisfying the PLR $V(\neg\mathbf{P}) \hat{=} 1$, and vice versa, whereas the negation of a vav-neutral ER **P**, i.e. of any one satisfying the PLR $V(\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\mathbf{P}\rangle$, is another vav-neutral ER ¬**P**, satisfying the PLR $V(\neg\mathbf{P}) \hat{=} \mathbf{i}_{\sim}|\neg\mathbf{P}\rangle \hat{=} 1 \hat{-} \mathbf{i}_{\sim}|\mathbf{P}\rangle$.
>
> ***BUTLOXM&DC2.*** The negation of a valid, or antivalid, ER (euautographic relation) or PLR (panlogographic relation) is respectively an antivalid, or valid, ER or PLR and vice versa, whereas the negation of a vav-neutral ER or PLR is another vav-neutral ER or PLR respectively.

Previously, I have repeatedly stated BUTLOXM&DC2 contextually without mentioning any name of it, whereas BUTLOXM&DC1 is a specification (restriction) of BUTLOXM&DC2. Comparison of the item 1.ii and BUTLOXM&DC1 shows that they are *semantically equivalent* (*semantically concurrent*). Therefore, BUTLOXM&DC1 can be regarded as a semi-verbal version of both the AlgLON (II.7.1γ) and the LogTLOXM (1.3), so that the two latter are *semantically equivalent* as well. By contrast, the item 1.i is, circularly, reduces to BUTLOXM&DC1 by (II.7.1γ), i.e. by that same BUTLOXM&DC1. In any case, in accordance with either one of the two biune law, *a vav-neutral ER and its negation coexist, not excluding and not contradicting each other, but excluding any other ER*. Therefore, the generic term "*law of excluded middle*" ("*LOXM*") that I have borrowed from DFL (dual formal logic) and have provided with the additional prepositive qualifier "*trial*" is a semantically adequate name of either theorem (1.2) or (1.3). Likewise, the generic name "*trial law of dual contradiction*", being a modification of the term "*law of contradiction*" of DFL seems to be a semantically adequate name of either of the subsequent theorems (1.7) and (1.8).

3) If the range (designatum) of '**P**' is restricted so as to include only valid and antivalid ER's then the panlogographic theorems (1.2) and (1.3) are *semantically*

(*mentally*) *restricted* to become respectively the *weak* and *strong dual laws of excluded middle* (*WDLOXM* and *SDLOXM*); and likewise the subsequent panlogographic theorems (1.7) and (1.8) are *semantically* (*mentally*) *restricted* to become respectively the *former dual law of contradiction* and the *latter dual law of contradiction* (*WDLOXM* and *SDLOXM*) In accordance with the semantic restriction of (1.2) and (1.3), the points 1.i.c and 1.ii.c should be omitted, while the point 1.i.a becomes:

1.i.a') if $V(\mathbf{P}) \hat{=} 0$, i.e. if an ER **P** is valid, then either $V(\neg\mathbf{P}) \hat{=} 0$ or $V(\neg\mathbf{P}) \hat{=} 1$, i.e. the ER ¬**P** is either valid or antivalid.

At the same time, BU*T*LOXM&*D*C1 and BU*T*LOXM&*D*C2 turn into the following BU*D*LOXM&C1 and BU*D*LOXM&C2, where "*D*" is an abbreviation of "*dual*":

BUDLOXM&D1. The negation of a valid ER **P**, i.e. of any one satisfying the PLR $V(\mathbf{P}) \hat{=} 0$, is the antivalid ER ¬**P**, i.e. the one satisfying the PLR $V(\neg\mathbf{P}) \hat{=} 1$, and vice versa.

BUTLOXM&DC2. The negation of a valid, or antivalid, ER or PLR is respectively an antivalid, or valid, ER or PLR and vice versa.

3) I recall that, in accordance with its semantic properties, the kernel-sign (logical connective) '∨', being the principal kernel-sign of (1.2), is called *inclusive or* (in Latin *vel*) in the sense that, when '∨' is applied to two operata, each of which that can assume (take on) the validity-values validity and antivalidity or the truth-values truth and antitruth (falsehood), then it is rendered into ordinary language as: «*either … or ***, or both*». By contrast, in accordance with its semantic properties, the kernel-sign '$\overline{\Leftrightarrow}$', being the principal kernel-sign of (1.3), is called *exclusive or* (in Latin *auf*) in the sense that, when '$\overline{\Leftrightarrow}$' is applied to two operata of the above kind then it is rendered into ordinary language as: «*either … or* ***», i.e. as «*either … or ***, but not both*». The points 1.i.a, 1.ii.a, and 1.i.a' are in agreement with the above properties '∨' and '$\overline{\Leftrightarrow}$', the understanding being that if the two operata of '∨' are ER's **P** and ¬**P** then the option «*or both*» in the semi-verbal expression «*either* **P** *or* ¬**P**, *or both*» for «**P**∨¬**P**» is eliminated only because the *disjuncts* '**P**' *and* '¬**P**' *are interrelated by (II.7.1γ)*, as was repeatedly stated previously.

4) In the literature on symbolic logic, the W*D*LOXM, homographic of the W*T*LOXM (1.2), is persistently written with one or another version of the kernel-sign

‘$\vee$’ of *inclusive or* and is called “*Law of excluded middle*” (or “*tertium non datur*” in Latin) instead of more naturally writing it with one or another version of the kernel-sign ‘$\overline{\Leftrightarrow}$’ of *exclusive or* (*antiequivalence*, *antibiimplication*) (see, e.g., Suppes [1957, p. 34] or Church [1956, p. 102, 15.0(10)]). The fact of employment of a kernel-sign “$\vee$” in the conventional *Law of excluded middle* can be explained as follows. In the traditional verbal form of that law, the conjunction (verbal sentential connective) “*or*” was likely understood in the exclusive sense of Latin “*auf*”, and not in the inclusive sense of Latin “*vel*”. However, when that law was incorporated into modern symbolic logic, a simple logographic sign “$\vee$”, being a stylized first letter of Latin “*vel*”, was employed instead of some appropriate logographic counterpart (as my sign ‘$\overline{\Leftrightarrow}$’) of Latin “*auf*”. A like confusion occurred when the traditional verbal form of *modus ponendo tollens* was incorporated into modern symbolic logic (see the item 6 of subsection 3.4 for greater detail).•

Cmt 1.2. 1) Besides (1.2) and (1.3), there are in A_0 (A_0 and $\mathbf{A}_0$) some *modified weak* (*lax*) and *strong* (*strict*) *laws of excluded middle*, e.g. these two:

$$[V(\mathbf{P})\hat{=}0]\vee[V(\mathbf{P})\hat{=}1], \tag{1.4}$$

$$S[V(\mathbf{P})\hat{=}0]\overline{\Leftrightarrow}[V(\mathbf{P})\hat{=}1], \tag{1.5}$$

which will be called the *modified WTLOXM* and the *modified STLOXM* respectively. The theorems (1.4) and (1.5) are proved by the following variants of (1.2_1) and (1.3_1) with $[V(\mathbf{P})\hat{=}0]$ and $[V(\mathbf{P})\hat{=}1]$ in place of $\mathbf{P}$ and $\neg\mathbf{P}$ respectively:

$$\begin{gathered} V([V(\mathbf{P})\hat{=}0]\vee[V(\mathbf{P})\hat{=}1])\hat{=}V(V(\mathbf{P})\hat{=}0)\hat{\cdot}V(V(\mathbf{P})\hat{=}1) \\ \hat{=}V(\mathbf{P})\hat{\cdot}[1\hat{-}V(\mathbf{P})]\hat{=}0, \end{gathered} \tag{1.4$_1$}$$

$$\begin{gathered} V([V(\mathbf{P})\hat{=}0]\overline{\Leftrightarrow}[V(\mathbf{P})\hat{=}1]) \\ \hat{=}1\hat{-}[V(V(\mathbf{P})\hat{=}0)\hat{+}V(V(\mathbf{P})\hat{=}1)\hat{-}2\hat{\cdot}V(V(\mathbf{P})\hat{=}0)\hat{\cdot}V(V(\mathbf{P})\hat{=}1)] \\ \hat{=}1\hat{-}[V(\mathbf{P})\hat{+}[1\hat{-}V(\mathbf{P})]\hat{-}2\hat{\cdot}V(\mathbf{P})\hat{\cdot}[1\hat{-}V(\mathbf{P})]]\hat{=}1\hat{-}1\hat{=}0, \end{gathered} \tag{1.5$_1$}$$

where use of theorems (II.6.19) and (II.6.20) has been made in developing the final expressions. The master-theorem (1.4_1) or (1.5_1) is respectively the *modified* AlgWTLOXM or AlgSTLOXM, which immediately infers (1.4) or (1.5), by the instance of (II.4.40a) with the respective one of the relations (1.4) and (1.5) in turn in place of **P**.

2) By Df II.1.10(13), it follows that

$$
\begin{aligned}
V\left(V(\mathbf{P})\,\overline{\hat{=}}\,0\right) &\hat{=} V\left(\neg\left[V(\mathbf{P})\hat{=}0\right]\right)\hat{=}1\,\hat{-}\,V\left(V(\mathbf{P})\hat{=}0\right)\\
&\hat{=}1\,\hat{-}\,V(\mathbf{P})\hat{=}V\left(V(\mathbf{P})\hat{=}1\right),\\
V\left(V(\mathbf{P})\,\overline{\hat{=}}\,1\right) &\hat{=} V\left(\neg\left[V(\mathbf{P})\hat{=}1\right]\right)\hat{=}1\,\hat{-}\,V\left(V(\mathbf{P})\hat{=}1\right)\\
&\hat{=}1\,\hat{-}\left[1\,\hat{-}\,V(\mathbf{P})\right]\hat{=}V(\mathbf{P})\hat{=}V\left(V(\mathbf{P})\hat{=}0\right).
\end{aligned}
\tag{1.6}
$$

Hence, the slave theorems (1.4) and (1.5) can be rewritten in analogy with (1.2) and (1.3) either by substituting $\neg\left[V(\mathbf{P})\hat{=}0\right]$, or $\left[V(\mathbf{P})\,\overline{\hat{=}}\,0\right]$, for $\left[V(\mathbf{P})\hat{=}1\right]$ or by substituting $\neg\left[V(\mathbf{P})\hat{=}1\right]$, or $\left[V(\mathbf{P})\,\overline{\hat{=}}\,1\right]$, for $\left[V(\mathbf{P})\hat{=}0\right]$.•

Th 1.3:** ***Two trial laws of dual contradiction.

$\mathbf{P}\wedge\!\!\!\!-\neg\mathbf{P}$. (*The former trial law of dual contradiction*) (1.7)

$\mathbf{P}\,\overline{\wedge}\,\neg\mathbf{P}$. (*The latter trial law of dual contradiction*) (1.8)

Proof: By (1.1_1) and (II.7.15γ), it follows from the instance of (II.7.5γ) and (II.7.9γ) with $\neg\mathbf{P}$ as $\mathbf{Q}$ that

$$
\begin{aligned}
V(\mathbf{P}\wedge\!\!\!\!-\neg\mathbf{P}) &\hat{=} V\left([\neg\mathbf{P}]\vee[\neg\neg\mathbf{P}]\right)\hat{=}V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\neg\mathbf{P})\\
&\hat{=}V(\neg\mathbf{P}\vee\mathbf{P})\hat{=}V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{P})\hat{=}0,
\end{aligned}
\tag{1.7$_1$}
$$

$$
\begin{aligned}
V(\mathbf{P}\,\overline{\wedge}\,\neg\mathbf{P}) &\hat{=} V\left(\neg[\mathbf{P}\wedge\neg\mathbf{P}]\right)\hat{=}1\,\hat{-}\,V(\mathbf{P}\wedge\neg\mathbf{P})\hat{=}V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\neg\mathbf{P})\\
&\hat{=}V(\neg\mathbf{P}\vee\mathbf{P})\hat{=}V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{P})\hat{=}0.
\end{aligned}
\tag{1.8$_1$}
$$

These two master-theorems are the former and latter *algebraic trial laws of dual contradiction* (*AlgTLODC*), which immediately infer the slave theorems (1.7) and (1.8), being the respective *logical trial laws of dual contradiction* (*LogTLODC*), by the instances of (II.4.40a) with each one of the relations (1.7) and (1.8) in place of **P**.•

Cmt 1.3. 1) In analogy with (1.4) and (1.5), the relations

$$\left[V(\mathbf{P})\hat{=}0\right]\overline{\wedge}\left[V(\mathbf{P})\hat{=}1\right], \tag{1.9}$$

$$\left[V(\mathbf{P})\hat{=}0\right]\wedge\!\!\!\!-\left[V(\mathbf{P})\hat{=}1\right], \tag{1.10}$$

are slave theorems, which are called the *modified former TLODC* the *modified latter TLODC* respectively and which are proved by the following variant of (1.7_1) with $\left[V(\mathbf{P})\hat{=}0\right]$ and $\left[V(\mathbf{P})\hat{=}1\right]$ in place of **P** and $\neg\mathbf{P}$ respectively:

$$
\begin{aligned}
V\left(\left[V(\mathbf{P})\hat{=}0\right]\overline{\wedge}\left[V(\mathbf{P})\hat{=}1\right]\right) &\hat{=} V\left(\left[V(\mathbf{P})\hat{=}0\right]\wedge\!\!\!\!-\left[V(\mathbf{P})\hat{=}1\right]\right)\\
\hat{=}1\,\hat{-}\,V\left(V(\mathbf{P})\hat{=}0\right)\hat{-}\,V\left(V(\mathbf{P})\hat{=}1\right)&\hat{+}\,V\left(V(\mathbf{P})\hat{=}0\right)\hat{\cdot}\,V\left(V(\mathbf{P})\hat{=}1\right)\\
&\hat{=}1\,\hat{-}\,V(\mathbf{P})\,\hat{-}\,V(\neg\mathbf{P})\,\hat{+}\,V(\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{P})\hat{=}0,
\end{aligned}
\tag{1.9$_1$}
$$

where use of theorems (II.6.19) and (II.6.20) has been made in developing the final expression. The master-theorem (1.9_1) is the *modified AlgTLODC*, which immediately infers both slave theorems (1.9) and (1.10) by the instances of (II.4.40a) with each one of the relations (1.9) and (1.10) in turn in place of **P**.

2) By (1.6), the slave-theorems (1.9) and (1.10) can be rewritten in analogy with (1.7) and (1.8) either by substituting $\neg[V(\mathbf{P})\hat{=}0]$, or $[V(\mathbf{P})\overline{\hat{=}}0]$, for $[V(\mathbf{P})\hat{=}1]$ or by substituting $\neg[V(\mathbf{P})\hat{=}1]$, or $[V(\mathbf{P})\overline{\hat{=}}1]$, for $[V(\mathbf{P})\hat{=}0]$.•

*__Th 1.4:__ ***Reflexive laws for*** $\Rightarrow$, $\Leftarrow$ ***and*** $\Leftrightarrow$**.**

$\mathbf{P}\Rightarrow\mathbf{P}$. (*The reflexive law for* $\Rightarrow$)

(1.11)

$\mathbf{P}\Leftarrow\mathbf{P}$. (*The reflexive law for* $\Leftarrow$)

(1.12)

$\mathbf{P}\Leftrightarrow\mathbf{P}$. (*The reflexive law for* $\Leftrightarrow$)

(1.13)

Proof: By (II.4.2), it follows from the instances of (II.7.3γ), (II.7.4γ), and (II.7.7γ) with **P** in place of **Q** that

$$V(\mathbf{P}\Rightarrow\mathbf{P})\hat{=}[1\hat{-}V(\mathbf{P})]\hat{\cdot}V(\mathbf{P})\hat{=}0, \tag{1.11$_1$}$$

$$V(\mathbf{P}\Leftarrow\mathbf{P})\hat{=}V(\mathbf{P})\hat{\cdot}[1\hat{-}V(\mathbf{P})]\hat{=}0, \tag{1.12$_1$}$$

$$V(\mathbf{P}\Leftrightarrow\mathbf{P})\hat{=}V(\mathbf{P}\Rightarrow\mathbf{P})\hat{+}V(\mathbf{P}\Leftarrow\mathbf{P})\hat{=}[V(\mathbf{P})\hat{-}V(\mathbf{P})]^2\hat{=}0, \tag{1.13$_1$}$$

which immediately infer (1.11)–(1.13) by the versions of (II.4.40a) with **P⇒P**, **P⇐P**, or **P⇔P** in turn in place of **P**. In agreement with Preliminary Remark 1.1, identity (1.13_1) is tantamount to the conjunction of identities (1.11_1) and (1.12_1) and conversely each one of the two latter identities follows from (1.13_1). However, in this case, I have stated all the three slave theorems (1.11)–(1.13) and their master theorems (1.11_1)–(1.13_1) for more clarity, because each one of the three former or the respective one of the tree latter expresses a certain inherent property of the pertinent logical connective in no connection with any other connectives.•

Cmt 1.4. By Df. II.1.10(3,4),

$$[\mathbf{P}\Rightarrow\mathbf{P}]\rightarrow[[\neg\mathbf{P}]\vee\mathbf{P}],\ [\mathbf{P}\Leftarrow\mathbf{P}]\rightarrow[\mathbf{P}\vee[\neg\mathbf{P}]]. \tag{1.14}$$

By (1.7) and (1.4), theorems (1.2), (1.3), (1.7), (1.8), (1.11), and (1.12) are equivalent.•

*__Th 1.5:__ ***The idempotent laws for*** $\vee$ ***and*** $\wedge$ ***relative to*** $\Leftrightarrow$**.**

$$[\mathbf{P}\vee\mathbf{P}]\Leftrightarrow\mathbf{P}. \tag{1.15}$$

$$[\mathbf{P} \wedge \mathbf{P}] \Leftrightarrow \mathbf{P}. \tag{1.16}$$

Proof: By (II.4.2), it follows from the instances of (II.7.2γ) and (II.7.6γ) with **P** in place of **Q** that

$$V(\mathbf{P} \vee \mathbf{P}) \hat{=} V(\mathbf{P}) \hat{\cdot} V(\mathbf{P}) \hat{=} V(\mathbf{P}), \tag{1.15$_1$}$$

$$V(\mathbf{P} \wedge \mathbf{P}) \hat{=} V(\mathbf{P}) \hat{+} V(\mathbf{P}) \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{P}) \hat{=} V(\mathbf{P}). \tag{1.16$_1$}$$

By (1.15$_1$) or (1.16$_1$), the instance of (II.7.7γ) with $[\mathbf{P} \vee \mathbf{P}]$ or $[\mathbf{P} \wedge \mathbf{P}]$ in turn in place of **P** and with **P** in place of **Q** yields

$$\begin{aligned} V([\mathbf{P} \vee \mathbf{P}] \Leftrightarrow \mathbf{P}) &\hat{=} V([\mathbf{P} \vee \mathbf{P}] \Rightarrow \mathbf{P}) \hat{+} V([\mathbf{P} \vee \mathbf{P}] \Leftarrow \mathbf{P}) \\ &\hat{=} [V(\mathbf{P} \vee \mathbf{P}) \hat{-} V(\mathbf{P})]^2 \hat{=} 0, \end{aligned} \tag{1.15$_2$}$$

$$\begin{aligned} V([\mathbf{P} \wedge \mathbf{P}] \Leftrightarrow \mathbf{P}) &\hat{=} V([\mathbf{P} \wedge \mathbf{P}] \Rightarrow \mathbf{P}) \hat{+} V([\mathbf{P} \wedge \mathbf{P}] \Leftarrow \mathbf{P}) \\ &\hat{=} [V(\mathbf{P} \wedge \mathbf{P}) \hat{-} V(\mathbf{P})]^2 \hat{=} 0, \end{aligned} \tag{1.16$_2$}$$

which immediately infer (1.15) and (1.16) by the versions of (II.4.40a) each one of the relations (1.15) and (1.16) in turn in place of **P**.•

Cmt 1.5. In accordance with (1.15$_2$) and (1.16$_2$) and in agreement with the pertinent general remark of Preliminary Remark 1.1, the kyrologies (1.15) and (1.16) are tantamount to the conjunctions of two pertinent implicative kyrologies each:

$$[\mathbf{P} \vee \mathbf{P}] \Rightarrow \mathbf{P} \text{ and } [\mathbf{P} \vee \mathbf{P}] \Leftarrow \mathbf{P}, \tag{1.15'}$$

$$[\mathbf{P} \wedge \mathbf{P}] \Rightarrow \mathbf{P} \text{ and } [\mathbf{P} \wedge \mathbf{P}] \Leftarrow \mathbf{P} \tag{1.16'}$$

respectively. From the standpoint of the pertinent BEADP's or BPLADP's this result is trivial and is not even worthy to be mention. IT is, however, mentioned, because the CFCL interpretand $[p \vee p] \Rightarrow p$ of the euautographic instance (interpretand) $[p \vee p] \Rightarrow p$ of the first panlogographic kyrology in (1.15') is the first axiom of the *known axiomatic propositional calculus system* $\mathrm{P_R}$ *due to Russell* [1908], which is called "principle of tautology" (see, e.g., Whitehead and Russell [1910, 1925; 1962, pp. 96, 97, item **∗1·2**]).•

°**Crl 1.1.** 1) The occurrences of the APLR (atomic panlogographic relations) '**P**' throughout any one of the theorems (1.1)–(1.16') can be interpreted (replaced) with occurrences of any valid, antivalid, or vav-neutral ER. Particularly, the following valid (kyrologous) EOR's are results of replacing occurrences of '**P**' throughout all theorems (1.1)–(1.16') with occurrences of *vav-neutral AER* (atomic euautographic relation) p:

$$\neg\neg p \Leftrightarrow p\,. \tag{1.1μ}$$

$$p \vee \neg p\,. \tag{1.2μ}$$

$$p \overline{\Leftrightarrow} \neg p\,. \tag{1.3μ}$$

$$[V(p) \hat{=} 0] \vee [V(p) \hat{=} 1]. \tag{1.4μ}$$

$$[V(p) \hat{=} 0] \overline{\Leftrightarrow} [V(p) \hat{=} 1]. \tag{1.5μ}$$

$$p \not\wedge \neg p\,. \tag{1.7μ}$$

$$p \overline{\wedge} \neg p\,. \tag{1.8μ}$$

$$[V(p) \hat{=} 0] \overline{\wedge} [V(p) \hat{=} 1]. \tag{1.9μ}$$

$$[V(p) \hat{=} 0] \not\wedge [V(p) \hat{=} 1]. \tag{1.10μ}$$

$$p \Rightarrow p\,. \tag{1.11μ}$$

$$p \Leftarrow p\,. \tag{1.12μ}$$

$$p \Leftrightarrow p\,. \tag{1.13μ}$$

$$[p \vee p] \Leftrightarrow p\,. \tag{1.15μ}$$

$$[p \wedge p] \Leftrightarrow p\,. \tag{1.16μ}$$

$$[p \vee p] \Rightarrow p \text{ and } [p \vee p] \Leftarrow p\,. \tag{1.15'μ}$$

$$[p \wedge p] \Rightarrow p \text{ and } [p \wedge p] \Leftarrow p\,. \tag{1.16'μ}$$ •

Cmt 1.6. 1) Consequently, the occurrences of the APLR '**P**' throughout any one of the theorems (1.1)–(1.16') can now be replaced with occurrences of any one of the valid ER's (1.1μ)–(1.16'μ) or with occurrences of any one of the negations of ER's (1.1μ)–(1.16'μ), being antivalid ER's. For example,

$$[\neg\neg p \Leftrightarrow p] \overline{\Leftrightarrow} \neg[\neg\neg p \Leftrightarrow p],\ [p \vee \neg p] \overline{\Leftrightarrow} \neg[p \vee \neg p],$$
$$[p \overline{\wedge} \neg p] \overline{\Leftrightarrow} \neg[p \overline{\wedge} \neg p],\ [p \not\vee \neg p] \overline{\Leftrightarrow} \neg[p \not\vee \neg p]$$

are some instances of (1.3), illustrating the above said.

2) Each one of the kyrologies (1.1μ)–(1.16'μ) is an insignificant, indivisible, and hence immediately ineffective valid ER. Indeed, as I have repeatedly pointed out previously, particularly in Cmt II.7.3(4), p is a vav-neutral AER, because its PVI (primary validity integron) $V(p)$ is irreducible. Therefore, I may not in principle assume that $V(p) \hat{=} 0$ or that $V(p) \hat{=} 1$. The ER's (1.1μ)–(1.16'μ) are nevertheless indispensable because their CFCL (conformal catlogographic) interpretands, which are made explicit in the next corollary, preserve their validity-value validity and at the

same time they are semantically significant, so that they are qualified *tautologous* (*tautological*).

3) When a PLR is used for mentioning a common (general) ER of its range, all *euautographic kernel-signs* (*EKS's*) (as some of those mentioned in Df I.1.2(3) or, particularly, as some of those listed in Df 1.1(3)), which occur in the PLR and which are applied *to the pertinent panlogographic operata autonymously* (*contactually*, *immediately*), are supposed to be, at the same time, applied *to the common euautographic operata*, *being common euautographic elements of the designata (ranges) of the panlogographic operata*, *xenonymously* (*slidingly*, *transitorily*). Therefore, a token of an EKS, which occurs in an assertive valid PLOR, has a wordy (verbal) counterpart indicated in Df 1.1(3) – in contrast to a token of the same EKS, which occur in an ER and particularly in an euautographic interpretand of the PLOR, e.g. in any one of the kyrologies (1.1μ)–(1.16'μ).•

+**Crl 1.2.** In accordance with Ax I.8.2, substitution of the ACLR (atomic catlogographic relation) *p* without any quotation marks for all occurrences of the AER *p* throughout any given one of the above kyrologous (valid) EOR's (1.1μ)–(1.16'μ) results in the *CFCL interpretand of the given EOR*, which is, like its *CFE* (*conformal euautographic*) *interpretans*, *kyrologous* (*valid*), but which is, unlike its CFE interpretans, *interpretable significantly*. by *mentally* (*psychically*) attaching it with the *formal tautologousness-value* (*f-tautologousness-value*) *f-tautologousness* (*universal f-truth*) and by simultaneously attaching its constituent APLR '*p*' with the *f-tautologousness-value f-ttatt-neutrality* (*f-ttatt-indeterminacy*), i.e. neutrality (indeterminacy) with respect to the *f-tautologousness-values f-tautologousness and f-antitautologousness* (*universal f-antitruth*, *universal f-falsehood*, *contradictoriness*) – *the quality of being neither f-tautologous* (*universally f-true*) *nor f-antitautologous* (*universally f-antitrue*, *universally f-false*). Then '*p*' can be attached with one of the three *formal veracity-values* (*f-veracity-values*): *f-veracity* (*accidental f-truth*), *f-antiveracity* (*accidental f-antitruth*, *accidental f-falsehood*), and *f-vravr-neutrality* (*f-vravr-indeterminacy*), i.e. neutrality (indeterminacy) with respect to the *f-veracity-values f-veracity and f-antiveracity – the quality of being neither f-veracious* (*accidentally f-true*) *nor f-antiveracious* (*accidentally f-antitrue*, *accidentally f-false*). If the ACLR '*p*' is decided to be f-veracious then it is replaceable with a *materially veracious* (*m-veracious*) English (e.g.) affirmative simple declarative sentence, i.e.

with one that is conformed to a certain complex nonlinguistic object as its *matter* that is called a *state of affairs* or *fact* and also a *case*, *event*, *phenomenon,* etc. Here follow *catlogographic tautologies*, being CFCL interpretands of the above euautographic kyrologies (1.1μ)–(1.16'μ):

$$\neg\neg p \Leftrightarrow p\,. \tag{1.1κ}$$

$$p \vee \neg p\,. \tag{1.2κ}$$

$$p \mathrel{\overline{\Leftrightarrow}} \neg p\,. \tag{1.3κ}$$

$$[V(p) \mathrel{\hat{=}} 0] \vee [V(p) \mathrel{\hat{=}} 1]. \tag{1.4κ}$$

$$[V(p) \mathrel{\hat{=}} 0] \mathrel{\overline{\Leftrightarrow}} [V(p) \mathrel{\hat{=}} 1]. \tag{1.5κ}$$

$$p \mathrel{\not\wedge} \neg p\,. \tag{1.7κ}$$

$$p \mathrel{\overline{\wedge}} \neg p\,. \tag{1.8κ}$$

$$[V(p) \mathrel{\hat{=}} 0] \mathrel{\overline{\wedge}} [V(p) \mathrel{\hat{=}} 1]. \tag{1.9κ}$$

$$[V(p) \mathrel{\hat{=}} 0] \mathrel{\not\wedge} [V(p) \mathrel{\hat{=}} 1]. \tag{1.10κ}$$

$$p \Rightarrow p\,. \tag{1.11κ}$$

$$p \Leftarrow p\,. \tag{1.12κ}$$

$$p \Leftrightarrow p\,. \tag{1.13κ}$$

$$[p \vee p] \Leftrightarrow p\,. \tag{1.15κ}$$

$$[p \wedge p] \Leftrightarrow p\,. \tag{1.16κ}$$

$$[p \vee p] \Rightarrow p \text{ and } [p \vee p] \Leftarrow p\,. \tag{1.15'κ}$$

$$[p \wedge p] \Rightarrow p \text{ and } [p \wedge p] \Leftarrow p\,. \tag{1.16'κ}$$

Tautologies (1.1κ)–(1.16'κ) are immediately inferred (deduced) from kyrologies (1.1μ)–(1.16'μ) by Ax I.8.2, being the pertinent rule of inference. Therefore, the former are, strictly speaking, metatheorems. Since, however, any catlogographic tautology is inferred from the respective euautographic kyrology by one application of a unique inference rule, therefore I regard a tautology as a corollary, i.e. as a relation that does not require any proof. This is why this article is introduced under the logical name "$^{+}$Crl 1.2" and not "$^{+}$Th 1.6", the understanding being that the prepositive superscript $^{+}$ *is the flag of CFCL interpretations or interpetands*. I reserve the abbreviation "$^{+}$Th" as a generic (classifying) logical name of *veracious catlogographic theorems*, which are proved from some *veracious catlogographic postulates – accidental* (*temporary*) *ones*, called *catlogographic hypotheses*, or *essential* (*permanent*) *ones*, called *catlogographic axioms*. If present, the latter will be marked by using the abbreviation "$^{+}$Ax" in their logical names.•

Cmt 1.7. For instance, (1.2κ), (1.3κ), (1.7κ), and (1.8κ) are respectively the following tautologous non-syllogistic *catlogographic* laws:

a) ***The weak trial law of excluded middle*:** $p \vee \neg p$, i.e. *either p or not p or both* (in Latin, *p vel non p*).

b) ***The strong law of excluded middle*:** $p \overline{\Leftrightarrow} \neg p$, i.e. *either p or not p but not both* (in Latin, *p auf non p*).

c) ***The former trial law of dual contradiction*:** $p \barwedge \neg p$, i.e. *not* [*p and not q*] (in Latin, *non* [*p et non p*]), or equivalently $\neg p \vee \neg\neg p$ and hence $p \vee \neg p$.

d) ***The latter trial law of dual contradiction*:** $\neg[p \wedge \neg p]$, i.e. in words the same as $p \barwedge \neg p$ or $\neg p \vee \neg p$.

The occurrence of the option «or both» in the point a is eliminated by (II.7.1κ), so that all the above four laws are mutually equivalent, in agreement with Cmts 1.1(1) and 1.4. The ACLR (atomic catlogographic relation) '*p*' can assume three f-veracity values: f-veracity (accidental f-truth), f-antiveracity (accidental f-antitruth¸ accidental f-falsehood), and vravr-neutrality (vravr-indeterminacy), i.e. neutrality (indeterminacy) with respect to both f-veracity and f-antiveracity. If the range of '*p*' is mentally restricted to the first two values then the laws a–d turn into the respective dual laws.•

1.3. Two panlogographic trial laws of excluded middle for implications and a few vav-neutral PLOR's of rank 3 and their PLMT's

Th 1.6:** ***Two trial laws of excluded middle for implications.

$$[\mathbf{P} \Rightarrow \mathbf{Q}] \vee [\mathbf{Q} \Rightarrow \mathbf{R}]. \tag{1.17}$$

$$[\mathbf{P} \Rightarrow \mathbf{Q}] \vee [\neg\mathbf{P} \Rightarrow \mathbf{R}]. \tag{1.18}$$

Kyrologies (2.6)–(2.9) are called *the first* and *second laws of excluded middle for implications*.

Proof: By the variant (1.2_1) with '**Q**' in place of '**P**', or by (1.2_1) itself, it follows from the version of (II.7.2γ) with '$[\mathbf{P} \Rightarrow \mathbf{Q}]$' in place of '**P**' and '$[\mathbf{Q} \Rightarrow \mathbf{R}]$', or '$[\neg\mathbf{P} \Rightarrow \mathbf{Q}]$', in place of '**Q**' that

$$\begin{aligned} V([\mathbf{P} \Rightarrow \mathbf{Q}] \vee [\mathbf{Q} \Rightarrow \mathbf{R}]) &\hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{\cdot} V(\mathbf{Q} \Rightarrow \mathbf{R}) \\ &\hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{R}) \hat{=} 0, \end{aligned} \tag{1.17_1}$$

$$V([\mathbf{P} \Rightarrow \mathbf{Q}] \vee [\neg\mathbf{P} \Rightarrow \mathbf{R}]) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{\cdot} V(\neg\mathbf{P} \Rightarrow \mathbf{R})$$
$$\hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\neg\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{=} 0, \tag{1.18$_1$}$$

respectively. By (2.17$_1$) and (2.18$_1$), the two pertinent versions of (II.7.50) immediately infer (2.8) and (2.9).•

Cmt 1.8. 1) The train of identities (1.2$_1$) is the PLADP of the kyrology (1.2), so that (1.2$_1$) includes, as its final identity, the PLMT (PLDT) of the PLST (1.2). Hence, the PLMT itself is *the basic weak law of excluded middle in the subjective* (*algebraic*) *form*. Since the pertinent variant of (1.2$_1$) or (1.2$_1$) itself predetermines respectively the PLMT that is proved by (1.17$_1$) or the PLMT that is proved by (1.18$_1$), I regard those PLMT's as two different *laws of excluded middle for implications in the subjective* (*algebraic*) *form*, and their PLST's as the respective *laws of excluded middle for implications in the objective* (*logical*) *form*.

2) From the pertinent versions of (II.7.6γ) and (II.7.7γ), it respectively follows that

$$V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\mathbf{Q} \Rightarrow \mathbf{R})$$
$$\hat{-} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{\cdot} V(\mathbf{Q} \Rightarrow \mathbf{R}) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\mathbf{Q} \Rightarrow \mathbf{R}), \tag{1.19}$$

$$V([\mathbf{P} \Rightarrow \mathbf{Q}] \Leftrightarrow [\mathbf{Q} \Rightarrow \mathbf{R}]) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\mathbf{Q} \Rightarrow \mathbf{R})$$
$$\hat{-} 2 \hat{\cdot} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{\cdot} V(\mathbf{Q} \Rightarrow \mathbf{R}) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\mathbf{Q} \Rightarrow \mathbf{R}), \tag{1.20}$$

by (1.17$_1$), and

$$V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\neg\mathbf{P} \Rightarrow \mathbf{R}]) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\neg\mathbf{P} \Rightarrow \mathbf{R})$$
$$\hat{-} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{\cdot} V(\neg\mathbf{P} \Rightarrow \mathbf{R}) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\neg\mathbf{P} \Rightarrow \mathbf{R}). \tag{1.21}$$

$$V([\mathbf{P} \Rightarrow \mathbf{Q}] \Leftrightarrow [\neg\mathbf{P} \Rightarrow \mathbf{R}]) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\neg\mathbf{P} \Rightarrow \mathbf{R})$$
$$\hat{-} 2 \hat{\cdot} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{\cdot} V(\neg\mathbf{P} \Rightarrow \mathbf{R}) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\neg\mathbf{P} \Rightarrow \mathbf{R}). \tag{1.22}$$

By the pertinent versions of (II.7.3γ), identities (1.19) and (1.20), or (1.21) and (1.22), infer that

$$U_1(\mathbf{P},\mathbf{Q},\mathbf{R}) \vec{\hat{=}} V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]) \hat{=} V([\mathbf{P} \Rightarrow \mathbf{Q}] \Leftrightarrow [\mathbf{Q} \Rightarrow \mathbf{R}])$$
$$\hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\mathbf{Q} \Rightarrow \mathbf{R}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{+} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{R}), \tag{1.23}$$

$$U_2(\mathbf{P},\mathbf{Q},\mathbf{R}) \vec{\hat{=}} V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\neg\mathbf{P} \Rightarrow \mathbf{R}]) \hat{=} V([\mathbf{P} \Rightarrow \mathbf{Q}] \Leftrightarrow [\neg\mathbf{P} \Rightarrow \mathbf{R}])$$
$$\hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\neg\mathbf{P} \Rightarrow \mathbf{R}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{+} V(\mathbf{P}) \hat{\cdot} V(\mathbf{R}). \tag{1.24}$$

At the same time, by (1.23) and (1.24), two pertinent versions of (II.7.12γ) yield:

$$V([\mathbf{P} \Rightarrow \mathbf{Q}] \overline{\Leftrightarrow} [\mathbf{Q} \Rightarrow \mathbf{R}]) \hat{=} 1 \hat{-} V([\mathbf{P} \Rightarrow \mathbf{Q}] \Leftrightarrow [\mathbf{Q} \Rightarrow \mathbf{R}])$$
$$\hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{-} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{R}) \hat{=} 1 \hat{-} U_1(\mathbf{P},\mathbf{Q},\mathbf{R}), \tag{1.25}$$

$$
\begin{aligned}
&V([\mathbf{P}\Rightarrow\mathbf{Q}]\overline{\Leftrightarrow}[\neg\mathbf{P}\Rightarrow\mathbf{R}])\mathrel{\hat{=}}1\mathbin{\hat{-}}V([\mathbf{P}\Rightarrow\mathbf{Q}]\Leftrightarrow[\neg\mathbf{P}\Rightarrow\mathbf{R}])\\
&\mathrel{\hat{=}}1\mathbin{\hat{-}}V(\neg\mathbf{P})\mathbin{\hat{\cdot}}V(\mathbf{Q})\mathbin{\hat{-}}V(\mathbf{P})\mathbin{\hat{\cdot}}V(\mathbf{R})\mathrel{\hat{=}}1\mathbin{\hat{-}}U_2(\mathbf{P},\mathbf{Q},\mathbf{R}).
\end{aligned}
\tag{1.26}
$$

By (1.23) and (1.24), the two pertinent versions of (II.7.50) immediately infer that

$$[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\Leftrightarrow[[\mathbf{P}\Rightarrow\mathbf{Q}]\Leftrightarrow[\mathbf{Q}\Rightarrow\mathbf{R}]], \tag{1.23a}$$

$$[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\neg\mathbf{P}\Rightarrow\mathbf{R}]]\Leftrightarrow[[\mathbf{P}\Rightarrow\mathbf{Q}]\Leftrightarrow[\neg\mathbf{P}\Rightarrow\mathbf{R}]]. \tag{1.24a}$$

At the same time, it follows from (1.25) and (1.26) that, in contrast to the PLR's (1.2) and (1.3), both of which are kyrologies the variants of kyrologies (1.17) and (1.18) with $\overline{\Leftrightarrow}$ in place $\vee$ are udeterologies.

3) In analogy with the separate items of Crl 1.1, the kyrologous (valid) EOR's

$$[p\Rightarrow q]\vee[q\Rightarrow r], \tag{1.17μ}$$

$$[p\Rightarrow q]\vee[\neg p\Rightarrow r] \tag{1.18μ}$$

are the concrete conformal euautographic interpretands (instances, corollaries) of the valid (kyrologous) PLOR's (1.17) and (1.18) respectively. Consequently, just as all separate CLR's (catlogographic relations) displayed in Crl 1.2, the CLR's

$$[p\Rightarrow q]\vee[q\Rightarrow r], \tag{1.17κ}$$

$$[p\Rightarrow q]\vee[\neg p\Rightarrow r], \tag{1.18κ}$$

being the *CFCL interpretands* of the above kyrologous (valid) EOR's, are kyrologous (valid) CLR's, which are significantly interpretable mentally (psychically), and which are therefore called tautologous (universally true) CLR's and also catlogographic tautologies.

3) Similarly, the trains of euautographic identities:

$$
\begin{aligned}
&U_1(p,q,r)\mathrel{\vec{\hat{=}}}V([p\Rightarrow q]\wedge[q\Rightarrow r])\mathrel{\hat{=}}V([p\Rightarrow q]\Leftrightarrow[q\Rightarrow r])\\
&\mathrel{\hat{=}}V(p\Rightarrow q)\mathbin{\hat{+}}V(q\Rightarrow r)\mathrel{\hat{=}}V(\neg p)\mathbin{\hat{\cdot}}V(q)\mathbin{\hat{+}}V(\neg q)\mathbin{\hat{\cdot}}V(r),
\end{aligned}
\tag{1.23μ}
$$

$$
\begin{aligned}
&U_2(p,q,r)\mathrel{\vec{\hat{=}}}V([p\Rightarrow q]\wedge[\neg p\Rightarrow r])\mathrel{\hat{=}}V([p\Rightarrow q]\Leftrightarrow[\neg p\Rightarrow r])\\
&\mathrel{\hat{=}}V(p\Rightarrow q)\mathbin{\hat{+}}V(\neg p\Rightarrow q)\mathrel{\hat{=}}V(\neg p)\mathbin{\hat{\cdot}}V(q)\mathbin{\hat{+}}V(p)\mathbin{\hat{\cdot}}V(r)
\end{aligned}
\tag{1.24μ}
$$

are the concrete CFE (conformal euautographic) interpretands (instances, corollaries) of the PLMT's (1.23) and (1.24) respectively, whereas

$$
\begin{aligned}
&U_1(p,q,r)\mathrel{\vec{\hat{=}}}V([p\Rightarrow q]\wedge[q\Rightarrow r])\mathrel{\hat{=}}V([p\Rightarrow q]\Leftrightarrow[q\Rightarrow r])\\
&\mathrel{\hat{=}}V(p\Rightarrow q)\mathbin{\hat{+}}V(q\Rightarrow r)\mathrel{\hat{=}}V(\neg p)\mathbin{\hat{\cdot}}V(q)\mathbin{\hat{+}}V(\neg q)\mathbin{\hat{\cdot}}V(r),
\end{aligned}
\tag{1.23κ}
$$

$$
\begin{aligned}
&U_2(p,q,r)\mathrel{\vec{\hat{=}}}V([p\Rightarrow q]\wedge[\neg p\Rightarrow r])\mathrel{\hat{=}}V([p\Rightarrow q]\Leftrightarrow[\neg p\Rightarrow r])\\
&\mathrel{\hat{=}}V(p\Rightarrow q)\mathbin{\hat{+}}V(\neg p\Rightarrow q)\mathrel{\hat{=}}V(\neg p)\mathbin{\hat{\cdot}}V(q)\mathbin{\hat{+}}V(p)\mathbin{\hat{\cdot}}V(r)
\end{aligned}
\tag{1.24κ}
$$

are the CFCL interpretands of (1.23μ) and (1.24μ) respectively. As I have repeatedly pointed out previously, particularly in Cmt II.7.3(4), *p*, *q*, and *r* are vav-neutral AER's (atomic euautographic relations), because their PVI's (primary validity integrons) $V(p)$, $V(q)$, and $V(r)$ are irreducible. I may not in principle assume, e.g., that $V(p)\hat{=}0$ or that $V(p)\hat{=}1$ (cf. Cmt 1.8(i)), and similarly with *q* or *r* in place of *p*. Consequently, the ultimate validity-integron $U_1(p,q,r)$ of either one of the ESR's

$$\text{‘}[p \Rightarrow q] \wedge [q \Rightarrow r]\text{’ and ‘}[p \Rightarrow q] \Leftrightarrow [q \Rightarrow r]\text{’} \qquad (1.23'\mu)$$

and the ultimate validity-integron $U_2(p,q,r)$ of either one of the ESR's

$$\text{‘}[p \Rightarrow q] \wedge [\neg p \Rightarrow r]\text{’ and ‘}[p \Rightarrow q] \Leftrightarrow [\neg p \Rightarrow r]\text{’} \qquad (1.24'\mu)$$

do not reduce either to 0 or to 1. Therefore, by the pertinent instances of (II.4.40c), the four ESR's are vav-neutral (vav-indeterminate) ones, i.e. euautographic udetorogies. Also, *p*, *q*, and *r* cannot assume any xenovalues including veracity-values and truth-values. Therefore, *the EDT's (1.23) and (1.24) prove and include as their final identities cannot be used for establishing any validity-tables, any veracity-tables, any truth-tables for their ESR's.*

4) Like the AER's *p*, *q*, and *r*, the conformal APLR's ‘**P**’, ‘**Q**’, and ‘**R**’ are vav-neutral. However, when ‘**P**’, e.g., is used xenonymously, it may assume (take on) any ER, – valid, antivalid, or vav-neutral, – as its accidental denotatum (denotation value). Therefore, in reference to potential denotata of ‘**P**’ in its range, I may assume either that $V(\mathbf{P})\hat{=}0$ or that $V(\mathbf{P})\hat{=}1$, or else that $V(\mathbf{P})\hat{=}\mathbf{i}_{\sim}|\mathbf{P}\rangle$, where $\mathbf{i}_{\sim}|\mathbf{P}\rangle$ is an IRNDEVI (irreducible non-digital euautographic validity-integron). Each one of the above three assumptions is a *condition* (*hypothesis*) that is imposed on **P**, i.e. on the ER's that are comprised in the respective restricted range of ‘**P**’ as its allowable concrete instances (accidental denotata) (cf. Cmt 7.3(5)). A like remark applies to ‘**Q**’, or ‘**R**’ in place of ‘**P**’.

5) Likewise, the ACLR's ‘*p*’, ‘*q*’, and ‘*r*’ are vav-neutral and hence ttatt-neutral (ttatt-indeterminate), i.e. neither tautologous (neither universally true) nor antitautilogous (nor universally antitrue). However, when ‘*p*’, e.g., is used xenonymously, it may assume (take on) any simple declarative sentence, – veracious (accidentally true), antiveracious (accidentally antitrue, accidentally false), or vravr-neutral (vravr-indeterminate), i.e. neither veracious nor antiveracious., – as its accidental denotatum (denotation value). Therefore, in reference to potential denotata

of ‘p’ in its range, I may assume either that $V(p) \hat{=} 0$ or that $V(p) \hat{=} 1$, or else that $V(p) \hat{=} \boldsymbol{i}_{\sim}|p\rangle$, where $\boldsymbol{i}_{\sim}|p\rangle$ is an IRNDCLVrI (irreducible non-digital catlogographic veracity-integron). Each one of the above three assumptions is a *condition* (*hypothesis*) that is imposed on p, i.e. on simple declarative sentences as potential material denotata of ‘p’ (see Cmt II.7.5 for greater detail). A like remark applies to ‘q’ or ‘r’ in place of ‘p’.

6) Validity-values validity (0) and antivalidity (1) of $U_1(\mathbf{P},\mathbf{Q},\mathbf{R})$ and $U_2(\mathbf{P},\mathbf{Q},\mathbf{R})$ versus validity-values validity (0) and antivalidity (1) of $V(\mathbf{P})$, $V(\mathbf{Q})$, and $V(\mathbf{R})$, and veracity-values veracity (0) and antiveracity (1) of $U_1(p,q,r)$ and $U_2(p,q,r)$ versus veracity-values veracity (0) and antiveracity (1) of $V(p)$, $V(q)$, and $V(r)$ are given in Table 1.1, which is analogous to Table II.7.1.

Table 1.1: ***The validity-antivalidity table for*** $U_1(\mathbf{P},\mathbf{Q},\mathbf{R})$ ***and*** $U_2(\mathbf{P},\mathbf{Q},\mathbf{R})$ ***and veracity-antiveracity table for*** $U_1(p,q,r)$ ***and*** $U_2(p,q,r)$

$V(\mathbf{P})$, $V(p)$	$V(\mathbf{Q})$, $V(q)$	$V(\mathbf{R})$, $V(r)$	$U_1(\mathbf{P},\mathbf{Q},\mathbf{R})$ $U_1(p,q,r)$	$U_2(\mathbf{P},\mathbf{Q},\mathbf{R})$ $U_2(p,q,r)$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	1
0	1	1	1	1
1	0	0	0	0
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

1.4. Valid one-premise panlogographic implications

***Th 1.7.**

1) *The law of addition*:

$$\mathbf{Q} \Rightarrow [\mathbf{P} \vee \mathbf{Q}]. \tag{1.27}$$

2) *The law of simplification*:

$$[\mathbf{P} \wedge \mathbf{Q}] \Rightarrow \mathbf{P}. \tag{1.28}$$

3) *The law of adjunction*:

$$[\mathbf{P} \wedge \mathbf{Q}] \Rightarrow [\mathbf{P} \vee \mathbf{Q}]. \tag{1.29}$$

4) *The law of assertion:*

$$\mathbf{P} \Rightarrow [[\mathbf{P} \Rightarrow \mathbf{Q}] \Rightarrow \mathbf{Q}]. \tag{1.30}$$

5) *The law of affirmation of the consequent*:

$$\mathbf{P} \Rightarrow [\mathbf{Q} \Rightarrow \mathbf{P}]. \tag{1.31}$$

6) *The law of denial of the antecedent*:

$$\neg\mathbf{P} \Rightarrow [\mathbf{P} \Rightarrow \mathbf{Q}]. \tag{1.32}$$

7) *The law of commutation*:

$$[\mathbf{R} \Rightarrow [\mathbf{P} \Rightarrow \mathbf{Q}]] \Rightarrow [\mathbf{P} \Rightarrow [\mathbf{R} \Rightarrow \mathbf{Q}]]. \tag{1.33}$$

8) *The self-distributive law of implication*:

$$[\mathbf{R} \Rightarrow [\mathbf{P} \Rightarrow \mathbf{Q}]] \Rightarrow [[\mathbf{R} \Rightarrow \mathbf{P}] \Rightarrow [\mathbf{R} \Rightarrow \mathbf{Q}]]. \tag{1.34}$$

9) *The law of summation*:

$$[\mathbf{Q} \Rightarrow \mathbf{R}] \Rightarrow [[\mathbf{P} \vee \mathbf{Q}] \Rightarrow [\mathbf{P} \vee \mathbf{R}]]. \tag{1.35}$$

Proof: All the above valid panlogographic implications follow from the pertinent versions of the pertinent PLMT's (PLDT's) of Th II.7.2, namely:

$$V(\mathbf{Q} \Rightarrow [\mathbf{P} \vee \mathbf{Q}]) \hat{=} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{P} \vee \mathbf{Q}) \hat{=} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{=} 0, \tag{1.27$_1$}$$

$$V([\mathbf{P} \wedge \mathbf{Q}] \Rightarrow \mathbf{P}) \hat{=} V(\neg[\mathbf{P} \wedge \mathbf{Q}]) \hat{\cdot} V(\mathbf{P}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{P}) \hat{=} 0, \tag{1.28$_1$}$$

$$\begin{gathered} V([\mathbf{P} \wedge \mathbf{Q}] \Rightarrow [\mathbf{P} \vee \mathbf{Q}]) \hat{=} V(\neg[\mathbf{P} \wedge \mathbf{Q}]) \hat{\cdot} V(\mathbf{P} \vee \mathbf{Q}) \\ \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{=} 0, \end{gathered} \tag{1.29$_1$}$$

$$\begin{gathered} V(\mathbf{P} \Rightarrow [[\mathbf{P} \Rightarrow \mathbf{Q}] \Rightarrow \mathbf{Q}]) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V([\mathbf{P} \Rightarrow \mathbf{Q}] \Rightarrow \mathbf{Q}) \\ \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\neg[\mathbf{P} \Rightarrow \mathbf{Q}]) \hat{\cdot} V(\mathbf{Q}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} [1 \hat{-} V(\mathbf{P} \Rightarrow \mathbf{Q})] \hat{\cdot} V(\mathbf{Q}) \\ \hat{=} V(\neg\mathbf{P}) \hat{\cdot} [1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q})] \hat{\cdot} V(\mathbf{Q}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} [1 \hat{-} V(\neg\mathbf{P})] \hat{\cdot} V(\mathbf{Q}) \\ \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{=} 0, \end{gathered} \tag{1.30$_1$}$$

$$V(\mathbf{P} \Rightarrow [\mathbf{Q} \Rightarrow \mathbf{P}]) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q} \Rightarrow \mathbf{P}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{P}) \hat{=} 0, \tag{1.31$_1$}$$

$$\begin{gathered} V(\neg\mathbf{P} \Rightarrow [\mathbf{P} \Rightarrow \mathbf{Q}]) \hat{=} V(\neg\neg\mathbf{P}) \hat{\cdot} V(\mathbf{P} \Rightarrow \mathbf{Q}) \\ \hat{=} V(\mathbf{P}) \hat{\cdot} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{=} 0, \end{gathered} \tag{1.32$_1$}$$

$$\begin{gathered} V([\mathbf{R} \Rightarrow [\mathbf{P} \Rightarrow \mathbf{Q}]] \Rightarrow [\mathbf{P} \Rightarrow [\mathbf{R} \Rightarrow \mathbf{Q}]]) \\ \hat{=} [1 \hat{-} V(\mathbf{R} \Rightarrow [\mathbf{P} \Rightarrow \mathbf{Q}])] \hat{\cdot} V(\mathbf{P} \Rightarrow [\mathbf{R} \Rightarrow \mathbf{Q}]) \\ \hat{=} [1 \hat{-} V(\neg\mathbf{R}) \hat{\cdot} V(\mathbf{P} \Rightarrow \mathbf{Q})] \hat{\cdot} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R} \Rightarrow \mathbf{Q}) \\ \hat{=} [1 \hat{-} V(\neg\mathbf{R}) \hat{\cdot} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q})] \hat{\cdot} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{R}) \hat{\cdot} V(\mathbf{Q}) \hat{=} 0, \end{gathered} \tag{1.33$_1$}$$

$$\begin{array}{c} V([\mathbf{R}\Rightarrow[\mathbf{P}\Rightarrow\mathbf{Q}]]\Rightarrow[[\mathbf{R}\Rightarrow\mathbf{P}]\Rightarrow[\mathbf{R}\Rightarrow\mathbf{Q}]]) \\ \hat{=}[1\hat{-}V(\mathbf{R}\Rightarrow[\mathbf{P}\Rightarrow\mathbf{Q}])]\hat{\cdot}V([\mathbf{R}\Rightarrow\mathbf{P}]\Rightarrow[\mathbf{R}\Rightarrow\mathbf{Q}]) \\ \hat{=}[1\hat{-}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{P}\Rightarrow\mathbf{Q})]\hat{\cdot}[1\hat{-}V(\mathbf{R}\Rightarrow\mathbf{P})]\hat{\cdot}V(\mathbf{R}\Rightarrow\mathbf{Q}) \\ \hat{=}[1\hat{-}V(\neg\mathbf{R})\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})]\hat{\cdot}[1\hat{-}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{P})]\hat{\cdot}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{Q}) \\ \hat{=}[1\hat{-}V(\neg\mathbf{R})\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{P})]\hat{\cdot}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{Q}) \\ \hat{=}[V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\neg\mathbf{R})\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\mathbf{P})] \\ \hat{=}[1\hat{-}V(\neg\mathbf{P})\hat{-}V(\mathbf{P})]\hat{\cdot}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{Q})\hat{=}[1\hat{-}1]\hat{\cdot}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{Q})\hat{=}0, \end{array} \tag{1.34$_1$}$$

$$\begin{array}{c} V([\mathbf{Q}\Rightarrow\mathbf{R}]\Rightarrow[[\mathbf{P}\vee\mathbf{Q}]\Rightarrow[\mathbf{P}\vee\mathbf{R}]]) \\ \hat{=}V(\neg[\mathbf{Q}\Rightarrow\mathbf{R}])\hat{\cdot}V([\mathbf{P}\vee\mathbf{Q}]\Rightarrow[\mathbf{P}\vee\mathbf{R}]) \\ \hat{=}[1\hat{-}V(\mathbf{Q}\Rightarrow\mathbf{R})]\hat{\cdot}V(\neg[\mathbf{P}\vee\mathbf{Q}])\hat{\cdot}V(\mathbf{P}\vee\mathbf{R}) \\ \hat{=}[1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R})]\hat{\cdot}[1\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})]\hat{\cdot}V(\mathbf{P})\hat{\cdot}V(\mathbf{R}) \\ \hat{=}[1\hat{-}V(\neg\mathbf{Q})]\hat{\cdot}[1\hat{-}V(\mathbf{Q})]\hat{\cdot}V(\mathbf{P})\hat{\cdot}V(\mathbf{R}) \\ \hat{=}V(\mathbf{Q})\hat{\cdot}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{P})\hat{\cdot}V(\mathbf{R})\hat{=}0. \end{array} \tag{1.35$_1$}$$

By (1.27$_1$)–(1.35$_1$), the pertinent instances of (II.4.40a) immediately infer (1.27)–(1.35) respectively.•

°**Crl 1.3.** The following valid (kyrologous) EOR's are concrete conformal euautographic interpretands (instances, corollaries) of the valid (kyrologous) PLOR's (1.27)–(1.35) in that order:

$$q\Rightarrow[p\vee q]. \tag{1.27μ}$$

$$[p\wedge q]\Rightarrow p. \tag{1.28μ}$$

$$[p\wedge q]\Rightarrow[p\vee q]. \tag{1.29μ}$$

$$p\Rightarrow[[p\Rightarrow q]\Rightarrow q]. \tag{1.30μ}$$

$$p\Rightarrow[q\Rightarrow p]. \tag{1.31μ}$$

$$\neg p\Rightarrow[p\Rightarrow q]. \tag{1.32μ}$$

$$[r\Rightarrow[p\Rightarrow q]]\Rightarrow[p\Rightarrow[r\Rightarrow q]]. \tag{1.33μ}$$

$$[r\Rightarrow[p\Rightarrow q]]\Rightarrow[[r\Rightarrow p]\Rightarrow[r\Rightarrow q]]. \tag{1.34μ}$$

$$[q\Rightarrow r]\Rightarrow[[p\vee q]\Rightarrow[p\vee r]]. \tag{1.35μ}$$ •

⁺**Crl 1.4.** In accordance with Ax I.8.2 and in analogy with ⁺Crls 1.3 and 1.4, here follow catlogographic tautologies, which are CFCL interpretands of the above euautographic kyrologies (1.27μ)–(1.35μ) in that order:

$$q\Rightarrow[p\vee q]. \tag{1.27κ}$$

$$[p \wedge q] \Rightarrow p. \tag{1.28κ}$$

$$[p \wedge q] \Rightarrow [p \vee q]. \tag{1.29κ}$$

$$p \Rightarrow [[p \Rightarrow q] \Rightarrow q]. \tag{1.30κ}$$

$$p \Rightarrow [q \Rightarrow p]. \tag{1.31κ}$$

$$\neg p \Rightarrow [p \Rightarrow q]. \tag{1.32κ}$$

$$[r \Rightarrow [p \Rightarrow q]] \Rightarrow [p \Rightarrow [r \Rightarrow q]]. \tag{1.33κ}$$

$$[r \Rightarrow [p \Rightarrow q]] \Rightarrow [[r \Rightarrow p] \Rightarrow [r \Rightarrow q]]. \tag{1.34κ}$$

$$[q \Rightarrow r] \Rightarrow [[p \vee q] \Rightarrow [p \vee r]]. \tag{1.35κ•}$$

1.5. A summary of the panlogographic transitive laws for $\Leftrightarrow$ and $\Rightarrow$ relative to $\Rightarrow$

The laws, which are mentioned in the above head and which have been established in section II.7 as (II.7.71a)–(II.7.76a), are valid panlogographic relations of implication. Therefore, for the sake of completeness and for convenience in the further discussion, those relations are restated below in that order:

$$[[\mathbf{P} \Leftrightarrow \mathbf{Q}] \wedge [\mathbf{Q} \Leftrightarrow \mathbf{R}]] \Rightarrow [\mathbf{P} \Leftrightarrow \mathbf{R}], \tag{1.36}$$

$$[\mathbf{P} \Leftrightarrow \mathbf{Q}] \wedge [\mathbf{Q} \Leftrightarrow \mathbf{R}] \Rightarrow [\mathbf{Q} \Leftrightarrow [\mathbf{P} \wedge \mathbf{R}]], \tag{1.37}$$

$$[[\mathbf{P} \Leftrightarrow \mathbf{Q}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]] \Rightarrow [\mathbf{P} \Rightarrow \mathbf{R}], \tag{1.38}$$

$$[[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{Q} \Leftrightarrow \mathbf{R}]] \Rightarrow [\mathbf{P} \Rightarrow \mathbf{R}], \tag{1.39}$$

$$[[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]] \Rightarrow [\mathbf{P} \Rightarrow \mathbf{R}], \tag{1.40}$$

$$[[\mathbf{P} \wedge \mathbf{Q}] \wedge [\mathbf{Q} \wedge \mathbf{R}]] \Rightarrow [\mathbf{P} \wedge \mathbf{R}]. \tag{1.41}$$

°**Crl 1.5.** The following valid (kyrologous) EOR's are concrete conformal euautographic interpretands (instances, corollaries) of the valid (kyrologous) PLOR's (1.36)–(1.41) in that order:

$$[p \Leftrightarrow q] \wedge [q \Leftrightarrow r] \Rightarrow [p \Leftrightarrow r]. \tag{1.36μ}$$

$$[p \Leftrightarrow q] \wedge [p \Leftrightarrow r] \Rightarrow [p \Leftrightarrow [q \wedge r]]. \tag{1.37μ}$$

$$[p \Leftrightarrow q] \wedge [q \Rightarrow r] \Rightarrow [p \Rightarrow r]. \tag{1.38μ}$$

$$[p \Rightarrow q] \wedge [q \Leftrightarrow r] \Rightarrow [p \Rightarrow r]. \tag{1.39μ}$$

$$[p \Rightarrow q] \wedge [q \Rightarrow r] \Rightarrow [p \Rightarrow r]. \tag{1.40μ}$$

$$[p \wedge q] \wedge [q \wedge r] \Rightarrow [p \wedge r]. \tag{1.41μ•}$$

+**Crl 1.6.** In accordance with Ax I.8.2 and in analogy with +Crls 1.3, 1.4, and 1.6, here follow catlogographic tautologies, which are CFCL interpretands of the above euautographic kyrologies (1.36μ)–(1.41μ) in that order:

$$[p \Leftrightarrow q] \wedge [q \Leftrightarrow r] \Rightarrow [p \Leftrightarrow r]. \qquad (1.36\kappa)$$

$$[p \Leftrightarrow q] \wedge [p \Leftrightarrow r] \Rightarrow [p \Leftrightarrow [q \wedge r]]. \qquad (1.37\kappa)$$

$$[p \Leftrightarrow q] \wedge [q \Rightarrow r] \Rightarrow [p \Rightarrow r]. \qquad (1.38\kappa)$$

$$[p \Rightarrow q] \wedge [q \Leftrightarrow r] \Rightarrow [p \Rightarrow r]. \qquad (1.39\kappa)$$

$$[p \Rightarrow q] \wedge [q \Rightarrow r] \Rightarrow [p \Rightarrow r]. \qquad (1.40\kappa)$$

$$[p \wedge q] \wedge [q \wedge r] \Rightarrow [p \wedge r]. \qquad (1.41\kappa)\bullet$$

2. Selected valid equivalence panlogographic relations and their master theorems

2.1. Valid equivalencies of two panlogographic relations, neither of which is an equivalence

Th 2.1:** ***De Morgan's laws **[*of duality for*** $\vee$ ***and*** $\wedge$**].**

$$\neg[\mathbf{P} \wedge \mathbf{Q}] \Leftrightarrow [\neg\mathbf{P} \vee \neg\mathbf{Q}]. \qquad (2.1)$$

$$\neg[\mathbf{P} \vee \mathbf{Q}] \Leftrightarrow [\neg\mathbf{P} \wedge \neg\mathbf{Q}]. \qquad (2.2)$$

Proof: The instance of (II.7.7γ) with '$\neg[\mathbf{P} \wedge \mathbf{Q}]$' and '$[\neg\mathbf{P} \vee \neg\mathbf{Q}]$', or that with '$\neg[\mathbf{P} \vee \mathbf{Q}]$' and '$[\neg\mathbf{P} \wedge \neg\mathbf{Q}]$', in place of '**P**' and '**Q**' respectively yields:

$$V(\neg[\mathbf{P} \wedge \mathbf{Q}] \Leftrightarrow [\neg\mathbf{P} \vee \neg\mathbf{Q}]) \hat{=} [V(\neg[\mathbf{P} \wedge \mathbf{Q}]) \hat{-} V(\neg\mathbf{P} \vee \neg\mathbf{Q})]^2 \hat{=} 0, \qquad (2.1_1)$$

$$V(\neg[\mathbf{P} \vee \mathbf{Q}] \Leftrightarrow [\neg\mathbf{P} \wedge \neg\mathbf{Q}]) \hat{=} [V(\neg[\mathbf{P} \vee \mathbf{Q}]) \hat{-} V(\neg\mathbf{P} \wedge \neg\mathbf{Q})]^2 \hat{=} 0, \qquad (2.2_1)$$

respectively, because

$$V(\neg[\mathbf{P} \wedge \mathbf{Q}]) \hat{=} V(\neg\mathbf{P} \vee \neg\mathbf{Q}), \qquad (2.1_2)$$

by (II.7.9γ), and also because it follows from comparison of the variant (II.7.6γ) with '$\neg\mathbf{P}$' and '$\neg\mathbf{Q}$' in place of '**P**' and '**Q**' respectively, subject to (1.1_1), and of (II.7.8γ) that

$$V(\neg[\mathbf{P} \vee \mathbf{Q}]) \hat{=} V(\neg\mathbf{P} \wedge \neg\mathbf{Q}). \qquad (2.2_2)$$

The master-theorems (2.1_1) and (2.2_1) immediately infer the kyrologies (2.1) and (2.2) respectively by the instances of (II.4.40a) with each one of the relations (2.1) and (2.2) in turn in place of **P**.•

Cmt 2.1. 1) Regarding *De Morgan's laws*, Wikipedia says in the article of the same name:

> The laws are named «after Augustus De Morgan (1806–1871) who introduced a formal version of the laws to classical propositional logic. De Morgan's formulation was influenced by algebraization of logic undertaken by George Boole, which later cemented De Morgan's claim to the find. Although a similar observation was made by Aristotle and was known to Greek and Medieval logicians (in the 14th century William of Ockham wrote down the words that would result by reading the laws out), De Morgan is given credit for stating the laws formally and incorporating them in to the language of logic. De Morgan's Laws can be proved easily, and may even seem trivial. Nonetheless, these laws are helpful in making valid inferences in proofs and deductive arguments.»

That is to say, De Morgan's laws belong to traditional sentential logic and not to modern logic, as one might have concluded from their name.

2) De Morgan's laws in panlogographic setting, (2.1) and (2.2), can be expressed in words thus: *The negation of the conjunction of two PLR's, or two ER's, is equivalent to the inclusive disjunction of the negations of the PLR's*, *or ER's; and similarly with* "conjunction" *and* "inclusive disjunction" *exchanged*. Accordingly, De Morgan's laws in catlogographic setting, (2.1κ) and (2.2κ), as given below in this section can be expressed in words thus: *The negation of the conjunction of two CLR's* (or, in general, *propositions*) *is equivalent to the inclusive disjunction of the negations of the CLR's* (correspondingly, *propositions*)*; and similarly with* "conjunction" *and* "inclusive disjunction" *exchanged*.

3) By (II.7.6γ),

$$V(\mathbf{P}\wedge\mathbf{Q})\,\hat{=}\,V(\neg[\neg\mathbf{P}\vee\neg\mathbf{Q}]). \qquad (2.1_3)$$

At the same time, from (II.7.2γ) and from the variant (II.7.9γ) with '$\neg\mathbf{P}$' and '$\neg\mathbf{Q}$' in place of '**P**' and '**Q**' respectively, subject to (1.1_1), it follows that

$$V(\mathbf{P}\vee\mathbf{Q})\,\hat{=}\,V(\neg[\neg\mathbf{P}\wedge\neg\mathbf{Q}]) \qquad (2.2_3)$$

(see also (II.7.61)). By (2.1_3), or (2.2_3), it immediately follows from the variant of (II.7.50) with '$[\mathbf{P}\wedge\mathbf{Q}]$' and '$\neg[\neg\mathbf{P}\vee\neg\mathbf{Q}]$', or from that with '$[\mathbf{P}\vee\mathbf{Q}]$' and '$\neg[\neg\mathbf{P}\wedge\neg\mathbf{Q}]$', in place of '**P**' and '**Q**' respectively that

$$[\mathbf{P} \wedge \mathbf{Q}] \Leftrightarrow \neg[\neg\mathbf{P} \vee \neg\mathbf{Q}], \tag{2.1'}$$

$$[\mathbf{P} \vee \mathbf{Q}] \Leftrightarrow \neg[\neg\mathbf{P} \wedge \neg\mathbf{Q}] \tag{2.2'}$$

(see also (see also (II.7.61')). These two kyrologies can be called *Modified De Morgan's laws* and can be expressed in words thus: *The conjunction of two PLR's, or two ER's, is equivalent to the negation of the inclusive disjunction of the negations of the PLR's, or ER's; and similarly with* "conjunction" *and* "inclusive disjunction" *exchanged*.•

Th 2.2:** ***Three bilateral laws of inclusive disjunction, conjunction, and exclusive disjunction, of two implications with the same antecedent**:**

$$[\mathbf{P} \Rightarrow \mathbf{Q}] \vee [\mathbf{P} \Rightarrow \mathbf{R}] \Leftrightarrow [\mathbf{P} \Rightarrow [\mathbf{Q} \vee \mathbf{R}]]. \tag{2.3}$$

$$[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Rightarrow \mathbf{R}] \Leftrightarrow [\mathbf{P} \Rightarrow [\mathbf{Q} \wedge \mathbf{R}]]. \tag{2.4}$$

$$[[\mathbf{P} \Rightarrow \mathbf{Q}] \overline{\Leftrightarrow} [\mathbf{P} \Rightarrow \mathbf{R}]] \Leftrightarrow [\neg[\mathbf{P} \Rightarrow [\mathbf{Q} \Leftrightarrow \mathbf{R}]]]. \tag{2.5}$$

Proof: 1) By the pertinent instances of (II.4.2), (II.7.1γ)–(II.7.3γ), (II.7.6γ), (II.7.7γ), and (II.7.12γ), it follows that

$$\begin{gathered} V([\mathbf{P} \Rightarrow \mathbf{Q}] \vee [\mathbf{P} \Rightarrow \mathbf{R}]) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{\cdot} V(\mathbf{P} \Rightarrow \mathbf{R}) \\ \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\mathbf{R}) \\ \hat{=} V(\mathbf{P} \Rightarrow [\mathbf{Q} \vee \mathbf{R}]), \end{gathered} \tag{2.3$_1$}$$

$$\begin{gathered} V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Rightarrow \mathbf{R}]) \hat{=} 1 \hat{-} V(\neg[\mathbf{P} \Rightarrow \mathbf{Q}]) \hat{\cdot} V(\neg[\mathbf{P} \Rightarrow \mathbf{R}]) \\ \hat{=} 1 \hat{-} [1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q})] \hat{\cdot} [1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R})] \\ \hat{=} 1 \hat{-} [1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{+} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\mathbf{R})] \\ \hat{=} V(\neg\mathbf{P}) \hat{\cdot} [V(\mathbf{Q}) \hat{+} V(\mathbf{R}) \hat{-} V(\mathbf{Q}) \hat{\cdot} V(\mathbf{R})] \hat{=} V(\mathbf{P} \Rightarrow [\mathbf{Q} \wedge \mathbf{R}]), \end{gathered} \tag{2.4$_1$}$$

$$\begin{gathered} V([\mathbf{P} \Rightarrow \mathbf{Q}] \overline{\Leftrightarrow} [\mathbf{P} \Rightarrow \mathbf{R}]) \\ \hat{=} 1 \hat{-} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{-} V(\mathbf{P} \Rightarrow \mathbf{R}) \hat{+} 2 \hat{\cdot} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{\cdot} V(\mathbf{P} \Rightarrow \mathbf{R}) \\ \hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{+} 2 \hat{\cdot} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \\ \hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} [V(\mathbf{Q}) \hat{+} V(\mathbf{R}) \hat{-} 2 \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\mathbf{R})] \\ \hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q} \Leftrightarrow \mathbf{R}) \hat{=} 1 \hat{-} V(\mathbf{P} \Rightarrow [\mathbf{Q} \Leftrightarrow \mathbf{R}]) \hat{=} V(\neg[\mathbf{P} \Rightarrow [\mathbf{Q} \Leftrightarrow \mathbf{R}]]). \end{gathered} \tag{2.5$_1$}$$

Kyrologies (2.3)–(2.5) immediately follow from the pertinent instances of (II.7.50) by (2.3$_1$)–(2.5$_1$) respectively.•

Th 2.3:** ***Two bilateral laws of absurdity.

$$[[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Rightarrow \neg\mathbf{Q}]] \Leftrightarrow \neg\mathbf{P}. \tag{2.6}$$

$$[\mathbf{P} \Rightarrow [\mathbf{Q} \wedge \neg\mathbf{Q}]] \Leftrightarrow \neg\mathbf{P}. \tag{2.7}$$

Kyrologies (2.6) and (2.7) are called *the first* and *second bilateral laws of absurdity* (*reductio ad absurdum, reductio ad impossibile*).

Proof: PLR (2.6) is the specific instance of kyrology (2.6) with ‘¬**Q**’ in place of ‘**R**’. Indeed, by the variant of (II.7.1γ) with **Q** as **P**, it follows from the instance of (II.7.6γ) with ‘**Q**’ and ‘¬**Q**’ in place of ‘**P**’ and ‘**Q**’ respectively that

$$V(\mathbf{Q}\wedge\neg\mathbf{Q})\,\hat{=}\,V(\mathbf{P}\,\bar{\wedge}\,\neg\mathbf{P})\,\hat{=}\,V(\mathbf{Q})\,\hat{+}\,V(\neg\mathbf{Q})\,\hat{-}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{=}\,1, \tag{2.6$_1$}$$

in agreement with the variant of (1.7_1) with **Q** as **P**. Consequently, the variant of (II.7.3γ) with ‘[**Q**∧¬**Q**]’ in place of ‘**Q**’ becomes:

$$\begin{gathered}V(\mathbf{P}\Rightarrow[\mathbf{Q}\wedge\neg\mathbf{Q}])\,\hat{=}\,V([\neg\mathbf{P}]\vee[\mathbf{Q}\wedge\neg\mathbf{Q}])\,\hat{=}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q}\wedge\neg\mathbf{Q})\\ \hat{=}\,V(\neg\mathbf{P})\,\hat{\cdot}\,1\,\hat{=}\,V(\neg\mathbf{P}).\end{gathered} \tag{2.7$_1$}$$

Hence, the variant of (2.4_1) with ‘¬**Q**’ in place of ‘**R**’ reduces to

$$V([\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{P}\Rightarrow\neg\mathbf{Q}])\,\hat{=}\,V(\neg\mathbf{P}). \tag{2.6$_2$}$$

By (2.6_2) and (2.7_1), the two pertinent versions of (II.7.50) immediately infer (2.6) and (2.7).•

Cmt 2.2. In accordance with (II.7.7γ) and in agreement with the pertinent general remark of Preliminaery Remark 1.1, kyrology (2.6) is equivalent to the conjunctions of these two:

$$[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{P}\Rightarrow\neg\mathbf{Q}]]\Rightarrow\neg\mathbf{P}, \tag{2.6'}$$

$$\neg\mathbf{P}\Rightarrow[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{P}\Rightarrow\neg\mathbf{Q}]], \tag{2.6''}$$

whereas kyrology (2.7) is equivalent to the conjunctions of these two:

$$[\mathbf{P}\Rightarrow[\mathbf{Q}\wedge\neg\mathbf{Q}]]\Rightarrow\neg\mathbf{P}, \tag{2.7'}$$

$$\neg\mathbf{P}\Rightarrow[\mathbf{P}\Rightarrow[\mathbf{Q}\wedge\neg\mathbf{Q}]]. \tag{2.7''}$$

The validity of the above four kyrologies can be demonstrated straightforwardly with the help of (2.6_2) and (2.7_1) thus:

$$\begin{gathered}V([[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{P}\Rightarrow\neg\mathbf{Q}]]\Rightarrow\neg\mathbf{P})\\ \hat{=}\,V(\neg[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{P}\Rightarrow\neg\mathbf{Q}]])\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{=}\,[1\,\hat{-}\,V(\neg\mathbf{P})]\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{=}\,0,\end{gathered} \tag{2.6$_3$}$$

$$\begin{gathered}V(\neg\mathbf{P}\Rightarrow[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{P}\Rightarrow\neg\mathbf{Q}]])\\ \hat{=}\,V(\neg\neg\mathbf{P})\,\hat{\cdot}\,V([\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{P}\Rightarrow\neg\mathbf{Q}])\,\hat{=}\,V(\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{=}\,0,\end{gathered} \tag{2.6$_4$}$$

$$\begin{gathered}V([\mathbf{P}\Rightarrow[\mathbf{Q}\wedge\neg\mathbf{Q}]]\Rightarrow\neg\mathbf{P})\,\hat{=}\,V(\neg[\mathbf{P}\Rightarrow[\mathbf{Q}\wedge\neg\mathbf{Q}]])\,\hat{\cdot}\,V(\neg\mathbf{P})\\ \hat{=}\,[1\,\hat{-}\,V(\neg\mathbf{P})]\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{=}\,0,\end{gathered} \tag{2.7$_2$}$$

$$\begin{gathered}V(\neg\mathbf{P}\Rightarrow[\mathbf{P}\Rightarrow[\mathbf{Q}\wedge\neg\mathbf{Q}]])\,\hat{=}\,V(\neg\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{P}\Rightarrow[\mathbf{Q}\wedge\neg\mathbf{Q}])\\ \hat{=}\,V(\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{P})\,\hat{=}\,0.\end{gathered} \tag{2.7$_3$}$$

In agreement with Preliminary Remark 1.1, kyrologies (2.6'), (2.6''), (2.7'), and (2.7'') are called the *first direct law of absurdity*, the *first converse law of absurdity*, the *second direct law of absurdity*, and the *second converse law of absurdity* in that order.•

*__Th 2.4:__ ***The bilateral law of contraposition.***

$$[\mathbf{P} \Rightarrow \mathbf{Q}] \Leftrightarrow [\neg\mathbf{Q} \Rightarrow \neg\mathbf{P}]. \tag{2.8}$$

Proof: From the pertinent versions of (II.7.1γ) and (II.7.3γ), it follows that

$$V(\neg\mathbf{Q} \Rightarrow \neg\mathbf{P}) \mathrel{\hat{=}} V(\neg\neg\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \mathrel{\hat{=}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} V(\mathbf{P} \Rightarrow \mathbf{Q}). \tag{2.8$_1$}$$

By (2.8$_1$), the two pertinent version of (II.7.50) immediately infers (2.8).•

Cmt 2.3 (*Analogous to Cmt 2.2***).** In accordance with (II.7.7γ) and in agreement with the pertinent general remark of Preliminary Remark 1.1, kyrology (2.8) is equivalent to the conjunctions of these two:

$$[\mathbf{P} \Rightarrow \mathbf{Q}] \Rightarrow [\neg\mathbf{Q} \Rightarrow \neg\mathbf{P}], \tag{2.8'}$$

$$[\neg\mathbf{Q} \Rightarrow \neg\mathbf{P}] \Rightarrow [\mathbf{P} \Rightarrow \mathbf{Q}], \tag{2.8''}$$

The validity of the above four kyrologies can be demonstrated straightforwardly with the help of (2.8$_1$) thus:

$$\begin{aligned} V([\mathbf{P} \Rightarrow \mathbf{Q}] \Rightarrow [\neg\mathbf{Q} \Rightarrow \neg\mathbf{P}]) &\mathrel{\hat{=}} V(\neg[\mathbf{P} \Rightarrow \mathbf{Q}]) \mathbin{\hat{\cdot}} V(\neg\mathbf{Q} \Rightarrow \neg\mathbf{P}) \\ &\mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\mathbf{P} \Rightarrow \mathbf{Q})] \mathbin{\hat{\cdot}} V(\neg\neg\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \\ &\mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q})] \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \\ &\mathrel{\hat{=}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathbin{\hat{-}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V \mathrel{\hat{=}} 0, \end{aligned} \tag{2.8$_2$}$$

$$\begin{aligned} V([\neg\mathbf{Q} \Rightarrow \neg\mathbf{P}] \Rightarrow [\mathbf{P} \Rightarrow \mathbf{Q}]) &\mathrel{\hat{=}} V(\neg[\neg\mathbf{Q} \Rightarrow \neg\mathbf{P}]) \mathbin{\hat{\cdot}} V(\mathbf{P} \Rightarrow \mathbf{Q}) \\ &\mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\neg\mathbf{Q} \Rightarrow \neg\mathbf{P})] \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \\ &\mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\neg\neg\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{P})] \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \\ &\mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q})] \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathbin{\hat{-}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} 0. \end{aligned} \tag{2.8$_3$}$$

In agreement with Preliminary Remark 1.1, kyrologies (2.8') and (2.8'') are called the *direct law of contraposition* and the *converse law of contraposition* respectively.•

*__Th 2.5:__ ***The bilateral modified laws of contraposition.***

$$[\mathbf{P} \Rightarrow \mathbf{Q}] \Leftrightarrow [[\mathbf{P} \Rightarrow \neg\mathbf{Q}] \Rightarrow \neg\mathbf{P}]. \tag{2.9}$$

$$[\mathbf{P} \Rightarrow \mathbf{Q}] \Leftrightarrow [[\mathbf{P} \wedge \neg\mathbf{Q}] \Rightarrow \neg\mathbf{P}]. \tag{2.10}$$

Proof: From the pertinent versions of (II.7.1γ), (II.7.3γ), and (II.7.6γ), it follows that

$$\begin{aligned} V([\mathbf{P} \Rightarrow \neg\mathbf{Q}] \Rightarrow \neg\mathbf{P}) &\mathrel{\hat{=}} V(\neg[\mathbf{P} \Rightarrow \neg\mathbf{Q}]) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \\ \mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\mathbf{P} \Rightarrow \neg\mathbf{Q})] \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) &\mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\neg\mathbf{Q})] \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \\ \mathrel{\hat{=}} [V(\neg\mathbf{P}) \mathbin{\hat{-}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\neg\mathbf{Q})] &\mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\neg\mathbf{Q})] \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \\ &\mathrel{\hat{=}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} V(\mathbf{P} \Rightarrow \mathbf{Q}), \end{aligned} \tag{2.9$_1$}$$

$$V([\mathbf{P} \wedge \neg\mathbf{Q}] \Rightarrow \neg\mathbf{P}) \hat{=} V(\neg[\mathbf{P} \wedge \neg\mathbf{Q}]) \hat{\cdot} V(\neg\mathbf{P})$$
$$\hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\neg\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{P}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}). \tag{2.10$_1$}$$

By (2.9$_1$) and (2.10$_1$), the two pertinent versions of (II.7.50) immediately infer (2.9) and (2.10).•

Th 2.6:** ***The bilateral law of exportation and importation.

$$[[\mathbf{P} \wedge \mathbf{Q}] \Rightarrow \mathbf{R}] \Leftrightarrow [\mathbf{P} \Rightarrow [\mathbf{Q} \Rightarrow \mathbf{R}]], \tag{2.11}$$

the understanding being that

$$[[\mathbf{P} \wedge \mathbf{Q}] \Rightarrow \mathbf{R}] \Rightarrow [\mathbf{P} \Rightarrow [\mathbf{Q} \Rightarrow \mathbf{R}]] \tag{2.11a}$$

is the *law of exportation*, whereas

$$[[\mathbf{P} \wedge \mathbf{Q}] \Rightarrow \mathbf{R}] \Leftarrow [\mathbf{P} \Rightarrow [\mathbf{Q} \Rightarrow \mathbf{R}]] \tag{2.11b}$$

is the *law of importation*.

Proof: From the pertinent versions of (II.7.1γ), (II.7.3γ), and (II.7.6γ), it follows that

$$V([\mathbf{P} \wedge \mathbf{Q}] \Rightarrow \mathbf{R}) \hat{=} V(\neg[\mathbf{P} \wedge \mathbf{Q}]) \hat{\cdot} V(\mathbf{R}) \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{R})$$
$$\hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q} \Rightarrow \mathbf{R}) \hat{=} V(\mathbf{P} \Rightarrow [\mathbf{Q} \Rightarrow \mathbf{R}]). \tag{2.11$_1$}$$

By (2.11$_1$), the pertinent version of (II.7.50) immediately infers (2.11).•

Th 2.7:** ***The law of a double (two-fold) implication.

$$[[\mathbf{P} \Rightarrow \mathbf{Q}] \Rightarrow \mathbf{Q}] \Leftrightarrow [\mathbf{P} \vee \mathbf{Q}]. \tag{2.12}$$

Proof: From the pertinent versions of (II.7.1γ)–(II.7.3γ), it follows that

$$V([\mathbf{P} \Rightarrow \mathbf{Q}] \Rightarrow \mathbf{Q}) \hat{=} V(\neg[\mathbf{P} \Rightarrow \mathbf{Q}]) \hat{\cdot} V(\mathbf{Q})$$
$$\hat{=} [1 \hat{-} V(\mathbf{P} \Rightarrow \mathbf{Q})] \hat{\cdot} V(\mathbf{Q}) \hat{=} [1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q})] \hat{\cdot} V(\mathbf{Q})$$
$$\hat{=} \left[V(\mathbf{Q}) \hat{-} V(\neg\mathbf{P}) \hat{\cdot} [V(\mathbf{Q})]^2\right] \hat{=} [1 \hat{-} V(\neg\mathbf{P})] \hat{\cdot} V(\mathbf{Q})$$
$$\hat{=} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{=} V(\mathbf{P} \vee \mathbf{Q}), \tag{2.12$_1$}$$

By (2.12$_1$), the pertinent version of (II.7.50) immediately infers (2.12).•

Th 2.8:** ***The bilateral law of consistency.

$$[[\mathbf{P} \Rightarrow \mathbf{Q}] \Rightarrow [\mathbf{P} \Rightarrow \neg\mathbf{Q}]] \Leftrightarrow \mathbf{P}. \tag{2.13}$$

Proof: From the pertinent versions of (II.7.1γ) and (II.7.3γ), it follows that

$$V([[\mathbf{P} \Rightarrow \mathbf{Q}] \Rightarrow [\mathbf{P} \Rightarrow \neg\mathbf{Q}]]) \hat{=} [1 \hat{-} V(\mathbf{P} \Rightarrow \mathbf{Q})] \hat{\cdot} V(\mathbf{P} \Rightarrow \neg\mathbf{Q})$$
$$\hat{=} [1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q})] \hat{\cdot} [1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q})]$$
$$\hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} [V(\mathbf{Q}) \hat{+} V(\neg\mathbf{Q})] \hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{=} V(\mathbf{P}). \tag{2.13$_1$}$$

By (2.13$_1$), the pertinent version of (II.7.50) immediately infers (2.13).•

Th 2.9:** ***The law of negation for implication.

$$\neg[\mathbf{P} \Rightarrow \mathbf{Q}] \Leftrightarrow [\mathbf{P} \wedge \neg\mathbf{Q}]. \tag{2.14}$$

Proof: From the pertinent versions of (II.7.1γ), (II.7.3γ), (II.7.6γ), and (1.1_1), it follows that

$$V(\neg[\mathbf{P} \Rightarrow \mathbf{Q}]) \hat{=} 1 \hat{-} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}), \tag{2.14_1}$$

$$V(\mathbf{P} \wedge \neg\mathbf{Q}) \hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\neg\mathbf{Q}) \hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}), \tag{2.14_2}$$

whence

$$V(\neg[\mathbf{P} \Rightarrow \mathbf{Q}]) \hat{=} V(\mathbf{P} \wedge \neg\mathbf{Q}) \hat{=} 1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}). \tag{2.14_3}$$

By (2.14_3), the pertinent version of (II.7.50) immediately infers (2.14).•

2.2. Valid equivalencies of two panlogographic relations, one of which is an equivalence

***Th 2.10.**

$$[\mathbf{P} \Leftrightarrow \mathbf{Q}] \Leftrightarrow [V(\mathbf{P}) \hat{=} V(\mathbf{Q})]. \tag{2.15}$$

Proof: By (II.4.2) and (II.7.7γ), axiom (II.4.3) can be developed thus:

$$V(V(\mathbf{P}) \hat{=} V(\mathbf{Q})) \hat{=} [V(\mathbf{P}) \hat{-} V(\mathbf{Q})]^2 \hat{=} V(\mathbf{P} \Leftrightarrow \mathbf{Q}), \tag{2.15_1}$$

which is the same as the train of identities (II.6.23). By (2.15_1), the version of (II.7.7γ) with '$[\mathbf{P} \Leftrightarrow \mathbf{Q}]$' in place of '**P**' and '$[V(\mathbf{P}) \hat{=} V(\mathbf{Q})]$' in place of '**Q**' yields:

$$\begin{gathered} V([\mathbf{P} \Leftrightarrow \mathbf{Q}] \Leftrightarrow [V(\mathbf{P}) \hat{=} V(\mathbf{Q})]) \hat{=} [V(\mathbf{P} \Leftrightarrow \mathbf{Q}) \hat{-} V(V(\mathbf{P}) \hat{=} V(\mathbf{Q}))]^2 \\ \hat{=} [V(\mathbf{P} \Leftrightarrow \mathbf{Q}) \hat{-} V(\mathbf{P} \Leftrightarrow \mathbf{Q})]^2 \hat{=} 0, \end{gathered} \tag{2.15_2}$$

which is the same as the train of identities ($\text{II}.7.57_1$) and which immediately infers relation (2.15) by the instance of (II.4.40a) with that relation in place of **P**.•

Th 2.11:** ***The law of equivalence of an equivalence relation and of the appropriate inclusive disjunction.

$$[\mathbf{P} \Leftrightarrow \mathbf{Q}] \Leftrightarrow [[\mathbf{P} \wedge \mathbf{Q}] \vee [\neg\mathbf{P} \wedge \neg\mathbf{Q}]]. \tag{2.16}$$

Proof: From the pertinent versions of (II.7.1γ), (II.7.2γ), (II.7.6γ), and (II.7.7γ), it follows that

$$\begin{gathered} V([\mathbf{P} \wedge \mathbf{Q}] \vee [\neg\mathbf{P} \wedge \neg\mathbf{Q}]) \hat{=} V(\mathbf{P} \wedge \mathbf{Q}) \hat{\cdot} V(\neg\mathbf{P} \wedge \neg\mathbf{Q}) \\ \hat{=} [1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q})] \hat{\cdot} [1 \hat{-} V(\neg\neg\mathbf{P}) \hat{\cdot} V(\neg\neg\mathbf{Q})] \\ \hat{=} [1 \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q})] \hat{\cdot} [1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q})] \\ \hat{=} 1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q}) \hat{=} V(\mathbf{P} \Leftrightarrow \mathbf{Q}), \end{gathered} \tag{2.16_1}$$

By (2.16), the pertinent version of (II.7.50) immediately infers (2.16).•

Th 2.12:** ***The law of equivalence of an equivalence relation and of the appropriate exclusive disjunction.

$$[\mathbf{P} \Leftrightarrow \mathbf{Q}] \Leftrightarrow [[\mathbf{P} \wedge \mathbf{Q}] \overline{\Leftrightarrow} [\neg\mathbf{P} \wedge \neg\mathbf{Q}]]. \tag{2.17}$$

Proof: From the pertinent versions of (II.7.12γ), (II.7.1γ), (II.7.6γ), and (II.7.7γ), it follows that

$$
\begin{aligned}
&V([[\mathbf{P}\wedge\mathbf{Q}]\overline{\Leftrightarrow}[\neg\mathbf{P}\wedge\neg\mathbf{Q}]])\hat{=}1\hat{-}[V(\mathbf{P}\wedge\mathbf{Q})\hat{-}V(\neg\mathbf{P}\wedge\neg\mathbf{Q})]^2\\
&\hat{=}1\hat{-}[V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{-}V(\neg\neg\mathbf{P})\hat{\cdot}V(\neg\neg\mathbf{Q})]^2\\
&\hat{=}1\hat{-}[V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})]^2\\
&\hat{=}1\hat{-}[V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})]^2\hat{-}[V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})]^2\\
&\hat{+}2\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\\
&\hat{=}1\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{=}V(\mathbf{P}\Leftrightarrow\mathbf{Q}).
\end{aligned}
\tag{2.17$_1$}
$$

By (2.17$_1$), the pertinent version of (II.7.50) immediately infers (2.17).•

Lemma 2.1.

$$V([V(\mathbf{P})\hat{=}0]\wedge[V(\mathbf{Q})\hat{=}0])\hat{=}V(\mathbf{P}\wedge\mathbf{Q}). \tag{2.18}$$

$$V([V(\mathbf{P})\hat{=}1]\wedge[V(\mathbf{Q})\hat{=}1])\hat{=}V(\neg\mathbf{P}\wedge\neg\mathbf{Q}). \tag{2.19}$$

Proof: From the pertinent versions of (II.7.6γ), (II.7.1γ), (II.6.19), and (II.6.20), it follows that

$$
\begin{aligned}
&V([V(\mathbf{P})\hat{=}0]\wedge[V(\mathbf{Q})\hat{=}0])\hat{=}1\hat{-}V(\neg[V(\mathbf{P})\hat{=}0])\hat{\cdot}V(\neg[V(\mathbf{Q})\hat{=}0])\\
&\hat{=}1\hat{-}[1\hat{-}V(V(\mathbf{P})\hat{=}0)]\hat{\cdot}[1\hat{-}V(V(\mathbf{Q})\hat{=}0)]\hat{=}1\hat{-}[1\hat{-}V(\mathbf{P})]\hat{\cdot}[1\hat{-}V(\mathbf{Q})]\\
&\hat{=}1\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{=}V(\mathbf{P}\wedge\mathbf{Q}),
\end{aligned}
\tag{2.18$_1$}
$$

$$
\begin{aligned}
&V([V(\mathbf{P})\hat{=}1]\wedge[V(\mathbf{Q})\hat{=}1])\hat{=}1\hat{-}V(\neg[V(\mathbf{P})\hat{=}1])\hat{\cdot}V(\neg[V(\mathbf{Q})\hat{=}1])\\
&\hat{=}1\hat{-}[1\hat{-}V(V(\mathbf{P})\hat{=}1)]\hat{\cdot}[1\hat{-}V(V(\mathbf{Q})\hat{=}1)]\\
&\hat{=}1\hat{-}[1\hat{-}[1\hat{-}V(\mathbf{P})]]\hat{\cdot}[1\hat{-}[1\hat{-}V(\mathbf{Q})]]\hat{=}1\hat{-}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{=}V(\neg\mathbf{P}\wedge\neg\mathbf{Q}).
\end{aligned}
\tag{2.19$_1$}
$$
•

Th 2.13.

$$[\mathbf{P}\Leftrightarrow\mathbf{Q}]\Leftrightarrow[[V(\mathbf{P})\hat{=}0]\wedge[V(\mathbf{Q})\hat{=}0]]\vee[[V(\mathbf{P})\hat{=}1]\wedge[V(\mathbf{Q})\hat{=}1]]. \tag{2.20}$$

$$[\mathbf{P}\Leftrightarrow\mathbf{Q}]\Leftrightarrow[[V(\mathbf{P})\hat{=}0]\wedge[V(\mathbf{Q})\hat{=}0]]\overline{\Leftrightarrow}[[V(\mathbf{P})\hat{=}1]\wedge[V(\mathbf{Q})\hat{=}1]]. \tag{2.21}$$

Proof: By (2.18) and (2.16$_1$), or by (2.19) and (2.17$_1$), it follows that

$$
\begin{aligned}
&V([[V(\mathbf{P})\hat{=}0]\wedge[V(\mathbf{Q})\hat{=}0]]\vee[[V(\mathbf{P})\hat{=}1]\wedge[V(\mathbf{Q})\hat{=}1]])\\
&\hat{=}V([V(\mathbf{P})\hat{=}0]\wedge[V(\mathbf{Q})\hat{=}0])\hat{\cdot}V([V(\mathbf{P})\hat{=}1]\wedge[V(\mathbf{Q})\hat{=}1])\\
&\hat{=}V(\mathbf{P}\wedge\mathbf{Q})\hat{\cdot}V(\neg\mathbf{P}\wedge\neg\mathbf{Q})\hat{=}V([\mathbf{P}\wedge\mathbf{Q}]\vee[\neg\mathbf{P}\wedge\neg\mathbf{Q}])\\
&\hat{=}V(\mathbf{P}\Leftrightarrow\mathbf{Q}),
\end{aligned}
\tag{2.20$_1$}
$$

$$
\begin{aligned}
&V([[V(\mathbf{P})\hat{=}0]\wedge[V(\mathbf{Q})\hat{=}0]]\overline{\Leftrightarrow}[[V(\mathbf{P})\hat{=}1]\wedge[V(\mathbf{Q})\hat{=}1]])\\
&\hat{=}1\hat{-}V([[V(\mathbf{P})\hat{=}0]\wedge[V(\mathbf{Q})\hat{=}0]]\Leftrightarrow[[V(\mathbf{P})\hat{=}1]\wedge[V(\mathbf{Q})\hat{=}1]])\\
&\hat{=}1\hat{-}[V([V(\mathbf{P})\hat{=}0]\wedge[V(\mathbf{Q})\hat{=}0])\hat{-}V([V(\mathbf{P})\hat{=}1]\wedge[V(\mathbf{Q})\hat{=}1])]^2\\
&\hat{=}1\hat{-}[V(\mathbf{P}\wedge\mathbf{Q})\hat{-}V(\neg\mathbf{P}\wedge\neg\mathbf{Q})]^2\hat{=}V([\mathbf{P}\wedge\mathbf{Q}]\vee[\neg\mathbf{P}\wedge\neg\mathbf{Q}])\\
&\hat{=}V(\mathbf{P}\Leftrightarrow\mathbf{Q}),
\end{aligned}
\tag{2.21$_1$}
$$

respectively. By (2.20$_1$) or (2.21$_1$), the two pertinent version of (II.7.50) immediately infers (2.20) or (2.21) respectively.•

2.3. Corollaries of the above theorems

°**Crl 2.1.** The following valid (kyrologous) EOR's are concrete conformal euautographic interpretands (instances, corollaries) of the valid (kyrologous) PLOR's having the same double position-numerals before the flag 'μ':

$$\neg[p \wedge q] \Leftrightarrow [\neg p \vee \neg q]. \tag{2.1μ}$$

$$\neg[p \vee q] \Leftrightarrow [\neg p \wedge \neg q]. \tag{2.2μ}$$

$$[p \wedge q] \Leftrightarrow \neg[\neg p \vee \neg q]. \tag{2.1'μ}$$

$$[p \vee q] \Leftrightarrow \neg[\neg p \wedge \neg q]. \tag{2.2'μ}$$

$$[p \Rightarrow q] \vee [p \Rightarrow r] \Leftrightarrow [p \Rightarrow [q \vee r]]. \tag{2.3μ}$$

$$[p \Rightarrow q] \wedge [p \Rightarrow r] \Leftrightarrow [p \Rightarrow [q \wedge r]]. \tag{2.4μ}$$

$$[[p \Rightarrow q] \overline{\Leftrightarrow} [p \Rightarrow r]] \Leftrightarrow [\neg[p \Rightarrow [q \Leftrightarrow r]]]. \tag{2.5μ}$$

$$[[p \Rightarrow q] \wedge [p \Rightarrow \neg q]] \Leftrightarrow \neg p. \tag{2.6μ}$$

$$[p \Rightarrow [q \wedge \neg q]] \Leftrightarrow \neg p. \tag{2.7μ}$$

$$[[p \Rightarrow q] \wedge [p \Rightarrow \neg q]] \Rightarrow \neg p. \tag{2.6'μ}$$

$$\neg p \Rightarrow [[p \Rightarrow q] \wedge [p \Rightarrow \neg q]]. \tag{2.6''μ}$$

$$[p \Rightarrow [q \wedge \neg q]] \Rightarrow \neg p. \tag{2.7'μ}$$

$$\neg p \Rightarrow [p \Rightarrow [q \wedge \neg q]]. \tag{2.7''μ}$$

$$[p \Rightarrow q] \Leftrightarrow [\neg q \Rightarrow \neg p]. \tag{2.8μ}$$

$$[p \Rightarrow q] \Rightarrow [\neg q \Rightarrow \neg p]. \tag{2.8'μ}$$

$$[\neg q \Rightarrow \neg p] \Rightarrow [p \Rightarrow q]. \tag{2.8''μ}$$

$$[p \Rightarrow q] \Leftrightarrow [[p \Rightarrow \neg q] \Rightarrow \neg p]. \tag{2.9μ}$$

$$[p \Rightarrow q] \Leftrightarrow [[p \wedge \neg q] \Rightarrow \neg p]. \tag{2.10μ}$$

$$[[p \wedge q] \Rightarrow r] \Leftrightarrow [p \Rightarrow [q \Rightarrow r]]. \tag{2.11μ}$$

$$[[p \Rightarrow q] \Rightarrow q] \Leftrightarrow [p \vee q]. \tag{2.12μ}$$

$$[[p \Rightarrow q] \Rightarrow [p \Rightarrow \neg q]] \Leftrightarrow p. \tag{2.13μ}$$

$$\neg[p \Rightarrow q] \Leftrightarrow [p \wedge \neg q]. \tag{2.14μ}$$

$$[p \Leftrightarrow q] \Leftrightarrow [V(p) \mathrel{\hat{=}} V(q)]. \tag{2.15μ}$$

$$[p \Leftrightarrow q] \Leftrightarrow [[p \wedge q] \vee [\neg p \wedge \neg q]]. \tag{2.16μ}$$

$$[p \Leftrightarrow q] \Leftrightarrow [[p \wedge q] \overline{\Leftrightarrow} [\neg p \wedge \neg q]]. \tag{2.17μ}$$

$$V([V(p) \mathrel{\hat{=}} 0] \wedge [V(q) \mathrel{\hat{=}} 0]) \mathrel{\hat{=}} V(p \wedge q). \tag{2.18μ}$$

$$V([V(p) \mathrel{\hat{=}} 1] \wedge [V(q) \mathrel{\hat{=}} 1]) \mathrel{\hat{=}} V(\neg p \wedge \neg q). \tag{2.19μ}$$

$$[p \Leftrightarrow q] \Leftrightarrow [[V(p) \mathrel{\hat{=}} 0] \wedge [V(q) \mathrel{\hat{=}} 0]] \vee [[V(p) \mathrel{\hat{=}} 1] \wedge [V(q) \mathrel{\hat{=}} 1]]. \tag{2.20μ}$$

$$[p \Leftrightarrow q] \Leftrightarrow [[V(p) \mathrel{\hat{=}} 0] \wedge [V(q) \mathrel{\hat{=}} 0]] \overline{\Leftrightarrow} [[V(p) \mathrel{\hat{=}} 1] \wedge [V(q) \mathrel{\hat{=}} 1]]. \tag{2.21μ•}$$

$^{+}$**Crl 2.2.** In accordance with Ax I.8.2 and in analogy with $^{+}$Crl 1.2, here follow catlogographic tautologies, which are CFCL interpretands of the above euautographic kyrologies (1.1μ)–(1.16'μ) in that order:

$$\neg[p \wedge q] \Leftrightarrow [\neg p \vee \neg q]. \tag{2.1κ}$$

$$\neg[p \vee q] \Leftrightarrow [\neg p \wedge \neg q]. \tag{2.2κ}$$

$$[p \wedge q] \Leftrightarrow \neg[\neg p \vee \neg q]. \tag{2.1'κ}$$

$$[p \vee q] \Leftrightarrow \neg[\neg p \wedge \neg q]. \tag{2.2'κ}$$

$$[p \Rightarrow q] \vee [p \Rightarrow r] \Leftrightarrow [p \Rightarrow [q \vee r]]. \tag{2.3κ}$$

$$[p \Rightarrow q] \wedge [p \Rightarrow r] \Leftrightarrow [p \Rightarrow [q \wedge r]]. \tag{2.4κ}$$

$$[[p \Rightarrow q] \overline{\Leftrightarrow} [p \Rightarrow r]] \Leftrightarrow [\neg[p \Rightarrow [q \Leftrightarrow r]]]. \tag{2.5κ}$$

$$[[p \Rightarrow q] \wedge [p \Rightarrow \neg q]] \Leftrightarrow \neg p. \tag{2.6κ}$$

$$[p \Rightarrow [q \wedge \neg q]] \Leftrightarrow \neg p. \tag{2.7κ}$$

$$[[p \Rightarrow q] \wedge [p \Rightarrow \neg q]] \Rightarrow \neg p. \tag{2.6'κ}$$

$$\neg p \Rightarrow [[p \Rightarrow q] \wedge [p \Rightarrow \neg q]]. \tag{2.6''κ}$$

$$[p \Rightarrow [q \wedge \neg q]] \Rightarrow \neg p. \tag{2.7'κ}$$

$$\neg p \Rightarrow [p \Rightarrow [q \wedge \neg q]]. \tag{2.7''κ}$$

$$[p \Rightarrow q] \Leftrightarrow [\neg q \Rightarrow \neg p]. \tag{2.8κ}$$

$$[p \Rightarrow q] \Rightarrow [\neg q \Rightarrow \neg p]. \tag{2.8'κ}$$

$$[\neg q \Rightarrow \neg p] \Rightarrow [p \Rightarrow q]. \tag{2.8''κ}$$

$$[p \Rightarrow q] \Leftrightarrow [[p \Rightarrow \neg q] \Rightarrow \neg p]. \tag{2.9κ}$$

$$[p \Rightarrow q] \Leftrightarrow [[p \wedge \neg q] \Rightarrow \neg p]. \tag{2.10κ}$$

$$[[p \wedge q] \Rightarrow r] \Leftrightarrow [p \Rightarrow [q \Rightarrow r]]. \tag{2.11κ}$$

$$[[p \Rightarrow q] \Rightarrow q] \Leftrightarrow [p \vee q]. \tag{2.12κ}$$

$$[[p \Rightarrow q] \Rightarrow [p \Rightarrow \neg q]] \Leftrightarrow p\,. \tag{2.13κ}$$

$$\neg[p \Rightarrow q] \Leftrightarrow [p \wedge \neg q]. \tag{2.14κ}$$

$$[p \Leftrightarrow q] \Leftrightarrow [V(p) \hat{=} V(q)]. \tag{2.15κ}$$

$$[p \Leftrightarrow q] \Leftrightarrow [[p \wedge q] \vee [\neg p \wedge \neg q]]. \tag{2.16κ}$$

$$[p \Leftrightarrow q] \Leftrightarrow [[p \wedge q] \overline{\Leftrightarrow} [\neg p \wedge \neg q]]. \tag{2.17κ}$$

$$V([V(p) \hat{=} 0] \wedge [V(q) \hat{=} 0]) \hat{=} V(p \wedge q). \tag{2.18κ}$$

$$V([V(p) \hat{=} 1] \wedge [V(q) \hat{=} 1]) \hat{=} V(\neg p \wedge \neg q). \tag{2.19κ}$$

$$[p \Leftrightarrow q] \Leftrightarrow [[V(p) \hat{=} 0] \wedge [V(q) \hat{=} 0]] \vee [[V(p) \hat{=} 1] \wedge [V(q) \hat{=} 1]]. \tag{2.20κ}$$

$$[p \Leftrightarrow q] \Leftrightarrow [[V(p) \hat{=} 0] \wedge [V(q) \hat{=} 0]] \overline{\Leftrightarrow} [[V(p) \hat{=} 1] \wedge [V(q) \hat{=} 1]]. \tag{2.21κ}$$ •

2.4. A summary of the commutative, anti-commutative, associative, and distributive laws for logical connectives relative to ⇔

The laws, which are mentioned in the above head and which have been established in section II.7, are valid panlogographic relations of equivalence. Therefore, for the sake of completeness and for convenience in the further discussion, these relations are summarized below:

1) *Commutative laws*:

$$[\mathbf{P\theta Q}] \Leftrightarrow [\mathbf{Q\theta P}] \text{ for each } \boldsymbol{\theta} \vec{\in} \{\not\vee, \vee, \not\wedge, \wedge, \Leftrightarrow, \overline{\vee}, \overline{\wedge}, \overline{\Leftrightarrow}\}, \tag{2.22}$$

which are the same as (II.7.64') subject to (II.7.63').

2) *Anti-commutative laws*:

$$[\mathbf{P} \Leftarrow \mathbf{Q}] \Leftrightarrow [\mathbf{Q} \Rightarrow \mathbf{P}], \tag{2.23}$$

$$[\mathbf{P} \overline{\Leftarrow} \mathbf{Q}] \Leftrightarrow [\mathbf{Q} \overline{\Rightarrow} \mathbf{P}], \tag{2.24}$$

which are the same as (II.7.65') and (II.7.66').

3) *Associative laws*:

$$[[\mathbf{P} \vee \mathbf{Q}] \vee \mathbf{R}] \Leftrightarrow [\mathbf{P} \vee [\mathbf{Q} \vee \mathbf{R}]], \tag{2.25}$$

$$[[\mathbf{P} \wedge \mathbf{Q}] \wedge \mathbf{R}] \Leftrightarrow [\mathbf{P} \wedge [\mathbf{Q} \wedge \mathbf{R}]], \tag{2.26}$$

which are the same as (II.7.67a) and (II.7.68a).

4) *Distributive laws*:

$$[\mathbf{P} \vee [\mathbf{Q} \wedge \mathbf{R}]] \Leftrightarrow [[\mathbf{P} \vee \mathbf{Q}] \wedge [\mathbf{P} \vee \mathbf{R}]], \tag{2.27}$$

$$[\mathbf{P} \wedge [\mathbf{Q} \vee \mathbf{R}]] \Leftrightarrow [[\mathbf{P} \wedge \mathbf{Q}] \vee [\mathbf{P} \wedge \mathbf{R}]], \tag{2.28}$$

which are the same as (II.7.69a) and (II.7.70a); (2.27) is *the distributive laws for* $\vee$ *over* $\wedge$ *relative to* $\Leftrightarrow$, and (2.27) is *the distributive laws for* $\wedge$ *over* $\vee$*, relative to* $\Leftrightarrow$*.*

°**Crl 2.3.** The following valid (kyrologous) EOR's are concrete conformal euautographic interpretands (instances, corollaries) of the valid (kyrologous) PLOR's (2.22)–(2.28) in that order:

$$[p\boldsymbol{\theta}q]\Leftrightarrow[q\boldsymbol{\theta}p] \text{ for each } \boldsymbol{\theta}\,\vec{\in}\,\{\not\vee,\vee,\not\wedge,\wedge,\Leftrightarrow,\overline{\vee},\overline{\wedge},\overline{\Leftrightarrow}\}. \qquad (2.22\mu)$$

$$[p\Leftarrow q]\Leftrightarrow[q\Rightarrow p]. \qquad (2.23\mu)$$

$$[p\,\overline{\Leftarrow}\,q]\Leftrightarrow[q\,\overline{\Rightarrow}\,p]. \qquad (2.24\mu)$$

$$[p\vee q]\vee r\Leftrightarrow p\vee[q\vee r]. \qquad (2.25\mu)$$

$$[p\wedge q]\wedge r\Leftrightarrow p\wedge[q\wedge r]. \qquad (2.26\mu)$$

$$p\vee[q\wedge r]\Leftrightarrow[p\vee q]\wedge[p\vee r]. \qquad (2.27\mu)$$

$$p\wedge[q\vee r]\Leftrightarrow[p\wedge q]\vee[p\wedge r]. \qquad (2.28\mu)\bullet$$

$^{+}$**Crl 2.4.** In accordance with Ax I.8.2 and in analogy with $^{+}$Crl 2.2, here follow catlogographic tautologies, which are CFCL interpretands of the above euautographic kyrologies (2.22μ)–(2.28μ) in that order:

$$[p\boldsymbol{\theta}q]\Leftrightarrow[q\boldsymbol{\theta}p] \text{ for each } \boldsymbol{\theta}\,\vec{\in}\,\{\not\vee,\vee,\not\wedge,\wedge,\Leftrightarrow,\overline{\vee},\overline{\wedge},\overline{\Leftrightarrow}\}. \qquad (2.22\kappa)$$

$$[p\Leftarrow q]\Leftrightarrow[q\Rightarrow p]. \qquad (2.23\kappa)$$

$$[p\,\overline{\Leftarrow}\,q]\Leftrightarrow[q\,\overline{\Rightarrow}\,p]. \qquad (2.24\kappa)$$

$$[p\vee q]\vee r\Leftrightarrow p\vee[q\vee r]. \qquad (2.25\kappa)$$

$$[p\wedge q]\wedge r\Leftrightarrow p\wedge[q\wedge r]. \qquad (2.26\kappa)$$

$$p\vee[q\wedge r]\Leftrightarrow[p\vee q]\wedge[p\vee r]. \qquad (2.27\kappa)$$

$$p\wedge[q\vee r]\Leftrightarrow[p\wedge q]\vee[p\wedge r]. \qquad (2.28\kappa)\bullet$$

3. Selected valid implicative panlogographic relations and their master theorems

3.1. Traditional sentential logic (TrSL) and conventional axiomatic sentential calculi (CASC'i) as slaves of A_0

Df 3.3. 1) In a broad historical prospective, *dual formal logic* (*DFL*) as a single whole field of study and discourse is divided into *traditional* (*classical*) *formal logic* (*TrFL*), *algebraic logic*, called also *old mathematical*, or *old symbolic, logic* (*OMhL* or *OSbL*), i.e. *symbolic logic of the middle of 19th century*, and *new* (*contemporary, modern*) *mathematical*, or *symbolic, logic* (*NMhL* or *NSbL*) *that arouse at the joint of 19*th *and 20*th *centuries and have been developing through 20*th *century*. TrFL is divided into *traditional deductive formal logic* (*TrDdFL*) and *traditional inductive*, or *Bacon-Mill's, formal logic* (*TrIdFL*). TrDdFL is, in turn, divided into *traditional sentential formal logic* (*TrSFL*) and *traditional predicate formal logic* (*TrPFL*), called also *predicate formal syllogistics* (*PFS*), *Aristotelian formal logic* (*AFL*), *Aristotelian formal syllogistics* (*AFS*), or *categorical formal syllogistics* (*CFS*). Of the last five synonymous names, the first two, "traditional *predicate* formal logic" ("Tr*P*FL") and "*predicate* formal syllogistics" ("*P*FS"), are descriptive of the fact that, from the standpoint of NSbL and especially from the standpoint of $\mathcal{A}_1$, any one of 19 *categorical syllogism-schemata* (*syllogism-forms, syllogism-rules*) comprised in TrPFL (PFS) is a *latent quantified predicate* (*functional*) *rule of deductive inference.* By contrast, from the same standpoint, TrSFL comprises *tautologous sentential forms*, *most* of which can be regarded (used) as *syllogism-forms*, i.e. as *syllogistic rules of deductive inference*, according to which, from a certain number 1 to 3 of *judgment-forms* (*f-veracious sentential forms*) as *premises*, another judgment-form is *immediately inferred* as *conclusion* – just as in the case of the categorical syllogism-forms. Accordingly, TrSFL can be divided into *sentential formal syllogistics* (*SFS*), comprising *sentential syllogism-forms* (*SSF's*), and *supplementary sentential formal logic* (*SSFL*), comprising few *non-syllogistic sentential forms* (*NSSF's*) that are not comprised in SFS. It is understood that all tautologous sentential forms, comprised in TrSFL, are expressed in terms of modern symbolic logic (NSbL). Therefore, any one of the tautologous sentential forms, comprised in SSFL, might have been resulted by inadequate incorporation of its original verbal or semi-verbal laws into NSbL (to be illustrated). In any case, for convenience in description and study in terms of NSbL, I regard SSFL as a part of *miscellaneous sentential formal logic* (*MscSFL*) that comprises, by definition, all NSSF's of SSFL and, in addition, *Law of double negation* ($\neg\neg\boldsymbol{P}\Leftrightarrow\boldsymbol{P}$), which MscSFL shares (has in common) with SFS (cf. subsection 1.2, being the panlogographic precursor of MscSFL). TrPFL (AFL) can be called a *semi-verbal formal logic* because the *primitive copulas* (*link-verbs*) (e.g. "*is*" or "*is not*") and the *quantifiers of universality and particularity* (e.g. "all" and "some" respectively), which are employed in this logic, are verbal expressions of a certain native language (as Greek, Latin, or English), into which the logic is incorporated.

2) The term "*traditional logic*" ("*TrL*") alone, without any additional qualifier, denotes a totality of *traditional logical theories* (*TrLT's*), each of which determines a *traditional FLS* (*TrFLS*) along with the respective *traditional MLS* (*TrMLS*), such as a certain system of *declarative sentences* (*DS's*) of a given *written native language* (*WNL*), e.g. written English. Consequently, in accordance with the previous item, TrL is divided into *traditional deductive logic* (*TrDdL*) and *traditional inductive*, or *Bacon-Mill's*, *logic* (*TrIL*). TrDdL is divided into *traditional sentential logic* (*TrSL*) and *traditional predicate logic* (*TrPL*), called also *predicate syllogistics* (*PS*), *Aristotelian logic* (*AL*), *Aristotelian syllogistics* (*AS*), or *categorical syllogistics* (*CS*). TrSL is divided into *sentential syllogistics* (*SS*) and *supplementary sentential logic* (*SSL*).•

Cmt 3.1. TrSFL comprises *tautologous sentential forms* (*schemata*), some of which, including De Morgan's laws, were invented by ancient Greek philosophers, pre-Aristotelian ones and post-Aristotelian ones (particularly, by Stoics), whereas the other ones were invented by medieval Scholastics, and all of which were later deduced in the *conventional axiomatic sentential*, or *propositional, calculi* (*CASC'i*) constituting a part of NMhL (NSbL). Most of these tautologous sentential forms, namely those comprised in SFS, are *tautologous sentential forms* (*schemata*, *rules*) *of deductive inference*, which can therefore be alternatively called *sentential syllogism-forms* (*syllogism-schemata*, *syllogism-rules*) or *formal*, or *schematic*, *sentential syllogisms* (briefly, *FSS's* or *SSS's*). At the same time, in authoritative explanatory dictionaries and in encyclopedias, the term "syllogism" is as rule defined in the narrow sense of "categorical syllogism". Therefore, for avoidance or confusion, I shall stick to the following definition of term "syllogism".•

Df 3.4. By Df 3.3(1), TrDdFL is a part of *dual* FL (*D*FL). However, in the following classification of individual inference rules of TrDdFL some metaterms of the *trial* FL (*T*FL) $\mathscr{A}_1$, belonging to the IML of the latter, are used because these metaterms are absent in the IML of any DFLS, in accordance with Df 3.2.

1) A *judgment* is an *m-veracious* (*accidentally m-true*) and hence *m-ttatt-neutral* (*m-ttatt-indeterminate*) *declarative sentence* (*DS*), i.e. a *one-sentence statement*, in any basic or rich written native language (WNL) as English, and vice

versa. Consequently, an *m-tautologous statement is not a judgment.* Accordingly, a *judgment-form* or *judgment-schema* is an *f-veracious* (*accidentally f-true*) and hence *f-ttatt-neutral* (*f-ttatt-indeterminate*) *sentential form* and vice versa.

2) A *syllogism-form* is an *f-true form of deductive inference* (*proof*) *of an f-veracious form,* i.e. *of a judgment-form,* called the *conclusion-form,* from one or more *f-veracious forms of known judgments,* called the *premise-forms*; "form" can be used interchangeably with "*schema*", "*pattern*", or "*rule*". Consequently, a *syllogism-instance* is an *m-true instance of the pertinent f-true syllogism-form, of deductive inference* (*proof*) *of the judgment,* being the pertinent *m-veracious conclusion-instances of the conclusion-form*, from *known judgments*, being the pertinent *m-veracious premise-instances of the premise-forms*.

3) A syllogism-form can either be f-tautologous (universally f-true), as any one of the sentential syllogism-forms or as any one of the 15 *tautologous* categorical syllogisms, or be f-veracious (accidentally f-true). as any one of the 4 *veracious* categorical syllogisms Bamalip, Barapti (former Darapti), Felapton, and Fesapo, indicated in Df I.7.1(6), Consequently, all syllogism-instances of an f-tautologous syllogism-form are m-tautologous, whereas all syllogism-instances of an f-veracious syllogism-form are m-veracious.

4) A *syllogism-form* or a *syllogism-instance* is indiscriminately called *a syllogism.* Consequently, the *premise-forms* or the *premise-instances* are indiscriminately called the *premises* and the *conclusion-form* or the *conclusion-instance* is indiscriminately called the *conclusion.*

5) A syllogism that has *n* judgments, subject to $n \geq 2$, i.e. *n*–1 *premises* and one *conclusion,* is said to be an *n-judgment syllogism* or an (*n*–1)-*premise syllogism*.

6) The generic term "*syllogism*" is derived from the Greek etymons "συλλογή" \silloγí\ *s.f.*, meaning *a collection, thought,* or *reflection,* and "συλλογισμός" \silloγismós\ *s.m.*, meaning *a reflection* or, tautologically, *a syllogism*.•

Cmt 3.2. 1) Each rule of TrSFL had been regarded as valid in its own right until all rules of TrSFL *were incorporated as tautologies* into every modern *conventional axiomatic sentential* (*propositional*) *calculus* (*CASC*), and hence into every modern *conventional axiomatic predicate calculus* [*of first order*] (*CAPC*, pl. "*CAPC'i*"). I have elementarily proved all these rules in the framework of $\mathcal{A}_0$, and hence in the framework of $\mathcal{A}_1$ containing $\mathcal{A}_0$ as its self-subsistent part, by the pertinent *algebraic decision procedures* (*ADP's*). In the result, any law (tautologous sentential form) of TrSFL exists now in various equivalent variants of two *syntactic forms*: *logographic form* and s*emi-verbal* (*logophonograpic, phonologographic*) *form*, while some rules of TrSFL are also expressible in pure *verbal* (*phonograaphic*) *form*. In this case, a tautologous sentential form is a *sentential syllogism-form* if it is either (a) an implication such as ***P***⇒***Q***, where the antecedent '***P***' is a placeholder of the premise or of the conjoined premises and the consequent '***Q***' is a placeholder of the conclusion; or (b) a biimplication (bihypothetical, equivalence) such as ***P***⇔***Q***, where '***P***' is a placeholder of the premise or of the conjoined premises and '***Q***' is a placeholder of the conclusion, or vice versa. A tautologous sentential form that does not have the form either of an implication or of biimplication is not a syllogism-schema.

2) At the same time, any syllogism-schema of TrSFL, i.e. of SFS, can be represented either in a *staccato form* (*style*) or in a *legato form* (*style*).

a) A *staccato form* of a sentential syllogism-schema is a *form*, in the framework of which all premise-schemata and the conclusion-schema are asserted separately from one another after the manner of simple declarative sentences. Hence, *a staccato form of a sentential syllogism-schema is necessarily either a semi-verbal one or a pure verbal one, but not necessarily vice versa*.

b) A *legato form* of a sentential syllogism-schema is a form, in the framework of which the syllogism-schema is represented a *single whole,* after the manner of a complex sentence. Hence, *a logographic form of a sentential syllogism-schema is necessarily a legato form, but not necessarily vice versa*.•

3) All sentential rules of TrSFL turn out to be *dualistic semantic* (*mental*) *restrictions* of the CFCL interpretands (corollaries) or their verbal or semi-verbal expressions in one of the written native languages (as English) of certain euautographic theorems of A_0, whereas the latter theorems are in turn euautographic interpretands (instances, corollaries) of certain panlogographic theorems of $\mathbf{A}_0$. In this case, all theorems of $\mathbf{A}_0$, which have been or will be formulated and proved *under traditional names of certain laws of TrSFL*, are the panlogographic interpretantia of, i.e. the PLR's that are syntactically (substitutionally) interpretable as, *the traditional laws carrying those names*.

4) Besides the traditional sentential laws that have been derived in the above way in the previous two sections, there are in TrSFL *four kinds of three-judgment*

sentential (*not predicate*) *conditional syllogisms*, called collectively *modi* and also *conditional sentential syllogistics* (*CSS*), namely *two hypothetical syllogisms*, called *modus ponendo ponens* and *modus tollendo tollens*, and *two disjunctive syllogisms*, called *modus tollendo ponens* and *modus ponendo tollens*; and there are also *four kinds of four-judgment sentential dilemmatic syllogisms*, called collectively *dilemmas* and also *dilemmatic sentential syllogistics* (*DSS*), namely, *the simple constructive dilemma*, *simple destructive dilemma*, *complex constructive dilemma*, and *complex destructive dilemma*. The valid PLR's, underlying the above traditional rules, are stated and proved below in this section under the conventional traditional names of the latter rules. Just as in all previous cases, the PLR's in question will be interpreted (replaced) by appropriate valid ER's, whereas the latter will, in turn, be interpreted (replaced) by the f-tautologous conformal CLR's, whose mental dualistic restrictions are the respective traditional sentential laws.

5) It is noteworthy that *modus ponendo ponens* is conventionally used as a primary rule of inference in all axiomatic systems of modern sentential (propositional) and predicate (functional) calculi, but this rule is not used in $\mathbf{A}_1$ and hence it is not used in $\mathbf{A}_0$.•

3.2. Panlogographic conditional syllogisms (modi) and panlogographic dilemmas

*Th 3.1: ***Panlogographic conditional syllogisms (modi).***

A) Hypothetical syllogisms

1) *Modus ponendo ponens*:

$$[[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge \mathbf{P}] \Rightarrow \mathbf{Q}. \tag{3.1}$$

2) *Modus tollendo tollens*:

$$[[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge \neg\mathbf{Q}] \Rightarrow \neg\mathbf{P}. \tag{3.2}$$

B) Disjunctive syllogisms

3) *Modus tollendo ponens*:

$$[[\mathbf{P} \vee \mathbf{Q}] \wedge \neg\mathbf{P}] \Rightarrow \mathbf{Q}, \tag{3.3}$$

$$[[\mathbf{P} \vee \mathbf{Q}] \wedge \neg\mathbf{Q}] \Rightarrow \mathbf{P}. \tag{3.3'}$$

4) *Modus ponendo tollens*:

$$[[\mathbf{P} \mathrel{\overline{\Leftrightarrow}} \mathbf{Q}] \wedge \mathbf{P}] \Rightarrow \neg\mathbf{Q}, \tag{3.4}$$

$$[[\mathbf{P} \mathrel{\overline{\Leftrightarrow}} \mathbf{Q}] \wedge \mathbf{Q}] \Rightarrow \neg\mathbf{P}. \tag{3.4'}$$

Proof: 1) By (II.7.3γ), it follows from the version (intrinsic interpretand) of (II.7.6γ) with ‘$[\mathbf{P} \Rightarrow \mathbf{Q}]$’ and ‘**P**’ in place of ‘**P**’ and ‘**Q**’ respectively that

$$\begin{gathered} V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge \mathbf{P}) \mathrel{\hat{=}} V(\mathbf{P}) \mathbin{\hat{+}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{P} \Rightarrow \mathbf{Q}) \\ \mathrel{\hat{=}} V(\mathbf{P}) \mathbin{\hat{+}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} V(\mathbf{P}) \mathbin{\hat{+}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} V(\mathbf{P} \wedge \mathbf{Q}), \end{gathered} \tag{3.1$_1$}$$

where use of the version of (II.4.2) with ‘**P**’ in place of ‘¬**P**’ and also use of (II.7.6γ) have been made in that order in developing the final result. By (3.1_1), it follows from the version of (II.7.3γ) with ‘$[[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge \mathbf{P}]$’ in place of ‘**P**’ that

$$\begin{gathered} V([[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge \mathbf{P}] \Rightarrow \mathbf{Q}) \mathrel{\hat{=}} [1 \mathbin{\hat{-}} V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge \mathbf{P})] \mathbin{\hat{\cdot}} V(\mathbf{Q}) \\ \mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\mathbf{P} \wedge \mathbf{Q})] \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\neg\mathbf{Q}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} 0 \mathrel{\hat{=}} 0, \end{gathered} \tag{3.1$_2$}$$

where use of (II.7.6γ) and also use of the version of (II.7.15γ) with ‘**Q**’ in place of ‘**P**’ have been made in developing the final result. By (3.1_2), the pertinent version of (II.4.40a) immediately infers (3.1).

2) By (II.7.3γ), it follows from the version of (II.7.6γ) with ‘$[\mathbf{P} \Rightarrow \mathbf{Q}]$’ and ‘**P**’ in place of ‘**P**’ and ‘**Q**’ respectively that

$$\begin{gathered} V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge \neg\mathbf{Q}) \mathrel{\hat{=}} V(\neg\mathbf{Q}) \mathbin{\hat{+}} V(\neg\neg\mathbf{Q}) \mathbin{\hat{\cdot}} V(\mathbf{P} \Rightarrow \mathbf{Q}) \\ \mathrel{\hat{=}} V(\neg\mathbf{Q}) \mathbin{\hat{+}} V(\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} V(\neg\mathbf{Q}) \mathbin{\hat{+}} V(\mathbf{Q}) \mathbin{\hat{-}} V(\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} V(\neg\mathbf{P} \wedge \neg\mathbf{Q}) \end{gathered} \tag{3.2$_1$}$$

(cf. (3.1_1)), where use of the versions of (II.4.2) and (II.7.1γ) with ‘**Q**’ in place of ‘**P**’ and also use of (II.7.6γ) with ‘¬**P**’ and ‘¬**Q**’ in place of ‘**P**’ and ‘**Q**’ respectively have been made in that order in developing the final result. By (3.2_1), it follows from the version of (II.7.3γ) with ‘$[[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge \neg\mathbf{Q}]$’ and ‘¬**P**’ in place of ‘**P**’ and ‘**Q**’ respectively that

$$\begin{gathered} V([[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge \neg\mathbf{Q}] \Rightarrow \neg\mathbf{P}) \mathrel{\hat{=}} [1 \mathbin{\hat{-}} V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge \neg\mathbf{Q})] \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \\ \mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\neg\mathbf{P} \wedge \neg\mathbf{Q})] \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \mathrel{\hat{=}} 0, \end{gathered} \tag{3.2$_2$}$$

where use of the version of (II.7.6γ) with ‘¬**P**’ and ‘¬**Q**’ in place of ‘**P**’ and ‘**Q**’ respectively and also use of of (II.7.15γ) have been made in developing the final result. By (3.2_2), the pertinent version of (II.4.40a) immediately infers (3.2).

3) By (II.7.2γ), it follows from the version of (II.7.6γ) with ‘$[\mathbf{P} \vee \mathbf{Q}]$’ and ‘¬**P**’ in place of ‘**P**’ and ‘**Q**’ respectively, it follows that

$$\begin{gathered} V([\mathbf{P} \vee \mathbf{Q}] \wedge \neg\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{P} \vee \mathbf{Q}) \mathbin{\hat{+}} V(\neg\mathbf{P}) \mathbin{\hat{-}} V(\mathbf{P} \vee \mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \\ \mathrel{\hat{=}} V(\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathbin{\hat{+}} V(\neg\mathbf{P}) \mathbin{\hat{-}} V(\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}) \mathrel{\hat{=}} V(\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathbin{\hat{+}} V(\neg\mathbf{P}) \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\mathbf{P}) \mathbin{\hat{+}} V(\mathbf{P}) \mathbin{\hat{\cdot}} V(\mathbf{Q}) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\mathbf{P}) \mathbin{\hat{\cdot}} V(\neg\mathbf{Q}), \end{gathered} \tag{3.3$_1$}$$

where use of the variant of (II.7.1γ) with ‘**Q**’ in place of ‘**P**’ has been made in developing the final result. By (3.3_1), it follows from the version of (II.7.3γ) with ‘$[[\mathbf{P}\vee\mathbf{Q}]\wedge\neg\mathbf{P}]$’ in place of ‘**P**’ that

$$\begin{aligned}V([[\mathbf{P}\vee\mathbf{Q}]\wedge\neg\mathbf{P}]\Rightarrow\mathbf{Q})&\,\hat{=}\,[1\,\hat{-}\,V([\mathbf{P}\vee\mathbf{Q}]\wedge\neg\mathbf{P})]\,\hat{\cdot}\,V(\mathbf{Q})\\&\,\hat{=}\,V(\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{=}\,0.\end{aligned}\tag{3.3$_2$}$$

By (3.3_2), the pertinent version of (II.4.40a) immediately infers (3.3). By (II.7.63) and (II.7.64), $V(\mathbf{P}\vee\mathbf{Q})\,\hat{=}\,V(\mathbf{Q}\vee\mathbf{P})$. Therefore, (3.3') follows from (3.3) by permutation of ‘**P**’ and ‘**Q**’.

4) By (II.7.12γ), it follows from the version of (II.7.6γ) with ‘$[\mathbf{P}\,\overline{\Leftrightarrow}\,\mathbf{Q}]$’ and ‘**P**’ in place of ‘**P**’ and ‘**Q**’ respectively that

$$\begin{aligned}V([\mathbf{P}\,\overline{\Leftrightarrow}\,\mathbf{Q}]\wedge\mathbf{P})&\,\hat{=}\,V(\mathbf{P})\,\hat{+}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{P}\,\overline{\Leftrightarrow}\,\mathbf{Q})\\&\,\hat{=}\,V(\mathbf{P})\,\hat{+}\,V(\neg\mathbf{P})\,\hat{\cdot}\,[V(\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{+}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{Q})]\\&\,\hat{=}\,V(\mathbf{P})\,\hat{+}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{=}\,V(\mathbf{P})\,\hat{+}\,V(\neg\mathbf{P})\,\hat{\cdot}\,[1\,\hat{-}\,V(\mathbf{Q})]\\&\,\hat{=}\,1\,\hat{-}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q}),\end{aligned}\tag{3.4$_1$}$$

where use has been made of the following identities in that order: (II.7.15γ), the version of (II.4.2) with ‘¬**P**’ in place of ‘**P**’, (II.7.1γ), and the variant of (II.7.1γ) with ‘**Q**’ in place of ‘**P**’. By (3.4_1), it follows from the version of (II.7.3γ) with ‘$[[\mathbf{P}\,\overline{\Leftrightarrow}\,\mathbf{Q}]\wedge\mathbf{P}]$’ and ‘¬**Q**’ in place of ‘**P**’ and ‘**Q**’ respectively that

$$\begin{aligned}V([[\mathbf{P}\,\overline{\Leftrightarrow}\,\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\neg\mathbf{Q})&\,\hat{=}\,[1\,\hat{-}\,V([\mathbf{P}\,\overline{\Leftrightarrow}\,\mathbf{Q}]\wedge\mathbf{P})]\,\hat{\cdot}\,V(\neg\mathbf{Q})\\&\,\hat{=}\,V(\neg\mathbf{P})\,\hat{\cdot}\,V(\mathbf{Q})\,\hat{\cdot}\,V(\neg\mathbf{Q})\,\hat{=}\,0.\end{aligned}\tag{3.4$_2$}$$

By (3.4_2), the pertinent version of (II.4.40a) immediately infers (3.4). By (II.7.63) and (II.7.64), $V(\mathbf{P}\,\overline{\Leftrightarrow}\,\mathbf{Q})\,\hat{=}\,V(\mathbf{Q}\,\overline{\Leftrightarrow}\,\mathbf{P})$. Therefore, (3.4') follows from (3.4) by permutation of ‘**P**’ and ‘**Q**’.•

Th 3.2: ***Panlogographic dilemmas.***

1) *Simple constructive dilemma*:

$$[[[\mathbf{P}\Rightarrow\mathbf{R}]\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\wedge[\mathbf{P}\vee\mathbf{Q}]]\Rightarrow\mathbf{R}\,.\tag{3.5}$$

2) *Simple destructive dilemma*:

$$[[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{P}\Rightarrow\mathbf{R}]]\wedge[\neg\mathbf{Q}\vee\neg\mathbf{R}]]\Rightarrow\neg\mathbf{P}\,.\tag{3.6}$$

3) *Complex constructive dilemma*:

$$[[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]\wedge[\mathbf{P}\vee\mathbf{R}]]\Rightarrow[\mathbf{Q}\vee\mathbf{S}].\tag{3.7}$$

4) *Complex destructive dilemma*:

$$[[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]\wedge[\neg\mathbf{Q}\vee\neg\mathbf{S}]]\Rightarrow[\neg\mathbf{P}\vee\neg\mathbf{R}].\tag{3.8}$$

Proof: 1) By the pertinent variants of (II.7.3γ), it follows from the version (intrinsic interpretand) of (II.7.6γ) with '$[\mathbf{P} \Rightarrow \mathbf{R}]$' in place of '**P**' and '$[\mathbf{Q} \Rightarrow \mathbf{R}]$' in place of '**Q**' that

$$\begin{gathered} V([\mathbf{P} \Rightarrow \mathbf{R}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]) \\ \hat{=} V(\mathbf{P} \Rightarrow \mathbf{R}) \hat{+} V(\mathbf{Q} \Rightarrow \mathbf{R}) \hat{-} V(\mathbf{P} \Rightarrow \mathbf{R}) \hat{\cdot} V(\mathbf{Q} \Rightarrow \mathbf{R}) \\ \hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{+} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{R}) \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{\cdot} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{R}) \\ \hat{=} [V(\neg\mathbf{P}) \hat{+} V(\neg\mathbf{Q}) \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q})] \hat{\cdot} V(\mathbf{R}) \\ \hat{=} [V(\neg\mathbf{P}) \hat{+} V(\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q})] \hat{\cdot} V(\mathbf{R}) \hat{=} V(\neg\mathbf{P} \wedge \neg\mathbf{Q}) \hat{\cdot} V(\mathbf{R}), \end{gathered} \tag{3.5$_1$}$$

where use of (II.7.6γ) with '$\neg$**P**' and '$\neg$**Q**' in place of '**P**' and '**Q**' respectively has been made in developing the final result. By (3.5$_1$), the version of (II.7.6γ) with '$[[\mathbf{P} \Rightarrow \mathbf{R}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]]$' in place of '**P**' and '$[\mathbf{P} \vee \mathbf{Q}]$' in place of '**Q**' yields:

$$\begin{gathered} V([[\mathbf{P} \Rightarrow \mathbf{R}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]] \wedge [\mathbf{P} \vee \mathbf{Q}]) \\ \hat{=} V(\mathbf{P} \vee \mathbf{Q}) \hat{+} V(\neg[\mathbf{P} \vee \mathbf{Q}]) \hat{\cdot} V([\mathbf{P} \Rightarrow \mathbf{R}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]) \\ \hat{=} V(\mathbf{P} \vee \mathbf{Q}) \hat{+} [1 \hat{-} V(\mathbf{P} \vee \mathbf{Q})] \hat{\cdot} V(\neg\mathbf{P} \wedge \neg\mathbf{Q}) \hat{\cdot} V(\mathbf{R}) \\ \hat{=} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{+} V(\neg\mathbf{P} \wedge \neg\mathbf{Q}) \hat{\cdot} V(\mathbf{R}) \\ \hat{=} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{+} [V(\neg\mathbf{P}) \hat{+} V(\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q})] \hat{\cdot} V(\mathbf{R}), \end{gathered} \tag{3.5$_2$}$$

because

$$V(\mathbf{P} \vee \mathbf{Q}) \hat{\cdot} V(\neg\mathbf{P} \wedge \neg\mathbf{Q}) \hat{=} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} [V(\neg\mathbf{P}) \hat{+} V(\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q})] \hat{=} 0, \tag{3.5$_3$}$$

by (II.7.2γ) and by the pertinent versions of (II.7.6γ) and (II.7.15γ). By (3.5$_2$), it follows from the version of (II.7.3γ) with '$[[\mathbf{P} \Rightarrow \mathbf{R}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]] \wedge [\mathbf{P} \vee \mathbf{Q}]$' in place of '**P**' and '**R**' in place of '**Q**' that

$$\begin{gathered} V([[[\mathbf{P} \Rightarrow \mathbf{R}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]] \wedge [\mathbf{P} \vee \mathbf{Q}]] \Rightarrow \mathbf{R}) \\ \hat{=} [1 \hat{-} V([[\mathbf{P} \Rightarrow \mathbf{R}] \wedge [\mathbf{Q} \Rightarrow \mathbf{R}]] \wedge [\mathbf{P} \vee \mathbf{Q}])] \hat{\cdot} V(\mathbf{R}) \\ \hat{=} [1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{-} [V(\neg\mathbf{P}) \hat{+} V(\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q})] \hat{\cdot} V(\mathbf{R})] \hat{\cdot} V(\mathbf{R}) \\ \hat{=} [1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{-} V(\neg\mathbf{P}) \hat{-} V(\mathbf{P}) \hat{\cdot} V(\neg\mathbf{Q})] \hat{\cdot} V(\mathbf{R}) \\ \hat{=} [1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{-} 1 \hat{+} V(\mathbf{P}) \hat{-} V(\mathbf{P}) \hat{\cdot} [1 \hat{-} V(\mathbf{Q})]] \hat{\cdot} V(\mathbf{R}) \hat{=} 0 \hat{\cdot} V(\mathbf{R}) \hat{=} 0. \end{gathered} \tag{3.5$_4$}$$

In developing the final result in (3.5$_4$), use of the pertinent instance of identity (II.5.10) or (II.5.10ε) has been made for eliminating the occurrence of $V(\mathbf{R})$ in the outer square brackets. Then the resulting expression in the inner square brackets has been reduced by (II.7.1γ) and by the variant of (II.7.1γ) with '**Q**' in place of '**P**'. By (3.5$_4$), the pertinent version of (II.4.40a) immediately infers (3.5).

2) By the pertinent variants of (II.7.3γ), it follows from the version of (II.7.6γ) with '$[\mathbf{P} \Rightarrow \mathbf{Q}]$' in place of '**P**' and '$[\mathbf{P} \Rightarrow \mathbf{R}]$' in place of '**Q**' that

$$
\begin{aligned}
&V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Rightarrow \mathbf{R}]) \\
&\hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} [\mathbf{P} \Rightarrow \mathbf{R}] \hat{-} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{\cdot} [\mathbf{P} \Rightarrow \mathbf{R}] \\
&\hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{+} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \\
&\hat{=} [V(\mathbf{Q}) \hat{+} V(\mathbf{R}) \hat{-} V(\mathbf{Q}) \hat{\cdot} V(\mathbf{R})] \hat{\cdot} V(\neg\mathbf{P}) \\
&\hat{=} [V(\mathbf{R}) \hat{+} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R})] \hat{\cdot} V(\neg\mathbf{P}) \hat{=} V(\mathbf{Q} \wedge \mathbf{R}) \hat{\cdot} V(\neg\mathbf{P}),
\end{aligned}
\tag{3.6$_1$}
$$

where use of (II.7.6γ) with ‘**Q**’ and ‘**R**’ in place of ‘**P**’ and ‘**Q**’ respectively has been made in developing the final result. By (3.6_1), the version of (II.7.6γ) with ‘$[[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Rightarrow \mathbf{R}]]$’ in place of ‘**P**’ and ‘$[\neg\mathbf{Q} \vee \neg\mathbf{R}]$’ in place of ‘**Q**’ yields:

$$
\begin{aligned}
&V([[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Rightarrow \mathbf{R}]] \wedge [\neg\mathbf{Q} \vee \neg\mathbf{R}]) \\
&\hat{=} V(\neg\mathbf{Q} \vee \neg\mathbf{R}) \hat{+} V(\neg[\neg\mathbf{Q} \vee \neg\mathbf{R}]) \hat{\cdot} V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Rightarrow \mathbf{R}]) \\
&\hat{=} V(\neg\mathbf{Q} \vee \neg\mathbf{R}) \hat{+} [1 \hat{-} V(\neg\mathbf{Q} \vee \neg\mathbf{R})] \hat{\cdot} V(\mathbf{Q} \wedge \mathbf{R}) \hat{\cdot} V(\neg\mathbf{P}) \\
&\hat{=} V(\neg\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{+} V(\mathbf{Q} \wedge \mathbf{R}) \hat{\cdot} V(\neg\mathbf{P}) \\
&\hat{=} V(\neg\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{+} [V(\mathbf{R}) \hat{+} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R})] \hat{\cdot} V(\neg\mathbf{P}),
\end{aligned}
\tag{3.6$_2$}
$$

because

$$
V(\neg\mathbf{Q} \vee \neg\mathbf{R}) \hat{\cdot} V(\mathbf{Q} \wedge \mathbf{R}) \hat{=} V(\neg\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{\cdot} [V(\mathbf{Q}) \hat{+} V(\neg\mathbf{Q}) \hat{\cdot} V(\mathbf{R})] \hat{=} 0, \tag{3.6$_3$}
$$

by the pertinent versions of (II.7.2γ), (II.7.6γ), and (II.7.15γ). By (3.6_2), it follows from the version of (II.7.3γ) with ‘$[[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Rightarrow \mathbf{R}]] \wedge [\neg\mathbf{Q} \vee \neg\mathbf{R}]$’ in place of ‘**P**’ and ‘¬**P**’ in place of ‘**Q**’ that

$$
\begin{aligned}
&V([[[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Rightarrow \mathbf{R}]] \wedge [\neg\mathbf{Q} \vee \neg\mathbf{R}]] \Rightarrow \neg\mathbf{P}) \\
&\hat{=} [1 \hat{-} V([[\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{P} \Rightarrow \mathbf{R}]] \wedge [\neg\mathbf{Q} \vee \neg\mathbf{R}])] \hat{\cdot} V(\neg\mathbf{P}) \\
&\hat{=} [1 \hat{-} V(\neg\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{-} [V(\mathbf{R}) \hat{+} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R})] \hat{\cdot} V(\neg\mathbf{P})] \hat{\cdot} V(\neg\mathbf{P}) \\
&\hat{=} [1 \hat{-} V(\neg\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{-} V(\mathbf{R}) \hat{-} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R})] \hat{\cdot} V(\neg\mathbf{P}) \\
&\hat{=} [1 \hat{-} [1 \hat{-} V(\mathbf{Q})] \hat{\cdot} [1 \hat{-} V(\mathbf{R})] \hat{-} V(\mathbf{R}) \hat{-} V(\mathbf{Q}) \hat{\cdot} [1 \hat{-} V(\mathbf{R})]] \hat{\cdot} V(\neg\mathbf{P}) \hat{=} 0.
\end{aligned}
\tag{3.6$_4$}
$$

In developing the final result in (3.6_4), use of the pertinent instance of identity (II.5.10) or (II.5.10ε) has been made for eliminating the occurrence of $V(\neg\mathbf{P})$ in the outer square brackets. Then the resulting expression in the inner square brackets has been reduced by the variants of (II.7.1γ) with ‘**Q**’ or ‘**R**’ in place of ‘**P**’. By (3.6_4), the pertinent version of (II.4.40a) immediately infers (3.6).

3) It follows from the version of (II.7.6γ) with ‘$[\mathbf{P} \Rightarrow \mathbf{Q}]$’ in place of ‘**P**’ and ‘$[\mathbf{R} \Rightarrow \mathbf{S}]$’ in place of ‘**Q**’ that

$$
\begin{aligned}
&V([\mathbf{P} \Rightarrow \mathbf{Q}] \wedge [\mathbf{R} \Rightarrow \mathbf{S}]) \\
&\hat{=} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{+} V(\mathbf{R} \Rightarrow \mathbf{S}) \hat{-} V(\mathbf{P} \Rightarrow \mathbf{Q}) \hat{\cdot} V(\mathbf{R} \Rightarrow \mathbf{S}) \\
&\hat{=} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{+} V(\neg\mathbf{R}) \hat{\cdot} V(\mathbf{S}) \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{\cdot} V(\mathbf{S}),
\end{aligned}
\tag{3.7$_1$}
$$

where use of (II.7.3γ) and of the variant of with '**R**' in place of '**P**' and '**S**' in place of '**Q**' has been made. By (3.7$_1$), the version of (II.7.6γ) with '$[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]$' in place of '**P**' and "$[\mathbf{P}\vee\mathbf{R}]$' in place of '**Q**' yields:

$$\begin{gathered} V([[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]\wedge[\mathbf{P}\vee\mathbf{R}]) \\ \hat{=} V(\mathbf{P}\vee\mathbf{R}) \hat{+} [1 \hat{-} V(\mathbf{P}\vee\mathbf{R})] \hat{\cdot} V([\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]) \\ \hat{=} V(\mathbf{P}\vee\mathbf{R}) \hat{+} V([\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]) \\ \hat{=} V(\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{+} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{+} V(\neg\mathbf{R}) \hat{\cdot} V(\mathbf{S}) \\ \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{\cdot} V(\mathbf{S}), \end{gathered} \tag{3.7$_2$}$$

because

$$\begin{gathered} V(\mathbf{P}\vee\mathbf{R}) \hat{\cdot} V([\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]) \\ \hat{=} V(\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{\cdot} [V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{+} V(\neg\mathbf{R}) \hat{\cdot} V(\mathbf{S}) \\ \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{\cdot} V(\mathbf{S})] \hat{=} 0, \end{gathered} \tag{3.7$_3$}$$

by (II.7.15γ) and by the variant of (II.7.15γ) with '**R**' in place of '**P**'. By (3.7$_2$), it follows from the version of (II.7.3γ) with '$[[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]\wedge[\mathbf{P}\vee\mathbf{R}]]$' in place of '**P**' and '$[\mathbf{Q}\vee\mathbf{S}]$' in place of '**Q**' that

$$\begin{gathered} V([[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]\wedge[\mathbf{P}\vee\mathbf{R}]]\Rightarrow[\mathbf{Q}\vee\mathbf{S}]) \\ \hat{=} [1 \hat{-} V([[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]\wedge[\mathbf{P}\vee\mathbf{R}]])] \hat{\cdot} V(\mathbf{Q}\vee\mathbf{S}) \\ \hat{=} [1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{-} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{-} V(\neg\mathbf{R}) \hat{\cdot} V(\mathbf{S}) \\ \hat{+} V(\neg\mathbf{P}) \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{\cdot} V(\mathbf{S})] \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\mathbf{S}) \\ \hat{=} [1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{-} V(\neg\mathbf{P}) \hat{-} V(\neg\mathbf{R}) \hat{+} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{R})] \hat{\cdot} V(\mathbf{Q}) \hat{\cdot} V(\mathbf{S}) \hat{=} 0. \end{gathered} \tag{3.7$_4$}$$

In developing the final result in (3.7$_4$), use of the pertinent instances of identity (II.5.10) or (II.5.10ε) has been made for eliminating the occurrences of $V(\mathbf{Q})$ and $V(\mathbf{S})$ in the square brackets. Then the resulting expression in the square brackets has been reduced by (II.7.1γ) and by the variants of (II.7.1γ) with '**R**' in place of '**P**' thus:

$$\begin{gathered} 1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{-} V(\neg\mathbf{P}) \hat{-} V(\neg\mathbf{R}) \hat{+} V(\neg\mathbf{P}) \hat{\cdot} V(\neg\mathbf{R}) \\ \hat{=} 1 \hat{-} V(\mathbf{P}) \hat{\cdot} V(\mathbf{R}) \hat{-} 1 \hat{+} V(\mathbf{P}) \hat{-} 1 \hat{+} V(\mathbf{R}) \hat{+} [1 \hat{-} V(\mathbf{P})] \hat{\cdot} [1 \hat{-} V(\mathbf{R})] \hat{=} 0. \end{gathered} \tag{3.7$_5$}$$

By (3.7$_4$), the pertinent version of (II.4.40a) immediately infers (3.7).

4) By (3.7$_1$), the version of (II.7.6γ) with '$[\mathbf{P}\Rightarrow\mathbf{Q}]$' in place of '**P**' and '$[\mathbf{P}\Rightarrow\mathbf{R}]$' in place of '**Q**' that with '$[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]$' in place of '**P**' and '$[\neg\mathbf{Q}\vee\neg\mathbf{S}]$' in place of '**Q**' yields:

$$
\begin{aligned}
&V([[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]\wedge[\neg\mathbf{Q}\vee\neg\mathbf{S}])\\
&\hat{=}V(\neg\mathbf{Q}\vee\neg\mathbf{S})\hat{+}[1\hat{-}V(\neg\mathbf{Q}\vee\neg\mathbf{S})]\hat{\cdot}V([\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}])\\
&\hat{=}V(\neg\mathbf{Q}\vee\neg\mathbf{S})\hat{+}V([\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}])\\
&\hat{=}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{S})\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{+}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{S})\\
&\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{S}),
\end{aligned}
\tag{3.8$_1$}
$$

because

$$
\begin{aligned}
&V(\neg\mathbf{Q}\vee\neg\mathbf{S})\hat{\cdot}V([\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}])\\
&\hat{=}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{S})\hat{\cdot}[V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{+}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{S})\\
&\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{S})]\hat{=}0,
\end{aligned}
\tag{3.8$_2$}
$$

by the variants of (II.7.15γ) with ‘**Q**’ or ‘**S**’ in place of ‘**P**’. By (3.8$_1$), it follows from the version of (II.7.3γ) with ‘$[[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]\wedge[\neg\mathbf{Q}\vee\neg\mathbf{S}]]$’ in place of ‘**P**’ and ‘$[\neg\mathbf{P}\vee\neg\mathbf{R}]$’ in place of ‘**Q**’ that

$$
\begin{aligned}
&V([[[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]\wedge[\neg\mathbf{Q}\vee\neg\mathbf{S}]]\Rightarrow[\neg\mathbf{P}\vee\neg\mathbf{R}])\\
&\hat{=}[1\hat{-}V([[\mathbf{P}\Rightarrow\mathbf{Q}]\wedge[\mathbf{R}\Rightarrow\mathbf{S}]]\wedge[\neg\mathbf{Q}\vee\neg\mathbf{S}])]\hat{\cdot}V(\neg\mathbf{P}\vee\neg\mathbf{R})\\
&\hat{=}[1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{S})\hat{-}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{-}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{S})\\
&\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{\cdot}V(\neg\mathbf{R})\hat{\cdot}V(\mathbf{S})]\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{R})\\
&\hat{=}[1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{S})\hat{-}V(\mathbf{Q})\hat{-}V(\mathbf{S})\hat{+}V(\mathbf{Q})\hat{\cdot}V(\mathbf{S})]\hat{\cdot}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{R})\hat{=}0,
\end{aligned}
\tag{3.8$_3$}
$$

In developing the final result in (3.8$_3$), use of the pertinent instances of identity (II.5.10) or (II.5.10ε) has been made for eliminating the occurrences of $V(\neg\mathbf{P})$ and $V(\neg\mathbf{R})$ in the square brackets. Then the resulting expression in the square brackets has been reduced by (II.7.1γ) and by the variants of (II.7.1γ) with ‘**Q**’ or ‘**S**’ in place of ‘**P**’ thus:

$$
\begin{aligned}
&1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{S})\hat{-}V(\mathbf{Q})\hat{-}V(\mathbf{S})\hat{+}V(\mathbf{Q})\hat{\cdot}V(\mathbf{S})\\
&\hat{=}1\hat{-}[1\hat{-}V(\mathbf{Q})]\hat{\cdot}[1\hat{-}V(\mathbf{S})]\hat{-}V(\mathbf{Q})\hat{-}V(\mathbf{S})\hat{+}V(\mathbf{Q})\hat{\cdot}V(\mathbf{S})\hat{=}0.
\end{aligned}
\tag{3.8$_4$}
$$

By (3.8$_3$), the pertinent version of (II.4.40a) immediately infers (3.8).•

3.3. Euautographic and catlogographic corollaries of Ths 3.1 and 3.2

°**Crl 3.1:** ***Euautographic conditional syllogisms (modi).***

A) Hypothetical syllogisms

1) *Modus ponendo ponens*:

$$[[p\Rightarrow q]\wedge p]\Rightarrow q\ . \tag{3.1μ}$$

2) *Modus tollendo tollens*:

$$[[p\Rightarrow q]\wedge\neg q]\Rightarrow\neg p\ . \tag{3.2μ}$$

B) Disjunctive syllogisms

1) *Modus tollendo ponens*:

$$[[p \vee q] \wedge \neg p] \Rightarrow q\ , \tag{3.3μ}$$

$$[[p \vee q] \wedge \neg q] \Rightarrow p\ . \tag{3.3'μ}$$

2) *Modus ponendo tollens*:

$$[[p\ \overline{\Leftrightarrow}\ q] \wedge p] \Rightarrow \neg q\ , \tag{3.4μ}$$

$$[[p\ \overline{\Leftrightarrow}\ q] \wedge q] \Rightarrow \neg p\ . \tag{3.4'μ}$$

°**Crl 3.2:** ***Euautographic dilemmas.***

1) *Simple constructive dilemma*:

$$[[[p \Rightarrow r] \wedge [q \Rightarrow r]] \wedge [p \vee q]] \Rightarrow r\ . \tag{3.5μ}$$

2) *Simple destructive dilemma*:

$$[[[p \Rightarrow q] \wedge [p \Rightarrow r]] \wedge [\neg q \vee \neg r]] \Rightarrow \neg p\ . \tag{3.6μ}$$

3) *Complex constructive dilemma*:

$$[[[p \Rightarrow q] \wedge [r \Rightarrow s]] \wedge [p \vee r]] \Rightarrow [q \vee s]. \tag{3.7μ}$$

4) *Complex destructive dilemma*:

$$[[[p \Rightarrow q] \wedge [r \Rightarrow s]] \wedge [\neg q \vee \neg s]] \Rightarrow [\neg p \vee \neg r]. \tag{3.8μ}$$

$^+$**Crl 3.3:** ***Conformal catlogographic conditional syllogisms (modi).***

A) Hypothetical syllogisms

1) *Modus ponendo ponens*:

$$[[p \Rightarrow q] \wedge p] \Rightarrow q\ . \tag{3.1κ}$$

2) *Modus tollendo tollens*:

$$[[p \Rightarrow q] \wedge \neg q] \Rightarrow \neg p\ . \tag{3.2κ}$$

B) Disjunctive syllogisms

1) *Modus tollendo ponens*:

$$[[p \vee q] \wedge \neg p] \Rightarrow q\ , \tag{3.3κ}$$

$$[[p \vee q] \wedge \neg q] \Rightarrow p\ . \tag{3.3'κ}$$

2) *Modus ponendo tollens*:

$$[[p\ \overline{\Leftrightarrow}\ q] \wedge p] \Rightarrow \neg q\ . \tag{3.4κ}$$

$$[[p\ \overline{\Leftrightarrow}\ q] \wedge q] \Rightarrow \neg p\ . \tag{3.4'κ}$$

$^+$**Crl 3.4:** ***Conformal catlogographic dilemmas.***

1) *Simple constructive dilemma*:

$$[[[p \Rightarrow r] \wedge [q \Rightarrow r]] \wedge [p \vee q]] \Rightarrow r\,. \qquad (3.5\kappa)$$

2) *Simple destructive dilemma*:

$$[[[p \Rightarrow q] \wedge [p \Rightarrow r]] \wedge [\neg q \vee \neg r]] \Rightarrow \neg p\,. \qquad (3.6\kappa)$$

3) *Complex constructive dilemma*:

$$[[[p \Rightarrow q] \wedge [r \Rightarrow s]] \wedge [p \vee r]] \Rightarrow [q \vee s]. \qquad (3.7\kappa)$$

4) *Complex destructive dilemma*:

$$[[[p \Rightarrow q] \wedge [r \Rightarrow s]] \wedge [\neg q \vee \neg s]] \Rightarrow [\neg p \vee \neg r]. \qquad (3.8\kappa)$$

3.4. A discussion of modi and dilemmas

1**)** I shall use the abbreviations: "MPP" for "modus ponendo ponens", "MTT" for "modus tollendo tollens", "MTP" for "modus tollendo ponens", "MPT" for "modus ponendo tollens", "SCD" for "simple constructive dilemma", "SDD" for "simple destructive dilemma", "CCD" for "complex constructive dilemma", and "CDD" for "complex destructive dilemma".

2) In the *verbal staccato form* (*style*), the CLR's (3.1κ)–(3.8κ) can be rendered into ordinary language thus:

Hypothetical syllogisms

MPP: If *p* then *q*. *p*. Therefore, *q*.

MTT: If *p* then *q*. Not *q*. Therefore, not *p*.

Disjunctive syllogisms

MTP: *p* or *q*. Not *p*. Therefore, *q*.

MPT: Either *p* or *q* but not both. *p*. Therefore, not *q*.

Dilemmas

SCD: If *p* then *r*. If *q* then *r*. *p* or *q*. Therefore, *r*.

SDD: If *p* then *q*. If *p* then *r*. Not *q* or not *r*. Therefore, not *p*.

CCD: If *p* then *q*. If *r* then *s*. *p* or *r*. Therefore, *q* or *s*.

CDD: If *p* then *q*. If *r* then *s*. Not *q* or not *s*. Therefore, not *p* or not *r*.

3) The modi and dilemmas of traditional logic as cited in Church [1956, pp. 104, 105] differ from the above verbal versions of the CLR's (3.1κ)–(3.8κ) in the following respects:

i) Church employs the letters 'A', 'B', 'C', 'D' in place of '*p*', '*q*', '*r*', '*s*' respectively.

ii) He abbreviates the names "*modus ponendo ponens*" and "*modus tolendo tollens*" as "*modus ponens*" and "*modus tollens*" respectively.

iii) In the major, disjunctive, premise of *his verbal version of modus [ponendo] tollens*, Church conventionally employs the *inclusive disjunctive conjunction* "or" (in Latin "vel") instead of the *exclusive disjunctive conjunction* **"**either … or *** but not both**"** (in Latin 'auf") that I employ in my verbal version *modus ponendo tollens* (*MPT*).

4) In accordance with the above point (iii), the panlogographic counterpart (interpretans) of Church's version of MTP should have the form:

$$\text{`}[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\neg\mathbf{Q}\text{'} \tag{3.10}$$

instead of (3.4). In this case, by (II.7.2γ), it follows from the version of (II.7.6γ) with '$[\mathbf{P}\vee\mathbf{Q}]$' and '$\mathbf{P}$' in place of '$\mathbf{P}$' and '$\mathbf{Q}$' respectively, it follows that

$$\begin{gathered} V([\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P})\hat{=}V(\mathbf{P})\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{P}\vee\mathbf{Q}) \\ \hat{=}V(\mathbf{P})\hat{+}V(\neg\mathbf{P})\hat{\cdot}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})\hat{=}V(\mathbf{P}), \end{gathered} \tag{3.10$_1$}$$

where use of identity has been made. By (3.10$_1$)¸ it follows from the version of (II.7.3γ) with '$[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]$' and '$\neg\mathbf{Q}$' in place of '$\mathbf{P}$' and '$\mathbf{Q}$' respectively that

$$\begin{gathered} V([[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q})\hat{=}[1\hat{-}V([\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P})]\hat{\cdot}V(\neg\mathbf{Q}) \\ \hat{=}[1\hat{-}V(\mathbf{P})]\hat{\cdot}[1\hat{-}V(\mathbf{Q})]\hat{=}1\hat{-}V(\mathbf{P})\hat{-}V(\mathbf{Q})\hat{+}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q}) \\ \hat{=}V(\neg\mathbf{P})\hat{\cdot}V(\neg\mathbf{Q})\hat{=}V(\neg\mathbf{P}\vee\neg\mathbf{Q}) \\ \hat{=}V(\mathbf{P}\Rightarrow\mathbf{Q})\hat{=}V(\mathbf{P}\Leftarrow\mathbf{Q})\hat{=}V(\mathbf{P}\curlywedge\mathbf{Q})\hat{=}V(\mathbf{P}\barwedge\mathbf{Q}), \end{gathered} \tag{3.10$_2$}$$

where use of (II.7.1γ), (II.7.3γ)–(II.7.5γ), and (II.7.9γ) has been made in developing the train of identities following '$1\hat{-}V(\mathbf{P})\hat{-}V(\mathbf{Q})\hat{+}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})$'. By (3.10$_2$), the version of (II.4.40c) with '$[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}$' and '$1\hat{-}V(\mathbf{P})\hat{-}V(\mathbf{Q})\hat{+}V(\mathbf{P})\hat{\cdot}V(\mathbf{Q})$' in place of '$\mathbf{P}$' and '$\mathbf{i}_{\sim}|\mathbf{P}\rangle$' respectively, indicates that the PLR (3.10) is a vav-neutral one, i.e. an ureterolodgy. At the same, the versions of (II.7.7γ) with '$[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}$' in place of '$\mathbf{P}$' and with '[$\neg\mathbf{P}\vee\neg\mathbf{Q}$]', '[$\mathbf{P}\vee\mathbf{Q}$]', '[$\mathbf{P}\vee\mathbf{Q}$]', '[$\mathbf{P}\curlywedge\mathbf{Q}$]', or '[$\mathbf{P}\barwedge\mathbf{Q}$]' in turn in place of '$\mathbf{Q}$' yield:

$$\begin{gathered} V([[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}]\Rightarrow[\neg\mathbf{P}\vee\neg\mathbf{Q}]) \\ \hat{=}V([[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{Q}]) \\ \hat{=}V([[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}]\Rightarrow[\mathbf{P}\Leftarrow\mathbf{Q}]) \\ \hat{=}V([[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}]\Rightarrow[\mathbf{P}\curlywedge\mathbf{Q}]) \\ \hat{=}V([[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}]\Rightarrow[\mathbf{P}\barwedge\mathbf{Q}]), \end{gathered} \tag{3.10$_3$}$$

whence

$$\begin{array}{c}[[[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}]\Rightarrow[\neg\mathbf{P}\vee\neg\mathbf{Q}]]\\ \Leftrightarrow[[[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}]\Rightarrow[\mathbf{P}\Rightarrow\mathbf{Q}]]\\ \Leftrightarrow[[[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}]\Rightarrow[\mathbf{P}\Leftarrow\mathbf{Q}]]\\ \Leftrightarrow[[[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}]\Rightarrow[\mathbf{P}\not\wedge\mathbf{Q}]]\\ \Leftrightarrow[[[[\mathbf{P}\vee\mathbf{Q}]\wedge\mathbf{P}]\Rightarrow\mathbf{Q}]\Rightarrow[\mathbf{P}\,\overline{\wedge}\,\mathbf{Q}]],\end{array} \tag{3.10$_4$}$$

by the pertinent instances of (II.4.40a).

5) Consequently, the ER

$$[[p\vee q]\wedge p]\Rightarrow\neg q\ , \tag{3.10μ}$$

being the conformal euautographic interpretand (instance, corollary) of the PLR (3.10), and the CLR

$$‘[[p\vee q]\wedge p]\Rightarrow\neg q’, \tag{3.10κ}$$

being the CFCL interpretand (instance, corollary) of the ER (3.10μ), are also *vav-neutral* (*vav-indeterminate*) ones, i.e. *udeterologies*. Thus, in contrast to (3.4κ), the CLR (3.10κ) is a *ttatt-neutral* one, and *not a tautology*. At the same time, the CLR (3.10κ) is a *logographic legato form* of the conventional verbal staccato form of traditional *modus ponendo tollens* as that cited by Church.

6) Susan K. Langer [1967, pp. 347, 348] seems to have been the first logician to notice that *modus ponendo tollens* of traditional logic is incompatible with the propositional calculus of *Principia Mathematica* and that *neither this syllogism nor modus tollendo ponens* appears in *Principia Mathematica*. She has also suggested that the reason for the above incompatibility is that the conjunction (verbal sentential connective) "or" occurring in the traditional verbal form of *modus ponendo tollens* was understood in the exclusive sense. I have therefore replaced the occurrence of the conjunction "or" in traditional *modus ponendo tollens* by "either ... or *** but not both" and, after all, by the logographic sentential logical connective '$\overline{\Leftrightarrow}$' defined by Df II.1.10(12). Incorporated into A_0 in this way, *modus ponendo tollens* has become a theorem of A_0, like all other inference rules of traditional sentential logic.

7) It is known that MPP, MTT, and MPT were Stoics' discovery, whereas MTP was expressed by means of the *inclusive* (*polyadic*) *or* (Latin *vel*) by Galen and was then used prevalently by the Schoolmen, who unlike the Stoics, did not use the *exclusive*(*m onadic*) *or* (Latin *auf*) at all.

8) In accordance with its verbal traditional counterpart, a panlogographic conditional syllogism is an implication, whose *antecedent* is the conjunction of two

premises, one of which is *conditional* that is called the *major premise*, while the other one is *categorical* (*unconditional*) that is called the *minor premise* of the syllogism; the *consequent* of the syllogism corresponds to the *conclusion* of its traditional semi-verbal counterpart. Although the premises of a syllogism are commutable, the major (conditional) premise is conventionally put first. A conditional syllogism is called a *hypothetical syllogism* if its major premise is a *hypothetical* (*implication*) and a *disjunctive syllogism* if its major premise is a *disjunction*, *inclusive* or *exclusive*. In the name of a conditional syllogism, the qualifier "*ponendo*" means that the minor premise of the syllogism is *affirmative*, whereas the qualifier "*tollendo*" means that the minor premise is *negative*. At the same time, the qualifier "*ponens*" means that the conclusion of the syllogism is *affirmative*, whereas the qualifier "*tollens*" means that the conclusion is *negative*. It is noteworthy that the major premise of *modus tollendo ponens* is an *inclusive disjunction*, whereas the major premise of *modus ponendo tollens* is an *exclusive disjunction*, i.e. *anti-equivalence*.

9) A verbal traditional dilemma, called also a *dilemmatic syllogism*, is a *four-sentence schema* of inference that consists of three premises and a conclusion; the first two premises of any dilemma are *hypothetical* (*implicative*) *declarative sentences.* A dilemma is said to be:

a) *simple* if its rank equals 3 and if its conclusion is *categorical*,

b) *complex* if its rank equals 4 and if its conclusion is an inclusive disjunction.

A *simple dilemma* is said to be:

a') a *simple constructive* one if its third premise is the inclusive disjunction of *simple affirmative clauses*, while its conclusion is a *simple affirmative categorical sentence*;

a'') a *simple destructive* one if its third premise is the *inclusive disjunction of simple negative clauses,* while its conclusion is a simple *negative categorical sentence*.

A *complex dilemma* is said to be:

b') a *complex constructive* one if both its third premise and its conclusion are *inclusive disjunctions of simple affirmative clauses*;

b'') a *complex destructive* one if both its third premise and its conclusion are *inclusive disjunctions of simple negative clauses*.•

10) In accordance with its verbal traditional counterpart, a panlogographic dilemma, called also a panlogographic *dilemmatic syllogism*, is an implication¸ whose *antecedent* is the conjunction of three premises, which correspond to the premises of the verbal dilemma, and whose *consequent* corresponds to the *conclusion* of the verbal dilemma. The first two premises of any panlogographic dilemma are *hypothetical* (*implicative*) PLR’s. A dilemma said to be:

a) *simple* if its rank equals 3 and if its conclusion is *categorical,*

b) *complex* if its rank equals 4 and if its conclusion is an inclusive disjunction.

A *simple dilemma* is said to be:

a') a *simple constructive* one if its third premise is the *inclusive disjunction of two APLR’s,* while its conclusion is an APLR;

a'') a *simple destructive* one if its third premise is the *inclusive disjunction of the negations of two APLR’s,* while its conclusion is the *negation of an APLR.*

A *complex dilemma* is said to be:

b') a *complex constructive* one if both its third premise and its conclusion are *inclusive disjunctions of APLR’s*;

b'') a *complex destructive* one if both its third premise and its conclusion are *inclusive disjunctions of the negations of APLR’s.*

11) Some identifying properties of the dilemmas of four different kinds are conditional. Namely, by Df I.I1.10(3) and by the theorem (1.1_1), if follows that

$$V(\mathbf{P} \vee \mathbf{Q}) \hat{=} V(\neg\mathbf{P} \Rightarrow \mathbf{Q}) \text{ (a); } V(\neg\mathbf{Q} \vee \neg\mathbf{R}) \hat{=} V(\mathbf{Q} \Rightarrow \neg\mathbf{R}) \text{ (b);}$$
$$V(\mathbf{P} \vee \mathbf{R}) \hat{=} V(\neg\mathbf{P} \Rightarrow \mathbf{R}) \text{ (c); } V(\neg\mathbf{Q} \vee \neg\mathbf{S}) \hat{=} V(\mathbf{Q} \Rightarrow \neg\mathbf{S}) \text{ (d);}$$
$$V(\mathbf{Q} \vee \mathbf{S}) \hat{=} V(\neg\mathbf{Q} \Rightarrow \mathbf{S}) \text{ (e); } V(\neg\mathbf{P} \vee \neg\mathbf{R}) \hat{=} V(\mathbf{P} \Rightarrow \neg\mathbf{R}) \text{ (f).}$$

Hence, in contrast to what is stated in the above item 9, the third premise of each one of the four dilemmas (3.5)–(3.8) and the conclusion of either one of the two complex dilemmas (3.7) and (3.8) can be represented as the respective implications (3.5)–(3.8).•

3.5. Theoremhood of the axioms of CASC’i

1) In the previous two sections of this chapter and in the preceding part of this section, I have demonstrated that all traditional sentential rules of inference are the CFCL interpretands of certain euautographic theorems of A_0. In agreement with subsection 3.1, I may also take for granted the following axiom.

Ax 3.1: Every axiom and hence every theorem of any known CASC is the CFCL interpretand of a certain theorem of A_0. •

This axiom is self-evident. However, there is a great many of CALCI' (see, e.g., Church [1956, chapters I and II, pp. 69–167; especially, §29, pp. 155-167]). It is therefore impossible to demonstrate the thesis that I state as Ax 3.1 in full. I shall therefore demonstrate the validity of Ax 3.1 on the example of the CASC'i P_R, P_1, and P_2 (in Church's nomenclature)

2) The CASC P_R is based on the following five axioms as stated in Whitehead and Russell [1910, 1925; 1962, pp. 96, 97]:

∗1·2. $[p \vee p] \Rightarrow p$. (*Principle of tautology*)

∗1·3. $q \Rightarrow [p \vee q]$. (*Principle of addition*)

∗1·4. $[p \vee q] \Rightarrow [q \vee p]$. (*Principle of permutation*)

∗1·5. $[p \vee [q \vee r]] \Rightarrow [q \vee [p \vee r]]$. (*Associative principle*)

∗1·6. $[q \Rightarrow r] \Rightarrow [[p \vee q] \Rightarrow [p \vee r]]$. (*Principle of summation*)

The parenthesized names have been given to the axioms by their authors.

3) The above axioms were for the first time published by Russell [1908], and were then used by Whitehead and Russell in the first edition of their *Principia Mathematica* in 1910. Somewhat later, Paul Bernays [1926] discovered the non-independence of axiom **∗1·5**. Accordingly, the propositional calculus system, which is obtained by deleting the redundant axiom, is called the *Russell-Bernays system*, and it is denoted by 'P_{RB}'. The calculi P_R and P_{RB} are discussed, e.g., in Hilbert and Ackermann [1950, §10, pp. 27–30] and in Church [*ibid.* §25, pp. 136–138]. The system of four axioms of P_{RB} is used in Burbaki [1960, §3, axioms S1-S4] as the groundwork of their set theory.

4) Axioms **∗1·2**, **∗1·3**, and **∗1·6** coincide with (1.15κ), (1.27κ), and (1.35κ), respectively; axiom **∗1·4** is the instance of (2.21κ) for $\vee$ as $\boldsymbol{\theta}$ and with $\Rightarrow$ in place of $\Leftrightarrow$; axiom **∗1·5** is the instance of (2.24κ) with $\Rightarrow$ in place of $\Leftrightarrow$, subject to the pertinent variant of the above instance of (2.21κ). Alternatively, axioms **∗1·4** and **∗1·5** can be deduced straightforwardly, e.g., as follows. From the instance of (II.7.3γ) with $[p \vee q]$ as **P** and $[q \vee p]$ as **Q**, it follows that

$$\begin{gathered} V([p \vee q] \Rightarrow [q \vee p]) \hat{=} V(\neg[p \vee q]) \hat{\cdot} V(q \vee p) \\ \hat{=} [1 \hat{-} V(p \vee q)] \hat{\cdot} V(q) \hat{\cdot} V(p) \hat{=} [1 \hat{-} V(p) \hat{\cdot} V(q)] \hat{\cdot} V(q) \hat{\cdot} V(p) \hat{=} 0. \end{gathered} \tag{3.11}$$

Likewise, from the`` instance of (II.7.3γ) with $[p \vee [q \vee r]]$ as **P** and $[q \vee [p \vee r]]$ as **Q**, it follows that

$$\begin{aligned} V([p \vee [q \vee r]] \Rightarrow [q \vee [p \vee r]]) &\hat{=} [1 \hat{-} V(p \vee [q \vee r])] \hat{\cdot} V(q \vee [p \vee r]) \\ &\hat{=} [1 \hat{-} V(p) \hat{\cdot} V(q) \hat{\cdot} V(r)] \hat{\cdot} V(q) \hat{\cdot} V(p) \hat{\cdot} V(r) \hat{=} 0. \end{aligned} \tag{3.12}$$

By (3.11) and (3.12), the pertinent instances of (II.4.40a) immediately infer that

$$[p \vee q] \Rightarrow [q \vee p], \tag{3.11a}$$

$$[p \vee [q \vee r]] \Rightarrow [q \vee [p \vee r]] \tag{3.12a}$$

respectively, whereas Russell's axioms**∗1·4** and **∗1·5** are the CFCL interpretands of (3.11a) and (3.12a).

5) The axioms †102 and †103 of P_1 in Church [1956, p. 72] coincide with corollaries (1.31κ) and (1.34κ), respectively. The axiom †104 of P_1 [*ibid.*] has the form:

$$[[p \Rightarrow f] \Rightarrow f] \Rightarrow p, \tag{3.13}$$

which Church calls the *law of double negation*. In this case, *f* is a primitive constant of P_1 [*ibid.*, p. 69], which should be regarded as a placeholder, whose range is the class of false (antitrue) simple negative declarative sentences, in accordance with the principal interpretation of P_1 [*ibid.*, pp. 73, 74]. Therefore, in order to adjust A_0 to P_1, I should supplement the list (I.5.2) with another AER's, say p_0 (because f is already employed as an atomic predicate-sign), such that

$$V(p_0) \hat{=} 1. \tag{3.14}$$

Therefore, by the pertinent instances of (II.7.3γ), it follows that

$$\begin{aligned} V([[p \Rightarrow p_0] \Rightarrow p_0] \Rightarrow p) &\hat{=} [1 \hat{-} V([p \Rightarrow p_0] \Rightarrow p_0)] \hat{\cdot} V(p) \\ &\hat{=} [1 \hat{-} [1 \hat{-} V(p \Rightarrow p_0)] \hat{\cdot} V(p_0)] \hat{\cdot} V(p) \\ &\hat{=} [1 \hat{-} [1 \hat{-} V(\neg p) \hat{\cdot} V(p_0)] \hat{\cdot} V(p_0)] \hat{\cdot} V(p) \\ &\hat{=} [1 \hat{-} [1 \hat{-} V(\neg p) \hat{\cdot} 1] \hat{\cdot} 1] \hat{\cdot} V(p) \hat{=} [1 \hat{-} [1 \hat{-} V(\neg p)]] \hat{\cdot} V(p) \\ &\hat{=} V(\neg p) \hat{\cdot} V(p) \hat{=} 0, \end{aligned} \tag{3.15}$$

whence

$$[[p \Rightarrow p_0] \Rightarrow p_0] \Rightarrow p, \tag{3.15a}$$

by the pertinent intstance of (II.4.40a). The CLR

$$[[p \Rightarrow p_0] \Rightarrow p_0] \Rightarrow p, \tag{3.15κ}$$

being the CFCL interpretand of the euautographic kyrology (3.15a), coincides with (3.13), up to the notation used.

6) The first two of three axioms of P_2¸ †202 and †203¸ in Church [*ibid.*, p.119] are the same as the first two axioms of P_1, †102 and †103¸ while the third axiom of P_2, †204, coincides with corollary (2.10"κ).•

Chapter IV. The main branches of A_1 and their pseudo-confined versions: the organons $\mathsf{A}_{1\in}$, $\bar{\mathbf{A}}_{1\in}$, $\mathsf{A}_{1\subseteq}$, $\bar{\mathbf{A}}_{1\subseteq}$ and $\mathsf{A}_{1=}$

1. The organon $\mathsf{A}_{1\in G}$

1.1. Basic definitions

†**Df 1.1.** 1) In accordance with Df I.7.1, the branch of A_1 that is [logographically] denoted by '$\mathsf{A}_{1\in}$' and is [phonographically] called the *Pseudo-Class Euautogographic Algebraico-Predicate* (*PCsEAPO*) has the atomic basis, denoted by '$\mathsf{B}_{1\in}$', which comprises the distinguished *primary binary atomic pseudo-constant ordinary predicate-sign* (*PBAPCOPS*) $\in$ that is indicated in the point c of item 8 of Ax II.5.1 and also all primary atomic euautographs that are indicated in items 1–7 and 9–12 of Ax II.5.1. The organon that is denoted by '$\mathsf{A}_{1\in G}$' and is called the *Ground PCsEAPO* (*GPCsEAPO*) is the branch $\mathsf{A}_{1\in}$ at the phase (stage) of its setup (development), at which its atomic basis, denoted by '$\mathsf{B}_{1\in G}$', includes all elements of $\mathsf{B}_{1\in}$ in the exclusion of the two APCOT's $\emptyset$ and $\emptyset'$ indicated in item 9 of Ax II.5.1, and at which $\mathsf{A}_{1\in}$ it has *no subject axioms other than those of* A_{1P}, but at which it has, thirteen *secondary binary primitive* (*elemental*, *atomic or molecular*) *pseudo-constant ordinary predicate-signs* (*SBPPCOPS's*) that are defined in terms $\in$ by the following *secondary formation rules* having the status of *meta-axioms*:

$$[\mathbf{x}\in\mathbf{u}]\to\in(\mathbf{x},\mathbf{u})\leftarrow\ni(\mathbf{u},\mathbf{x})\leftarrow[\mathbf{u}\ni\mathbf{x}]. \tag{1.1}$$

$$\begin{aligned}&[\mathbf{x}\,\overline{\in}\,\mathbf{u}]\to\neg[\mathbf{x}\in\mathbf{u}]\to\overline{\in}(\mathbf{x},\mathbf{u})\to\neg\,\overline{\in}(\mathbf{x},\mathbf{u})\\&\leftarrow\neg\ni(\mathbf{u},\mathbf{x})\leftarrow\overline{\ni}(\mathbf{u},\mathbf{x})\leftarrow\neg[\mathbf{u}\,\overline{\ni}\,\mathbf{x}]\leftarrow[\mathbf{u}\,\overline{\ni}\,\mathbf{x}].\end{aligned} \tag{1.2}$$

$$[\mathbf{u}\subseteq\mathbf{v}]\to\subseteq(\mathbf{u},\mathbf{v})\to\textstyle\bigwedge_{\mathbf{x}}[[\mathbf{x}\in\mathbf{u}]\Rightarrow[\mathbf{x}\in\mathbf{v}]]\leftarrow\supseteq(\mathbf{v},\mathbf{u})\leftarrow[\mathbf{v}\supseteq\mathbf{u}]. \tag{1.3}$$

$$\begin{aligned}&[\mathbf{u}\,\overline{\subseteq}\,\mathbf{v}]\to\neg[\mathbf{u}\subseteq\mathbf{v}]\to\overline{\subseteq}(\mathbf{u},\mathbf{v})\to\neg\,\overline{\subseteq}(\mathbf{u},\mathbf{v})\\&\leftarrow\neg\,\overline{\supseteq}(\mathbf{v},\mathbf{u})\leftarrow\overline{\supseteq}(\mathbf{v},\mathbf{u})\leftarrow\neg[\mathbf{v}\,\overline{\supseteq}\,\mathbf{u}]\leftarrow[\mathbf{v}\,\overline{\supseteq}\,\mathbf{u}].\end{aligned} \tag{1.4}$$

$$[\mathbf{u}=\mathbf{v}]\to=(\mathbf{u},\mathbf{v})\to[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{u}]]. \tag{1.5}$$

$$[\mathbf{u}\equiv\mathbf{v}]\to\neg[\mathbf{u}=\mathbf{v}]\to\neg=(\mathbf{u},\mathbf{v})\leftarrow\equiv(\mathbf{u},\mathbf{v}). \tag{1.6}$$

$$[\mathbf{u}\subset\mathbf{v}]\to\subset(\mathbf{u},\mathbf{v})\to[[\mathbf{u}\subseteq\mathbf{v}]\wedge\neg[\mathbf{v}\subseteq\mathbf{u}]]\leftarrow\supset(\mathbf{v},\mathbf{u})\leftarrow[\mathbf{v}\supset\mathbf{u}]. \tag{1.7}$$

$$\begin{gathered}[\mathbf{u} \bar{\subset} \mathbf{v}] \rightarrow \neg[\mathbf{u} \subset \mathbf{v}] \rightarrow \bar{\subset}(\mathbf{u}, \mathbf{v}) \rightarrow \neg \subset(\mathbf{u}, \mathbf{v}) \\ \leftarrow \neg \supset(\mathbf{v}, \mathbf{u}) \leftarrow \bar{\supset}(\mathbf{v}, \mathbf{u}) \leftarrow \neg[\mathbf{v} \bar{\supset} \mathbf{u}] \leftarrow [\mathbf{v} \bar{\supset} \mathbf{u}].\end{gathered} \tag{1.8}$$

Accordingly, the calculus $\mathbf{A}_1$ being at this stage the calculus of placeholders of formulas of $\mathsf{A}_{1\in G}$, is denoted by '$\mathbf{A}_{1\in G}$' and is called the *Ground Pseudo-Class Panlogographic Algebraico-Predicate Organon* (*GPCsPLAPO*).

2) In stating definitions (1.1)–(1.8), use has been made of the pertinent instances of the *contextual* definition schema

$$[\mathbf{x}\mathbf{F}^2\mathbf{y}] \rightarrow \mathbf{F}^2(\mathbf{x}, \mathbf{y}) \tag{1.9}$$

of a *binary* predicate-operator '$[\ \mathbf{F}^2\ ]$' in terms of the respective *binary* predicate-operator '$\mathbf{F}^2(\ ,\ )$', which is applicable with any *binary* predicate-sign $\mathbf{F}^2$ (cf. (II.1.18))**.** It will be recalled that a binary relation schema '$\mathbf{F}^2(\mathbf{x}, \mathbf{y})$' and any of its instances or variants are said to be given (written) in the *nonlinear*, or *inhomogeneous*, or *Clairaut-Euler's*, *form*, whereas the binary relation schema '$[\mathbf{x}\mathbf{F}^2\mathbf{y}]$' and any of its instances or variants are said to be given (written) in the *bilinear*, or less explicitly *homogeneous*, *form*; the word "*form*" in any of the above terms can be used interchangeably with the words "*notation*" and "*representation*". In the sequel, I shall, as a rule, employ only those definienda of definitions (1.1)–(1.8), which are given in the *homogeneous* (*bilinear*) *form* (*notation*, *representation*).

3) *The definition of the equality sign*, (1.5), is analogous to the *axiom of extension*, or *extensionality*, of set theory (see, e.g., Halmos [1960, p. 2] or Fraenkel *et al* [1973, p. 27]) and therefore it will be called the *definition of pseudo-extension*, or *pseudo-extensionality*. Still, all *definitions* (1.1)–(1.8), including (1.5), are *axiomatic ones*, i.e. they are at the same time *axioms of incidence*. The fact that these definitions allow proving from them a variety of *theorems* evidences of their axiomatic status.

4) I shall give the proper names: "*the direct sign of membership*" to $\in$, "*the direct sign of inclusion*" to $\subseteq$, "*the direct strict sign of inclusion*" to $\subset$, "*the direct sign of non-membership*" to $\bar{\in}$, "*the direct sign of non-inclusion*" to $\bar{\subseteq}$, "*the direct sign of non-inclusion*" to $\bar{\subset}$, "*the ordinary sign of equality*" to =, and "*the ordinary sign of anti-equality*" to $\equiv$, the understanding being that in contrast to the conventional mathematical *symmetric* signs: ≤, ≥, <, and >, which are *asymmetric* (*unilateral*, *one-sided*), the *anti-equality* sign $\equiv$ is *symmetric* (*bilateral*, *two-sided*). The signs $\ni$, $\bar{\ni}$, $\supseteq$, $\bar{\supseteq}$, $\supset$, $\bar{\supset}$ will be termed by the variants of the first six of the above names with

"*converse*" in place of "*direct*". Thus, the signs (kernel-signs, predicate-signs,predicates) $\in$, $\subseteq$, $\subset$, $\overline{\in}$, $\overline{\subseteq}$, $\overline{\subset}$ are said to be *direct*; $\ni$, $\overline{\ni}$, $\supseteq$, $\overline{\supseteq}$, $\supset$, $\overline{\supset}$ are said to be *converse*; the signs = and $\equiv$ are *symmetric* so that they can be relegated to the direct ones and to the converse ones simultaneously; $\in$, $\subseteq$, =, $\subset$, $\ni$, $\supseteq$, $\supset$ are said to be *atomic*; $\overline{\in}$, $\overline{\subseteq}$, $\equiv$, $\overline{\subset}$, $\overline{\ni}$, $\overline{\supseteq}$, $\overline{\supset}$ are said to be *molecular* for the following reason (cf. Df A3.1(1c)). Just as in the items 8–13 of Df I.1.10(8–13), the overbar of an adjustable length, ¯, in any of definitions (1.2), (1.4), (1.6), and (1.8) is defined as a *secondary atomic* pseudo-constant sign, which can be regarded as the sign of negation of the sign over which it is put, and which is therefore a *synonym* of the universal sign of negation, $\neg$, being another *secondary atomic* pseudo-constant sign to be defined; that is, $\overline{\in}\to\neg\in$, $\overline{\subseteq}\to\neg\subseteq$, etc. Keeping in mind definition (1.9), the qualifiers "direct" and "converse" can be used interchangeably with "*rightward*" and "*leftward*" respectively. An atomic or molecular sign is indiscriminately called a *primitive*, or *elemental*, *sign*. As long as the BPPCOPS's (BOPDP's) apply to [*atomic*] *euautographic ordinary terms* (*AtEOT's* or *EOT's*) or to *structural atomic panlogographic ordinary terms* (*StAtPLOT's*), being their panlogographic placeholders, they are euautographs themselves, and therefore they cannot be read verbally, but rather they can be mentioned by using their proper verbal names. Once, however, the BPPCOPS's apply to the *conformal catlogographic* (*CFCL*) *interpretands* of EOT's, they become catlogographs that can be read verbally. Proper names of the BPPCOPS's and verbal equivalents of their *catlogographic homonymous* (*cathomographs*) are given in Table 1.1.

5) A relation involving any of the above fourteen kernel-signs will be termed by the version of the name of the sign with "relation" in place of "sign". That is to say, I shall use the names: "*direct relation of membership*", "*direct relation of inclusion*" "*a direct strict relation of inclusion*", "*the direct relation of non-membership*", "*direct relation of non-inclusion*", "*direct relation of strict non-inclusion*", and the synonymous names with "rightward" in place of "direct" for relations involving the signs $\in$, $\subseteq$, $\subset$, $\overline{\in}$, $\overline{\subseteq}$, $\overline{\subset}$ respectively; the versions of the above names with "converse" and "leftward" in place of "direct" and "rightward" for mentioning relations involving the signs: $\ni$, $\overline{\ni}$, $\supseteq$, $\overline{\supseteq}$, $\supset$, $\overline{\supset}$ respectively; "*ordinary relation of equality*" and "*ordinary relation of anti-equality*", or briefly "*ordinary equality*" and "*ordinary anti-equality*", for mentioning relations involving = and $\equiv$

respectively. I shall also give the names: "*the member-term*" and "*the class-term*" respectively to **x** and **u** occurring in the definiens or in any definientia of (1.1)–(1.3); any of the relations; "*the subclass-term*" and "*the superclass-term*" respectively to **u** and **v** occurring in the definiens or in any definientia of any train of definitions (1.3)–(1.5), (1.7), or (1.8).•

Table 1.1: ***Proper names of the euautographic BPPCOPS's and wordy equivalents of their cathomographs (catlogographic homonyms)***

Sign	A proper name of a sign	Verbal equivalents of the cathomograph of a sign
$\in$	The direct sign of membership	belongs to, is a member of
$\subseteq$	The direct sign of inclusion	is a part of, is a strict part or whole of, is included in, is a subclass of
$=$	The ordinary sign of equality	equals, is equal to
$\subset$	The direct sign of strict inclusion	is a strict part of, is strictly included in, is a strict subclass of
$\ni$	The converse sign of membership	is a class of, contains
$\supseteq$	The converse sign of inclusion	is a whole of, includes, is a superclass of
$\supset$	The converse sign of inclusion	is a strict whole of, strictly includes, is a strict superclass of
$\overline{\in}$	The direct sign of non-membership	does not belong to, is not a member of
$\overline{\subseteq}$	The direct sign of [strict] non-inclusion	is not a lax part of, is neither a strict part nor a whole of, is not included in, is not a subclass of
$\overline{\subset}$	The direct sign of strict non-inclusion	is not a strict part of, is not strictly included in, is not a strict subclass of
$\overline{=}$	The ordinary symmetric sign of inequality, the ordinary sign of anti-equality	does not equal, is not equal
$\overline{\ni}$	The direct sign of non-membership	is not a class of, does not contain
$\overline{\supseteq}$	The converse sign of inclusion	is not a whole of, does not laxly include, is not a superclass of

$\overline{\supset}$	The converse sign of strict non-inclusion (anti-inclusion)	is not a strict whole of, does not strictly include, is not a strict superclass of

Cmt 1.1. 1) In compliance with the *negative logical connectives* and with the *special sign of anti-equality*, $\hat{\overline{=}}$, which have been introduced in Df 1.10(8–13), a bar, $^{-}$, of an adjustable length over a base kernel-sign is used in Df 1.1 instead of the more common slant upright to the left stroke, /, across the base sign for the sake of universality in the consequence of typographical difficulties.•

Df 1.2. In accordance with Dfs I.5.2 and 1.1, $\mathrm{A}_{1\in\mathrm{G}}$ has four sets of main atomic ordinary euautographs: τ^{pv}, σ, κ^{pv}, and $\mathrm{K}_{\in}^{\mathrm{pc}}$, and the set of all 14 binary primitive (atomic and molecular) pseudo-constant predicate-signs: $\hat{\mathrm{K}}_{\in}^{2\mathrm{pc}}$, which are formally defined, partitioned, and united as follows.

$$\tau^{\mathrm{pv}} \to \{u,v,w,x,y,z,u_1,v_1,w_1,x_1,y_1,z_1,u_2,v_2,w_2,x_2,y_2,z_2,\ldots\}. \tag{1.10}$$

$$\sigma \to \{p,q,r,s,p_1,q_1,r_1,s_1,p_2,q_2,r_2,s_2,\ldots\}. \tag{1.11}$$

$$\begin{gathered}\kappa^{\mathrm{pv}} \to \bigcup_{m=1}^{\infty}\kappa^{m\mathrm{pv}},\\ \kappa^{m\mathrm{pv}} \to \{f^m,g^m,h^m,f_1^m,g_1^m,h_1^m,f_2^m,g_2^m,h_2^m,\ldots\}\text{for each } m \mathrel{\vec{\in}} \omega_1.\end{gathered} \tag{1.12}$$

$$\begin{gathered}\mathrm{K}_{\in}^{\mathrm{pc}} \to \mathrm{K}_{\in}^{2\mathrm{pc}} \to \{\in,\subseteq,\subset,=\},\\ \hat{\mathrm{K}}_{\in}^{2\mathrm{pc}} \to \vec{\mathrm{K}}_{\in}^{2\mathrm{pc}} \cup \bar{\mathrm{K}}_{\in}^{2\mathrm{pc}} \leftrightarrow \{\in,\subseteq,\subset,\overline{\in},\overline{\subseteq},\overline{\subset},=,\equiv,\ni,\supseteq,\supset,\overline{\ni},\overline{\supseteq},\overline{\supset}\},\\ \vec{\mathrm{K}}_{\in}^{2\mathrm{pc}} \to \{\in,\subseteq,\subset,\overline{\in},\overline{\subseteq},\overline{\subset},=,\equiv\},\ \bar{\mathrm{K}}_{\in}^{2\mathrm{pc}} \to \{\ni,\supseteq,\supset,\overline{\ni},\overline{\supseteq},\overline{\supset},=,\equiv\},\\ \hat{\mathrm{K}}_{=}^{2\mathrm{pc}} \to \vec{\mathrm{K}}_{\in}^{2\mathrm{pc}} \cap \bar{\mathrm{K}}_{\in}^{2\mathrm{pc}} \leftrightarrow \{=,\equiv\}.\end{gathered} \tag{1.13}$$

$$\begin{gathered}\mathrm{K}_{\in} \to \kappa^{\mathrm{pv}} \cup \mathrm{K}_{\in}^{\mathrm{pc}} \to \bigcup_{m=1}^{\infty}\mathrm{K}_{\in}^{m},\\ \mathrm{K}_{\in}^{m} \to \kappa^{m\mathrm{pv}} \text{ for each } m \mathrel{\vec{\in}} \omega_1 - \{2\},\ \mathrm{K}_{\in}^{2} \to \kappa^{2\mathrm{pv}} \cup \mathrm{K}_{\in}^{\mathrm{pc}} \to \kappa^{2\mathrm{pv}} \cup \mathrm{K}_{\in}^{2\mathrm{pc}}.\end{gathered} \tag{1.14}$$

It will be recalled that 'pv' is an abbreviation for "pseudo-variable" and 'pc' for "pseudo-constant".•

Df 1.3. 1) Each logograph on each one of the following lists:

$$\text{'}\mathbf{F}^1\text{'},\ \text{'}\mathbf{G}^1\text{'},\ \text{'}\mathbf{H}^1\text{'},\ \text{'}\mathbf{F}_1^1\text{'},\ \text{'}\mathbf{G}_1^1\text{'},\ \text{'}\mathbf{H}_1^1\text{'},\ \text{'}\mathbf{F}_2^1\text{'},\ \text{'}\mathbf{G}_2^1\text{'},\ \text{'}\mathbf{H}_2^1\text{'},\ \ldots, \tag{1.15^1}$$

$$\text{'}\mathbf{F}^2\text{'},\ \text{'}\mathbf{G}^2\text{'},\ \text{'}\mathbf{H}^2\text{'},\ \text{'}\mathbf{F}_1^2\text{'},\ \text{'}\mathbf{G}_1^2\text{'},\ \text{'}\mathbf{H}_1^2\text{'},\ \text{'}\mathbf{F}_2^2\text{'},\ \text{'}\mathbf{G}_2^2\text{'},\ \text{'}\mathbf{H}_2^2\text{'},\ \ldots, \tag{1.15^2}$$

$$\text{'}\mathbf{F}^3\text{'},\ \text{'}\mathbf{G}^3\text{'},\ \text{'}\mathbf{H}^3\text{'},\ \text{'}\mathbf{F}_1^3\text{'},\ \text{'}\mathbf{G}_1^3\text{'},\ \text{'}\mathbf{H}_1^3\text{'},\ \text{'}\mathbf{F}_2^3\text{'},\ \text{'}\mathbf{G}_2^3\text{'},\ \text{'}\mathbf{H}_2^3\text{'},\ \ldots, \tag{1.15^3}$$

etc is a *secondary structural atomic panlogographic ordinary predicate-sign* (*SStAPLOPS*), whose range is ‘$\mathrm{K}_{\in}^{1}$’, ‘$\mathrm{K}_{\in}^{2}$’, ‘$\mathrm{K}_{\in}^{3}$’, etc, so that, e.g., for each $m \vec{\in} \omega_1$ $\mathbf{F}^m \vec{\in} \mathrm{K}_{\in}^{m}$, and similarly with any other SStAPLOPS of the list (1.15^m).

2) Each logograph on the following list:

$$\text{‘}\mathbf{F}\text{’, ‘}\mathbf{G}\text{’, ‘}\mathbf{H}\text{’, ‘}\mathbf{F}_1\text{’, ‘}\mathbf{G}_1\text{’, ‘}\mathbf{H}_1\text{’, ‘}\mathbf{F}_2\text{’, ‘}\mathbf{G}_2\text{’, ‘}\mathbf{H}_2\text{’}, \ldots \tag{1.15}$$

is an SStAPLOPS, whose range is $\mathrm{K}_{\in}$, so that, e.g., $\mathbf{F} \vec{\in} \mathrm{K}_{\in}$, and similarly with any other SStAPLOPS of the list (1.15).

3) Just as in the case of PStAPLOPS’s (see the items 3–5 and 7 of Df I.5.1), at any place, the range $\mathrm{K}_{\in}^{2}$ of any given SStAPLOPS on the list (1.15^2) (or the range $\mathrm{K}_{\in}$ of any given SStAPLOPS on the list (1.15)) may at any place be restricted either to $\kappa^{2\mathrm{pv}}$ or to $\mathrm{K}_{\in}^{2\mathrm{pc}}$ (correspondingly, either to κ^{pv} or to $\mathrm{K}_{\in}^{2\mathrm{pc}}$) by the appropriate statement in the metalanguage or by furnishing the given SStAPLOPS with the corresponding superscript ‘$^{\mathrm{pv}}$’ or ‘$^{\mathrm{pc}}$’, which is put after the digital superscript if present. For instance, the range of any SStAPLOPS of the list:

$$\text{‘}\mathbf{F}^{2\mathrm{pv}}\text{’, ‘}\mathbf{G}^{2\mathrm{pv}}\text{’, ‘}\mathbf{H}^{2\mathrm{pv}}\text{’, ‘}\mathbf{F}_1^{2\mathrm{pv}}\text{’, ‘}\mathbf{G}_1^{2\mathrm{pv}}\text{’, ‘}\mathbf{H}_1^{2\mathrm{pv}}\text{’, ‘}\mathbf{F}_2^{2\mathrm{pv}}\text{’}, \ldots \tag{1.15^{2pv}}$$

is $\kappa^{2\mathrm{pv}}$, while the range of any SStAPLOPS of the list:

$$\text{‘}\mathbf{F}^{2\mathrm{pc}}\text{’, ‘}\mathbf{G}^{2\mathrm{pc}}\text{’, ‘}\mathbf{H}^{2\mathrm{pc}}\text{’, ‘}\mathbf{F}_1^{2\mathrm{pc}}\text{’, ‘}\mathbf{G}_1^{2\mathrm{pc}}\text{’, ‘}\mathbf{H}_1^{2\mathrm{pc}}\text{’, ‘}\mathbf{F}_2^{2\mathrm{pc}}\text{’}, \ldots \tag{1.15^{2pc}}$$

is $\mathrm{K}_{\in}^{2\mathrm{pc}}$; that is, $\mathbf{F}^{2\mathrm{pv}} \vec{\in} \kappa^{2\mathrm{pv}}$ and $\mathbf{F}^{\mathrm{pc}} = \mathbf{F}^{2\mathrm{pc}} \vec{\in} \mathrm{K}_{\in}^{2\mathrm{pc}}$, and similarly with any other SStAPLOPS of either list (1.15^2) or (1.15).

4) In general, ‘**F**’ (e.g.) can be defined so as to have *ad hoc* any desired range by including in the pertinent statement the appropriate one of the defining (quantifying) logographic clauses such as $\mathbf{F} \vec{\in} \widehat{\mathrm{K}}_{\in}^{2\mathrm{pc}}$, $\mathbf{F} \vec{\in} \vec{\mathrm{K}}_{\in}^{2\mathrm{pc}}$, $\mathbf{F} \vec{\in} \overleftarrow{\mathrm{K}}_{\in}^{2\mathrm{pc}}$, $\mathbf{F} \vec{\in} \{\underline{\subseteq}, \overline{\subset}\}$, etc, and similarly with any other SStAPLOPS of either list (1.15) or (1.15^2). •

Cmt 1.2. In accordance with the rules of CFCL interpretation of formulas of A_1 by analo-homolographic expressions, the euautographic relation-formula [*x*∈*y*] (e.g.) should be replaced with the analo-homolographic relation-formula ‘[*x*∈*y*]’, the understanding being that, within A_1, the formula [*x*∈*y*] can be used interchangeably with the formula [‘*x*’∈‘*y*’] subject to the additional condition that the photographic quotations are used autonymously, and not for mentioning their interiors. In this case, the formulas ‘[*x*∈*y*]’ and [‘*x*’∈‘*y*’] are effective under two different mental attitudes of the interpreter and hence in two different scopes. In a scope of interpretation of A_1, the denotata of '*x*' and '*y*' are *x* and *y* respectively. Accordingly, *x*, i.e. the CFCL interpretand of the member-term x of the euautographic relation $[x \in y]$ is said to be a *member*, or *element*, in *y*, while *y*, i.e. the CFCL interpretand of the class-term y of that relation is called a *class*. In some cases, a class is called a *set*, the understanding being that *a set is a class, but not necessarily vice versa*.

In order to be members (elements) of a class, self-subsisting but not necessarily coexisting entities, physical (real) or psychical (ideal), should have a certain property in common, sensorial (sensory, sensational) or conceptual, with respect to a given sapient subject (as me), and also each of the entities should be denoted by the same common name, which is therefore connotative of the common property and hence of the class, but the entities should not necessarily be self-subsisting or coexisting (existing simultaneously). Intuitively, a class is called a set if and only if all its members are regarded by the sapient subject both as self-subsisting and as coexisting (existing simultaneously).

For instance, my library is the class of my books. This class is the current set of my books until I bye some new books or get rid of some old ones. Once I do either of the two acts, the set turns into another set of my books, which exists within another time interval, but it is still my library, i.e. the class of my books. Thus, the class of my books is fact the class of equivalence of the different sets of my books existing in the different successive time intervals. *Homo sapiens* or *man* (without any article) is a *species*, i.e. a *specific class*, of an indefinite number of sapient mammals that lived, lives, and will live on the Earth. This class is not a set. On the other hand, a subclass of men that are gathered together in a certain room at the same time is the set (or group or aggregation) of those men. The class of integers (strictly positive, strictly negative, and zero), the class of rational numbers, the class of real numbers, the class of complex numbers, and the class of points and class of vectors of the three-dimensional (or, generally, *n*-dimensional) affine space over the field of real numbers, etc. are sets of the same names. The English alphabet consisting of the twenty-six recepts (percept-classes, isotoken classes, memory images) of a sapient subject is a set with respect to the subject, while an indefinite number of isotokens or phonic paratokens tokens of these twenty-six recepts is a class not being a set. Likewise, the English lexicon (vocabulary) is a class simply because it is impossible to collect together an indefinite number of English linguistic forms being in use in any given time, to say nothing of a long historical period. Autonyms and xenonyms that are used in any given field of study and discourse form two different complementary classes, but these are not sets because any xenonym can, at any moment, be mentally turned by any given or its interpreters (as myself) into an autonym, and vice versa.

Thus, a class whose members are fixed so that none of them can appear or disappear is called a set. In mathematics and particularly in set theory, the word “set” is used as a technical term rather than “class” with the above connotation in view. In this exposition, I use the words “class” and “set” in accordance with the above remarks.•

1.2. Straightforward Implications of the basic definitions

Preliminary Remark 1.1. By (II.4.31) and (II.7.59), the trains of definitions (1.1)–(1.8) imply the respective trains of identities and the respective trains of equivalemces. Particularly, by (II.7.59), any given train of definitions is immediately turned into the respective train of valid formal equivalencies by replacing all occurrences of each of the definition signs $\rightarrow$ and $\leftarrow$ throughout the former train with occurrences of the formal equivalence sign $\Leftrightarrow$. The purely mechanical procedure of turning a train of valid formal equivalencies into the respective train of identities is self-evident.

After turning the trains of definitions (1.1)–(1.8) into the respective trains of identities, the latter can be used for proving various important theorems before imposing any explicit specific axioms on $\in$. In this and the next subsection, some most conspicuous theorems of this kind will be stated and proved. In deducing the implications of definitions (1.1)–(1.8) in the form of identities or valid formal equivalencies by the rule (II.4.31) or (II.7.59), the rule will not, most often, be mentioned explicitly.•

***Lemma 1.1.**

$$[\mathbf{u} \subseteq \mathbf{v}] \Leftrightarrow \bigwedge_{\mathbf{x}} [[\mathbf{x} \in \mathbf{u}] \Rightarrow [\mathbf{x} \in \mathbf{v}]]. \tag{1.16}$$

$$[\mathbf{u} \overline{\subseteq} \mathbf{v}] \Leftrightarrow \neg[\mathbf{u} \subseteq \mathbf{v}] \Leftrightarrow \neg \bigwedge_{\mathbf{x}} [[\mathbf{x} \in \mathbf{u}] \Rightarrow [\mathbf{x} \in \mathbf{v}]]. \tag{1.17}$$

$$[\mathbf{u} = \mathbf{v}] \Leftrightarrow [[\mathbf{u} \subseteq \mathbf{v}] \wedge [\mathbf{v} \subseteq \mathbf{u}]]. \tag{1.18}$$

$$[\mathbf{u} \equiv \mathbf{v}] \Leftrightarrow \neg[\mathbf{u} = \mathbf{v}] \Leftrightarrow \neg[[\mathbf{u} \subseteq \mathbf{v}] \wedge [\mathbf{v} \subseteq \mathbf{u}]]. \tag{1.19}$$

$$[\mathbf{u} \subset \mathbf{v}] \Leftrightarrow [[\mathbf{u} \subseteq \mathbf{v}] \wedge \neg[\mathbf{v} \subseteq \mathbf{u}]]. \tag{1.20}$$

$$[\mathbf{u} \overline{\subset} \mathbf{v}] \Leftrightarrow \neg[\mathbf{u} \subset \mathbf{v}] \Leftrightarrow \neg[[\mathbf{u} \subseteq \mathbf{v}] \wedge \neg[\mathbf{v} \subseteq \mathbf{u}]]. \tag{1.21}$$

Proof: (1.16)–(1.21) immediately follow from (1.3)–(1.8) respectively by (II.7.59).•

*__Lemma 1.2.__

$$V(\mathbf{u}\subseteq\mathbf{v})\mathrel{\hat{=}}1\mathbin{\hat{-}}\hat{\prod}_{\mathbf{x}}\left[1\mathbin{\hat{-}}V(\neg[\mathbf{x}\in\mathbf{u}])\mathbin{\hat{\cdot}}V(\mathbf{x}\in\mathbf{v})\right]. \tag{1.22}$$

$$V(\mathbf{u}\mathrel{\overline{\subseteq}}\mathbf{v})\mathrel{\hat{=}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\mathrel{\hat{=}}\hat{\prod}_{\mathbf{x}}\left[1\mathbin{\hat{-}}V(\neg[\mathbf{x}\in\mathbf{u}])\mathbin{\hat{\cdot}}V(\mathbf{x}\in\mathbf{v})\right]. \tag{1.23}$$

$$V(\mathbf{u}=\mathbf{v})\mathrel{\hat{=}}V([\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{u}])\mathrel{\hat{=}}1\mathbin{\hat{-}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\mathbin{\hat{\cdot}}V(\neg[\mathbf{v}\subseteq\mathbf{u}]). \tag{1.24}$$

$$V(\mathbf{u}\equiv\mathbf{v})\mathrel{\hat{=}}V(\neg[\mathbf{u}=\mathbf{v}])\mathrel{\hat{=}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\mathbin{\hat{\cdot}}V(\neg[\mathbf{v}\subseteq\mathbf{u}]). \tag{1.25}$$

$$\begin{gathered}V(\mathbf{u}\subset\mathbf{v})\mathrel{\hat{=}}V([\mathbf{u}\subseteq\mathbf{v}]\wedge\neg[\mathbf{v}\subseteq\mathbf{u}])\\ \mathrel{\hat{=}}1\mathbin{\hat{-}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\mathbin{\hat{\cdot}}V(\neg\neg[\mathbf{v}\subseteq\mathbf{u}])\mathrel{\hat{=}}1\mathbin{\hat{-}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\mathbin{\hat{\cdot}}V(\mathbf{v}\subseteq\mathbf{u}).\end{gathered} \tag{1.26}$$

$$V(\mathbf{u}\mathrel{\overline{\subset}}\mathbf{v})\mathrel{\hat{=}}V(\neg[\mathbf{u}\subset\mathbf{v}])\mathrel{\hat{=}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\mathbin{\hat{\cdot}}V(\mathbf{v}\subseteq\mathbf{u}). \tag{1.27}$$

Proof: The trains of equivalences (1.16)–(1.21) immediately turn into the trains of identities for PVI's (primary validity integrons) of the PLR's (panlogographic relations) occurring in the former trains by (II.7.50). The latter trains can alternatively be immediately deduced from definitions (1.3)–(1.8) by (II.4.31).The pertinent links of the tains trains of identities are developed to yield (1.22)–(1.27).as follows. Follow. By the pertinent instance (II.7.3γ), it follows from (1.16) that

$$\begin{gathered}V(\mathbf{u}\subseteq\mathbf{v})\mathrel{\hat{=}}V\left(\bigwedge\nolimits_{\mathbf{x}}[[\mathbf{x}\in\mathbf{u}]\Rightarrow[\mathbf{x}\in\mathbf{v}]]\right)\\ \mathrel{\hat{=}}1\mathbin{\hat{-}}\hat{\prod}_{\mathbf{x}}\left[1\mathbin{\hat{-}}V([\mathbf{x}\in\mathbf{u}]\Rightarrow[\mathbf{x}\in\mathbf{v}])\right]\\ \mathrel{\hat{=}}1\mathbin{\hat{-}}\hat{\prod}_{\mathbf{x}}\left[1\mathbin{\hat{-}}V(\neg[\mathbf{x}\in\mathbf{u}])\mathbin{\hat{\cdot}}V(\mathbf{x}\in\mathbf{v})\right],\end{gathered} \tag{1.22$_1$}$$

which proves (1.22). Identity (1.23) follows from (1.17) by the pertinent instance of (II.7.1γ) and by (1.22). Identities (1.24)–(1.27) follow from (1.18)–1.21) by the pertinent instances of (II.7.6γ) and (II.7.1γ).•

Cmt 1.3. Besides the definiens of definition (1.3), there are some other expressions having the same validity-index as $V(\mathbf{u}\subseteq\mathbf{v})$ given by (1.17), e.g.

$$\begin{gathered}V\left(\neg\bigvee\nolimits_{\mathbf{x}}[[\mathbf{x}\in\mathbf{u}]\wedge\neg[\mathbf{x}\in\mathbf{v}]]\right)\\ \mathrel{\hat{=}}V\left(\neg\bigvee\nolimits_{\mathbf{x}}[[\mathbf{x}\in\mathbf{u}]\wedge\neg[[\mathbf{x}\in\mathbf{u}]\Rightarrow[\mathbf{x}\in\mathbf{v}]]]\right)\\ \mathrel{\hat{=}}V\left(\bigwedge\nolimits_{\mathbf{x}}[[\mathbf{x}\in\mathbf{u}]\Rightarrow[\mathbf{x}\in\mathbf{v}]]\right)\\ \mathrel{\hat{=}}1\mathbin{\hat{-}}\hat{\prod}_{\mathbf{x}}\left[1\mathbin{\hat{-}}V(\neg[\mathbf{x}\in\mathbf{u}])\mathbin{\hat{\cdot}}V(\mathbf{x}\in\mathbf{v})\right]\mathrel{\hat{=}}V(\mathbf{u}\subseteq\mathbf{v}).\end{gathered} \tag{1.22$_2$}$$

Indeed, by the pertinent versions of (II.7.1γ) and (II.7.3γ), and (II.7.6γ), it follows that

$$\begin{gathered}V(\neg[\mathbf{Q}\Rightarrow\mathbf{R}])\mathrel{\hat{=}}1\mathbin{\hat{-}}V(\mathbf{Q}\Rightarrow\mathbf{R})\mathrel{\hat{=}}1\mathbin{\hat{-}}V(\neg\mathbf{Q})\mathbin{\hat{\cdot}}V(\mathbf{R})\\ \mathrel{\hat{=}}1\mathbin{\hat{-}}V(\mathbf{Q}\wedge\neg\mathbf{R}),\end{gathered} \tag{1.22$_3$}$$

$$V(\mathbf{Q}\wedge\neg[\mathbf{Q}\Rightarrow\mathbf{R}])\hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\neg[\mathbf{Q}\Rightarrow\mathbf{R}])$$
$$\hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{Q}\Rightarrow\mathbf{R})\hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R}) \tag{1.22$_4$}$$
$$\hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R})\hat{=}V(\mathbf{Q}\wedge\neg\mathbf{R}).$$

The train of identities follows from the pertinent instances of (II.4.23) and (II.8.2) by (1.22$_3$) and (1.22$_4$) with $[\mathbf{x}\in\mathbf{u}]$ as **Q** and $[\mathbf{x}\in\mathbf{v}]$ as **R**.•

***Th 1.1.**

$$V([\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{u}=\mathbf{v}])\hat{=}V(\mathbf{u}=\mathbf{v}), \tag{1.28}$$

$$V([\mathbf{u}\subseteq\mathbf{v}]\wedge\neg[\mathbf{u}=\mathbf{v}])\hat{=}V(\mathbf{u}\subset\mathbf{v}). \tag{1.29}$$

Proof: In analogy with (1.24) (e.g.), it follows that

$$V([\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{u}=\mathbf{v}])\hat{=}1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{u}=\mathbf{v}])$$
$$\hat{=}1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}[V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{u}])] \tag{1.28$_1$}$$
$$\hat{=}1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{u}])\hat{=}V(\mathbf{u}=\mathbf{v}),$$

where in developing the final result use of the following identities in that order has been made: (1.25), the instance of (II.4.2) with the '$\neg[\mathbf{u}\subseteq\mathbf{v}]$' in place of '**P**', and (1.24). Likewise,

$$V([\mathbf{u}\subseteq\mathbf{v}]\wedge\neg[\mathbf{u}=\mathbf{v}])\hat{=}1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg\neg[\mathbf{u}=\mathbf{v}])$$
$$\hat{=}1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{u}=\mathbf{v})\hat{=}1\hat{-}V(\neg[\mathbf{u}\subset\mathbf{v}])\hat{=}V(\mathbf{u}\subset\mathbf{v}), \tag{1.29$_1$}$$

because

$$V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{u}=\mathbf{v})$$
$$\hat{=}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}[1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{u}])]$$
$$\hat{=}[V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{u}])] \tag{1.29$_1$}$$
$$\hat{=}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}[1\hat{-}V(\neg[\mathbf{v}\subseteq\mathbf{u}])]$$
$$\hat{=}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{v}\subseteq\mathbf{u})\hat{=}V(\neg[\mathbf{u}\subset\mathbf{v}]),$$

where use of the instance of (II.4.2) with the '$\neg[\mathbf{u}\subseteq\mathbf{v}]$' in place of '**P**' and of (1.28$_1$) in that order has been made in developing the final result.•

Cmt 1.4. Identities (1.28) and (1.29) imply the following kyrological (valid) equivalencies:

$$[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{u}=\mathbf{v}]]\Leftrightarrow[\mathbf{u}=\mathbf{v}], \tag{1.28a}$$

$$[\mathbf{u}\subset\mathbf{v}]\Leftrightarrow[[\mathbf{u}\subseteq\mathbf{v}]\wedge\neg[\mathbf{v}\subseteq\mathbf{u}]]\Leftrightarrow[[\mathbf{u}\subseteq\mathbf{v}]\wedge\neg[\mathbf{u}=\mathbf{v}]]. \tag{1.29a}$$ •

***Th 1.2.**

$$V(\mathbf{u}\subset\mathbf{v})\hat{=}V(\mathbf{u}\subseteq\mathbf{v})\hat{+}V(\neg[\mathbf{u}=\mathbf{v}]). \tag{1.30}$$

Proof: It follows from (1.26) that

$$
\begin{aligned}
V(\mathbf{u}\subset\mathbf{v}) &\hat{=} 1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{v}\subseteq\mathbf{u})\\
&\hat{=} 1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}[1\hat{-}V(\neg[\mathbf{v}\subseteq\mathbf{u}])]\\
&\hat{=} 1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{+}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{u}])\\
&\hat{=} 1\hat{+}[1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])]\hat{-}[1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{u}])]\\
&\hat{=} 1\hat{+}V(\mathbf{u}\subseteq\mathbf{v})\hat{-}V(\mathbf{u}=\mathbf{v})\hat{=}V(\mathbf{u}\subseteq\mathbf{v})\hat{+}V(\neg[\mathbf{u}=\mathbf{v}]),
\end{aligned}
\tag{1.30$_1$}
$$

where use of (1.24) has been made. The identity (1.30) thus proved immediately implies (1.31). QED.•

***Th 1.3.**

$$V(\neg[\mathbf{u}\subset\mathbf{v}])\hat{=}V(\mathbf{u}=\mathbf{v})\hat{-}V(\mathbf{u}\subseteq\mathbf{v}). \tag{1.31}$$

$$V(\mathbf{u}=\mathbf{v})\hat{=}V(\mathbf{u}\subseteq\mathbf{v})\hat{+}V(\neg[\mathbf{u}\subset\mathbf{v}]). \tag{1.32}$$

Proof: The above identities follow from (1.30) by the pertinent instances of (II.7.1γ) and of (II.5.3) subject to (II.5.4).•

***Th 1.4.**

$$V([\mathbf{u}\subseteq\mathbf{v}]\Rightarrow[[\mathbf{u}\subset\mathbf{v}]\vee[\mathbf{u}=\mathbf{v}]])\hat{=}0. \tag{1.33}$$

$$V([\mathbf{u}\subset\mathbf{v}]\Rightarrow[\mathbf{u}\subseteq\mathbf{v}])\hat{=}0. \tag{1.34}$$

Proof: By (1.21), it follows from the two pertinent versions of (II.7.3γ) that

$$
\begin{aligned}
V([\mathbf{u}\subset\mathbf{v}]\Rightarrow[\mathbf{u}\subseteq\mathbf{v}]) &\hat{=} V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{u}\subseteq\mathbf{v})\\
&\hat{=} [V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{v}\subseteq\mathbf{u})]\hat{\cdot}V(\mathbf{u}\subseteq\mathbf{v})\\
&\hat{=} V(\mathbf{v}\subseteq\mathbf{u})\hat{\cdot}[V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{u}\subseteq\mathbf{v})]\hat{=}V(\mathbf{v}\subseteq\mathbf{u})\hat{\cdot}0\hat{=}0,
\end{aligned}
\tag{1.33$_1$}
$$

$$
\begin{aligned}
V([\mathbf{u}\subseteq\mathbf{v}]\Rightarrow[[\mathbf{u}\subset\mathbf{v}]\vee[\mathbf{u}=\mathbf{v}]]) &\hat{=} V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V([\mathbf{u}\subset\mathbf{v}]\vee[\mathbf{u}=\mathbf{v}])\\
&\hat{=} V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{u}\subset\mathbf{v})\hat{\cdot}V(\mathbf{u}=\mathbf{v})\\
&\hat{=} V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{u}\subset\mathbf{v})\hat{\cdot}[V(\mathbf{u}\subseteq\mathbf{v})\hat{+}V(\neg[\mathbf{u}\subset\mathbf{v}])]\hat{=}0.
\end{aligned}
\tag{1.34$_1$}
$$

In developing the final identity in (1.34$_1$), use of (1.32) has been made. QED.•

Cmt 1.5. Identities (1.28) and (1.29) imply the following valid equivalencies:

$$[\mathbf{u}\subseteq\mathbf{v}]\Rightarrow[[\mathbf{u}\subset\mathbf{v}]\vee[\mathbf{u}=\mathbf{v}]], \tag{1.33a}$$

$$[\mathbf{u}\subset\mathbf{v}]\Rightarrow[\mathbf{u}\subseteq\mathbf{v}]. \tag{1.34a}$$

To say nothing of the converse of (1.34), the converse of (1.33) is an udeterology, and not a kyrology. Indeed, it follows in analogy with (1.33$_1$) that

$$
\begin{aligned}
&V([[\mathbf{u} \subset \mathbf{v}] \vee [\mathbf{u} = \mathbf{v}]] \Rightarrow [\mathbf{u} \subseteq \mathbf{v}]) \\
&\hat{=} V(\neg[[\mathbf{u} \subset \mathbf{v}] \vee [\mathbf{u} = \mathbf{v}]]) \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{v}) \\
&\hat{=} [1 \hat{-} V(\mathbf{u} \subset \mathbf{v}) \hat{\cdot} V(\mathbf{u} = \mathbf{v})] \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{v}) \\
&\hat{=} [1 \hat{-} V(\mathbf{u} \subset \mathbf{v}) \hat{\cdot} [V(\mathbf{u} \subseteq \mathbf{v}) \hat{+} V(\neg[\mathbf{u} \subset \mathbf{v}])]] \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{v}) \\
&\hat{=} [1 \hat{-} V(\mathbf{u} \subset \mathbf{v}) \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{v})] \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{v}) \hat{=} V(\neg[\mathbf{u} \subset \mathbf{v}]) \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{v}) \\
&\hat{=} V(\neg[\mathbf{u} \subset \mathbf{v}]) \hat{\cdot} [V(\mathbf{u} = \mathbf{v}) \hat{-} V(\neg[\mathbf{u} \subset \mathbf{v}])] \\
&\hat{=} V(\neg[\mathbf{u} \subset \mathbf{v}]) \hat{\cdot} [V(\mathbf{u} = \mathbf{v}) \hat{-} 1],
\end{aligned}
\tag{1.33$_2$}
$$

where use of both (1.32) and (1.31) has been made in that order. Consequently, by (II.7.7γ), (1.33), and (1.33_2), it follows that

$$
\begin{aligned}
&V([\mathbf{u} \subseteq \mathbf{v}] \Leftrightarrow [[\mathbf{u} \subset \mathbf{v}] \vee [\mathbf{u} = \mathbf{v}]]) \\
&\hat{=} V([\mathbf{u} \subseteq \mathbf{v}] \Rightarrow [[\mathbf{u} \subset \mathbf{v}] \vee [\mathbf{u} = \mathbf{v}]]) \hat{+} V([[\mathbf{u} \subset \mathbf{v}] \vee [\mathbf{u} = \mathbf{v}]] \Rightarrow [\mathbf{u} \subseteq \mathbf{v}]) \\
&\hat{=} V([[\mathbf{u} \subset \mathbf{v}] \vee [\mathbf{u} = \mathbf{v}]] \Rightarrow [\mathbf{u} \subseteq \mathbf{v}]),
\end{aligned}
\tag{1.33$_3$}
$$

so that ‘$[\mathbf{u} \subseteq \mathbf{v}] \Leftrightarrow [[\mathbf{u} \subset \mathbf{v}] \vee [\mathbf{u} = \mathbf{v}]]$’ is also an udeterology.●

*__Th 1.5:__ ***Reflexivity laws for*** ⊆, =, and $\overline{\subset}$.

$$V(\mathbf{u} \subseteq \mathbf{u}) \hat{=} 0. \tag{1.35}$$

$$V(\mathbf{u} = \mathbf{u}) \hat{=} 0. \tag{1.36}$$

$$V(\mathbf{u} \,\overline{\subset}\, \mathbf{u}) \hat{=} 0. \tag{1.37}$$

Proof: The instances of (1.22), (1.24), and (1.27) with **u** in place of **v** yield:

$$
\begin{aligned}
&V(\mathbf{u} \subseteq \mathbf{u}) \hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{x}} [1 \hat{-} V(\neg[\mathbf{x} \in \mathbf{u}]) \hat{\cdot} V(\mathbf{x} \in \mathbf{u})] \\
&\hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{x}} [1 \hat{-} 0] \hat{=} 1 \hat{-} 1 \hat{=} 0.
\end{aligned}
\tag{1.35$_1$}
$$

$$
\begin{aligned}
&V(\mathbf{u} = \mathbf{u}) \hat{=} 1 \hat{-} V(\neg[\mathbf{u} \subseteq \mathbf{u}]) \hat{\cdot} V(\neg[\mathbf{u} \subseteq \mathbf{u}]) \hat{=} 1 \hat{-} V(\neg[\mathbf{u} \subseteq \mathbf{u}]) \\
&\hat{=} 1 \hat{-} V(\mathbf{u} \subseteq \mathbf{u}) \hat{=} 0.
\end{aligned}
\tag{1.36$_1$}
$$

$$
\begin{aligned}
&V(\mathbf{u} \,\overline{\subset}\, \mathbf{u}) \hat{=} V(\neg[\mathbf{u} \subset \mathbf{u}]) \hat{=} V(\neg[\mathbf{u} \subseteq \mathbf{u}]) \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{u}) \\
&\hat{=} V(\neg[\mathbf{u} \subseteq \mathbf{u}]) \hat{\cdot} 0 \hat{=} 0.
\end{aligned}
\tag{1.37$_1$}
$$ ●

Cmt 1.6. Identity (1.37_1) is tantamount to

$$V(\mathbf{u} \subset \mathbf{u}) \hat{=} 1, \tag{1.37$_2$}$$

which can be called *Antireflexivity law for* ⊂.●

*__Th 1.6:__ ***Symmetry law for*** =.

$$V(\mathbf{u} = \mathbf{v}) \hat{=} V(\mathbf{v} = \mathbf{u}). \tag{1.38}$$

Proof: (1.38) immediately follows from (1.24) and from the variant of (1.24) with **u** and **v** exchanged.●

*__Lemma 1.3.__

$$
\begin{aligned}
V([\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]) &\hat{=} 1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{w}])\\
&\hat{=} 1\hat{-}\hat{\bullet}_{\mathbf{x}}[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{-}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}V(\mathbf{x}\in\mathbf{w})]\\
&\hat{=} 1\hat{-}\hat{\bullet}_{\mathbf{x}}[V(\mathbf{x}\in\mathbf{u})\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{+}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{x}\in\mathbf{w}])].
\end{aligned}
\tag{1.39}
$$

Proof: By (1.22) and by the variant of (1.22) with **v** and **w** in place of **u** and **v** respectively, it follows that

$$
\begin{aligned}
V([\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]) &\hat{=} 1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{w}])\\
&\hat{=} 1\hat{-}\left[\hat{\bullet}_{\mathbf{x}}[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})]\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}[1\hat{-}V(\neg[\mathbf{y}\in\mathbf{v}])\hat{\cdot}V(\mathbf{y}\in\mathbf{w})]\right]\\
&\hat{=} 1\hat{-}\hat{\bullet}_{\mathbf{x}}[[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})]\hat{\cdot}[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}V(\mathbf{x}\in\mathbf{w})]]\\
&\hat{=} 1\hat{-}\hat{\bullet}_{\mathbf{x}}[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{-}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}V(\mathbf{x}\in\mathbf{w})].
\end{aligned}
\tag{1.39$_1$}
$$

In developing this train of identities, use of the appropriate instance of the Fusion Law (II.4.29) and also use of the identity

$$V(\mathbf{x}\in\mathbf{v})\hat{\cdot}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{=}0, \tag{1.39$_2$}$$

have been made. With the help of the identities:

$$V(\neg[\mathbf{x}\in\mathbf{u}])\hat{=}1\hat{-}V(\mathbf{x}\in\mathbf{u}), \tag{1.39$_3$}$$

$$V(\mathbf{x}\in\mathbf{w})\hat{=}1\hat{-}V(\neg[\mathbf{x}\in\mathbf{w}]), \tag{1.39$_4$}$$

$$V(\mathbf{x}\in\mathbf{v})\hat{+}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{=}1, \tag{1.39$_5$}$$

the operatum of the operator $\hat{\bullet}_{\mathbf{x}}$ in (1.39$_1$) can be transformed thus:

$$
\begin{aligned}
&1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{-}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}V(\mathbf{x}\in\mathbf{w})\\
&\hat{=} 1\hat{-}[1\hat{-}V(\mathbf{x}\in\mathbf{u})]\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{-}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{w}])]\\
&\hat{=} 1\hat{-}V(\mathbf{x}\in\mathbf{v})\hat{-}V(\neg[\mathbf{x}\in\mathbf{v}])\\
&\quad\hat{+}V(\mathbf{x}\in\mathbf{u})\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{+}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{x}\in\mathbf{w}])\\
&\hat{=} V(\mathbf{x}\in\mathbf{u})\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{+}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{x}\in\mathbf{w}]).
\end{aligned}
\tag{1.39$_6$}
$$

The identities (1.39$_1$) and (1.39$_6$) prove (1.39).•

***Th 1.7.**

$$
\begin{aligned}
V(\mathbf{u}=\mathbf{v}) &\hat{=} V([\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{u}])\\
&\hat{=} V\left(\bigwedge\nolimits_{\mathbf{x}}[[[\mathbf{x}\in\mathbf{u}]\Rightarrow[\mathbf{x}\in\mathbf{v}]]\wedge[[\mathbf{x}\in\mathbf{v}]\Rightarrow[\mathbf{x}\in\mathbf{u}]]]\right)\\
&\hat{=} V\left(\bigwedge\nolimits_{\mathbf{x}}[[\mathbf{x}\in\mathbf{u}]\Leftrightarrow[\mathbf{x}\in\mathbf{v}]]\right)\\
&\hat{=} 1\hat{-}\hat{\bullet}_{\mathbf{x}}[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{-}V(\mathbf{x}\in\mathbf{u})\hat{\cdot}V(\neg[\mathbf{x}\in\mathbf{v}])]\\
&\hat{=} 1\hat{-}\hat{\bullet}_{\mathbf{x}}[V(\mathbf{x}\in\mathbf{u})\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{+}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\neg[\mathbf{x}\in\mathbf{v}])]\\
&\hat{=} 1\hat{-}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}[V(\mathbf{x}\in\mathbf{u})\hat{-}V(\mathbf{x}\in\mathbf{v})]^2\right].
\end{aligned}
\tag{1.40}
$$

Proof: By the instance of (1.39) with **u** in place of **w**, it follows from (1.24) that

$$\begin{aligned} V(\mathbf{u}=\mathbf{v}) &\hat{=} V([\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{u}]) \hat{=} 1 \hat{-} V(\neg[\mathbf{u}\subseteq\mathbf{v}]) \hat{\cdot} V(\neg[\mathbf{v}\subseteq\mathbf{u}]) \\ &\hat{=} 1 \hat{-} \mathop{\hat{\cdot}}_{\mathbf{x}} [1 \hat{-} V(\neg[\mathbf{x}\in\mathbf{u}]) \hat{\cdot} V(\mathbf{x}\in\mathbf{v}) \hat{-} V(\mathbf{x}\in\mathbf{u}) \hat{\cdot} V(\neg[\mathbf{x}\in\mathbf{v}])] \\ &\hat{=} 1 \hat{-} \mathop{\hat{\cdot}}_{\mathbf{x}} [V(\mathbf{x}\in\mathbf{u}) \hat{\cdot} V(\mathbf{x}\in\mathbf{v}) \hat{-} V(\neg[\mathbf{x}\in\mathbf{u}]) \hat{\cdot} V(\neg[\mathbf{x}\in\mathbf{v}])] \\ &\hat{=} 1 \hat{-} \mathop{\hat{\cdot}}_{\mathbf{x}} [1 \hat{-} V([\mathbf{x}\in\mathbf{u}] \Leftrightarrow [\mathbf{x}\in\mathbf{v}])] \hat{=} V\left(\bigwedge_{\mathbf{x}} [[\mathbf{x}\in\mathbf{u}] \Leftrightarrow [\mathbf{x}\in\mathbf{v}]]\right). \end{aligned} \tag{1.40$_1$}$$

In developing the final result in (1.40$_1$), use of the instance of the train (II.7.7γ) with $[\mathbf{x}\in\mathbf{u}]$ as **P** and $[\mathbf{x}\in\mathbf{v}]$ as **Q** has been made. By the same train, the EVI (euautographic validity-integron) $V([\mathbf{x}\in\mathbf{u}]\Leftrightarrow[\mathbf{x}\in\mathbf{v}])$ can be represented in various algebraic forms, three of which are written down in (1.40). QED.•

Cmt 1.7. The first three identities in the train (1.40) imply the following kyrological (valid) equivalences:

$$\begin{aligned} &[\mathbf{u}=\mathbf{v}] \Leftrightarrow [\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{u}] \\ &\Leftrightarrow [[[\mathbf{x}\in\mathbf{u}]\Rightarrow[\mathbf{x}\in\mathbf{v}]]\wedge[[\mathbf{x}\in\mathbf{v}]\Rightarrow[\mathbf{x}\in\mathbf{u}]]] \\ &\Leftrightarrow \bigwedge_{\mathbf{x}} [[\mathbf{x}\in\mathbf{u}]\Leftrightarrow[\mathbf{x}\in\mathbf{v}]]. \end{aligned} \tag{1.40a}$$ •

***Th 1.8.**

$$\begin{aligned} V(\mathbf{u}\subset\mathbf{v}) &\hat{=} V(\mathbf{u}\subseteq\mathbf{v}) \hat{+} V(\neg[\mathbf{u}=\mathbf{v}]) \\ &\hat{=} 1 \hat{-} \mathop{\hat{\cdot}}_{\mathbf{x}} [1 \hat{-} V(\neg[\mathbf{x}\in\mathbf{u}]) \hat{\cdot} V(\mathbf{x}\in\mathbf{v})] \\ &\hat{+} \mathop{\hat{\cdot}}_{\mathbf{x}} [1 \hat{-} V(\neg[\mathbf{x}\in\mathbf{u}]) \hat{\cdot} V(\mathbf{x}\in\mathbf{v}) \hat{-} V(\mathbf{x}\in\mathbf{u}) \hat{\cdot} V(\neg[\mathbf{x}\in\mathbf{v}])] \\ &\hat{=} 1 \hat{-} \mathop{\hat{\cdot}}_{\mathbf{x}} [1 \hat{-} V(\neg[\mathbf{x}\in\mathbf{u}]) \hat{\cdot} V(\mathbf{x}\in\mathbf{v})] \\ &\hat{+} \mathop{\hat{\cdot}}_{\mathbf{x}} [V(\mathbf{x}\in\mathbf{u}) \hat{\cdot} V(\mathbf{x}\in\mathbf{v}) \hat{+} V(\neg[\mathbf{x}\in\mathbf{u}]) \hat{\cdot} V(\neg[\mathbf{x}\in\mathbf{v}])] \\ &\hat{=} 1 \hat{-} \mathop{\hat{\cdot}}_{\mathbf{x}} [1 \hat{-} V(\neg[\mathbf{x}\in\mathbf{u}]) \hat{\cdot} V(\mathbf{x}\in\mathbf{v})] \hat{+} \mathop{\hat{\cdot}}_{\mathbf{x}} \left[1 \hat{-} [V(\mathbf{x}\in\mathbf{u}) \hat{-} V(\mathbf{x}\in\mathbf{v})]^2\right] \end{aligned} \tag{1.41}$$

Proof: (1.41) follows from (1.30) by (1.22) and (1.40).•

1.3. Transitivity and incidence laws

Th 1.9.** ***Transitivity law for $\subseteq$.

$$\begin{aligned} &V([[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]]\Rightarrow[\mathbf{u}\subseteq\mathbf{w}]) \\ &\hat{=} V([[\neg[\mathbf{u}\subseteq\mathbf{v}]]\vee[\neg[\mathbf{v}\subseteq\mathbf{w}]]]\vee[\mathbf{u}\subseteq\mathbf{w}]) \\ &\hat{=} [V(\neg[\mathbf{u}\subseteq\mathbf{v}]) \hat{\cdot} V(\neg[\mathbf{v}\subseteq\mathbf{w}])] \hat{\cdot} V(\mathbf{u}\subseteq\mathbf{w}) \hat{=} 0. \end{aligned} \tag{1.42}$$

Proof: I proceed from the train of identities:

$$\begin{aligned} &V([[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]]\Rightarrow[\mathbf{u}\subseteq\mathbf{w}]) \\ &\hat{=} V([\neg[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]]]\vee[\mathbf{u}\subseteq\mathbf{w}]) \\ &\hat{=} V([[\neg[\mathbf{u}\subseteq\mathbf{v}]]\vee[\neg[\mathbf{v}\subseteq\mathbf{w}]]]\vee[\mathbf{u}\subseteq\mathbf{w}]) \\ &\hat{=} [V(\neg[\mathbf{u}\subseteq\mathbf{v}]) \hat{\cdot} V(\neg[\mathbf{v}\subseteq\mathbf{w}])] \hat{\cdot} V(\mathbf{u}\subseteq\mathbf{w}) \\ &\hat{=} V(\neg[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]]) \hat{\cdot} [1 \hat{-} V(\neg[\mathbf{u}\subseteq\mathbf{w}])], \end{aligned} \tag{1.42$_1$}$$

which is developed straightforwardly by the pertinent rules of Th II.7.2. In this case, by (1.39) and by the variant of (1.22) with **y** and **w** in place of **x** and **v** respectively, and also by the pertinent variant of the Fusion Law (4.29), it follows to that

$$\begin{gathered}
V(\neg[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]])\hat{\cdot}V(\neg[\mathbf{u}\subseteq\mathbf{w}]) \\
\hat{=}\left[\hat{\bullet}_{\mathbf{x}}[V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{-}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}V(\mathbf{x}\in\mathbf{w})]\right] \\
\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}[1\hat{-}V(\neg[\mathbf{y}\in\mathbf{u}])\hat{\cdot}V(\mathbf{y}\in\mathbf{w})]\right] \\
\hat{=}\hat{\bullet}_{\mathbf{x}}[[V(\mathbf{x}\in\mathbf{u})\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{-}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{x}\in\mathbf{w}])] \\
\hat{\cdot}[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{w})]] \\
\hat{=}\hat{\bullet}_{\mathbf{x}}[V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\hat{-}V(\neg[\mathbf{x}\in\mathbf{v}])\hat{\cdot}V(\mathbf{x}\in\mathbf{w})] \\
\hat{=}V(\neg[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]]).
\end{gathered}\tag{1.42$_2$}$$

In developing the final expression in the train (1.42$_2$), use of the following identities has been made:

$$V(\mathbf{x}\in\mathbf{u})\hat{\cdot}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{=}0,\tag{1.42$_3$}$$

$$V(\mathbf{x}\in\mathbf{w})\hat{\cdot}V(\neg[\mathbf{x}\in\mathbf{w}])\hat{=}0.\tag{1.42$_4$}$$

By (1.42$_2$), the identity (1.42$_1$) reduces to

$$\begin{gathered}
V([[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]]\Rightarrow[\mathbf{u}\subseteq\mathbf{w}]) \\
\hat{=}V(\neg[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]])\hat{\cdot}[1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{w}])]. \\
\hat{=}V(\neg[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]])\hat{-}V(\neg[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]])\hat{=}0.
\end{gathered}\tag{1.42$_5$}$$

The identities (1.42$_1$) and (1.42$_5$) prove (1.42).•

Th 1.10:** ***Transitivity law for =.

$$\begin{gathered}
V([[\mathbf{u}=\mathbf{v}]\wedge[\mathbf{v}=\mathbf{w}]]\Rightarrow[\mathbf{u}=\mathbf{w}]) \\
\hat{=}V([[\neg[\mathbf{u}=\mathbf{v}]]\vee[\neg[\mathbf{v}=\mathbf{w}]]]\vee[\mathbf{u}=\mathbf{w}]) \\
\hat{=}[V(\neg[\mathbf{u}=\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}=\mathbf{w}])]\hat{\cdot}V(\mathbf{u}=\mathbf{w})\hat{=}0.
\end{gathered}\tag{1.43}$$

Proof: By the pertinent rules of Th II.7.2, it follows that

$$\begin{gathered}
V([[\mathbf{u}=\mathbf{v}]\wedge[\mathbf{v}=\mathbf{w}]]\Rightarrow[\mathbf{u}=\mathbf{w}]) \\
\hat{=}V([\neg[[\mathbf{u}=\mathbf{v}]\wedge[\mathbf{v}=\mathbf{w}]]]\vee[\mathbf{u}=\mathbf{w}]) \\
\hat{=}V([[\neg[\mathbf{u}=\mathbf{v}]]\vee[\neg[\mathbf{v}=\mathbf{w}]]]\vee[\mathbf{u}=\mathbf{w}]) \\
\hat{=}[V(\neg[\mathbf{u}=\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}=\mathbf{w}])]\hat{\cdot}V(\mathbf{u}=\mathbf{w}) \\
\hat{=}[V(\neg[\mathbf{u}=\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}=\mathbf{w}])]\hat{\cdot}[1\hat{-}V(\neg[\mathbf{u}=\mathbf{w}])] \\
\hat{=}V(\neg[\mathbf{u}=\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}=\mathbf{w}])\hat{-}\mathbf{i}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle,
\end{gathered}\tag{1.43$_1$}$$

subject to

$$
\begin{aligned}
\mathbf{i}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle &\overset{\rightarrow}{\hat{=}} V(\neg[[\mathbf{u}=\mathbf{v}]\wedge[\mathbf{v}=\mathbf{w}]])\,\hat{\cdot}\,V(\neg[\mathbf{u}=\mathbf{w}]) \\
&\hat{=} V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}=\mathbf{w}])\,\hat{\cdot}\,V(\neg[\mathbf{u}=\mathbf{w}]) \\
&\hat{=} V(\neg[\mathbf{u}\subseteq\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{u}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{w}])\,\hat{\cdot}\,V(\neg[\mathbf{w}\subseteq\mathbf{v}]) \\
&\quad\hat{\cdot}\,V(\neg[\mathbf{u}\subseteq\mathbf{w}])\,\hat{\cdot}\,V(\neg[\mathbf{w}\subseteq\mathbf{u}]),
\end{aligned}
\tag{1.43$_2$}
$$

where the final expression for the definiens has been developed with the help of (1.25) and of its two pertinent variants. By (1.42) and by the variant of (1.42) with **w**, **u**, and **v** in place of **u**, **v**, and **w**, it follows that

$$
\begin{aligned}
&V(\neg[\mathbf{u}\subseteq\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{w}])\,\hat{\cdot}\,V(\neg[\mathbf{u}\subseteq\mathbf{w}]) \\
&\hat{=} V(\neg[\mathbf{u}\subseteq\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{w}])\,\hat{\cdot}\,[1\,\hat{-}\,V(\mathbf{u}\subseteq\mathbf{w})] \\
&\hat{=} V(\neg[\mathbf{u}\subseteq\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{w}]),
\end{aligned}
\tag{1.43$_3$}
$$

$$
\begin{aligned}
&V(\neg[\mathbf{w}\subseteq\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{u}])\,\hat{\cdot}\,V(\neg[\mathbf{w}\subseteq\mathbf{u}]) \\
&\hat{=} V(\neg[\mathbf{w}\subseteq\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{u}])\,\hat{\cdot}\,[1\,\hat{-}\,V(\mathbf{w}\subseteq\mathbf{u})] \\
&\hat{=} V(\neg[\mathbf{w}\subseteq\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{u}]),
\end{aligned}
\tag{1.43$_4$}
$$

respectively. Consequently, (1.43_2) reduces to:

$$
\begin{aligned}
\mathbf{i}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle &\hat{=} V(\neg[\mathbf{u}\subseteq\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{w}])\,\hat{\cdot}\,V(\neg[\mathbf{w}\subseteq\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{u}]) \\
&\hat{=} [V(\neg[\mathbf{u}\subseteq\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subseteq\mathbf{u}])]\,\hat{\cdot}\,[V(\neg[\mathbf{v}\subseteq\mathbf{w}])\,\hat{\cdot}\,V(\neg[\mathbf{w}\subseteq\mathbf{v}])] \\
&\hat{=} V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}=\mathbf{w}]).
\end{aligned}
\tag{1.43$_5$}
$$

where use of (1.25) and of the variant of (1.25) with **v** and **w** in place of **u** and **v** respectively has been made. Hence, (1.43_1) becomes:

$$
\begin{aligned}
&V([[\mathbf{u}=\mathbf{v}]\wedge[\mathbf{v}=\mathbf{w}]]\Rightarrow[\mathbf{u}=\mathbf{w}]) \\
&\hat{=} V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}=\mathbf{w}])\,\hat{-}\,V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}=\mathbf{w}])\,\hat{=}\,0.
\end{aligned}
\tag{1.43$_6$}
$$

The identities (1.43_1) and (1.43_6) prove (1.43).•

Th 1.11:** ***Transitivity law for $\subset$.

$$
\begin{aligned}
&V([[\mathbf{u}\subset\mathbf{v}]\wedge[\mathbf{v}\subset\mathbf{w}]]\Rightarrow[\mathbf{u}\subset\mathbf{w}]) \\
&\hat{=} V([[\neg[\mathbf{u}\subset\mathbf{v}]]\vee[\neg[\mathbf{v}\subset\mathbf{w}]]]\vee[\mathbf{u}\subset\mathbf{w}]) \\
&\hat{=} [V(\neg[\mathbf{u}\subset\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\subset\mathbf{w}])]\,\hat{\cdot}\,V(\mathbf{u}\subset\mathbf{w})\,\hat{=}\,0.
\end{aligned}
\tag{1.44}
$$

Proof: By the pertinent rules of Th II.7.2 (cf. the proof of Th 1.10), it follows that

$$
\begin{aligned}
&V([[\mathbf{u}\subset\mathbf{v}]\wedge[\mathbf{v}\subset\mathbf{w}]]\Rightarrow[\mathbf{u}\subset\mathbf{w}])\\
&\mathrel{\hat{=}} V([\neg[[\mathbf{u}\subset\mathbf{v}]\wedge[\mathbf{v}\subset\mathbf{w}]]]\vee[\mathbf{u}\subset\mathbf{w}])\\
&\mathrel{\hat{=}} V([[\neg[\mathbf{u}\subset\mathbf{v}]]\vee[\neg[\mathbf{v}\subset\mathbf{w}]]]\vee[\mathbf{u}\subset\mathbf{w}])\\
&\mathrel{\hat{=}} V(\neg[\mathbf{u}\subset\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subset\mathbf{w}])\hat{\cdot}V(\mathbf{u}\subset\mathbf{w})\\
&\mathrel{\hat{=}} V(\neg[\mathbf{u}\subset\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subset\mathbf{w}])\hat{\cdot}[1\hat{-}V(\neg[\mathbf{u}\subset\mathbf{w}])]\\
&\mathrel{\hat{=}} V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{v}\subseteq\mathbf{u})\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{w}])\hat{\cdot}V(\mathbf{w}\subseteq\mathbf{v})\\
&\quad\hat{\cdot}[1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{w}])\hat{\cdot}V(\mathbf{w}\subseteq\mathbf{u})].
\end{aligned}
\tag{1.44$_1$}
$$

where the final expression has been developed with the help of (1.26) and (1.27) and of their pertinent variants. Making use of the identity:

$$V(\mathbf{w}\subseteq\mathbf{u})\mathrel{\hat{=}}1\hat{-}V(\neg[\mathbf{w}\subseteq\mathbf{u}]),\tag{1.44$_2$}$$

the last factor in that expression can be transformed thus:

$$
\begin{aligned}
1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{w}])\hat{\cdot}V(\mathbf{w}\subseteq\mathbf{u}) &\mathrel{\hat{=}} 1\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{w}])\hat{\cdot}[1\hat{-}V(\neg[\mathbf{w}\subseteq\mathbf{u}])]\\
&\mathrel{\hat{=}} V(\mathbf{u}\subseteq\mathbf{w})\hat{-}V(\neg[\mathbf{u}\subseteq\mathbf{w}])\hat{\cdot}V(\neg[\mathbf{w}\subseteq\mathbf{u}]).
\end{aligned}
\tag{1.44$_3$}
$$

The result thus obtained can be reduced with the help of the identity

$$V(\neg[\mathbf{w}\subseteq\mathbf{u}])\hat{\cdot}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{w}\subseteq\mathbf{v})\mathrel{\hat{=}}0,\tag{1.44$_4$}$$

which is the variant of (1.42) with **w**, **u**, and **v** in place of **u**, **v**, and **w** respectively. Thus, the final expression in the train (1.44$_1$) is successively reduced as follows:

$$
\begin{aligned}
&V([[\mathbf{u}\subset\mathbf{v}]\wedge[\mathbf{v}\subset\mathbf{w}]]\Rightarrow[\mathbf{u}\subset\mathbf{w}])\\
&\mathrel{\hat{=}} V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{v}\subseteq\mathbf{u})\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{w}])\hat{\cdot}V(\mathbf{w}\subseteq\mathbf{v})\\
&\quad\hat{\cdot}[V(\mathbf{u}\subseteq\mathbf{w})\hat{-}V(\neg[\mathbf{w}\subseteq\mathbf{u}])].\\
&\mathrel{\hat{=}} [V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{w}])\hat{\cdot}V(\mathbf{u}\subseteq\mathbf{w})]\hat{\cdot}V(\mathbf{v}\subseteq\mathbf{u})\hat{\cdot}V(\mathbf{w}\subseteq\mathbf{v})\\
&\hat{-}[V(\neg[\mathbf{w}\subseteq\mathbf{u}])\hat{\cdot}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\mathbf{w}\subseteq\mathbf{v})]\hat{\cdot}V(\mathbf{v}\subseteq\mathbf{u})\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{w}])\\
&\mathrel{\hat{=}} 0\hat{\cdot}V(\mathbf{v}\subseteq\mathbf{u})\hat{\cdot}V(\mathbf{w}\subseteq\mathbf{v})\hat{-}0\hat{\cdot}V(\mathbf{v}\subseteq\mathbf{u})\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{w}])\mathrel{\hat{=}}0,
\end{aligned}
\tag{1.44$_5$}
$$

where use of (1.42) and (1.44$_4$) has been made. The identities (1.44$_1$) and (1.44$_5$) prove (1.44). •

°**Crl 1.1.**

$$
\begin{aligned}
&V([[u\subseteq v]\wedge[v\subseteq w]]\Rightarrow[u\subseteq w])\\
&\mathrel{\hat{=}} V([[\neg[u\subseteq v]]\vee[\neg[v\subseteq w]]]\vee[u\subseteq w])\\
&\mathrel{\hat{=}} [V(\neg[u\subseteq v])\hat{\cdot}V(\neg[v\subseteq w])]\hat{\cdot}V(u\subseteq w)\mathrel{\hat{=}}0.
\end{aligned}
\tag{1.42μ}
$$

$$
\begin{aligned}
&V([[u=v]\wedge[v=w]]\Rightarrow[u=w])\\
&\mathrel{\hat{=}} V([[\neg[u=v]]\vee[\neg[v=w]]]\vee[u=w])\\
&\mathrel{\hat{=}} [V(\neg[u=v])\hat{\cdot}V(\neg[v=w])]\hat{\cdot}V(u=w)\mathrel{\hat{=}}0.
\end{aligned}
\tag{1.43μ}
$$

$$
\begin{aligned}
&V([[u\subset v]\wedge[v\subset w]]\Rightarrow[u\subset w])\\
&\mathrel{\hat{=}} V([[\neg[u\subset v]]\vee[\neg[v\subset w]]]\vee[u\subset w])\\
&\mathrel{\hat{=}} V(\neg[u\subset v])\hat{\cdot}V(\neg[v\subset w])\hat{\cdot}V(u\subset w)\mathrel{\hat{=}}0.
\end{aligned}
\tag{1.44μ}
$$

Proof: The euautographic kyrologies (identities) (1.42μ)–(1.44μ) are conformal (analo-euautographic) concrete euautographic instances (interpretands) of the panlogographic kyrologies (identities) (1.42)–(1.44) subject to (II.4.47).•

+**Th 1.12.**

$$\begin{gathered} V([[u \subseteq v] \wedge [v \subseteq w]] \Rightarrow [u \subseteq w]) \\ \hat{=} V([[\neg[u \subseteq v]] \vee [\neg[v \subseteq w]]] \vee [u \subseteq w]) \\ \hat{=} [V(\neg[u \subseteq v]) \hat{\vee} V(\neg[v \subseteq w])] \hat{\vee} V(u \subseteq w) \hat{=} 0. \end{gathered} \tag{1.42κ}$$

$$\begin{gathered} V([[u = v] \wedge [v = w]] \Rightarrow [u = w]) \\ \hat{=} V([[\neg[u = v]] \vee [\neg[v = w]]] \vee [u = w]) \\ \hat{=} [V(\neg[u = v]) \hat{\vee} V(\neg[v = w])] \hat{\vee} V(u = w) \hat{=} 0. \end{gathered} \tag{1.43κ}$$

$$\begin{gathered} V([[u \subset v] \wedge [v \subset w]] \Rightarrow [u \subset w]) \\ \hat{=} V([[\neg[u \subset v]] \vee [\neg[v \subset w]]] \vee [u \subset w]) \\ \hat{=} [V(\neg[u \subset v]) \hat{\vee} V(\neg[v \subset w])] \hat{\vee} V(u \subset w) \hat{=} 0. \end{gathered} \tag{1.44κ}$$

Proof: The trains of identities (1.42κ)–(1.44κ) are variants of (1.42μ)–1.42μ) under the rule (I.8.20) of analo-homolographic substitutions, being one of the rules of CFCL interpretations of DdER's A_1, which are comprised in Ax I.8.1.•

Cmt 1.8. 1) The three trains of identities (1.42μ)–(1.44μ), or (1.42κ)–(1.44κ), are similar to one another. Therefore in what follows, I shall, for the sake of being specific, discuss (1.43μ) and (1.43κ).

1) The kyrology

$$V([[\neg[u = v]] \vee [\neg[v = w]]] \vee [u = w]) \hat{=} 0 \tag{1.43μ_1}$$

is equivalent to

$$[[V(\neg[u = v]) \hat{=} 0] \vee [V(\neg[v = w]) \hat{=} 0]] \vee [V(v = w) \hat{=} 0]. \tag{1.43μ_2}$$

Indeed, application of the validity-operator V to (1.43μ_1) yields:

$$\begin{gathered} V([[V(\neg[u = v]) \hat{=} 0] \vee [V(\neg[v = w]) \hat{=} 0]] \vee [V(v = w) \hat{=} 0]) \\ \hat{=} V([V(\neg[u = v]) \hat{=} 0] \vee [V(\neg[v = w]) \hat{=} 0]) \hat{\vee} V(V(v = w) \hat{=} 0) \\ \hat{=} [V(V(\neg[u = v]) \hat{=} 0) \hat{\vee} V(V(\neg[v = w]) \hat{=} 0)] \hat{\vee} V(V(v = w) \hat{=} 0) \\ \hat{=} [V(\neg[u = v]) \hat{\vee} V(\neg[v = w])] \hat{\vee} V(u = w) \hat{=} 0, \end{gathered} \tag{1.43μ_3}$$

where use has been made of the following instances of (II.6.19):

$$\begin{gathered} V(V(\neg[u = v]) \hat{=} 0) \hat{=} V(\neg[u = v]), \\ V(V(\neg[v = w]) \hat{=} 0) \hat{=} V(\neg[v = w]), V(V(u = w) \hat{=} 0) \hat{=} V(u = w). \end{gathered} \tag{1.43μ_4}$$

2) There is a temptation to conclude from (1.43μ) that

$$V(\neg[u = v]) \hat{=} 0 \text{ or } V(\neg[v = w]) \hat{=} 0 \text{ or } V(u = w) \hat{=} 0. \tag{1.43μ_5}$$

However, since the EVI's (euautographic validity-intrgrons) $V(u=v)$, $V(v=w)$, and $V(u=w)$ are *irreducible*, therefore it follows from (1.43μ$_4$) that the ER's $\neg[u=v]$, $\neg[v=w]$, and $u=w$ are *vav-neutral* (*vav-indeterminate*), – like the AOR's p, q, etc. Consequently, each of the three ER's (1.43μ$_5$) is also *vav-neutral.* The assumption that some one or some two or all the three of these equalities are valid would contradict the above fundamental fact of the AEADM. In the framework of A_1 in general and of $A_{1\in}$ in particular, the only possible interpretations of its formulas are *intrinsic substitutional* (*syntactic*) *interpretations*. For instance, replacement of the occurrences of v in (1.43μ) with occurrences of u or w results in

$$\begin{aligned} &V([[u=u]\wedge[u=w]]\Rightarrow[u=w]) \\ &\hat{=}[V(\neg[u=u])\hat{\cdot}V(\neg[u=w])]\hat{\cdot}V(u=w) \\ &\hat{=}V(\neg[u=u])\hat{\cdot}[V(\neg[u=w])\hat{\cdot}V(u=w)]\hat{=}1\hat{\cdot}0\hat{=}0 \end{aligned} \qquad (1.43\mu_6)$$

or

$$\begin{aligned} &V([u=w]\wedge[w=w]\Rightarrow[u=w]) \\ &\hat{=}[V(\neg[u=w])\hat{\cdot}V(\neg[w=w])]\hat{\cdot}V(u=w) \\ &\hat{=}V(\neg[w=w])\hat{\cdot}[V(\neg[u=w])\hat{\cdot}V(u=w)]\hat{=}1\hat{\cdot}0\hat{=}0, \end{aligned} \qquad (1.43\mu_7)$$

respectively. In developing (1.43μ$_6$) and (1.43μ$_7$), use of the identities u=u and w=w, following from (1.36), has been made.

3) At the same time, every DdER (decided ER) of $A_{1\in}$ can be subjected to the respective CFCL (conformal catlogographic) interpretation. In this case, the CFCL interpretand of a DdER preserves the validity-value of the DdER being its CFE (conformal euautographic) interpretans. Consequently, the identity:

$$\begin{aligned} &V([[u=v]\wedge[v=w]]\Rightarrow[u=w]) \\ &\hat{=}[V(\neg[u=v])\hat{\cdot}V(\neg[v=w])]\hat{\cdot}V(u=w)\hat{=}0, \end{aligned} \qquad (1.43\kappa_1)$$

being the CFCL interpretand of the *valid* (*kyrologous*) *ER* (1.43μ$_1$), is a *valid* (*kyrologous*) and hence *tautologous* (*universally true*) *CFCLR* (*CFCL relation*), whereas the CFCL interpretands of the *vav-neutral ER's* (1.43μ$_5$):

$$\text{‘}V(\neg[u=v])\hat{=}0\text{’},\ \text{‘}V(\neg[v=w])\hat{=}0\text{’},\ \text{‘}V(u=w)\hat{=}0\text{’} \qquad (1.43\kappa_2)$$

are *vav-veutral* and hence *ttatt-neutral CFCLR's*, i.e.

$$\dagger[V(\neg[u=v])\hat{=}0],\dagger[V(\neg[v=w])\hat{=}0],\dagger[V(u=w)\hat{=}0]. \qquad (1.43\kappa_3)$$

In contrast to u, v, and w, being APVOT's, ‘*u*’, ‘*v*’, and ‘*w*’ are CFCL AVOT's that may take on some distinct *classes* (or particularly *sets*) as their accidental denotata. In this case, the following detachment procedures for the separate multipliers occurring in ($1.43\kappa_1$) are legitimate.

a) If $\models[V(u=v)\hat{=}V(v=w)\hat{=}0]$, which means that both '$u=v$' and '$v=w$' are assumed to be *veracious* (*accidentally true*), i.e. that $\models[u=v]$ and $\models[v=w]$, – or, what comes to the same thing, if $\models[V(\neg[u=v])\hat{=}V(\neg[v=w])\hat{=}1]$, which means that both '$\neg[u=v]$' and '$\neg[v=w]$' are assumed to be *antiveracious* (*accidentally antiatrue, accidentally false*), , i.e. $\dashv\neg[u=v]$ and $\dashv\neg[v=w]$, – then it follows from ($1.43\kappa_1$) that $\models[V(u=w)\hat{=}0]$, whence $\models[u=w]$. Thus, not coming into a conflict with the AEADM, the vav-neutral (vav-indeterminate, udeterogical) *relation-formula* $V(u=w)\hat{=}0$ involving the euautographic APVOT's u and w has been transduced into the *catlogographic condition* $\models[V(u=w)\hat{=}0]$ on accidental denotata of the CFCL AVOT's '*u*' and '*w*' in a certain range.

b) If $\models[[V(\neg[u=v])\hat{\vee}V(\neg[v=w])\hat{=}0]]$, which means that it is assumed that at least one of the equalities '$u=v$' and '$v=w$' is *antiveracious* (*accidentally antitrue, accidentally false*), i.e. that $\dashv[u=v]$ or $\dashv[v=w]$ or both, then the *non-digital validity-integron* $V(u=w)$ becomes a *non-digital veracity-integron*, so that the CFCLR '$u=w$' is *neither veracious nor antiveracious*, i.e. *vravr-neutral* (*vravr-indeterminate*) – symbolically, $\not\models[u=w]$.

4) The catlogographic interpretand of any of the three transitive laws (1.42)–(1.44) is analogous to the Aristotelian syllogism, which denoted as 'Barbara(*u*,*v*,*w*)', the understanding being that '*v*' is its *middle term*. In this case, '**u**', '**v**', and '**w**' are analogous respectively to '*u*', '*v*', '*w*', the first conjunct $[\mathbf{u}\subseteq\mathbf{v}]$, $[\mathbf{u}=\mathbf{v}]$, or $[\mathbf{u}\subset\mathbf{v}]$ and the second one $[\mathbf{v}\subseteq\mathbf{w}]$, $[\mathbf{v}=\mathbf{w}]$, or $[\mathbf{v}\subset\mathbf{w}]$ of the pertinent antecedent $[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{w}]]$, $[[\mathbf{u}=\mathbf{v}]\wedge[\mathbf{v}=\mathbf{w}]]$, or $[[\mathbf{u}\subset\mathbf{v}]\wedge[\mathbf{v}\subset\mathbf{w}]]$ are analogous respectively to the *minor* and *major premises* of Barbara, while the pertinent consequent $[\mathbf{u}\subseteq\mathbf{w}]$, $[\mathbf{u}=\mathbf{w}]$, or $[\mathbf{u}\subset\mathbf{w}]$ is analogous to the conclusion of Barbara(*u*,*v*,*w*). As was already pointed out in Df I.7.1(6), I shall, in Chapter V of the treatise, define several sets of 19 *ordinary ER's* (*OER's, EOR's*) *of* $\mathsf{A}_{1\in}$ in each set, each ER being called a *euautographic syllogistic implication* (*ESI*) because it has the form an *implicative transitive law* of the same structure as a certain one of 19 categorical syllogisms of Aristotelian logic. Therefore, separate ESI's are

distinguished by the same catchwords as those identifying separate categorical syllogisms, e.g. “Barbara”, “Bamalip”, etc, but these are set in the Roman Arial Narrow Font, and are furnished with various *additional subscripts* distinguishing the different sets of ESI’s. Also, for the purpose of a certain auxiliary unconventional convenient classification of the ESI’s and of the categorical syllogisms, being their CFCL interpretands, I have replaced the conventional catchword “Darapti” with “Barapti”. The organon $\mathsf{A}_{1\in}$, that is augmented by the definitions of ESI’s is denoted by ‘$\mathsf{A}_{1\in\mathsf{A}}$’ or ‘$\mathsf{A}_{1\mathsf{A}}$’ and is called the *Aristotelian*, or *Syllogistic*, *EAPO* (*AEAPO* or *SEAPO*). The purpose of $\mathsf{A}_{1\mathsf{A}}$ is to apply D_1 to all defined ESI’s and to *calculate* their *validity indices* (*VID’s*), which are tantamount to their validity-values. In this way, I have proved that 15 categorical syllogisms, other than Bamalip, Barapti (former Darapti), Felapton, and Fesapo, are *universally true* (*tautologous)*, because they are the CFCL interpretands of the respective *valid* (*kyrologous*) ESI’s, whereas the latter four categorical syllogisms are the CFCL interpretands of the respective *vav-neutral* (*vav-indeterminate*, *vav-udeterologous*) *ESI’s*, which are *veracious* (*accidentally true*) because they are subjected to a certain additional catlogographic (semantic) axiom. As was already mentioned in Df I.7.1(6), This result is in agreement with the finding of Hilbert and Ackermann [1950, pp. 48–54, 53ff] that all categorical syllogisms in the exclusion of the above four are deducible from Boolean algebra.•

****Th 1.13:** ***Incidence laws for operands of*** $\subseteq$, $=$, ***and*** $\bar{\subset}$.

$$V\left(\bigvee_{\mathbf{u}}[\mathbf{u}\subseteq\mathbf{v}]\right)\hat{=}0. \tag{1.45}$$

$$V\left(\bigvee_{\mathbf{v}}[\mathbf{u}\subseteq\mathbf{v}]\right)\hat{=}0. \tag{1.46}$$

$$V\left(\bigvee_{\mathbf{u}}[\mathbf{u}=\mathbf{v}]\right)\hat{=}0. \tag{1.47}$$

$$V\left(\bigvee_{\mathbf{v}}[\mathbf{u}=\mathbf{v}]\right)\hat{=}0. \tag{1.48}$$

$$V\left(\bigvee_{\mathbf{u}}[\mathbf{u}\,\bar{\subset}\,\mathbf{v}]\right)\hat{=}0. \tag{1.49}$$

$$V\left(\bigvee_{\mathbf{v}}[\mathbf{u}\,\bar{\subset}\,\mathbf{v}]\right)\hat{=}0. \tag{1.50}$$

Proof: With the help of the pertinent instances of the Emission Law (II.4.28)), the expressions on the left-hand sides of identities (1.45)–(1.50) can be developed thus:

$$\begin{gathered}V\left(\bigvee_{\mathbf{u}}[\mathbf{u}\subseteq\mathbf{v}]\right)\hat{=}\hat{\cdot}_{\mathbf{u}}V(\mathbf{u}\subseteq\mathbf{v})\\ \hat{=}V(\mathbf{v}\subseteq\mathbf{v})\hat{\cdot}\hat{\cdot}_{\mathbf{u}}V(\mathbf{u}\subseteq\mathbf{v})\hat{=}0\hat{\cdot}\hat{\cdot}_{\mathbf{u}}V(\mathbf{u}\subseteq\mathbf{v})\hat{=}0.\end{gathered} \tag{1.45$_1$}$$

$$V\left(\bigvee_{\mathbf{v}}[\mathbf{u} \subseteq \mathbf{v}]\right) \hat{=} \mathop{\hat{\bullet}}_{\mathbf{v}} V(\mathbf{u} \subseteq \mathbf{v})$$
$$\hat{=} V(\mathbf{u} \subseteq \mathbf{u}) \hat{\cdot} \mathop{\hat{\bullet}}_{\mathbf{v}} V(\mathbf{u} \subseteq \mathbf{v}) \hat{=} 0 \hat{\cdot} \mathop{\hat{\bullet}}_{\mathbf{v}} V(\mathbf{u} \subseteq \mathbf{v}) \hat{=} 0. \tag{1.46$_1$}$$

$$V\left(\bigvee_{\mathbf{u}}[\mathbf{u} = \mathbf{v}]\right) \hat{=} \mathop{\hat{\bullet}}_{\mathbf{u}} V(\mathbf{u} = \mathbf{v})$$
$$\hat{=} V(\mathbf{v} = \mathbf{v}) \hat{\cdot} \mathop{\hat{\bullet}}_{\mathbf{u}} V(\mathbf{u} = \mathbf{v}) \hat{=} 0 \hat{\cdot} \mathop{\hat{\bullet}}_{\mathbf{u}} V(\mathbf{u} = \mathbf{v}) \hat{=} 0. \tag{1.47$_1$}$$

$$V\left(\bigvee_{\mathbf{v}}[\mathbf{u} = \mathbf{v}]\right) \hat{=} \mathop{\hat{\bullet}}_{\mathbf{v}} V(\mathbf{u} = \mathbf{v})$$
$$\hat{=} V(\mathbf{u} = \mathbf{u}) \hat{\cdot} \mathop{\hat{\bullet}}_{\mathbf{v}} V(\mathbf{u} = \mathbf{v}) \hat{=} 0 \hat{\cdot} \mathop{\hat{\bullet}}_{\mathbf{v}} V(\mathbf{u} = \mathbf{v}) \hat{=} 0. \tag{1.48$_1$}$$

$$V\left(\bigvee_{\mathbf{u}}[\mathbf{u} \mathrel{\bar{\subset}} \mathbf{v}]\right) \hat{=} V\left(\bigvee_{\mathbf{u}} \neg[\mathbf{u} \subset \mathbf{v}]\right) \hat{=} \mathop{\hat{\bullet}}_{\mathbf{u}} V(\neg[\mathbf{u} \subset \mathbf{v}])$$
$$\hat{=} V(\neg[\mathbf{v} \subset \mathbf{v}]) \hat{\cdot} \mathop{\hat{\bullet}}_{\mathbf{u}} V(\neg[\mathbf{u} \subset \mathbf{v}]) \hat{=} 0 \hat{\cdot} \mathop{\hat{\bullet}}_{\mathbf{u}} V(\neg[\mathbf{u} \subset \mathbf{v}]) \hat{=} 0. \tag{1.49$_1$}$$

$$V\left(\bigvee_{\mathbf{v}}[\mathbf{u} \mathrel{\bar{\subset}} \mathbf{v}]\right) \hat{=} V\left(\bigvee_{\mathbf{v}} \neg[\mathbf{u} \subset \mathbf{v}]\right) \hat{=} \mathop{\hat{\bullet}}_{\mathbf{v}} V(\neg[\mathbf{u} \subset \mathbf{v}])$$
$$\hat{=} V(\neg[\mathbf{u} \subset \mathbf{u}]) \hat{\cdot} \mathop{\hat{\bullet}}_{\mathbf{v}} V(\neg[\mathbf{u} \subset \mathbf{v}]) \hat{=} 0 \hat{\cdot} \mathop{\hat{\bullet}}_{\mathbf{v}} V(\neg[\mathbf{u} \subset \mathbf{v}]) \hat{=} 0. \tag{1.50$_1$}$$

In this case use of the following identities has also been made: (1.35) in (1.45$_1$) and (1.46$_1$), (1.36) in (1.47$_1$) and (1.48$_1$), and (1.37) in (1.49$_1$) and (1.50$_1$).•

1.4. A summary of the basic laws

By (II.4.40a), algebraic (special) identities (1.35)–(1.38) and (1.45)–(1.50) are equivalent to the following logical (ordinary) kyrologies.

1) Laws for $\subseteq$

$\mathbf{u} \subseteq \mathbf{u}$. (Reflexivity law) (1.35a)

$[[\mathbf{u} \subseteq \mathbf{v}] \wedge [\mathbf{v} \subseteq \mathbf{w}]] \Rightarrow [\mathbf{u} \subseteq \mathbf{w}]$. (Transitivity law) (1.42a)

$\bigvee_{\mathbf{u}}[\mathbf{u} \subseteq \mathbf{v}]$. (First incidence law) (1.45a)

$\bigvee_{\mathbf{v}}[\mathbf{u} \subseteq \mathbf{v}]$. (Second incidence law) (1.46a)

2) Laws for =

$\mathbf{u} = \mathbf{u}$. (Reflexivity law) (1.36a)

$[\mathbf{u} = \mathbf{v}] \Leftrightarrow [\mathbf{v} = \mathbf{u}]$. (Symmetry law) (1.38a)

$[[\mathbf{u} = \mathbf{v}] \wedge [\mathbf{v} = \mathbf{w}]] \Rightarrow [\mathbf{u} = \mathbf{w}]$. (Transitivity law) (1.43a)

$\bigvee_{\mathbf{u}}[\mathbf{u} = \mathbf{v}]$. (First incidence law) (1.47a)

$\bigvee_{\mathbf{v}}[\mathbf{u} = \mathbf{v}]$. (Second incidence law) (1.48a)

3) Laws for $\subset$ and $\bar{\subset}$

$\mathbf{u} \mathrel{\bar{\subset}} \mathbf{u}$. (Reflexivity law) (1.37a)

$[[\mathbf{u} \subset \mathbf{v}] \wedge [\mathbf{v} \subset \mathbf{w}]] \Rightarrow [\mathbf{u} \subset \mathbf{w}]$. (Transitivity law) (1.44a)

$\bigvee_{\mathbf{u}}[\mathbf{u} \mathrel{\bar{\subset}} \mathbf{v}]$. (First incidence law) (1.49a)

$\bigvee_{\mathbf{v}}[\mathbf{u} \,\overline{\subset}\, \mathbf{v}]$. (Second incidence law) (1.50a)

Like kyrological equivalencies (1.16)–(1.21), all the above laws hold independent of any individualizing axioms that will be imposed on $\in$ in the sequel. Kyrologies (1.18), (1.35a), and (1.42a) mean that once operata of the predicate $\subseteq$ are interpreted, this predicate is the *order relation in intension* on the class of interpretands of its operata. In this case, (1.18) is the *antisymmetry law for the predicate* $\subseteq$. At the same time, kyrologies (1.36a), (1.38a), and (1.43a) mean that once operata of the predicate = are interpreted, this predicate is the *equivalence relation in intension* but again on the class of interpretands of its operata.

The specific subject axioms for $\in$ that allow calculating the validity indices of the relations schemata '$\bigvee_{\mathbf{u}}\neg[\mathbf{u} \subseteq \mathbf{v}]$', '$\bigvee_{\mathbf{v}}\neg[\mathbf{u} \subseteq \mathbf{v}]$', and '$\bigvee_{\mathbf{u}}\neg[\mathbf{u} = \mathbf{v}]$' (or '$\bigvee_{\mathbf{v}}\neg[\mathbf{u} = \mathbf{v}]$', – see (1.38)), $\bigvee_{\mathbf{u}}[\mathbf{u} \subset \mathbf{v}]$', and '$\bigvee_{\mathbf{v}}[\mathbf{u} \subset \mathbf{v}]$' which belong to $\mathbf{A}_1$, and hence the validity indices of any concrete instances of these schemata, which belong to A_1, will be laid down in $\mathsf{A}_{1\in\mathrm{D}}$. Along with a certain additional subject axiom that will be laid in $\mathsf{A}_{1\in}$ for $\emptyset$, the above mentioned axioms will also allow calculating the validity indices of the relations schemata '$\bigvee_{\mathbf{u}}[\mathbf{u} \subset \mathbf{v}]$' and '$\bigvee_{\mathbf{v}}[\mathbf{u} \subset \mathbf{v}]$'. In the framework of $\mathsf{A}_{1\in\mathrm{G}}$, all the above-mentioned schemata and all their concrete instances are udeterologies.•

1.5. Implications of reflexivity laws

***Lemma 1.4.** Let $\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle$ be any relation of A_1 containing APVOT's $\mathbf{x}$ and $\mathbf{y}$, such that

$$V(\mathbf{R}\langle\mathbf{x},\mathbf{x}\rangle) \hat{=} V(\mathbf{R}\langle\mathbf{y},\mathbf{y}\rangle) \hat{=} 0\,, \tag{1.51}$$

i.e. $\mathbf{R}\langle\mathbf{x},\mathbf{x}\rangle$ and hence $\mathbf{R}\langle\mathbf{y},\mathbf{y}\rangle$ are *valid reflexive ER's* (*reflexive kyrologies*) *of* A_1. If $\mathbf{P}$ and $\mathbf{Q}$ are two arbitrary relations of A_1 then

$$\begin{aligned} V\left(\bigvee_{\mathbf{x}|\mathbf{P}}\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\right) &\hat{=} V\left(\bigvee_{\mathbf{x}|\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle}\mathbf{P}\right) \vec{\hat{=}} V\left(\bigvee_{\mathbf{x}}[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle \wedge \mathbf{P}]\right) \\ &\hat{=} V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}|\right) \hat{\cdot} V\left(\bigvee_{\mathbf{x}}[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle \wedge \mathbf{P}]\right), \end{aligned} \tag{1.52}$$

$$\begin{aligned} V\left(\bigvee_{\mathbf{y}|\mathbf{Q}}\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\right) &\hat{=} V\left(\bigvee_{\mathbf{y}|\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle}\mathbf{Q}\right) \vec{\hat{=}} V\left(\bigvee_{\mathbf{y}}[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle \wedge \mathbf{Q}]\right) \\ &\hat{=} V\left(\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\mathbf{Q}|\right) \hat{\cdot} V\left(\bigvee_{\mathbf{y}}[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle \wedge \mathbf{Q}]\right), \end{aligned} \tag{1.53}$$

$$
\begin{aligned}
&V\left(\bigwedge_{\mathbf{x}|\mathbf{P}}\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{=}V\left(\bigwedge_{\mathbf{x}|\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle}\neg\mathbf{P}\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\neg[\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{P}]\right)\\
&\hat{=}V\left(\bigvee_{\mathbf{y}}\neg[\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{Q}]\right)\hat{=}V\left(\bigwedge_{\mathbf{y}|\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle}\neg\mathbf{Q}\right)\hat{=}V\left(\bigwedge_{\mathbf{y}|\mathbf{Q}}\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{=}0,
\end{aligned}
\tag{1.54}
$$

which incorporate the pertinent versions of definitions (II.9.1) and (II.9.2) and of theorem (II.9.25). It will be recalled that $\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}|$, e.g., is the relation resulting by substitution of **y** for **x** throughout **P**; if **x** does not occur in **P** then $\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}|$ is **P**.

Proof: Making use of the pertinent instances of the Emission Law (II.4.28) with substitution of **y** for **x** or vice versa in the respective emitted term yields:

$$
\begin{aligned}
&V\left(\bigvee_{\mathbf{x}}[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{P}]\right)\hat{=}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{P})\right]\\
&\hat{=}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{y},\mathbf{y}\rangle)\hat{\cdot}V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\neg\mathbf{P}|\right)\right]\hat{\cdot}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{P})\right]\\
&\hat{=}V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}|\right)\hat{\cdot}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{P})\right]\\
&\hat{=}V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}|\right)\hat{\cdot}V\left(\bigvee_{\mathbf{x}}[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{P}]\right),
\end{aligned}
\tag{1.52$_1$}
$$

$$
\begin{aligned}
&V\left(\bigvee_{\mathbf{y}}[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{Q}]\right)\hat{=}\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{Q})\right]\\
&\hat{=}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x},\mathbf{x}\rangle)\hat{\cdot}V\left(\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\neg\mathbf{Q}|\right)\right]\hat{\cdot}\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{Q})\right]\\
&\hat{=}V\left(\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\mathbf{Q}|\right)\hat{\cdot}\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{Q})\right]\\
&\hat{=}V\left(\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\mathbf{Q}|\right)\hat{\cdot}V\left(\bigvee_{\mathbf{y}}[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{Q}]\right),
\end{aligned}
\tag{1.53$_1$}
$$

because

$$
1\hat{-}V(\neg\mathbf{R}\langle\mathbf{y},\mathbf{y}\rangle)\hat{\cdot}V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\neg\mathbf{P}|\right)\hat{=}1\hat{-}1\hat{\cdot}V\left(\neg\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}|\right)\hat{=}V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}|\right),\tag{1.52$_2$}
$$

$$
1\hat{-}V(\neg\mathbf{R}\langle\mathbf{x},\mathbf{x}\rangle)\hat{\cdot}V\left(\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\neg\mathbf{Q}|\right)\hat{=}1\hat{-}1\hat{\cdot}V\left(\neg\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\mathbf{Q}|\right)\hat{=}V\left(\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\mathbf{Q}|\right),\tag{1.53$_2$}
$$

by (1.51). Analogously,

$$
\begin{aligned}
&V\left(\bigvee_{\mathbf{x}}\neg[\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{P}]\right)\hat{=}\hat{\bullet}_{\mathbf{x}}V\left(\neg[\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{P}]\right)\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[V(\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{P})\right]\\
&\hat{=}\left[V(\mathbf{R}\langle\mathbf{y},\mathbf{y}\rangle)\hat{\cdot}V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\neg\mathbf{P}|\right)\right]\hat{\cdot}\hat{\bullet}_{\mathbf{x}}\left[V(\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{P})\right]\\
&\hat{=}\left[0\hat{\cdot}V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\neg\mathbf{P}|\right)\right]\hat{\cdot}\hat{\bullet}_{\mathbf{x}}\left[V(\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle)\hat{\cdot}V(\neg\mathbf{P})\right]\hat{=}0,
\end{aligned}
\tag{1.54$_1$}
$$

where use of (1.51) has been made again. $V\left(\bigvee_{\mathbf{y}}\neg[\neg\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{Q}]\right)$ is transformed similarly by exchanging '**x**' and '**y**' in all occurrences throughout (1.54$_1$) except those in $\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle$.•

***Th 1.14.**

$$
V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}|\Rightarrow\bigvee_{\mathbf{x}}[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{P}]\right)\hat{=}V\left(\neg\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}|\right)\hat{\cdot}V\left(\bigvee_{\mathbf{x}}[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle\wedge\mathbf{P}]\right)\hat{=}0\,.\tag{1.55}
$$

$$V\left(\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\mathbf{Q}\right| \Rightarrow \bigvee_{\mathbf{y}}\left[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle \wedge \mathbf{Q}\right]\right) \hat{=} V\left(\neg\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\mathbf{Q}\right|\right) \hat{\cdot} V\left(\bigvee_{\mathbf{y}}\left[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle \wedge \mathbf{Q}\right]\right) \hat{=} 0. \quad (1.56)$$

Proof: (1.66) and (1.67) can be rewritten as:

$$\left[1 \hat{-} V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}\right|\right)\right] \hat{\cdot} V\left(\bigvee_{\mathbf{x}}\left[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle \wedge \mathbf{P}\right]\right)$$
$$\hat{=} V\left(\neg\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}\right|\right) \hat{\cdot} V\left(\bigvee_{\mathbf{x}}\left[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle \wedge \mathbf{P}\right]\right) \hat{=} 0, \quad (1.52_3)$$

$$\left[1 \hat{-} V\left(\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\mathbf{Q}\right|\right)\right] \hat{\cdot} V\left(\bigvee_{\mathbf{y}}\left[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle \wedge \mathbf{Q}\right]\right)$$
$$\hat{=} V\left(\neg\mathrm{S}_{\mathbf{x}}^{\mathbf{y}}\mathbf{Q}\right|\right) \hat{\cdot} V\left(\bigvee_{\mathbf{y}}\left[\mathbf{R}\langle\mathbf{x},\mathbf{y}\rangle \wedge \mathbf{Q}\right]\right) \hat{=} 0, \quad (1.53_2)$$

which prove (1.55) and (1.56). Alternatively, substitution of (1.52) into (1.55) and of (1.53) into (1.56) demonstrates that (1.55) and (1.56) hold because

$$V\left(\neg\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}\right|\right) \hat{\cdot} V\left(\mathrm{S}_{\mathbf{y}}^{\mathbf{x}}\mathbf{P}\right|\right) \hat{=} 0, \quad (1.55_1)$$

and similarly with ‘**Q**’ in place of ‘**P**’ and ‘**x**’ and ‘**y**’ exchanged.•

Cmt 1.9. Lemma 1.4 and Th 1.14 comprise general kyrology schemata, which are not directly relevant to any special definitions such as Df 1.1. However, comparison of (1.51) with (1.35)–(1.37) shows that the above articles are particularly applicable with ‘**u**’ and ‘**v**’ in place of ‘**x**’ and ‘**y**’ respectively and with ‘$[\mathbf{u} \subseteq \mathbf{v}]$’, ‘$[\mathbf{u} = \mathbf{v}]$’, or ‘$[\mathbf{u} \,\bar{\subset}\, \mathbf{v}]$’ as ‘$\mathbf{R}\langle\mathbf{u},\mathbf{v}\rangle$’, i.e. with

$$\mathbf{R}\langle\mathbf{u},\mathbf{v}\rangle \,\vec{\in}\, \{[\mathbf{u} \subseteq \mathbf{v}], [\mathbf{u} = \mathbf{v}], [\mathbf{u} \,\bar{\subset}\, \mathbf{v}]\}. \quad (1.57)$$

Thus, for example,

$$V\left(\bigvee_{\mathbf{u}}\left[[\mathbf{u} \subseteq \mathbf{v}] \wedge \mathbf{P}\right]\right) \hat{=} V\left(\mathrm{S}_{\mathbf{v}}^{\mathbf{u}}\mathbf{P}\right|\right) \hat{\cdot} V\left(\bigvee_{\mathbf{u}}\left[[\mathbf{u} \subseteq \mathbf{v}] \wedge \mathbf{P}\right]\right), \quad (1.52\varepsilon)$$

$$V\left(\bigvee_{\mathbf{v}}\left[[\mathbf{u} \subseteq \mathbf{v}] \wedge \mathbf{Q}\right]\right) \hat{=} V\left(\mathrm{S}_{\mathbf{u}}^{\mathbf{v}}\mathbf{Q}\right|\right) \hat{\cdot} V\left(\bigvee_{\mathbf{v}}\left[[\mathbf{u} \subseteq \mathbf{v}] \wedge \mathbf{Q}\right]\right), \quad (1.53\varepsilon)$$

$$V\left(\bigvee_{\mathbf{u}}\neg\left[\neg[\mathbf{u} \subseteq \mathbf{v}] \wedge \mathbf{P}\right]\right) \hat{=} V\left(\bigvee_{\mathbf{v}}\neg\left[\neg[\mathbf{u} \subseteq \mathbf{v}] \wedge \mathbf{Q}\right]\right) \hat{=} 0, \quad (1.54\varepsilon)$$

$$V\left(\widehat{\mathrm{S}}_{\mathbf{v}}^{\mathbf{u}}\mathbf{P}\right| \Rightarrow \bigvee_{\mathbf{u}}\left[[\mathbf{u} \subseteq \mathbf{v}] \wedge \mathbf{P}\right]\right) \hat{=} V\left(\neg\mathrm{S}_{\mathbf{v}}^{\mathbf{u}}\mathbf{P}\right|\right) \hat{\cdot} V\left(\bigvee_{\mathbf{u}}\left[[\mathbf{u} \subseteq \mathbf{v}] \wedge \mathbf{P}\right]\right) \hat{=} 0, \quad (1.55\varepsilon)$$

$$V\left(\widehat{\mathrm{S}}_{\mathbf{u}}^{\mathbf{v}}\mathbf{Q}\right| \Rightarrow \bigvee_{\mathbf{v}}\left[[\mathbf{u} \subseteq \mathbf{v}] \wedge \mathbf{Q}\right]\right) \hat{=} V\left(\neg\mathrm{S}_{\mathbf{u}}^{\mathbf{v}}\mathbf{Q}\right|\right) \hat{\cdot} V\left(\bigvee_{\mathbf{v}}\left[[\mathbf{u} \subseteq \mathbf{v}] \wedge \mathbf{Q}\right]\right) \hat{=} 0 \quad (1.56\varepsilon)$$

are some pertinent specifications of (1.52)–(1.56), which immediately apply with = or $\bar{\subset}$ in place of $\subseteq$.•

1.6. Specifications of sorting panlogographic placeholders in $\mathrm{A}_{1\in}$

1) In Dfs I.6.3(5a) and II.1.7, I have introduced an infinite number of so-called sorting, or descriptive, analytical molecular panlogographic formulas (StgAnMPLF or DAnMPLF’s), or molecular formulary panlogographs (StgAnMFPL’s or

DAnMFPL's), of euautographic formulas – panlogographs such as '$\mathbf{P}\langle\mathbf{x}\rangle$', '$\mathbf{P}\langle\mathbf{y}\rangle$', '$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$', '$\mathbf{P}\langle\mathbf{y},\mathbf{z}\rangle$', '$\mathbf{i}\langle\mathbf{x}\rangle$', '$\mathbf{i}\langle\mathbf{y}\rangle$', '$\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle$', '$\mathbf{i}\langle\mathbf{y},\mathbf{z}\rangle$', etc. With the help of such panlogographs, I have stated and proved various fundamental formulas of A_1 and $\boldsymbol{\mathsf{A}}_1$, while these panlogographs have acquired some additional properties that have been sanctions by their use. Therefore, in what follows, I shall, for convenience in further discussion, summarize the most essential properties of the StgAnMFPL's, both initial ones and acquired ones.

2) Different atomic panlogographic ordinary terms (APLOT's) '**u**' to '**z**', alone or furnished with some Arabic digital subscripts $_1$, $_2$, etc that occur in the same formula or generally in the same complete fragment of the treatise, within which they preserve they recognizable identities, are supposed *to assume* (*take on*, *accidentally denote*) *different atomic pseudo-variable ordinary terms* (*APVOT's*) of the set that comprises u to w, alone or furnished with some Arabic digital subscripts $_1$, $_2$, etc. In order to satisfy the above condition, different APLOT's, which are arranged in the alphabetic order should, when desired, be replaced by any different APVOT's also taken either in the alphabetic order, successively or not, – particularly, by conformal (analo-homolographic) APVOT's.

3) Under the above assumption, if the PLR's (panlogographic relations) '$\mathbf{P}\langle\mathbf{x}\rangle$', '$\mathbf{P}\langle\mathbf{y}\rangle$', and '$\mathbf{P}\langle\mathbf{z}\rangle$', e.g., that occur in the same larger formula or generally in the same fragment of the treatise, within which they preserve their recognizable identities, then it is assumed that $\mathbf{P}\langle\mathbf{x}\rangle$, e.g., contains an AEOT **x** and perhaps some other AOET's but it does not contain either **y** or **z**, whereas

$$\mathbf{P}\langle\mathbf{y}\rangle \to \widehat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{y}}\mathbf{P}\langle\mathbf{x}\rangle\Big|,\ \mathbf{P}\langle\mathbf{z}\rangle \to \widehat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{z}}\mathbf{P}\langle\mathbf{x}\rangle\Big| \tag{1.58}$$

(cf. (II.1.5)); $\widehat{\mathrm{S}}^{\mathbf{x}}_{\mathbf{y}}\mathbf{P}\langle\mathbf{x}\rangle\Big|$, e.g., is the ER resulting by substitution of **y** for **x** throughout $\mathbf{P}\langle\mathbf{x}\rangle$. From the first definition (1.58), e.g., it follows that:

$$V\big(\mathbf{P}\langle\mathbf{x}\rangle\big) \hat{=} 0 \text{ if and only if } V\big(\mathbf{P}\langle\mathbf{y}\rangle\big) \hat{=} 0, \tag{a}$$

$$V\big(\mathbf{P}\langle\mathbf{x}\rangle\big) \hat{=} 1 \text{ if and only if } V\big(\mathbf{P}\langle\mathbf{y}\rangle\big) \hat{=} 1, \tag{b}$$

(1.59)

$$V\big(\mathbf{P}\langle\mathbf{x}\rangle\big) \hat{=} \mathbf{i}_{\sim}\big|\mathbf{P}\langle\mathbf{x}\rangle\big\rangle \text{ if and only if } V\big(\mathbf{P}\langle\mathbf{y}\rangle\big) \hat{=} \mathbf{i}_{\sim}\big|\mathbf{P}\langle\mathbf{y}\rangle\big\rangle, \tag{c}$$

and similarly with any other pair of letters selected out of ‘**x**’, ‘**y**’, and ‘**z**’ in place of ‘**x**’ and ‘**y**’. That is to say, owing to definitions (1.58), $\mathbf{P}\langle\mathbf{x}\rangle$, $\mathbf{P}\langle\mathbf{y}\rangle$, and $\mathbf{P}\langle\mathbf{z}\rangle$ are simultaneously either kyrologies or antikyrologies or else udeterologies.

4) Unless stated otherwise, the occurrences of **x** in an ER $\mathbf{P}\langle\mathbf{x}\rangle$ are supposed to be free. In this case, the PLR ‘$\mathbf{P}\langle\mathbf{x}\rangle$’ can be specified, e.g., as:

$$\mathbf{P}\langle\mathbf{x}\rangle \rhd \mathbf{F}^2(\mathbf{x},\mathbf{x}), \tag{1.60}$$

where $\mathbf{F}^2 \vec{\in} \kappa^{2pv} \cup \widehat{K}^{2pc}_{\in}$ subject to (1.12) and (1.13), and similarly with ‘**y**’ or ‘**z**’ in place of ‘**x**’. In $A_{1\in G}$, it follows from (1.35)–(1.37) subject to (1.3), (1.4), and (1.7)–(1.9) that

$$V(\mathbf{F}^2(\mathbf{x},\mathbf{x})) \hat{=} 0 \text{ if } \mathbf{F}^2 \vec{\in} \{\subseteq, =, \overline{\subset}, \supseteq, \overline{\supset}\}, \tag{a}$$

$$V(\mathbf{F}^2(\mathbf{x},\mathbf{x})) \hat{=} 1 \text{ if } \mathbf{F}^2 \vec{\in} \{\subset, \overline{\subseteq}, \equiv, \supset, \overline{\supseteq}\}, \tag{b} \quad (1.61)$$

$$V(\mathbf{F}^2(\mathbf{x},\mathbf{x})) \hat{=} \mathbf{i}_{\sim}|\mathbf{F}^2(\mathbf{x},\mathbf{x})\rangle \text{ if } \mathbf{F}^2 \vec{\in} \kappa^{2pv} \cup \{\in, \overline{\in}, \ni, \overline{\ni}\}, \tag{c}$$

subject to (1.12). In $A_{1\in D}$ and hence in $A_{1\in S}$, the domains of applicability in the relations (1.61a)–(1.61c) are altered owing to theorem (IV.2.5), so that (1.61) will turn into

$$V(\mathbf{F}^2(\mathbf{x},\mathbf{x})) \hat{=} 0 \text{ if } \mathbf{F}^2 \in \{\subseteq, =, \overline{\in}, \overline{\subset}, \supseteq, \overline{\ni}, \overline{\supset}\}, \tag{a}$$

$$V(\mathbf{F}^2(\mathbf{x},\mathbf{x})) \hat{=} 1 \text{ if } \mathbf{F}^2 \in \{\in, \subset, \overline{\subseteq}, \equiv, \ni, \supset, \overline{\supseteq}\}, \tag{b} \quad (1.62)$$

$$V(\mathbf{F}^2(\mathbf{x},\mathbf{x})) \hat{=} \mathbf{i}_{\sim}|\mathbf{F}^2(\mathbf{x},\mathbf{x})\rangle \text{ if } \mathbf{F}^2 \vec{\in} \kappa^{2pv} \cup \{\in, \ni\}, \tag{c}$$

subject to (1.12).

5) In analogy with the item 4, if the PLR’s ‘$\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$’, ‘$\mathbf{P}\langle\mathbf{u},\mathbf{w}\rangle$’, ‘$\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle$’, and ‘$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$’, e.g., occur in the same larger formula or generally in the same fragment of the treatise, within which they preserve their recognizable identities, then it is assumed that $\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle$, e.g., contains an AEOT’s **u** and **v** and perhaps some other AOET’s but it does not contain either **y** or **z**, whereas

$$\mathbf{P}\langle\mathbf{u},\mathbf{x}\rangle \to \mathrm{S}^{\mathbf{x}}_{\mathbf{v}}\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle\big|, \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle \to \mathrm{S}^{\mathbf{x}}_{\mathbf{u}}\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle\big|, \mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle \to \mathrm{S}^{\mathbf{y}}_{\mathbf{v}}\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\big|,$$
$$\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle \to \mathrm{S}^{\mathbf{x}}_{\mathbf{y}}\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\big|, \mathbf{P}\langle\mathbf{y},\mathbf{y}\rangle \to \mathrm{S}^{\mathbf{y}}_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\big|, \text{etc} \qquad (1.63)$$

(cf. (II.1.5)). In analogy with (1.59), it follows from definitions (1.63) that

$$V(\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} 0 \text{ if and only if } V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle) \hat{=} 0, \qquad (a)$$

$$V(\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} 1 \text{ if and only if } V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle) \hat{=} 1, \qquad (b) \quad (1.64)$$

$$V(\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} \mathbf{i}_{\sim}\big|\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle\big\rangle \text{ if and only if } V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle) \hat{=} \mathbf{i}_{\sim}\big|\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\big\rangle, \qquad (c)$$

and similarly with any other pairs of two different letters selected out of ‘**u**’, ‘**v**’, ‘**x**’, and ‘**y**’ in place of ⟨‘**u**’,‘**v**’⟩ and ⟨‘**x**’,‘**y**’⟩. That is to say, owing to definitions (1.63), $\mathbf{P}\langle\mathbf{u},\mathbf{v}\rangle, \mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$, $\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle$, $\mathbf{P}\langle\mathbf{u},\mathbf{y}\rangle$, etc are simultaneously either kyrologies or antikyrologies or else udeterologies. In this case, $\mathbf{P}\langle\mathbf{u},\mathbf{u}\rangle$ and $\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle$, e.g., satisfy the variant of the schema (1.59) with ‘⟨**u**,**u**⟩’ and ‘⟨**x**,**x**⟩’ in place of ‘⟨**x**⟩’ and ‘⟨**x**⟩’ respectively.

6) Unless stated otherwise, the occurrences of **x** and **y** in an ER $\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$ are supposed to be free. In this case, the PLR’s ‘$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$’ and ‘$\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle$’ can be specified, e.g., as:

$$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle \rhd \mathbf{F}^2(\mathbf{x},\mathbf{y}), \mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle \rhd \mathbf{F}^2(\mathbf{x},\mathbf{x}) \qquad (1.65)$$

subject to $\mathbf{F}^2 \vec{\in} \kappa^{2pv} \cup \hat{\mathrm{K}}^{2pc}_{\in}$ (cf. (1.60)), and similarly with any two different of the four letters ‘**u**’, ‘**v**’, ‘**x**’, and ‘**y**’ in place of ‘**x**’ and ‘**y**’, whereas ‘$\mathbf{F}^2$’ can be specified or particularized as indicated in Df 1.3(3). In $\mathrm{A}_{1\in\mathrm{G}}$, it follows from (1.22)–(1.27) that

$$V(\mathbf{F}^2(\mathbf{x},\mathbf{y})) \hat{=} \mathbf{i}_{\sim}\big|\mathbf{F}^2(\mathbf{x},\mathbf{y})\big\rangle \text{ for each } \mathbf{F}^2 \in \mathrm{K}^2_{\in}, \qquad (1.66)$$

i.e. all ER’s $\mathbf{F}^2(\mathbf{x},\mathbf{y})$ with mutually different **x** and **y** are udeterologies. In this case, ‘$\mathbf{F}^2(\mathbf{x},\mathbf{x})$’ satisfies the schema (1.61) in $\mathrm{A}_{1\in\mathrm{G}}$ or the schema (1.62) in $\mathrm{A}_{1\in\mathrm{D}}$ and hence in $\mathrm{A}_{1\in\mathrm{S}}$.

7) Definitions (1.58) and (1.63) are obviously generalized to StgAnMPLF’s such as ‘$\mathbf{P}\langle\mathbf{u},\mathbf{v},\mathbf{w}\rangle$’, ‘$\mathbf{P}\langle\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{x}\rangle$’, etc and their appropriate variants.

1.7. Infrafunctional and functional pseudo-quantifiers

Preliminary Remark 1.2. 1) If an ER $\mathbf{P}\langle\mathbf{x}\rangle$ contains **x** and does not contain **y**, **z**, **v**, and **w** then, in accordance with Df II.2.1(5,6),

$$\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{x}\rangle \to \bigwedge\nolimits_{\mathbf{x}}\bigwedge\nolimits_{\mathbf{y}}[\![[\mathbf{P}\langle\mathbf{x}\rangle \wedge \mathbf{P}\langle\mathbf{y}\rangle] \Rightarrow [\mathbf{x}=\mathbf{y}]]\!], \tag{1.67}$$

$$\left\lfloor \bigvee\nolimits^{1}_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle \right\rfloor \to \left\lfloor\!\left\lfloor \overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle \right\rfloor \wedge \left\lfloor \bigvee\nolimits_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle \right\rfloor\!\right\rfloor, \tag{1.68}$$

subject to (1.58) and also subject to the variants of (1.58) with ‘**v**’ or ‘**w**’ in place of ‘**y**’ or ‘**z**’. Since the set τ^{pv}, defined by (1.10) in accordance with Ax II.5.1(5) and being the range of each one of the PLOT”s ‘**x**’, ‘**y**’, ‘**z**’, ‘**v**’, and ‘**w**’, is infinite, therefore the above condition can always be satisfied.

2) By Df II.2.4(5,6), an ER $\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$ of A_1 is called *the infrafunctional disjunction over* **z** *of* $\mathbf{P}\langle\mathbf{z}\rangle$ or less explicitly *an infrafunctional contraction* (without the postpositive qualifier “over **z** of $\mathbf{P}\langle\mathbf{z}\rangle$”), whereas $\bigvee^{1}_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle$ is called *the functional disjunction over* **v** *of* $\mathbf{P}\langle\mathbf{v}\rangle$ or less explicitly *a functional disjunction*. Accordingly, by Df II.2.5(5,6), a euautographic operator (kernel-sign) $\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}}$ (i.e. $\overset{\frown}{\bigvee}{}^{1}_{u}$ to $\overset{\frown}{\bigvee}{}^{1}_{z}$, $\overset{\frown}{\bigvee}{}^{1}_{u_1}$ to $\overset{\frown}{\bigvee}{}^{1}_{z_1}$, etc) is called *the infrafunctional disjunctive contractor over* **z** or less explicitly *an infrafunctional contractor* (without the postpositive qualifier “over **z**”), whereas $\bigvee^{1}_{\mathbf{v}}$ is called *the functional contractor over* **v** or less explicitly *a functional contractor*. In accordance with Df II.2.6, the following expressions (e.g.) can be used interchangeably: “*infrafunctional disjunctive contraction*” and “*infrafunctional disjunction*”; “*functional disjunctive contraction*”, “*functional disjunction*”, and “*strict exclusive disjunction*”; “*infrafunctional disjunctive contractor over*” and “*infrafunctional pseudo-qualifier of*”; “*functional disjunctive contractor over*”, “*strict exclusive disjunctive contractor over*”, “*functional pseudo-qualifier of*”, and “*strict existential pseudo-qualifier of*”. Also, as opposed to quasi-quantifiers $\overset{\frown}{\bigvee}{}^{1}_{\mathbf{z}}$ and $\bigvee^{1}_{\mathbf{v}}$, which are qualified *infrafunctional* and *functional*, a lax (weak) inclusive existential pseudo-quantifier $\bigvee_{\mathbf{x}}$, a strict (strong) existential pseudo-quantifier $\overset{\smile}{\bigvee}_{\mathbf{z}}$, and a universal pseudo-quantifier $\bigwedge_{\mathbf{x}}$ are collectively called *suprafunctional* pseudo-quantifiers.

3) The general properties of infrafunctional and functional pseudo-quantifiers, i.e. their properties relative to a general (common) vav-neutral ER of A_1, $\mathbf{P}\langle\mathbf{z}\rangle$, have been established in Ths II.8.6–II.8.9, Cmts II.8.3, II.8.4, and II.8.6–II.8.8, and Df II.8.1. Particularly, it has been proved in Ths II.8.6 and II.8.8 that

$$\begin{aligned} V\left(\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right) &\hat{=} V\left(\bigwedge_x \bigwedge_y \left[\left[\mathbf{P}(\mathbf{x}) \wedge \mathbf{P}\langle\mathbf{y}\rangle\right] \Rightarrow [\mathbf{x}=\mathbf{y}]\right]\right) \\ &\hat{=} 1 \hat{-} \hat{\bullet}_{\mathbf{x}} \hat{\bullet}_{\mathbf{y}} \left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right) \hat{\cdot} V(\mathbf{x}=\mathbf{y})\right], \end{aligned} \tag{1.69}$$

$$\begin{aligned} V\left(\bigvee{}^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right) &\hat{=} V\left(\left[\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right] \wedge \left[\bigvee_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right]\right) \hat{=} V\left(\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right) \hat{+} V\left(\bigvee_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right) \\ &\hat{=} 1 \hat{-} \hat{\bullet}_{\mathbf{x}} \hat{\bullet}_{\mathbf{y}} \left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{x}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{y}\rangle\right) \hat{\cdot} V(\mathbf{x}=\mathbf{y})\right] \hat{+} \hat{\bullet}_{\mathbf{w}} V\left(\mathbf{P}\langle\mathbf{w}\rangle\right), \end{aligned} \tag{1.70}$$

which are tokens of the PLMT's (panlogographic master-theorems) (II.8.12) and (II.8.14) for the PLSR's (panlogographic slave-relations) '$\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$' and $\bigvee{}^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle$. Some specific properties that the infrafunctional and functional pseudo-quantifiers have *in* $\mathsf{A}_{1\in\mathrm{G}}$ are established below in this subsection.•

***Th 1.15.**

$$V\left(\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}[\mathbf{z}=\mathbf{u}]\right) \hat{=} 0. \tag{1.71}$$

$$V\left(\bigvee{}^1_{\mathbf{v}}[\mathbf{v}=\mathbf{u}]\right) \hat{=} V\left(\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}[\mathbf{z}=\mathbf{u}]\right) \hat{+} V\left(\bigvee_{\mathbf{w}}[\mathbf{w}=\mathbf{u}]\right) \hat{=} V\left(\bigvee_{\mathbf{w}}[\mathbf{w}=\mathbf{u}]\right) \hat{=} 0. \tag{1.72}$$

Proof: With '$[\mathbf{z}=\mathbf{u}]$', '$[\mathbf{x}=\mathbf{u}]$', '$[\mathbf{y}=\mathbf{u}]$', '$[\mathbf{v}=\mathbf{u}]$', and '$[\mathbf{w}=\mathbf{u}]$' in place of '$\mathbf{P}\langle\mathbf{z}\rangle$', '$\mathbf{P}\langle\mathbf{x}\rangle$', '$\mathbf{P}\langle\mathbf{y}\rangle$', '$\mathbf{P}\langle\mathbf{v}\rangle$', and '$\mathbf{P}\langle\mathbf{w}\rangle$' respectively, the identity (1.69), i.e. (II.8.12), becomes:

$$\begin{aligned} V\left(\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}[\mathbf{z}=\mathbf{u}]\right) &\hat{=} 1 \hat{-} \hat{\bullet}_{\mathbf{x}} \hat{\bullet}_{\mathbf{y}} \left[1 \hat{-} V\left(\neg[\mathbf{x}=\mathbf{u}]\right) \hat{\cdot} V\left(\neg[\mathbf{y}=\mathbf{u}]\right) \hat{\cdot} V(\mathbf{x}=\mathbf{y})\right] \\ &\hat{=} 1 \hat{-} \hat{\bullet}_{\mathbf{x}} \hat{\bullet}_{\mathbf{y}} 1 \hat{=} 1 \hat{-} 1 \hat{=} 0, \end{aligned} \tag{1.71$_1$}$$

whereas the identity (1.70), i.e. (II.8.14), turns into (1.72). The train of identities (1.71) has been developed by the following arguments. First,

$$\begin{aligned} &V\left(\neg[\mathbf{x}=\mathbf{u}]\right) \hat{\cdot} V\left(\neg[\mathbf{y}=\mathbf{u}]\right) \hat{\cdot} V(\mathbf{x}=\mathbf{y}) \\ &\hat{=} V\left(\neg[\mathbf{x}=\mathbf{u}]\right) \hat{\cdot} V\left(\neg[\mathbf{u}=\mathbf{y}]\right) \hat{\cdot} V(\mathbf{x}=\mathbf{y}) \hat{=} 0, \end{aligned} \tag{1.71$_2$}$$

where use of the variant of (1.38) with '**x**' and '**u**' in place of '**u**' and '**v**' respectively and then use of the variant of (1.43) with '**x**', '**u**', and '**y**' in place of '**u**, '**v**', and '**w**' respectively have been made. After transformation (1.71$_2$), the final result in (1.71$_1$) is obtained by making use of the second identity (II.4.24$\gamma\iota_1$). Thus, (1.71) is established.

The final result in (1.72) is obtained by (1.71) and by the variant of (1.47) with '**w**' and '**u**' in place of '**u**' and '**v**' respectively. QED.•

***Th 1.16.**

$$V\left(\widehat{\bigvee}_{\mathbf{z}}^{1}\left[[\mathbf{z}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{z}\rangle\right]\right)\hat{=}0. \tag{1.73}$$

$$\begin{aligned} V\left(\bigvee_{\mathbf{v}}^{1}\left[[\mathbf{v}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{v}\rangle\right]\right)&\hat{=}V\left(\widehat{\bigvee}_{\mathbf{z}}^{1}\left[[\mathbf{z}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{z}\rangle\right]\right)\hat{+}V\left(\bigvee_{\mathbf{w}}\left[[\mathbf{w}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{w}\rangle\right]\right)\\ &\hat{=}V\left(\bigvee_{\mathbf{w}}\left[[\mathbf{w}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{w}\rangle\right]\right)\hat{=}\hat{\prod}_{\mathbf{w}}V\left([\mathbf{w}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{w}\rangle\right). \end{aligned} \tag{1.74}$$

Proof: With

'$[\mathbf{z}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{z}\rangle$', '$[\mathbf{x}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{x}\rangle$', '$[\mathbf{y}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{y}\rangle$',

'$[\mathbf{v}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{v}\rangle$', and '$[\mathbf{w}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{w}\rangle$'

in place of '$\mathbf{P}\langle\mathbf{z}\rangle$', '$\mathbf{P}\langle\mathbf{x}\rangle$', '$\mathbf{P}\langle\mathbf{y}\rangle$', '$\mathbf{P}\langle\mathbf{v}\rangle$', and '$\mathbf{P}\langle\mathbf{w}\rangle$' respectively, the identity (1.69), i.e. (II.8.12), becomes:

$$V\left(\widehat{\bigvee}_{\mathbf{x}}^{1}\left[[\mathbf{x}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{x}\rangle\right]\right)\hat{=}1\hat{-}\hat{\prod}_{\mathbf{x}}\hat{\prod}_{\mathbf{y}}\left[1\hat{-}\mathbf{J}\langle\mathbf{x},\mathbf{u}\rangle\hat{\cdot}\mathbf{J}\langle\mathbf{y},\mathbf{u}\rangle\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right] \tag{1.73$_1$}$$

subject to

$$\begin{aligned} \mathbf{J}\langle\mathbf{x},\mathbf{u}\rangle&\stackrel{\rightarrow}{\hat{=}}V\left(\neg\left[[\mathbf{x}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{x}\rangle\right]\right)\hat{=}V(\neg[\mathbf{x}=\mathbf{u}])\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x}\rangle),\\ \mathbf{J}\langle\mathbf{y},\mathbf{u}\rangle&\hat{=}V(\neg[\mathbf{y}=\mathbf{u}])\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\hat{=}V(\neg[\mathbf{y}=\mathbf{u}])\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle), \end{aligned} \tag{1.73$_2$}$$

whereas the identity (1.70), i.e. (II.8.14), turns into (1.74). By (1.71$_1$) and (1.73$_2$), it follows that

$$\begin{aligned} &\mathbf{J}\langle\mathbf{x},\mathbf{u}\rangle\hat{\cdot}\mathbf{J}\langle\mathbf{y},\mathbf{u}\rangle\hat{\cdot}V(\mathbf{x}=\mathbf{y})\\ &\hat{=}\left[V(\neg[\mathbf{x}=\mathbf{u}])\hat{\cdot}V(\neg[\mathbf{y}=\mathbf{u}])\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right]\hat{\cdot}\left[V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\right]\\ &\hat{=}0\hat{\cdot}\left[V(\neg\mathbf{P}\langle\mathbf{x}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{y}\rangle)\right]\hat{=}0. \end{aligned} \tag{1.73$_3$}$$

Hence, by the second identity (II.4.24γι$_1$), identity (1.73$_1$) reduces to:

$$V\left(\widehat{\bigvee}_{\mathbf{x}}^{1}\left[[\mathbf{x}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{x}\rangle\right]\right)\hat{=}1\hat{-}\hat{\prod}_{\mathbf{x}}\hat{\prod}_{\mathbf{y}}1\hat{=}1\hat{-}1\hat{=}0, \tag{1.73$_4$}$$

which proves (1.73). The final result in (1.74) is obtained by (1.73). QED.•

Cmt 1.10. By (II.4.40a), identities (1.71) and (1.72) are tantamount to

$$\widehat{\bigvee}_{\mathbf{x}}^{1}[\mathbf{x}=\mathbf{u}], \tag{1.71a}$$

$$\bigvee_{\mathbf{x}}^{1}[\mathbf{x}=\mathbf{u}], \tag{1.72a}$$

and (1.47a), whereas identities (1.73) and (1.74) are tantamount to

$$\widehat{\bigvee}_{\mathbf{z}}^{1}\left[[\mathbf{z}=\mathbf{u}]\wedge\mathbf{P}\langle\mathbf{z}\rangle\right], \tag{1.73a}$$

$$\bigvee_{\mathbf{x}}^{1}\left[[\mathbf{x}=\mathbf{u}]\wedge \mathbf{P}\langle\mathbf{x}\rangle\right]\Leftrightarrow \bigvee_{\mathbf{z}}\left[[\mathbf{z}=\mathbf{u}]\wedge \mathbf{P}\langle\mathbf{z}\rangle\right], \tag{1.74a}$$

respectively. Also, by the variant of (1.38) with '**z**' and '**u**' in place of '**u**' and '**v**' respectively, kyrologies (1.71a) and (1.72a) can be restated as:

$$\widehat{\bigvee}_{\mathbf{z}}^{1}[\mathbf{u}=\mathbf{z}], \tag{1.71b}$$

$$\bigvee_{\mathbf{x}}^{1}[\mathbf{u}=\mathbf{x}]. \tag{1.72b}$$ •

1.8. Application of the General Law of Nonexistence of Russell's Paradox to $\mathsf{A}_{1\in}$

1) In Th II.8.10, I have established the following *master*, or *decision*, *theorem* (*MT* or *DT*) *of* A_1 in terms of '$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$' and '$\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle$':

$$V\left(\neg\bigwedge_{\mathbf{x}}[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle]\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\neg[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle]\right)\hat{=}0, \tag{1.84}$$

which is a token of (I.8.23) and which is, in accordance with the terminology that has been introduced in Df II.8.2, called the [two-fold] *Weak*, or *Unbound*, *General Law of Denial of Russell's Paradox* (briefly *Weak GLDRP* or *WGLDRP*) *in the objective* (or *algebraic*) *form*. By (4.40a), the train of identities (1.84) is concurrent (tantamount, equivalent) to either one of following two *subjective* (or *logical*) *forms* of the WGLDRP:

$$\bigvee_{\mathbf{x}}\neg[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle]. \tag{1.84'}$$

$$\neg\bigwedge_{\mathbf{x}}[\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle], \tag{1.84''}$$

which are tokens of (II.8.23') and (II.8.23'). Since (1.84') and (1.84") are *kyrologies* (*valid relations*), therefore the three-fold *Strong*, or *Bound*, *GLDRP* (*SGLDRP*) *in* A_1 in the objective form, (II.8.24), and the respective three subjective forms of the SGLDRP, (II.8.24')–(II.8.24'''), are trivial implications of (II.8.23), i.e. of (1.84).

2) In any phase $\mathsf{A}_{1\in G}$, $\mathsf{A}_{1\in D}$, or $\mathsf{A}_{1\in S}$, '$\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$' and '$\mathbf{P}\langle\mathbf{x},\mathbf{x}\rangle$' can be specified as indicated by (1.65), so that to yield:

$$V\left(\neg\bigwedge_{\mathbf{x}}[\mathbf{F}^2(\mathbf{x},\mathbf{y})\wedge\neg\mathbf{F}^2(\mathbf{x},\mathbf{x})]\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\neg[\mathbf{F}^2(\mathbf{x},\mathbf{y})\wedge\neg\mathbf{F}^2(\mathbf{x},\mathbf{x})]\right)\hat{=}0, \tag{1.85}$$

which is the pertinent *specific* instance of (II.8.23), i.e. of (1.84);

$$\begin{aligned}V\left(\neg\bigvee_{\mathbf{y}}\bigwedge_{\mathbf{x}}[\mathbf{F}^2(\mathbf{x},\mathbf{y})\wedge\neg\mathbf{F}^2(\mathbf{x},\mathbf{x})]\right)&\hat{=}V\left(\bigwedge_{\mathbf{y}}\neg\bigwedge_{\mathbf{x}}[\mathbf{F}^2(\mathbf{x},\mathbf{y})\wedge\neg\mathbf{F}^2(\mathbf{x},\mathbf{x})]\right)\\&\hat{=}V\left(\neg\bigwedge_{\mathbf{y}}\bigwedge_{\mathbf{x}}[\mathbf{F}^2(\mathbf{x},\mathbf{y})\wedge\neg\mathbf{F}^2(\mathbf{x},\mathbf{x})]\right)\hat{=}0,\end{aligned} \tag{1.86}$$

which is the pertinent *specific* instance of (II.8.24), and

$$\bigvee_{\mathbf{x}}\neg\left[\mathbf{F}^2(\mathbf{x},\mathbf{y})\wedge\neg\mathbf{F}^2(\mathbf{x},\mathbf{x})\right]. \tag{1.85'}$$

$$\neg\bigwedge_{\mathbf{x}}\left[\mathbf{F}^2(\mathbf{x},\mathbf{y})\wedge\neg\mathbf{F}^2(\mathbf{x},\mathbf{x})\right], \tag{1.85''}$$

$$\neg\bigvee_{\mathbf{y}}\bigwedge_{\mathbf{x}}\left[\mathbf{F}^2(\mathbf{x},\mathbf{y})\wedge\neg\mathbf{F}^2(\mathbf{x},\mathbf{x})\right], \tag{1.86'}$$

$$\bigwedge_{\mathbf{y}}\neg\bigwedge_{\mathbf{x}}\left[\mathbf{F}^2(\mathbf{x},\mathbf{y})\wedge\neg\mathbf{F}^2(\mathbf{x},\mathbf{x})\right], \tag{1.86''}$$

$$\neg\bigwedge_{\mathbf{y}}\bigwedge_{\mathbf{x}}\left[\mathbf{F}^2(\mathbf{x},\mathbf{y})\wedge\neg\mathbf{F}^2(\mathbf{x},\mathbf{x})\right], \tag{1.86'''}$$

which are the pertinent *specific* instances of (II.8.23')–(II.8.24'''). That is to say, the train (1.85) of PLS'ta of ER's is the pertinent specific instance of the *two-fold* WGLDRP in the objective (or algebraic) form; the train (1.86) is the pertinent specific instance of the *three-fold* SGLDRP in the objective (or algebraic) form; the valid ordinary (logical) relations (1.85') and (1.85'') are the pertinent specific instances of two subjective forms of the WGLDRP, and the valid ordinary relations (1.86')–(1.86''') are the pertinent specific instances of three subjective forms of the SGLDRP.

3) Although (1.85) and hence (1.86) and (1.85')–(1.86''') are valid for every $\mathbf{F}^2\in\mathrm{K}^2_{\in}$, the proof of (1.85) in the case of (1.61a), or (1.62a), differs somewhat from the proof of (1.85) in the cases of (1.61b) and (1.61c), or (1.62b) and (1.62c), respectively. The difference can be seen from the proof of Th II.8.10, i.e. of (1.84).

4) Any of the PLS'ta (1.85), (1.86), and (1.85')–(1.86''') can be *specified* further in various ways, for instance, by substitution of '$\mathbf{F}^{2\mathrm{pv}}$' or '$\mathbf{F}^{2\mathrm{pc}}$' for both occurrences of '$\mathbf{F}^2$' in the PLS. Also, any of these PLS'ta can be *concretized* by an infinite number of ways by replacing of occurrences '**x**' and '**y**' throughout the PLS with occurrences of any two different AEOT's either of the set τ^{pv} (defined by (1.10)) in either phase $\mathsf{A}_{1\in\mathsf{G}}$ or $\mathsf{A}_{1\in\mathsf{D}}$ or of the set $\tau^{\mathrm{pv}}\cup\{\emptyset,\emptyset'\}$ in the phase $\mathsf{A}_{1\in\mathsf{S}}$, and also by replacing both occurrences of '$\mathbf{F}^2$' either with occurrences of any BAPVOPS (as f^2, g^2, etc) of the set $\kappa^{2\mathrm{pv}}$ (defined in (1.12)) or with occurrences of any one of the fourteen BAPCOPS's (as $\in$, $\subseteq$, $\subset$, etc) of the set $\mathrm{K}^{2\mathrm{pc}}_{\in}$, in any phase of $\mathsf{A}_{1\in}$; in the latter case, every ER $\mathbf{F}^{2\mathrm{pc}}(\mathbf{x},\mathbf{y})$ or $\mathbf{F}^{2\mathrm{pc}}(\mathbf{x},\mathbf{x})$ subject to $\mathbf{F}^{2\mathrm{pc}}\in\mathrm{K}^{2\mathrm{pc}}_{\in}$ (as $\in(x,y)$ or $\in(x,x)$) is supposed to be written in the bilinear form (as $[x\in y]$ or $[x\in x]$), in accordance with definition (1.9). Thus, for instance, (1.85) can be concretized as:

$$V\left(\neg\bigwedge_x\left[f^2(x,y)\wedge\neg f^2(x,x)\right]\right)\hat{=}V\left(\bigvee_x\neg\left[f^2(x,y)\wedge\neg f^2(x,x)\right]\right)\hat{=}0 \tag{1.85μ_1}$$

and similarly with any $\mathbf{F}^{2\mathrm{pv}}\in\kappa^{2\mathrm{pv}}$ in place of f^2 or as:

$$V\left(\neg \bigwedge_x \left[\left[x \in y\right] \wedge \neg\left[x \in x\right]\right]\right) \hat{=} V\left(\bigvee_x \neg\left[\left[x \in y\right] \wedge \neg\left[x \in x\right]\right]\right) \hat{=} 0 \qquad (1.85\mu_2)$$

and similarly with any $\mathbf{F}^{2pc} \in \mathrm{K}_{\in}^{2pc}$ in place of $\in$. Consequently, (1.85') and (1.85") can be concretized respectively thus:

$$\bigvee_x \neg\left[f^2(x,y) \wedge \neg f^2(x,x)\right], \qquad (1.85'\mu_1)$$

$$\neg \bigwedge_x \left[f^2(x,y) \wedge \neg f^2(x,x)\right], \qquad (1.85''\mu_1)$$

or thus:

$$\bigvee_x \neg\left[\left[x \in y\right] \wedge \neg\left[x \in x\right]\right], \qquad (1.85'\mu_2)$$

$$\neg \bigwedge_x \left[\left[x \in y\right] \wedge \neg\left[x \in x\right]\right]. \qquad (1.85''\mu_2)$$

5) In accordance with the previous item, Th II.8.10 guaranties that any formalized language based on the CFCL interpretation of $\mathsf{A}_{1\in}$ will be free of Russell's paradox.•

2. The organon $\mathsf{A}_{1\in D}$

2.1. Preliminaries

Df 2.1. 1) The subject axioms of A_{1P} together with the meta-axioms (rules) of inference and decision determine the AEADM (Advanced Euautographic Algebraic Decision Method) of A_1. Therefore, those subject axioms are called the *general* or *typical*, and also *underlying* or *fundamental*, *axioms of* A_1. Any additional explicit subject axiom, which is imposed *either* on any one of the three *primary binary atomic pseudo-constant ordinary predicate-signs* (*PBAPCOPS's*) $\in$, $\subseteq$, and = relative to *atomic pseudo-variable ordinary terms* (*APVOT's*) *of the set* τ^{pv} (see (1.11)) *or*, in the presence of $\subseteq$ or $\in$ as the PBAPCOPS, on either of the two *atomic pseudo-constant ordinary terms* (*APCOT's*) $\emptyset$ and $\emptyset'$ relative to $\subseteq$ or $\in$ and also relative to APVOT's of the set τ^{pv}, is called a *specific* or *atypical*, and also an *individualizing* or *individuating*, *axiom of* $\mathsf{A}_{1\in}$, $\mathsf{A}_{1\subseteq}$, or $\mathsf{A}_{1=}$ respectively.

2) A subject theorem of any given one of the organons $\mathsf{A}_{1\in}$, $\mathsf{A}_{1\subseteq}$, and $\mathsf{A}_{1=}$ is called a *specific* or *atypical*, one if it is proved either from one or more axiomatic definitions of the organon (as (1.1)–(1.8)) or from one or more specific (atypical) subject axioms of the organon or from both, and it is called a *general*, or *typical*, one if otherwise, i.e. if it is proved from general (typical) subject axioms of A_{1P}.

3) A specific subject axiom of $\mathsf{A}_{1\in}$, e.g., is an ER of $\mathsf{A}_{1\in}$, which is taken for granted to be *valid* and which necessarily has as its constituent formula at least one *free PVOT* (*APVOT*), so that it is *effective* in the sense that a variety of specific subject theorems can be proved from it by the AEADM. An ER of $\mathsf{A}_{1\in}$, which does not involve any free PVOT and which is therefore a pseudo-constant ER, cannot be employed as a specific axiom of $\mathsf{A}_{1\in}$, because such a pseudo-constant axiom would have been an *ineffective thing-in-itself* that does not allow proving any theorem from it. Still, a CFCL (conformal catlogographic) interpretand of a vav-neutral pseudo-constant ER of $\mathsf{A}_{1\in}$ can be taken for granted to be *veracious* (*accidentally true*) and to become thus an effective catlogographic (semantic) axiom that allows proving some catlogographic theorems (to be demonstrated).•

†**Df 2.2.** The calculus $\mathsf{A}_{1\in\mathsf{D}}$, called the *Deficient Pseudo-Class Euautogographic Algebraico-Predicate Organon* (*DPCsEAPO*), is obtained by supplementing $\mathsf{A}_{1\in\mathsf{G}}$ with two *specific* (*atypical*) *subject axioms*, which are imposed on $\in$ relative to APVOT's and from which a wide variety of important specific (atypical) subject theorems is proved by means of D_1. The two additional specific axioms, which determine $\mathsf{A}_{1\in\mathsf{D}}$ and which are briefly called the $\in$*-axioms*, are stated below under the single logical heading "Ax 2.1". The organon $\mathsf{A}_{1\in\mathsf{D}}$ is qualified *deficient* because it has the same atomic basis $\mathsf{B}_{1\in\mathsf{G}}$ as that of $\mathsf{A}_{1\in\mathsf{G}}$, i.e. because it does not have the APCOT's $\emptyset$ and $\emptyset'$.•

Cmt 2.1. In contrast to Df 1.1 and also in contrast to the specific (atypical) subject theorems, which are proved from that definition in section 1 of this chapter, in stating the two specific (atypical) $\in$-axioms, which are included under the logical heading "Ax 2.1" and which determine $\mathsf{A}_{1\in\mathsf{D}}$, and also in stating and proving a group of specific (atypical) subject theorems that most straightforwardly follow from those axioms, I shall, for more clarity, employ various APVOT's of the set τ^{pv} (defined in (1.11)), and not their PLPH's (panlogographic placeholders). In accordance with the pertinent rules of substitution of A_1, both a variant and an intrinsic interpretand of a given euautographic kyrology are also euautographic kyrologies. At the same time, any given euautographic kyrology of $\mathsf{A}_{1\in\mathsf{D}}$ can, when desired, be turned into a PLS (panlogographic schema), belonging to $\mathbf{A}_{1\in\mathsf{D}}$, of an infinite number of euautographic kyrologies of $\mathsf{A}_{1\in\mathsf{D}}$ by replacing the occurrences of the APVOT's in the former with the analo-homolographic (conformal) bold-faced StAtPLOT's (structural atomic panlogographic ordinary terms), i.e. by replacing u, v, w, x, y, and z, alone or as base letters furnished with Arabic numeral subscripts, with '**u**', '**v**', '**w**', '**x**', '**y**' and '**z**' respectively. In any palace, where the schematic panlogographic method seems to be more convenient than the concrete euautographic one, I shall employ the former without any further comments.•

2.2. Two specific axioms $\mathrm{A}_{1\in\mathrm{D}}$ and their straightforward implications

°**Ax 2.1:** ***Two basic laws for*** $\in$ ***in subjective (logical) form.***

1) *Asymmetry law.*

$$\neg[[x \in y] \wedge [y \in x]]. \tag{2.1}$$

2) *Incidence law with respect to the class-term.*

$$\bigvee_u [x \in u]. \tag{2.2}$$

•

°**Th 2.1:** ***Two basic laws for*** $\in$ ***in (objective) algebraic form.***

1) *Asymmetry law.*

$$\begin{aligned} &V(\neg[[x \in y] \wedge [y \in x]]) \hat{=} V([x \in y] \Rightarrow \neg[y \in x]) \\ &\hat{=} V(\neg[x \in y] \vee \neg[y \in x]) \hat{=} V(\neg[x \in y]) \hat{\cdot} V(\neg[y \in x]) \hat{=} 0. \end{aligned} \tag{2.3}$$

2) *Incidence law with respect to the class-term.*

$$V(\bigvee_u [x \in u]) \hat{=} \hat{\bullet}_u V(x \in u) \hat{=} 0\,. \tag{2.4}$$

Proof: (2.3) and (2.4) follow straightforwardly from (2.1) and (2.2) by the pertinent items of Th II.7.1.•

Cmt 2.1. It follows from (2.3) that either of the following two kyrologies can be used as an alternative asymmetry law instead of (2.1):

$$[x \in y] \Rightarrow \neg[y \in x]. \tag{2.1a}$$

$$\neg[x \in y] \vee \neg[y \in x]). \tag{2.1b}$$

•

°**Th 2.2.** ***Antireflexivity law for*** $\in$**–** ***Reflexivity law for*** $\overline{\in}$.

$$V(\neg[x \in x]) \hat{=} 1 \hat{-} V(x \in x) \hat{=} 0 \text{ or } V(x \in x) \hat{=} 1. \tag{2.5}$$

Proof: The instance of (2.3) with x in place of y yields:

$$\begin{aligned} &0 \hat{=} V(\neg[[x \in x] \wedge [x \in x]]) \hat{=} V(\neg[[x \in x] \wedge [x \in x]]) \\ &\hat{=} V(\neg[x \in x]) \hat{\cdot} V(\neg[x \in x]) \hat{=} V(\neg[x \in x]). \end{aligned} \tag{2.5$_1$}$$

•

Cmt 2.2. By (II.4.40a), it immediately follows from (2.5) that

$$\neg[x \in x]. \tag{2.5a}$$

•

°**Th 2.3:** ***Incidence laws for [operata of]*** $\overline{\in}$.

$$V\left(\bigvee_u \neg[x \in u]\right) \hat{=} \hat{\cdot}_u V(\neg[x \in u]) \hat{=} \hat{\cdot}_u [1 \hat{-} V(x \in u)] \hat{=} 0. \quad (2.6)$$

$$V\left(\bigvee_x \neg[x \in u]\right) \hat{=} \hat{\cdot}_x V(\neg[x \in u]) \hat{=} \hat{\cdot}_x [1 \hat{-} V(x \in u)] \hat{=} 0. \quad (2.7)$$

Proof: Making use of the appropriate instances of the Emission Law (II.4.28) and of the pertinent variants of (2.5) in this order yields:

$$\begin{gathered} V\left(\bigvee_u \neg[x \in u]\right) \hat{=} \hat{\cdot}_u V(\neg[x \in u]) \\ \hat{=} V(\neg[x \in x]) \hat{\cdot} \hat{\cdot}_u V(\neg[x \in u]) \hat{=} 0 \hat{\cdot} \hat{\cdot}_u V(\neg[x \in u]) \hat{=} 0, \end{gathered} \quad (2.6_1)$$

$$\begin{gathered} V\left(\bigvee_x \neg[x \in u]\right) \hat{=} \hat{\cdot}_x V(\neg[x \in u]) \\ \hat{=} V(\neg[u \in u]) \hat{\cdot} \hat{\cdot}_x V(\neg[x \in u]) \hat{=} 0 \hat{\cdot} \hat{\cdot}_z V(\neg[x \in u]) \hat{=} 0. \end{gathered} \quad (2.7_1)\bullet$$

Cmt 2.3. 1) By (6.6) and (6.7), it follows that

$$\begin{gathered} V\left(\neg \bigwedge_u [x \in u]\right) \hat{=} 1 \hat{-} V\left(\bigwedge_u [x \in u]\right) \\ \hat{=} 1 \hat{-} \left[1 \hat{-} \hat{\cdot}_u V(\neg[x \in u])\right] \hat{=} \hat{\cdot}_u V(\neg[x \in u]) \hat{=} 0, \end{gathered} \quad (2.8)$$

$$\begin{gathered} V\left(\neg \bigwedge_x [x \in u]\right) \hat{=} 1 \hat{-} V\left(\bigwedge_x [x \in u]\right) \\ \hat{=} 1 \hat{-} \left[1 \hat{-} \hat{\cdot}_x V(\neg[x \in u])\right] \hat{=} \hat{\cdot}_x V(\neg[x \in u]) \hat{=} 0. \end{gathered} \quad (2.9)$$

Hence,

$$\begin{gathered} V\left(\neg \bigvee_u \bigwedge_x [x \in u]\right) \hat{=} 1 \hat{-} V\left(\bigvee_u \bigwedge_x [x \in u]\right) \\ \hat{=} 1 \hat{-} \hat{\cdot}_u V\left(\bigwedge_x [x \in u]\right) \hat{=} 1 \hat{-} \hat{\cdot}_u 1 \hat{=} 1 \hat{-} 1 \hat{=} 0, \end{gathered} \quad (2.10)$$

$$\begin{gathered} V\left(\neg \bigvee_x \bigwedge_u [x \in u]\right) \hat{=} 1 \hat{-} V\left(\bigvee_x \bigwedge_u [x \in u]\right) \\ \hat{=} 1 \hat{-} \hat{\cdot}_x V\left(\bigwedge_u [x \in u]\right) \hat{=} 1 \hat{-} \hat{\cdot}_x 1 \hat{=} 1 \hat{-} 1 \hat{=} 0. \end{gathered} \quad (2.11)$$

2) By (II.4.40a), identities (2.6)–(2.11) are tantamount to the following logical kyrologies:

$$\bigvee_u \neg[x \in u]. \quad (2.6a)$$

$$\bigvee_x \neg[x \in u]. \quad (2.7a)$$

$$\neg \bigwedge_u [x \in u]. \quad (2.8a)$$

$$\neg \bigwedge_x [x \in u]. \quad (2.9a)$$

$$\neg \bigvee_u \bigwedge_x [x \in u]. \quad (2.10a)$$

$$\neg \bigvee_x \bigwedge_u [x \in u]. \quad (2.11a)$$

Kyrologies (2.6a)–(2.9a) are *pseudo-variable relations* in the sense that each of them involves one free APVOT. By contrast, kyrologies (2.10a) and (2.11a) do not involve free APVOT's and are, in these sense, *pseudo-constants relations*, i.e. *pseudo-sentences*.

3) In accordance with the general rules of CFCL (conformal catlogographic interpretation) of formulas of A_1, in order to provide the kyrologies (valid ER's), which have been established above, with CFCL interpretands, all tokens of x, y, and z, occurring in them, should be replaced with tokens of '*x*', '*y*', and '*z*' respectively. In this case, (2.10) turns the following *catlogographic identity*:

$$V\left(\neg\bigvee_z\bigwedge_x[x\in z]\right)\hat{=}1\dot{-}V\left(\bigvee_z\bigwedge_x[x\in z]\right)\hat{=}1\dot{-}\hat{\curlyvee}_z V\left(\bigwedge_x[x\in z]\right)\hat{=}0, \qquad (2.10_1)$$

which is, from the standpoint of semantic analysis, a *tautology*, i.e. a *universally true relation*.

4) Let

$$z \text{ is a universal class} \to \bigwedge_x[x\in z]. \qquad (2.10_2)$$

Consequently, (2.10_1) can be asserted in words thus:

A universal class does not exist. (2.10_3)

In this case, each of the kyrologies (2.8)–(2.10) and (2.8a)–(2.10a) is a version of the general law which can be called *Law of non-existence of a universal class*. At the same time, statement (2.10_2) is a verbal interpretand of the catlogographic formula-relation (2.10_1), which is in turn the *CFCL interpretand of the euautographic relation-formula (2.10) of* $\mathsf{A}_{1\in\mathsf{D}}$. In this connection, the following remark should be made.

5) Any self-contained and self-consistent system of objects has the property that the universal class of the objects, i.e. the class of all objects of the system, does not belong to the system itself. For instance, the class (set) of all natural numbers (positive natural integers, including zero) is not a natural number; both a vector (linear) space and the underlying class (set) of all vectors of the space are not vectors; the species *Homo sapiens,* i.e. the specific class of men, is not a man; etc. Therefore, a universal class of substantive objects, which can serve as accidental psychical (mental) denotata of the CFCL interpretand of an APVOT of $\mathsf{A}_{1\in\mathsf{D}}$, should be a psychical (mental) denotatum of the CFCL interpretand of a certain euautographic pseudo-constant term, which does not belong either $\mathsf{A}_{1\in\mathsf{D}}$ or to $\mathsf{A}_{1\in}$. Consequently, statement (2.10_2) should, more precisely, be replaced with the following definition of the term "universal class" relevant to this treatise.•

Df 2.3. A *universal class* is the *psychical* (*mental*) denotatum of the CFCL interpretand of an *extraordinary atomic pseudo-constant* (*euautographic*) *term* (EXAPCOT) U, briefly called the *universal term*, which does not belong to the atomic basis of A_1 but which is associated with $\mathsf{A}_{1\in}$ by relating it to every AEOT of the set

$\tau_\in$, defined as $\tau_\in \to \tau^{pv} \cup \{\emptyset\}$ (see (1.11) for τ^{pv}), and to itself by the kernel-signs of the set $K_\in$ (defined in (1.14)) through certain individual formation rules and certain individual subject axioms in the framework of the AEADM of A_1. The basic principles of the organon $\bar{A}_{1\in}$ having the universal term U will be made explicit in subsection 3.2.•

Cmt 2.4. The kyrologies, which have been comprised in Ax 2.1 and Ths 2.1–2.3, allow establishing an indefinite number of other kyrologies. For instance, by (2.4), it follows that

$$V(\vee_u \vee_x [x \in u]) \hat{=} V(\vee_x \wedge_u [x \in u]) \hat{=} V(\vee_x \vee_u [x \in u]) \hat{=} 0, \tag{2.12}$$

because

$$V(\vee_x \vee_u [x \in u]) \hat{=} \hat{\vee}_x \hat{\vee}_u V(x \in u) \hat{=} \hat{\vee}_x 0 \hat{=} 0, \tag{2.12$_1$}$$

$$V(\wedge_x \vee_u [x \in u]) \hat{=} 1 \hat{-} \hat{\vee}_x [1 \hat{-} \hat{\vee}_u V(x \in u)] \hat{=} 1 \hat{-} \hat{\vee}_x 1 \hat{=} \hat{\vee}_x 0 \hat{=} 0, \tag{2.12$_2$}$$

$$V(\vee_u \vee_x [x \in u]) \hat{=} \hat{\vee}_u \hat{\vee}_x V(x \in u) \hat{=} \hat{\vee}_x \hat{\vee}_u V(x \in u) \hat{=} \hat{\vee}_x 0 \hat{=} 0. \tag{2.12$_3$}$$

It is noteworthy that

$$V(\vee_x [x \in u]) \hat{=} \hat{\vee}_x V(x \in u), \tag{2.12$_4$}$$

whereas the EVI $\hat{\vee}_x V(x \in u)$ is irreducible. Therefore, the EVI $V(\vee_x [x \in u])$ does not reduce either to 0 or to 1 and hence the ER $\vee_x [x \in u]$ is *vav-neutral*, because it has not been postulated otherwise. Nevertheless, $V(\vee_u \vee_x [x \in u])$ does reduce to 0 as shown by (2.12$_3$). However, the latter hides the fundamental difference between $\vee_x [x \in u]$ and $\vee_u [x \in u]$.•

2.3. Relations between = and ∈ under the asymmetry law for ∈

***Lemma 2.1.**

$$\begin{aligned} V(\neg[\mathbf{u} = \mathbf{v}]) &\hat{=} V(\mathbf{u} \in \mathbf{v}) \hat{\vee} V(\neg[\mathbf{u} = \mathbf{v}]) \hat{=} V(\mathbf{v} \in \mathbf{u}) \hat{\vee} V(\neg[\mathbf{u} = \mathbf{v}]) \\ &\hat{=} V(\mathbf{u} \in \mathbf{v}) \hat{\vee} V(\mathbf{v} \in \mathbf{u}) \hat{\vee} V(\neg[\mathbf{u} = \mathbf{v}]). \end{aligned} \tag{2.14}$$

Proof: By the instance of (II.7.1γ) with ‘$[\mathbf{u} = \mathbf{v}]$’ in place of ‘**P**’, it follows from (1.40) that

$$V(\neg[\mathbf{u} = \mathbf{v}]) \hat{=} \hat{\vee}_\mathbf{x} [V(\mathbf{x} \in \mathbf{u}) \hat{\vee} V(\mathbf{x} \in \mathbf{v}) \hat{+} V(\neg[\mathbf{x} \in \mathbf{u}]) \hat{\vee} V(\neg[\mathbf{x} \in \mathbf{v}])]. \tag{2.14$_1$}$$

Making use of two instances of the Emission Law (II.4.27) with

$$[V(\mathbf{x} \in \mathbf{u}) \hat{\vee} V(\mathbf{x} \in \mathbf{v}) \hat{+} V(\neg[\mathbf{x} \in \mathbf{u}]) \hat{\vee} V(\neg[\mathbf{x} \in \mathbf{v}])]$$

as $\mathbf{i}\langle\mathbf{x}\rangle$ and with $\mathbf{u}$ or $\mathbf{v}$ as $\mathbf{y}$ and then making use of the pertinent versions of (2.5) yields:

$$\begin{aligned} V(\neg[\mathbf{u}=\mathbf{v}]) &\,\hat{=}\, [V(\mathbf{u}\in\mathbf{u})\,\hat{\cdot}\,V(\mathbf{u}\in\mathbf{v})\,\hat{+}\,V(\neg[\mathbf{u}\in\mathbf{u}])\,\hat{\cdot}\,V(\neg[\mathbf{u}\in\mathbf{v}])] \\ &\quad \hat{\cdot}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}[V(\mathbf{x}\in\mathbf{u})\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{v})\,\hat{+}\,V(\neg[\mathbf{x}\in\mathbf{u}])\,\hat{\cdot}\,V(\neg[\mathbf{x}\in\mathbf{v}])] \\ &\,\hat{=}\, [1\,\hat{\cdot}\,V(\mathbf{u}\in\mathbf{v})\,\hat{+}\,0\,\hat{\cdot}\,V(\neg[\mathbf{u}\in\mathbf{v}])]\,\hat{\cdot}\,V(\neg[\mathbf{u}=\mathbf{v}]) \\ &\,\hat{=}\, V(\mathbf{u}\in\mathbf{v})\,\hat{\cdot}\,V(\neg[\mathbf{u}=\mathbf{v}]), \end{aligned} \tag{2.14$_1$}$$

$$\begin{aligned} V(\neg[\mathbf{u}=\mathbf{v}]) &\,\hat{=}\, [V(\mathbf{v}\in\mathbf{u})\,\hat{\cdot}\,V(\mathbf{v}\in\mathbf{v})\,\hat{+}\,V(\neg[\mathbf{v}\in\mathbf{u}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\in\mathbf{v}])] \\ &\quad \hat{\cdot}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}[V(\mathbf{x}\in\mathbf{u})\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{v})\,\hat{+}\,V(\neg[\mathbf{x}\in\mathbf{u}])\,\hat{\cdot}\,V(\neg[\mathbf{x}\in\mathbf{v}])] \\ &\,\hat{=}\, [V(\mathbf{v}\in\mathbf{u})\,\hat{\cdot}\,1\,\hat{+}\,V(\neg[\mathbf{v}\in\mathbf{u}])\,\hat{\cdot}\,0]\,\hat{\cdot}\,V(\neg[\mathbf{u}=\mathbf{v}]) \\ &\,\hat{=}\, V(\mathbf{v}\in\mathbf{u})\,\hat{\cdot}\,V(\neg[\mathbf{u}=\mathbf{v}]), \end{aligned} \tag{2.14$_2$}$$

respectively. Combination of (2.14$_1$) and (2.14$_2$) yields

$$V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{=}\,V(\mathbf{u}\in\mathbf{v})\,\hat{\cdot}\,V(\mathbf{v}\in\mathbf{u})\,\hat{\cdot}\,V(\neg[\mathbf{u}=\mathbf{v}]). \tag{2.14$_3$}$$

QED.•

*__Lemma 2.2.__

$$\begin{aligned} V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{u}\in\mathbf{v}]) &\,\hat{=}\, V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\in\mathbf{u}]) \\ &\,\hat{=}\, V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,[V(\neg[\mathbf{u}\in\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\in\mathbf{u}])]. \end{aligned} \tag{2.15}$$

Proof: By the pertinent instances of (II.5.3) subject to (II.5.4), it follows from (2.14$_1$)– (2.14$_3$) that

$$V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,[1\,\hat{-}\,V(\mathbf{u}\in\mathbf{v})]\,\hat{=}\,V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{u}\in\mathbf{v}])\,\hat{=}\,0, \tag{2.15$_1$}$$

$$V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,[1\,\hat{-}\,V(\mathbf{v}\in\mathbf{u})]\,\hat{=}\,V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\in\mathbf{u}])\,\hat{=}\,0, \tag{2.15$_2$}$$

$$\begin{aligned} &V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,[1\,\hat{-}\,V(\mathbf{u}\in\mathbf{v})\,\hat{\cdot}\,V(\mathbf{v}\in\mathbf{u})] \\ &\,\hat{=}\, V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[[\mathbf{u}\in\mathbf{v}]\vee[\mathbf{v}\in\mathbf{u}]])\,\hat{=}\,0, \end{aligned} \tag{2.15$_3$}$$

respectively. QED.•

Cmt 2.5. By (1.38), it follows from the instance of (II.7.1γ) with ‘$[\mathbf{u}=\mathbf{v}]$’ in place of ‘**P**’ that

$$V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{=}\,1\,\hat{-}\,V(\mathbf{u}=\mathbf{v})\,\hat{=}\,1\,\hat{-}\,V(\mathbf{v}=\mathbf{u})\,\hat{=}\,V(\neg[\mathbf{v}=\mathbf{u}]). \tag{2.14$_4$}$$

Hence, (2.14$_2$) follows from the variant of (2.14$_1$) with **u** and **v** exchanged by (2.14$_4$). Likewise, (2.15$_2$) follows from the variant of (2.15$_1$) with **u** and **v** exchanged by (1.38).•

*__Th 2.4.__

$$\begin{aligned} V(\neg[[\mathbf{u}=\mathbf{v}]\wedge[\mathbf{u}\in\mathbf{v}]]) &\,\hat{=}\, V(\neg[\mathbf{u}=\mathbf{v}]\vee\neg[\mathbf{u}\in\mathbf{v}]) \\ &\,\hat{=}\, V([\mathbf{u}=\mathbf{v}]\Rightarrow\neg[\mathbf{u}\in\mathbf{v}])\,\hat{=}\,V([\mathbf{u}\in\mathbf{v}]\Rightarrow\neg[\mathbf{u}=\mathbf{v}]) \\ &\,\hat{=}\, V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{u}\in\mathbf{v}])\,\hat{=}\,0. \end{aligned} \tag{2.16}$$

$$
\begin{aligned}
&V(\neg[[\mathbf{u}=\mathbf{v}]\wedge[\mathbf{v}\in\mathbf{u}]])\,\hat{=}\,V(\neg[\mathbf{u}=\mathbf{v}]\vee\neg[\mathbf{v}\in\mathbf{u}])\\
&\hat{=}\,V([\mathbf{u}=\mathbf{v}]\Rightarrow\neg[\mathbf{v}\in\mathbf{u}])\,\hat{=}\,V([\mathbf{v}\in\mathbf{u}]\Rightarrow\neg[\mathbf{u}=\mathbf{v}])\\
&\hat{=}\,V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}\in\mathbf{u}])\,\hat{=}\,0.
\end{aligned}
\tag{2.17}
$$

$$
\begin{aligned}
&V(\neg[\mathbf{u}=\mathbf{v}]\vee\neg[[\mathbf{u}\in\mathbf{v}]\vee[\mathbf{v}\in\mathbf{u}]])\\
&\hat{=}\,V([\mathbf{u}=\mathbf{v}]\Rightarrow\neg[[\mathbf{u}\in\mathbf{v}]\vee[\mathbf{v}\in\mathbf{u}]])\\
&\hat{=}\,V([[\mathbf{u}\in\mathbf{v}]\vee[\mathbf{v}\in\mathbf{u}]]\Rightarrow\neg[\mathbf{u}=\mathbf{v}])\\
&\hat{=}\,V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[[\mathbf{u}\in\mathbf{v}]\vee[\mathbf{v}\in\mathbf{u}]])\,\hat{=}\,0.
\end{aligned}
\tag{2.18}
$$

Proof: The trains of identities (2.16)–(2.18) follow from the pertinent items of Th II.7.2 by (2.15). In agreement with Cmt 2.5, (2.17) is the variant of (2.17) with $[\mathbf{v}\in\mathbf{u}]$ in place of $[\mathbf{u}\in\mathbf{v}]$.•

2.4. Incidence laws implied by the specific subject axioms

Th 2.5:** ***The incidence law for $\subseteq$ with respect to a subclass term.

$$
V(\textstyle\bigvee_{\mathbf{u}}\neg[\mathbf{u}\subseteq\mathbf{v}])\,\hat{=}\,\overset{\frown}{\cdot}_{\mathbf{u}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\,\hat{=}\,\overset{\frown}{\cdot}_{\mathbf{u}}[1\,\hat{-}\,V(\mathbf{u}\subseteq\mathbf{v})]\,\hat{=}\,0\,. \tag{2.19}
$$

Proof: By (II.4.23) and (1.23), the initial expression in the train of equalities (2.19) can be developed thus:

$$
\begin{aligned}
&V(\textstyle\bigvee_{\mathbf{u}}\neg[\mathbf{u}\subseteq\mathbf{v}])\,\hat{=}\,\overset{\frown}{\cdot}_{\mathbf{u}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\,\hat{=}\,\overset{\frown}{\cdot}_{\mathbf{u}}\overset{\frown}{\cdot}_{\mathbf{x}}[1\,\hat{-}\,V(\neg[\mathbf{x}\in\mathbf{u}])\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{v})]\\
&\hat{=}\,\overset{\frown}{\cdot}_{\mathbf{u}}[[1\,\hat{-}\,V(\neg[\mathbf{v}\in\mathbf{u}])\,\hat{\cdot}\,V(\mathbf{v}\in\mathbf{v})]\,\hat{\cdot}\,\overset{\frown}{\cdot}_{\mathbf{x}}[1\,\hat{-}\,V(\neg[\mathbf{x}\in\mathbf{u}])\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{v})]]\\
&\hat{=}\,\overset{\frown}{\cdot}_{\mathbf{u}}[[1\,\hat{-}\,V(\neg[\mathbf{v}\in\mathbf{u}])\,\hat{\cdot}\,1]\,\hat{\cdot}\,\overset{\frown}{\cdot}_{\mathbf{x}}[1\,\hat{-}\,V(\neg[\mathbf{x}\in\mathbf{u}])\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{v})]]\\
&\hat{=}\,\overset{\frown}{\cdot}_{\mathbf{u}}[V(\mathbf{v}\in\mathbf{u})\,\hat{\cdot}\,\overset{\frown}{\cdot}_{\mathbf{x}}[1\,\hat{-}\,V(\neg[\mathbf{x}\in\mathbf{u}])\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{v})]]\\
&\hat{=}\,[\overset{\frown}{\cdot}_{\mathbf{u}_1}[V(\mathbf{v}\in\mathbf{u}_1)]]\,\hat{\cdot}\,[\overset{\frown}{\cdot}_{\mathbf{u}_2}\overset{\frown}{\cdot}_{\mathbf{x}}[1\,\hat{-}\,V(\neg[\mathbf{x}\in\mathbf{u}_2])\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{v})]]\\
&\hat{=}\,0\,\hat{\cdot}\,\overset{\frown}{\cdot}_{\mathbf{u}_2}\overset{\frown}{\cdot}_{\mathbf{x}}[1\,\hat{-}\,V(\neg[\mathbf{x}\in\mathbf{u}_2])\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{v})]\,\hat{=}\,0.
\end{aligned}
\tag{2.19$_1$}
$$

In this case, use of the following rules of inference has been made in that order.

i) The pertinent instance of the Emission Law (II.4.28) of Ax II.4.11, according to which the instance with **v** as **x** of the operatum of $\overset{\frown}{\cdot}_{\mathbf{x}}$ has been emitted (taken out) of the scope of $\overset{\frown}{\cdot}_{\mathbf{x}}$ as an additional idempotent factor.

ii) The above factor has been reduced to $V(\mathbf{v}\in\mathbf{u})$ by the variant of (2.5) with **v** in place of x.

iii) The pertinent instance of the Fission Law (II.4.29), according to which the operand (scope) of $\overset{\frown}{\cdot}_{\mathbf{u}}$ has been represented as the product of the appropriate operands (scopes) of $\overset{\frown}{\cdot}_{\mathbf{u}_1}$ and $\overset{\frown}{\cdot}_{\mathbf{u}_2}$.

iv) The identity

$$\hat{\cdot}_{\mathbf{u}_1}\left[V(\mathbf{v}\in\mathbf{u_1})\right]\hat{=}0,\qquad (2.19_2)$$

which is the variant of (2.4) with **v** in place *x* and $\mathbf{u}_1$ in place of *u*.•

***Th 2.6.**

$$V\left(\bigvee_{\mathbf{v}}\neg[\mathbf{u}\subseteq\mathbf{v}]\right)\hat{=}\hat{\cdot}_{\mathbf{v}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}\hat{\cdot}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u})\hat{=}V\left(\bigvee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}]\right).\qquad (2.20)$$

Proof: In analogy with (2.19), by (II.4.23) and (1.23), the expression on the left-hand side of the equality (2.19) can be developed thus:

$$\begin{aligned}
V\left(\bigvee_{\mathbf{v}}\neg[\mathbf{u}\subseteq\mathbf{v}]\right)&\hat{=}\hat{\cdot}_{\mathbf{v}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}\hat{\cdot}_{\mathbf{v}}\hat{\cdot}_{\mathbf{x}}\left[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\hat{\cdot}_{\mathbf{v}}\left[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[\left[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{x})\right]\hat{\cdot}\hat{\cdot}_{\mathbf{v}}\left[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[\left[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}1\right]\hat{\cdot}\hat{\cdot}_{\mathbf{v}}\left[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\right]\right]\qquad (2.20_1)\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V(\mathbf{x}\in\mathbf{u})\hat{\cdot}\hat{\cdot}_{\mathbf{v}}\left[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\hat{\cdot}_{\mathbf{v}}\left[V(\mathbf{x}\in\mathbf{u})\hat{\cdot}\left[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\mathbf{v})\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\hat{\cdot}_{\mathbf{v}}V(\mathbf{x}\in\mathbf{u})\hat{=}\hat{\cdot}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u}).
\end{aligned}$$

In this case, after exchanging the order of the pseudo-multipliers $\hat{\cdot}_{\mathbf{v}}$ and $\hat{\cdot}_{\mathbf{x}}$, of the following rules of inference has been made in that order.

i) The pertinent instance of the Emission Law (II.4.28) of Ax II.4.11, according to which the instance with **x** as **v** of the operatum of $\hat{\cdot}_{\mathbf{v}}$, has been taken out of the scope of $\hat{\cdot}_{\mathbf{v}}$ as an additional idempotent factor.

ii) The above factor has been reduced to $V(\mathbf{x}\in\mathbf{u})$ by the variant of (2.5) with **x** in place of *x*.

iii) The pertinent instance of the Transparency Law (II.4.29) for $\hat{\cdot}_{\mathbf{v}}$, according to which the factor $V(\mathbf{x}\in\mathbf{u})$, which is independent of **v**, has been introduced into the operand (scope) of $\hat{\cdot}_{\mathbf{v}}$.

iv) The identity

$$V(\mathbf{x}\in\mathbf{u})\hat{\cdot}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{=}0,\qquad (2.20_2)$$

which is the instance of (II.7.15γ) with $[\mathbf{x}\in\mathbf{u}]$ as **P**.

v) The Idleness Law (II.4.24) with **v** as **x** and $V(\mathbf{x}\in\mathbf{u})$ as **i**.•

Cmt 2.6. The APCOT ∅ is not available in $\mathsf{A}_{1\in\mathrm{D}}$. However, (2.20) is also a subject theorem of $\mathsf{A}_{1\in\mathrm{D}}$. At the same time, once ∅ is introduced, it is automatically included into the range of every free PLOT, including ‘**u**’. Therefore, (2.20) can be particularized by substitution of $\emptyset$ for **u**. In this case,

$$V(\mathbf{x}\in\emptyset)\hat{=}1 \text{ and } V(\emptyset\subseteq\mathbf{v})\hat{=}0 \qquad (2.20_3)$$

are specific subject theorems of $\mathsf{A}_{1\in}$, which follow from the specific subject *axiom of indivisibility of* $\emptyset$: $\neg[\mathbf{x}\in\emptyset]$, belonging to $\mathsf{A}_{1\in}$. Hence, at $\emptyset$ for **u** (2.20) becomes:

$$V(\vee_{\mathbf{v}}\neg[\emptyset\subseteq\mathbf{v}])\hat{=}\hat{\cdot}_{\mathbf{v}}V(\neg[\emptyset\subseteq\mathbf{v}])\hat{=}\hat{\cdot}_{\mathbf{x}}V(\mathbf{x}\in\emptyset)\hat{=}V(\vee_{\mathbf{x}}[\mathbf{x}\in\emptyset])\hat{=}1. \qquad (2.20_4)$$

This example justifies the fact that the PLS ‘$\vee_{\mathbf{v}}\neg[\mathbf{u}\subseteq\mathbf{v}]$’ is, by (2.20), *vav-neutral*, – in contrast to the PLS ‘$\vee_{\mathbf{u}}\neg[\mathbf{u}\subseteq\mathbf{v}]$’, which is, by (2.19), *valid*. At the same time, both theorems (2.19) and (2.20) turn out to be *compatible* with the axiom of indivisibility of $\emptyset$, although they are established before $\emptyset$ is introduced in $\mathsf{A}_{1\in D}$ in order to turn it into $\mathsf{A}_{1\in}$. Therefore, besides the General Law of Nonexistence of Russell’s Paradox, I regard Ths 2.5 and 2.6 as most amazing results, which are obtained pure formally with the help of the AEADM of A_1.•

Th 2.7:** ***Incidence law for anti-equalities.

$$V(\vee_{\mathbf{u}}\neg[\mathbf{u}=\mathbf{v}])\hat{=}\hat{\cdot}_{\mathbf{u}}V(\neg[\mathbf{u}=\mathbf{v}])\hat{=}\hat{\cdot}_{\mathbf{u}}[1\hat{-}V(\mathbf{u}=\mathbf{v})]\hat{=}0. \qquad (2.21)$$

$$V(\vee_{\mathbf{v}}\neg[\mathbf{u}=\mathbf{v}])\hat{=}\hat{\cdot}_{\mathbf{v}}V(\neg[\mathbf{u}=\mathbf{v}])\hat{=}\hat{\cdot}_{\mathbf{v}}[1\hat{-}V(\mathbf{u}=\mathbf{v})]\hat{=}0. \qquad (2.21_+)$$

Proof: By (II.4.23) and (1.25), it follows that

$$\begin{aligned}&V(\vee_{\mathbf{u}}\neg[\mathbf{u}=\mathbf{v}])\hat{=}\hat{\cdot}_{\mathbf{u}}V(\neg[\mathbf{u}=\mathbf{v}])\hat{=}\hat{\cdot}_{\mathbf{u}}[V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{u}])]\\&\hat{=}\left[\hat{\cdot}_{\mathbf{u_1}}V(\neg[\mathbf{u_1}\subseteq\mathbf{v}])\right]\hat{\cdot}\left[\hat{\cdot}_{\mathbf{u_2}}V(\neg[\mathbf{v}\subseteq\mathbf{u_2}])\right]\hat{=}0\,\hat{\cdot}\left[\hat{\cdot}_{\mathbf{u_2}}V(\neg[\mathbf{v}\subseteq\mathbf{u_2}])\right]\hat{=}0,\end{aligned} \qquad (2.21_1)$$

where use has been made of the following two identities in that order: (i) the pertinent instance of the Fission Law (II.4.29), according to which the operand (scope) of $\hat{\cdot}_{\mathbf{u}}$ has been represented as the product of two appropriate operands (scopes) of $\hat{\cdot}_{\mathbf{u_1}}$ and $\hat{\cdot}_{\mathbf{u_2}}$ (cf. item iii in the proof of (2.19_1)); (ii) the variant of identity (2.19) with ‘$\mathbf{u}_1$’ inplace of ‘**u**’. Identity (2.18) is established likewise. Alternatively, (2.21_+) is the variant of (2.21) with **u** and **v** exchanged subject to (2.14_4).•

Cmt 2.7. 1) By (II.4.40a), algebraic (special) identities (2.19)–(2.21) and (2.21_+) are tantamount to the following logical (ordinary) kyrologies:

$$\vee_{\mathbf{u}}\neg[\mathbf{u}\subseteq\mathbf{v}], \qquad (2.19a)$$

$$\vee_{\mathbf{v}}\neg[\mathbf{u}\subseteq\mathbf{v}]\Leftrightarrow\vee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}], \qquad (2.20a)$$

$$\bigvee_{\mathbf{u}}\neg[\mathbf{u}=\mathbf{v}], \tag{2.21a}$$

$$\bigvee_{\mathbf{v}}\neg[\mathbf{u}=\mathbf{v}], \tag{2.21$_+$a}$$

the understanding being that (2.19a), (2.21a), and (2.21$_+$a) are inferred from (2.19), (2.21), and (2.21$_+$) by the pertinent instances of (II.4.40a), whereas (2.20a) is inferred from (2.20) by the pertinent instance of (II.7.50). In this case, $\hat{:}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u})$ is an *irreducible non-digital* (*non-numeral*) *euautographic*, or, depending on a viewpoint (mental attitude), *panlogographic*, *validity-integron*, – briefly an *IRNDEVI* (*IRNNEVI*), or, correspondingly, an *IRNDPLVI* (*IRNNPLVI*). Therefore, it follows from (2.20) that the validity integron $V(\bigvee_{\mathbf{v}}\neg[\mathbf{u}\subseteq\mathbf{v}])$ is an NDEVI, which does not reduce either to 0 or to 1. Consequently, like $\bigvee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}]$, an ER, or the PLR, $\bigvee_{\mathbf{v}}\neg[\mathbf{u}\subseteq\mathbf{v}]$ is an udeterology, i.e. *no incidence law for* $\sqsubseteq$ *with respect to the superclass term exists either in* $\mathsf{A}_{1\in\mathrm{D}}$ *or in* $\mathsf{A}_{1\in}$ (see Cmt 2.6).

2) By (II.4.23) and (II.8.2), it follows from (2.19) and (2.20), in analogy with (2.12), that

$$V(\bigvee_{\mathbf{u}}\bigvee_{\mathbf{v}}\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}V(\bigwedge_{\mathbf{v}}\bigvee_{\mathbf{u}}\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}V(\bigvee_{\mathbf{v}}\bigvee_{\mathbf{u}}\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}0, \tag{2.22}$$

because successively:

$$V(\bigvee_{\mathbf{v}}\bigvee_{\mathbf{u}}\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}\hat{:}_{\mathbf{v}}\hat{:}_{\mathbf{u}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}\hat{:}_{\mathbf{v}}0\hat{=}0, \tag{2.22$_1$}$$

$$V(\bigwedge_{\mathbf{v}}\bigvee_{\mathbf{u}}\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}1\hat{-}\hat{:}_{\mathbf{v}}[1\hat{-}\hat{:}_{\mathbf{u}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])]\hat{=}1\hat{-}\hat{:}_{\mathbf{v}}1\hat{=}\hat{:}_{\mathbf{v}}0\hat{=}0, \tag{2.22$_2$}$$

$$V(\bigvee_{\mathbf{u}}\bigvee_{\mathbf{v}}\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}\hat{:}_{\mathbf{u}}\hat{:}_{\mathbf{v}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}\hat{:}_{\mathbf{v}}\hat{:}_{\mathbf{u}}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{=}\hat{:}_{\mathbf{v}}0\hat{=}0. \tag{2.22$_3$}$$

However, identity (2.22$_3$) hides the fundamental difference between $\bigvee_{\mathbf{v}}\neg[\mathbf{u}\subseteq\mathbf{v}]$ and $\bigvee_{\mathbf{u}}\neg[\mathbf{u}\subseteq\mathbf{v}]$ (cf. (2.12$_3$)).•

Cmt 2.8. a) It follows from the master-theorems (II.8.12) and (II.8.14) that if

$$V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}0, \tag{2.23}$$

i.e. if $\mathbf{P}\langle\mathbf{x}\rangle$ is an *valid common ER* (*common euautographic kyrology*), then

$$\begin{aligned}V(\bar{\bigvee}^{1}_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle)&\hat{=}1\hat{-}\hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}[1\hat{-}1\hat{\cdot}1\hat{\cdot}V(\mathbf{x}=\mathbf{y})]\hat{=}1\hat{-}\hat{:}_{\mathbf{x}}\hat{:}_{\mathbf{y}}V(\neg[\mathbf{x}=\mathbf{y}])\\&\hat{=}1\hat{-}\hat{:}_{\mathbf{x}}[\hat{:}_{\mathbf{y}}V(\neg[\mathbf{x}=\mathbf{y}])]\hat{=}1\hat{-}\hat{:}_{\mathbf{x}}0\hat{=}1\hat{-}0\hat{=}1,\end{aligned} \tag{2.24}$$

$$\begin{aligned}V(\bigvee^{1}_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle)&\hat{=}V(\bar{\bigvee}^{1}_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle)\hat{+}V(\bigvee_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle)\\&\hat{=}1\hat{+}\hat{:}_{\mathbf{w}}V(\mathbf{P}\langle\mathbf{w}\rangle)\hat{=}1\hat{+}\hat{:}_{\mathbf{w}}0\hat{=}1\hat{+}0\hat{=}1.\end{aligned} \tag{2.25}$$

In developing (2.24) and (2.25), use of the following premises and of the following rules of inference has been made. From the instance of (II.4.36) with '1', '**x**', and '**y**' or '**w**' in place of '*m*', '$\mathbf{x}_1$', and '$\mathbf{y}_1$' respectively, it follows that the identity (2.23) holds with '**y**' or '**w**' in place of '**x**'. Consequently, the identity (II.8.12) has been particularized by (2.23) and by the variant of (2.23) with '**y**' in place of '**x**' as indicated in (2.24) and then use of the variant of (2.21_{+}) with **x** and **y** as **u** and **v** respectively and also use of the first identity (II.4.24$\gamma\iota_1$) have been made in that order. Making use of (2.24) and of the variants of the identity (II.4.23) and of the first identity (4.24$\gamma\iota_1$) with **w** as **x**, the identity (II.8.14) can be developed as indicated in (2.25). Thus, both $\overset{\frown}{\bigvee}{}^1_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle$ and $\bigvee^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle$ are antikyrologies.

b) According to Th II.8.9, it follows from the master-theorems (II.8.12) and (II.8.14) (see also (2.15) and (2.16)) that if

$$V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}1, \tag{2.26}$$

i.e. if $\mathbf{P}\langle\mathbf{x}\rangle$ is an *antivalid common ER* (*common euautographic antikyrology*), then

$$V\left(\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{=}0, \tag{2.27}$$

$$V\left(\bigvee^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right)\hat{=}1, \tag{2.28}$$

i.e. $\overset{\frown}{\bigvee}{}^1_{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle$ is a kyrology and $\bigvee^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle$ is an antikyrology, *independent of the sign* =.

c) If

$$V(\mathbf{P}\langle\mathbf{x}\rangle)\hat{=}\mathbf{i}_{\sim}|\mathbf{P}\langle\mathbf{x}\rangle\rangle, \tag{2.29}$$

i.e. if $\mathbf{P}\langle\mathbf{x}\rangle$ is a *vav-neutral common ER* (*common euautographic udeterology*), then (II.8.12) and (II.8.14) become:

$$V\left(\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{=}1\hat{-}\hat{\cdot}_{\mathbf{x}}\hat{\cdot}_{\mathbf{y}}\left[1\hat{-}\mathbf{i}_{\sim}|\neg\mathbf{P}\langle\mathbf{x}\rangle\rangle\hat{\cdot}\mathbf{i}_{\sim}|\neg\mathbf{P}\langle\mathbf{y}\rangle\rangle\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right], \tag{2.30}$$

$$\begin{aligned} V\left(\bigvee^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle\right)&\hat{=}V\left(\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle\right)\hat{+}V\left(\bigvee_{\mathbf{w}}\mathbf{P}\langle\mathbf{w}\rangle\right)\\ &\hat{=}1\hat{-}\hat{\cdot}_{\mathbf{x}}\hat{\cdot}_{\mathbf{y}}\left[1\hat{-}\mathbf{i}_{\sim}|\neg\mathbf{P}\langle\mathbf{x}\rangle\rangle\hat{\cdot}\mathbf{i}_{\sim}|\neg\mathbf{P}\langle\mathbf{y}\rangle\rangle\hat{\cdot}V(\mathbf{x}=\mathbf{y})\right]\hat{+}\hat{\cdot}_{\mathbf{w}}\mathbf{i}_{\sim}|\mathbf{P}\langle\mathbf{w}\rangle\rangle. \end{aligned} \tag{2.31}$$

Thus, $\overset{\frown}{\bigvee}{}^1_{\mathbf{z}}\mathbf{P}\langle\mathbf{z}\rangle$ and $\bigvee^1_{\mathbf{v}}\mathbf{P}\langle\mathbf{v}\rangle$ are euautographic udeterologies, so that their CFCL interpretands can be *veracious* (*accidentally true*), *antiveracious* (*accidentally antitrue*, or *vravr-neutral*, i.e. *neither veracious nor anti-veracious.*•

2.3. Miscellaneous theorems

Lemma 2.3.

$$\begin{aligned} &V\left(\bigvee_y [[x \in z] \wedge [y \in z]]\right) \hat{=} \overset{\frown}{\bullet}_y V([x \in z] \wedge [y \in z]) \\ &\hat{=} \overset{\frown}{\bullet}_y [1 \hat{-} V(\neg[x \in z]) \hat{\cdot} V(\neg[y \in z])] \hat{=} V(x \in z). \end{aligned} \tag{2.32}$$

$$\begin{aligned} &V\left(\bigvee_x [[x \in z] \wedge [y \in z]]\right) \hat{=} \overset{\frown}{\bullet}_x V([x \in z] \wedge [y \in z]) \\ &\hat{=} \overset{\frown}{\bullet}_x [1 \hat{-} V(\neg[x \in z]) \hat{\cdot} V(\neg[y \in z])] \hat{=} V(y \in z), \end{aligned} \tag{2.33}$$

Proof: Making use of the pertinent instance of the Emission Law (II.4.27) with substitution of x for y in the emitted term yields:

$$\begin{aligned} &\overset{\frown}{\bullet}_y [1 \hat{-} V(\neg[x \in z]) \hat{\cdot} V(\neg[y \in z])] \\ &\hat{=} [1 \hat{-} V(\neg[x \in z]) \hat{\cdot} V(\neg[x \in z])] \hat{\cdot} \overset{\frown}{\bullet}_y [1 \hat{-} V(\neg[x \in z]) \hat{\cdot} V(\neg[y \in z])] \\ &\hat{=} V(x \in z) \hat{\cdot} \overset{\frown}{\bullet}_y [1 \hat{-} V(\neg[x \in z]) \hat{\cdot} V(\neg[y \in z])] \\ &\hat{=} \overset{\frown}{\bullet}_y [V(x \in z) \hat{\cdot} [1 \hat{-} V(\neg[x \in z]) \hat{\cdot} V(\neg[y \in z])]] \hat{=} V(x \in z), \end{aligned} \tag{2.32$_1$}$$

where use of the following identities has also been made:

$$1 \hat{-} V(\neg[x \in z]) \hat{\cdot} V(\neg[x \in z]) \hat{=} 1 \hat{-} V(\neg[x \in z]) \hat{=} V(x \in z), \tag{2.32$_2$}$$

$$V(x \in z) \hat{\cdot} V(\neg[x \in z]) \hat{=} 0. \tag{2.32$_3$}$$

Consequently,

$$\begin{aligned} &\overset{\frown}{\bullet}_x [1 \hat{-} V(\neg[x \in z]) \hat{\cdot} V(\neg[y \in z])] \\ &\hat{=} \overset{\frown}{\bullet}_x [1 \hat{-} V(\neg[y \in z]) \hat{\cdot} V(\neg[x \in z])] \hat{=} V(y \in z) \end{aligned} \tag{2.33$_1$}$$

because the next to last term is this train of identities is the variant of the next to last term in the train (2.32$_1$) with x and y exchanged. QED.•

Cmt 2.9. 1) Identities (2.32) and (2.33) are *simplification laws* for the relations $\bigvee_x [[x \in z] \wedge [y \in z]]$ and $\bigvee_y [[x \in z] \wedge [y \in z]]$, which prove that these relations are *udeterologies* and which are useful in proving some theorems involving either of these relations as a constituent part.

2) In contrast to (2.32) and (2.33), application of the pertinent instance of the Emission Law (II.4.27) in the train of identities:

$$\begin{aligned} &V\left(\bigvee_z [[x \in z] \wedge [y \in z]]\right) \hat{=} \overset{\frown}{\bullet}_z V([x \in z] \wedge [y \in z]) \\ &\hat{=} \overset{\frown}{\bullet}_z [1 \hat{-} V(\neg[x \in z]) \hat{\cdot} V(\neg[y \in z])] \end{aligned} \tag{2.34}$$

with substitution either of x or of y for z in the emitted term leaves that expression unchanged. For instance,

$$
\begin{aligned}
&\hat{\cdot}_z\left[1\hat{-}V(\neg[x\in z])\hat{\cdot}V(\neg[y\in z])\right]\\
&\hat{=}\left[1\hat{-}V(\neg[x\in x])\hat{\cdot}V(\neg[y\in z])\right]\hat{\cdot}\hat{\cdot}_z\left[1\hat{-}V(\neg[x\in z])\hat{\cdot}V(\neg[y\in z])\right]\\
&\hat{=}\left[1\hat{-}0\right]\hat{\cdot}\hat{\cdot}_z\left[1\hat{-}V(\neg[x\in z])\hat{\cdot}V(\neg[y\in z])\right]\\
&\hat{=}\hat{\cdot}_z\left[1\hat{-}V(\neg[x\in z])\hat{\cdot}V(\neg[y\in z])\right],
\end{aligned}
\tag{2.34$_1$}
$$

where use of (2.5) has been made. Hence, the relation $\bigvee_z[[x\in z]\wedge[y\in z]]$ is also an udeterology.•

Lemma 2.4.

$$
\begin{aligned}
&V\left(\neg\bigwedge_y[[x\in z]\wedge[y\in z]]\right)\hat{=}1\hat{-}V\left(\bigwedge_y[[x\in z]\wedge[y\in z]]\right)\\
&\hat{=}\hat{\cdot}_yV(\neg[[x\in z]\wedge[y\in z]])\hat{=}\hat{\cdot}_y\left[V(\neg[x\in z])\hat{\cdot}V(\neg[y\in z])\right]\\
&\hat{=}V(\neg[x\in z])\hat{\cdot}\hat{\cdot}_yV(\neg[y\in z])\hat{=}V(\neg[x\in z])\hat{\cdot}0\hat{=}0.
\end{aligned}
\tag{2.35}
$$

$$
\begin{aligned}
&V\left(\neg\bigwedge_x[[x\in z]\wedge[y\in z]]\right)\hat{=}1\hat{-}V\left(\bigwedge_x[[x\in z]\wedge[y\in z]]\right)\\
&\hat{=}\hat{\cdot}_xV(\neg[[x\in z]\wedge[y\in z]])\hat{=}\hat{\cdot}_x\left[V(\neg[x\in z])\hat{\cdot}V(\neg[y\in z])\right]\\
&\hat{=}V(\neg[y\in z])\hat{\cdot}\hat{\cdot}_xV(\neg[x\in z])\hat{=}V(\neg[y\in z])\hat{\cdot}0\hat{=}0.
\end{aligned}
\tag{2.36}
$$

Proof: In developing the final result in each of the above two trains of identities, use of the pertinent instance of the Idleness Law (II.4.24) and use of the pertinent variant of (2.5) in this order have been made.•

Cmt 2.10. By theorem (II.7.50), identities (2.32) and(2.33) are tantamount to the following logical kyrologies:

$$\bigvee_y[[x\in z]\wedge[y\in z]]\Leftrightarrow[x\in z], \tag{2.32a}$$

$$\bigvee_x[[x\in z]\wedge[y\in z]]\Leftrightarrow[y\in z], \tag{2.33a}$$

whereas by axiom (II.4.40a), identities (2.35) and (2.36) are tantamount to the following logical kyrologies:

$$\neg\bigwedge_y[x\in z]\wedge[y\in z], \tag{2.35a}$$

$$\neg\bigwedge_x[x\in z]\wedge[y\in z]. \tag{2.36a}$$

Cmt 2.11. Lemmas 2.3 and 2.4 allow proving various useful theorems. Here follow some simple examples.

1) Making use of (2.32) and (2.4) in this order yields:

$$V\left(\bigvee_z\bigvee_y[[x\in z]\wedge[y\in z]]\right)\hat{=}\hat{\cdot}_z\hat{\cdot}_yV([x\in z]\wedge[y\in z])\hat{=}\hat{\cdot}_zV(x\in z)\hat{=}0, \tag{2.37}$$

$$
\begin{aligned}
&V\left(\bigvee_y\bigvee_z[[x\in z]\wedge[y\in z]]\right)\hat{=}\hat{\cdot}_y\hat{\cdot}_zV([x\in z]\wedge[y\in z])\\
&\hat{=}\hat{\cdot}_z\hat{\cdot}_yV([x\in z]\wedge[y\in z])\hat{=}\hat{\cdot}_zV(x\in z)\hat{=}0,
\end{aligned}
\tag{2.38}
$$

and similarly with $\bigvee_x$ and $\hat{\cdot}_x$ in place of $\bigvee_y$ and $\hat{\cdot}_y$, and hence with y in place of x in the final identity. Consequently,

$$\begin{gathered} V\left(\bigvee_x \bigvee_y \bigvee_z [[x \in z] \wedge [y \in z]]\right) \\ \hat{=} \hat{\cdot}_x V\left(\bigvee_y \bigvee_z [[x \in z] \wedge [y \in z]]\right) \hat{=} \hat{\cdot}_x 0 \hat{=} 0, \end{gathered} \tag{2.39}$$

$$\begin{gathered} V\left(\bigwedge_x \bigvee_y \bigvee_z [[x \in z] \wedge [y \in z]]\right) \\ \hat{=} 1 \hat{-} \hat{\cdot}_x \left[1 \hat{-} V\left(\bigvee_y \bigvee_z [[x \in z] \wedge [y \in z]]\right)\right] \hat{=} 1 \hat{-} \hat{\cdot}_x 1 \hat{=} 1 \hat{-} 1 \hat{=} 0; \end{gathered} \tag{2.40}$$

(2.39) holds under any permutation of the pseudo-quantifiers $\bigvee_x$, $\bigvee_y$, $\bigvee_z$.

2) Making use of (2.35) yields:

$$\begin{gathered} V\left(\neg \bigwedge_x \bigwedge_y [[x \in z] \wedge [y \in z]]\right) \hat{=} 1 \hat{-} V\left(\bigwedge_x \bigwedge_y [[x \in z] \wedge [y \in z]]\right) \\ \hat{=} \hat{\cdot}_x V\left(\neg \bigwedge_y [[x \in z] \wedge [y \in z]]\right) \hat{=} \hat{\cdot}_x 0 \hat{=} 0. \end{gathered} \tag{2.41}$$

Hence, by (2.41),

$$\begin{gathered} V\left(\neg \bigvee_z \bigwedge_x \bigwedge_y [[x \in z] \wedge [y \in z]]\right) \\ \hat{=} 1 \hat{-} V\left(\bigvee_z \bigwedge_x \bigwedge_y [[x \in z] \wedge [y \in z]]\right) \\ \hat{=} 1 \hat{-} \hat{\cdot}_z V\left(\bigwedge_x \bigwedge_y [[x \in z] \wedge [y \in z]]\right) \hat{=} 1 \hat{-} \hat{\cdot}_z 1 \hat{=} 1 \hat{-} 1 \hat{=} 0. \end{gathered} \tag{2.42}$$

Taking into account that $\neg \bigwedge_{\mathbf{x}}$ and $\bigvee_{\mathbf{x}} \neg$ can, in accordance with the definition $\bigwedge_{\mathbf{x}} \rightarrow \neg \bigvee_{\mathbf{x}} \neg$ (see (II.8.2)), be used interchangeably, a great many of other identities can be inferred with the help of Lemmas 2.3 and 2.4.

The most important property of the identities (2.32), (2.33), (2.35), and (2.36) is that they can be incorporated into the respective *recursive schemata* of identities, which are made explicit in the next subsection. Still, in order to develop these schemata with complete rigor, use of the method of mathematical induction should be made. This method is not, however, among the rules of inference of A_1. Also, the schemata in question turn out to be cumbersome somewhat. Therefore, the reader may omit the next subsection without prejudice to understanding the subsequent matter. •

2.4. Recursive theorem schemata of $\mathsf{A}_{1 \in \mathrm{D}}$

By (II.7.6γ), it follows that

$$V(\mathbf{P}_1 \wedge \mathbf{P}_2) \hat{=} 1 \hat{-} V(\neg \mathbf{P}_1) \hat{\cdot} V(\neg \mathbf{P}_2) \hat{=} 1 \hat{-} V(\neg \mathbf{P}_2) \hat{\cdot} V(\neg \mathbf{P}_1) \hat{=} V(\mathbf{P}_2 \wedge \mathbf{P}_1), \tag{2.43}$$

$$\begin{gathered} V(\mathbf{P}_1 \wedge \mathbf{P}_2 \wedge \mathbf{P}_3) \hat{=} V(\mathbf{P}_1 \wedge [\mathbf{P}_2 \wedge \mathbf{P}_3]) \hat{=} 1 \hat{-} V(\neg \mathbf{P}_1) \hat{\cdot} V(\neg [\mathbf{P}_2 \wedge \mathbf{P}_3]) \\ \hat{=} 1 \hat{-} V(\neg \mathbf{P}_1) \hat{\cdot} [1 \hat{-} V(\mathbf{P}_2 \wedge \mathbf{P}_3)] \hat{=} 1 \hat{-} V(\neg \mathbf{P}_1) \hat{\cdot} V(\neg \mathbf{P}_2) \hat{\cdot} V(\neg \mathbf{P}_3), \end{gathered} \tag{2.44}$$

etc, so that in general

$$V\left(\mathbf{P}_1 \wedge \mathbf{P}_2 \wedge \ldots \wedge \mathbf{P}_n\right) \hat{=} 1 \hat{-} V\left(\neg \mathbf{P}_1\right) \hat{\cdot} V\left(\neg \mathbf{P}_2\right) \hat{\cdot} \ldots \hat{\cdot} V\left(\neg \mathbf{P}_n\right). \tag{2.45}$$

where '$\mathbf{P}_n$' is a meta-syntactic placeholder whose immediate range is the set of the syntactic relation-valued placeholders '$\mathbf{P}_1$', '$\mathbf{P}_2$', '$\mathbf{P}_3$', etc; i.e. **'$\mathbf{P}_n$'** *is* any of latter placeholders. Consequently,

$$\begin{gathered} V\left(\left[x_1 \in z\right] \wedge \left[x_2 \in z\right] \wedge \ldots \wedge \left[x_n \in z\right]\right) \\ \hat{=} 1 \hat{-} V\left(\neg\left[x_1 \in z\right]\right) \hat{\cdot} V\left(\neg\left[x_2 \in z\right]\right) \hat{\cdot} \ldots \hat{\cdot} V\left(\neg\left[x_n \in z\right]\right), \end{gathered} \tag{2.46}$$

where 'x_n' is a metalogographic placeholder (MLPH) whose range is the set of x_1, x_2, ...

Making use of (2.46), the trains of identities (2.32) and (2.35), e.g., can be generalized thus.

$$\begin{gathered} V\left(\bigvee_{x_2} \bigvee_{x_3} \cdots \bigvee_{x_n} \left[\left[x_1 \in z\right] \wedge \left[x_2 \in z\right] \wedge \ldots \wedge \left[x_n \in z\right]\right]\right) \\ \hat{=} \hat{\boldsymbol{\cdot}}_{x_2} \hat{\boldsymbol{\cdot}}_{x_3} \ldots \hat{\boldsymbol{\cdot}}_{x_n} \left[1 \hat{-} V\left(\neg\left[x_1 \in z\right]\right) \hat{\cdot} V\left(\neg\left[x_2 \in z\right]\right) \hat{\cdot} \ldots \hat{\cdot} V\left(\neg\left[x_n \in z\right]\right)\right] \\ \hat{=} \left[1 \hat{-} V\left(\neg\left[x_1 \in z\right]\right) \hat{\cdot} V\left(\neg\left[x_1 \in z\right]\right) \hat{\cdot} \ldots \hat{\cdot} V\left(\neg\left[x_1 \in z\right]\right)\right] \\ \hat{\cdot} \hat{\boldsymbol{\cdot}}_{x_2} \hat{\boldsymbol{\cdot}}_{x_3} \ldots \hat{\boldsymbol{\cdot}}_{x_n} \left[1 \hat{-} V\left(\neg\left[x_1 \in z\right]\right) \hat{\cdot} V\left(\neg\left[x_2 \in z\right]\right) \hat{\cdot} \ldots \hat{\cdot} V\left(\neg\left[x_n \in z\right]\right)\right] \\ \hat{=} V\left(x_1 \in z\right) \hat{\cdot} \hat{\boldsymbol{\cdot}}_{x_2} \hat{\boldsymbol{\cdot}}_{x_3} \ldots \hat{\boldsymbol{\cdot}}_{x_n} \left[1 \hat{-} V\left(\neg\left[x_1 \in z\right]\right)\right. \\ \left. \hat{\cdot} V\left(\neg\left[x_2 \in z\right]\right) \hat{\cdot} \ldots \hat{\cdot} V\left(\neg\left[x_n \in z\right]\right)\right] \hat{=} V\left(x_1 \in z\right). \end{gathered} \tag{2.47}$$

$$\begin{gathered} V\left(\neg \bigwedge_{x_n} \left[\left[x_1 \in z\right] \wedge \left[x_2 \in z\right] \wedge \ldots \wedge \left[x_n \in z\right]\right]\right) \\ \hat{=} 1 \hat{-} V\left(\bigwedge_{x_n} \left[\left[x_1 \in z\right] \wedge \left[x_2 \in z\right] \wedge \ldots \wedge \left[x_n \in z\right]\right]\right) \\ \hat{=} \hat{\boldsymbol{\cdot}}_{x_n} V\left(\neg\left[\left[x_1 \in z\right] \wedge \left[x_2 \in z\right] \wedge \ldots \wedge \left[x_n \in z\right]\right]\right) \\ \hat{=} \hat{\boldsymbol{\cdot}}_{x_n} \left[V\left(\neg\left[x_1 \in z\right]\right) \hat{\cdot} V\left(\neg\left[x_2 \in z\right]\right) \hat{\cdot} \ldots \hat{\cdot} V\left(\neg\left[x_n \in z\right]\right)\right] \\ \hat{=} \left[V\left(\neg\left[x_1 \in z\right]\right) \hat{\cdot} V\left(\neg\left[x_2 \in z\right]\right) \hat{\cdot} \ldots \hat{\cdot} V\left(\neg\left[x_{n-1} \in z\right]\right)\right] \hat{\cdot} \hat{\boldsymbol{\cdot}}_{x_n} V\left(\neg\left[x_n \in z\right]\right) \\ \hat{=} \left[V\left(\neg\left[x_1 \in z\right]\right) \hat{\cdot} V\left(\neg\left[x_2 \in z\right]\right) \hat{\cdot} \ldots \hat{\cdot} V\left(\neg\left[x_{n-1} \in z\right]\right)\right] \hat{\cdot} 0 \hat{=} 0. \end{gathered} \tag{2.48}$$

In order to develop the final result in (2.47), I have first made use the Emission Law (II.4.27) *n*–1 times with substitutions of x_1 for each x_n, x_{n-1}, ..., x_3, x_2 in sequence in the emitted term, and then I have utilized the following two trains of self-evident transformations, which are analogous to (2.32_2) and (2.32_3):

$$\begin{gathered} 1 \hat{-} V\left(\neg\left[x_1 \in z\right]\right) \hat{\cdot} V\left(\neg\left[x_1 \in z\right]\right) \hat{\cdot} \ldots \hat{\cdot} V\left(\neg\left[x_1 \in z\right]\right) \\ \hat{=} 1 \hat{-} V\left(\neg\left[x_1 \in z\right]\right) \hat{=} V\left(x_1 \in z\right), \end{gathered} \tag{2.49}$$

$$\begin{aligned} &V(x_1 \in z) \hat{\cdot} \hat{\bullet}_{x_2} \hat{\bullet}_{x_3} \ldots \hat{\bullet}_{x_n} [1 \hat{-} V(\neg[x_1 \in z]) \\ &\quad \hat{\cdot} V(\neg[x_2 \in z]) \hat{\cdot} \ldots \hat{\cdot} V(\neg[x_n \in z])] \\ &\hat{=} \hat{\bullet}_{x_2} \hat{\bullet}_{x_3} \ldots \hat{\bullet}_{x_n} [V(x_1 \in z) \hat{-} [V(x_1 \in z) \hat{\cdot} V(\neg[x_1 \in z])] \\ &\quad \hat{\cdot} V(\neg[x_2 \in z]) \hat{\cdot} \ldots \hat{\cdot} V(\neg[x_n \in z])] \\ &\hat{=} \hat{\bullet}_{x_2} \hat{\bullet}_{x_3} \ldots \hat{\bullet}_{x_n} V(x_1 \in z) \hat{=} V(x_1 \in z). \end{aligned} \tag{2.50}$$

Incidentally, At $n \triangleright 2$, the train (2.47) reduces to:

$$\begin{aligned} &V\left(\bigvee\nolimits_{x_2} [[x_1 \in z] \wedge [x_2 \in z]]\right) \\ &\hat{=} \hat{\bullet}_{x_2} [1 \hat{-} V(\neg[x_1 \in z]) \hat{\cdot} V(\neg[x_2 \in z])] \hat{=} V(x_1 \in z), \end{aligned} \tag{2.47$_1$}$$

which is the variant of (2.32) with x_1 and x_2 in place of x and y respectively. In developing the final result in (2.48), I have made use first of the pertinent instance of the Transparency Law (II.4.24) for the algebraic contractor $\hat{\bullet}_{x_n}$ with respect to the factor $[V(\neg[x_1 \in z]) \hat{\cdot} V(\neg[x_2 \in z]) \hat{\cdot} \ldots \hat{\cdot} V(\neg[x_{n-1} \in z])]$ and then I have made use of the identity:

$$\hat{\bullet}_{x_n} V(\neg[x_n \in z]) \hat{=} 0, \tag{2.51}$$

which is the schematic variant of (2.7) with x_n in place of x.

Making use of (2.47) and (2.4) in this order yields:

$$\begin{aligned} &V\left(\bigvee\nolimits_{x_2} \cdots \bigvee\nolimits_{x_n} \bigvee\nolimits_{z} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]\right) \\ &\hat{=} V\left(\bigvee\nolimits_{z} \bigvee\nolimits_{x_2} \cdots \bigvee\nolimits_{x_n} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]\right) \\ &\hat{=} \hat{\bullet}_{x_2} \ldots \hat{\bullet}_{x_n} \hat{\bullet}_{z} [1 \hat{-} V(\neg[x_1 \in z]) \hat{\cdot} V(\neg[x_2 \in z]) \hat{\cdot} \ldots \hat{\cdot} V(\neg[x_n \in z])] \\ &\hat{=} \hat{\bullet}_{z} \hat{\bullet}_{x_2} \ldots \hat{\bullet}_{x_n} [1 \hat{-} V(\neg[x_1 \in z]) \hat{\cdot} V(\neg[x_2 \in z]) \hat{\cdot} \ldots \hat{\cdot} V(\neg[x_n \in z])] \\ &\hat{=} \hat{\bullet}_{z} V(x_1 \in z) \hat{=} 0, \end{aligned} \tag{2.52}$$

which is a generalization of (2.37) and (2.38). Hence, by (2.52),

$$\begin{aligned} &V\left(\bigvee\nolimits_{x_1} \bigvee\nolimits_{x_2} \cdots \bigvee\nolimits_{x_n} \bigvee\nolimits_{z} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]\right) \\ &\hat{=} V\left(\bigvee\nolimits_{z} \bigvee\nolimits_{x_1} \bigvee\nolimits_{x_2} \cdots \bigvee\nolimits_{x_n} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]\right) \\ &\hat{=} V\left(\bigvee\nolimits_{x_1} \bigvee\nolimits_{z} \bigvee\nolimits_{x_2} \cdots \bigvee\nolimits_{x_n} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]\right) \\ &\hat{=} \hat{\bullet}_{x_1} \hat{\bullet}_{x_2} \ldots \hat{\bullet}_{x_n} \hat{\bullet}_{z} [1 \hat{-} V(\neg[x_1 \in z]) \hat{\cdot} V(\neg[x_2 \in z]) \hat{\cdot} \ldots \hat{\cdot} V(\neg[x_n \in z])] \\ &\hat{=} \hat{\bullet}_{z} \hat{\bullet}_{x_1} \hat{\bullet}_{x_2} \ldots \hat{\bullet}_{x_n} [1 \hat{-} V(\neg[x_1 \in z]) \hat{\cdot} V(\neg[x_2 \in z]) \hat{\cdot} \ldots \hat{\cdot} V(\neg[x_n \in z])] \\ &\hat{=} \hat{\bullet}_{x_1} \hat{\bullet}_{z} \hat{\bullet}_{x_2} \ldots \hat{\bullet}_{x_n} [1 \hat{-} V(\neg[x_1 \in z]) \hat{\cdot} V(\neg[x_2 \in z]) \hat{\cdot} \ldots \hat{\cdot} V(\neg[x_n \in z])] \\ &\hat{=} \hat{\bullet}_{x_1} 0 \hat{=} 0, \end{aligned} \tag{2.53}$$

$$\begin{aligned}
&V\left(\bigwedge_{x_1}\bigvee_{z}\bigvee_{x_2}\cdots\bigvee_{x_n}\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\right)\\
&\hat{=}\,1\,\hat{-}\,\hat{\cdot}_{x_1}\left[1\,\hat{-}\,\hat{\cdot}_{z}\left[1\,\hat{-}\,V\left(\bigvee_{x_2}\cdots\bigvee_{x_n}\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\right)\right]\right]\\
&\hat{=}\,1\,\hat{-}\,\hat{\cdot}_{x_1}\left[1\,\hat{-}\,\hat{\cdot}_{z}V\left(x_1\in z\right)\right]\hat{=}\,1\,\hat{-}\,\hat{\cdot}_{x_1}\left[1\,\hat{-}\,\hat{\cdot}_{z}0\right]\hat{=}\,1\,\hat{-}\,1\,\hat{=}\,0,
\end{aligned}\qquad(2.54)$$

which are generalizations of (2.39) and (2.40) respectively. It goes without saying that (2.52) holds under any permutation of the pseudo-quantifiers (logical contractors) $\bigvee_{z}$, $\bigvee_{x_2}$, …, $\bigvee_{x_n}$ or of the pseudo-multipliers (algebraic contractors) $\hat{\cdot}_{z}$, $\hat{\cdot}_{x_2}$, …, $\hat{\cdot}_{x_n}$ and that hence (2.53) holds under any permutation of the pseudo-quantifiers $\bigvee_{z}$, $\bigvee_{x_1}$, $\bigvee_{x_2}$, …, $\bigvee_{x_n}$ or of the pseudo-multipliers $\hat{\cdot}_{z}$, $\hat{\cdot}_{x_1}$, $\hat{\cdot}_{x_2}$, …, $\hat{\cdot}_{x_n}$.

Making repeatedly use of *n*–1 pertinent instances of (II.8.2) and then making use of *n*–1 variants of (2.7) with x_2, x_3, …, or x_n in place of x yield:

$$\begin{aligned}
&V\left(\neg\bigwedge_{x_2}\bigwedge_{x_3}\cdots\bigwedge_{x_n}\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\right)\\
&\hat{=}\,\hat{\cdot}_{x_2}V\left(\neg\bigwedge_{x_2}\bigwedge_{x_3}\cdots\bigwedge_{x_n}\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\right)\\
&\hat{=}\ldots\hat{=}\,\hat{\cdot}_{x_2}\hat{\cdot}_{x_2}\ldots\hat{\cdot}_{x_n}V\left(\neg\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\right)\\
&\hat{=}\,\hat{\cdot}_{x_2}\hat{\cdot}_{x_2}\ldots\hat{\cdot}_{x_n}\left[V\left(\neg\left[x_1\in z\right]\right)\hat{\cdot}V\left(\neg\left[x_2\in z\right]\right)\hat{\cdot}\ldots\hat{\cdot}V\left(\neg\left[x_n\in z\right]\right)\right]\\
&\hat{=}\,V\left(\neg\left[x_1\in z\right]\right)\hat{\cdot}\left[\hat{\cdot}_{x_2}V\left(\neg\left[x_2\in z\right]\right)\right]\hat{\cdot}\ldots\hat{\cdot}\left[\hat{\cdot}_{x_n}V\left(\neg\left[x_n\in z\right]\right)\right]\\
&\hat{=}\,V\left(\neg\left[x_1\in z\right]\right)\hat{\cdot}\left[0\,\hat{\cdot}\ldots\hat{\cdot}\,0\right]\hat{=}\,V\left(\neg\left[x_1\in z\right]\right)\hat{\cdot}\,0\,\hat{=}\,0,
\end{aligned}\qquad(2.55)$$

Hence,

$$\begin{aligned}
&V\left(\neg\bigwedge_{x_1}\bigwedge_{x_2}\cdots\bigwedge_{x_n}\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\right)\\
&\hat{=}\,\hat{\cdot}_{x_1}V\left(\neg\bigwedge_{x_2}\bigwedge_{x_3}\cdots\bigwedge_{x_n}\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\right)\\
&\hat{=}\,\hat{\cdot}_{x_1}0\,\hat{=}\,0,
\end{aligned}\qquad(2.56)$$

which is a generalization of (2.41). Consequently,

$$\begin{aligned}
&V\left(\neg\bigvee_{z}\bigwedge_{x_1}\bigwedge_{x_2}\cdots\bigwedge_{x_n}\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\right)\\
&\hat{=}\,1\,\hat{-}\,V\left(\bigvee_{z}\bigwedge_{x_1}\bigwedge_{x_2}\cdots\bigwedge_{x_n}\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\right)\\
&\hat{=}\,1\,\hat{-}\,\hat{\cdot}_{z}V\left(\bigwedge_{x_1}\bigwedge_{x_2}\cdots\bigwedge_{x_n}\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\right)\\
&\hat{=}\,1\,\hat{-}\,\hat{\cdot}_{z}1\,\hat{=}\,0,
\end{aligned}\qquad(2.57)$$

which is a generalization of (2.42).

By (II.7.50), identity (2.47) is tantamount to the following logical kyrology:

$$\bigvee_{x_2}\bigvee_{x_3}\cdots\bigvee_{x_n}\left[\left[x_1\in z\right]\wedge\left[x_2\in z\right]\wedge\ldots\wedge\left[x_n\in z\right]\right]\Leftrightarrow\left[x_1\in z\right],\qquad(2.47a)$$

whereas by (II.7.50), identities (2.48) and (2.52)–(2.57) are tantamount to the following ones:

$$\neg \bigwedge_{x_n} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]. \tag{2.48a}$$

$$\bigvee_z \bigvee_{x_2} \cdots \bigvee_{x_n} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]. \tag{2.52a}$$

$$\bigvee_{x_1} \bigvee_z \bigvee_{x_2} \cdots \bigvee_{x_n} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]. \tag{2.53a}$$

$$\bigvee_{x_1} \bigwedge_z \bigvee_{x_2} \cdots \bigvee_{x_n} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]. \tag{2.54a}$$

$$\neg \bigwedge_{x_2} \bigwedge_{x_3} \cdots \bigwedge_{x_n} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]. \tag{2.55a}$$

$$\neg \bigwedge_{x_1} \bigwedge_{x_2} \cdots \bigwedge_{x_n} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]. \tag{2.56a}$$

$$\neg \bigvee_z \bigwedge_{x_1} \bigwedge_{x_2} \cdots \bigwedge_{x_n} [[x_1 \in z] \wedge [x_2 \in z] \wedge \ldots \wedge [x_n \in z]]. \tag{2.57a}$$

Intuitively, kyrology (2.52a), e.g., means via its CFCL interpretand that there is a class (set) that contains at least *n*–1 elements, the understanding being that *n* is not bounded from above. That is to say, there is an *infinite set*.•

2.5. Some catlogographic axioms of $A_{1\in S}$

Preliminary Remark 2.1. 1) The calculus $A_{1\in}$ is uninterpreted logical calculus, while set theory, e.g., is an interpreted mathematical theory. Therefore, expressive means of $A_{1\in}$ are not sufficient for stating a complete set of axioms of set theory. Axioms of $A_{1\in}$ are udeterologies, i.e. vav-neutral ER's of A_{1P}, which are taken for granted to be valid and which thus turn into kyrologies. Likewise, axioms of class, or set, theory, and generally of any branch of mathematics, are extrinsic interpretands of some udeterologies of $A_{1\in}$ – interpertands, which are taken for granted to be *veracious* (*accidentally true*). This study is not, and cannot be, a full-scale set theory, but rather it is an explication of the most fundamental principles underlying any system of logical reasoning. The main subject of the study is to demonstrate how various rules of the AEADM should be used in order to get valid relations from some other valid relations. Still, in order to avoid the danger of arriving at a contradiction, any vav-neutral relation of $A_{1\in}$, which is supposed to be turned into an axiom, should be scrutinized from the standpoint of its subsequent interpretands before letting it pass as a euautographic axiom. Relations (2.1) and (2.2) seem to satisfy this requirement.

2) Once some udeterologies (vav-neutral ER's) of $A_{1\in G}$ are laid down as subject axioms, i.e. as primary kyrologies, of $A_{1\in S}$ (e.g.), a kyrology (valid ER) of $A_{1\in D}$ that is proved from those axioms is a subject theorem of both $A_{1\in D}$ and $A_{1\in S}$ and therefore it cannot be a subject axiom of either of two organons. Moreover, if I

anticipate that a certain udeterology of $\mathsf{A}_{1\in D}$ (and hence that of $\mathsf{A}_{1\in G}$) will be a subject theorem (and hence a kyrology) of $\mathsf{A}_{1\in S}$, then it is counterproductive to lay down the former as a subject axiom of $\mathsf{A}_{1\in D}$. For instance, in $\mathsf{A}_{1\in D}$, the identity

$$V\left(\bigvee_u \bigwedge_z \neg[z \in u]\right) \hat{=} \dot{\curlyvee}_u \left[1 \hat{-} \dot{\curlyvee}_z V(z \in u)\right], \tag{2.58}$$

which is inferred straightforwardly by making use of the pertinent instances of (II.4.23), (II.8.2), and (II.7.1γ), cannot be developed further and therefore it is the EDT (euautographic decision theorem) of the ESR (euautographic slave-relation)

$$\bigvee_u \bigwedge_z \neg[z \in u], \tag{2.58a}$$

so that the latter is an udeterology of $\mathsf{A}_{1\in D}$. The CFCL interpretand

$$\bigvee_u \bigwedge_z \neg[z \in u] \tag{2.58aκ}$$

of the ER (2.58a) can be rendered into ordinary language thus: "There exists an object *u* such that for every object *z*: *z* is not an element of *u*" or briefly thus: "There exists a class or set that has no members (elements)". Since the *memberless set* (*memberless regular class*), called also the *empty set* or the *empty class* or the *empty individual* (*empty indivisible being*), is known to exist in any consistent set or class theory, therefore it seems at first glance natural to take the ER (2.58a) for granted as another axiom of $\mathsf{A}_{1\in D}$, so that (2.58) turns into

$$V\left(\bigvee_u \bigwedge_z \neg[z \in u]\right) \hat{=} \dot{\curlyvee}_u \left[1 \hat{-} \dot{\curlyvee}_z V(z \in u)\right] \hat{=} 0. \tag{2.58$_1$}$$

However, both APVOT's u and z occurring in (2.58a) and (2.58$_1$) are bound. Therefore, as an axiom, (2.58a) is *ineffective* in the sense that (2.58$_1$), being *its master-theorem*, cannot be used for proving any other theorem. At the same time, $\neg[z \in \emptyset]$ is a concrete euautographic instance of the PLR '$\neg[\mathbf{z} \in \emptyset]$', which is taken for granted as a subject axiom of $\mathbf{A}_{1\in S}$. Therefore, by assumption,

$$V(\neg[\mathbf{z} \in \emptyset]) \hat{=} 1 \hat{-} V(\mathbf{z} \in \emptyset) \hat{=} 0, \tag{2.59}$$

whence

$$V(\neg[z \in \emptyset]) \hat{=} 1 \hat{-} V(z \in \emptyset) \hat{=} 0. \tag{2.59μ}$$

Consequently, (2.58$_1$) is a theorem, which has the following proof from (2.59μ):

$$\begin{gathered} V\left(\bigvee_u \bigwedge_z \neg[z \in u]\right) \hat{=} \dot{\curlyvee}_u \left[1 \hat{-} \dot{\curlyvee}_z V(z \in u)\right] \\ \hat{=} \left[1 \hat{-} \dot{\curlyvee}_z V(z \in \emptyset)\right] \hat{\cdot} \dot{\curlyvee}_u \left[1 \hat{-} \dot{\curlyvee}_z V(z \in u)\right] \hat{=} \left[1 \hat{-} \dot{\curlyvee}_z 1\right] \hat{\cdot} \dot{\curlyvee}_u \left[1 \hat{-} \dot{\curlyvee}_z V(z \in u)\right] \\ \hat{=} 0 \hat{\cdot} \dot{\curlyvee}_u \left[1 \hat{-} \dot{\curlyvee}_z V(z \in u)\right] \hat{=} 0, \end{gathered} \tag{2.58$_2$}$$

where use has been made of the instance of the Emission Law (II.4.27) with u, $\emptyset$, and $\lfloor 1 \mathbin{\hat{-}} \hat{\cdot}_z V(z \in u) \rfloor$ as **x**, **y,** and $\mathbf{i}\langle \mathbf{x} \rangle$ respectively.

3) In what follows, I shall, by way of example, state two catlogographic axioms of $\mathcal{A}_{1\in\mathrm{D}}$ as CFCL interpretands of the two pertinent vav-neutral ER’s of $\mathsf{A}_{1\in\mathsf{D}}$. The catlogographic axioms are well-established axioms of set theory, but I present them in algebraic form in accordance with the pertinent extension $\mathcal{D}_1$ of the AEADM D_1.•

***Lemma 2.5.**

$$V\left(\bigwedge_{\mathbf{z}}\left[[\mathbf{z} \subseteq \mathbf{u}] \Leftrightarrow [\mathbf{z} \in \mathbf{w}]\right]\right) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_{\mathbf{z}} \left[1 \mathbin{\hat{-}} V\left([\mathbf{z} \subseteq \mathbf{u}] \Leftrightarrow [\mathbf{z} \in \mathbf{w}]\right)\right] \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_{\mathbf{z}} \left[1 \mathbin{\hat{-}} \left[V(\mathbf{z} \subseteq \mathbf{u}) \mathbin{\hat{-}} V(\mathbf{z} \in \mathbf{w})\right]^2\right] \tag{2.60}$$

$$V\left(\bigwedge_{\mathbf{z}}\left[[\mathbf{z} = \mathbf{v}] \vee [\mathbf{z} = \mathbf{w}]\right] \Leftrightarrow [\mathbf{z} \in \mathbf{u}]\right) \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_{\mathbf{z}} \left[1 \mathbin{\hat{-}} V\left(\left[[\mathbf{z} = \mathbf{v}] \vee [\mathbf{z} = \mathbf{w}]\right] \Leftrightarrow [\mathbf{z} \in \mathbf{u}]\right)\right] \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_{\mathbf{z}} \left[1 \mathbin{\hat{-}} \left[V\left([\mathbf{z} = \mathbf{v}] \vee [\mathbf{z} = \mathbf{w}]\right) \mathbin{\hat{-}} V(\mathbf{z} \in \mathbf{w})\right]^2\right] \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_{\mathbf{z}} \left[1 \mathbin{\hat{-}} \left[V(\mathbf{z} = \mathbf{v}) \mathbin{\hat{\cdot}} V(\mathbf{z} = \mathbf{w}) \mathbin{\hat{-}} V(\mathbf{z} \in \mathbf{w})\right]^2\right]. \tag{2.61}$$

Proof: The trains of identities (2.60) and (2.61) are inferred straightforwardly by the pertinent instances of (II.7.2γ), (II.7.6γ), (II.7.7γ), and (II.8.2).•

°**Crl 2.1.**

$$V\left(\bigwedge_z\left[[z \subseteq u] \Leftrightarrow [z \in w]\right]\right) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_z \left[1 \mathbin{\hat{-}} \left[V(z \subseteq u) \mathbin{\hat{-}} V(z \in w)\right]^2\right]. \tag{2.60μ}$$

$$V\left(\bigwedge_z\left[\left[[z = v] \vee [z = w]\right] \Leftrightarrow [z \in u]\right]\right) \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_z \left[1 \mathbin{\hat{-}} \left[V(z = v) \mathbin{\hat{\cdot}} V(z = w) \mathbin{\hat{-}} V(z \in u)\right]^2\right]. \tag{2.61μ}$$

+**Ax 2.2.**

$$V\left(\bigwedge_z\left[[z \subseteq u] \Leftrightarrow [z \in w]\right]\right) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_z \left[1 \mathbin{\hat{-}} \left[V(z \subseteq u) \mathbin{\hat{-}} V(z \in w)\right]^2\right] \mathrel{\hat{=}} 0. \tag{2.60κ}$$

$$V\left(\bigwedge_z\left[\left[[z = v] \vee [z = w]\right] \Leftrightarrow [z \in u]\right]\right) \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_z \left[1 \mathbin{\hat{-}} \left[V(z = v) \mathbin{\hat{\cdot}} V(z = w) \mathbin{\hat{-}} V(z \in u)\right]^2\right] \mathrel{\hat{=}} 0. \tag{2.61κ}$$

+**Th 2.8.**

$$V\left(\bigwedge_z\left[[z = v] \Leftrightarrow [z \in u]\right]\right) \mathrel{\hat{=}} V\left(\bigwedge_z\left[\left[[z = v] \vee [z = v]\right] \Leftrightarrow [z \in u]\right]\right) \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_z \left[1 \mathbin{\hat{-}} \left[V(z = v) \mathbin{\hat{\cdot}} V(z = v) \mathbin{\hat{-}} V(z \in u)\right]^2\right] \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} \hat{\cdot}_z \left[1 \mathbin{\hat{-}} \left[V(z = v) \mathbin{\hat{-}} V(z \in u)\right]^2\right] \mathrel{\hat{=}} 0. \tag{2.62κ}$$

Proof: (2.62κ) follows from (2.61κ) at w=v.•

Cmt 2.12. 1) The train of identities occurring in (2.60κ), or that occurring in (2.61κ), to the left of the predicate '$\hat{=}0$' is a CFCL theorem that follows from Crl 2.1 by Ax I.8.2. The predicate '$\hat{=}0$' signifies that either term of each of the two trains is taken for granted to equal 0, which imposes certain restrictions on the range of the bound catlogographic variable '*z*', while the denotata of all free variables are supposed to be given.

2) By the pertinent extension of (II.4.40a), it follows from (2.60κ)–(2.62κ) that

$$\bigwedge_z [[z \subseteq u] \Leftrightarrow [z \in w]], \tag{2.60κa}$$

$$\bigwedge_z [[[z = v] \vee [z = w]] \Leftrightarrow [z \in u]], \tag{2.61κa}$$

$$\bigwedge_z [[z = v] \Leftrightarrow [z \in u]], \tag{2.62κa}$$

i.e. that the above CLR's (catlogographic relations) are asserted, because they are taken for granted to be veracious (accidentally true) vav-neutral CLR's. It is understood that (2.60κa) is a catlogographic (semantic) *axiom of the power set w of the set u*; (2.61κa) is a catlogographic *axiom of the set u of two sets v and w*, i.e. *one of the unordered pair u of v and w*; (2.62κa) is a catlogographic *theorem of the singleton u of v*. In this case, the identities (2.60κ) and (2.61κ) should be understood as explicated in the following two items, whereas (2.61κ) can be explicated likewise.

3) Given a class *u*, let $\models [V(z \subseteq u) \hat{=} 0]$. Then

$$\begin{aligned} V\left(\bigwedge_z [[z \subseteq u] \Leftrightarrow [z \in w]]\right) &\hat{=} 1 \hat{-} \hat{\Pi}_{z|V(z\subseteq u)\hat{=}0}\left[1 \hat{-} [0 \hat{-} V(z \in w)]^2\right] \\ &\hat{=} 1 \hat{-} \hat{\Pi}_{z|V(z\subseteq u)\hat{=}0}[1 \hat{-} V(z \in w)] \hat{=} 0. \end{aligned} \tag{2.60κ$_1$}$$

The above identity holds if and only if $\models [V(z \in w) \hat{=} 0]$, so that

$$\begin{aligned} V\left(\bigwedge_z [[z \subseteq u] \Leftrightarrow [z \in w]]\right) &\hat{=} 1 \hat{-} \hat{\Pi}_{z|V(z\subseteq u)\hat{=}0}[1 \hat{-} 0] \\ &\hat{=} 1 \hat{-} \hat{\Pi}_{z|V(z\subseteq u)\hat{=}0} 1 \hat{=} 1 \hat{-} 1 \hat{=} 0. \end{aligned} \tag{2.60κ$_2$}$$

Conversely, given a class *w*, let $\models [V(z \in w) \hat{=} 0]$. Then

$$\begin{aligned} V\left(\bigwedge_z [[z \subseteq u] \Leftrightarrow [z \in w]]\right) &\hat{=} 1 \hat{-} \hat{\Pi}_{z|V(z\in w)\hat{=}0}\left[1 \hat{-} [V(z \subseteq u) \hat{-} 0]^2\right] \\ &\hat{=} 1 \hat{-} \hat{\Pi}_{z|V(z\in w)\hat{=}0}[1 \hat{-} V(z \subseteq u)] \hat{=} 0. \end{aligned} \tag{2.60κ$_3$}$$

The above identity holds if and only if $\models [V(z \subseteq u) \hat{=} 0]$, so that

$$\begin{aligned} V\left(\bigwedge_z [[z \subseteq u] \Leftrightarrow [z \in w]]\right) &\hat{=} 1 \hat{-} \hat{\Pi}_{z|V(z\in w)\hat{=}0}[1 \hat{-} 0] \\ &\hat{=} 1 \hat{-} \hat{\Pi}_{z|V(z\in w)\hat{=}0} 1 \hat{=} 1 \hat{-} 1 \hat{=} 0. \end{aligned} \tag{2.60κ$_4$}$$

4) Given a class *u*, let $\models [V(z \in u) \hat{=} 0]$. Then

$$V\left(\bigwedge_z [[[z=v]\vee[z=w]]\Leftrightarrow[z\in u]]\right)$$
$$\hat{=}\,1\,\hat{-}\,\mathop{\hat{\bullet}}_{z|V(z\in u)\hat{=}0}\left[1\,\hat{-}\,[V(z=v)\,\hat{\cdot}\,V(z=w)\,\hat{-}\,0]^2\right] \qquad (2.61_1)$$
$$\hat{=}\,1\,\hat{-}\,\mathop{\hat{\bullet}}_{z|V(z\in u)\hat{=}0}\left[1\,\hat{-}\,V(z=v)\,\hat{\cdot}\,V(z=w)\right]\hat{=}\,0,$$

because

$$[V(z=v)\,\hat{\cdot}\,V(z=w)]^2\,\hat{=}\,V(z=v)\,\hat{\cdot}\,V(z=w). \qquad (2.61\kappa_2)$$

The identity (2.61κ_1) holds if and only if $\models[V(z=v)\,\hat{\cdot}\,V(z=w)\,\hat{=}\,0]$, so that

$$V\left(\bigwedge_z [[[z=v]\vee[z=w]]\Leftrightarrow[z\in u]]\right)$$
$$\hat{=}\,1\,\hat{-}\,\mathop{\hat{\bullet}}_{z|V(z\in u)\hat{=}0}[1\,\hat{-}\,0]\,\hat{=}\,1\,\hat{-}\,1\,\hat{=}\,0. \qquad (2.61\kappa_3)$$

Conversely, given classes v and w, let $\models[V(z=v)\,\hat{\cdot}\,V(z=w)\,\hat{=}\,0]$, so that

$$V\left(\bigwedge_z [[[z=v]\vee[z=w]]\Leftrightarrow[z\in u]]\right)$$
$$\hat{=}\,1\,\hat{-}\,\mathop{\hat{\bullet}}_{z|V(z=v)\hat{\cdot}V(z=w)\hat{=}0}\left[1\,\hat{-}\,[0\,\hat{-}\,V(z\in u)]^2\right] \qquad (2.61\kappa_4)$$
$$\hat{=}\,1\,\hat{-}\,\mathop{\hat{\bullet}}_{z|V(z=v)\hat{\cdot}V(z=w)\hat{=}0}[1\,\hat{-}\,V(z\in u)]\,\hat{=}\,0.$$

The above identity holds if and only if $\models[V(z\in u)\,\hat{=}\,0]$, so that

$$V\left(\bigwedge_z [[[z=v]\vee[z=w]]\Leftrightarrow[z\in u]]\right)$$
$$\hat{=}\,1\,\hat{-}\,\mathop{\hat{\bullet}}_{z|V(z=v)\hat{\cdot}V(z=w)\hat{=}0}[1\,\hat{-}\,0]\,\hat{=}\,1\,\hat{-}\,1\,\hat{=}\,0. \qquad (2.61\kappa_5)\bullet$$

3. The organons $\mathsf{A}_{1\in}$ and $\bar{\mathbf{A}}_{1\in}$

3.1. The ordinary zero-term $\emptyset$ and the organon $\mathsf{A}_{1\in}$

†**Df 3.1.** The calculus, which is logographically denoted by '$\mathsf{A}_{1\in}$' and also, redundantly, by '$\mathsf{A}_{1\in S}$' and which is phonographically called the *Pseudo-Class Euautogographic Algebraico-Predicate Organon* (*PCsEAPO*) and also, redundantly, the *Sufficient PCsEAPO* (*SPCsEAPO*), is obtained by supplementing the atomic basis of $\mathsf{A}_{1\in D}$ with two *atomic pseudo-constant ordinary terms* (*APCOT's*) $\emptyset$ and $\emptyset'$ in accordance with Ax I.5.1(9) and by imposing two similar PLS'ta of *specific* (*atypical*) *subject euautographic axioms* on $\emptyset$ and $\emptyset'$ relative to $\in$ and also relative to AEOT's including $\emptyset$ and $\emptyset'$ themselves, namely '$\neg[\mathbf{x} \in \emptyset]$' and '$\neg[\mathbf{y} \in \emptyset']$'. These PLS'ta are called the *primary laws of pseudo-indivisibility and pseudo-emptiness of* $\emptyset$ *and* $\emptyset'$ or briefly the $\emptyset$*-axiom schema* and the $\emptyset'$*-axiom schema* respectively. With the help of the pertinent AEADP, it will be proved from the two subject axiom schemata as a subject theorem of $\mathsf{A}_{1\in S}$ that $\emptyset = \emptyset'$, so that $\emptyset'$ will be eliminated from $\mathsf{A}_{1\in}$. The CFCL interpretand of $\emptyset$ is '∅', which denotes the *empty class*, called also the *empty set* and the *empty individual*, and which is used without quotation marks for mentioning its denotatum ∅.•

Corollary 3.1. As compared to $\mathsf{A}_{1\in G}$ and $\mathsf{A}_{1\in D}$, the calculus $\mathsf{A}_{1\in}$ has the APCOT $\emptyset$. In accordance with Ax II.4.18.1(1), the ER that results by replacing all occurrences of a free APVOT throughout a valid ER of A_1 (particularly of $\mathsf{A}_{1\in G}$ or $\mathsf{A}_{1\in D}$) with occurrences $\emptyset$ is another valid ER of A_1. Consequently, the PLR that results by replacing of all occurrences a free APLOT throughout a valid PLR of $\mathbf{A}_1$ (particularly of $\mathbf{A}_{1\in G}$ or $\mathbf{A}_{1\in D}$) with $\emptyset$ is another valid PLR of $\mathbf{A}_1$. A valid ER or PLR thus obtained is a corollary and therefore it does not require any proof.

For instance, $\emptyset$ satisfies the identities:

$$V(\emptyset \subseteq \emptyset) \hat{=} 0, \tag{3.1}$$

$$V(\emptyset = \emptyset) \hat{=} 0, \tag{3.2}$$

$$V(\neg[\emptyset \subset \emptyset]) \hat{=} 0, \tag{3.3}$$

$$V(\vee_{\mathbf{u}} \neg[\mathbf{u} \subset \emptyset]) \hat{=} 0 \tag{3.4}$$

$$V(\vee_{\mathbf{v}} \neg[\emptyset \subset \mathbf{v}]) \hat{=} 0, \tag{3.5}$$

$$V\left(\widehat{\bigvee}_{\mathbf{v}}^{1}[\mathbf{v}=\emptyset]\right)\hat{=}V\left(\bigvee_{\mathbf{z}}^{1}[\mathbf{z}=\emptyset]\right)\hat{=}V\left(\bigvee_{\mathbf{w}}[\mathbf{w}=\emptyset]\right)\hat{=}0, \tag{3.6}$$

$$V(\neg[\emptyset\in\emptyset])\hat{=}0; \tag{3.7}$$

The identities (3.1)–(3.3), (3.5), and (3.6) are respectively the instances of (1.35)–(1.37), (1.50), and (1.72) with $\emptyset$ as **u**, (3.4) is the instance of (1.49) with $\emptyset$ as **v**, and (3.7) is the variant of (2.5) with $\emptyset$ in place of x. Therefore, (3.1)–(3.6) are corollaries of $\mathsf{A}_{1\in C}$, whereas (3.7) is a corollary of $\mathsf{A}_{1\in D}$. Identities (3.1)–(3.4) are tantamount to the following logical kyrologies:

$$\emptyset\subseteq\emptyset, \tag{3.1a}$$

$$\emptyset=\emptyset, \tag{3.2a}$$

$$\neg[\emptyset\subset\emptyset], \tag{3.3a}$$

$$\bigvee_{\mathbf{u}}\neg[\mathbf{u}\subset\emptyset], \tag{3.4a}$$

$$\bigvee_{\mathbf{v}}\neg[\emptyset\subset\mathbf{v}], \tag{3.5a}$$

$$\widehat{\bigvee}_{\mathbf{v}}^{1}[\mathbf{x}=\emptyset], \bigvee_{\mathbf{z}}^{1}[\mathbf{z}=\emptyset], \bigvee_{\mathbf{w}}[\mathbf{w}=\emptyset], \tag{3.6a}$$

$$\neg[\emptyset\in\emptyset], \tag{3.7a}$$

respectively.•

Ax 3.1:** ***Axiom schema, or Law, of pseudo-indivisibility and pseudo-emptiness of $\emptyset$**.**

$$V(\neg[\mathbf{x}\in\emptyset])\hat{=}1\hat{-}V(\mathbf{x}\in\emptyset)\hat{=}0. \tag{3.8}$$

In contrast to 0, which is called the *zero integron* or the *special zero-term* (*SZT*), $\emptyset$ is called the *euautographic ordinary zero-term* (*EOZT*), in agreement with Ax 5.1(9).•

Cmt 3.1. By (3.7), $\neg[\emptyset\in\emptyset]$ is a theorem. Therefore, (3.8) postulates that

$$V\left(\neg\left[\mathbf{x}^{\mathrm{pv}}\in\emptyset\right]\right)\hat{=}1\hat{-}V\left(\mathbf{x}^{\mathrm{pv}}\in\emptyset\right)\hat{=}0, \tag{3.8$_1$}$$

i.e. that, for instance,

$$V(\neg[x\in\emptyset])\hat{=}1\hat{-}V(x\in\emptyset)\hat{=}0. \tag{3.8μ}$$ •

***Th 3.1.**

$$V(\emptyset\subseteq\mathbf{v})\hat{=}0. \tag{3.9}$$

$$V(\mathbf{u}\,\overline{\subset}\,\emptyset)\hat{=}0. \tag{3.10}$$

$$V\left(\bigwedge_{\mathbf{v}}[\emptyset\subseteq\mathbf{v}]\right)\hat{=}0. \tag{3.11}$$

$$V\left(\bigwedge_{\mathbf{u}}[\mathbf{u}\,\overline{\subset}\,\emptyset]\right)\hat{=}0. \tag{3.12}$$

$$V\left(\neg\bigvee_{\mathbf{x}}[\mathbf{x}\in\emptyset]\right)\hat{=}V\left(\bigwedge_{\mathbf{x}}\neg[\mathbf{x}\in\emptyset]\right)\hat{=}0. \tag{3.13}$$

$$V\left(\bigvee_{\mathbf{x}}[\mathbf{x} \subseteq \mathbf{v}]\right) \hat{=} 0. \tag{3.14}$$

$$V\left(\bigvee_{\mathbf{v}} \bigwedge_{\mathbf{x}} \neg[\mathbf{x} \in \mathbf{v}]\right) \hat{=} 0. \tag{3.15}$$

Identity (3.9) will be called *Theorem of emptiness of* $\emptyset$.

Proof: By (3.8), it follows from (1.22) with $\emptyset$ as **u** that:

$$\begin{aligned} V(\emptyset \subseteq \mathbf{v}) &\hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{x}}[1 \dot{-} V(\neg[\mathbf{x} \in \emptyset]) \dot{\cdot} V(\mathbf{x} \in \mathbf{v})] \\ &\hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{x}}[1 \dot{-} 0 \dot{\cdot} V(\mathbf{x} \in \mathbf{v})] \hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{x}} 1 \hat{=} 1 \dot{-} 1 \hat{=} 0, \end{aligned} \tag{3.9$_1$}$$

which proves (3.9). Then, by the variant of (3.9) with '**u**' in place of '**v**', it follows from (1.27) with $\emptyset$ as **v** that:

$$\begin{aligned} V(\mathbf{u} \overline{\subset} \emptyset) &\hat{=} V(\neg[\mathbf{u} \subset \emptyset]) \hat{=} V(\neg[\mathbf{u} \subseteq \emptyset]) \dot{\cdot} V(\emptyset \subseteq \mathbf{u}) \\ &\hat{=} V(\neg[\mathbf{u} \subseteq \emptyset]) \dot{\cdot} 0 \hat{=} 0, \end{aligned} \tag{3.10$_1$}$$

which proves (3.10). Identities (3.11) and (3.12) are proved from the pertinent specific instances of (II.8.2) by making use of (3.9) and (3.10) respectively as follows:

$$V\left(\bigwedge_{\mathbf{v}}[\emptyset \subseteq \mathbf{v}]\right) \hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{v}}[1 \dot{-} V(\emptyset \subseteq \mathbf{v})] \hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{v}} 1 \hat{=} 1 \dot{-} 1 \hat{=} 0, \tag{3.11$_1$}$$

$$V\left(\bigwedge_{\mathbf{u}}[\mathbf{u} \overline{\subset} \emptyset]\right) \hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{u}}[1 \dot{-} V(\mathbf{u} \overline{\subset} \emptyset)] \hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{u}} 1 \hat{=} 1 \dot{-} 1 \hat{=} 0. \tag{3.12$_1$}$$

Identities (3.13) are proved by the following two trains of identities, in which use of (3.8) is made:

$$V\left(\neg \bigvee_{\mathbf{x}}[\mathbf{x} \in \emptyset]\right) \hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{x}} V(\mathbf{x} \in \emptyset) \hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{x}} 1 \hat{=} 1 \dot{-} 1 \hat{=} 0, \tag{3.13$_1$}$$

$$V\left(\bigwedge_{\mathbf{x}} \neg[\mathbf{x} \in \emptyset]\right) \hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{x}} V(\neg\neg[\mathbf{x} \in \emptyset]) \hat{=} 1 \dot{-} \hat{\cdot}_{\mathbf{x}} V(\mathbf{x} \in \emptyset) \hat{=} 0. \tag{3.13$_2$}$$

Identity (3.14) is proved thus:

$$\begin{aligned} V\left(\bigvee_{\mathbf{x}}[\mathbf{x} \subseteq \mathbf{v}]\right) &\hat{=} \hat{\cdot}_{\mathbf{x}} V(\mathbf{x} \subseteq \mathbf{v}) \hat{=} V(\emptyset \subseteq \mathbf{v}) \dot{\cdot} \left[\hat{\cdot}_{\mathbf{x}} V(\mathbf{x} \subseteq \mathbf{v})\right] \\ &\hat{=} 0 \dot{\cdot} \left[\hat{\cdot}_{\mathbf{x}} V(\mathbf{x} \subseteq \mathbf{v})\right] \hat{=} 0, \end{aligned} \tag{3.14$_1$}$$

where use of the appropriate instance of the EL and then use of (3.9) have been made. Identity (3.15), which is a generalization of (2.58$_1$), is proved by the following train of identities:

$$\begin{aligned} &V\left(\bigvee_{\mathbf{v}} \bigwedge_{\mathbf{x}} \neg[\mathbf{x} \in \mathbf{v}]\right) \hat{=} \hat{\cdot}_{\mathbf{v}}\left[1 \dot{-} \hat{\cdot}_{\mathbf{x}} V(\mathbf{x} \in \mathbf{v})\right] \\ &\hat{=} \left[1 \dot{-} \hat{\cdot}_{\mathbf{x}} V(\mathbf{x} \in \emptyset)\right] \dot{\cdot} \hat{\cdot}_{\mathbf{v}}\left[1 \dot{-} \hat{\cdot}_{\mathbf{x}} V(\mathbf{x} \in \mathbf{v})\right] \hat{=} 0 \dot{\cdot} \hat{\cdot}_{\mathbf{v}}\left[1 \dot{-} \hat{\cdot}_{\mathbf{x}} V(\mathbf{x} \in \mathbf{v})\right] \hat{=} 0. \end{aligned} \tag{3.15$_1$}$$

In developing this train, use was made of the instance of the Emission Law (II.4.27) with **v**, $\emptyset$, and $\left[1 \dot{-} \hat{\cdot}_{\mathbf{x}} V(\mathbf{x} \in \mathbf{v})\right]$ as **x**, **y**, and $\mathbf{i}\langle\mathbf{x}\rangle$ respectively – the instance, according to which the instance of the operatum of the operator $\hat{\cdot}_{\mathbf{v}}$ with $\emptyset$ as **v** was taken (emitted) out the scope of that operator; and then use of (3.8) was made.•

Cmt 3.2. By (II.4.40a), identities (3.8)–(3.15) are tantamount to the following logical kyrologies:

$$\neg[\mathbf{x} \in \emptyset], \tag{3.8a}$$

$$\emptyset \subseteq \mathbf{v}\,. \tag{3.9a}$$

$$\mathbf{u} \,\bar{\sqsubset}\, \emptyset\,. \tag{3.10a}$$

$$\bigwedge_{\mathbf{v}}[\emptyset \subseteq \mathbf{v}]. \tag{3.11a}$$

$$\bigwedge_{\mathbf{u}}[\mathbf{u} \,\bar{\sqsubset}\, \emptyset]. \tag{3.12a}$$

$$\neg\bigvee_{\mathbf{x}}[\mathbf{x} \in \emptyset], \bigwedge_{\mathbf{x}}\neg[\mathbf{x} \in \emptyset]. \tag{3.13a}$$

$$\bigvee_{\mathbf{x}}[\mathbf{x} \subseteq \mathbf{v}]. \tag{3.14a}$$

$$\bigvee_{\mathbf{v}}\bigwedge_{\mathbf{x}}\neg[\mathbf{x} \in \mathbf{v}]. \tag{3.15a}$$ •

Th 3.2:** ***The theorem of uniqueness of $\emptyset$**.** Let $\emptyset'$ be an APCOT, which is introduced into $\mathsf{A}_{1\in}$ in accordance with Ax II.5.1(9), and which satisfies the variant of the axiom schema (3.8) with $\emptyset'$ in place of $\emptyset$, i.e.

$$V(\neg[\mathbf{y} \in \emptyset']) \hat{=} 1 \hat{-} V(\mathbf{y} \in \emptyset') \hat{=} 0\,. \tag{3.8'}$$

Then

$$\emptyset = \emptyset'\,. \tag{3.16}$$

Proof: By (3.8) and (3.8'), it follows from the instance of (1.40) with $\emptyset$ as $\mathbf{u}$ and $\emptyset'$ as $\mathbf{v}$ that

$$\begin{aligned} V(\emptyset = \emptyset') &\hat{=} 1 \hat{-} \hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1 \hat{-} \left[V(\mathbf{x} \in \emptyset) \hat{-} V(\mathbf{x} \in \emptyset')\right]^2\right] \\ &\hat{=} 1 \hat{-} \hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1 \hat{-} [1 \hat{-} 1]^2\right] \hat{=} 1 \hat{-} \hat{\boldsymbol{\cdot}}_{\mathbf{x}} 1 \hat{=} 0, \end{aligned} \tag{3.16$_1$}$$

which is tantamount to (3.16).•

Cmt 3.3. Under (3.8'), all theorems that have been proved from (3.8), remain valid with $\emptyset'$ in place of $\emptyset$. Hence, particularly,

$$V(\emptyset' \subseteq \mathbf{w}) \hat{=} 0\,. \tag{3.9'}$$

Identity (3.9) with $\mathbf{v} \triangleright \emptyset'$ and (3.9') with $\mathbf{w} \triangleright \emptyset$ become:

$$V(\emptyset' \subseteq \emptyset) \hat{=} V(\emptyset \subseteq \emptyset') \hat{=} 0\,. \tag{3.17}$$

Consequently, (1.24) with $\mathbf{u} \triangleright \emptyset$ and $\mathbf{v} \triangleright \emptyset'$ yields:

$$V(\emptyset = \emptyset') \hat{=} 1 \hat{-} V(\neg[\emptyset \subseteq \emptyset']) \hat{\cdot} V(\neg[\emptyset' \subseteq \emptyset]) \hat{=} 1 \hat{-} 1 \hat{\cdot} 1 \hat{=} 0\,, \tag{3.16$_2$}$$

which is another proof of (3.16). This one is instructive because it is effective also in the framework of $\mathsf{A}_{1\subseteq}$, while the proof (3.16$_1$) is inapplicable in $\mathsf{A}_{1\subseteq}$. •

***Th 3.3.**

$$V(\neg[\mathbf{u}\subseteq\emptyset])\hat{=}V(\neg[\mathbf{u}=\emptyset])\hat{=}V(\emptyset\subset\mathbf{u})\hat{=}V\left(\bigvee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}]\right)\hat{=}\hat{:}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u}) \tag{3.18}$$

or concurrently

$$\begin{gathered}V(\mathbf{u}\subseteq\emptyset)\hat{=}V(\mathbf{u}=\emptyset)\hat{=}V(\neg[\emptyset\subset\mathbf{u}])\\ \hat{=}V\left(\neg\bigvee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}]\right)\hat{=}V\left(\bigwedge_{\mathbf{x}}\neg[\mathbf{x}\in\mathbf{u}]\right)\hat{=}1\hat{-}\hat{:}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u}).\end{gathered} \tag{3.19}$$

Proof: From the instances of (1.22) and (1.40) with $\emptyset$ as $\mathbf{v}$, it follows that

$$\begin{gathered}V(\neg[\mathbf{u}\subseteq\emptyset])\hat{=}\hat{:}_{\mathbf{x}}[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\mathbf{x}\in\emptyset)]\\ \hat{=}\hat{:}_{\mathbf{x}}[1\hat{-}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}1]\hat{=}\hat{:}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u})\hat{=}V\left(\bigvee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}]\right),\end{gathered} \tag{3.18$_1$}$$

$$\begin{gathered}V(\neg[\mathbf{u}=\emptyset])\hat{=}\hat{:}_{\mathbf{x}}[V(\mathbf{x}\in\mathbf{u})\hat{\cdot}V(\mathbf{x}\in\emptyset)\hat{+}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}V(\neg[\mathbf{x}\in\emptyset])]\\ \hat{=}\hat{:}_{\mathbf{x}}[V(\mathbf{x}\in\mathbf{u})\hat{\cdot}1\hat{+}V(\neg[\mathbf{x}\in\mathbf{u}])\hat{\cdot}0]\hat{=}\hat{:}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u}),\end{gathered} \tag{3.18$_2$}$$

where use of (3.8) has been made. At the same time, by (3.9), identity (1.30) with $\emptyset$ as $\mathbf{u}$ yields:

$$\begin{gathered}V(\emptyset\subset\mathbf{v})\hat{=}V(\emptyset\subseteq\mathbf{v})\hat{+}V(\neg[\emptyset=\mathbf{v}])\hat{=}V(\neg[\emptyset=\mathbf{v}])\\ \hat{=}V(\neg[\mathbf{v}=\emptyset])\hat{=}\hat{:}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{v}),\end{gathered} \tag{3.18$_3$}$$

where use of the instance of (1.38) with $\emptyset$ as $\mathbf{u}$ and of the variant of (3.18$_2$) with '$\mathbf{v}$' in place of '$\mathbf{u}$' has been made. Identities (3.18$_1$) and (3.18$_2$), and the variant of (3.18$_3$) with '$\mathbf{u}$' in place of '$\mathbf{v}$' prove (3.18).•

Cmt 3.4. $\hat{:}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u})$ is an irreducible non-digital, or non-numeral, euautographic validity-integron (IRNDEVI or IRNNEVI) independent of $\emptyset$. Therefore, it follows from (3.18) and (3.19) that $V(\neg[\mathbf{u}\subseteq\emptyset])$, $V(\neg[\mathbf{u}=\emptyset])$, $V(\emptyset\subset\mathbf{u})$, $V(\mathbf{u}\subseteq\emptyset)$, $V(\mathbf{u}=\emptyset)$, and $V(\neg[\emptyset\subset\mathbf{u}])$ are also NDEVI's, which do not reduce either to 0 or to 1. Consequently, $\neg[\mathbf{u}\subseteq\emptyset]$, $\neg[\mathbf{u}=\emptyset]$, $\emptyset\subset\mathbf{u}$, $\mathbf{u}\subseteq\emptyset$, $\mathbf{u}=\emptyset$, $\neg[\emptyset\subset\mathbf{u}]$, $\bigvee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}]$, and $\bigwedge_{\mathbf{x}}\neg[\mathbf{x}\in\mathbf{u}]$ are udeterologies (vav-neutral relations).•

Th 3.4.

$$\begin{gathered}V\left([\mathbf{u}\subseteq\emptyset]\vee\bigvee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}]\right)\hat{=}V\left([\mathbf{u}=\emptyset]\vee\bigvee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}]\right)\\ \hat{=}V\left(\neg[\emptyset\subset\mathbf{u}]\vee\bigvee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}]\right)\hat{=}0.\end{gathered} \tag{3.20}$$

Proof: By (3.19), it follows that

$$\begin{gathered}V\left([\mathbf{u}\subseteq\emptyset]\vee\bigvee_{\mathbf{x}}[\mathbf{x}\in\mathbf{u}]\right)\hat{=}V(\mathbf{u}\subseteq\emptyset)\hat{\cdot}\hat{:}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u})\\ \hat{=}[1\hat{-}\hat{:}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u})]\hat{\cdot}\hat{:}_{\mathbf{x}}V(\mathbf{x}\in\mathbf{u})\hat{=}0\end{gathered} \tag{3.20$_1$}$$

and similarly with '$\mathbf{u}=\emptyset$' or '$\neg[\emptyset\subset\mathbf{u}]$' in place of '$\mathbf{u}\subseteq\emptyset$'. QED.•

***Th 3.5.**

$$V\left(\left[\emptyset \subset \mathbf{v}\right] \Rightarrow \bigvee_{\mathbf{u}}\left[\mathbf{u} \subset \mathbf{v}\right]\right) \hat{=} V\left(\neg\left[\emptyset \subset \mathbf{v}\right]\right) \hat{\cdot} \left[\hat{\cdot}_{\mathbf{u}} V\left(\mathbf{u} \subset \mathbf{v}\right)\right] \hat{=} 0. \tag{3.21}$$

Proof: By the pertinent instances of (II.4.23) and of the Emission Law (II.4.28), it follows that

$$V\left(\bigvee_{\mathbf{u}}\left[\mathbf{u} \subset \mathbf{v}\right]\right) \hat{=} \hat{\cdot}_{\mathbf{u}} V\left(\mathbf{u} \subset \mathbf{v}\right) \hat{=} V\left(\emptyset \subset \mathbf{v}\right) \hat{\cdot} \left[\hat{\cdot}_{\mathbf{u}} V\left(\mathbf{u} \subset \mathbf{v}\right)\right] \tag{3.21$_1$}$$

Substitution of (3.21$_1$) into (3.21) proves the latter because

$$V\left(\neg\left[\emptyset \subset \mathbf{v}\right]\right) \hat{\cdot} V\left(\emptyset \subset \mathbf{v}\right) \hat{=} 0. \tag{3.21$_2$}$$

Alternatively, identity (3.21$_1$) can be rewritten as:

$$\left[1 \hat{-} V\left(\emptyset \subset \mathbf{v}\right)\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{u}} V\left(\mathbf{u} \subset \mathbf{v}\right)\right] \hat{=} V\left(\neg\left[\emptyset \subset \mathbf{v}\right]\right) \hat{\cdot} \left[\hat{\cdot}_{\mathbf{u}} V\left(\mathbf{u} \subset \mathbf{v}\right)\right] \hat{=} 0. \tag{3.21$_3$}$$ •

*__Th 3.6.__

$$V\left(\neg\left[\mathbf{v} \subseteq \emptyset\right] \Rightarrow \bigvee_{\mathbf{u}}\left[\mathbf{u} \subset \mathbf{v}\right]\right) \hat{=} V\left(\mathbf{v} \subseteq \emptyset\right) \hat{\cdot} \left[\hat{\cdot}_{\mathbf{u}} V\left(\mathbf{u} \subset \mathbf{v}\right)\right] \hat{=} 0, \tag{3.22}$$

$$V\left(\neg\left[\mathbf{v} = \emptyset\right] \Rightarrow \bigvee_{\mathbf{u}}\left[\mathbf{u} \subset \mathbf{v}\right]\right) \hat{=} V\left(\mathbf{v} = \emptyset\right) \hat{\cdot} \left[\hat{\cdot}_{\mathbf{u}} V\left(\mathbf{u} \subset \mathbf{v}\right)\right] \hat{=} 0, \tag{3.23}$$

$$V\left(\bigvee_{\mathbf{x}}\left[\mathbf{x} \in \mathbf{v}\right] \Rightarrow \bigvee_{\mathbf{u}}\left[\mathbf{u} \subset \mathbf{v}\right]\right) \hat{=} V\left(\neg \bigvee_{\mathbf{x}}\left[\mathbf{x} \in \mathbf{v}\right]\right) \hat{\cdot} V\left(\bigvee_{\mathbf{u}}\left[\mathbf{u} \subset \mathbf{v}\right]\right) \hat{=} 0. \tag{3.24}$$

Proof: (3.22)–(3.24) immediately follow from (3.21) by the variant of (3.19) with ‘**v**’ in place of ‘**u**’.•

Cmt 3.5. By (II.4.40a), identities (3.18)–(3.15) are tantamount to the following logical kyrologies:

$$\neg\left[\mathbf{u} \subseteq \emptyset\right] \Leftrightarrow \neg\left[\mathbf{u} = \emptyset\right] \Leftrightarrow \left[\emptyset \subset \mathbf{u}\right] \Leftrightarrow \bigvee_{\mathbf{x}}\left[\mathbf{x} \in \mathbf{u}\right], \tag{3.18a}$$

$$\left[\mathbf{u} \subseteq \emptyset\right] \Leftrightarrow \left[\mathbf{u} = \emptyset\right] \Leftrightarrow \neg\left[\emptyset \subset \mathbf{u}\right] \Leftrightarrow \neg \bigvee_{\mathbf{x}}\left[\mathbf{x} \in \mathbf{u}\right] \Leftrightarrow \bigwedge_{\mathbf{x}} \neg\left[\mathbf{x} \in \mathbf{u}\right], \tag{3.19a}$$

$$\left[\mathbf{u} \subseteq \emptyset\right] \vee \bigvee_{\mathbf{x}}\left[\mathbf{x} \in \mathbf{u}\right], \tag{3.20a}$$

$$\left[\mathbf{u} = \emptyset\right] \vee \bigvee_{\mathbf{x}}\left[\mathbf{x} \in \mathbf{u}\right], \tag{3.20b}$$

$$\neg\left[\emptyset \subset \mathbf{u}\right] \vee \bigvee_{\mathbf{x}}\left[\mathbf{x} \in \mathbf{u}\right], \tag{3.20c}$$

$$\left[\emptyset \subset \mathbf{v}\right] \Rightarrow \bigvee_{\mathbf{u}}\left[\mathbf{u} \subset \mathbf{v}\right], \tag{3.21a}$$

$$\neg\left[\mathbf{v} \subseteq \emptyset\right] \Rightarrow \bigvee_{\mathbf{u}}\left[\mathbf{u} \subset \mathbf{v}\right], \tag{3.22a}$$

$$\neg\left[\mathbf{v} = \emptyset\right] \Rightarrow \bigvee_{\mathbf{u}}\left[\mathbf{u} \subset \mathbf{v}\right], \tag{3.23a}$$

$$\bigvee_{\mathbf{x}}\left[\mathbf{x} \in \mathbf{v}\right] \Rightarrow \bigvee_{\mathbf{u}}\left[\mathbf{u} \subset \mathbf{v}\right]. \tag{3.24a}$$ •

Cmt 3.6. 1) In accordance with Df I.8.1(1), $\emptyset \to$‘∅’ so that all occurrences of $\emptyset$ in a *euautographic* or *panlogographic formula of* $\mathsf{A}_{1\in}$ (i.e. of $\mathsf{A}_{1\in}$ or $\mathbf{A}_{1\in}$ respectively) can be replaced with an occurrence of the HAQ (homoloautographic, or photographic, quotation) ‘∅’. However, in accordance with Ax 8.1(2), in any pertinent *panlogographic formula*, being the CFCL interpretand of a certain euautographic formula, '$\varnothing$' should occur without quotation marks, – like all other atomic catlogographs of $A_{1\in}$. In such an occurrence '$\varnothing$' is used for mentioning its denotatum $\varnothing$, which is called the *empty class* or *empty set* or *empty individual*. These names are justified by the properties of $\varnothing$ as explicated below.•

2) The euautographic identities:

$$V(\neg[x \in \emptyset]) \hat{=} 1 \dot{-} V(x \in \emptyset) \hat{=} 0, \tag{3.8μ}$$

$$V(\emptyset \subseteq v) \hat{=} 0 \tag{3.9μ}$$

are the analo-euautographic instances (interpretands, denotata) of (3.8) and (3.9), whereas the catlogographic identities:

$$V(\neg[x \in \varnothing]) \hat{=} 1 \dot{-} V(x \in \varnothing) \hat{=} 0, \tag{3.8κ}$$

$$V(\varnothing \subseteq v) \hat{=} 0 \tag{3.9κ}$$

are, in accordance with Ax I.8.1, the CFCL interpretnds of (3.8μ) and (3.9μ) respectively. By (II.4.40a), the identities (3.8μ) and (3.9μ) are tantamount to the logical kyrologies:

$$\neg[x \in \emptyset], \tag{3.8μa}$$

$$\emptyset \subseteq v, \tag{3.9μa}$$

and accordingly the identities (3.8κ) and (3.9κ) are tantamount to the logical tautologies:

$$\neg[x \in \varnothing], \tag{3.8κa}$$

$$\varnothing \subseteq v, \tag{3.9κa}$$

which are the CFCL interpretnds of (3.8μa) and (3.9μa), respectively. Either tautology (3.8κ) or (3.8κa) means that $\varnothing$ is a *class*, because its name stands to the right of the class-membership predicate $\in$, and that at the same time this class is *empty* (*memberless*) and is hence an *indivisible entity*, i.e. an *individual*. In agreement with the above fact, either tautology (3.8κ) or (3.9κa) means that $\varnothing$ is a *universal subclass of every class including itself*. In this case, Ths 3.2 implies that *no universal nonempty individuals can exist*.

3) The vav-neutral (udeterological) PLR's (panlogographic relations) '$\mathbf{x}\in\mathbf{u}$' and '$\bigvee_{\mathbf{v}}[\mathbf{u} \subset \mathbf{v}]$', e.g., can be interpreted euautographically, e.g., by the vav-neutral conformal ER's $x \in u$ and $\bigvee_u [u \subset v]$, whereas the latter two can in turn be interpreted

by the vav-neutral conformal catlogographic relations (CFCLR's) '$x \in u$' and '$\bigvee_u [u \subset v]$', respectively. In this case, 'x', 'u', and 'v' are ordinary mathematical variables that may assume *accidental denotata*. Therefore, '$x \in u$' and '$\bigvee_u [u \subset v]$' can be interpreted verbally as follows.

a) If u is a nonempty class then the assumption that '$x \in u$' is *veracious* (*accidentally true*), i.e. symbolically that $\models [x \in u]$ or, equivalently, that $\models [V(x \in u) \hat{=} 0]$, is a condition that is imposed on the range of the variable 'x'. If $x = \varnothing$ then yhe pertinent CFCLR '$x \in \varnothing$' is *antitautologous* (*universally antitrue, universally false*).

b) If v is a nonempty class then the vav-neutral CFCLR $\bigvee_u [u \subset v]$ is *veracious*. If, however, $v = \varnothing$ then the pertinent CFCLR $\bigvee_u [u \subset \varnothing]$ is *antiveracious*.•

3.2. The universal term *U* and the organon $\bar{A}_{1\in}$

In accordance with theorem (2.10), which follows from axiom (2.1), there is no universal class of interpretands of APVOT's of $A_{1\in}$ (see Cmt 2.3(4)). Still, this universal class can be introduced axiomatically via a certain euautographic *extraordinary* atomic pseudo-constant term that will be called the *universal term* and also self-referentially the *U-term*. In accordance with the latter name, I shall, as the *U*-term, employ the letter '*U*', the understanding being that *U*→'*U*', whereas *U*, i.e. the object that is denoted by '*U*' and that is hencea mentioned by using '*U*', will be called *the universal class associated with* $A_{1\in}$. *U* doeas not belong to the atomic basis of $A_{1\in}$ that is introduced by Ax I.5.1. Consequently, in contrast to $\emptyset$, which is an APC*O*T (atomic pseudo-constant *ordinary* term), *U* is the *extraordinary* atomic pseudo-constant term, which is not subjugated in advance to any of the formation rules, subject axioms, and rules of inference and decision of A_1 in general and of $A_{1\in}$ in particular. The formation rule for the formulas involving *U*, the subject axioms for *U*, and the rules of inference and decision for relations involving *U* will be called the *formation U-rules*, the *subject U-axioms* or *axioms of universality*, and the *U-rules of inference and decision* respectively. In accordance with Cmt I.7.6(1), the organon, which is obtained by supplementing $A_{1\in}$ with the *U*-term and with all the above-mentioned *U*-rules and subject *U*-axioms, is denoted logographically by '$\bar{A}_{1\in}$' and is called phonographically (verbally) the *Pseudo-Restricted*, or *Pseudo-Confined, Pseudo-Class EAPO* (*PCsEAPO*).

†**Ax 3.2:** ***The primary formation rules of*** $\bar{A}_{1\in}$**.** 1) A formula $\mathbf{\Phi}$ of $A_{1\in}$ is a formula of $\bar{A}_{1\in}$. Particularly, an ordinary atomic term $\mathbf{x}$, pseudo-variable or pseudo-constant, an integron (special term) $\mathbf{I}$, and a relation $\mathbf{P}$, of $A_{1\in}$ are those of $\bar{A}_{1\in}$. The dichotomy of the formulas or relations of $A_{1\in}$ into ordinary ones and special ones retains with the proviso that "non-special" is not a synonym of "ordinary" anymore (see the next item).

2) If $\mathbf{\Phi}\langle\mathbf{x}\rangle$ is a formula of $A_{1\in}$ that contains $\mathbf{x}$ (e.g., u, v, w, x, y, z, u_1, etc.) as a *free* APVOT then $\mathbf{\Phi}\langle U\rangle$ subject to $\mathbf{\Phi}\langle U\rangle \to \mathrm{S}_U^{\mathbf{x}}\mathbf{\Phi}\langle\mathbf{x}\rangle\rfloor$ is a formula of $\bar{A}_{1\in}$. Particularly, if $\mathbf{I}\langle\mathbf{x}\rangle$ is an integron (special term) of $A_{1\in}$ that contains $\mathbf{x}$ as a *free* APVOT then $\mathbf{I}\langle U\rangle$ subject to $\mathbf{I}\langle U\rangle \to \mathrm{S}_U^{\mathbf{x}}\mathbf{I}\langle\mathbf{x}\rangle\rfloor$ is an integron (special term) of $\bar{A}_{1\in}$ and if $\mathbf{P}\langle\mathbf{x}\rangle$ is a relation of $A_{1\in}$ that contains $\mathbf{x}$ as a *free* APVOT then $\mathbf{P}\langle U\rangle$ subject to $\mathbf{P}\langle U\rangle \to \mathrm{S}_U^{\mathbf{x}}\mathbf{P}\langle\mathbf{x}\rangle\rfloor$ is a relation of $\bar{A}_{1\in}$. In this case, the relation $\mathbf{P}\langle U\rangle$ is said to be *extraordinary* if $\mathbf{P}\langle\mathbf{x}\rangle$ is *ordinary* and *extraspecial* if $\mathbf{P}\langle\mathbf{x}\rangle$ is *special*. Accordingly, the qualifier "non-special" now means "*either ordinary or extraordinary*".•

Ax 3.3:** ***The subject U***-axioms (axioms of universality)*** $\bar{A}_{1\in}$ ***ian logical form.***

$$\neg[U \in U]. \tag{3.25}$$

$$\mathbf{x} \in U. \tag{3.26}$$

$$\neg[U \in \mathbf{x}]. \tag{3.27}$$
•

†**Ax 3.4:** ***The rules of inference and decision of*** $\bar{A}_{1\in}$**.** The rules of inference and decision of $\bar{A}_{1\in}$ are the same as those of $A_{1\in}$ with the proviso the Emission and Absorption Law (II.4.28) is not applicable in the case, where U occurs in place of $\mathbf{y}$.•

Th 3.7:** ***The subject U***-axioms (axioms of universality) of*** $\bar{A}_{1\in}$ ***in algebraic form.***

$$V(\neg[U \in U]) \hat{=} 1 \hat{-} V(U \in U) \hat{=} 0, \text{ i.e. } V(U \in U) \hat{=} 1. \tag{3.28}$$

$$V(\mathbf{x} \in U) \hat{=} 0. \tag{3.29}$$

$$V(\neg[U \in \mathbf{x}]) \hat{=} 1 \hat{-} V(U \in \mathbf{x}) \hat{=} 0, \text{ i.e. } V(U \in \mathbf{x}) \hat{=} 1. \tag{3.30}$$

Proof: (3.28)–(3.30) immediately follow from (3.25)–(3.27) by the pertinent instances of axiom (II.4.40a).•

***Th 3.8.**

$$V(U \subseteq U) \hat{=} 0. \tag{3.31}$$

$$V(\mathbf{u} \subseteq U) \hat{=} 0. \tag{3.32}$$

$$V(\neg[U \subseteq \mathbf{u}]) \hat{=} 0. \tag{3.33}$$

$$V(U = U) \hat{=} 0. \tag{3.34}$$

$$V(\neg[\mathbf{u} = U]) \hat{=} 0. \tag{3.35}$$

$$V(\mathbf{u} \subset U) \hat{=} 0. \tag{3.36}$$

$$V(\neg[U \subset \mathbf{u}]) \hat{=} 0. \tag{3.37}$$

$$V\left(\bigwedge_{\mathbf{u}}[\mathbf{u} \in U]\right) \hat{=} V\left(\bigvee_{\mathbf{u}}[\mathbf{u} \in U]\right) \hat{=} 0. \tag{3.38}$$

$$V\left(\bigwedge_{\mathbf{u}}\neg[U \in \mathbf{u}]\right) \hat{=} V\left(\bigvee_{\mathbf{u}}\neg[U \in \mathbf{u}]\right) \hat{=} 0. \tag{3.39}$$

$$V\left(\bigwedge_{\mathbf{u}}[\mathbf{u} \subseteq U]\right) \hat{=} V\left(\bigvee_{\mathbf{u}}[\mathbf{u} \subseteq U]\right) \hat{=} 0. \tag{3.40}$$

$$V\left(\bigwedge_{\mathbf{u}}\neg[U \subseteq \mathbf{u}]\right) \hat{=} V\left(\bigvee_{\mathbf{u}}\neg[U \subseteq \mathbf{u}]\right) \hat{=} 0. \tag{3.41}$$

$$V\left(\bigwedge_{\mathbf{u}}\neg[\mathbf{u} = U]\right) \hat{=} V\left(\bigvee_{\mathbf{u}}\neg[\mathbf{u} = U]\right) \hat{=} 0. \tag{3.42}$$

$$V\left(\bigwedge_{\mathbf{u}}[\mathbf{u} \subset U]\right) \hat{=} V\left(\bigvee_{\mathbf{u}}[\mathbf{u} \subset U]\right) \hat{=} 0. \tag{3.43}$$

$$V\left(\bigwedge_{\mathbf{u}}\neg[U \subset \mathbf{u}]\right) \hat{=} V\left(\bigvee_{\mathbf{u}}\neg[U \subset \mathbf{u}]\right) \hat{=} 0. \tag{3.44}$$

Identities (3.33)–(3.37) hold with ∅ as **u**. (3.45)

Proof: In accordance with Ax 3.2(1), the equality, which results by substitution of U for each occurrence of **u** or **v** or both throughout any of the *identities* (*valid equalities*) (1.22)–(1.27) is an identity of $\bar{A}_{1\in}$. Accordingly, if U is substituted for each occurrence of one of the placeholders '**u**' and '**v**' throughout a certain one of the identities (1.22)–(1.27), while the occurrences of the other placeholder remain unaltered or are replaced either with occurrences of another APLOT of the list (II.5.6) or with occurrences of any APVOT of the list (II.5.1), or else with occurrences of the APCOT ∅, then the resulting equality is also an identity. Thus, by (3.28)–(3.30), it follows from the pertinent instances of (1.22)–(1.27) as indicated above that:

$$\begin{aligned} V(U \subseteq U) &\hat{=} 1 \hat{-} \hat{\wedge}_{\mathbf{x}}[1 \hat{-} V(\neg[\mathbf{x} \in U]) \hat{\cdot} V(\mathbf{x} \in U)] \\ &\hat{=} 1 \hat{-} \hat{\wedge}_{\mathbf{x}}[1 \hat{-} 0 \hat{\cdot} 1] \hat{=} 1 \hat{-} \hat{\wedge}_{\mathbf{x}} 1 \hat{=} 1 \hat{-} 1 \hat{=} 0. \end{aligned} \tag{3.31$_1$}$$

$$\begin{aligned} V(\mathbf{u} \subseteq U) &\hat{=} 1 \hat{-} \hat{\wedge}_{\mathbf{x}}[1 \hat{-} V(\neg[\mathbf{x} \in \mathbf{u}]) \hat{\cdot} V(\mathbf{x} \in U)] \\ &\hat{=} 1 \hat{-} \hat{\wedge}_{\mathbf{x}}[1 \hat{-} V(\neg[\mathbf{x} \in \mathbf{u}]) \hat{\cdot} 0] \hat{=} 1 \hat{-} \hat{\wedge}_{\mathbf{x}} 1 \hat{=} 1 \hat{-} 1 \hat{=} 0. \end{aligned} \tag{3.32$_1$}$$

$$\begin{aligned} V(\neg[U \subseteq \mathbf{u}]) &\hat{=} \hat{\wedge}_{\mathbf{x}}[1 \hat{-} V(\neg[\mathbf{x} \in U]) \hat{\cdot} V(\mathbf{x} \in \mathbf{u})] \\ &\hat{=} \hat{\wedge}_{\mathbf{x}}[1 \hat{-} 1 \hat{\cdot} V(\mathbf{x} \in \mathbf{u})] \hat{=} \hat{\wedge}_{\mathbf{x}} V(\neg[\mathbf{x} \in \mathbf{u}]) \hat{=} 0. \end{aligned} \tag{3.33$_1$}$$

$$\begin{aligned} &V(U=U)\,\hat{=}\,V([U\subseteq U]\wedge[U\subseteq U])\\ &\hat{=}\,1\,\hat{-}\,V(\neg[U\subseteq U])\,\hat{\cdot}\,V(\neg[U\subseteq U])\,\hat{=}\,1\,\hat{-}\,1\,\hat{\cdot}\,1\,\hat{=}\,1\,\hat{-}\,1\,\hat{=}\,0.\end{aligned} \tag{3.34$_1$}$$

$$\begin{aligned} &V(\neg[\mathbf{u}=U])\,\hat{=}\,V(\neg[[\mathbf{u}\subseteq U]\wedge[U\subseteq \mathbf{u}]])\\ &\hat{=}\,V(\neg[\mathbf{u}\subseteq U])\,\hat{\cdot}\,V(\neg[U\subseteq \mathbf{u}])\,\hat{=}\,1\,\hat{\cdot}\,0\,\hat{=}\,0.\end{aligned} \tag{3.35$_1$}$$

$$V(\mathbf{u}\subset U)\,\hat{=}\,1\,\hat{-}\,V(\neg[\mathbf{u}\subseteq U])\,\hat{\cdot}\,V(U\subseteq \mathbf{u})\,\hat{=}\,1\,\hat{-}\,1\,\hat{\cdot}\,1\,\hat{=}\,0\,. \tag{3.36$_1$}$$

$$V(\neg[U\subset \mathbf{u}])\,\hat{=}\,V(\neg[U\subseteq \mathbf{u}])\,\hat{\cdot}\,V(\mathbf{u}\subseteq U)\,\hat{=}\,0\,\hat{\cdot}\,1\,\hat{=}\,0\,. \tag{3.37$_1$}$$

$$\begin{aligned} &V\left(\bigwedge\nolimits_{\mathbf{u}}[\mathbf{u}\in U]\right)\hat{=}\,1\,\hat{-}\,\hat{\cdot}_{\mathbf{u}}[1\,\hat{-}\,V(\mathbf{u}\in U)]\,\hat{=}\,1\,\hat{-}\,\hat{\cdot}_{\mathbf{u}}[1\,\hat{-}\,0]\,\hat{=}\,1\,\hat{-}\,\hat{\cdot}_{\mathbf{u}}1\,\hat{=}\,0,\\ &V\left(\bigvee\nolimits_{\mathbf{u}}[\mathbf{u}\in U]\right)\hat{=}\,\hat{\cdot}_{\mathbf{u}}V(\mathbf{u}\in U)\,\hat{=}\,\hat{\cdot}_{\mathbf{u}}0\,\hat{=}\,0,\end{aligned} \tag{3.38$_1$}$$

which prove (3.31)–(3.38) respectively. In developing the final result in (3.33$_1$) that proves (3.33), use of (2.7) has been made. In developing the train (3.34$_1$), use of the final result of the train has been made¸ whereas in developing each one of the trains (3.35$_1$)–(3.37$_1$), use of the final results of the preceding trains (3.32$_1$) and (3.33$_1$) has been made. Identity (3.38$_1$) that proves (3.38) has been developed by making use of (3.29). Likewise, in order to prove (3.39)–(3.44), the corresponding trains of identities analogous to (3.38$_1$) can be developed straightforwardly by making use of (3.30) and (3.33)–(3.37) instead of (3.29), respectively. Statement (3.45) is an instance of the SSR1, Ax II.4.18(1). QED.•

Cmt 3.7. By the pertinent instances of (II.4.40a), identities (3.31)–(3.45) are tantamount to the following logical kyrologies in that order:

$$U\subseteq U\,. \tag{3.31a}$$

$$\mathbf{u}\subseteq U\,. \tag{3.32a}$$

$$\neg[U\subseteq \mathbf{u}]. \tag{3.33a}$$

$$U=U\,. \tag{3.34a}$$

$$\neg[\mathbf{u}=U]. \tag{3.35a}$$

$$\mathbf{u}\subset U\,. \tag{3.36a}$$

$$\neg[U\subset \mathbf{u}]. \tag{3.37a}$$

$$\bigwedge\nolimits_{\mathbf{u}}[\mathbf{u}\in U],\ \bigvee\nolimits_{\mathbf{u}}[\mathbf{u}\in U]. \tag{3.38a}$$

$$\bigwedge\nolimits_{\mathbf{u}}\neg[U\in \mathbf{u}],\ \bigvee\nolimits_{\mathbf{u}}\neg[U\in \mathbf{u}]. \tag{3.39a}$$

$$\bigwedge\nolimits_{\mathbf{u}}[\mathbf{u}\subseteq U],\ \bigvee\nolimits_{\mathbf{u}}[\mathbf{u}\subseteq U]. \tag{3.40a}$$

$$\bigwedge\nolimits_{\mathbf{u}}\neg[U\subseteq \mathbf{u}],\ \bigvee\nolimits_{\mathbf{u}}\neg[U\subseteq \mathbf{u}]. \tag{3.41a}$$

$$\bigwedge\nolimits_{\mathbf{u}}\neg[\mathbf{u}=U],\ \bigvee\nolimits_{\mathbf{u}}\neg[\mathbf{u}=U]. \tag{3.42a}$$

$$\bigwedge_{\mathbf{u}}[\mathbf{u} \subset U],\ \bigvee_{\mathbf{u}}[\mathbf{u} \subset U]. \tag{3.43a}$$

$$\bigwedge_{\mathbf{u}}\neg[U \subset \mathbf{u}],\ \bigvee_{\mathbf{u}}\neg[U \subset \mathbf{u}]. \tag{3.44a}$$

Identities (3.33a)–(3.37a) hold with $\emptyset$ as **u**. (3.45a)•

Cmt 3.8. A concrete *euautographic interpretand,* or in other words *euautographic instance*¸ of any one of the *panlogographic kyrologies*, i.e. *valid panlogographic relations* (*PLR's*), (3.26), (3.27), (3.29), (3.30), (3.32), (3.33), (3.35)–(3.44), and also (3.32a), (3.33a), and (3.35a)–(3.44a) can be written down immediately as their corollaries, i.e. without any proofs, by substituting any one of the APVOT's of the list (II.5.1) or the APCOT $\emptyset$ for the every occurrence of '**x**' or '**u**' in the PLR. For the sake of being specific, let '**x**' and '**u**' be replaced either with x and u respectively or with $\emptyset$ both. The CFCL interpretands of the concrete ER's thus obtained and also of the ER's (3.25), (3.28), (3.31), (3.34), (3.31a), and (3.34a) are, in agreement with Ax II.8.2, obtained by replacing occurrences of U, x, u, and $\emptyset$ throughout the ER's with occurrences of 'U', 'x', 'u', and '$\varnothing$' respectively without any quotation marks. Thus, for instance,

$$V(\neg[U \in U]) \hat{=} 0, \tag{3.28κ}$$

$$V(x \in U) \hat{=} 0, \tag{3.29κ}$$

$$V(\neg[U \in x]) \hat{=} 0, \tag{3.30κ}$$

$$V(U \subseteq U) \hat{=} 0, \tag{3.31κ}$$

$$V(u \subseteq U) \hat{=} 0, \tag{3.32κ}$$

$$V(\neg[U \subseteq u]) \hat{=} 0, \tag{3.33κ}$$

$$V(U = U) \hat{=} 0, \tag{3.34κ}$$

$$V(\neg[u = U]) \hat{=} 0, \tag{3.35κ}$$

$$V(u \subset U) \hat{=} 0, \tag{3.36κ}$$

$$V(\neg[U \subset u]) \hat{=} 0 \tag{3.37κ}$$

are the CFCL interpretands of their euautographic interpretantia with U, x, and u in place of 'U', 'x', and 'u' respectively. The *catlogographic relations* (*CLR's*) (3.28κ)–(3.37κ) are *catlogographic master*, or *decision*, *theorems* (*CLMT's* or *CLDT's*) *of their catlogographic slave-relations* (*CLSR's*), parenthesized as an argument of the operator V, the understanding being that both a CLMT (CLDT) and its CLSR are *tautologies*, i.e. *tautologous* (*universally true*), and hence *valid* (*kyrologous*) CLR's, whereas the EMT's (EDT's) and ESR's, being euautographic interpretantia of the CLMT (CLDT) and CLSR respectively, are *kyrologies*, i.e. semantically uninterpreted valid (kyrologous) ER's.•

Cmt 3.9. 1) If a relation $\mathbf{P}\langle\mathbf{x}\rangle$ has been decided (postulated or proved) in $\mathsf{A}_{1\in\mathsf{D}}$ or $\mathsf{A}_{1\in}$ to be valid or antivalid or vav-neutral then the relation $\mathbf{P}\langle U\rangle$ should not necessarily have the same validity-value. For instance, the relations: $x\in y$, $y\in x$, $\neg[x\in y]$, and $\neg[y\in x]$ are postulated both in $\mathsf{A}_{1\in\mathsf{D}}$ and in $\mathsf{A}_{1\in}$ to be vav-neutral, whereas $\neg[x\in x]$ is proved in $\mathsf{A}_{1\in}$ to be valid (see Th 2.2). At the same time, by (3.25)–(3.27), the relations $\mathrm{S}_U^x\neg[x\in x]\rfloor$, $\mathrm{S}_U^y[x\in y]\rfloor$, and $\mathrm{S}_U^y\neg[y\in x]\rfloor$ are postulated to be valid in $\bar{\mathsf{A}}_{1\in}$, whereas the relations $\mathrm{S}_U^y[y\in x]\rfloor$ and $\mathrm{S}_U^y\neg[x\in y]\rfloor$ are proved to be antivalid in $\bar{\mathsf{A}}_{1\in}$. Also, in accordance with Ax 3.3, given a validity-integron $\mathbf{i}\langle\mathbf{x}\rangle$, the validity-integron $\mathbf{i}\langle U\rangle\,\hat{\cdot}\,\hat{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle$ is not tantamount to $\hat{\cdot}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x}\rangle$. That is to say, U cannot be incorporated into the Emission and Absorption Law, Ax II.4.11. Therefore, if a relation $\mathbf{P}\langle\mathbf{x}\rangle$ has been proved in $\mathsf{A}_{1\in\mathsf{D}}$ or $\mathsf{A}_{1\in}$ with the help of the Emission Law to be valid or antivalid or vav-neutral, that proof is not effective for $\mathbf{P}\langle U\rangle$ even if $\mathbf{P}\langle U\rangle$ turns out to have the same validity-value owing to axioms (3.25)–(3.27). For instance, by (1.45) and (1.46), both relations $\bigvee_u[u\subseteq v]$ and $\bigvee_v[u\subseteq v]$ have been proved with the help of the Emission Law to be valid in $\mathsf{A}_{1\in\mathsf{D}}$ and hence in $\mathsf{A}_{1\in}$. At the same time, $\bigvee_u[u\subseteq U]$, i.e. $\mathrm{S}_U^v\bigvee_u[u\subseteq v]\rfloor$, turns out to be valid by (3.26), while $\bigvee_v[U\subseteq v]$, i.e. $\mathrm{S}_U^u\bigvee_v[u\subseteq v]\rfloor$, turns out to be antivalid by (3.27). $\bar{\mathsf{A}}_{1\in}$

2) In connection of the above-said, Th II.8.10, in which the Weak General Law of Denial of Russell's Paradox (WGLDRP) has been proved for A_1, should be revised if it involves the extraordinary term U. It will be recalled that there are two somewhat different proofs of the WGLDRP, one of which applies to a *reflexive* ER and the other one to a *non-reflexive* ER. According to (1.35), an ER $\mathbf{x}\subseteq\mathbf{y}$ is *reflexive*, whereas according (2.5), an ER $\mathbf{x}\in\mathbf{y}$ is *antireflexive* and hence *non-reflexive*. Therefore, the proofs of theorem (II.8.23), i.e. of the WGLDRP, for $\mathbf{x}\subseteq\mathbf{y}$ and for $\mathbf{x}\in\mathbf{y}$ as $\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$ are the following trains of identities:

$$
\begin{aligned}
&V\left(\bigvee_{\mathbf{x}}\neg[[\mathbf{x}\subseteq\mathbf{y}]\wedge\neg[\mathbf{x}\subseteq\mathbf{x}]]\right)\hat{=}V\left(\neg\bigwedge_{\mathbf{x}}[[\mathbf{x}\subseteq\mathbf{y}]\wedge\neg[\mathbf{x}\subseteq\mathbf{x}]]\right)\\
&\hat{=}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V(\neg[\mathbf{x}\subseteq\mathbf{y}])\,\hat{\cdot}\,V(\mathbf{x}\subseteq\mathbf{x})\right]\hat{=}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V(\neg[\mathbf{x}\subseteq\mathbf{y}])\,\hat{\cdot}\,0\right]\hat{=}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}0\,\hat{=}\,0,
\end{aligned}
\tag{3.46}
$$

$$
\begin{aligned}
&V\left(\bigvee_{\mathbf{x}}\neg[[\mathbf{x}\in\mathbf{y}]\wedge\neg[\mathbf{x}\in\mathbf{x}]]\right)\hat{=}V\left(\neg\bigwedge_{\mathbf{x}}[[\mathbf{x}\in\mathbf{y}]\wedge\neg[\mathbf{x}\in\mathbf{x}]]\right)\\
&\hat{=}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V(\neg[\mathbf{x}\in\mathbf{y}])\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{x})\right]\hat{=}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V(\neg[\mathbf{x}\in\mathbf{y}])\,\hat{\cdot}\,1\right]\hat{=}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V(\neg[\mathbf{x}\in\mathbf{y}])\\
&\hat{=}V(\neg[\mathbf{y}\in\mathbf{y}])\,\hat{\cdot}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V(\neg[\mathbf{x}\in\mathbf{y}])\,\hat{=}\,0\,\hat{\cdot}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}V(\neg[\mathbf{x}\in\mathbf{y}])\,\hat{=}\,0,
\end{aligned}
\tag{3.47}
$$

which are the pertinent instances of the proofs a and b of that theorem respectively.

a) The final result in (3.46) is predetermined by the reflexivity law:

$$V(\mathbf{x}\subseteq\mathbf{x})\,\hat{=}\,0, \tag{3.46$_1$}$$

being a variant of (1.35). Therefore, the variant (3.46) with U in place of '**y**' remain valid.

b) By contrast, in developing (3.47), use of the following three relations is made in that order:

i) the antireflexivity law

$$V(\mathbf{x}\in\mathbf{x})\,\hat{=}\,1, \tag{3.47$_1$}$$

ii) the instance of the Emission Law (II.4.28) with $V(\neg[\mathbf{x}\in\mathbf{y}])$ as $\mathbf{i}\langle\mathbf{x},\mathbf{y}\rangle$,

iii) the antireflexivity law

$$V(\neg[\mathbf{y}\in\mathbf{y}])\,\hat{=}\,0\,. \tag{3.47$_2$}$$

Since the proof (3.47) involves the Emission Law, therefore that proof is inapplicable if **y** is replaced with U. In this case, the pertinent valid proof is the following one:

$$
\begin{aligned}
&V\left(\bigvee_{\mathbf{x}}\neg[[\mathbf{x}\in U]\wedge\neg[\mathbf{x}\in\mathbf{x}]]\right)\hat{=}V\left(\neg\bigwedge_{\mathbf{x}}[[\mathbf{x}\in U]\wedge\neg[\mathbf{x}\in\mathbf{x}]]\right)\\
&\hat{=}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V(\neg[\mathbf{x}\in U])\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{x})\right]\hat{=}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}[1\,\hat{\cdot}\,1]\,\hat{=}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}1\,\hat{=}\,1,
\end{aligned}
\tag{3.48}
$$

where use of (3.26) has been made instead of the Emission Law (II.4.28). Thus, either of the two equivalent slave-relations:

'$\bigvee_{\mathbf{x}}\neg[[\mathbf{x}\in U]\wedge\neg[\mathbf{x}\in\mathbf{x}]]$' and '$\neg\bigwedge_{\mathbf{x}}[[\mathbf{x}\in U]\wedge\neg[\mathbf{x}\in\mathbf{x}]]$'

of the respective two equivalent master-theorems that are proved by (3.48) turn out to be *antivalid*, i.e. they are *slave-antitheorem*. Consequently, the pertinent slave-theorem

$$\bigwedge_{\mathbf{x}}[[\mathbf{x}\in U]\wedge\neg[\mathbf{x}\in\mathbf{x}]] \tag{3.49}$$

is proved by the following master-theorem (decision-theorem):

$$
\begin{aligned}
&V\left(\bigwedge_{\mathbf{x}}[[\mathbf{x}\in U]\wedge\neg[\mathbf{x}\in\mathbf{x}]]\right)\hat{=}\,1\,\hat{-}\,V\left(\neg\bigwedge_{\mathbf{x}}[[\mathbf{x}\in U]\wedge\neg[\mathbf{x}\in\mathbf{x}]]\right)\\
&\hat{=}\,1\,\hat{-}\,\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V(\neg[\mathbf{x}\in U])\,\hat{\cdot}\,V(\mathbf{x}\in\mathbf{x})\right]\hat{=}\,1\,\hat{-}\,1\,\hat{=}\,0.
\end{aligned}
\tag{3.50}
$$

3) In general, an AEADP of any given relation that involves a pseudo-quantifier over $\mathbf{x}$ of any one of the relations $[\mathbf{x}\in U]$, $\neg[\mathbf{x}\in U]$, $[U\in\mathbf{x}]$, and $\neg[U\in\mathbf{x}]$ unavoidably involves, at a certain stage, the respective one of the integrons $\hat{\cdot}_{\mathbf{x}}V(\mathbf{x}\in U)$, $\hat{\cdot}_{\mathbf{x}}V(\neg[\mathbf{x}\in U])$, $\hat{\cdot}_{\mathbf{x}}V(U\in\mathbf{x})$, and $\hat{\cdot}_{\mathbf{x}}V(\neg[U\in\mathbf{x}])$, which is immediately reduceds by means of (3.29) and (3.30) as:

$$\begin{gathered}\hat{\cdot}_{\mathbf{x}}V(\mathbf{x}\in U)\,\hat{=}\,\hat{\cdot}_{\mathbf{x}}0\,\hat{=}\,0,\hat{\cdot}_{\mathbf{x}}V(\neg[\mathbf{x}\in U])\,\hat{=}\,\hat{\cdot}_{\mathbf{x}}1\,\hat{=}\,1,\\ \hat{\cdot}_{\mathbf{x}}V(U\in\mathbf{x})\,\hat{=}\,\hat{\cdot}_{\mathbf{x}}1\,\hat{=}\,1,\hat{\cdot}_{\mathbf{x}}V(\neg[U\in\mathbf{x}])\,\hat{=}\,\hat{\cdot}_{\mathbf{x}}0\,\hat{=}\,0\end{gathered} \tag{3.51}$$

instead of Ax II.4.11.•

4. The organons $\mathrm{A}_{1\subseteq}$, $\bar{\mathbf{A}}_{1\subseteq}$, and $\mathrm{A}_{1=}$

4.1. The organons $\mathrm{A}_{1\subseteq}$

4.1.1. Basic definitions

†**Df 4.1.** 1) Besides the *infinite set* κ^{pv} *of atomic pseudo-variable ordinary predicate-signs* (*APVOPS's*) *of all weights from 1 to infinity* that are defined by (1.12) in accordance with Ax I.5.1(7), the branch of A_1, which is [logographically] denoted by '$\mathrm{A}_{1\subseteq}$' and is [phonographically] called the *Pseudo-Mass EAPO* (*PMsEAPO*), meaning the *EAPO of Pseudo-Masses*, has a single *distinguished primary binary atomic pseudo-constant ordinary predicate-sign* (*PBAPCOPS*) $\subseteq$ that is introduced in accordance with Ax I.5.1(8b) and it also has *nine secondary binary primitive* (*elemental*, *atomic or molecular*) *pseudo-constant ordinary predicate-signs* (*SBPPCOPS's*) that are defined in terms $\subseteq$ by the following *secondary formation rules*:

$$[\mathbf{u}\subseteq\mathbf{v}]\to\subseteq(\mathbf{u},\mathbf{v})\leftarrow\supseteq(\mathbf{v},\mathbf{u})\leftarrow[\mathbf{v}\supseteq\mathbf{u}], \tag{4.1}$$

$$\begin{gathered}[\mathbf{u}\,\overline{\subseteq}\,\mathbf{v}]\to\neg[\mathbf{u}\subseteq\mathbf{v}]\to\overline{\subseteq}(\mathbf{u},\mathbf{v})\to\neg\,\overline{\subseteq}(\mathbf{u},\mathbf{v})\\ \leftarrow\neg\,\overline{\supseteq}(\mathbf{v},\mathbf{u})\leftarrow\overline{\supseteq}(\mathbf{v},\mathbf{u})\leftarrow\neg[\mathbf{v}\,\overline{\supseteq}\,\mathbf{u}]\leftarrow[\mathbf{v}\,\overline{\supseteq}\,\mathbf{u}],\end{gathered} \tag{4.2}$$

$$[\mathbf{u}=\mathbf{v}]\to=(\mathbf{u},\mathbf{v})\to[[\mathbf{u}\subseteq\mathbf{v}]\wedge[\mathbf{v}\subseteq\mathbf{u}]], \tag{4.3}$$

$$[\mathbf{u}\equiv\mathbf{v}]\to\neg[\mathbf{u}=\mathbf{v}]\to\neg=(\mathbf{u},\mathbf{v})\leftarrow\equiv(\mathbf{u},\mathbf{v}), \tag{4.4}$$

$$[\mathbf{u}\subset\mathbf{v}]\to\subset(\mathbf{u},\mathbf{v})\to[[\mathbf{u}\subseteq\mathbf{v}]\wedge\neg[\mathbf{v}\subseteq\mathbf{u}]]\leftarrow\supset(\mathbf{v},\mathbf{u})\leftarrow[\mathbf{v}\supset\mathbf{u}], \tag{4.5}$$

$$\begin{gathered}[\mathbf{u}\,\overline{\subset}\,\mathbf{v}]\to\neg[\mathbf{u}\subset\mathbf{v}]\to\overline{\subset}(\mathbf{u},\mathbf{v})\to\neg\subset(\mathbf{u},\mathbf{v})\\ \leftarrow\neg\supset(\mathbf{v},\mathbf{u})\leftarrow\overline{\supset}(\mathbf{v},\mathbf{u})\leftarrow\neg[\mathbf{v}\,\overline{\supset}\,\mathbf{u}]\leftarrow[\mathbf{v}\,\overline{\supset}\,\mathbf{u}],\end{gathered} \tag{4.6}$$

which have the status of *meta-axioms*. In this case, the train of definitions (4.1) is formally a part of the train (1.3), whereas the secondary formation rules (4.2)–(4.6) are formally identical with the secondary formation rules (1.3)–(1.8) of $\mathsf{A}_{1\in}$ in that order. It goes without saying that definitions (4.1)–(4.6) are in agreement with the general definition schema (1.9).

2) The predicate-sign $\subseteq$ is subjugated to three specific (atypical) subject axioms (laws): the *axiom of reflexivity*, the *axiom of transitivity*, and also the *axiom of incidence for a lax* (*weak*) *anti-inclusion relation* $[\mathbf{u} \sqsubseteq \mathbf{v}]$, defined by (4.3), *relative to a pseudo-submass term* **u**. The former two specific axioms have homographic specific (atypical) theorems in $\mathsf{A}_{1\in\mathsf{G}}$, whereas the latter specific axiom has homographic specific theorem in $\mathsf{A}_{1\in\mathsf{D}}$. The conventional *laws of reflexivity, symmetry, and transitivity of the sign* =, defined by (4.3), and the *incidence law for an anti-identity* [*relation*] $[\mathbf{u} \equiv \mathbf{v}]$, defined by (4.4), turn out to be specific theorems of $\mathsf{A}_{1\subseteq}$, which have homographic specific theorems in $\mathsf{A}_{1\in\mathsf{G}}$ and in $\mathsf{A}_{1\in\mathsf{D}}$ respectively.

3) Besides an *infinite set* τ^{pv} *of atomic pseudo-variable ordinary terms* (*APVOT's, PVOT's*), defined by (1.10) in accordance with Ax I.5.1(5), $\mathsf{A}_{1\subseteq}$ has *two atomic pseudo-variable ordinary terms* (*APCOT's, PCOT's*) $\emptyset$ and $\emptyset'$, the first of which is called the *euautographic empty pseudo-mass* (*EEPMs*) or *euautographic empty pseudo-individual* (*EEPIl*), while the second one is called the *subsidiary EEPMs* or *subsidiary EEPIl*. Thus, $\mathsf{A}_{1\subseteq}$ is a *one-pseudo-individual pseudo-mass theory*. The *euautographic ordinary terms* (*EOT's*), i.e. *PCOT's and PVOT's*, of $\mathsf{A}_{1\subseteq}$ are alternatively called *pseudo-masses*, because they are interrelated by the predicate-sign $\subseteq$ and are not interrelated by the predicate-sign $\in$, which is not available. The PCOT's $\emptyset$ and $\emptyset'$ are subjugated to two similar specific *axioms of pseudo-emptiness*: $\emptyset \subseteq \mathbf{u}$ and $\emptyset' \subseteq \mathbf{v}$, which allow proving that $\emptyset' = \emptyset$. The two axioms have homographic specific theorems in $\mathsf{A}_{1\in\mathsf{S}}$, i.e. in $\mathsf{A}_{1\in}$.

4) In analogy with phasing of $\mathsf{A}_{1\in}$, the three consecutive phases (stages) of the setup of $\mathsf{A}_{1\subseteq}$, which are indicated in the above items 1–3, can be called the *Ground PMsEAPO* (*GPMsEAPO*), the *Deficient PMsEAPO* (*DPMsEAPO*), and the *Sufficient PMsEAPO* (*SPMsEAPO*), and be denoted logographically by '$\mathsf{A}_{1\subseteq\mathsf{G}}$', '$\mathsf{A}_{1\subseteq\mathsf{D}}$', and '$\mathsf{A}_{1\subseteq\mathsf{S}}$' in that order, the understanding being that $\mathsf{A}_{1\subseteq\mathsf{S}}$ is identical with $\mathsf{A}_{1\subseteq}$. However, an attempt to distinguish between $\mathsf{A}_{1\subseteq\mathsf{G}}$ and $\mathsf{A}_{1\subseteq\mathsf{D}}$ turns out to be counterproductive for the following reason. Theorems (1.35) and (1.42) of $\mathsf{A}_{1\in\mathsf{G}}$ and theorem (2.19) of $\mathsf{A}_{1\in\mathsf{D}}$ will be taken for granted as axioms of $\mathsf{A}_{1\subseteq\mathsf{D}}$. Consequently, the theorems of $\mathsf{A}_{1\in\mathsf{G}}$,

which are proved in section 1 from theorems (1.35) and (1.42), become theorems of $A_{1\subseteq D}$, whereas all other theorems, which are proved in section 1 without involving the predicate $\in$, become theorems of $A_{1\subseteq G}$. Therefore, for avoidance of confusion, I shall not regard $A_{1\subseteq G}$ as a separate phase of $A_{1\subseteq}$ and I shall begin with setting up $A_{1\subseteq D}$ that includes $A_{1\subseteq G}$ from the very beginning.

5) By way of emphatic comparison with the extension of $A_{1\subseteq}$, which will be discussed in subsection 4.2 of this section and which will be denoted by '$\bar{A}_{1\subseteq}$' and be called the *Pseudo-Confined PMsEAPO*¸ $A_{1\subseteq}$ can alternatively be called the *Pseudo-Unconfined PMsEAPO.* A certain part of this IML (this treatise), with the help of which and within which $A_{1\subseteq}$ is developed (set up and executed), is called the *Euautographic Pseudo-Mass Theory* (*EPMsT*) or alternatively and more precisely the *Pseudo-Unconfined EPMsT*.•

4.1.2. The organon $A_{1\subseteq D}$

Ax 4.1:** ***The basic laws for $\subseteq$ in subjective (logical) form.

$$\mathbf{u} \subseteq \mathbf{u}\,. \qquad \text{(Reflexivity law)} \quad (4.7)$$

$$[[\mathbf{u} \subseteq \mathbf{v}] \wedge [\mathbf{v} \subseteq \mathbf{w}]] \Rightarrow [\mathbf{u} \subseteq \mathbf{w}]. \qquad \text{(Transitivity law)} \quad (4.8)$$

$$\vee_{\mathbf{u}} \neg[\mathbf{u} \subseteq \mathbf{v}]. \qquad \text{(Incidence law for anti-inclusions)} \quad (4.9)$$

The last law can alternatively be called *Law of non-triviality of* $A_{1\subseteq}$.•

Th 4.1:** ***The basic laws for $\subseteq$ in objective (algebraic) form.

$$V(\mathbf{u} \subseteq \mathbf{u}) \hat{=} 0\,. \qquad \text{(Reflexivity law)} \quad (4.10)$$

$$[V(\neg[\mathbf{u} \subseteq \mathbf{v}]) \hat{\cdot} V(\neg[\mathbf{v} \subseteq \mathbf{w}])] \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{w}) \hat{=} 0\,. \qquad \text{(Transitivity law)} \quad (4.11)$$

$$V(\vee_{\mathbf{u}} \neg[\mathbf{u} \subseteq \mathbf{v}]) \hat{=} \hat{\prod}_{\mathbf{u}} V(\neg[\mathbf{u} \subseteq \mathbf{v}]) \hat{=} \hat{\prod}_{\mathbf{u}} [1 \hat{-} V(\mathbf{u} \subseteq \mathbf{v})] \hat{=} 0.$$

$$\text{(Incidence law for anti-inclusions)} \quad (4.12)$$

Proof: The theorem follows from Ax 4.1 by axiom (II.4.40a). In this case, (4.11) is proved by the following train of identities (valid equalities):

$$\begin{aligned} &V([[\mathbf{u} \subseteq \mathbf{v}] \wedge [\mathbf{v} \subseteq \mathbf{w}]] \Rightarrow [\mathbf{u} \subseteq \mathbf{w}]) \\ &\hat{=} V(\neg[[\mathbf{u} \subseteq \mathbf{v}] \wedge [\mathbf{v} \subseteq \mathbf{w}]]) \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{w}) \\ &\hat{=} [1 \hat{-} V([\mathbf{u} \subseteq \mathbf{v}] \wedge [\mathbf{v} \subseteq \mathbf{w}])] \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{w}) \\ &\hat{=} [V(\neg[\mathbf{u} \subseteq \mathbf{v}]) \hat{\cdot} V(\neg[\mathbf{v} \subseteq \mathbf{w}])] \hat{\cdot} V(\mathbf{u} \subseteq \mathbf{w}), \end{aligned} \qquad (4.11_1)$$

where use of the pertinent instances of theorems (II.7.3γ), (II.7.1γ), and (II.7.6γ) has been made in that order. The train of identities (4.12) is developed by the pertinent instances of (II.4.23) and (II.7.1γ). The final result ($\hat{=} 0$) in each one of the relations (4.10)–(4.12) is taken for granted.•

Cmt 4.1. 1) Axioms (4.9)–(4.11) are homographs of theorems (1.35), (1.42), and (2.19) respectively.

2) To say nothing of equivalences (1.16) and (1.17) of Lemma 1.1 and of identities (1.22) and (1.23) of Lemma 1.2, the proofs of the following theorems of the organon $A_{1\in G}$, which are given in section 1 of this chapter, are based on the fact that an identity proved is or is supposed to be explicitly expressed in terms of $\in$ by a certain version of identity (1.22) or (1.23) of Lemma 1.2, rather than to be expressed just in terms of $\subseteq$ in no explicit connection with $\in$: identity (1.35) of Th 1.5, Lemma 1.3, and Ths 1.7–1.9. Therefore, these theorems of $A_{1\in G}$ are not theorems of $A_{1\subseteq D}$. The rest of theorems of the section 1, namely equivalences (1.18)–(1.21) of Lemma 1.1, identities (1.24)–(1.27) of Lemma 1.2, and Ths 1.1–1.6 and 1.10–1.16 can be regarded as ones that are proved from definitions (1.4)–(1.8) and theorems (1.35) and (1.42) of $A_{1\in G}$ or from the respective conformal definitions (4.2)–(4.6) and respective conformal axioms (4.7)–(4.9) of $A_{1\subseteq D}$.

3) A like remark applies to the theorems of $A_{1\in D}$ proved in section 2 of this chapter. Namely, the theorems of $A_{1\in D}$, which have been proved in terms of $\in$, are not theorems of $A_{1\subseteq D}$, whereas the theorems of $A_{1\in D}$, which have been proved in terms of $\subseteq$ in no explicit connection with $\in$, are alss theorems of $A_{1\subseteq D}$. For instance, Th 2.5, i.e. identity (2.19), has been proved in terms of $\in$, and therefore that identity is now taken for granted as axiom (2.12) of $A_{1\subseteq D}$. By contrast, Th 2.7, i.e. identities (2.21) and (2.21_+) for = have been proved straightforwardly in terms of $\subseteq$, and therefore these identities are theorems of $A_{1\subseteq D}$.

4) Thus, particularly, the identities (1.36), (1.38), (1.43), and (2.21), which are master-theorems of the basic laws in subjective (logical) form for the sign = of $A_{1\in}$, defined by definition (1.5), and which are themselves those same basic laws in objective (algebraic) form, are also master-theorems of the basic laws in subjective (logical) form for the *homographic* sign = of $A_{1\subseteq}$, defined by definition (4.3), and are themselves those same basic laws in objective form. For convenience in the further

discussion, the above master theorems and their slave theorems are cited below under the appropriate heads respectively as (4.13)–(4.16) and as (4.13a)–(4.16a) in that order.•

*Th 4.2: ***The basic laws for the sign*** = ***of*** $\mathsf{A}_{1\in}$ ***and for the homographic sign*** = ***of*** $\mathsf{A}_{1\subseteq}$ ***in objective (algebraic) form.***

$$V(\mathbf{u}=\mathbf{u})\,\hat{=}\,0. \qquad \text{(Reflexivity law)} \qquad (4.13)$$

$$V(\mathbf{u}=\mathbf{v})\,\hat{=}\,V(\mathbf{v}=\mathbf{u}). \qquad \text{(Symmetry law)} \qquad (4.14)$$

$$[V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{\cdot}\,V(\neg[\mathbf{v}=\mathbf{w}])]\,\hat{\cdot}\,V(\mathbf{u}=\mathbf{w})\,\hat{=}\,0. \qquad \text{(Transitivity law)} \qquad (4.15)$$

$$V(\bigvee_{\mathbf{u}}\neg[\mathbf{u}=\mathbf{v}])\,\hat{=}\,\hat{\prod}_{\mathbf{u}}V(\neg[\mathbf{u}=\mathbf{v}])\,\hat{=}\,\hat{\prod}_{\mathbf{u}}[1\,\hat{-}\,V(\mathbf{u}=\mathbf{v})]\,\hat{=}\,0.$$

(Incidence law for anti-equalities) (4.16)

Proof: See proofs of (1.36), (1.38), (1.43), and (2.21) respectively.•

*Th 4.3: ***The basic laws for the sign*** = ***of*** $\mathsf{A}_{1\in}$ ***and for the homographic sign*** = ***of*** $\mathsf{A}_{1\subseteq}$ ***in subjective (algebraic) form.***

$$\mathbf{u}=\mathbf{u}. \qquad \text{(Reflexivity law)} \qquad (4.13a)$$

$$[\mathbf{u}=\mathbf{v}]\Leftrightarrow[\mathbf{v}=\mathbf{u}]. \qquad \text{(Symmetry law)} \qquad (4.14a)$$

$$[[\mathbf{u}=\mathbf{v}]\wedge[\mathbf{v}=\mathbf{w}]]\Rightarrow[\mathbf{u}=\mathbf{w}]. \qquad \text{(Transitivity law)} \qquad (4.15a)$$

$$\bigvee_{\mathbf{u}}\neg[\mathbf{u}=\mathbf{v}]. \qquad \text{(Incidence law for anti-equalities)} \qquad (4.16a)$$

Proof: By (II.4.40a), algebraic (special) identities (4.13)–(4.16) and the respective logical (ordinary) kyrologies (4.13a)–(4.16a), being the same as (1.36a), (1.38a), (1.43a), and (2.21a) in that order, are mutually equivalent.•

C**mt 4.2.** By the pertinent instances of (II.7.50) and (II.7.7γ), Th 1.8 that has been proved for $\mathsf{A}_{1\in G}$ can be written as:

$$\begin{aligned}&V([\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]]\Leftrightarrow[\mathbf{u}=\mathbf{v}])\\ &\hat{=}\,V([\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]]\Rightarrow[\mathbf{u}=\mathbf{v}])\\ &\hat{+}\,V([\mathbf{u}=\mathbf{v}]\Rightarrow\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]])\,\hat{=}\,0.\end{aligned} \qquad (4.17)$$

In what follows, the direct part of (4.17) is proved as a theorem of $\mathsf{A}_{1\subseteq G}$, i.e. as a theorem of $\mathsf{A}_{1\subseteq D}$ that is irrelevant to any of the axioms (4.7)–(4.9). However, the converse part of (4.17) cannot be proved in the framework of $\mathsf{A}_{1\subseteq G}$.•

***Th 4.4.**

$$V([\bigwedge_{\mathbf{z}}[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\Rightarrow[\mathbf{u}=\mathbf{v}])\,\hat{=}\,0, \qquad (4.18)$$

whence, by (II.4.40a),

$$\left[\bigwedge_{\mathbf{z}}[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]\right]\Rightarrow[\mathbf{u}=\mathbf{v}]. \tag{4.18a}$$

That is to say, the PLR and hence every ER of its range is valid, whereas the converse PLR and hence every ER of its range is vav-neutral.

Proof: By the pertinent instances of (II.7.1γ), (II.7.7γ), and (II.8.2), it follows that

$$\begin{aligned}
&V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right)\hat{=}\hat{\bullet}_{\mathbf{z}}V(\neg[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]])\\
&\hat{=}\hat{\bullet}_{\mathbf{z}}[1\hat{-}V([\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}])]\\
&\hat{=}\hat{\bullet}_{\mathbf{z}}[1\hat{-}V(\mathbf{z}\subseteq\mathbf{u})\hat{-}V(\mathbf{z}\subseteq\mathbf{v})\hat{+}2\hat{\cdot}V(\mathbf{z}\subseteq\mathbf{u})\hat{\cdot}V(\mathbf{z}\subseteq\mathbf{v})]\\
&\hat{=}[1\hat{-}V(\mathbf{u}\subseteq\mathbf{u})\hat{-}V(\mathbf{u}\subseteq\mathbf{v})\hat{+}2\hat{\cdot}V(\mathbf{u}\subseteq\mathbf{u})\hat{\cdot}V(\mathbf{u}\subseteq\mathbf{v})]\\
&\hat{\cdot}\hat{\bullet}_{\mathbf{z}}[1\hat{-}V(\mathbf{z}\subseteq\mathbf{u})\hat{-}V(\mathbf{z}\subseteq\mathbf{v})\hat{+}2\hat{\cdot}V(\mathbf{z}\subseteq\mathbf{u})\hat{\cdot}V(\mathbf{z}\subseteq\mathbf{v})]\\
&\hat{=}[1\hat{-}0\hat{-}V(\mathbf{u}\subseteq\mathbf{v})\hat{+}2\hat{\cdot}0\hat{\cdot}V(\mathbf{u}\subseteq\mathbf{v})]\hat{\cdot}\hat{\bullet}_{\mathbf{z}}V(\neg[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]])]\\
&\hat{=}V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right),
\end{aligned} \tag{4.18$_1$}$$

where use of the pertinent version of the Emission Law (II.4.27) with substitution of **u** for **z** in the emitted term has been made in developing the final result. Making use of the like version of the Emission Law with substitution of **v** for **z** in the emitted term yields:

$$\begin{aligned}
&V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right)\\
&\hat{=}V(\neg[\mathbf{v}\subseteq\mathbf{u}])\hat{\cdot}V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right).
\end{aligned} \tag{4.18$_2$}$$

By (1.25), combination of (4.18$_1$) and (4.18$_2$) yields:

$$\begin{aligned}
&V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right)\\
&\hat{=}[V(\neg[\mathbf{u}\subseteq\mathbf{v}])\hat{\cdot}V(\neg[\mathbf{v}\subseteq\mathbf{u}])]\hat{\cdot}V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right)\\
&\hat{=}V(\neg[\mathbf{u}=\mathbf{v}])\hat{\cdot}V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right).
\end{aligned} \tag{4.18$_3$}$$

Hence, by (4.18$_3$), it follows from the pertinent instance of (II.7.3γ) that

$$\begin{aligned}
&V\left(\left[\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\Rightarrow[\mathbf{u}=\mathbf{v}]\right]\right)\\
&\hat{=}V(\mathbf{u}=\mathbf{v})\hat{\cdot}V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right)\\
&\hat{=}V(\mathbf{u}=\mathbf{v})\hat{\cdot}\left[V(\neg[\mathbf{u}=\mathbf{v}])\hat{\cdot}V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right)\right]\\
&\hat{=}[V(\mathbf{u}=\mathbf{v})\hat{\cdot}V(\neg[\mathbf{u}=\mathbf{v}])]\hat{\cdot}V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right)\\
&\hat{=}0\hat{\cdot}V\left(\neg\bigwedge_{\mathbf{z}}[[\mathbf{z}\subseteq\mathbf{u}]\Leftrightarrow[\mathbf{z}\subseteq\mathbf{v}]]\right)\hat{=}0,
\end{aligned} \tag{4.18$_4$}$$

which proves (4.18).•

4.1.3. The organon $A_{1\subseteq S}$

*__Ax 4.1.__

$$V(\emptyset \subseteq \mathbf{v}) \hat{=} 0, \tag{4.19}$$

whence, by the pertinent instance axiom (II.4.40a),

$$\emptyset \subseteq \mathbf{v}. \tag{4.19a}$$

Cmt 4.3. 1) Axiom (4.19) is a homograph of theorem (3.9), which has been proved by the train of identities (4.19_1) in terms of $\in$.

2) The ER $\emptyset \subseteq \emptyset$ is a concrete instance and hence a corollary axiom (4.7) and therefore it is not required to postulate it one more. Consequently, (4.19) postulates that

$$V(\emptyset \subseteq \mathbf{v}^{pv}) \hat{=} 0, \tag{4.19$_1$}$$

i.e. that, for instance,

$$V(\emptyset \subseteq v) \hat{=} 0 \tag{4.19μ}$$

(cf. Cmt 3.1)).

3) In accordance with Ax II.4.18.1(1), the ER that results by replacing all occurrences of a free APVOT throughout a valid ER of $A_{1\subseteq D}$ is another valid ER of $A_{1\subseteq D}$. Consequently, the PLR that results by replacing of all occurrences a free APLOT throughout a valid PLR of $\mathbf{A}_{1\subseteq D}$ with $\emptyset$ is another valid PLR of $\mathbf{A}_1$. A valid ER or PLR thus obtained is a corollary and therefore it does not require any proof. Particularly, all ER's and all PLR's, which have been given in Corollary 3.1 and which do not involve the predicate-sign $\in$, are homographs of valid relations of $A_{1\subseteq D}$

4) All theorems of $\mathbf{A}_{1\in S}$, which have been proved in subsection 3.1 from theorem (3.9), and not directly from Ax 3.1, i.e. from identity (3.8), remain theorems of $\mathbf{A}_{1\subseteq S}$, which are provable from the homographic axiom (4.19). Th 3.2, i.e. identity (3.16), $\emptyset = \emptyset'$, is the most fundamental theorem, which has been proved from identity (3.8) and from its variant with $\emptyset'$ in place of $\emptyset$. However, it has been indicated in Cmt 3.3 that this theorem can alternatively be proved from identity (3.9) and from its variant with $\emptyset'$ in place of $\emptyset$, so that this proof is applicable also in $\mathbf{A}_{1\subseteq S}$. The pertinent theorem of $\mathbf{A}_{1\subseteq S}$ is stated and proved below.•

Th 4.5:** ***The theorem of uniqueness of $\emptyset$**.** Let $\emptyset'$ be an APCOT, which is introduced into $A_{1\in}$ in accordance with Ax II.5.1(9), and which satisfies the variant of the a````xiom schema (4.19) with $\emptyset'$ in place of $\emptyset$, i.e.

$$V(\emptyset' \subseteq \mathbf{w}) \hat{=} 0. \tag{4.19'}$$

Then

$$\emptyset = \emptyset' . \tag{4.20}$$

Proof: Identity (4.19) with $\mathbf{v} \rhd \emptyset'$ and (4.19') with $\mathbf{w} \rhd \emptyset$ become:

$$V(\emptyset' \subseteq \emptyset) \hat{=} V(\emptyset \subseteq \emptyset') \hat{=} 0 . \tag{4.21}$$

Consequently, (1.24) with $\mathbf{u} \rhd \emptyset$ and $\mathbf{v} \rhd \emptyset'$ yields:

$$V(\emptyset = \emptyset') \hat{=} 1 \hat{-} V(\neg[\emptyset \subseteq \emptyset']) \hat{\cdot} V(\neg[\emptyset' \subseteq \emptyset]) \hat{=} 1 \hat{-} 1 \hat{\cdot} 1 \hat{=} 0 . \tag{4.20$_1$}$$ •

4.2. The organon $\bar{\mathrm{A}}_{1\subseteq}$

†**Df 4.2.** In accordance with Cmt I.7.6(1) and in analogy with $\bar{\mathrm{A}}_{1\in}$ (see subsection 3.2 of this chapter) the organon, which is obtained by supplementing $\mathrm{A}_{1\subseteq}$ with the U-term and with all relevant formation rules, subject axioms, and rule of inference and decision, is denoted logographically by '$\bar{\mathrm{A}}_{1\subseteq}$' and is called phonographically (verbally) the *Pseudo-Restricted*, or *Pseudo-Confined*, *Pseudo-Mass EAPO* (*PMsEAPO*). The formation rules and rules of inference and decision of $\bar{\mathrm{A}}_{1\subseteq}$ are expressed verbatim by the variants of Axs 3.2 and 3.4 with $\subseteq$ in place of $\in$, whereas the following three-fold axiom of universality and three-fold theorem of universality of $\bar{\mathrm{A}}_{1\subseteq}$ comes instead of Ax 3.3 and Th 3.7¸ being the three-fold axiom of universality and three-fold theorem of universality of $\bar{\mathrm{A}}_{1\in}$.•

Ax 4.2:** ***The subject U***-axioms (axioms of universality) of*** $\bar{\mathrm{A}}_{1\subseteq}$ ***in logical form.***

$$U \subseteq U . \tag{4.21}$$

$$\mathbf{u} \subseteq U . \tag{4.22}$$

$$\neg[U \subseteq \mathbf{u}]. \tag{4.23}$$ •

Th 4.6:** ***The subject U***-axioms (axioms of universality) of*** $\bar{\mathrm{A}}_{1\subseteq}$ ***in algebraic form.***

$$V(U \subseteq U) \hat{=} 0 . \tag{4.24}$$

$$V(\mathbf{u} \subseteq U) \hat{=} 0 . \tag{4.25}$$

$$V(\neg[U \subseteq \mathbf{u}]) \hat{=} 1 \hat{-} V(U \subseteq \mathbf{u}) \hat{=} 0 \text{, i.e. } V(U \subseteq \mathbf{u}) \hat{=} 1. \tag{4.26}$$

Proof: (4.24)–(4.26) immediately follow from (4.21)–(4.23) by the pertinent instances of axiom (II.4.40a).•

Cmt 4.4. 1) Axioms (4.21)–(4.23) are homographs of theorems (3.31a)–(3.33a) in that order, whereas theorems (4.24)–(4.26) are homographs of theorems (3.31)–(3.33) in that order. In this case, theorems (3.31a)–(3.33a) (3.34a) have been *immediately inferred* from the respective theorems (3.31)–(3.33), and theorems

(4.24)–(4.26) have been *immediately inferred* from the respective axioms (4.21)–(4.23) by the pertinent instances of axiom (II.4.40a). At the same time, theorems (3.31)-(3.33) have been proved (inferred) from axioms (3.25)–(3.27) by the respective trains of identities (3.31_1)–(3.33_1) in terms of the predicate-sign $\in$. Since the latter sign is not available in $\bar{A}_{1\subseteq}$, therefore theorems (3.31)–(3.33) of $\bar{A}_{1\in}$ are actually taken for granted in $\bar{A}_{1\subseteq}$ as theorems (4.24)–(4.26), because the latter are *immediate counterparts* of axioms (4.21)–(4.23). Consequently, all theorems of $\bar{A}_{1\in}$, which have been proved in section 3 from theorem (3.31)–(3.33), – namely, theorems (3.34)–(3.45) and (3.34a)–(3.45a), – are at the same time theorems of $\bar{A}_{1\subseteq}$, which are provable in the same way from theorems (4.24)–(4.26). It is understood that, for instance, the CLR's (3.31κ)–(3.37κ), being the CFCL interpretands of theorems (3.31)–(3.37) remain are can be used as homographic CFCL interpretands of the respective homographic theorems of theorems $\bar{A}_{1\subseteq}$.

2) Cmt 3.9 applies, *mutatis mutandis*, to $\bar{A}_{1\subseteq}$ as well. Particularly, in this case, the following general remark can be made in analogy with item 3 of Cmt 3.9. an AEADP of any given relation that involves a *pseudo-qualifier* over **u** of any one of the relations [**u**⊆*U*], ¬[**u**⊆*U*], [*U*⊆**u**], and ¬[*U*⊆**u**] unavoidably involves, at a certain stage, the respective one of the integrons $\hat{\smile}_{\mathbf{u}}V(\mathbf{u}\subseteq U)$, $\hat{\smile}_{\mathbf{u}}V(\neg[\mathbf{u}\subseteq U])$, $\hat{\smile}_{\mathbf{u}}V(U\subseteq \mathbf{u})$, and $\hat{\smile}_{\mathbf{u}}V(\neg[U\subseteq \mathbf{u}])$, which is immediately reduced by means of (4.25) and (4.26) as:

$$\begin{gathered}\hat{\smile}_{\mathbf{u}}V(\mathbf{u}\subseteq U)\hat{=}\hat{\smile}_{\mathbf{u}}0\hat{=}0, \hat{\smile}_{\mathbf{u}}V(\neg[\mathbf{u}\subseteq U])\hat{=}\hat{\smile}_{\mathbf{u}}1\hat{=}1,\\ \hat{\smile}_{\mathbf{u}}V(U\subseteq \mathbf{u})\hat{=}\hat{\smile}_{\mathbf{u}}1\hat{=}1, \hat{\smile}_{\mathbf{u}}V(\neg[U\subseteq \mathbf{u}])\hat{=}\hat{\smile}_{\mathbf{u}}0\hat{=}0\end{gathered} \tag{4.27}$$

instead of Ax II.4.11.•

4.3. The organon $A_{1=}$

†**Df 4.3.** 1) In accordance with Df I.7.1(3), the branch of A_1 that is denoted logographically by '$A_{1=}$' and is called (denoted phonographically by) the *Egalitarian*, or *Pseudo Nonempty-Individual*, *EAPO* (*EgEAPO* or *PNEIEAPO*), has the atomic basis, denoted by '$B_{1=}$', which comprises the distinguished *primary binary atomic pseudo-constant ordinary predicate-sign* (*PBAPCOPS*) = that is indicated in the point a of item 8 of Ax II.5.1 and also all primary atomic euautographs that are indicated in items 1–7 and 10–12 of Ax II.5.1. Thus, as compared with the atomic bases $B_{1\in}$ of $A_{1\in}$ and $B_{1\subseteq}$ of $A_{1\subseteq}$, the atomic basis $B_{1=}$ of $A_{1=}$ contains = instead of ∈ or ⊆

respectively and it does not contain the PCOT's (pseudo-constant ordinary terms) $\emptyset$ and $\emptyset'$.

2) The mental (psychical) accidental denotata

$$u \text{ to } z,\ u_1 \text{ to } z_1,\ u_2 \text{ to } z_2, \text{ etc} \tag{4.28}$$

of the CFCL (conformal catlogographic) interpretands

$$`u\text{' to }`z\text{'},\ `u_1\text{' to }`z_1\text{'},\ `u_2\text{' to }`z_2\text{'}, \text{ etc} \tag{4.29}$$

of the PVOT's (pseudo-variable ordinary terms)

$$\mathsf{u} \text{ to } \mathsf{z},\ \mathsf{u}_1 \text{ to } \mathsf{z}_1,\ \mathsf{u}_2 \text{ to } \mathsf{z}_2, \text{ etc} \tag{4.30}$$

of A_1 are called *classes*, *masses*, or *nonempty pseudo-individuals*, – and the PVOT's are called *pseudo-classes*, *pseudo-masses*, or *pseudo nonempty-individuals*, – if A_1 is restricted to $\mathsf{A}_{1\in}$, $\mathsf{A}_{1\subseteq}$, or $\mathsf{A}_{1=}$ respectively. The second proper name of $\mathsf{A}_{1=}$, indicated above, is descriptive of this property of the PVOT's $\mathsf{A}_{1=}$.

3) In $\mathsf{A}_{1=}$, either one of the two tantamount (concurrent) sets (conjunctions) of four theorems (4.13)–(4.16) and (4.13a)–(4.16a) is taken for granted as the defining set (conjunction) of subject axioms, whereas the other set becomes the set of subject theorems of $\mathsf{A}_{1=}$, which are immediately inferred from the set of axioms by the pertinent instances of axiom (II.4.40a).

4) All subject theorems, which have been proved from theorems (4.13)–(4.16) in $\mathsf{A}_{1\in}$ or $\mathsf{A}_{1\subseteq}$, are homographic subject theorems of $\mathsf{A}_{1=}$, which are provable in the same way from the homographic tokens of theorems (4.13)–(4.16) as axioms of $\mathsf{A}_{1=}$.•

Cmt 4.5. 1) In ER's of $\mathsf{A}_{1\in}$ or $\mathsf{A}_{1\subseteq}$, the PCOT $\emptyset$ can stand on either side of each one of the predicate-signs $\in$ and $\subseteq$, and so does the CFCL interpretand '$\varnothing$' of $\emptyset$, which is a logographic name of the *empty individual* $\varnothing$. Therefore, the latter is alternatively called the *empty class* or *empty set* if it is associated with $\emptyset$ employed $\mathsf{A}_{1\in}$ and the *empty mass* if it is associated with $\emptyset$ employed $\mathsf{A}_{1\subseteq}$. Consequently, $\emptyset$ is called the *pseudo empty-class* in the former case or the *pseudo empty-mass* in the latter case, or else the *pseudo empty-individual* in both cases indiscriminately.

2) I have already indicated in Cmt I.8.1 that in the English translations of Aristotle [350 BCE, *Categories*] by Edghill (referred to as [ACE]) and by Owen (referred to as [ACO]) and in studies of that treatise (e.g., in Studtmann [2008]), the terms "*primasry substances*" and "*secondary substances*" are used for denoting the *entities* (*beings* – τᾶ ὄντα \tá ónta\, singular "τό ὀντότης" \tó ontótis\ *s. f.*), which are respectively called "*nonempty individuals*" and "*classes*" in the presently common

terminology. In Aristotelianism, immediate classes of nonempty individuals are called *species*, whereas a *superclass* (*whole*) *of a species* is called a *genus*.

3) A_1 allows distinguishing formally (axiomatically) between classes (including sets) and masses. Therefore, I divide the Aristotelian subcategory of *secondary substances* into two distinct narrower subcategories: *classes* and *masses*, so that a class or a mass, nonempty one or empty is an Aristotelian secondary substance. At the same time, an Aristotelian primary substance is a nonempty individual and vice versa. A primary substance *is not predicable of a subject* in the sense that its name cannot be used as a *predicative* (see Cmt I.8.1 for greater detail). Accordingly, a nonempty individual can be an element, i.e. a member, of a class, but it cannot be either a *subclass*, i.e. a part of a class, or a *submass*, i.e. a part of a mass. Formally, in contrast to the CCLOT (constant catlogographic ordinary term) '$\varnothing$', which can stand on either side of each one of the predicate-signs $\in$ and $\subseteq$, a logographic name of a nonempty individual (primary substance) can stand to the left of the sign $\in$, but it cannot stand to the right of it sign, and also that name cannot stand on either side of the sign $\subseteq$. Consequently, every PVOT of the list (I.5.1) or (4.30) has the same formal (syntactic) properties. These properties do not manifest themselves in the organon $\mathsf{A}_{1=}$ because the latter has neither $\in$ nor $\subseteq$. However, if one wishes to construct an uninterpreted calculus that meets these properties then it is necessary to supplement the list (I.5.1) or (4.30) by introduce another infinite set of PVOT's and to lay down formation rules, which incorporate PVOT's of both lists and according to which a PVOT of the new list can stand on each side of either $\in$ nor $\subseteq$. At the same time, it is unlikely that such a calculus may have any algebraic decision method after the manner of D_1.•

Chapter V. The organon A_{1A}

1. Aristotelian logic

1.1. Introduction

Df 1.1. 1) *Aristotelian logic* (*AL*) or *Aristotelian syllogistics* (*AS*) has two physically inseparable *psychical aspects*, one of which is its *form*, called *Aristotelian formal logic* (*AFL*) or *Aristotelian formal syllogistics* (*AFS*), while the other one is its *matter*, called *Aristotelian material logic* (*AML*) or *Aristotelian material syllogistics* (*AMS*). *AFL* (*AFS*) is *a system of 19 formal deductive three-judgment rules, of inference,* called *formal categorical syllogisms* (*FCS's*) or, more precisely, *categorical syllogism schemata,* or *forms,* i.e. *schemata,* or *forms, of categorical syllogism instances*, the first 14 of which Aristotle laid down himself in his «*Prior Analytics*» (Aristotle [350 BCE, *ditto*], referred to as [APrAJ]) on the basis of his «*Categories*» (Aristotle [350 BCE, *ditto*] referred to as [ACE]) and «*On Interpretations*» (Aristotle [350 BCE, *ditto*] referred to as [AIE]), while the remaining 5 were discovered later reputably by *Galen of Pergamum* (A.D. c130–c200). *AML* (*AMS*) comprises concrete *material instances* (*matters*) of the FCS's, which are expressed in a certain native language (as English), into which AFL is incorporated, and which are called *material categorical syllogisms* (*MCS's*).

2) A *formal*, or *schematic*, *categorical* (*unconditional*) *syllogism* (*FCS* or *SchCS*), called also a *categorical syllogism schema*, or *form*, is *one of 19 three-judgment three-term formal* (*schematic*) *rules of deductive inference of a categorical judgment*, called the *conclusion, from two known categorical judgments,* called the *premises*. A *judgment* is a *veracious* (*accidentally true*) *simple extended declarative sentence* of a certain one of four standard forms as specified below. An FCS is composed of two *premise schemata* (*forms*) and a *conclusion schema* (*form*), which are certain instances of certain three *judgment-schema*, or *judgment-form*, *placeholders* (*JSPH's* or *JFPH's*), i.e. *placeholders of judgment schemata* (*JS'ta*), or *judgment forms* (*JF's*), that are selected out of the following four placeholders:

"All ***u*** are ***v***", "All ***u*** are not ***v***", "Some ***u*** are ***v***", "Some ***u*** are not ***v***", (1.1)

which will be called, in that order, *universal affirmative*, *universal negative*, *particular affirmative*, and *particular negative JSPH's* in the sense that they are *placeholders* (*PH's*) *of schemata* (*S*) *of universal affirmative*, *universal negative*, *particular affirmative*, and *particular negative judgments* (*J*) respectively. Consequently, either bold-faced italic letter ‘***u***’ or ‘***v***’ is a *placeholder*, whose range is the set of any three light-faced italic letters as specified, e.g., ‘*u*’, ‘*v*’, and ‘*w*’, or ‘*A*’, ‘*B*’, and ‘*C*’, or ‘*P*’ (“*P*redicate”), ‘*S*’ (“*S*ubject”), and ‘*M*’ (“*M*iddle term”), the understanding being that each of the light-faced letters is, in turn, *a placeholder, whose range is the class of English count names in the plural number form*. Therefore, given a set of such three light-faced placeholders, say {‘*u*’,‘*v*’,‘*w*’}, the instances of any one of the above four logographic or wordy JSPH’s, has six instances that corresponds to six possible instances of the ordered pair (***u***,***v***), namely

$$(\boldsymbol{u},\boldsymbol{v}) \in \{(u,v),(u,w),(v,w),(v,u),(w,u),(w,v)\}. \tag{1.2}$$

3) The JSPH’s (1.1) will be denoted logographically by ‘ $\mathrm{A}(\boldsymbol{u},\boldsymbol{v})$’, ‘ $\mathrm{E}(\boldsymbol{u},\boldsymbol{v})$’, ‘ $\mathrm{I}(\boldsymbol{u},\boldsymbol{v})$’, and ‘ $\mathrm{O}(\boldsymbol{u},\boldsymbol{v})$’ in that order, i.e.

$$\begin{gathered}\mathrm{A}(\boldsymbol{u},\boldsymbol{v}) \to [\text{All } \boldsymbol{u} \text{ are } \boldsymbol{v}],\ \mathrm{E}(\boldsymbol{u},\boldsymbol{v}) \to [\text{All } \boldsymbol{u} \text{ are not } \boldsymbol{v}],\\ \mathrm{I}(\boldsymbol{u},\boldsymbol{v}) \to [\text{Some } \boldsymbol{u} \text{ are } \boldsymbol{v}],\ \mathrm{O}(\boldsymbol{u},\boldsymbol{v}) \to [\text{Some } \boldsymbol{u} \text{ are not } \boldsymbol{v}],\end{gathered} \tag{1.3}$$

where the round brackets are mentioned, while the square brackets are used but not mentioned. The four capital letters ‘A’, ‘E’, ‘I’, ‘O’ and the corresponding small ones ‘a’, ‘e’, ‘i’, ‘o’ are *conventional* (*traditional*) *code* (*catch*) *letters* for any JS’ta (JF’s), called also *propositional schemata* (*PS’ta*) or *propositional forms* (*PF’s*), that are associated with the four JSPH’s of the list (1.1). The code letters were derived from the two Latin words “***a**ff**i**rmo*” and “*n**e**g**o***”. However, in contrast to the conventional use of these code letters, I employ the *capital code letters* as *logical predicates of the JSPH’s* and as *logical predicates of the JS’ta* in the ranges of the JSPH’s.

4) The traditional form of an FCS, in which the premises and conclusion are stated as three separate JF’s, each of which ends with a full stop, can be called a *verbal staccato form* (*VSF*) *of the FCS*. Alternatively, an FCS can be asserted in the form of a *hypothetical statement schema*, in which the antecedent is the conjunction of two premises and the consequent is the conclusion. This form of an FCS can be called the *logographic legato form* (*LLF*) *of the FCS* and also a *formal hypothetico-categorical syllogism* (*FHCS*), *formal quantified transitive law* (*FQTL*), or *formal syllogistic implication* (*FSI).* It is essential that in passage from the staccato form of an FCS to its legato form, the premises and conclusion do not alter and hence they remain *categorical* (*unconditional*), i.e. *neither hypothetical nor disjunctive*. Therefore, both forms are equivalent, while the legato form is preferable because it is

naturally incorporated into logistic systems. For instance, the first one of the 19 FCS's, which is *mnemonically* denoted as "B*a*rb*a*r*a*(u,w,v)", has the verbal staccato form:

$$\text{« A}(w,v)\text{. A}(u,w)\text{. Therefore, A}(u,v)\text{.»} \tag{1.4}$$

or the tantamount logographic legato form:

$$\text{«}[\mathrm{A}(w,v)\wedge \mathrm{A}(u,w)]\Rightarrow \mathrm{A}(u,v)\text{»}, \tag{1.5}$$

where '$\wedge$' stands for "and" and '… $\Rightarrow$ ***' for "if … then ***" respectively.

5) A *judgment instantiating* a certain JF (JS) is called a *material instance*, or *material interpretand* (i.e. the result of interpreting), or *matter* (*content*) *of the JF* (*JS*). Accordingly, an instance of the FCS is called a *material interpretand*, or *matter* (*content*) of the FCS, and also, less explicitly, a *material categorical syllogism* (*MCS*). There exists an indefinite number of material interpretands of any FCS. For instance, the FCS 'Barbara(*u*,*w*,*v*)' can be materialized (instantiated) thus: Barbara(mammals, vertebrates, animals), i.e. «All vertebrates are animals. All mammals are vertebrates. Therefore, all mammals are animals.»

6) The words occurring in (1.1) have been derived as translations into English of the corresponding Greek words employed by Aristotle in the pertinent treatises of «Organon», primarily in «On Interpretation» and «Prior Analytics» or of the Latin words employed in translations of «Organon» into Latin. The *plural number form of the quantifiers*, occurring in the placeholders of the list (1.1), is predetermined by the fact that names of *nonempty individuals*, i.e. of *primary substances* in Aristotelian coinage, are rejected in AL (see, e.g., Łukasiewicz [1951] and Lamontagne and Woo [2008]). Therefore, there are *no singular judgment*s in AL at all. AL is often introduced by stating the following argument as a typical example of categorical syllogisms:

«All men are mortal. Socrates is a man. Therefore, Socrates is mortal.»

This argument is not, however, an Aristotelian syllogism. An appropriate example of categorical syllogisms would be the following:

«All men are mortal [beings]. All Greeks are men. Therefore, all Greeks are mortal [beings].»

The reason for excluding primary substances, called also *individual subjects* or *singular terms*, in Aristotelian syllogistics is that subjects and predicatives ("predicates" in the Aristotelian terminology) must be exchangeable in the sense that the subject of one proposition (judgment) can be the predicative of another proposition and vice versa. But a primary substance cannot be predicated of ("said-of") any other substance, and therefore it is not admissible in Aristotelian syllogistics.

7) Still, the basic property of AL not to deal with nonempty individual (primary substances) remains unaltered if the JSPH's (1.1) are represented in *the equivalent single number form* as:

"Every ***u*** is a ***v***", "Every ***u*** is not a ***v***", "Some ***u*** is a ***v***", "Some ***u*** is not a ***v***". (1.6)

However, unlike English, both Ancient and Modern Greek have *no indefinite article*, whereas Latin has *no articles at all*, either definite or indefinite. In this respect, Hebrew, e.g., is similar to Greek, whereas Russian, e.g., is similar to Latin. Therefore, the classification of the English words occurring in the JSPH's (1.6) should differ somewhat from the classification of the words occurring in the Greek or Latin counterparts of those placeholders.

8) The range of either term placeholder '***u***' or '***v***' occurring in the JSPH's (1.4) is the same as that of the homographic term placeholder occurring in the JSPH's (1.1), e.g. the set {'*u*','*v*','*w*'}. However, the term placeholders '*u*', '*v*', and '*w*' are now replaceable with *count names in the singular number form*. Therefore, the logical predicates, which are associated with the JSPH's (1.6), are distinct from the logical predicates, which are associated with the JSPH's (1.1), and which have been denoted by 'A', 'E', 'I', and 'O'. In order to maintain the distinction between the plural and singular logical predicates symbolically, the former can be redenoted as 'A_p', 'E_p', 'I_p', and 'O_p', while the latter are denoted as 'A_s', 'E_s', 'I_s', and 'O_s', the understanding being that the subscript '$_p$' stands for "plural" and '$_s$' for "singular". At the same time, in the general discussion of categorical syllogisms, I may use 'A(***u***,***v***)', e.g., for equivocally mentioning both 'A_p(***u***,***v***)' and 'A_s(***u***,***v***)', and I may likewise use 'A(*u*,*v*)' for equivocally mentioning, e.g., both 'A_p(*u*,*v*)' and 'A_s(*u*,*v*)'. In this case, once I substitute concrete count names for '*u*' and '*v*', either in the plural or in the singular, the subscript to 'A' can immediately be restored from the *number form* of the names. For instance, "A(squares, polygons)" stands for "A_p(squares, polygons)" and hence for "All squares are polygons", whereas "A(square, polygon)" stands for "A_s(square, polygon)" and hence for "Every square is a polygon". Likewise, the FCS 'Barbara($\underline{u},\underline{w},\underline{v}$)' can be materialized (instantiated), either thus: Barbara(squares, rectangles,polygons), i.e. «All rectangles are polygons. All squares are rectangles.

Therefore, all squares are polygons.», or thus: Barbara(square,rectangle,polygon), i.e. «Every rectangle is a polygon. Every square is a rectangle. Therefore, every square is a polygons.»

9) The placeholders "All ***u*** are not ***v***" and "Every ***u*** is not a ***v***" are conventionally written in the literature in the shorter equivalent forms "No ***u*** are ***v***" and "No ***u*** is a ***v***" respectively, but I shall not employ the latter forms, because they introduce undesirable asymmetry in classification of the similarly positioned parts of the four placeholders (1.1) or (1.6).•

Cmt 1.1. 1) According to *Aristotelianism* (*Aristotelian philosophy*), every corporeal entity (being) is a *biune* one that consists of *two inherent principles*, namely a *primordial* (*primary*) *potential one* that is called *matter* and a *secondary actual*, or *entelechial*, *one* that is called *form*. This doctrine is called *hylomorphism* or, more specifically, *Aristotelian hylomorphism*. The term "hylomorphism" originates from two Greek nouns: "ύλη" \íli\ (pl. "ύλαι" \íle\), meaning *a matter*, and "μορφή" \morfí\ (dual "μορφά" \morfá\, pl. "μορφαί" \morfé\), meaning *a form*. The English nouns "*matter*" and "*form*" are in turn derived respectively from the Latin nouns "mātĕrĭia", meaning *matter*, *material*, *stuff of which anything is composed* (besides having some other meanings), and "forma", meaning *form*, *figure*, *shape* (see Simpson [1959]). According to hylomorphism, matter and form are two conceptual aspects of an entity, which can be distinguished and contrasted, but which cannot be separated from each other.

2) A form and its matter or a matter and its form are *epistemologically relativistic entities*, not only in the sense that the two are inseparable from each other, but also in the sense that the form of one entity can become or be treated as the matter of another entity. For instance, I have posited above that an FCS can be regarded as the form (schema, placeholder) of any concrete instance of it, while the latter can be regarded as a matter of the FCS, which it is called an MCS. At the same time, any FCS can be deduced as an interpretand of a certain valid or vav-neutral euautographic relation (ER) of A_{1A}, so that the former is a matter of the latter, while the latter is the form of the former. The following simple examples from arithmetic and algebra illustrate relationship between form and matter of a judgment or between formal logic and material logic, and they also illustrate the *epistemologically relativistic character* of relationship between form and its matter or between matter and its form.

3) For instance, the *veracious* relation “1+1=2” can be regarded as *formally veracious* (*f-veracious*) one that is analogous to the f-veracious relation (judgment form) “All *u* are *v*”. Indeed, the latter is the form of any of the judgments (judgment instances) such as: “All squares are quadrangles”, “All quadrangles are polygons”, “All men are mammals”, “All mammals are vertebrates”, etc. Likewise, the relation “1+1=2” expresses the abstract *form* of the operation of adding 1 entity of any class to 1 other entity of the same class, without reference to any matter (content) of the operation, which is associated with *dimension* of the numbers added. Therefore, possible matters of the operation are expressed by the judgments (judgment instances) such as “1 square + 1 square = 2 squares”, “1 man + 1 man = 2 men”, etc. At the same time, any one of the abstract veracious relations “1+1=2”, “1+2=3”, “2+2=4”, etc has the *form* of the more abstract *vav-neutral* (*neither valid nor antivalid*) *algebraic relation* “*a*+*b*=*c*” and it is therefore a matter of the latter algebraic relation. Consequently, the *abstract integral domain*, e.g., is the form of *arithmetic of natural integers* and conversely the latter is a matter of that form. Likewise, the *field* (*algebraic system*) *of abstract scalars* is the form of the *field of specific scalars* as the *field of rational numbers*, the *field of real numbers*, etc, and conversely the latter fields are matters of the former abstract field. At the same time, the field of rational numbers is the form of book-keeping.•

1.2. Propositions

Aristotle’s title of *founder of logic* rests mainly on AFL that was developed in his «*Prior Analytics*», [APrAJ], on the basis of his «*Categories*», [ACE], and «*On Interpretation*», [AIE].

1) «*Categories*» deals with semantic properties of separate *words*; «*On Interpretation*» (also known as «*On Propositions*» and as «*Propositions*») deals with syntactic relations among words, which are called *propositions*, or, to use the modern terminology, *truth-valued declarative sentences*; «*Prior Analytics*» deals *with 4 kinds of two-term judgments*, i.e. *true propositions*, viz. *universal affirmative*, *universal negative*, *particular affirmative*, and *particular negative*, which are organized in 3 *figures* and 14 *moods of deductive three-judgment inference of the conclusion from two premises*, traditionally called *categorical*, or *unconditional*, *syllogisms*, – as opposed to *conditional*, i.e. *hypothetical* or *disjunctive*, *syllogisms*. AFL was

supplemented with the additional forth figure, comprising five moods of categorical syllogisms, about half a millennium after Aristotle' death by Galen.

2) In «*Categories*» («*Categoriae*» in Latin), [ACE], Aristotle's introduces a *fourfold* taxonomy of [*the*] *beings* ("τα όντα" \ónta\, pl. of "το ον", Dict. A2.1), or *entities*, in relation to their properties of *predictability* of other beings and he also introduces a *tenfold* taxonomy of beings themselves. Each of the *taxa* (*taxonomic classes*) and, apparently, each of the *taxonyms* (*taxonomic names*), Aristotle equivocally and enigmatically call a *category* ("κατηγορία" \katiγoría\, pl. "κατηγορίαι" \katiγoríe\). Aristitle's etymon "κατηγορία" of the English noun "category" is kindred of the nouns "κατηγορούμενον" (pl. "κατηγορούμενοντα" \katiγorúmenonta\), meaning *a predicate*, and "κατηγόρημα" \katiγórima\ (pl. "κατηγορήματα" \katiγórimata\), meaning *a predicative*. In this connection, it should be recalled that at present the English noun "predicate" is usually used in the sense of the noun "grammatical predicate", whereas a predicative is the complementary part of the link-verb in a compound grammatical predicate. By contrast, Aristotle uses "κατηγορούμενον" in the sense of "κατηγόρημα". In addition, Aristotle does not maintain, either terminologically or at least phraseologically, *the distinction between use and mention of graphic expressions*, so that one can think that this is an intended (voluntary and hence conscious) result of his doctrine of universals and particulars. Thus, most straightforwardly, any one of Aristotle's categories is either a *kind* (*class*) *of predicability* (*predicative ability*) of an entity, or of its name, – i.e. a kind of ability of an entity or its name to predicate of another entity or of its name, – or a kind of an entity as whole, or of its name. Still, the fact that a category itself is a *universal* (*class*) and hence a being (entity) is blurred.

3) In «*On Interpretation*» («*De Interpretatione*» in Latin and «Περὶ Ερμηνείας» \peri erminías\ in ancient Greek), [AIE], Aristotle deals with linguistic foundations of his *analytics*. In this treatise, consisting of 14 short chapters (sections), Aristotle defines basic linguistic forms of his analytics and discusses their syntactic and semantic properties. He begins with defining the terms "noun", "verb", "affirmation", "denial", "sentence", and "proposition". A general discussion of these terms occupies the first five chapters of his treatise. Aristotle defines a *proposition* (πρότασις \prótasis\, pl. πρότασεις \prótasís\, or ἀποφὰνσις\, pl. ἀπόφανσεις \apófansis\) as a declarative sentence (πρόταση \prótasi\, pl. πρότασαι \prótase\) that is either true or false. Therefore, not every sentence is a proposition; e.g. a prayer is a

sentence, but not a proposition. In his study in question, Aristotle dismisses sentences of all other types but propositions, and leaves the others to the study of rhetoric and poetry. Then Aristotle confines his inquiry to *simple propositions*, so that "proposition" is understood as an abbreviation of "simple proposition". Since propositions are the main objects of «On Interpretation», this title is misleading. Therefore, in post-Aristotelian antiquity that treatise was sometimes entitled, more adequately, "Περὶ Πρότασεις" \peri prótasis\, and accordingly some modern English-speaking scholars employ the respective English title "*On Propositions*" or simply "*Propositions*" in place of "On Interpretation" (see, e.g., Durant [1926, p. 46]).

4) The meaning of many English expressions, which are used as *parasynonyms* (*translations*) of the corresponding original Aristotle's terms, differs from the meanings of the same expressions occurring in the writings on contemporary symbolic logic. For instance, it is said in [AIE, chap. 5]:

> «Let us, moreover, consent to call a noun or a verb an expression only, and not a proposition, since it is not possible for a man to speak in this way when he is expressing something, in such a way as to make a statement, whether his utterance is an answer to a question or an act of his own initiation. »

In contrast to the above citation and in contrast to the pertinent English version of Aristotle's terminology that is used in translations of Aristotle's works into English, in contemporary philosophical English language, the *sense* (*concept*, *class-concept*) *of a categorematic expression* is said to *express* (or *connote*) its *sense*, i.e. a *concept*, or *class-concept, of its denotatum* (*denotation value*), – no matter whether that expression is a nounal name (particularly a sole noun) or whether it is a declarative sentence. Also, the noun "proposition" is typically used in the English metalanguage of contemporary symbolic logic for denoting the *sense of a declarative sentence that is or can be either true or antitrue* (*false*). Such a sentence is said to be *truth-functional* or *propositional*. The above English terminology is discussed, e.g., in Church [1956, pp. 4–7, 23–27]. The reader should, however, be warned that Church is a Platonic realist and that he employs the term "proposition" as a parasynonym (translation) of the German term "*Gedanke*" ofdd Frege [1892], being another Platonic realist. Accordingly, in order to explain the ontological status of *a proposition*, as he understands it, he cites the saying of Frege about the ontological status of *ein Gedanke*: «nicht das subjective Thun des Denkens, sondern dessen objectiven Inhalt, der fähig ist, gemeinsames Eigenthum von Vielen zu sein»; that is, in my own translation, «*not the subjective activity of thought, but the objective content capable of being property of many*». Also, the main postulate of *the Frege-Church theory of the meaning of truth-functional proper declarative sentences* that a *true* proper declarative sentence *denotes the truth-value truth* and that a *false* proper declarative sentence *denotes the truth-value falsehood* is obscure. In fact, a true declarative sentence denotes a certain complex object that is called a *state of affairs* and also a *fact*, *case*, *event*, *relation*, etc, whereas a false declarative sentence denotes nothing, but rather it just *expresses its sense*. This is why the latter sentence *cannot be asserted* (*used assertively*).

5) In accordance with (1.1), there are four types of Aristotelian syllogistic propositions, which are adequately translated into English by simple extended declarative English sentences of the following four *plural* number forms:

“All *u* are *v*”, “All *u* are not *v*”, “Some *u* are *v*”, “Some *u* are not *v*”. (1.7)

Any one of the sentences consists of a *grammatical subject* «All *u*» or «Some *u*» and a *grammatical predicate* «are *v*» or «are not *v*». At the same time, in accordance with Aristotle’s terminology, *u* is the [*effectual*] *subject* and *v* is the [*effectual*] *predicate*, of a proposition. Hence, the grammatical subject of a proposition comprises the subject *u* and a *quantity*, i.e. a *logical quantifier* in the modern terminology, to it. Accordingly, the grammatical subject is said to be *universal* if it has the *universal quantity* (*quantifier*) “*all*” and *particular* if it has the *particular*, i.e. *existential* in the modern terminology, *quantity* (*quantifier*) “*some*”. By contrast, the grammatical predicate of a proposition is always *universal*. However, the grammatical predicate of a proposition comprises the predicate *v*, which is known in English as the *predicative* (see the item 2 above in this subsection), and a *quality*, i.e. a *verbal qualifier* (*modifier*), to it. Accordingly, the grammatical predicate is said to be *affirmative* if it has the affirmative copula “*are*” and *negative* if it has the negative copula “*are not*”. The subject or the predicate of a proposition is indiscriminately called a *term* of the proposition, so that, conversely, a term of a proposition is either its subject or its predicate (predicative The quantity of the grammatical subject of a proposition is at the same time *the quantity of the proposition*, and the quality of the grammatical predicate of a proposition is at the same time *the quality of the proposition*. Consequently, a proposition is said to be *universal* if its grammatical subject is universal, *particular* if its grammatical subject is particular, *affirmative* if its grammatical predicate is affirmative, and *negative* if its grammatical predicate is negative. The quantity or the quality of a proposition is indiscriminately called a *property* of the proposition, Thus, an Aristotelian syllogistic proposition has one subject, one predicate, and two properties: quantity and quality; quantity can be universal or particular and quality affirmative or negative.

6) As I have already mentioned above in item 7 of Df 1.1, Aristotle excluded from his syllogistics any singular propositions, i.e. any propositions, which have quantity *one* and which he describes in chapter 5 of [ACE] thus:

> «We call those propositions single which indicate a single fact, or the conjunction of the parts of which results in unity: those propositions, on the other hand, are separate and many in number, which indicate many facts, or whose parts have no conjunction.»

Incidentally, *singular*, i.e. *of quantity one*, is not only a proposition such as "Socrates is a man" or "Aristotle is the founder of logic», whose subject is a proper and hence singular name of a nonempty individual, but also a proposition such as "Water is a fluid', "Dark-red is red", "Red is color", or "Color is quality", i.e. a proposition whose subject has an unquantified single number form because it is a mass name. Consequently, *Aristotelian syllogistics is logic of classes, which does not deal either with nonempty individuals or with masses.*

7) Chapter 6 and a part of chapter 7 of [AIE] are concerned with relationships among four types of *plural propositions with the same terms*: *universal affirmative*, *universal negative*, *particular* (*existential*) *affirmative*, and *particular* (*existential*) *negative*. Any two of such propositions are said to be *opposite propositions* or *opposites*. Specifically, two plural propositions with the same terms are said to be:

a) *contradictory ones* or *contradictories* if they differ both in quantity and in quality,

b) *contrary ones* or *contraries* if they differ in quality,

c) *alternate ones* or *alterns* if they differ in quantity.

Consequently, A(u,v) and O(u,v) or E(u,v) and I(u,v) are contradictories, A(u,v) and E(u,v) or I(u,v) and O(u,v) are contraries, and A(u,v) and I(u,v) or E(u,v) and O(u,v) are alterns. More specifically, A(u,v) and E(u,v) are called *supercontraries*, and I(u,v) and O(u,v) *subcontraries*. The above relations among the four opposites are conventionally illustrated in the form of a so-called *square of oppositions*, whose vertices are labeled either with the JFPH's 'A($\boldsymbol{u}$, $\boldsymbol{v}$)', 'E($\boldsymbol{u}$, $\boldsymbol{v}$)', 'I($\boldsymbol{u}$, $\boldsymbol{v}$)', and 'O($\boldsymbol{u}$, $\boldsymbol{v}$)' or with the JF's 'A(u, v)', 'E(u, v)', 'I(u, v)', and 'O(u, v)' (e.g.) in that order in the clockwise direction starting from the upper left vertex. In this case, each diagonal joins two contradictories, each vertical side joins two alterns, and each horizontal side joins two contraries, the upper one joining two supercontraries and the lower one two subcontraries. The square of oppositions is implied by Aristotle's fourfold taxonomic schema of simple non-singular propositions, but that square was not drawn by Aristotle himself.

8) It will be rigorously proved with the help of $\mathsf{A}_{1\mathsf{A}}$ and its CFCL interpretation that

$$\mathrm{O}(u,v) \Leftrightarrow \neg\mathrm{A}(u,v), \mathrm{I}(u,v) \Leftrightarrow \neg\mathrm{E}(u,v), \tag{1.8}$$

$$\mathrm{A}(u,v) \Rightarrow \mathrm{I}(u,v), \mathrm{E}(u,v) \Rightarrow \mathrm{O}(u,v), \tag{1.9}$$

where '¬' stands for "not", '⇔' for "if and only if", and **'**… ⇒ *****'** stands for **"**if … then *****"**. That is to say, relations (1.8) and (1.9) are *tautologies*.

9) In agreement with (1.8), two contradictories cannot be either both true or both antitrue (false) simultaneouslys. Unlike either pair of contradictories, the supercontraries A(u,v) and E(u,v) can be both antitrue but they cannot be both true, whereas the subcontraries I(u,v) and O(u,v) can be both true but they cannot be both antitrue. At the same time, in agreement with (1.9), if A(u,v) is true then I(u,v), being its altern, is also true, and likewise if E(u,v) is true then O(u,v), being its altern, is also true. According to Aristotle, propositions I(u,v) and O(u,v) are not semantically equivalent. This means that the logical quantifier (quantity) "*some*" is used in Aristotelian syllogistics in *inclusive sense*, i.e. as an abbreviation of the expression "*strictly some or all*" or of the sense-concurrent expression "*less than all or all*".

1.3. The equivocal operators "is", "is not", "are", and "are not"

1) In ordinary non-technical English, "is" or "are" ("εστί" \estí\ or "είναι" \íne\ in Greek and "est" or "sunt" in Latin, respectively), i.e. third person singular or plural Present Tense of the copula (link-verb) "to be" ("είμαι" \íme\ in Greek and "esse" in Latin), *along with the indefinite or definite article following it*, if this article is required by the English grammar (Greek has no indefinite article and Latin has no articles at all), may *equivocally* denote any one of a great many different *relations in intension* (*predicates*) such as: the *class-membership* (*member-of-class, object-to-*

property) *relation*; a *class-inclusion* (*part-to-whole*, *subclass-to-superclass*, *species-to-genus*) *relation*, an *equivalence relation* and particularly the *identity relation*, the *logical entailment relation*, etc. In this case, a class-inclusion relation can, in turn, be either a *strict* (*strong*) one or a *lax* (*weak*) one, because the word "*part*" can be understood either as "*strict part*", i.e. "*part but not the whole*", or as "*lax part*" i.e. "*strict part or the whole*". Accordingly, the class-inclusion relation is a strict (strong) one in the former case and a lax (weak) one in the latter case. Like remarks apply to the *parasynonyms* (*counterparts*) of "is" and "are" in any of a great many of phonemic (alphabetic or polysyllabic) languages (as Greek, Latin, Russian, Hebrew, Japanese, etc). Owing to the ambiguity of the copulas "is" and "are" in English and of their parasynonyms in other languages, a class-membership predicate was confused with a class-inclusion predicate for hundreds of years.

2) Due to Italian logician and mathematician *Giuseppe Peano* (1858–1932), it was recognized that in English, e.g., any of the expressions "*is a*", "*is an*", and "*is the*", or just "*is*", which is immediately followed by a *numerable* (*count*) *name*, denotes the *relation of membership in the multipleton* (*many-member class*) *denoted by that name*. Also, after Peano, the *class-membership relation in intension* is denoted by the special *lexigraph* (*atomic logograph*, *atomic logographic symbol*) "$\in$", which is a stylized acronym of the Greek link-verb "εστί" \estí\ meaning *is*. By contrast, the strict (strong) class-inclusion relation [in intension] and the lax (weak) one are, after German logician and mathematician *Friedrich Ludwig Gottlob Frege* (1848–1925), commonly denoted by the signs "$\subset$" and "$\subseteq$", respectively.

3) Thus, according to Peano, the statement «Socrates is a man», e.g., can be restated as «Socrates $\in$ man» and the statement «A man is a mammal» can be restated as «A man $\in$ mammal», the understanding being that a singular count name without any singular quantifier (as "one" or "a") denotes a class. At the same time, according to Frege, using the terms "class" and "subclass" in a broad sense, the statement «Species man is a subclass of class Mammalia» can be restated «Man $\subset$ Mammalia» or as «*Homo sapiens* $\subset$ Mammalia», and also as «Man $\subseteq$ Mammalia» or as «*Homo sapiens* $\subseteq$ Mammalia». Likewise, the statement «The singleton of Socrates is a subclass of species man» can be restated as «Socrates $\subset$ man» or as «Socrates $\subset$ *Homo sapiens*», and also as «Socrates $\subseteq$ man» or as «Socrates $\subseteq$ *Homo sapiens*».

4) In contrast to Latin, Italian has both the definite article and the indefinite article, although their grammatical properties essentially differ from the grammatical properties of the respective English articles. Like the English indefinite article, “a” or “an”, the Italian indefinite article (*l'articolo indeterminativo*) is used with singular nouns and noun constructions, and in addition it denotes the number one. Unlike the English indefinite article, the Italian one has different forms depending on the gender and initial one or two letters of the word (noun or adjective) that it precedes and modifies. In contrast to the English definite article having only one form, “the”, the Italian definite article (*l'articolo determinativo*) has different forms depending on the gender, number, and initial one or two letters of the noun or adjective it precedes and modifies.

5) An Italian article agrees in gender and number with the name that it modifies, so that Italian has a paradigm of indefinite articles and a paradigm of definite articles. Also, an Italian article is repeated before each name in a conjunction or disjunction. Still, owing to the fact that both Italian and English are two-article languages, Peano’s discovery is immediately applicable to English. For instance, one may assert that, according to Peano, “is a” or “is an” denotes the class-membership relation, in spite of the fact that Peano expressed his discovery in Italian. At the same time, one cannot make such a straightforward assertion in regard to any article-less or indefinite-article-less language as Greek, Latin, Hebrew, or Russian, although Peano’s suggestion to employ the character “$\in$” as the class-membership relation is applicable in any language. In application to any of a great many of native languages, Peano’s discovery has the following aspects.

i) A count name can be used either as a proper class-name (i.e. as a singular collective name in the terminology of Mill [1843]) or as a common individual name.

ii) If the *grammatical predicate* of an affirmative simple declarative sentence comprises the *copula* analogous to the English copula “is” and a *predicative* in the form of count (numerable) name, which is used as a proper class-name, then that copula is used as the class-membership predicate.

iii) If the language in question has either a definite article or both a definite and an indefinite article, and if an article occurs in the predicative mentioned in the previous item, then that article just serves as an *index* (*indicator*) that the copula and the count name are used in the above-mentioned way.

6) In application to English, Peano's "is a"-rule can be extended as follows. The expressions "*is the*" or just "*is*" if "the" is omitted in accordance with the pertinent rule of the English grammar (as, e.g., in the grammatical predicate "is captain of the ship"), which is immediately followed by a *numerable* (*count*) *name*, denotes the *relation of membership in the singleton* (*one-member class*) *denoted by the name*. For instance, the statements «Socrates is the husband of Xantippe» and «Aristotle is the founder of logic» can be restated as «Socrates $\in$ husband of Xantippe» and «Aristotle $\in$ founder of logic». At the same time, in accordance with the equivocality of "is", which have been indicated at the beginning of this subsection, the former two statements can also be restated either as the identities of the individuals: «Socrates = the husband of Xantippe» and «Aristotle = the founder of logic» or as the identities of the singletons of individuals: «Socrates = husband of Xantippe» and «Aristotle = founder of logic» or else as the weak class-inclusion relations between identical singletons: «Socrates $\subseteq$ husband of Xantippe» and «Aristotle $\subseteq$ founder of logic».

7) After creation of *set theory*, – at first of the naive one by Georg Cantor (1845–1918) during the years 1878-84, and then of the axiomatic one by Ernst Zermelo [1908], – certain special classes were called *sets*, although it took some time with mathematicians and logicians to realize that a set was a class, but not necessarily vice versa. It will be recalled that, in outline, a set is a class, which has permanent member population and which can serve as a domain of definition of the linear order relation $\leq$. Therefore, I alternatively call a set "a regular class" and a class that is not a set "an irregular class". In the literature, the classes of the two kinds are distinguished by the names "small class" and "proper class" in that order.

8) For the reasons that have been explicated in Df 1.1(6), Aristotle does not discuss quantified propositions, which could be translated into English by simple extended declarative English sentences of the four *singular* number forms:

«Every *u* is a *v*», «Every *u* is not a *v*», «Some *u* is a *v*», «Some *u* is not a *v*» (1.10)

(cf. (1.6)). However, as has been demonstrated above, properties, i.e. quantities and qualities, of English propositions (sentences) of the singular number forms (1.10) are the same as those of English propositions of the respective plural number forms (1.7), provided that quality (verbal qualifier) of any one of the former propositions comprises the pertinent copula "is" or "is not" and the indefinite article "a".

9) About eight and half centuries after Aristotle's death, his «*Organon*» was translated into Latin by Roman philosopher Anicius Manlius Severinus Boethius (AD 475?–524). Historically, Latin was the most important cultural language of Western Europe until the end of the 17th century. Therefore, Aristotelianism became known in Western Europe primarily owing to translations of Aristotle's works into Latin. However, owing to the different article vocabularies of Greek and Latin, the latter is one of the least appropriate languages for treating of Greek-related philosophy, especially of the part of it dealing with logic of universals (classes).

1.4. Categorical syllogisms

1) A categorical syllogism is a *latent quantified predicate* (*functional*) *rule of inference* that is composed of two premises and a conclusion. The premises and conclusion are *veracious*, i.e. *accidentally true, propositions* of one of the four standard forms (1.1) each; "*accidentally*" means *not universally*, i.e. *not tautologously*. Such a veracious proposition (and not just an undecided one) is conveniently call here a *judgment*, although the Greek parasynonym of the English noun "*judgment*", namely "κρίσις" \krísis\ (from the kindred verb "κρίνω" \kríno\ meaning [*I*] *judge*, *deem*, *consider*, or *decide*) is not an Aristotelian term. The premises of an MCS are *veracious* either *by assumption* or *by the previous knowledge*, while the conclusion of the MCS is *veracious by inference* (*deduction*) *of the MCS itself*.

2) An FCS has three terms, each of which was given a proper name by Aristotle as follows. The predicate of the conclusion is called the *major term of the FCS*, while the subject of the conclusion is called the *minor term of the FCS*. The two premises have a term in common, which is called the *middle term of the FCS*, while the two other terms of the premises *extreme terms of the FCS*, the understanding being that one of the extreme terms is major and the other one is minor. The premise that contains the major term is called the *major premise of the FCS* and the premise that contains the minor term is called the *minor premise of the FCS*. The order of the premises is not important. However, in stating an FCS, the major premise is always *conventionally* stated first, the minor premise is stated second, and the conclusion comes last.

3) The order, in which the major term, the minor term, and the middle term of an FCS are arranged in the conventionally arranged premises of the FCS, is called the *figure of the FCS* or less explicitly a *syllogistic figure*. There are *four syllogistic figures* altogether, which are represented by the *definientia* of the following four definition schemata:

$$\begin{aligned}
&(1\mathbf{UVW})(u,w,v)\to[\mathbf{U}(w,v)\wedge\mathbf{V}(u,w)\Rightarrow\mathbf{W}(u,v)],\\
&(2\mathbf{UVW})(u,w,v)\to[\mathbf{U}(v,w)\wedge\mathbf{V}(u,w)\Rightarrow\mathbf{W}(u,v)],\\
&(3\mathbf{UVW})(u,w,v)\to[\mathbf{U}(w,v)\wedge\mathbf{V}(w,u)\Rightarrow\mathbf{W}(u,v)],\\
&(4\mathbf{UVW})(u,w,v)\to[\mathbf{U}(v,w)\wedge\mathbf{V}(w,u)\Rightarrow\mathbf{W}(u,v)],
\end{aligned}\tag{1.11}$$

where each of the letters ‘**U**’, ‘**V**’, and ‘**W**’ is a placeholder whose range is the set of four letters ‘A’, ‘E’, ‘I’, and ‘O’ without single quotation marks. It is understood that *v* is the major term, *u* is the minor term, and *w* is the middle term of each of the 19 FCS’s, which fits in exactly one of the above four definition schemata. Namely, the strings ‘(1**UVW**)’, ‘(2**UVW**)’, ‘(3**UVW**)’, and ‘(4**UVW**)’ are placeholders having the following ranges:

$$\begin{aligned}
&(1\mathbf{UVW})\,\vec{\in}\,\{(1\mathrm{AAA}),(1\mathrm{EAE}),(1\mathrm{AII}),(1\mathrm{EIO})\},\\
&(2\mathbf{UVW})\,\vec{\in}\,\{(2\mathrm{AEE}),(2\mathrm{EAE}),(2\mathrm{EIO}),(2\mathrm{AOO})\},\\
&(3\mathbf{UVW})\,\vec{\in}\,\{(3\mathrm{AAI}),(3\mathrm{AII}),(3\mathrm{IAI}),(3\mathrm{EAO}),(3\mathrm{EIO}),(3\mathrm{OAO})\},\\
&(4\mathbf{UVW})\,\vec{\in}\,\{(4\mathrm{AAI}),(4\mathrm{AEE}),(4\mathrm{IAI}),(4\mathrm{EAO}),(4\mathrm{EIO})\}.
\end{aligned}\tag{1.12}$$

Any concrete string in the range of each one of these four placeholders is called the *logographic logical predicate* (*LLP*) of the corresponding FCS, the understanding being that the latter is uniquely determined by the former. In this case, the given number from 1 to 4, occurring in the LLP of an FCS, indicates the figure of the FCS, whereas the sequence of three code letters selected out of the four ones ‘A’, ‘E’, ‘I’, and ‘O’ indicates the *sequence of qualities of the major premise, minor premise, and conclusion of the FCS* (in that order) – the sequence, which is called the *mood of the FCS* or less explicitly a *syllogistic mood*. In accordance with (1.12), there is the following *ten* syllogistic moods altogether:

AAA, AAI, AII, IAI, AEE, EAE, EAO, EIO, AOO, OAO. (1.13)

Thus, *every FCS and hence its LLP are uniquely determined by its figure and its mood*.

4) It is seen from (1.12) that the successive syllogistic figures 1 to 4 contain 4, 4, 6, and 5 FCS’s respectively, in accordance with the same numbers of syllogistic moods in those figures. The 14 FCS’s comprised in the first three figures were laid

down by Aristotle himself, whereas the 5 FCS's comprised in the fourth figure, in which the middle term is the predicate of the major premise and the subject in the minor, was reputedly added by Galen of Pergamum (A.D. c130–c200), a prominent Roman (of Greek ethnicity) physician, surgeon, and philosopher, who gathered up and systematized ancient knowledge of medicine and anatomy and remained the supreme authority in these fields for more than a thousand years. Galen also wrote about logic and philosophy. The fourth syllogistic figure is sometimes called the *Galenian figure*, whereas the version that Galen was the first scholar to use and possibly to discover it is explicitly supported, e.g., in the article **[1]figure** of WTNID. Still, according to the article **Prior Analytics** of Wikipedia, the fourth figure was added after Aristotle's death by Theophrastus (c372–c287 B.C.), a student and close associate of Aristotle, who succeeded him on his retirement as scholarch (head) of the Lyceum in Athens and who led the school for more than three decades.

5) In his «*Prior Analytics*», [APrAJ], Aristotle divides syllogisms into two groups: *perfect syllogisms* and *imperfect syllogisms*. The perfect syllogisms are those four, which have the first figure. Aristotle takes them for granted as *valid axioms*. Then he uses these axioms to prove the imperfect syllogisms from some intuitive considerations based on *conversions of judgments* (*true propositions*). A *conversion* is the act of altering a proposition by exchanging its subject and its predicate, while preserving its quality. There are two types of conversions: a *simple conversion* when the quantity of the converted proposition is kept unaltered and a *conversion per accidens* when the universal quantity of the converted proposition is changed to the particular one. Some conversions do not exist. Also, in accordance with the previous item, only the first three syllogistic figures of FCS's are original Aristotelian ones, while the fourth figure is Galenian or somebody else's one, which comprises five more improper syllogisms. Aristotle's conversions are discussed in detail in Lamontagne and Woo [2008].

6) In accordance with (1.11) and (1.12), each one of the 19 FCS's is identified by its LLP. Alternatively, each given FCS is identified by the respective *conventional three-syllable catchword*, whose vowels are selected out of the four small letters 'a', 'e', 'i', and 'o' in a certain order so as to indicate the mood of the FCS. In this case, the sequence of consonants occurring in a catchword and hence the catchword as a whole are uniquely associated with the figure of the FCS. Hence, the catchword of an FCS is concurrent to its LLP, and it will therefore be called the *phonographic*, or

verbal, logical predicate (*PhLP* or *VLP*) *of the FCS*. The latter predicates are defined as follows:

List 1.1: ***The catchwords of the 19 FCS's.***

Figure 1:

Barbara→1AAA, Celarent→1EAE, Darii→1AII, Ferio→1EIO.

Figure 2:

Baroco→2AOO, Camestres→2AEE, Cesare→2EAE, Festimo→2EIO.

Figure 3:

Barapti→3AAI, Bocardo→3OAO, Datisi→3AII, Disamis→3IAI,
Felapton→3EAO, Feriso→3EIO.

Figure 4:

Bamalip→4AAI, Calemes→4AEE, Dimatis→4IAI, Fesapo→4EAO,
Fresison→4EIO.

7) For convenience in subsequent calculation of *validity-values* of separate FCS's, it is convenient to divide the 19 FCS's into *mood-related groups*, each of which contains all FCS's, whose LLP's involve certain two first code letters in either order. In accordance with (1.13), there is six such mood-related groups altogether, namely:

AAA&AAI, AII&IAI, AEE&EAE, EAO, EIO, AOO&OAO. (1.14)

Each mood-related group contains FCS's of all pertinent moods independent of their figures. At the same time, the consonant letters occurring in the catchword of an FCS are irrelevant to the mood of the FCS. However, it turns out that *all catchwords of the FCS's of each one of the six mood-related groups of FCS's begin with the same consonant letter*, provided that the conventional catchword "*Darapti*" is replaced with "*Barapti*". Consequently, the catchword "Barapti" of my own is used in this treatise instead of the conventional catchword "Darapti". In the following definition, the FCS's are given in accordance by their mood-related groups.•

Df 1.2.

1°) Group AAA&AAI

1) $\text{Barbara}(u,w,v) \to (1\text{AAA})(u,w,v) \leftrightarrow [\text{A}(w,v) \wedge \text{A}(u,w) \Rightarrow \text{A}(u,v)]$.

2) $\text{Barapti}(u,w,v) \to (3\text{AAI})(u,w,v) \to [\text{A}(w,v) \wedge \text{A}(w,u) \Rightarrow \text{I}(u,v)]$.

3) $\text{Bamalip}(u,w,v) \to (4\text{AAI})(u,w,v) \leftrightarrow [\text{A}(v,w) \wedge \text{A}(w,u) \Rightarrow \text{I}(u,v)]$.

2°) Group AII&IAI

4) $\text{Darii}(u,w,v) \to (1\text{AII})(u,w,v) \leftrightarrow [\text{A}(w,v) \wedge \text{I}(u,w) \Rightarrow \text{I}(u,v)]$.

5) $\text{Datisi}(u,w,v) \to (3\text{AII})(u,w,v) \leftrightarrow [\text{A}(w,v) \wedge \text{I}(w,u) \Rightarrow \text{I}(u,v)]$.

6) $\text{Disamis}(u,w,v) \to (3\text{IAI})(u,w,v) \leftrightarrow [\text{I}(w,v) \wedge \text{A}(w,u) \Rightarrow \text{I}(u,v)]$.

7) $\text{Dimatis}(u,w,v) \to (4\text{IAI})(u,w,v) \leftrightarrow [\text{I}(v,w) \wedge \text{A}(w,u) \Rightarrow \text{I}(u,v)]$.

3°) Group EAE&AEE

8) $\text{Celarent}(u,w,v) \to (1\text{EAE})(u,w,v) \leftrightarrow [\text{E}(w,v) \wedge \text{A}(u,w) \Rightarrow \text{E}(u,v)]$.

9) $\text{Camestres}(u,w,v) \to (2\text{AEE})(u,w,v) \leftrightarrow [\text{A}(v,w) \wedge \text{E}(u,w) \Rightarrow \text{E}(u,v)]$.

10) $\text{Cesare}(u,w,v) \to (2\text{EAE})(u,w,v) \leftrightarrow [\text{E}(v,w) \wedge \text{A}(u,w) \Rightarrow \text{E}(u,v)]$.

11) $\text{Calemes}(u,w,v) \to (4\text{AEE})(u,w,v) \leftrightarrow [\text{A}(v,w) \wedge \text{E}(w,u) \Rightarrow \text{E}(u,v)]$.

4°) Group EAO

12) $\text{Felapton}(u,w,v) \to (3\text{EAO})(u,w,v) \leftrightarrow [\text{E}(w,v) \wedge \text{A}(w,u) \Rightarrow \text{O}(u,v)]$.

13) $\text{Fesapo}(u,w,v) \to (4\text{EAO})(u,w,v) \leftrightarrow [\text{E}(v,w) \wedge \text{A}(w,u) \Rightarrow \text{O}(u,v)]$.

5°) Group EIO

14) $\text{Ferio}(u,w,v) \to (1\text{EIO})(u,w,v) \leftrightarrow [\text{E}(w,v) \wedge \text{I}(u,w) \Rightarrow \text{O}(u,v)]$.

15) $\text{Festino}(u,w,v) \to (2\text{EIO})(u,w,v) \leftrightarrow [\text{E}(v,w) \wedge \text{I}(u,w) \Rightarrow \text{O}(u,v)]$.

16) $\text{Feriso}(u,w,v) \to (3\text{EIO})(u,w,v) \leftrightarrow [\text{E}(w,v) \wedge \text{I}(w,v) \Rightarrow \text{O}(u,v)]$.

17) $\text{Fresison}(u,w,v) \to (4\text{EIO})(u,w,v) \leftrightarrow [\text{E}(v,w) \wedge \text{I}(w,u) \Rightarrow \text{O}(u,v)]$.

6°) Group AOO&OAO

18) $\text{Baroco}(u,w,v) \to (2\text{AOO})(u,w,v) \leftrightarrow [\text{A}(v,w) \wedge \text{O}(u,w) \Rightarrow \text{O}(u,v)]$.

19) $\text{Bocardo}(u,w,v) \to (3\text{OAO})(u,w,v) \leftrightarrow [\text{O}(w,v) \wedge \text{A}(w,u) \Rightarrow \text{O}(u,v)]$.•

Cmt 1.2. In principle, an MCS may contain three or more premises, but such an MCS is always a combined one that can be reduced to a sequence of Aristotelian MCS's. For instance, the following three-premise syllogism, due to Lewis Carroll (*Symbolic Logic*, Part I, 1896), is given and illustrated by the pertinent *Venn diagrams* in Lipschutz [1964, pp. 225, 226]:

«Babies are illogical. Nobody is despised who can manage a crocodile. Illogical people are despised. Hence, babies cannot manage crocodiles.»

This unconventional MCS is however reduced to the following sequence of two conventional MCS's:

a) Barbara(illogical people, babies, despised people): All illogical people are despised ones. All babies are illogical people. Hence, all babies are despised people.

b) Celarent(despised people, people who can manage crocodiles, babies): All despised people are not people who can manage crocodiles. All babies are despised people. Hence, all babies are not people who can manage crocodiles.

Incidentally, the above two conventional MCS's can be restated as the following equivalent singular MCS's:

a') Barbara(illogical man, baby, despised man): Every illogical man is a despised one. Every baby is an illogical man. Hence, every baby is a despised man.

b') Celarent(despised man, man who can manage crocodile, baby): Every despised man is not a man who can manage a crocodile. Every baby is a despised man. Hence, every baby is not a man who can manage a crocodile.•

1.5. Interpretations of categorical syllogisms

1) In connection with the properties of the premises and conclusion of MCS, which have been explicated in subsection 1.3, the following two questions can be raised:

i) Class-relations of which one of the four kinds: class-membership ones (determined by the copula $\in$), lax class-inclusion ones (determined by the copula $\subseteq$), strict class-inclusion ones (determined by the copula $\subset$), or class-identity ones (determined by the copula $=$) are comprised in the ranges of the premise schemata (forms) and conclusion schema (form) of any given FCS?

ii) Given an FCS, is it tautologous (tautological, universally true) or veracious (accidentally true)?

However, to say nothing of their VSF's (verbal staccato forms), the logographic legato forms (LLF's) of the FCS's, which are defined by Df 1.2 and which are also called FHCS's (formal hypothetico-categorical syllogisms), FQTL's (formal quantified transitive laws), and FSI's (formal syllogistic implications), are in fact *semi-formal*, because the premise and conclusion schemata of the FSI's involve certain ones of the equivocal verbal operators discussed in subsection 1.3. Therefore, no straightforward answers to the above two questions exist within Aristotelian syllogistics itself. The answers to those two questions that will be given in this chapter are multitudinous and hence conditional (not universal) and also they are mutually relative in the sense that the answer to the second question in regard to some specific FCS's depends on an answer to the first question. In general outlook, the answers are described below.

3) Let either bold-faced Roman letter '**u**' or '**v**' be a placeholder, whose range is the set of three light-faced italic letters u, v, and w. Let for each $\mathbf{F} \vec{\in} \{\in, \subseteq, \subset, =\}$:

$$\begin{gathered} \mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \rightarrow \textstyle\bigwedge_{\mathbf{z}}[[\mathbf{zFu}] \Rightarrow [\mathbf{zFv}]], \mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \rightarrow \neg \mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \\ \mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \rightarrow \textstyle\bigwedge_{\mathbf{z}}[[\mathbf{zFu}] \Rightarrow \neg[\mathbf{zFv}]], \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \rightarrow \neg \mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}), \end{gathered} \tag{1.15}$$

$$\begin{gathered} \mathsf{I}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \rightarrow \textstyle\bigvee_{\mathbf{z}}[[\mathbf{zFu}] \wedge [\mathbf{zFv}]], \mathsf{E}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \rightarrow \neg \mathsf{I}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}), \\ \mathsf{O}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \rightarrow \textstyle\bigvee_{\mathbf{z}}[[\mathbf{zFu}] \wedge \neg[\mathbf{zFv}]], \mathsf{A}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \rightarrow \neg \mathsf{O}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}), \end{gathered} \tag{1.16}$$

$$\begin{gathered} \mathsf{I}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \rightarrow \textstyle\bigvee_{\mathbf{z}}[[\mathbf{zFu}] \wedge [[\mathbf{zFu}] \Rightarrow [\mathbf{zFv}]]], \mathsf{E}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \rightarrow \neg \mathsf{I}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}), \\ \mathsf{O}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \rightarrow \textstyle\bigvee_{\mathbf{z}}[[\mathbf{zFu}] \wedge \neg[[\mathbf{zFu}] \Rightarrow [\mathbf{zFv}]]], \mathsf{A}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \rightarrow \neg \mathsf{O}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}). \end{gathered} \tag{1.17}$$

The three quadruples:

$$\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}),\ \mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}),\ \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}),\ \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}) \text{ with } n \vec{\in} \{1,2,3\}, \tag{1.18}$$

thus defined in terms of a given **F**, are ones of standard *euautographic syllogistic judgments* (*ESJ's*), which I shall introduce in the course of further purposeful development of $\mathsf{A}_{1\in}$ ($\mathsf{A}_{1\in}$ and $\mathbf{A}_{1\in}$). The EMT's (EDT's) for the ESJ's have the form for each $n \vec{\in} \{1,2,3\}$:

$$\begin{gathered} V(\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} 1 \hat{-} V(\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \\ \hat{=} \hat{\vee}_{\mathbf{z}}[1 \hat{-} V(\neg[\mathbf{zFu}]) \hat{\vee} V(\mathbf{zFv})], \end{gathered} \tag{1.19}$$

$$\begin{gathered} V(\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} 1 \hat{-} V(\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \\ \hat{=} \hat{\vee}_{\mathbf{z}}[1 \hat{-} V(\neg[\mathbf{zFu}]) \hat{\vee} V(\neg[\mathbf{zFv}])], \end{gathered} \tag{1.20}$$

so that all ESJ's are vav-neutral. Also, it follows from (1.19) and (1.20) that

$$\begin{gathered} V(\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} V(\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})), V(\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} V(\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})), \\ V(\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} V(\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})), V(\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} V(\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})), \\ \text{for each } n \vec{\in} \{2,3\}, \end{gathered} \tag{1.21}$$

$$V(\mathsf{E}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})) \hat{=} V(\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})), V(\mathsf{I}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})) \hat{=} V(\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})). \tag{1.22}$$

There are many ESJ's, which satisfy the identities (1.19)–(1.22), but I confine myself only to the three simplest of them.

4) After the manner of (1.11) and (1.12), I shall introduce three four-figure systems of 19 *euautographic syllogistic implications* (*ESI's*) along with their code names (in each system) by the following four definition schemata for each $\mathbf{F} \vec{\in} \{\in,\subseteq,\subset,=\}$ and each $n \vec{\in} \{1,2,3\}$:

$$\begin{gathered} (\mathsf{1UVW})_{\mathbf{F}n}(u,w,v) \to [\mathsf{U}_{\mathbf{F}n}(w,v) \wedge \mathsf{V}_{\mathbf{F}n}(u,w) \Rightarrow \mathsf{W}_{\mathbf{F}n}(u,v)], \\ (\mathsf{2UVW})_{\mathbf{F}n}(u,w,v) \to [\mathsf{U}_{\mathbf{F}n}(v,w) \wedge \mathsf{V}_{\mathbf{F}n}(u,w) \Rightarrow \mathsf{W}_{\mathbf{F}n}(u,v)], \\ (\mathsf{3UVW})_{\mathbf{F}n}(u,w,v) \to [\mathsf{U}_{\mathbf{F}n}(w,v) \wedge \mathsf{V}_{\mathbf{F}n}(w,u) \Rightarrow \mathsf{W}_{\mathbf{F}n}(u,v)], \\ (\mathsf{4UVW})_{\mathbf{F}n}(u,w,v) \to [\mathsf{U}_{\mathbf{F}n}(v,w) \wedge \mathsf{V}_{\mathbf{F}n}(w,u) \Rightarrow \mathsf{W}_{\mathbf{F}n}(u,v)], \end{gathered} \tag{1.23}$$

subject to

$$\begin{gathered} (\mathsf{1UVW})_{\mathbf{F}n} \vec{\in} \{(\mathsf{1AAA})_{\mathbf{F}n},(\mathsf{1EAE})_{\mathbf{F}n},(\mathsf{1AII})_{\mathbf{F}n},(\mathsf{1EIO})_{\mathbf{F}n}\}, \\ (\mathsf{2UVW})_{\mathbf{F}n} \vec{\in} \{(\mathsf{2AEE})_{\mathbf{F}n},(\mathsf{2EAE})_{\mathbf{F}n},(\mathsf{2EIO})_{\mathbf{F}n},(\mathsf{2AOO})_{\mathbf{F}n}\}, \\ (\mathsf{3UVW})_{\mathbf{F}n} \vec{\in} \{(\mathsf{3AAI})_{\mathbf{F}n},(\mathsf{3AII})_{\mathbf{F}n},(\mathsf{3IAI})_{\mathbf{F}n},(\mathsf{3EAO})_{\mathbf{F}n},(\mathsf{3EIO})_{\mathbf{F}n},(\mathsf{3OAO})_{\mathbf{F}n}\}, \\ (\mathsf{4UVW})_{\mathbf{F}n} \vec{\in} \{(\mathsf{4AAI})_{\mathbf{F}n},(\mathsf{4AEE})_{\mathbf{F}n},(\mathsf{4IAI})_{\mathbf{F}n},(\mathsf{4EAO})_{\mathbf{F}n},(\mathsf{4EIO})_{\mathbf{F}n}\}. \end{gathered} \tag{1.24}$$

Each of the letters 'U', 'V', and 'W' is a *metalogographic* (*metalinguistic logographic*) *placeholder* (*MLPH*), whose range is the set of four letters 'A', 'E', 'I', and 'O', without single quotation marks, in the light-faced upright Gothic (sans serif) font, called

Light-Faced Roman Arial Narrow Font (LFRANF). Any concrete string in the range of each one of the four MLPH's $(\mathsf{1UVW})_{\mathbf{F}n}$ to $(\mathsf{4UVW})_{\mathbf{F}n}$ is called the *logographic logical predicate* (*LLP*) of the corresponding ESI, the understanding being that the latter is uniquely determined by the former, – just as in the case of an FCS. Alternatively, each given ESI $(\mathsf{mUVW})_{\mathbf{F}n}$ can be identified by its *phonographic*, or *verbal*, *logical predicate* (*PhLP* or *VLP*), which consists of the pertinent catchword in LFRANF (as 'Barbara', 'Barapti', etc to 'Bocardo') and of the subscript $_{\mathbf{F}n}$ on it.

5) The solution of the vavn-decision problem for the ESI's defined by (1.23) subject to (1.24) can be summarized as follows. aa

a) Under only *typical axioms* and *theorems* of A_1, for each $\mathbf{F} \vec{\in} \{\in, \subseteq, \subset, =\}$ and each $n \vec{\in} \{1,2,3\}$:

$$\begin{gathered} V((\mathsf{1UVW})_{\mathbf{F}n}(u,w,v)) \mathrel{\hat{=}} V(\neg \mathsf{U}_{\mathbf{F}n}(w,v)) \mathbin{\hat{\cdot}} V(\neg \mathsf{V}_{\mathbf{F}n}(u,w)) \mathbin{\hat{\cdot}} V(\mathsf{W}_{\mathbf{F}n}(u,v)) \mathrel{\hat{=}} 0 \\ \text{for each } (\mathsf{1UVW})_{\mathbf{F}n} \vec{\in} \{(\mathsf{1AAA})_{\mathbf{F}n}, (\mathsf{1EAE})_{\mathbf{F}n}, (\mathsf{1AII})_{\mathbf{F}n}, (\mathsf{1EIO})_{\mathbf{F}n}\}, \end{gathered} \tag{1.25}$$

$$\begin{gathered} V((\mathsf{2UVW})_{\mathbf{F}n}(u,w,v)) \mathrel{\hat{=}} V(\neg \mathsf{U}_{\mathbf{F}n}(v,w)) \mathbin{\hat{\cdot}} V(\neg \mathsf{V}_{\mathbf{F}n}(u,w)) \mathbin{\hat{\cdot}} V(\mathsf{W}_{\mathbf{F}n}(u,v)) \mathrel{\hat{=}} 0 \\ \text{for each } (\mathsf{2UVW})_{\mathbf{F}n} \vec{\in} \{(\mathsf{2AEE})_{\mathbf{F}n}, (\mathsf{2EAE})_{\mathbf{F}n}, (\mathsf{2EIO})_{\mathbf{F}n}, (\mathsf{2AOO})_{\mathbf{F}n}\}, \end{gathered} \tag{1.26}$$

$$\begin{gathered} V((\mathsf{3UVW})_{\mathbf{F}n}(u,w,v)) \mathrel{\hat{=}} V(\neg \mathsf{U}_{\mathbf{F}n}(w,v)) \mathbin{\hat{\cdot}} V(\neg \mathsf{V}_{\mathbf{F}n}(w,u)) \mathbin{\hat{\cdot}} V(\mathsf{W}_{\mathbf{F}n}(u,v)) \mathrel{\hat{=}} 0 \\ \text{for each } (\mathsf{3UVW})_{\mathbf{F}n} \vec{\in} \{(\mathsf{3AII})_{\mathbf{F}n}, (\mathsf{3IAI})_{\mathbf{F}n}, (\mathsf{3EIO})_{\mathbf{F}n}, (\mathsf{3OAO})_{\mathbf{F}n}\}, \end{gathered} \tag{1.27}$$

$$\begin{gathered} V((\mathsf{4UVW})_{\mathbf{F}n}(u,w,v)) \mathrel{\hat{=}} V(\neg \mathsf{U}_{\mathbf{F}n}(v,w)) \mathbin{\hat{\cdot}} V(\neg \mathsf{V}_{\mathbf{F}n}(w,u)) \mathbin{\hat{\cdot}} V(\mathsf{W}_{\mathbf{F}n}(u,v)) \mathrel{\hat{=}} 0 \\ \text{for each } (\mathsf{4UVW})_{\mathbf{F}n} \vec{\in} \{(\mathsf{4AEE})_{\mathbf{F}n}, (\mathsf{4IAI})_{\mathbf{F}n}, (\mathsf{4EIO})_{\mathbf{F}n}\}, \end{gathered} \tag{1.28}$$

$$\begin{gathered} V(\mathsf{Barapti}_{\mathbf{F}n}(u,w,v)) \mathrel{\hat{=}} V((\mathsf{3AAI})_{\mathbf{F}n}(u,w,v)) \\ \mathrel{\hat{=}} V(\neg \mathsf{A}_{\mathbf{F}n}(w,v)) \mathbin{\hat{\cdot}} V(\neg \mathsf{A}_{\mathbf{F}n}(w,u)) \mathbin{\hat{\cdot}} V(\mathsf{I}_{\mathbf{F}n}(u,v)) \\ \mathrel{\hat{=}} \left[\hat{\bullet}_x V(x\mathbf{F}w)\right] \mathbin{\hat{\cdot}} \left[\hat{\bullet}_y \left[1 \mathbin{\hat{-}} V(\neg[y\mathbf{F}u]) \mathbin{\hat{\cdot}} V(\neg[y\mathbf{F}v])\right]\right] \\ \mathrel{\hat{=}} V\left(\bigvee_x [x\mathbf{F}w]\right) \mathbin{\hat{\cdot}} V\left(\bigvee_y [y\mathbf{F}u] \wedge [y\mathbf{F}u]\right) \overleftarrow{\hat{=}} \mathsf{J}^1_{\mathbf{F}}(u,w,v), \end{gathered} \tag{1.29}$$

$$\begin{gathered} V(\mathsf{Bamalip}_{\mathbf{F}n}(u,w,v)) \mathrel{\hat{=}} V((\mathsf{4AAI})_{\mathbf{F}n}(u,w,v)) \\ \mathrel{\hat{=}} V(\neg \mathsf{A}_{\mathbf{F}n}(v,w)) \mathbin{\hat{\cdot}} V(\neg \mathsf{A}_{\mathbf{F}n}(w,u)) \mathbin{\hat{\cdot}} V(\mathsf{I}_{\mathbf{F}n}(u,v)) \\ \mathrel{\hat{=}} \left[\hat{\bullet}_x V(x\mathbf{F}v)\right] \mathbin{\hat{\cdot}} \left[\hat{\bullet}_y \left[1 \mathbin{\hat{-}} V(y\mathbf{F}u) \mathbin{\hat{\cdot}} V(\neg[y\mathbf{F}w])\right]\right] \\ \mathrel{\hat{=}} V\left(\bigvee_x [x\mathbf{F}v]\right) \mathbin{\hat{\cdot}} V\left(\bigvee_y [\neg[y\mathbf{F}u] \wedge [y\mathbf{F}w]]\right) \overleftarrow{\hat{=}} \mathsf{J}^2_{\mathbf{F}}(u,w,v), \end{gathered} \tag{1.30}$$

$$
\begin{aligned}
&V\left(\mathsf{Fesapo}_{\mathbf{F}n}(u,w,v)\right)\hat{=}V\left(\mathsf{Felapton}_{\mathbf{F}n}(u,w,v)\right)\\
&\hat{=}V\left((\mathsf{4EAO})_{\mathbf{F}n}(u,w,v)\right)\hat{=}V\left((\mathsf{3EAO})_{\mathbf{F}n}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathsf{E}_{\mathbf{F}n}(v,w)\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\mathbf{F}n}(w,u)\right)\hat{\cdot}V\left(\mathsf{O}_{\mathbf{F}n}(u,v)\right)\\
&\hat{=}V\left(\neg\mathsf{E}_{\mathbf{F}n}(w,v)\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\mathbf{F}n}(w,u)\right)\hat{\cdot}V\left(\mathsf{O}_{\mathbf{F}n}(u,v)\right)\\
&\hat{=}\left[\hat{\cdot}_x V(x\mathbf{F}w)\right]\hat{\cdot}\left[\hat{\cdot}_y\left[1\hat{-}V(\neg[y\mathbf{F}u])\hat{\cdot}V(y\mathbf{F}v)\right]\right]\\
&\hat{=}V\left(\bigvee_x[x\mathbf{F}w]\right)\hat{\cdot}V\left(\bigvee_y[[y\mathbf{F}u]\wedge\neg[y\mathbf{F}v]]\right)\hat{\hat{=}}\mathsf{J}^3_{\mathbf{F}}(u,w,v).
\end{aligned}
\tag{1.31}
$$

The catchwords of all pertinent ESI's can be recovered with the help of Df 1.2. By (1.21), it follows from definitions (1.23) subject to (1.24) that for each $\mathsf{m}\vec{\in}\{1,2,3,4\}$ and each $n\vec{\in}\{2,3\}$:

$$V\left((\mathsf{mUVW})_{\mathbf{F}n}(u,w,v)\right)\hat{=}V\left((\mathsf{mUVW})_{\mathbf{F}1}(u,w,v)\right). \tag{1.32}$$

Therefore, in making statements about ESI's or about their *conformal catlogographic* (*CFCL*) *interpretands*, I shall, without loss of generality, substitute '$_1$' for '$_n$', with the understanding that the statements apply also with '$_2$' or '$_3$' in place of '$_1$'.

b) According to the pertinent instances of *atypical* (*specific*) theorems (IV.1.45) and (IV.1.47),

$$V\left(\bigvee_x[x\subseteq w]\right)\hat{=}0,\ V\left(\bigvee_x[x\subseteq v]\right)\hat{=}0, \tag{1.33}$$

$$V\left(\bigvee_x[x=w]\right)\hat{=}0,\ V\left(\bigvee_x[x=v]\right)\hat{=}0. \tag{1.34}$$

Hence, the instances of (1.29)–(1.31) with $\subseteq$ (e.g.) in place of '**F**' become:

$$
\begin{aligned}
&V\left(\mathsf{Barapti}_{\subseteq n}(u,w,v)\right)\hat{=}V\left((\mathsf{3AAI})_{\subseteq n}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathsf{A}_{\subseteq n}(w,v)\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\subseteq n}(w,u)\right)\hat{\cdot}V\left(\mathsf{I}_{\subseteq n}(u,v)\right)\hat{=}0,
\end{aligned}
\tag{1.35}
$$

$$
\begin{aligned}
&V\left(\mathsf{Bamalip}_{\subseteq n}(u,w,v)\right)\hat{=}V\left((\mathsf{4AAI})_{\subseteq n}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathsf{A}_{\subseteq n}(v,w)\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\subseteq n}(w,u)\right)\hat{\cdot}V\left(\mathsf{I}_{\subseteq n}(u,v)\right)\hat{=}0,
\end{aligned}
\tag{1.36}
$$

$$
\begin{aligned}
&V\left(\mathsf{Fesapo}_{\subseteq n}(u,w,v)\right)\hat{=}V\left(\mathsf{Felapton}_{\subseteq n}(u,w,v)\right)\\
&\hat{=}V\left((\mathsf{4EAO})_{\subseteq n}(u,w,v)\right)\hat{=}V\left((\mathsf{3EAO})_{\subseteq n}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathsf{E}_{\subseteq n}(v,w)\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\subseteq n}(w,u)\right)\hat{\cdot}V\left(\mathsf{O}v(u,v)\right)\\
&\hat{=}V\left(\neg\mathsf{E}_{\subseteq n}(w,v)\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\subseteq n}(w,u)\right)\hat{\cdot}V\left(\mathsf{O}_{\subseteq n}(u,v)\right)\hat{=}0,
\end{aligned}
\tag{1.37}
$$

and similarly with = in place of $\subseteq$. By contrast, for each $\mathbf{F}\vec{\in}\{\in,\subset\}$ and each $m\vec{\in}\{1,2,3\}$, the integron $\mathsf{J}^m_{\mathbf{F}}(u,w,v)$ cannot be reduced either to 0 or to 1. Therefore, the instances of (1.29)–(1.31) with $\in$ (e.g.) in place of '**F**' become:

$$
\begin{aligned}
&V\left(\mathrm{Barapti}(u,w,v)\right)\hat{=}V\left((\mathrm{3AAI})_{\in n}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathrm{A}_{\in n}(w,v)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\in n}(w,u)\right)\hat{\cdot}V\left(\mathrm{I}_{\in n}(u,v)\right)\\
&\hat{=}\left[\hat{\bullet}_x V(x\in w)\right]\hat{\cdot}\left[\hat{\bullet}_y\left[1\hat{-}V(\neg[y\in u])\hat{\cdot}V(\neg[y\in v])\right]\right]\\
&\hat{=}V\left(\bigvee_x[x\in w]\right)\hat{\cdot}V\left(\bigvee_y[[y\in u]\wedge[y\in v]]\right)\overleftarrow{\hat{=}}\mathrm{J}^1_{\in}(u,w,v),
\end{aligned}
\tag{1.38}
$$

$$
\begin{aligned}
&V\left(\mathrm{Bamalip}_{\in n}(u,w,v)\right)\hat{=}V\left((\mathrm{4AAI})_{\in n}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathrm{A}_{\in n}(v,w)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\in n}(w,u)\right)\hat{\cdot}V\left(\mathrm{I}_{\in n}(u,v)\right)\\
&\hat{=}\left[\hat{\bullet}_x V(x\in v)\right]\hat{\cdot}\left[\hat{\bullet}_y\left[1\hat{-}V(y\in u)\hat{\cdot}V(\neg[y\in w])\right]\right]\\
&\hat{=}V\left(\bigvee_x[x\in v]\right)\hat{\cdot}V\left(\bigvee_y[\neg[y\in u]\wedge[y\in w]]\right)\overleftarrow{\hat{=}}\mathrm{J}^2_{\in}(u,w,v),
\end{aligned}
\tag{1.39}
$$

$$
\begin{aligned}
&V\left(\mathrm{Fesapo}_{\in n}(u,w,v)\right)\hat{=}V\left(\mathrm{Felapton}_{\in n}(u,w,v)\right)\\
&\hat{=}V\left((\mathrm{4EAO})_{\in n}(u,w,v)\right)\hat{=}V\left((\mathrm{3EAO})_{\in n}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{\in n}(v,w)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\in n}(w,u)\right)\hat{\cdot}V\left(\mathrm{O}_{\in n}(u,v)\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{\in n}(w,v)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\in n}(w,u)\right)\hat{\cdot}V\left(\mathrm{O}_{\in n}(u,v)\right)\\
&\hat{=}\left[\hat{\bullet}_x V(x\in w)\right]\hat{\cdot}\left[\hat{\bullet}_y\left[1\hat{-}V(\neg[y\in u])\hat{\cdot}V(y\in v)\right]\right]\\
&\hat{=}V\left(\bigvee_x[x\in w]\right)\hat{\cdot}\left[V\left(\bigvee_y[[y\in u]\wedge\neg[y\in v]]\right)\right]\overleftarrow{\hat{=}}\mathrm{J}^3_{\in}(u,w,v),
\end{aligned}
\tag{1.40}
$$

and similarly with $\subset$ in place of $\in$.

c) Thus, given $n\,\vec{\in}\,\{1,2,3\}$, for each $\mathbf{F}\,\vec{\in}\,\{\in,\subset\}$, the fifteen ESI's, other than the following four:

$$
\begin{aligned}
&\mathrm{Barapti}_{\mathbf{F}n}(u,w,v),\ \mathrm{Bamalip}_{\mathbf{F}n}(u,w,v),\ \mathrm{Felapton}_{\mathbf{F}n}(u,w,v),\ \mathrm{Fesapo}_{\mathbf{F}n}(u,w,v),\\
&\text{i.e. }(\mathrm{3AAI})_{\mathbf{F}n}(u,w,v),\ (\mathrm{4AAI})_{\mathbf{F}n}(u,w,v),\ (\mathrm{3EAO})_{\mathbf{F}n}(u,w,v),\ (\mathrm{4EAO})_{\mathbf{F}n}(u,w,v),
\end{aligned}
\tag{1.41}
$$

and for each $\mathbf{F}\,\vec{\in}\,\{\subseteq,=\}$, all nineteen ESI's without any exception are *valid* (*kyrologous*), whereas the above four exceptional ESI's are *vav-neutral* (*vav-indeterminate*, *udeterologous*).

6) The EMT (EDT) of an ESI is alternatively called a *euautographic syllogistic master,* or *decision, theorem* (*ESMT* or *ESDT*). I shall use the abbreviations: "*HESMT*" (or "*HEMDT*") for "*homogeneous* ESMT" (or "*homogeneous* ESDT") and "*IHESMT*" (or "*IHEMDT*") for "*inhomogeneous* ESMT" (or "*inhomogeneous* ESDT"). Since the conjuncts of the antecedent and the consequent of the ESI (euautographic syllogistic implication), being the ESR (euautographic slave relation) of a certain HESMT, are vav-neutral ER's, therefore *neither the HESMT nor its ESR is a rule of inference*, i.e. *neither of the two is a categorical syllogism*, – to say nothing of an IHESMT and its ESR.

7) In accordance with Ax I.8.1(2), Df I.8.4(2), and Cmt II.7.5(7), the *analo-catlogographic* (and hence *analo-homolographic*) substitutions (I.8.18) and (II.7.32a), i.e.

$$u \mapsto \mathsf{u},\ v \mapsto \mathsf{v},\ w \mapsto \mathsf{w},\ x \mapsto \mathsf{x},\ y \mapsto \mathsf{y},\ z \mapsto \mathsf{z}, \tag{1.42}$$

$$V \mapsto \mathsf{V}, \tag{1.43}$$

without any quotation marks, throughout a *euautographic master*, or *decision*, *theorem* (*EMT* or *EDT*) and hence throughout its *euautographic slave relation* (*ESR*), such as e.g. as an ESJ (euautographic syllogistic judgment) or an ESI (euautographic syllogistic implication), result in the *catlogographic relations* (*CLR's*), which are respectively called the *conformal catlogographic* (*CFCL*) *interpretand of the EMT* (*EDT*) and the *CFCL of the ESR* or, more generally, a *CFCL master*, or *decision*, *theorem* (*CFCLMT* or *CFCLDT*) and a *catlogographic slave relation* (*CLSR*). Consequently, the CFCL interpretand of an ESJ is called a *catlogographic syllogistic judgment schema* (*CLSJS*, pl. *CLSJS'ta*), or *form* (*CLSJF*), and similarly the CFCL interpretand of an ESI is called a *catlogographic syllogistic implication schema* (*CLSIS*, pl. *CLSIS'ta*), or *form* (*CLSIF*).

8) In accordance with Ax 8.1**(**6), a CLR, i.e. the CFCL interpretand of a vavn-decided ER, preserves the validity-value of the ER and acquires the respective *tautologousness-value* that is compatible with its validity-value so that the CLR is said to be *tautologous* (*universally true*) or *antitautologous* (*universally antitrue*, *universally false*, *contradictory*) or else *ttatt-neutral* (*neutral with respect to tautologousness and antitautologousness*, *neither tautologous nor antitatologous*) if and only if it is *valid* (*kyrologous*) or *antivalid* (*antikyrologous*) or *vav-neutral neutral with respect to validity and antivalidity* (*neither valid nor antivalid, udeterologous*) respectively.

9) A *ttatt-neutral* CLR, i.e. the CFCL interpretand of a vav-neutral ER, is called a *transformative CLR* (*TCLR*) and also the *transformative CFCL* (*TCFCL*) *interpretand of the ER* in either one of the following two cases:

a) The CLR is *assumed* (*postulated*, *taken for granted*) to be *veracious*, i.e. *accidentally true*, in the sense that it is *conformable to a certain fact of interrelation of classes*.

b) The CLR is a CFCLMT (CFCLDT), which is developed further with allowance for the *catlogographic postulates* (see the previous item) so as to result in the *transformed*, or *transformative*, *CFCLMT* (*TCFCLMT*), according to which its CLSR is unambiguously decided to be one of the following three kinds: *veracious* (*accidentally true*), *antiveracious* (*accidentally antitrue*, *accidentally false*), or *vravr-neutral* (*vravr-indeterminate*, *neither veracious nor antiveracious*).

A CLR, i.e. the CFCL interpretand of an ER, is said to be a *conservative CLR* (*CCLR*) and also the *conservative CFCL* (*TCFCL*) *interpretand of the ER* if it is not transformative. Particularly, a CLSR is said to be a conservative one if and only if it is a *ttatt-determinate* (*ttatt-unnutral*, *tautologous* or *antitautologous*) *CLR*, i.e. the CFCL interpretand of a *vav-determinate* (*vav-unnutral*, *valid* or *antivalid*) *ER*, or else a *suspended ttatt-neutral* (*ttatt-indeterminate*) *CLR*. Likewise, a CFCLMT (CFCLMT) is called a *conservative one* (*CCFCLMT* or *CCFCLMT*) if it is not transformative.

10) The *tautologousness-value tautologousness* (*universal truth*) or the *veracity-value veracity* (*accidental truth*) is indiscriminately called the *truth-value truth*; whereas the *tautologousness-value antitautologousness* (*universal antitruth, universal falsity, contradictoriness*) or the *veracity-value antiveracity* (*accidental antitruth, accidental falsity*) is indiscriminately called the *truth-value antitruth* (*falsity*); the veracity-value vravr-neutrality is alternatively called the *truth-value truth-antitruth neutrality* (*tat-neutrality*) and vice versa. Hence, the tautologousness-values tautologousness and antitautologousness are at the same time the truth-values truth and antitruth respectively, but not vice versa.

11) In accordance with the above item 7, a CLSIS (CLSIF) has the same LLP (logographic logical predicate) and the same VLP (verbal logical predicate) as those of the ESI being its conformal interpretans. For instance, for each $\mathbf{F} \,\vec{\in}\, \{\in, \subseteq, \subset, =\}$ and each $n \,\vec{\in}\, \{1,2,3\}$,

$$\begin{gathered}\text{Barapti}_{\mathbf{F}n}(u,w,v), \text{Bamalip}_{\mathbf{F}n}(u,w,v), \text{Felapton}_{\mathbf{F}n}(u,w,v), \text{Fesapo}_{\mathbf{F}n}(u,w,v),\\ \text{i.e.}\ (\text{3AAI})_{\mathbf{F}n}(u,w,v), (\text{4AAI})_{\mathbf{F}n}(u,w,v), (\text{3EAO})_{\mathbf{F}n}(u,w,v), (\text{4EAO})_{\mathbf{F}n}(u,w,v),\end{gathered} \tag{1.41κ}$$

are CFCL interpretands of ESI's (1.41), the understanding being that

$$\begin{gathered}V\left(\text{Barapti}_{\subseteq n}(u,w,v)\right) \hat{=} V\left((\text{3AAI})_{\subseteq n}(u,w,v)\right)\\ \hat{=} V\left(\neg \text{A}_{\subseteq n}(w,v)\right) \hat{\vee} V\left(\neg \text{A}_{\subseteq n}(w,u)\right) \hat{\vee} V\left(\text{I}_{\subseteq n}(u,v)\right) \hat{=} 0,\end{gathered} \tag{1.35κ}$$

$$\begin{gathered}V\left(\text{Bamalip}_{\subseteq n}(u,w,v)\right) \hat{=} V\left((\text{4AAI})_{\subseteq n}(u,w,v)\right)\\ \hat{=} V\left(\neg \text{A}_{\subseteq n}(v,w)\right) \hat{\vee} V\left(\neg \text{A}_{\subseteq n}(w,u)\right) \hat{\vee} V\left(\text{I}_{\subseteq n}(u,v)\right) \hat{=} 0,\end{gathered} \tag{1.36κ}$$

$$\begin{aligned} &V\left(\mathsf{Fesapo}_{\subseteq n}(u,w,v)\right) \hat{=} V\left(\mathsf{Felapton}_{\subseteq n}(u,w,v)\right) \\ &\hat{=} V\left((\mathsf{4EAO})_{\subseteq n}(u,w,v)\right) \hat{=} V\left((\mathsf{3EAO})_{\subseteq n}(u,w,v)\right) \\ &\hat{=} V\left(\neg\mathsf{E}_{\subseteq n}(v,w)\right) \hat{\cdot} V\left(\neg\mathsf{A}_{\subseteq n}(w,u)\right) \hat{\cdot} V\left(\mathsf{O}_{\subseteq n}(u,v)\right) \\ &\hat{=} V\left(\neg\mathsf{E}_{\subseteq n}(w,v)\right) \hat{\cdot} V\left(\neg\mathsf{A}_{\subseteq n}(w,u)\right) \hat{\cdot} V\left(\mathsf{O}_{\subseteq n}(u,v)\right) \hat{=} 0, \end{aligned} \tag{1.37κ}$$

and similarly with = in place of ⊆, and also that

$$\begin{aligned} &V\left(\mathsf{Barapti}_{\in n}(u,w,v)\right) \hat{=} V\left((\mathsf{3AAI})_{\in n}(u,w,v)\right) \\ &\hat{=} V\left(\neg\mathsf{A}_{\in n}(w,v)\right) \hat{\cdot} V\left(\neg\mathsf{A}_{\in n}(w,u)\right) \hat{\cdot} V\left(\mathsf{I}_{\in n}(u,v)\right) \\ &\hat{=} \left[\hat{\cdot}_x V(x \in w)\right] \hat{\cdot} \left[\hat{\cdot}_y \left[1 \hat{-} V(\neg[y \in u]) \hat{\cdot} V(\neg[y \in v])\right]\right] \\ &\hat{=} V\left(\bigvee_x [x \in w]\right) \hat{\cdot} V\left(\bigvee_y [[y \in u] \wedge [y \in v]]\right) \overset{\leftarrow}{\hat{=}} \mathsf{J}^1_{\in}(u,w,v), \end{aligned} \tag{1.38κ}$$

$$\begin{aligned} &V\left(\mathsf{Bamalip}_{\in n}(u,w,v)\right) \hat{=} V\left((\mathsf{4AAI})_{\in n}(u,w,v)\right) \\ &\hat{=} V\left(\neg\mathsf{A}_{\in n}(v,w)\right) \hat{\cdot} V\left(\neg\mathsf{A}_{\in n}(w,u)\right) \hat{\cdot} V\left(\mathsf{I}_{\in n}(u,v)\right) \\ &\hat{=} \left[\hat{\cdot}_x V(x \in v)\right] \hat{\cdot} \left[\hat{\cdot}_y \left[1 \hat{-} V(y \in u) \hat{\cdot} V(\neg[y \in w])\right]\right] \\ &\hat{=} V\left(\bigvee_x [x \in v]\right) \hat{\cdot} V\left(\bigvee_y [\neg[y \in u] \wedge [y \in w]]\right) \overset{\leftarrow}{\hat{=}} \mathsf{J}^2_{\in}(u,w,v), \end{aligned} \tag{1.39κ}$$

$$\begin{aligned} &V\left(\mathsf{Fesapo}_{\in n}(u,w,v)\right) \hat{=} V\left(\mathsf{Felapton}_{\in n}(u,w,v)\right) \\ &\hat{=} V\left((\mathsf{4EAO})_{\in n}(u,w,v)\right) \hat{=} V\left((\mathsf{3EAO})_{\in n}(u,w,v)\right) \\ &\hat{=} V\left(\neg\mathsf{E}_{\in n}(v,w)\right) \hat{\cdot} V\left(\neg\mathsf{A}_{\in n}(w,u)\right) \hat{\cdot} V\left(\mathsf{O}_{\in n}(u,v)\right) \\ &\hat{=} V\left(\neg\mathsf{E}_{\in n}(w,v)\right) \hat{\cdot} V\left(\neg\mathsf{A}_{\in n}(w,u)\right) \hat{\cdot} V\left(\mathsf{O}_{\in n}(u,v)\right) \\ &\hat{=} \left[\hat{\cdot}_x V(x \in w)\right] \hat{\cdot} \left[\hat{\cdot}_y \left[1 \hat{-} V(\neg[y \in u]) \hat{\cdot} V(y \in v)\right]\right] \\ &\hat{=} V\left(\bigvee_x [x \in w]\right) \hat{\cdot} \left[V\left(\bigvee_y [[y \in u] \wedge \neg[y \in v]]\right)\right] \overset{\leftarrow}{\hat{=}} \mathsf{J}^3_{\in}(u,w,v), \end{aligned} \tag{1.40κ}$$

and similarly with ⊂ in place of ∈.

12) The CFCLMT (CFCLDT) of a PLSIS (PLSIF) is alternatively called a *CFCL syllogistic master*, or *decision, theorem* (*CFCLSMT* or *CFCLSDT*). I shall use the abbreviations: "*HCFCLSMT*" (or "*HCFCLDT*") for "*homogeneous* CFCLSMT" (or "*homogeneous* CFCLSDT"), "*IHCFCLSMT*" (or "*IHCFCLSDT*") for "*inhomogeneous* CFCLSMT" (or "*inhomogeneous* CFCLSDT"), and also similar self-explanatory abbreviations with "CCFCL" or "TCFCL" in place of "CCFCL".

13) According to item 6 of this definition, an HESMT is not an inference rule. By contrast, an HCCFCLSMT is a *catlogographic inference rule*. For instance, it follows from (1.35κ) that

$$\text{if } V\left(\mathsf{A}_{\subseteq n}(w,v)\right) \hat{=} 0 \text{ and } V\left(\mathsf{A}_{\subseteq n}(w,u)\right) \hat{=} 0 \text{ then } V\left(\mathsf{I}_{\subseteq n}(u,v)\right) \hat{=} 0, \tag{1.44}$$

i.e.

$$\text{if '}\mathsf{A}_{\subseteq n}(w,v)\text{' and '}\mathsf{A}_{\subseteq n}(w,u)\text{' are veracious then '}\mathsf{A}_{\subseteq n}(u,v)\text{' is veracious} \tag{1.44'}$$

or simply

$$\text{if } \mathsf{A}_{\subseteq n}(w,v) \text{ and } \mathsf{A}_{\subseteq n}(w,u) \text{ then } \mathsf{A}_{\subseteq n}(u,v), \tag{1.44''}$$

which is a semi-verbal form of $\mathsf{Barapti}_{\subseteq n}(u,w,v)$, i.e. of $(\mathsf{3AAI})_{\subseteq n}(u,w,v)$. Consequently, (1.44'') can be used as an interpretand of the FCS $\text{Barapti}(u,w,v)$, i.e. $(\text{3AAI})(u,w,v)$, in accordance with the formal definition:

$$\begin{gathered} \text{Barapti}(u,w,v) \leftrightarrow (\text{3AAI})(u,w,v) \\ \to [\text{A}(w,v) \wedge \text{A}(w,u) \Rightarrow \text{I}(u,v)] \to [\mathsf{A}_{\subseteq n}(w,v) \wedge \mathsf{A}_{\subseteq n}(w,u) \Rightarrow \mathsf{I}_{\subseteq n}(u,v)] \\ \to (\mathsf{3AAI})_{\subseteq n}(u,w,v) \leftrightarrow \mathsf{Barapti}_{\subseteq n}(u,w,v). \end{gathered} \tag{1.45}$$

Similarly, (1.36κ) and (1.37κ) imply that

$$\begin{gathered} \text{Bamalip}(u,w,v) \leftrightarrow (\text{4AAI})(u,w,v) \\ \to [\text{A}(v,w) \wedge \text{A}(w,u) \Rightarrow \text{I}(u,v)] \to [\mathsf{A}_{\subseteq n}(v,w) \wedge \mathsf{A}_{\subseteq n}(w,u) \Rightarrow \mathsf{I}_{\subseteq n}(u,v)] \\ \to (\mathsf{4AAI})_{\subseteq n}(u,w,v) \leftrightarrow \mathsf{Bamalip}_{\subseteq n}(u,w,v), \end{gathered} \tag{1.46}$$

$$\begin{gathered} \text{Felapton}(u,w,v) \leftrightarrow (\text{3EAO})(u,w,v) \\ \to [\text{E}(w,v) \wedge \text{A}(w,u) \Rightarrow \text{O}(u,v)] \to [\mathsf{E}_{\subseteq n}(w,v) \wedge \mathsf{A}_{\subseteq n}(w,u) \Rightarrow \mathsf{O}_{\subseteq n}(u,v)] \\ \to (\mathsf{3EAO})_{\subseteq n}(u,w,v) \leftrightarrow \mathsf{Felapton}_{\subseteq n}(u,w,v), \end{gathered} \tag{1.47}$$

$$\begin{gathered} \text{Fesapo}(u,w,v) \leftrightarrow (\text{4EAO})(u,w,v) \\ \to [\text{E}(v,w) \wedge \text{A}(w,u) \Rightarrow \text{O}(u,v)] \to [\mathsf{E}_{\subseteq n}(v,w) \wedge \mathsf{A}_{\subseteq n}(w,u) \Rightarrow \mathsf{O}_{\subseteq n}(u,v)] \\ \to (\mathsf{4EAO})_{\subseteq n}(u,w,v) \leftrightarrow \mathsf{Fesapo}_{\subseteq n}(u,w,v). \end{gathered} \tag{1.48}$$

It is understood that definitions (1.45)–(1.48) apply with = in place of $\subseteq$.

14) Since the CCFCLMT's (1.38κ)–(1.40κ) are inhomogeneous, therefore they are not catlogographic inference rules. At the same time, any one of the following catlogographic postulates, – *permanent ones*, called *catlogographic axioms*, or *ad hoc ones*, called *catlogographic hypotheses*, – can be adopted (taken for granted):

$$\begin{gathered} \mathsf{J}_{\in}^{1}(u,w,v) \stackrel{\rightarrow}{\hat{=}} V\left(\bigvee_x [x \in w]\right) \hat{\cdot} V\left(\bigvee_y [[y \in u] \wedge [y \in v]]\right) \\ \hat{=} \left[\hat{\bullet}_x V(x \in w)\right] \hat{\cdot} \left[\hat{\bullet}_y [1 \hat{-} V(\neg[y \in u]) \hat{\cdot} V(\neg[y \in v])]\right] \hat{=} 0, \end{gathered} \tag{1.49}$$

i.e.

$$\bigvee_x [x \in w] \text{ or } \bigvee_y [[y \in u] \wedge [y \in v]], \tag{1.49a}$$

which means that the class *w* is not empty or the classes *u* and *v* are not empty, or else all the three classes are not empty;

$$\begin{gathered} \mathsf{J}_{\in}^{2}(u,w,v) \stackrel{\rightarrow}{\hat{=}} V\left(\bigvee_x [x \in v]\right) \hat{\cdot} V\left(\bigvee_y [\neg[y \in u] \wedge [y \in w]]\right) \\ \hat{=} \left[\hat{\bullet}_x V(x \in v)\right] \hat{\cdot} \left[\hat{\bullet}_y [1 \hat{-} V(y \in u) \hat{\cdot} V(\neg[y \in w])]\right] \hat{=} 0, \end{gathered} \tag{1.50}$$

i.e.

$$\bigvee_x [x \in v] \text{or} \bigvee_y [\neg[y \in u] \wedge [y \in w]], \tag{1.50a}$$

which means that the class *v* is not empty or the class *w* is not empty and is not intersected with the class *u*, or else both conditions are satisfied simultaneously;

$$\begin{aligned} \mathsf{J}^3_{\in}(u,w,v) &\vec{\hat{=}} V\left(\bigvee_x [x \in w]\right) \hat{\cdot} \left[V\left(\bigvee_y [[y \in u] \wedge \neg[y \in v]]\right)\right] \\ &\hat{=} \left[\hat{\boldsymbol{\cdot}}_x V(x \in w)\right] \hat{\cdot} \left[\hat{\boldsymbol{\cdot}}_y [1 \hat{-} V(\neg[y \in u]) \hat{\cdot} V(y \in v)]\right] \hat{=} 0, \end{aligned} \tag{1.51}$$

i.e.

$$\bigvee_x [x \in w] \text{or} \bigvee_y [[y \in u] \wedge \neg[y \in v]], \tag{1.51a}$$

which means that the class *w* is not empty or the class *u* is not empty and is not intersected with the class *v*, or else both conditions are satisfied simultaneously. Under postulates (1.49)–(1.51), the *IHC*CFCLSMT's (1.38κ)–(1.40κ) turn into the following *HT*CFCLSMT's:

$$\begin{aligned} &V(\mathsf{Barapti}_{\in n}(u,w,v)) \hat{=} V((\mathsf{3AAI})_{\in n}(u,w,v)) \\ &\hat{=} V(\neg\mathsf{A}_{\in n}(w,v)) \hat{\cdot} V(\neg\mathsf{A}_{\in n}(w,u)) \hat{\cdot} V(\mathsf{I}_{\in n}(u,v)) \hat{=} 0, \end{aligned} \tag{1.49b}$$

$$\begin{aligned} &V(\mathsf{Bamalip}_{\in n}(u,w,v)) \hat{=} V((\mathsf{4AAI})_{\in n}(u,w,v)) \\ &\hat{=} V(\neg\mathsf{A}_{\in n}(v,w)) \hat{\cdot} V(\neg\mathsf{A}_{\in n}(w,u)) \hat{\cdot} V(\mathsf{I}_{\in n}(u,v)) \hat{=} 0, \end{aligned} \tag{1.50b}$$

$$\begin{aligned} &V(\mathsf{Fesapo}_{\in n}(u,w,v)) \hat{=} V(\mathsf{Felapton}_{\in n}(u,w,v)) \\ &\hat{=} V((\mathsf{4EAO})_{\in n}(u,w,v)) \hat{=} V((\mathsf{3EAO})_{\in n}(u,w,v)) \\ &\hat{=} V(\neg\mathsf{E}_{\in n}(v,w)) \hat{\cdot} V(\neg\mathsf{A}_{\in n}(w,u)) \hat{\cdot} V(\mathsf{O}_{\in n}(u,v)) \\ &\hat{=} V(\neg\mathsf{E}_{\in n}(w,v)) \hat{\cdot} V(\neg\mathsf{A}_{\in n}(w,u)) \hat{\cdot} V(\mathsf{O}_{\in n}(u,v)) \hat{=} 0, \end{aligned} \tag{1.51b}$$

respectively, which are *veracious* and not tautologous. Consequently, the CLSR's (catlogographic slave relations) of (1.49b)–(1.51b) are *veracious ttatt-neutral CLSI's* (*catlogographic syllogistic implications*), which can be used as alternative interpretands of the pertinent FCS's in accordance with the variants of definitions (1.45)–(1.48) with $\in$ in place of $\subseteq$, namely:

$$\begin{aligned} &\mathrm{Barapti}(u,w,v) \leftrightarrow (\mathrm{3AAI})(u,w,v) \\ &\rightarrow [\mathrm{A}(w,v) \wedge \mathrm{A}(w,u) \Rightarrow \mathrm{I}(u,v)] \rightarrow [\mathsf{A}_{\in n}(w,v) \wedge \mathsf{A}_{\in n}(w,u) \Rightarrow \mathsf{I}_{\in n}(u,v)] \\ &\rightarrow (\mathsf{3AAI})_{\in n}(u,w,v) \leftrightarrow \mathsf{Barapti}_{\in n}(u,w,v), \end{aligned} \tag{1.45$_1$}$$

$$\begin{aligned} &\mathrm{Bamalip}(u,w,v) \leftrightarrow (\mathrm{4AAI})(u,w,v) \\ &\rightarrow [\mathrm{A}(v,w) \wedge \mathrm{A}(w,u) \Rightarrow \mathrm{I}(u,v)] \rightarrow [\mathsf{A}_{\in n}(v,w) \wedge \mathsf{A}_{\in n}(w,u) \Rightarrow \mathsf{I}_{\in n}(u,v)] \\ &\rightarrow (\mathsf{4AAI})_{\in n}(u,w,v) \leftrightarrow \mathsf{Bamalip}_{\in n}(u,w,v), \end{aligned} \tag{1.46$_1$}$$

$$\begin{aligned} &\mathrm{Felapton}(u,w,v) \leftrightarrow (\mathrm{3EAO})(u,w,v) \\ &\rightarrow [\mathrm{E}(w,v) \wedge \mathrm{A}(w,u) \Rightarrow \mathrm{O}(u,v)] \rightarrow [\mathsf{E}_{\in n}(w,v) \wedge \mathsf{A}_{\in n}(w,u) \Rightarrow \mathsf{O}_{\in n}(u,v)] \\ &\rightarrow (\mathsf{3EAO})_{\in n}(u,w,v) \leftrightarrow \mathsf{Felapton}_{\in n}(u,w,v), \end{aligned} \tag{1.47$_1$}$$

$$\begin{gathered}\text{Fesapo}(u,w,v)\leftrightarrow(\text{4EAO})(u,w,v)\\ \to[\text{E}(v,w)\wedge\text{A}(w,u)\Rightarrow\text{O}(u,v)]\to[\mathsf{E}_{\in n}(v,w)\wedge\mathsf{A}_{\in n}(w,u)\Rightarrow\mathsf{O}_{\in n}(u,v)]\\ \to(\mathsf{4EAO})_{\in n}(u,w,v)\leftrightarrow\mathsf{Fesapo}_{\in n}(u,w,v).\end{gathered}\quad(1.48_1)$$

15) The previous item applies, *mutatis mutandis*, with $\subset$ in place of $\in$.

16) In the exclusion of four FCS's having the PLP's (phonographic logical predicates)

Barapti (former Darapti), Bamalip, Felapton, and Fesapo, (1.52)

and the respective LLP's (logographic logical predicates)

(3AAI), (4AAI), (3EAO), (4EAO), (1.53)

each one of the other fifteen FCS's is equivocally interpreted by 12 *tautologous* ESI's with each $\mathbf{F}\mathrel{\vec{\in}}\{\in,\subseteq,\subset,=\}$ and each $n\mathrel{\vec{\in}}\{1,2,3\}$, without any exception.

17) Hilbert and Ackermann [1950, pp. 48–54, 53ff] have demonstrated that all FCS's in the exclusion of those of the list (1.52) *are deducible from Boolean algebra*.

2. Panlogographic and euautographic syllogistic bases

2.1. Basic panlogographic nomenclature

Ax 2.1. The following assumptions are based on Df II.1.7. In stating them, all pertinent panlogographic ordinary terms (PLOT's) and panlogographic ordinary relations (PLOR's) of $\mathbf{A}_1$ are used xenonymously as panlogographic placeholders (PLPH's) for mentioning respectively common (general, certain, concrete but not concretized) euautographic ordinary terms (EOT's) and common euautographic relations of A_1, of their ranges. In accordance with Df I.4.1(3a), a common member of the range of a PLPH is the range itself that I use in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* the range as *my as if extramental* (*exopsychical*) *object*.

1) $\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$ is a relation of A_1 that contains two different free APVOT's **z** and **u**.

2) **v**, **w**, **x**, and **y** are four different APVOT's, other than **z** and **u**, which do not occur in $\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$, so that

$$\begin{gathered}\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\to\mathrm{S}^{\mathbf{u}}_{\mathbf{v}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle|,\mathbf{P}\langle\mathbf{z},\mathbf{w}\rangle\to\mathrm{S}^{\mathbf{u}}_{\mathbf{w}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle|,\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\to\mathrm{S}^{\mathbf{z}}_{\mathbf{x}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle|,\\ \mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\to\mathrm{S}^{\mathbf{z}}_{\mathbf{y}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle|,\ \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\to\mathrm{S}^{\mathbf{u}}_{\mathbf{v}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle|,\text{ etc}\end{gathered}\quad(2.1)$$

and conversely

$$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \leftrightarrow \mathrm{S}_{\mathbf{u}}^{\mathbf{v}}\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\Big|, \mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \leftrightarrow \mathrm{S}_{\mathbf{u}}^{\mathbf{w}}\mathbf{P}\langle\mathbf{z},\mathbf{w}\rangle\Big|, \mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \leftrightarrow \mathrm{S}_{\mathbf{z}}^{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\Big|,$$
$$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \leftrightarrow \mathrm{S}_{\mathbf{z}}^{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\Big|,\ \mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \leftrightarrow \mathrm{S}_{\mathbf{u}}^{\mathbf{v}}\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\Big|, \text{etc} \tag{2.2}$$

where $\mathrm{S}_{\mathbf{v}}^{\mathbf{u}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Big|$, e.g., is the relation which results by of substitution of **v** for each occurrence of **u** throughout $\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$.

3) The PLOT's '**u**', '**v**', '**w**', '**x**', '**y**', and '**z**' preserve their recognizable identities, unless stated otherwise.•

Cnv 2.1. In accordance with Ax 2.1, all different PLOT's that occur in the same panlogographic formula of $\mathbf{A}_1$ are supposed to take on mutually different euautographic EOT's of A_1, unless stated otherwise.•

2.2. General underlying definitions

Df 2.1. 1)

$$\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \to \bigwedge\nolimits_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle], \mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle \to \neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle,$$
$$\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \to \bigwedge\nolimits_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle], \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle \to \neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle. \tag{2.3}$$

$$\mathsf{I}_2\langle\mathbf{u},\mathbf{v}\rangle \to \bigvee\nolimits_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle], \mathsf{E}_2\langle\mathbf{u},\mathbf{v}\rangle \to \neg\mathsf{I}_2\langle\mathbf{u},\mathbf{v}\rangle$$
$$\mathsf{O}_2\langle\mathbf{u},\mathbf{v}\rangle \to \bigvee\nolimits_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle], \mathsf{A}_2\langle\mathbf{u},\mathbf{v}\rangle \to \neg\mathsf{O}_2\langle\mathbf{u},\mathbf{v}\rangle. \tag{2.4}$$

$$\mathsf{I}_3\langle\mathbf{u},\mathbf{v}\rangle \to \bigvee\nolimits_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge [\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]],$$
$$\mathsf{O}_3\langle\mathbf{u},\mathbf{v}\rangle \to \bigvee\nolimits_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \neg[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]], \tag{2.5}$$
$$\mathsf{A}_3\langle\mathbf{u},\mathbf{v}\rangle \to \neg\mathsf{O}_3\langle\mathbf{u},\mathbf{v}\rangle, \mathsf{E}_3\langle\mathbf{u},\mathbf{v}\rangle \to \neg\mathsf{I}_3\langle\mathbf{u},\mathbf{v}\rangle.$$

$$\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \to \bigwedge\nolimits_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle], \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle \to \neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle,$$
$$\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle \to \mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle, \mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle \to \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle. \tag{2.6}$$

2) The letters 'A', 'E', 'I', and 'O' in the definienda of separate definitions comprised in the definition groups (2.3)–(2.6) indicate that the relations of $\mathbf{A}_1$ serving as the respective definientia and that euautographic instances of those relations, belonging to A_1, are regarded as analogues of the *verbal syllogistic judgments* (*VSJ's*) of the classes (kinds), which are conventionally denoted by the letters "A", "E", "I", and "O" or "a", "e", "i", and "o" respectively. The digital subscripts $_1$, $_2$, $_3$, $_4$ on the letters 'A', 'E', 'I', and 'O' are labels identifying different definienda of the different definientia of the same class.•

Df 2.2. 1) Df 2.1 applies under the substitutions:

$$\tilde{\mathsf{A}} \mapsto \mathsf{A},\ \tilde{\mathsf{E}} \mapsto \mathsf{E},\ \tilde{\mathsf{I}} \mapsto \mathsf{I},\ \tilde{\mathsf{O}} \mapsto \mathsf{O}, \tag{2.7}$$

$$\langle\mathbf{u},\mathbf{z}\rangle \mapsto \langle\mathbf{z},\mathbf{u}\rangle,\ \langle\mathbf{v},\mathbf{z}\rangle \mapsto \langle\mathbf{z},\mathbf{v}\rangle. \tag{2.8}$$

2) All statements that are made below on the basis of Df 2.1 are supposed to apply under substitutions (2.7) and (2.8) as well, unless stated otherwise.

3) The variant of Df 2.1, which results by substitutions (2.7) and (2.8), or the like variant of any expression that is based on Df 2.1 will be qualified *transposed* and it will, when convenient, be referred to by the same double position numeral (if it has one) adjoined with the letter "t". Particularly the transposed variants of separate definition groups (2.3)–(2.6) will be referred to as (2.3t)–(2.6t), while the transposed variant of the entire Df 2.1 will be will be referred to as Df 2.1t.•

Cmt 2.1. 1) The definition sign $\rightarrow$ occurring in definitions (2.3)–(2.6) should be understood as applied *contactually* to the graphonyms, between which it stands. Thus, for instance, in the first one of definitions (2.3), the panlogograph '$\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle$' is the definiendum, whereas the panlogograph '$\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]$' is the definiens. In this case, the definiens is a panlogographic relation (PLR) of $\mathbf{A}_1$ and at the same time a panlogographic schema (PLS) of euautographic relations (ER's) of A_1, while the definiendum is an *analytical molecular PLR* (*AnMlPLR*) *of* $\mathbf{A}_1$, i.e. it is *not schematic* (*not patterned*), and therefore it can be used for mentioning an ER of A_1 in the range of its patterned (schematic) definiens '$\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]$' only *transitively via the definiens, and not directly*. Thus, **'**$\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle$**'** *is* (*stands for*) *for the PLR* '$\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]$' (and not, say, for '$\bigwedge_{\mathbf{z}}[\mathbf{Q}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{Q}\langle\mathbf{z},\mathbf{v}\rangle]$'), whereas $\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle$ (without quotation marks) *is* (*stands for*) *the PLR* $\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]$ (without quotation marks), and at the same time it may *transitively* stand for any ER of A_1, in the range '$\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]$', i.e. it *is, equivocally, any ER* $\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]$.

2) The second one of definitions (2.3), e.g., should be understood as the train of definitions:

$$\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\rightarrow\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\rightarrow\neg\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle], \tag{2.9}$$

so that the actual (detailed) ultimate definiens of '$\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle$' is '$\neg\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]$', being another PLR of $\mathbf{A}_1$ and another PLS of ER's of A_1, while the definiendum '$\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle$' is another AnMlPLR of $\mathbf{A}_1$ and another non-schematic PLPH of ER's of A_1.The definition $\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\rightarrow\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle$ and all other like definitions are made in this manner for the sake of brevity and also for indicating the most immediate relationships, which exist among the AnMlPLR's '$\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle$', '$\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle$', etc to '$\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle$' from the very beginning owing to their definitions.

3) After the manner of the phraseology that I have employed in the item 1 above in this Comment, if it is desirable to indicate symbolically that I use the definiendum of a given separate definition that is selected out of definitions (2.3)–(2.6) for mentioning its ultimate patterned (detailed) panlogographic definiens of $\mathbf{A}_1$ then I shall, in accordance with the method of *quasi-autonymous quotations*, enclose the definiendum between bold-faced single quotation marks with the understanding that, once the interior of the quasi-autonymous quotation is replaced with its definiens, the bold-faced single quotation marks should be replaced by light-faced ones. In this case, I may, for instance, state that

$$\text{‘}\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\text{’ } \textit{is} \text{ the PLR ‘} \bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]\text{’ of } \mathbf{A}_1$$

rather than to state that

$$\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \textit{ is} \text{ the PLR } \bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle] \text{ of } \mathbf{A}_1,$$

thus using $\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle$ (without quotation marks) equivocally. Accordingly, the first one of definitions (12.3) can be understood as:

$$\text{‘}\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\text{’} \rightarrow \text{‘}\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]\text{’}.$$

Like remarks apply to all other definitions in (12.3)–(12.6), and generally to all analogous definitions to be stated in the sequel. Still, the method of quasi-autonymous quotations is an epistemologically relativistic (ad hoc) one, so that it is impossible to apply it systematically in all cases, which may result in confusion.•

Df 2.3. 1) Any one of the four quadruples

$$(\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle, \mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle, \mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle, \mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle) \text{ at } n \vec{\in} \{1,2,3,4\} \qquad (2.10)$$

will be called a *panlogographic syllogistic basis* (*PSB*), whereas any PLR being a member of a certain quadruple, i.e. any of the PLR's:

$$\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle] \text{ to } \neg\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle], \qquad (2.10_+)$$

being the ultimate definientia of definitions (2.3)–(2.6), will be called a *panlogographic syllogistic judgment* (*PSJ*).

2) An ER of A_1, being an instance (interpretand) of a PSJ, is called a *euautographic syllogistic judgment* (*ESJ*), whereas a euautographic instance

(interpretand) of a PSB is called will be called a *euautographic syllogistic basis* (*ESB*).•

Cmt 2.2. 1) The PLR's

$$‘\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right]’,\ ‘\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right]’, \tag{2.11}$$

which occurs in the definientia of definitions (2.5), are specifications (specific instances) of the PLR's:

$$‘\mathbf{Q}\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]’,\ ‘\mathbf{Q}\wedge\neg[\mathbf{Q}\Rightarrow\mathbf{R}]’ \tag{2.12}$$

respectively, subject to the substitutions:

$$\mathbf{Q}\rhd\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle,\mathbf{R}\rhd\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle. \tag{2.13}$$

The primary validity integrons (PVI's) of the PLR's (2.12) can be reduced thus:

$$\begin{gathered}V(\mathbf{Q}\wedge[\mathbf{Q}\Rightarrow\mathbf{R}])\hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\neg[\mathbf{Q}\Rightarrow\mathbf{R}])\\ \hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}[1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R})]\hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{+}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R})\\ \hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}[1\hat{-}V(\mathbf{R})]\hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{R})\hat{=}V(\mathbf{Q}\wedge\mathbf{R}),\end{gathered} \tag{2.14}$$

$$\begin{gathered}V(\mathbf{Q}\wedge\neg[\mathbf{Q}\Rightarrow\mathbf{R}])\hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\neg[\mathbf{Q}\Rightarrow\mathbf{R}])\\ \hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{Q}\Rightarrow\mathbf{R})\hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R})\\ \hat{=}1\hat{-}V(\neg\mathbf{Q})\hat{\cdot}V(\mathbf{R})\hat{=}V(\mathbf{Q}\wedge\neg\mathbf{R}).\end{gathered} \tag{2.15}$$

Under substitutions (2.13), the PLR's (2.14) and (2.15) become:

$$V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{=}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right), \tag{2.16}$$

$$V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{=}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right). \tag{2.17}$$

Hence, it follows from (2.4) and (2.5) that

$$\begin{gathered}V\left(\mathsf{A}_3\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_3\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{A}_2\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_2\langle\mathbf{u},\mathbf{v}\rangle\right),\\ V\left(\mathsf{E}_3\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}\left(\neg\mathsf{I}_3\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{E}_2\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}\left(\neg\mathsf{I}_2\langle\mathbf{u},\mathbf{v}\rangle\right).\end{gathered} \tag{2.18}$$

It also follows from (2.16) that if

$$\begin{gathered}\mathsf{A}_5\langle\mathbf{u},\mathbf{v}\rangle\to\bigwedge\nolimits_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right],\\ \mathsf{O}_5\langle\mathbf{u},\mathbf{v}\rangle\to\neg\mathsf{A}_5\langle\mathbf{u},\mathbf{v}\rangle,\end{gathered} \tag{2.19}$$

then

$$V\left(\mathsf{A}_5\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_5\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right). \tag{2.20}$$

2) The PLR's $\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$ and $\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$ are instructive objects of this study, but in contrast to $\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle$ and $\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle$ at $n\mathrel{\vec{\in}}\{1,2,3\}$ they will turn out to be irrelevant to verbal Aristotelian logic. I shall not therefore introduce and discuss any PSJ that is equivalent to $\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$ or to $\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$. Particularly, owing to (2.20), I have not,

included definitions (2.19) under the head "Df 2.1", and I shall not use '$\mathsf{A}_5\langle\mathbf{u},\mathbf{v}\rangle$' and '$\mathsf{O}_5\langle\mathbf{u},\mathbf{v}\rangle$' in the sequel. In this case, the following two remarks regarding Df 2.1 will be in order. First, the fact that I have introduced *four* PSB'es is accidental. Particularly, this fact is irrelevant to the fact that there are four PSJ's in each PSB. The number of PSJ's of each class is actually infinite if one counts equivalent PSJ's as different. Second, it will be demonstrated in the next section by Th 3.1 that $\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle$ and $\neg\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle$, or $\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle$ and $\neg\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle$, have the same validity-indices and are hence equivalent at all $n \vec{\in} \{1,2,3\}$, and not only those at $n \vec{\in} \{2,3\}$, as stated by (2.18). Therefore, the fact that, in definitions (2.6), I have supplemented the pair of PLR's $\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$ and $\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$ with the pair of PLR's $\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle$ and $\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle$, and not with either pair $\mathsf{E}_2\langle\mathbf{u},\mathbf{v}\rangle$ and $\mathsf{I}_2\langle\mathbf{u},\mathbf{v}\rangle$ or $\mathsf{E}_3\langle\mathbf{u},\mathbf{v}\rangle$ and $\mathsf{I}_3\langle\mathbf{u},\mathbf{v}\rangle$, does not lead to any loss of generality.

3) It is also noteworthy that the archetypal PLR '$\mathbf{Q}\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]$' mentioned above has the form of the conjoined antecedent of *modus ponendo ponens*:

$$[\mathbf{Q}\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\Rightarrow\mathbf{R}. \tag{2.21}$$

The latter is a kyrology (valid relation) in accordance with its BEADP:

$$\begin{gathered} V([\mathbf{Q}\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\Rightarrow\mathbf{R}) \mathrel{\hat{=}} V(\neg[\mathbf{Q}\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]) \mathbin{\hat{\cdot}} V(\mathbf{R}) \\ \mathrel{\hat{=}} [1 \mathbin{\hat{-}} V(\mathbf{Q}\wedge\mathbf{R})] \mathbin{\hat{\cdot}} V(\mathbf{R}) \mathrel{\hat{=}} [V(\neg\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{R})] \mathbin{\hat{\cdot}} V(\mathbf{R}) \mathrel{\hat{=}} 0, \end{gathered} \tag{2.22}$$

where use of (2.14) and of the variant of (II.7.15γ) with '$\mathbf{R}$' in place of '$\mathbf{P}$' has been made. Incidentally, by (2.14), *modus ponendo ponens* reduces to the *simplification law*:

$$[\mathbf{Q}\wedge\mathbf{R}]\Rightarrow\mathbf{R}. \tag{2.23}$$

Also, besides (2.14), it follows by the pertinent rules of the BEADM that

$$\begin{gathered} V(\mathbf{Q}\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\neg\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg[\mathbf{Q}\Leftrightarrow\mathbf{R}]) \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\neg\mathbf{Q}) \mathbin{\hat{\cdot}} [V(\neg\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{R}) \mathbin{\hat{+}} V(\mathbf{Q}) \mathbin{\hat{\cdot}} V(\mathbf{R})] \\ \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\neg\mathbf{Q}) \mathbin{\hat{\cdot}} V(\neg\mathbf{R}) \mathrel{\hat{=}} V(\mathbf{Q}\wedge\mathbf{R}). \end{gathered} \tag{2.24}$$

Hence, by (2.14) and (2.24),

$$[\mathbf{Q}\wedge[\mathbf{Q}\Rightarrow\mathbf{R}]]\Leftrightarrow[\mathbf{Q}\wedge[\mathbf{Q}\Leftrightarrow\mathbf{R}]]\Leftrightarrow[\mathbf{Q}\wedge\mathbf{R}]. \tag{2.25}$$ •

Th 2.1.

$$\begin{gathered} \mathsf{A}_2\langle\mathbf{u},\mathbf{v}\rangle \leftrightarrow \textstyle\bigwedge_{\mathbf{z}|\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle}\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle, \mathsf{O}_2\langle\mathbf{u},\mathbf{v}\rangle \leftrightarrow \textstyle\bigvee_{\mathbf{z}|\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle}\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle, \\ \mathsf{E}_2\langle\mathbf{u},\mathbf{v}\rangle \leftrightarrow \neg\textstyle\bigvee_{\mathbf{z}|\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle}\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle, \mathsf{I}_2\langle\mathbf{u},\mathbf{v}\rangle \leftrightarrow \textstyle\bigvee_{\mathbf{z}|\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle}\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle. \end{gathered} \tag{2.26}$$

$$\begin{aligned}
\mathsf{A}_3\langle\mathbf{u},\mathbf{v}\rangle &\leftrightarrow \bigwedge_{\mathbf{z}|\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle],\\
\mathsf{O}_3\langle\mathbf{u},\mathbf{v}\rangle &\leftrightarrow \bigvee_{\mathbf{z}|\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle}\neg[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle],\\
\mathsf{E}_3\langle\mathbf{u},\mathbf{v}\rangle &\leftrightarrow \neg\bigvee_{\mathbf{z}|\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle],\\
\mathsf{I}_3\langle\mathbf{u},\mathbf{v}\rangle &\leftrightarrow \bigvee_{\mathbf{z}|\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle].
\end{aligned} \tag{2.27}$$

Proof: For convenience in further reasoning, here follow variants of item 2 and 3 of Df II.2.2 with ‘**z**’, ‘**Q**’, and ‘**R**’ in place of ‘**x**’, ‘**R**’, and ‘**P**’ respectively:

$$\bigvee_{\mathbf{z}|\mathbf{Q}\langle\mathbf{z}\rangle}\mathbf{R}\langle\mathbf{z}\rangle \to \bigvee_{\mathbf{z}}[\mathbf{Q}\langle\mathbf{z}\rangle \wedge \mathbf{R}\langle\mathbf{z}\rangle], \tag{2.28}$$

$$\bigwedge_{\mathbf{z}|\mathbf{Q}\langle\mathbf{z}\rangle}\mathbf{R}\langle\mathbf{z}\rangle \to \neg\bigvee_{\mathbf{z}}[\mathbf{Q}\langle\mathbf{z}\rangle \wedge \neg\mathbf{R}\langle\mathbf{z}\rangle]. \tag{2.29}$$

Relations (2.26) immediately follow from the respective definitions (2.4) by the variants of (2.28) and (2.29) with ‘$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$’ in place of ‘$\mathbf{Q}\langle\mathbf{z}\rangle$’ and ‘$\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle$’ or ‘$\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle$’ (in the case of ‘$\mathsf{O}_2\langle\mathbf{u},\mathbf{v}\rangle$’ only) in place of ‘$\mathbf{R}\langle\mathbf{z}\rangle$’. Relations (2.27) follow from the respective definitions (2.5) by the variants of (2.28) and (2.29) with ‘$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$’ in place of ‘$\mathbf{Q}\langle\mathbf{z}\rangle$’ and ‘$[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]$’ or ‘$\neg[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]$’ (in the case of ‘$\mathsf{O}_3\langle\mathbf{u},\mathbf{v}\rangle$’ only) in place of ‘$\mathbf{R}\langle\mathbf{z}\rangle$’.•

2.3. Binary structural PSJ’s (BStPSJ’s) and binary structural PSB’es (BStPSB’es)

Preliminary Remark 2.2. Under Ax 2.1, the PLR’s ‘$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$’, ‘$\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle$’, and ‘$\mathbf{P}\langle\mathbf{z},\mathbf{w}\rangle$’, and their variants with ‘**x**’ or ‘**y**’ in place of ‘**z**’, and also the transposed variants of all the above PLR’s can be concretized by ER’s of A_1 arbitrarily in an infinite number of ways. Each such specification results in the respective specifications of the definientia of the separate definitions in (2.3)–(2.6), however the forms of the definienda of those definitions remain the same. Therefore, a definiendum can be used for mentioning ER’s of the range of its patterned (schematic) definiens only transitively via the latter, and not directly. In what follows, I shall describe a certain formal systematic way of simultaneously specifying both the definiendum and the definiens of each separate definition in (2.3)–(2.6), in the result of which the form of the specified definiendum and the form of the specified definiens turn out to stand in a bijective (one-to-one) correspondence with each other.•

Df 2.4: ***Binary structural specification of Df 2.1.*** 1) Df 2.1 applies under the following substitutions

$$\mathbf{P} \triangleright \mathbf{F}^2, \tag{2.30}$$

$$\langle\ \rangle \mapsto (\ ), \tag{2.31}$$

$$\mathsf{A}_n \triangleright \mathsf{A}_{\mathbf{F}^2 n}, \mathsf{E}_n \triangleright \mathsf{E}_{\mathbf{F}^2 n}, \mathsf{I}_n \triangleright \mathsf{I}_{\mathbf{F}^2 n},\ \mathsf{O}_n \triangleright \mathsf{O}_{\mathbf{F}^2 n} \text{ for each } n \vec{\in} \{1,2,3,4\}, \tag{2.32}$$

subject to Df IV.1.3; and similarly with any other *binary StAPLOPS* (*BStAPLOPS*) of the list (IV.1.15²) in place of '$\mathbf{F}^2$'.

2) '$\mathbf{F}^2$' will hereafter be abbreviated as '**F**'. In general, any SStAPLOPS of the list (IV.1.15) will hereafter be used as a StAPLOPS of weight 2, unless stated otherwise.

3) Under the above convention, substitutions (2.30)–(2.32) throughout definitions (2.3)–(2.6) yield:

$$\begin{gathered}\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \to \bigwedge\nolimits_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})], \mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \to \neg\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \\ \mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \to \bigwedge\nolimits_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \neg\mathbf{F}(\mathbf{z},\mathbf{v})], \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \to \neg\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}).\end{gathered} \tag{2.33}$$

$$\begin{gathered}\mathsf{I}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \to \bigvee\nolimits_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \mathbf{F}(\mathbf{z},\mathbf{v})], \mathsf{E}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \to \neg\mathsf{I}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}), \\ \mathsf{O}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \to \bigvee\nolimits_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \neg\mathbf{F}(\mathbf{z},\mathbf{v})], \mathsf{A}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \to \neg\mathsf{O}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}).\end{gathered} \tag{2.34}$$

$$\begin{gathered}\mathsf{I}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \to \bigvee\nolimits_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge [\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})]], \\ \mathsf{E}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \to \neg\mathsf{I}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}), \\ \mathsf{O}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \to \bigvee\nolimits_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \neg[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})]], \\ \mathsf{A}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \to \neg\mathsf{O}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}).\end{gathered} \tag{2.35}$$

$$\begin{gathered}\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \to \bigwedge\nolimits_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \mathbf{F}(\mathbf{z},\mathbf{v})], \mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \to \neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}), \\ \mathsf{E}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \to \mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}), \mathsf{I}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \to \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}).\end{gathered} \tag{2.36}$$

4) Any one of the four quadruples

$$(\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}),\ \mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}),\ \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}),\ \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \text{ at } n \vec{\in} \{1,2,3,4\} \tag{2.37}$$

will be called a *binary structural panlogographic syllogistic basis* (*BStPSB*), whereas any PLR being a member of a certain quadruple, i.e. any of the PLR's:

$$\bigwedge\nolimits_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})] \text{ to } \neg\bigwedge\nolimits_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \neg\mathbf{F}(\mathbf{z},\mathbf{v})], \tag{2.37 +}$$

being the ultimate definientia of definitions (2.33)–(2.36), will be called a *binary structural panlogographic syllogistic judgment* (*BStPSJ*). Consequently, an ER of A_1, being an instance of a certain BStPSJ, will be called a *binary euautographic syllogistic jusdgment* (*BESJ*), while the euautographic instance (interpretand) of any one of the four BStPLB'es (2.37) will be called a *binary euautographic syllogistic basis* (*BESB*).•

Cmt 2.3. 1) Besides substitution (2.30) and its variants with any other SStAPLOPS (secondary structural atomic panlogographic ordinary predicate-sign) of the list (IV.1.15$_2$), the occurrences of the placeholder '**P**' in the definientia of any one of definition groups (2.3)–(2.6) can be replaced with occurrences of any one of the SStAPLOPS of the lists (IV.1.15), (IV.1.15$_3$), (IV.1.15$_4$), etc, for instance with occurrences of '**F**' (not subjugated to Df 2.4(2)), '$\mathbf{F}^3$', or '$\mathbf{F}^4$'. However, substitution (2.30) implies substitution (2.31) so that,

$$\mathbf{F}^2\langle\mathbf{z},\mathbf{u}\rangle \leftrightarrow \mathbf{F}^2(\mathbf{z},\mathbf{u}),\ \mathbf{F}^2\langle\mathbf{u},\mathbf{z}\rangle \leftrightarrow \mathbf{F}^2(\mathbf{u},\mathbf{z}). \qquad (2.38)$$

By contrast, the placeholders '$\mathbf{F}\langle\mathbf{z},\mathbf{u}\rangle$', '$\mathbf{F}^3\langle\mathbf{z},\mathbf{u}\rangle$', and '$\mathbf{F}^4\langle\mathbf{z},\mathbf{u}\rangle$' are ambiguous. For instance, '$\mathbf{F}^3\langle\mathbf{z},\mathbf{u}\rangle$' may mean any variant of '$\mathbf{F}^3(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$', in which some two of the three PLOT's (panlogographic ordinary terms) '$\mathbf{x}_1$', '$\mathbf{x}_2$', and '$\mathbf{x}_3$' are replaced with '**z**' and '**u**' in that order; i.e. '$\mathbf{F}^3\langle\mathbf{z},\mathbf{u}\rangle$' may stand for any one any of these three placeholders: '$\mathbf{F}^3(\mathbf{z},\mathbf{u},\mathbf{x}_3)$', '$\mathbf{F}^3(\mathbf{x}_1,\mathbf{z},\mathbf{u})$', and '$\mathbf{F}^3(\mathbf{z},\mathbf{x}_2,\mathbf{u})$', while '$\mathbf{F}^3\langle\mathbf{u},\mathbf{z}\rangle$' *is* any one of these three: '$\mathbf{F}^3(\mathbf{u},\mathbf{z},\mathbf{x}_3)$', '$\mathbf{F}^3(\mathbf{x}_1,\mathbf{u},\mathbf{z})$', and '$\mathbf{F}^3(\mathbf{u},\mathbf{x}_2,\mathbf{z})$'.

2) Univocal definienda of the definitions that results by the substitution $\mathbf{P} \rhd \mathbf{F}^m$ with certain $m \vec{\in} \{3,4,...\}$ into the definientia of Df 2.1 and 2.1t, those definienda should be written in the form:

$$\begin{gathered}\mathsf{A}^{(i,j)}_{\mathbf{F}^m n}\langle\mathbf{u},\mathbf{v}\rangle, \mathsf{E}^{(i,j)}_{\mathbf{F}^m n}\langle\mathbf{u},\mathbf{v}\rangle, \mathsf{I}^{(i,j)}_{\mathbf{F}^m n}\langle\mathbf{u},\mathbf{v}\rangle, \mathsf{O}^{(i,j)}_{\mathbf{F}^m n}\langle\mathbf{u},\mathbf{v}\rangle \\ \text{at } i \vec{\in} \{1,2,...,m\},\ j \vec{\in} \{i+1, i+2,...,m\},\ m \vec{\in} \{3,4,...\},\ n \vec{\in} \{1,2,3,4\},\end{gathered} \qquad (2.39)$$

where '*i*' and '*j*' are placeholders, whose values indicate the ordinal numbers of th e two PLOT's in the direction from left to right in the relation '$\mathbf{F}^m(\mathbf{x}_1,\mathbf{x}_2,...,\mathbf{x}_m)$', the first of which is replaced with '**z**' and the second with '**u**' or '**v**' respectively. Thus, for instance,

$$\begin{gathered}\text{if } \mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \rhd \mathbf{F}^4(\mathbf{x}_1,\mathbf{z},\mathbf{x}_3,\mathbf{u}) \text{ and } \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle \rhd \mathbf{F}^4(\mathbf{x}_1,\mathbf{z},\mathbf{x}_3,\mathbf{v}) \\ \text{then } i \rhd 2 \text{ and } j \rhd 4,\end{gathered} \qquad (2.40)$$

so that '$\mathsf{A}^{(2,4)}_{\mathbf{F}^4 n}\langle\mathbf{u},\mathbf{v}\rangle$', e.g., is the respective instance of the symbol '$\mathsf{A}^{(i,j)}_{\mathbf{F}^m n}(\mathbf{u},\mathbf{v})$'. When necessary or desired, the remaining two free PLOT's can be indicated explicitly by using the symbol '$\mathsf{A}^{(2,4)}_{\mathbf{F}^4 n}(\mathbf{u},\mathbf{v};\mathbf{x}_1,\mathbf{x}_3)$' instead of '$\mathsf{A}^{(2,4)}_{\mathbf{F}^4 n}\langle\mathbf{u},\mathbf{v}\rangle$'.

3) In contrast to the angle brackets, the round and square brackets are elements of the primitive basis of A_1. Particularly, in accordance with the formation rules of A_1,

the placeholders ‘$\mathbf{F}^2(\mathbf{x}_1,\mathbf{x}_2)$’, ‘$\mathbf{F}^3(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$’, ‘$\mathbf{F}^4(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)$’, etc (e.g.), belonging to A_1, are schemata of *binary*, *ternary*, *quaternary*, etc, *euautogographic molecular ordinary relations* (*EMlOR’s*) *of* A_1, i.e. EMlOR’s *of weight 2, 3, 4,* etc. These placeholders cannot be abbreviated as ‘$\mathbf{F}^2$’, ‘$\mathbf{F}^3$’, ‘$\mathbf{F}^4$’, etc, because the latter are placeholders having completely different ranges. In this connection, the following remark should be made.

4) Under substitution $\mathbf{P} \triangleright \mathbf{F}$ (e.g.), not subjugated to Df 2.4(2), all occurrences of ‘**P**’ in the definientia of definitions (2.3)–(2.6) are replaced with occurrences of ‘**F**’. However, the symbol ‘$\mathbf{F}\langle\mathbf{z},\mathbf{u}\rangle$’, e.g., essentially differs from the symbol ‘$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$’. The former can never be abbreviated by omission of the string ‘$\langle\mathbf{z},\mathbf{u}\rangle$’ because ‘**F**’ and ‘$\mathbf{F}\langle\mathbf{z},\mathbf{u}\rangle$’ have incomparable ranges. A like remark applies with any of the placeholders ‘$\mathbf{F}^2$’, ‘$\mathbf{F}^3$’, etc in place of ‘**F**’. By contrast, the ranges of ‘**P**’ and ‘$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$’, e.g., are comparable, so that ‘$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$’ can be abbreviated by omission of the string ‘$\langle\mathbf{z},\mathbf{u}\rangle$’ in every case where there is no need to indicate explicitly the fact that **P** contains two certain EOT’s.•

Cmt 2.4. In accordance with items 1 and 2 of Df 2.4, definition (2.26) and (2.27) become

$$\mathsf{A}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \leftrightarrow \bigwedge\nolimits_{\mathbf{z}|\mathbf{F}(\mathbf{z},\mathbf{u})}\mathbf{F}(\mathbf{z},\mathbf{v}),\ \mathsf{O}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \leftrightarrow \bigvee\nolimits_{\mathbf{z}|\mathbf{F}(\mathbf{z},\mathbf{u})}\neg\mathbf{F}(\mathbf{z},\mathbf{v})$$
$$\mathsf{E}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \leftrightarrow \neg\bigvee\nolimits_{\mathbf{z}|\mathbf{F}(\mathbf{z},\mathbf{u})}\mathbf{F}(\mathbf{z},\mathbf{v}),\ \mathsf{I}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \leftrightarrow \bigvee\nolimits_{\mathbf{z}|\mathbf{F}(\mathbf{z},\mathbf{u})}\mathbf{F}(\mathbf{z},\mathbf{v}), \tag{2.41}$$

$$\mathsf{A}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \leftrightarrow \bigwedge\nolimits_{\mathbf{z}|\mathbf{F}(\mathbf{z},\mathbf{u})}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})],$$
$$\mathsf{O}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \leftrightarrow \bigvee\nolimits_{\mathbf{z}|\mathbf{F}(\mathbf{z},\mathbf{u})}\neg[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})],$$
$$\mathsf{E}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \leftrightarrow \neg\bigvee\nolimits_{\mathbf{z}|\mathbf{F}(\mathbf{z},\mathbf{u})}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})], \tag{2.42}$$
$$\mathsf{I}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \leftrightarrow \bigvee\nolimits_{\mathbf{z}|\mathbf{F}(\mathbf{z},\mathbf{u})}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})],$$

which are tantamount to definitions (2.34) and (2.35) respectively.•

2.4. Binary ESJ’s and ESB’es (BESJ’s and BESB’es)

Preliminary Remark 2.1. 1) Of all structural (detailed) specifications of definitions (2.3)–(2.6) described in the previous subsection, the binary ones, which have been defined in Df 2.4, are the simplest and also most important ones because the CFCL (conformal catlogographic) interpretands of some BESJ’s and BESB’es are immediately relevant to verbal Aristotelian logic (VAL), while BESJ’s and BESB’es are euautographic interpretands (instances, corollaries) of BStPSJ's and BStPSB'es that do not require any proofs.

2) All possible euautographic interpretands of definitions (2.33)–(2.36) are determined by Df IV.1.2 subject to Dfs 2.4(2), IV.1.2, and IV.3.1. Namely, in accordance with the last three definitions, the ranges of the PLPH's occurring in definitions (2.33)–(2.36) are defined as follows:

$$\mathbf{F}\,\vec{\in}\,\mathrm{K}_{\in}^{2}, \tag{2.43}$$

$$\mathbf{u}\,\vec{\in}\,\tau,\ \mathbf{v}\,\vec{\in}\,\tau - \{\mathbf{u}\},\ \mathbf{z}\,\vec{\in}\,\tau^{\mathrm{pv}} - \{\mathbf{u},\mathbf{v}\}, \tag{2.44}$$

subject to

$$\mathrm{K}_{\in}^{2} \to \kappa^{2\mathrm{pv}} \cup \mathrm{K}_{\in}^{\mathrm{pc}} \to \kappa^{2\mathrm{pv}} \cup \mathrm{K}_{\in}^{2\mathrm{pc}}, \tag{2.45}$$

$$\kappa^{2\mathrm{pv}} \to \left\{f^{2}, g^{2}, h^{2}, f_{1}^{2}, g_{1}^{2}, h_{1}^{2}, f_{2}^{2}, g_{2}^{2}, h_{2}^{2}, \ldots\right\}, \tag{2.46}$$

$$\mathrm{K}_{\in}^{\mathrm{pc}} \to \mathrm{K}_{\in}^{2\mathrm{pc}} \to \{\in, \subseteq, \subset, =\}, \tag{2.47}$$

$$\tau \to \tau^{\mathrm{pv}} \cup \{\emptyset\}, \tag{2.48}$$

$$\tau^{\mathrm{pv}} \to \{u, v, w, x, y, z, u_1, v_1, w_1, x_1, y_1, z_1, u_2, v_2, w_2, x_2, y_2, z_2, \ldots\}; \tag{2.49}$$

the sign '-' denotes the operation of subtraction of sets, according to which *A*-*B* is the set of all elements of the set *A* that are not elements of the set *B*. The conjunction of relations (2.44) is a convenient formalization of the relation:

$$\mathbf{u}\,\vec{\in}\,\tau,\ \mathbf{v}\,\vec{\in}\,\tau,\ \text{and}\ \mathbf{z}\,\vec{\in}\,\tau^{\mathrm{pv}},\ \text{subject to Cnv 2.1.} \tag{2.44a}$$

3) If

$$\mathbf{F} \rhd \mathbf{F}^{\mathrm{pc}}\,\vec{\in}\,\vec{\mathrm{K}}_{\in}^{\mathrm{pc}} \tag{2.50}$$

then, in accordance with Df IV.1.1(2),

$$\left[\mathbf{z}\mathbf{F}^{\mathrm{pc}}\mathbf{u}\right] \to \mathbf{F}^{\mathrm{pc}}(\mathbf{z},\mathbf{u}). \tag{2.51}$$

and similarly with '**v**' in place of '**u**'. It will be recalled that the form of a relation such as that of the definiens of definition (2.51) is called the *Clairaut-Euler*, or *nonlinear*, or *inhomogeneous, form* of the relation, whereas the form of the relation such as that of the definiendum of of definition (2.51) is called the *bilinear*, or less explicitly *homogeneous, form* of the relation; the word "*form*" in any of the above terms can be used interchangeably with either of the words "*notation*" and "*representation*". I shall give preference to $\left[\mathbf{z}\mathbf{F}^{\mathrm{pc}}\mathbf{u}\right]$ over $\mathbf{F}^{\mathrm{pc}}(\mathbf{z},\mathbf{u})$. However, in the case where the range of '**F**' is restricted to $\kappa^{2\mathrm{pv}}$, i.e. if $\mathbf{F} \rhd \mathbf{F}^{\mathrm{pv}}\,\vec{\in}\,\kappa^{2\mathrm{pv}}$, I shall employ the original Clairaut-Euler representation, $\mathbf{F}^{\mathrm{pv}}(\mathbf{z},\mathbf{u})$. Thus, I shall conventionally write $z \in u$, $z \subseteq u$, or $z = u$ (e.g.) instead of or interchangeably with $\in(z,u)$, $\subseteq(z,u)$, or $=(z,u)$ respectively, while writing (e.g.) $f^2(z,u)$ and not $[zf^2u]$.

4) Once all occurrences of '**F**' in any separate definition of the definition quadruples (2.33)–(2.36) are replaced with occurrences of a certain *euatogographic ordinary predicate-sign* (*EOPS*) *of* $\mathsf{A}_{1\in}$, either condition (2.44) or (2.44a) will be satisfied if the APLOT's (atomic panlogographic ordinary terms) '**u**', '**v**', and '**z**' are replaced throughout the definition with any three different APVOT's that are selected out of the set τ^{pv} in *alphabetic* order, in succession or not. Accordingly, this method of euautographic interpretation of the APLOT's will be called an *alphabetic* one. The simplest, mnemonically justifiable way to perform an alphabetic euautographic interpretation of the APLOT's is to make the *analo-euautographic* substitutions:

$$\mathbf{u} \triangleright u,\ \mathbf{v} \triangleright v,\ \mathbf{z} \triangleright z. \tag{2.52}$$

I employ the qualifier "*analo-autographic*" to a euautographic token of an APLOT as an abbreviation of "*analogous euautographic*" and as antonym of either of the synonymous qualifiers "homolographic" and "photographic", meaning *proportional* or, particularly, *congruent*. Accordingly, the latter instance of the alphabetic method of euautographic interpretation of the APLOT's will be called an *analo-eutographic* one, and it is the one that be adopted hereafter and that will be generalized in the next item. It is understood that the alphabetic interpretation of '**u**' and '**v**' in general and their analo-autographic interpretation in particular should be supplemented with either of the following two alternative substitutions

$$\mathbf{u} \vec{\in} \{\emptyset\} \text{ or } \mathbf{v} \vec{\in} \{\emptyset\}, \text{ i.e. } \mathbf{u} \triangleright \emptyset \text{ or } \mathbf{v} \triangleright \emptyset, \tag{2.53}$$

but not both. Consequently, after performing the analo-autographic substitutions (2.52) throughout (2.33)–(2.36), u or v, but not both, can be replaced with $\emptyset$.

5) In the sequel, I shall need to interpret euautographically the PLR's (panlogographic relations) of $\mathbf{A}_1$, which will be called *structural panlogographic syllogistic implications* (*StPSI's*), and the identities for the validity integrons of the StPSI's, which will be called *structural panlogographic syllogistic master*, or *decision, theorems* (*StPSMT's* or *StPSDT's*) *of the StPSI's*. Any StPSI is composed of three interrelated BPSJ's in such a way that it contains occurrences of six *binary structural panlogographic molecular relations* (*BStMlPLR's*):

$$\text{'}\mathbf{F}(\mathbf{x},\mathbf{u})\text{', '}\mathbf{F}(\mathbf{x},\mathbf{v})\text{', '}\mathbf{F}(\mathbf{y},\mathbf{v})\text{', '}\mathbf{F}(\mathbf{y},\mathbf{w})\text{', '}\mathbf{F}(\mathbf{z},\mathbf{u})\text{', '}\mathbf{F}(\mathbf{z},\mathbf{w})\text{'} \tag{2.54}$$

or of their variants with two arguments exchanged. In this case, the pertinent specification of Cnv 2.1 after the manner of (2.44) would be unreadable and hence unpractical. At the same time, the analo-euautographic substitutions (2.52), which are as such applicable only to the two specific BStMlPLR's '$\mathbf{F}(\mathbf{z},\mathbf{u})$' and '$\mathbf{F}(\mathbf{z},\mathbf{v})$' or to their transposed variants, can be generalized as follows. Once all occurrences of '**F**' in a given StPSI or its StPSMT (StPSDT) are replaced with occurrences of a certain EOPS of $\mathsf{A}_{1\in}$ in order to satisfy Cnv 2.1, the APLOT's '**u**', '**v**', '**w**', '**x**', '**y**', and '**z**' should be replaced throughout the given StPSI or StPSMT (StPSDT) with any six different APVOT's selected out of the set τ^{pv} in *alphabetic* order, in succession or not. Accordingly, this method of euautographic interpretation of the APLOT's is as before called an *alphabetic* one. The simplest, mnemonically justifiable way to perform an alphabetic euautographic interpretation of the APLOT's, – in fact without loss of generality in this case, – is again to make the *analo-euautographic* substitutions:

$$\mathbf{u} \triangleright \mathsf{u}, \mathbf{v} \triangleright \mathsf{v},\ \mathbf{w} \triangleright \mathsf{w},\ \mathbf{x} \triangleright \mathsf{x},\ \mathbf{y} \triangleright \mathsf{y},\ \mathbf{z} \triangleright \mathsf{z}. \qquad (2.55)$$

Accordingly, the latter instance of the alphabetic method of euautographic interpretation of the APLOT's is, as before, called an *analo-euautographic* one, and it is the one to be employed in the sequel. It is understood that after performing the analo-autographic substitutions (2.55) throughout a given StPSI or its StPSMT (StPSDT), any one and only one of the three APVOT's u, v, and w can be replaced with the APCOT $\emptyset$.

6) In the most general case, replacements (2.55) should be understood, not only as replacements of the particular six APLOT's with the particular analo-euautographic instances in question, but also as replacements of the base letters of all indexed APLOT's on the list (I.5.6) with the base letters of the respective indexed APVOT's of the list (I.5.1), i.e of the set τ^{pv}, (2.49), – e.g., as $\mathbf{u}_1 \triangleright \mathsf{u}_1$, $\mathbf{v}_1 \triangleright \mathsf{v}_1$, etc.

7) No specific (atypical) axioms (see Df IV.2.1(2)) are imposed on any APVPS of the set $\kappa^{2\mathrm{pv}}$ (see (2.46)). Therefore, members of these set are functionally indistinguishable and mutually independent. By contrast, the members of the set $\mathrm{K}_{\in}^{2\mathrm{pc}}$ (see (2.47)) are interrelated and are subjugated to certain specific axioms.•

$^{\circ}$**Crl 2.1:** ***Particularization of definitions (2.33)–(2.36) for any*** $\mathbf{F} \triangleright \mathbf{F}^{pv} \vec{\in} \kappa^{2pv}$**.** Under the simultaneous substitutions $\mathbf{F} \triangleright f^2$ and (2.52), deinitions (2.33)–(2.36) become:

$$\begin{gathered} \mathsf{A}_{f^2 1}(u,v) \to \bigwedge_z \left[f^2(z,u) \Rightarrow f^2(z,v)\right], \mathsf{O}_{f^2 1}(u,v) \to \neg \mathsf{A}_{f^2 1}(u,v), \\ \mathsf{E}_{f^2 1}(u,v) \to \bigwedge_z \left[f^2(z,u) \Rightarrow \neg f^2(z,v)\right], \mathsf{I}_{f^2 1}(u,v) \to \neg \mathsf{E}_{f^2 1}(u,v), \end{gathered} \tag{2.33μ0}$$

$$\begin{gathered} \mathsf{I}_{f^2 2}(u,v) \to \bigvee_z \left[f^2(z,u) \wedge f^2(z,v)\right], \mathsf{E}_{f^2 2}(u,v) \to \neg \mathsf{I}_{f^2 2}(u,v), \\ \mathsf{O}_{f^2 2}(u,v) \to \bigvee_z \left[f^2(z,u) \wedge \neg f^2(z,v)\right], \mathsf{A}_{f^2 2}(u,v) \to \neg \mathsf{O}_{f^2 2}(u,v), \end{gathered} \tag{2.34μ0}$$

$$\begin{gathered} \mathsf{I}_{f^2 3}(u,v) \to \bigvee_z \left[f^2(z,u) \wedge \left[f^2(z,u) \Rightarrow f^2(z,v)\right]\right], \\ \mathsf{E}_{f^2 3}(u,v) \to \neg \mathsf{I}_{f^2 3}(u,v), \\ \mathsf{O}_{f^2 3}(u,v) \to \bigvee_z \left[f^2(z,u) \wedge \neg \left[f^2(z,u) \Rightarrow f^2(z,v)\right]\right], \\ \mathsf{A}_{f^2 3}(u,v) \to \neg \mathsf{O}_{f^2 3}(u,v), \end{gathered} \tag{2.35μ0}$$

$$\begin{gathered} \mathsf{A}_{f^2 4}(u,v) \to \bigwedge_z \left[f^2(z,u) \wedge f^2(z,v)\right], \mathsf{O}_{f^2 4}(u,v) \to \neg \mathsf{A}_{f^2 4}(u,v), \\ \mathsf{E}_{f^2 4}(u,v) \to \mathsf{E}_{f^2 1}(u,v), \mathsf{I}_{f^2 4}(u,v) \to \mathsf{I}_{f^2 1}(u,v), \end{gathered} \tag{2.36μ0}$$

and similarly with any $\mathbf{F} \vec{\in} \kappa^{2pv}$ subject to (2.46) in place of f^2.•

Crl 2.1:** ***Specification of definitions (2.33)–(2.36) for each $\mathbf{F} \vec{\in} \{\in, \subseteq, =, \subset\}$**.** In accordance with (2.50) and (2.51), definitions (2.33)–(2.36) become:

$$\begin{gathered} \mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \to \bigwedge_{\mathbf{z}} \left[[\mathbf{zFu}] \Rightarrow [\mathbf{zFv}]\right], \mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \to \neg \mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \\ \mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \to \bigwedge_{\mathbf{z}} \left[[\mathbf{zFu}] \Rightarrow \neg[\mathbf{zFv}]\right], \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \to \neg \mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}), \end{gathered} \tag{2.33ε}$$

$$\begin{gathered} \mathsf{I}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \to \bigvee_{\mathbf{z}} \left[[\mathbf{zFu}] \wedge [\mathbf{zFv}]\right], \mathsf{E}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \to \neg \mathsf{I}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}), \\ \mathsf{O}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \to \bigvee_{\mathbf{z}} \left[[\mathbf{zFu}] \wedge \neg[\mathbf{zFv}]\right], \mathsf{A}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}) \to \neg \mathsf{O}_{\mathbf{F}2}(\mathbf{u},\mathbf{v}), \end{gathered} \tag{2.34ε}$$

$$\begin{gathered} \mathsf{I}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \to \bigvee_{\mathbf{z}} \left[[\mathbf{zFu}] \wedge \left[[\mathbf{zFu}] \Rightarrow [\mathbf{zFv}]\right]\right], \mathsf{E}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \to \neg \mathsf{I}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}), \\ \mathsf{O}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \to \bigvee_{\mathbf{z}} \left[[\mathbf{zFu}] \wedge \neg\left[[\mathbf{zFu}] \Rightarrow [\mathbf{zFv}]\right]\right], \mathsf{A}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}) \to \neg \mathsf{O}_{\mathbf{F}3}(\mathbf{u},\mathbf{v}). \end{gathered} \tag{2.35ε}$$

$$\begin{gathered} \mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \to \bigwedge_{\mathbf{z}} \left[[\mathbf{zFu}] \wedge [\mathbf{zFv}]\right], \mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \to \neg \mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}), \\ \mathsf{E}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \to \mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}), \mathsf{I}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \to \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}), \end{gathered} \tag{2.36ε}$$

$^{\circ}$**Crl 2.3:** ***Particularization of definitions (2.33)–(2.36) for any*** $\mathbf{F} \triangleright \mathbf{F}^{pc} \vec{\in} \vec{\mathrm{K}}^{pc}_{\in}$. Under the simultaneous substitutions $\mathbf{F} \triangleright \in$ and (2.53), definitions (2.33)–(2.36) become:

$$\begin{gathered} \mathsf{A}_{\in 1}(u,v) \to \bigwedge_z \left[[z \in u] \Rightarrow [z \in v]\right], \mathsf{O}_{\in 1}(u,v) \to \neg \mathsf{A}_{\in 1}(u,v), \\ \mathsf{E}_{\in 1}(u,v) \to \bigwedge_z \left[[z \in u] \Rightarrow \neg[z \in v]\right], \mathsf{I}_{\in 1}(u,v) \to \neg \mathsf{E}_{\in 1}(u,v), \end{gathered} \tag{2.33μ1}$$

$$\mathsf{I}_{\in 2}(u,v) \to \bigvee_z [[z \in u] \wedge [z \in v]], \mathsf{E}_{\in 2}(u,v) \to \neg \mathsf{I}_{\in 2}(u,v),$$
$$\mathsf{O}_{\in 2}(u,v) \to \bigvee_z [[z \in u] \wedge \neg [z \in v]], \mathsf{A}_{\in 2}(u,v) \to \neg \mathsf{O}_{\in 2}(u,v), \tag{2.34μ1}$$

$$\mathsf{I}_{\in 3}(u,v) \to \bigvee_z [[z \in u] \wedge [[z \in u] \Rightarrow [z \in v]]],$$
$$\mathsf{E}_{\in 3}(u,v) \to \neg \mathsf{I}_{\in 3}(u,v),$$
$$\mathsf{O}_{\in 3}(u,v) \to \bigvee_z [[z \in u] \wedge \neg [[z \in u] \Rightarrow [z \in v]]],$$
$$\mathsf{A}_{\in 3}(u,v) \to \neg \mathsf{O}_{\in 3}(u,v), \tag{2.35μ1}$$

$$\mathsf{A}_{\in 4}(u,v) \to \bigwedge_x [[z \in u] \wedge [z \in v]], \mathsf{O}_{\in 4}(u,v) \to \neg \mathsf{A}_{\in 4}(u,v),$$
$$\mathsf{E}_{\in 4}(u,v) \to \mathsf{E}_{\in 1}(u,v), \mathsf{I}_{\in 4}(u,v) \to \mathsf{I}_{\in 1}(u,v), \tag{2.36μ1}$$

and similarly with any of the three predicate-signs $\subseteq$, =, and $\subset$ in place of $\in$.•

Cmt 2.5. By definition (IV.1.3), it follows from definition (2.33μ1) of $\mathsf{A}_{\in 1}(u,v)$ that

$$\mathsf{A}_{\in 1}(u,v) \leftrightarrow [u \subseteq v]. \tag{2.36μ1+}$$ •

3. Validity indices of the PSJ's

3.1. Preliminaries

1) A relation between a selected PSJ (as '$\mathsf{A}_1\langle \mathbf{u}, \mathbf{v} \rangle$') on the one hand and, on the other hand, its constituent PLR's (as '$\mathbf{P}\langle \mathbf{z}, \mathbf{u} \rangle$' and '$\mathbf{P}\langle \mathbf{z}, \mathbf{v} \rangle$') or some other PSJ's (as '$\mathsf{I}_1\langle \mathbf{u}, \mathbf{v} \rangle$' or '$\mathsf{A}_4\langle \mathbf{u}, \mathbf{v} \rangle$') or both is called:

i) a *categotical*, or *unconditional*, *property* of the former PSJ if that relation is or can be expressed in the form of an identity or of a train of identities, belonging to $\mathbf{A}_1$, among the pertinent validity-integrons,

ii) a *conditional property* of the former PSJ if that relation is expressed verbally by means of a conditional (hypothetical or disjunctive) sentence or sentences belonging to the IML (inclusive meta-language) of $\mathbf{A}_1$.

2) The APLADM allows calculating the validity-indices of any PLR's of $\mathbf{A}_1$ including those, which may after all turn out to be irrelevant to the main object of this discourse – deducing Aristotelian logic from A_1. Therefore, in order not to lose this object among unessential digressions and not to turn this discourse into a collection of exercises on calculation of validity-indices of various PLR's, I shall, in choosing the properties of PSJ's that I make explicit, intuitively follow the principle of *Ockham's razor*, according to which entities should not be multiplied unnecessarily.•

3.2. Categorical properties of the PSJ's

***Th 3.1.**

$$
\begin{gathered}
V\left(\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle\right)\\
\hat{=}V\left(\bigwedge_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{=}V\left(\neg\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\\
\hat{=}V\left(\neg\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right]\right)\\
\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{z}}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\\
\text{for each } n\,\vec{\in}\,\{1,2,3\}.
\end{gathered}
\tag{3.1}
$$

$$
\begin{gathered}
V\left(\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle\right)\\
\hat{=}V\left(\neg\bigwedge_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{=}V\left(\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\\
\hat{=}V\left(\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right]\right)\\
\hat{=}\hat{\bullet}_{\mathbf{z}}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\\
\text{for each } n\,\vec{\in}\,\{1,2,3\}.
\end{gathered}
\tag{3.2}
$$

$$
\begin{gathered}
V\left(\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle\right)\\
\hat{=}V\left(\bigwedge_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{=}V\left(\neg\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\\
\hat{=}V\left(\neg\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right]\right)\\
\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{z}}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\\
\text{for each } n\,\vec{\in}\,\{1,2,3,4\}.
\end{gathered}
\tag{3.3}
$$

$$
\begin{gathered}
V\left(\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle\right)\\
\hat{=}V\left(\neg\bigwedge_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{=}V\left(\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\\
\hat{=}V\left(\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right]\right)\\
\hat{=}\hat{\bullet}_{\mathbf{z}}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\\
\text{for each } n\,\vec{\in}\,\{1,2,3,4\}.
\end{gathered}
\tag{3.4}
$$

$$
\begin{gathered}
V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\bigwedge_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\\
\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right].
\end{gathered}
\tag{3.5}
$$

$$
\begin{gathered}
V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{z}}\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\\
\hat{=}\hat{\bullet}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\hat{=}\left[\hat{\bullet}_{\mathbf{z}_1}V\left(\neg\mathbf{P}\langle\mathbf{z}_1,\mathbf{u}\rangle\right)\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{z}_2}V\left(\neg\mathbf{P}\langle\mathbf{z}_2,\mathbf{v}\rangle\right)\right]\\
\hat{=}V\left(\bigvee_{\mathbf{z}_1}\neg\mathbf{P}\langle\mathbf{z}_1,\mathbf{u}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{z}_2}\neg\mathbf{P}\langle\mathbf{z}_2,\mathbf{v}\rangle\right).
\end{gathered}
\tag{3.6}
$$

Proof: Applying the validity operator V to the definiens of each of the separate definitions in (2.3)-(2.6) and then reducing the expressions thus obtained with the help of the pertinent instances of items of Th II.7.2 also with the help of (2.18) yields:

$$
\begin{gathered}
V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\bigwedge_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\\
\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\\
\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\hat{=}V\left(\bigwedge_{\mathbf{z}}\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right).
\end{gathered}
\tag{3.1$_1$}
$$

$$V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,1\,\hat{-}\,V(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)$$
$$\hat{=}\,\mathop{\hat{\cdot}}_{\mathbf{z}}[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)]\,\hat{=}\,V(\textstyle\bigvee_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]). \tag{3.2$_1$}$$

$$V(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\textstyle\bigwedge_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle])$$
$$\hat{=}\,1\,\hat{-}\,\mathop{\hat{\cdot}}_{\mathbf{z}}V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\Rightarrow\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle) \tag{3.3$_1$}$$
$$\hat{=}\,1\,\hat{-}\,\mathop{\hat{\cdot}}_{\mathbf{z}}[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)]\,\hat{=}\,V(\textstyle\bigwedge_{\mathbf{z}}\neg[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]).$$

$$V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,1\,\hat{-}\,V(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)$$
$$\hat{=}\,\mathop{\hat{\cdot}}_{\mathbf{z}}[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)]\,\hat{=}\,V(\textstyle\bigvee_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]). \tag{3.4$_1$}$$

$$V(\mathsf{O}_2\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\mathsf{O}_3\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\textstyle\bigvee_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle])$$
$$\hat{=}\,\mathop{\hat{\cdot}}_{\mathbf{z}}V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\,\hat{=}\,\mathop{\hat{\cdot}}_{\mathbf{z}}[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)]. \tag{3.2$_2$}$$

$$V(\mathsf{A}_2\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{O}_2\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\mathsf{A}_3\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{O}_3\langle\mathbf{u},\mathbf{v}\rangle)$$
$$\hat{=}\,1\,\hat{-}\,V(\mathsf{O}_2\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,1\,\hat{-}\,\mathop{\hat{\cdot}}_{\mathbf{z}}[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)] \tag{3.1$_2$}$$
$$\hat{=}\,V(\textstyle\bigwedge_{\mathbf{z}}\neg[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]).$$

$$V(\mathsf{I}_2\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\mathsf{I}_3\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\textstyle\bigvee_{\mathbf{z}}[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle])$$
$$\hat{=}\,\mathop{\hat{\cdot}}_{\mathbf{z}}V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\,\hat{=}\,\mathop{\hat{\cdot}}_{\mathbf{z}}[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)]. \tag{3.4$_2$}$$

$$V(\mathsf{E}_2\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{I}_2\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\mathsf{E}_3\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{I}_3\langle\mathbf{u},\mathbf{v}\rangle)$$
$$\hat{=}\,1\,\hat{-}\,V(\mathsf{I}_2\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,1\,\hat{-}\,\mathop{\hat{\cdot}}_{\mathbf{z}}[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)] \tag{3.3$_2$}$$
$$\hat{=}\,V(\textstyle\bigwedge_{\mathbf{z}}\neg[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle]).$$

$$V(\mathsf{E}_4)\,\hat{=}\,V(\mathsf{E}_1). \tag{3.3$_3$}$$

$$V(\mathsf{I}_4)\,\hat{=}\,V(\mathsf{I}_1). \tag{3.4$_3$}$$

Particularly, use of (2.18) has been made in developing (3.2$_2$), (3.1$_2$), (3.4$_2$), and (3.3$_2$). Identities (3.1$_1$) and (3.1$_2$) prove (3.1); (3.2$_1$) and (3.2$_2$) prove (3.2); (3.3$_1$)–(3.3$_3$) prove (3.3); (3.4$_1$)–(3.4$_3$) prove (3.4). The trains of identities (3.5) and (3.6) are self-proving. In developing the next to last term in (3.6), use of the Fission Law, (II.4.29), has been made. It is understood that substitution of '$V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle)$' as given by (3.6) into (3.5) gives the pertinent alternative expressions for '$V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)$'. QED.•

Cmt 3.1. Owing to (3.1)–(3.6), the only syllogistic figures to be studied are those formed with the help of the PSB's (2.10) at $n \triangleright 1$ (e.g.) and $n \triangleright 4$. Also, relations (3.3) and (3.4) serve as the very demonstration that has been mentioned in item 2 of Cmt 2.3.•

Cnv 3.1. In accordance with Th 3.1 and Cmt 3.1, henceforth, the subscript 'n' in the code names of PSJ's 'A_n', 'E_n', 'I_n', 'O_n' either is explicitly particularized by its two values '1' and '4' or, when retained, is supposed to assume the above two values, unless stated otherwise.•

***Th 3.2.** For each $n \vec{\in} \{1,2,3\}$:

$$
\begin{aligned}
&V(\mathrm{O}_n\langle\mathbf{u},\mathbf{v}\rangle) \mathrel{\hat{=}} V(\neg\mathrm{A}_n\langle\mathbf{u},\mathbf{v}\rangle) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\mathrm{A}_n\langle\mathbf{u},\mathbf{v}\rangle) \\
&\mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}|V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\hat{=}0} V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle) \mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}|V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\hat{=}1} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle),
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
&V(\mathrm{I}_n\langle\mathbf{u},\mathbf{v}\rangle) \mathrel{\hat{=}} V(\neg\mathrm{E}_n\langle\mathbf{u},\mathbf{v}\rangle) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\mathrm{E}_n\langle\mathbf{u},\mathbf{v}\rangle) \\
&\mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}|V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\hat{=}0} V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle) \mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}|V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\hat{=}0} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle).
\end{aligned}
\tag{3.8}
$$

Proof: Relations (3.7) and (3.8) are identical with relations (3.2) and (3.4) respectively as demonstrated below.

With '$V(\mathbf{Q}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} 0$' in place of '$\mathbf{Q}\langle\mathbf{z}\rangle$', definition (2.28) becomes:

$$\vee_{\mathbf{z}|V(\mathbf{Q}(\mathbf{z}))\hat{=}0} \mathbf{R}\langle\mathbf{z}\rangle \rightarrow \vee_{\mathbf{z}} [[V(\mathbf{Q}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} 0] \wedge \mathbf{R}\langle\mathbf{z}\rangle]. \tag{3.9}$$

By the law of algebraization of $\vee_{\mathbf{z}}$, (II.4.23), it follows from (2.18) and (3.11) that

$$
\begin{aligned}
&\hat{\cdot}_{\mathbf{z}|\mathbf{Q}\langle\mathbf{x}\rangle} V(\mathbf{R}\langle\mathbf{z}\rangle) \mathrel{\vec{\hat{=}}} V(\vee_{\mathbf{z}|\mathbf{Q}\langle\mathbf{x}\rangle} \mathbf{R}\langle\mathbf{x}\rangle) \mathrel{\hat{=}} V(\vee_{\mathbf{z}} [\mathbf{Q}\langle\mathbf{z}\rangle \wedge \mathbf{R}\langle\mathbf{z}\rangle]) \\
&\mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}} V(\mathbf{Q}\langle\mathbf{z}\rangle \wedge \mathbf{R}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}} [1 \mathbin{\hat{-}} V(\neg\mathbf{Q}\langle\mathbf{z}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{R}\langle\mathbf{z}\rangle)],
\end{aligned}
\tag{3.10}
$$

$$
\begin{aligned}
&\hat{\cdot}_{\mathbf{z}|V(\mathbf{Q}\langle\mathbf{z}\rangle)\hat{=}0} V(\mathbf{R}\langle\mathbf{z}\rangle) \mathrel{\vec{\hat{=}}} V(\vee_{\mathbf{z}|V(\mathbf{Q}\langle\mathbf{z}\rangle)\hat{=}0} \mathbf{R}\langle\mathbf{z}\rangle) \\
&\mathrel{\hat{=}} V(\vee_{\mathbf{z}} [[V(\mathbf{Q}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} 0] \wedge \mathbf{R}\langle\mathbf{z}\rangle]) \mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}} V([V(\mathbf{Q}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} 0] \wedge \mathbf{R}\langle\mathbf{z}\rangle) \\
&\mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}} [1 \mathbin{\hat{-}} V(\neg[V(\mathbf{Q}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} 0]) \mathbin{\hat{\cdot}} V(\neg\mathbf{R}\langle\mathbf{z}\rangle)] \\
&\mathrel{\hat{=}} \hat{\cdot}_{z} [1 \mathbin{\hat{-}} V(\neg\mathbf{Q}\langle\mathbf{z}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{R}\langle\mathbf{z}\rangle)],
\end{aligned}
\tag{3.11}
$$

In developing the final expression in (3.10), use of the identity

$$V(\mathbf{Q}\langle\mathbf{z}\rangle \wedge \mathbf{R}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\neg\mathbf{Q}\langle\mathbf{z}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{R}\langle\mathbf{z}\rangle), \tag{3.12}$$

which is the variant of (II.7.6γ) with '$\mathbf{Q}\langle\mathbf{z}\rangle$' and '$\mathbf{R}\langle\mathbf{z}\rangle$' in place of '**P**' and '**Q**' respectively, has been made. In developing the final expression in (3.11), use the variant of (3.12) with '$V(\mathbf{Q}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} 0$' in place of '$\mathbf{Q}\langle\mathbf{z}\rangle$' and also use of the identities

$$V(\neg[V(\mathbf{Q}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} 0]) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(V(\mathbf{Q}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} 0) \mathrel{\hat{=}} 1 \mathbin{\hat{-}} V(\mathbf{Q}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} V(\neg\mathbf{Q}\langle\mathbf{z}\rangle) \tag{3.13}$$

(see (II.6.19)) have been made. Comparison of (3.10) and (3.11) shows that

$$
\begin{aligned}
&\hat{\cdot}_{\mathbf{z}|\mathbf{Q}\langle\mathbf{z}\rangle} V(\mathbf{R}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}|V(\mathbf{Q}\langle\mathbf{z}\rangle)\hat{=}0} V(\mathbf{R}\langle\mathbf{z}\rangle) \\
&\mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}} V(\mathbf{Q}\langle\mathbf{z}\rangle \wedge \mathbf{R}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}} [1 \mathbin{\hat{-}} V(\neg\mathbf{Q}\langle\mathbf{z}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{R}\langle\mathbf{z}\rangle)] \\
&\mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}|V(\mathbf{R}\langle\mathbf{z}\rangle)\hat{=}0} V(\mathbf{Q}\langle\mathbf{z}\rangle) \mathrel{\hat{=}} \hat{\cdot}_{\mathbf{z}|\mathbf{R}\langle\mathbf{z}\rangle} V(\mathbf{Q}\langle\mathbf{z}\rangle).
\end{aligned}
\tag{3.14}
$$

The variant of (3.14) with '$\neg\mathbf{R}\langle\mathbf{z}\rangle$' in place of '$\mathbf{R}\langle\mathbf{z}\rangle$' reduces to

$$\begin{gathered}\hat{\bullet}_{\mathbf{z}|\mathbf{Q}\langle\mathbf{z}\rangle} V(\neg\mathbf{R}\langle\mathbf{z}\rangle) \hat{=} \hat{\bullet}_{\mathbf{z}|V(\mathbf{Q}\langle\mathbf{z}\rangle)\hat{=}0} V(\neg\mathbf{R}\langle\mathbf{z}\rangle) \\ \hat{=} \hat{\bullet}_{\mathbf{z}} V(\mathbf{Q}\langle\mathbf{z}\rangle \wedge \neg\mathbf{R}\langle\mathbf{z}\rangle) \hat{=} \hat{\bullet}_{\mathbf{z}} [1 \hat{-} V(\neg\mathbf{Q}\langle\mathbf{z}\rangle) \hat{\cdot} V(\mathbf{R}\langle\mathbf{z}\rangle)] \\ \hat{=} \hat{\bullet}_{\mathbf{z}|V(\mathbf{R}\langle\mathbf{z}\rangle)\hat{=}1} V(\mathbf{Q}\langle\mathbf{z}\rangle) \hat{=} \hat{\bullet}_{\mathbf{z}|\neg\mathbf{R}\langle\mathbf{z}\rangle} V(\mathbf{Q}\langle\mathbf{z}\rangle).\end{gathered} \tag{3.15}$$

Relations (3.7) and (3.8) follow from (3.2) and (3.4) respectively by the pertinent versions of (3.14) and (3.15). QED.•

Cmt 3.2. Relations (3.5) and (3.6) cannot be written in analogy with relations (3.7)–(3.10) because they are in fact disjunctive with respect to $\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$ and $\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle$, and not conjunctive. In general, as compared to $\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle$ and $\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle$ at $n \vec{\in} \{1,2,3\}$, $\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$ and $\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$ have various peculiar properties and therefore they will not, after all, be interpreted by any VSJ's. Still, the study of the latter relations in parallel with the former will be instructive.•

***Th 3.3.**

$$\begin{gathered}V(\mathsf{E}_m\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{I}_m\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{I}_n\langle\mathbf{v},\mathbf{u}\rangle) \hat{=} V(\mathsf{E}_n\langle\mathbf{v},\mathbf{u}\rangle), \\ V(\mathsf{I}_m\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{E}_m\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{E}_n\langle\mathbf{v},\mathbf{u}\rangle) \hat{=} V(\mathsf{I}_n\langle\mathbf{v},\mathbf{u}\rangle) \\ \text{for each } m \vec{\in} \{1,2,3,4\} \text{ and each } n \vec{\in} \{1,2,3,4\};\end{gathered} \tag{3.16}$$

$$\begin{gathered}V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{O}_4\langle\mathbf{v},\mathbf{u}\rangle) \hat{=} V(\mathsf{A}_4\langle\mathbf{v},\mathbf{u}\rangle), \\ V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{A}_4\langle\mathbf{v},\mathbf{u}\rangle) \hat{=} V(\mathsf{O}_4\langle\mathbf{v},\mathbf{u}\rangle).\end{gathered} \tag{3.17}$$

Proof: Identities (3.16) follow from (3.2)–(3.4), whereas identities (3.17) follow from (3.5) and (3.6).•

Cmt 3.3. By the pertinent instances of (II.7.50), the trains of identities (3.16) and (3.17) can equivalently be stated as the following trains of equivalences:

$$\begin{gathered}\mathsf{E}_m\langle\mathbf{u},\mathbf{v}\rangle \Leftrightarrow \neg\mathsf{I}_m\langle\mathbf{u},\mathbf{v}\rangle \Leftrightarrow \neg\mathsf{I}_n\langle\mathbf{v},\mathbf{u}\rangle \Leftrightarrow \mathsf{E}_n\langle\mathbf{v},\mathbf{u}\rangle, \\ \mathsf{I}_m\langle\mathbf{u},\mathbf{v}\rangle \Leftrightarrow \neg\mathsf{E}_m\langle\mathbf{u},\mathbf{v}\rangle \Leftrightarrow \neg\mathsf{E}_n\langle\mathbf{v},\mathbf{u}\rangle \Leftrightarrow \mathsf{I}_n\langle\mathbf{v},\mathbf{u}\rangle, \\ \text{for each } m \vec{\in} \{1,2,3,4\} \text{ and each } n \vec{\in} \{1,2,3,4\};\end{gathered} \tag{3.16a}$$

$$\begin{gathered}\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Leftrightarrow \neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle \Leftrightarrow \neg\mathsf{O}_4\langle\mathbf{v},\mathbf{u}\rangle \Leftrightarrow \mathsf{A}_4\langle\mathbf{v},\mathbf{u}\rangle, \\ \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle \Leftrightarrow \neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Leftrightarrow \neg\mathsf{A}_4\langle\mathbf{v},\mathbf{u}\rangle \Leftrightarrow \mathsf{O}_4\langle\mathbf{v},\mathbf{u}\rangle.\end{gathered} \tag{3.17a}$$
•

Df 3.1. 1) The PSJ's:

$$\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle \text{ and } \mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle \text{ at } n \vec{\in} \{1,2,3\} \tag{3.18}$$

are called *asymmetric ones* in the sense that, in accordance with (3.1), their validity-indices are *not invariant* under the permutation of '**u**' and '**v**'. By contrast, all other PSJ's selected out of the list (2.10), i.e.

$$\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle \text{ and } \mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle \text{ at } n \,\vec{\in}\, \{1,2,3,4\}, \text{ and } \mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \text{ and } \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle, \tag{3.19}$$

are called *symmetric PSJ's* in the sense that their validity-indices are *invariant* under the permutation of '**u**' and '**v**', in accordance with Th 3.3. Consequently, any of the three PSB's given by (2.10) at $n \,\vec{\in}\, \{1,2,3\}$ is called an *asymmetric* one, while that with $n \triangleright 4$ is called a *symmetric* one.

2) Any ESJ in the range of a given PSJ is called an *asymmetric ESJ* if the PSJ is an asymmetric one and a *symmetric ESJ* if the PSJ is a symmetric one. Any ESB in the range of a given PSB is called an *asymmetric ESB* if the PSB is an asymmetric one and a *symmetric ESB* if the PSB is a symmetric one.•

***Th 3.4.**

$$V(\mathsf{A}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\mathsf{O}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, 0 \text{ for each } n \,\vec{\in}\, \{1,2,3\}. \tag{3.20}$$

$$V(\mathsf{O}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\mathsf{A}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, 1 \text{ for each } n \,\vec{\in}\, \{1,2,3\}. \tag{3.21}$$

$$\begin{gathered} V(\mathsf{E}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\mathsf{I}_n\langle\mathbf{u},\mathbf{u}\rangle) \\ \hat{=}\, 1 \,\hat{-}\, \hat{\vee}_{\mathbf{z}} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\textstyle\bigvee_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \text{ for each } n \,\vec{\in}\, \{1,2,3,4\}. \end{gathered} \tag{3.22}$$

$$\begin{gathered} V(\mathsf{I}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\mathsf{E}_n\langle\mathbf{u},\mathbf{u}\rangle) \\ \hat{=}\, \hat{\vee}_{\mathbf{z}} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \,\hat{=}\, V(\textstyle\bigvee_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \text{ for each } n \,\vec{\in}\, \{1,2,3,4\}. \end{gathered} \tag{3.23}$$

$$V(\mathsf{A}_4\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, 1 \,\hat{-}\, \hat{\vee}_{\mathbf{z}} V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \,\hat{=}\, V(\textstyle\bigwedge_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle). \tag{3.24}$$

$$V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \,\hat{=}\, V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle) \,\hat{=}\, \hat{\vee}_{\mathbf{z}} V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \,\hat{=}\, V(\textstyle\bigvee_{\mathbf{z}}\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle). \tag{3.25}$$

Proof: It follows from the instances of (3.1)–(3.6) with '**u**' in place of '**v**' that

$$\begin{gathered} V(\mathsf{A}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\mathsf{O}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, 1 \,\hat{-}\, \hat{\vee}_{\mathbf{z}}[1 \,\hat{-}\, V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \,\hat{\cdot}\, V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)] \\ \hat{=}\, 1 \,\hat{-}\, \hat{\vee}_{\mathbf{z}} 1 \,\hat{=}\, 1 \,\hat{-}\, 1 \,\hat{=}\, 0 \text{ at } n \,\vec{\in}\, \{1,2,3\}, \end{gathered} \tag{3.20$_1$}$$

$$\begin{gathered} V(\mathsf{O}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\mathsf{A}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, \hat{\vee}_{\mathbf{z}}[1 \,\hat{-}\, V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \,\hat{\cdot}\, V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)] \\ \hat{=}\, \hat{\vee}_{\mathbf{z}} 1 \,\hat{=}\, 1 \text{ at } n \,\vec{\in}\, \{1,2,3\}, \end{gathered} \tag{3.21$_1$}$$

$$\begin{gathered} V(\mathsf{E}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\mathsf{I}_n\langle\mathbf{u},\mathbf{u}\rangle) \\ \hat{=}\, 1 \,\hat{-}\, \hat{\vee}_{\mathbf{z}}[1 \,\hat{-}\, V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \,\hat{\cdot}\, V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)] \,\hat{=}\, 1 \,\hat{-}\, \hat{\vee}_{\mathbf{z}}[1 \,\hat{-}\, V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)] \\ \hat{=}\, 1 \,\hat{-}\, \hat{\vee}_{\mathbf{z}} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\textstyle\bigvee_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \text{ at } n \,\vec{\in}\, \{1,2,3,4\}, \end{gathered} \tag{3.22$_1$}$$

$$\begin{gathered} V(\mathsf{I}_n\langle\mathbf{u},\mathbf{u}\rangle) \,\hat{=}\, V(\neg\mathsf{E}_n\langle\mathbf{u},\mathbf{u}\rangle) \\ \hat{=}\, \hat{\vee}_{\mathbf{z}}[1 \,\hat{-}\, V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \,\hat{\cdot}\, V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)] \,\hat{=}\, \hat{\vee}_{\mathbf{z}}[1 \,\hat{-}\, V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)] \\ \hat{=}\, \hat{\vee}_{\mathbf{z}} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \,\hat{=}\, V(\textstyle\bigvee_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \text{ at } n \,\vec{\in}\, \{1,2,3,4\}, \end{gathered} \tag{3.23$_1$}$$

$$V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{u}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{u}\rangle\right)\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\right]$$
$$\hat{=}1\hat{-}\hat{\bullet}_{\mathbf{z}}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{=}V\left(\bigwedge_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right),\tag{3.24$_1$}$$

$$V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}\hat{\bullet}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\right]$$
$$\hat{=}\hat{\bullet}_{\mathbf{z}}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{z}}\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right).\tag{3.25$_1$}$$

QED.•

Cmt 3.4. By (3.20), all euautographic relations

$$\mathsf{A}_n\langle\mathbf{u},\mathbf{u}\rangle \text{ and } \neg\mathsf{O}_n\langle\mathbf{u},\mathbf{u}\rangle \text{ at } n\,\vec{\in}\,\{1,2,3\}\tag{3.26}$$

are kyrologies (valid relations), or, more specifically, theorems, of A_1 and accordingly, by (3.21), all euautographic relations

$$\mathsf{O}_n\langle\mathbf{u},\mathbf{u}\rangle \text{ and } \neg\mathsf{A}_n\langle\mathbf{u},\mathbf{u}\rangle \text{ at } n\,\vec{\in}\,\{1,2,3\}\tag{3.27}$$

are antikyrologies (antivalid relations), or, more specifically, antitheorems, of A_1. At the same time, the ultimate values of the validity integrons

$$V\left(\neg\bigvee_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right),\ V\left(\bigvee_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right),\ V\left(\bigwedge_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right),\ V\left(\bigvee_{\mathbf{z}}\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right),\tag{3.28}$$

occurring in (3.22)–(3.25) respectively, depend on a concrete euautographic relation, which is meant by '$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$', and on the axioms, which are imposed on the ordinary atomic euautographs involved in that concrete relation, $\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$.•

***Th 3.5.**

$$V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)$$
$$\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\tag{3.29}$$
$$\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{=}0.$$

$$V\left(\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\Rightarrow\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)$$
$$\hat{=}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\hat{\cdot}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\tag{3.30}$$
$$\hat{=}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{=}0.$$

$$V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)$$
$$\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\tag{3.31}$$
$$\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{=}0.$$

$$V\left(\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)$$
$$\hat{=}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\hat{\cdot}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\tag{3.32}$$
$$\hat{=}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{=}0.$$

$$
\begin{aligned}
&V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\
&\hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} \left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right] \hat{=} 0.
\end{aligned}
\tag{3.33}
$$

$$
\begin{aligned}
&V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right] \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} \left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right] \hat{=} 0.
\end{aligned}
\tag{3.34}
$$

$$
\begin{aligned}
&V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{=} V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right) \hat{=} 0.
\end{aligned}
\tag{3.35}
$$

$$
\begin{aligned}
&V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{=} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right) \hat{=} 0.
\end{aligned}
\tag{3.36}
$$

Proof: In developing each of the following trains of identities use of the pertinent version of the Emission Law (EL), (II.4.27), and of the algebraic law of excluded middle (ALEM), (II.7.15γ), is made:

$$
\begin{aligned}
&V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\
&\hat{=} V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{z}} V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right] \hat{\cdot} \left[1 \hat{-} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{z}} V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\\
&\hat{\cdot} \left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right) \hat{\cdot} \left[1 \hat{-} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{z}} V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right] \hat{\cdot} 0 \hat{=} 0,
\end{aligned}
\tag{3.29$_1$}
$$

$$
\begin{aligned}
&V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\
&\hat{=} V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{z}} V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right] \hat{\cdot} \left[1 \hat{-} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{z}} V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\\
&\hat{\cdot} \left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right) \hat{\cdot} \left[1 \hat{-} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{z}} V\left(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right] \hat{\cdot} 0 \hat{=} 0,
\end{aligned}
\tag{3.31$_1$}
$$

$$
\begin{aligned}
&V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\
&\hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{z}} V\left(\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\right] \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{z}} V\left(\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\right]\\
&\hat{\cdot} \left[V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{z}} \neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right] \hat{\cdot} 0 \hat{=} 0,
\end{aligned}
\tag{3.33$_1$}
$$

$$
\begin{aligned}
&V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \mathbin{\hat{\cdot}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\\
&\hat{=} \left[\hat{\cdot}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \mathbin{\hat{\cdot}} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right] \mathbin{\hat{\cdot}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\\
&\hat{=} \left[\hat{\cdot}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \mathbin{\hat{\cdot}} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\mathbin{\hat{\cdot}} \left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \mathbin{\hat{\cdot}} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right) \mathbin{\hat{\cdot}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\\
&\hat{=} \left[\hat{\cdot}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \mathbin{\hat{\cdot}} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right] \mathbin{\hat{\cdot}} 0 \hat{=} 0.
\end{aligned}
\tag{3.35$_1$}
$$

These relations prove (3.29), (3.31), (3.33), and a part of (3.35). The rest of the relations comprised in the theorem are proved elementarily as follows:

$$
\begin{aligned}
&V\left(\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right] \Rightarrow \mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right) \mathbin{\hat{\cdot}} V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right) \mathbin{\hat{\cdot}} V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right) \hat{=} 0,
\end{aligned}
\tag{3.30$_1$}
$$

$$
\begin{aligned}
&V\left(\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right] \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right) \mathbin{\hat{\cdot}} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right) \mathbin{\hat{\cdot}} V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right) \hat{=} 0,
\end{aligned}
\tag{3.32$_1$}
$$

$$
\begin{aligned}
&V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right] \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\neg\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right) \mathbin{\hat{\cdot}} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right) \mathbin{\hat{\cdot}} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right) \hat{=} 0,
\end{aligned}
\tag{3.34$_1$}
$$

$$
V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{A}_4\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right) \hat{=} 0\,,
\tag{3.35$_2$}
$$

$$
\begin{aligned}
&V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \mathbin{\hat{\cdot}} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \mathbin{\hat{\cdot}} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{=} 0,
\end{aligned}
\tag{3.36$_1$}
$$

$$
V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{v},\mathbf{u}\rangle\right) \hat{=} 0.
\tag{3.36$_2$}
$$

QED.•

Cmt 3.5. By the pertinent versions of (II.4.40a), identities (3.29)–(3.36) can be restated as the corresponding implications, e.g.:

$$
\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right],
\tag{3.29a}
$$

$$
\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right] \Rightarrow \mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle,
\tag{3.30a}
$$

etc subject to definitions (2.3)–(2.6).•

Cmt 3.6. From the variant of (II.7.3γ) with '¬**S**' and '¬**R**' in place of '**P**' and '**Q**' respectively, it follows that

$$V(\neg\mathbf{S}\Rightarrow\neg\mathbf{R})\hat{=}V(\neg\neg\mathbf{S})\hat{\vee}V(\neg\mathbf{R})\hat{=}V(\neg\mathbf{R})\hat{\vee}V(\mathbf{S})\hat{=}V(\mathbf{R}\Rightarrow\mathbf{S}), \tag{3.37}$$

whence, by the pertinent version of (II.4.40a),

$$[\neg\mathbf{S}\Rightarrow\neg\mathbf{R}]\Leftrightarrow[\mathbf{R}\Rightarrow\mathbf{S}]. \tag{3.37a}$$

Consequently, the trains of identities (3.30_1), (3.32_1), and (3.34_1) are the pertinent instances of the train (3.37) in the case where

$$V(\mathbf{R}\Rightarrow\mathbf{S})\hat{=}0, \tag{3.38}$$

i.e. where $\mathbf{R}\Rightarrow\mathbf{S}$ is a kyrology. The trains of identities forming any of the following pairs: (3.29) and (3.30), (3.31) and (3.32), (3.33) and (3.34), (3.35) and (3.36) are identical because they satisfy (3.37) and (3.38).•

Cmt 3.7. The panlogographic identities (3.29)–(3.34) are schemata of euautographic identities of their ranges. Therefore, the following statements, belonging to the IML (inclusive metalanguage) of A_1, express hypothetical conditions that are imposed on the *euautographic slave-relations* (*ESR's*) comprised in the ranges of the *panlogographic slave-relations* (*PLSR's*) that are involved in the above panlogographic identities, of which (3.29) and (3.30), (3.31) and (3.32), or (3.33) and (3.34) are mutually equivalent, as indicated in the previous comment.

i) If $V(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}V(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}0$, i.e. if $V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}1$, then $V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\vee}V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{=}0$.

ii) If $V(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}V(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}1$, i.e. if $V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}0$, then $V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\vee}V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)$ is undecided.

iii) If $[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\vee}V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]\hat{=}1$ then $V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}0$.

iv) If $[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\vee}V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]\hat{=}0$ then both $V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)$ and $V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)$ are undecided.

v) If $V(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}V(\neg\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}0$, i.e. if $V(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}1$, then $V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\vee}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{=}0$.

vi) If $V(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}V(\neg\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}1$, i.e. if $V(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}0$, then $V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\vee}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)$ is undecided.

vii) If $[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\vee}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]\hat{=}1$ then $V(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\hat{=}0$.

viii) If $\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{=}0$ then both $V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)$ and $V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)$ are undecided.

ix) If $V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}0$, i.e. if $V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1$, then $V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{=}1$.

x) If $V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1$, i.e. if $V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}0$, then $V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)$ is undecided.

xi) If $\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{=}0$ then $V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}0$.

xii) If $\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{=}0$ then both $V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)$ and $V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)$ are undecided.•

Cmt 3.8. By (3.1)–(3.6), identities (3.29)–(3.34) can be rewritten as:

$$\begin{aligned}&V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\&\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\&\hat{=}V\left(\neg\bigwedge_{\mathbf{z}}\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{\cdot}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\&\hat{=}V\left(\left[\bigwedge_{\mathbf{z}}\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right]\Rightarrow\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\hat{=}0.\end{aligned}\tag{3.29'}$$

$$\begin{aligned}&V\left(\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\Rightarrow\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\&\hat{=}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\hat{\cdot}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\&\hat{=}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\hat{\cdot}V\left(\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\\&\hat{=}V\left(\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\Rightarrow\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{=}0.\end{aligned}\tag{3.30'}$$

$$\begin{aligned}&V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\&\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\&\hat{=}V\left(\neg\bigwedge_{\mathbf{z}}\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{\cdot}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\\&\hat{=}V\left(\left[\bigwedge_{\mathbf{z}}\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right]\Rightarrow\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\hat{=}0.\end{aligned}\tag{3.31'}$$

$$\begin{aligned}&V\left(\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\hat{\cdot}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\&\hat{=}V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\hat{\cdot}V\left(\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\\&\hat{=}V\left(\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\Rightarrow\bigvee_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{=}0.\end{aligned}\tag{3.32'}$$

$$\begin{aligned}&V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\hat{=}V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\\&\hat{=}V\left(\neg\bigwedge_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\\&\hat{=}V\left(\left[\bigwedge_{\mathbf{z}}\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right]\Rightarrow\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right)\hat{=}0.\end{aligned}\tag{3.33'}$$

$$
\begin{aligned}
V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) &\mathrel{\hat{=}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}} V\left(\textstyle\bigvee_{\mathbf{z}}\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\\
&\mathrel{\hat{=}} V\left(\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\Rightarrow\textstyle\bigvee_{\mathbf{z}}\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right]\right)\mathrel{\hat{=}}0.
\end{aligned}
\tag{3.34'}
$$

2) At the same time, from the pertinent versions of (II.7.3γ), it follows that

$$
\begin{aligned}
V\left(\left[\textstyle\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\Rightarrow\mathbf{Q}\langle\mathbf{x}\rangle\right) &\mathrel{\hat{=}} V\left(\neg\textstyle\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right)\mathbin{\hat{\cdot}} V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\\
\mathrel{\hat{=}}\left[\hat{\cdot}_{\mathbf{z}}V\left(\neg\mathbf{Q}\langle\mathbf{z}\rangle\right)\right]\mathbin{\hat{\cdot}} V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right) &\mathrel{\hat{=}}\left[\hat{\cdot}_{\mathbf{z}}V\left(\neg\mathbf{Q}\langle\mathbf{z}\rangle\right)\right]\mathbin{\hat{\cdot}}\left[V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\mathbin{\hat{\cdot}} V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]\\
&\mathrel{\hat{=}}\left[\hat{\cdot}_{\mathbf{z}}V\left(\neg\mathbf{Q}\langle\mathbf{z}\rangle\right)\right]\mathbin{\hat{\cdot}}0\mathrel{\hat{=}}0,
\end{aligned}
\tag{3.39}
$$

$$
\begin{aligned}
V\left(\mathbf{Q}\langle\mathbf{x}\rangle\Rightarrow\left[\textstyle\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\right) &\mathrel{\hat{=}} V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\mathbin{\hat{\cdot}} V\left(\textstyle\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right)\\
\mathrel{\hat{=}} V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\mathbin{\hat{\cdot}}\left[\hat{\cdot}_{\mathbf{z}}V\left(\mathbf{Q}\langle\mathbf{z}\rangle\right)\right] &\mathrel{\hat{=}}\left[V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\mathbin{\hat{\cdot}} V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]\mathbin{\hat{\cdot}}\left[\hat{\cdot}_{\mathbf{z}}V\left(\mathbf{Q}\langle\mathbf{z}\rangle\right)\right]\\
&\mathrel{\hat{=}}0\mathbin{\hat{\cdot}}\left[\hat{\cdot}_{\mathbf{z}}V\left(\mathbf{Q}\langle\mathbf{z}\rangle\right)\right]\mathrel{\hat{=}}0.
\end{aligned}
\tag{3.40}
$$

In developing the final results in these trains of identities, use of the pertinent versions of the Emission Law (II.4.27) and of the ALEM (algebraic law of excluded middle) (II.7.15γ) has been made. With '$\neg\mathbf{Q}$' in place of '$\mathbf{Q}$', relations (3.39) and (3.40) exchange:

$$
\begin{aligned}
V\left(\left[\textstyle\bigwedge_{\mathbf{z}}\neg\mathbf{Q}\langle\mathbf{z}\rangle\right]\Rightarrow\neg\mathbf{Q}\langle\mathbf{x}\rangle\right) &\mathrel{\hat{=}} V\left(\neg\textstyle\bigwedge_{\mathbf{z}}\neg\mathbf{Q}\langle\mathbf{z}\rangle\right)\mathbin{\hat{\cdot}} V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\\
\mathrel{\hat{=}} V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\mathbin{\hat{\cdot}} V\left(\textstyle\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right) &\mathrel{\hat{=}}\left[\hat{\cdot}_{\mathbf{z}}V\left(\neg\mathbf{Q}\langle\mathbf{z}\rangle\right)\right]\mathbin{\hat{\cdot}} V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\\
&\mathrel{\hat{=}} V\left(\mathbf{Q}\langle\mathbf{x}\rangle\Rightarrow\left[\textstyle\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\right)\mathrel{\hat{=}}0,
\end{aligned}
\tag{3.39a}
$$

$$
\begin{aligned}
V\left(\neg\mathbf{Q}\langle\mathbf{x}\rangle\Rightarrow\left[\textstyle\bigvee_{\mathbf{z}}\neg\mathbf{Q}\langle\mathbf{z}\rangle\right]\right) &\mathrel{\hat{=}} V\left(\neg\neg\mathbf{Q}\langle\mathbf{x}\rangle\right)\mathbin{\hat{\cdot}} V\left(\textstyle\bigvee_{\mathbf{z}}\neg\mathbf{Q}\langle\mathbf{z}\rangle\right)\\
\mathrel{\hat{=}} V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\mathbin{\hat{\cdot}} V\left(\neg\neg\textstyle\bigvee_{\mathbf{z}}\neg\mathbf{Q}\langle\mathbf{z}\rangle\right) &\mathrel{\hat{=}} V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\mathbin{\hat{\cdot}} V\left(\neg\textstyle\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right)\\
&\mathrel{\hat{=}} V\left(\left[\textstyle\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\Rightarrow\mathbf{Q}\langle\mathbf{x}\rangle\right)\mathrel{\hat{=}}0.
\end{aligned}
\tag{3.40a}
$$

Comparison of (3.29')–(3.34') on the one hand and (3.39), (3.40), (3.39a), and (3.40a) on the other hand shows that any one of the former six relations is an instance of a certain one of the latter four relations under a certain one of the four pairs of substitutions:

$$\mathbf{Q}\langle\mathbf{z}\rangle\rhd\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right],\ \mathbf{Q}\langle\mathbf{x}\rangle\rhd\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right], \tag{3.41$_1$}$$

$$\mathbf{Q}\langle\mathbf{z}\rangle\rhd\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right],\ \mathbf{Q}\langle\mathbf{x}\rangle\rhd\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right], \tag{3.41$_2$}$$

$$\mathbf{Q}\langle\mathbf{z}\rangle\rhd\neg\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right],\ \mathbf{Q}\langle\mathbf{x}\rangle\rhd\neg\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right], \tag{3.41$_3$}$$

$$\mathbf{Q}\langle\mathbf{z}\rangle\rhd\left[\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right],\ \mathbf{Q}\langle\mathbf{x}\rangle\rhd\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]. \tag{3.41$_4$}$$

To be specific, relations (3.29'), (3.31'), and (3.33') are instances of (3.39) subject to (3.41$_1$), (3.41$_3$), and (3.41$_4$) respectively, while relations (3.30'), (3.32'), and (3.34') are instances of (3.40) subject to (3.41$_2$), (3.41$_4$), and (3.41$_3$) respectively.•

Cmt 3.9. Here follow some properties of relations (3.39) and (3.40), which becomes under substitutions (3.41_1)–(3.41_4), properties of relations (3.29)–(3.34)

1) By the pertinent versions of (II.4.23) and (II.8.2), it follows from (3.39) and (3.40), it follows that

$$
\begin{aligned}
&V\left(\bigvee_{\mathbf{x}}\left[\left[\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\Rightarrow\mathbf{Q}\langle\mathbf{x}\rangle\right]\right)\hat{=}\mathop{\hat{\bullet}}_{\mathbf{x}}\left[V\left(\neg\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]\hat{=}\mathop{\hat{\bullet}}_{\mathbf{x}}0\,\hat{=}\,0\\
&\hat{=}V\left(\neg\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right)\mathbin{\hat{\cdot}}\left[\mathop{\hat{\bullet}}_{\mathbf{x}}V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]\hat{=}V\left(\neg\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right)\mathbin{\hat{\cdot}}\left[V\left(\bigvee_{\mathbf{x}}\mathbf{Q}\langle\mathbf{x}\rangle\right)\right]\\
&\hat{=}V\left(\left[\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\Rightarrow\bigvee_{\mathbf{x}}\mathbf{Q}\langle\mathbf{x}\rangle\right),
\end{aligned}
\tag{3.42}
$$

$$
\begin{aligned}
&V\left(\bigwedge_{\mathbf{x}}\left[\left[\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\Rightarrow\mathbf{Q}\langle\mathbf{x}\rangle\right]\right)\hat{=}1\mathbin{\hat{-}}\mathop{\hat{\bullet}}_{\mathbf{x}}V\left(\neg\left[\left[\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\Rightarrow\mathbf{Q}\langle\mathbf{x}\rangle\right]\right)\\
&\hat{=}1\mathbin{\hat{-}}\mathop{\hat{\bullet}}_{\mathbf{x}}1\,\hat{=}\,1\mathbin{\hat{-}}1\,\hat{=}\,0,
\end{aligned}
\tag{3.43}
$$

$$
V\left(\bigvee_{\mathbf{x}}\left[\mathbf{Q}\langle\mathbf{x}\rangle\Rightarrow\left[\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\right]\right)\hat{=}\mathop{\hat{\bullet}}_{\mathbf{x}}V\left(\mathbf{Q}\langle\mathbf{x}\rangle\Rightarrow\left[\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\right)\hat{=}\mathop{\hat{\bullet}}_{\mathbf{x}}0\,\hat{=}\,0,
\tag{3.44}
$$

$$
\begin{aligned}
&V\left(\bigwedge_{\mathbf{x}}\left[\mathbf{Q}\langle\mathbf{x}\rangle\Rightarrow\left[\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\right]\right)\hat{=}1\mathbin{\hat{-}}\mathop{\hat{\bullet}}_{\mathbf{x}}V\left(\neg\left[\mathbf{Q}\langle\mathbf{x}\rangle\Rightarrow\left[\bigvee_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\right]\right)\\
&\hat{=}1\mathbin{\hat{-}}\mathop{\hat{\bullet}}_{\mathbf{x}}1\,\hat{=}\,1\mathbin{\hat{-}}1\,\hat{=}\,0.
\end{aligned}
\tag{3.45}
$$

2) If an occurrence of an APLOT (atomic panlogographic ordinary term), e.g. ‘**z**’, is bound then its range is the set τ^{pv} defined in (2.49), so that any euautographic instance (interpretand) of the APLOT in that occurrence is a bound APVOT (atomic pseudo-variable ordinary term) of that set. If however an occurrence of an APLOT, e.g. ‘**x**’, is free then its range is the set τ defined in (2.48), so that a euautographic instance (interpretand) of the APLOT in that occurrence is either a free APVOT of the set τ^{pv} or the APCOT (atomic pseudo-constant ordinary term) $\emptyset$. In order to indicate that the range of an APLOT in a given occurrence either is τ^{pv} or is $\{\emptyset\}$, the APLOT can be provided with the corresponding superscripts ‘$^{\mathrm{pv}}$’ (meaning *pseudo-variable*) or ‘$^{\mathrm{pc}}$’ (meaning *pseudo-constant*). For instance, ‘$\mathbf{z}^{\mathrm{pv}}$’ and ‘$\mathbf{x}^{\mathrm{pv}}$’ are APLOT’s whose range is τ^{pv}, whereas and ‘$\mathbf{z}^{\mathrm{pc}}$’ and ‘$\mathbf{x}^{\mathrm{pc}}$’ are APLOT’s whose range is $\{\emptyset\}$. Either one of the trains of identities (3.39) and (3.40) is semantically invariant under the replacement of ‘**z**’ with ‘$\mathbf{z}^{\mathrm{pv}}$’, but it is not semantically invariant under the replacement of ‘**x**’ with ‘$\mathbf{x}^{\mathrm{pv}}$’ or ‘$\mathbf{x}^{\mathrm{pc}}$’. That is to say, the relation schema (3.39), e.g., is semantically concurrent to the relation schema:

$$
V\left(\left[\bigwedge_{\mathbf{z}^{\mathrm{pv}}}\mathbf{Q}\left\langle\mathbf{z}^{\mathrm{pv}}\right\rangle\right]\Rightarrow\mathbf{Q}\langle\mathbf{x}\rangle\right)\hat{=}V\left(\neg\bigwedge_{\mathbf{z}^{\mathrm{pv}}}\mathbf{Q}\left\langle\mathbf{z}^{\mathrm{pv}}\right\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathbf{Q}\langle\mathbf{x}\rangle\right)\hat{=}\,0,
\tag{3.39_1}
$$

and it is also semantically concurrent to the conjunction of the two relation schemata:

$$
V\left(\left[\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\Rightarrow\mathbf{Q}\left\langle\mathbf{x}^{\mathrm{pv}}\right\rangle\right)\hat{=}V\left(\neg\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathbf{Q}\left\langle\mathbf{x}^{\mathrm{pv}}\right\rangle\right)\hat{=}\,0,
\tag{3.39_2}
$$

$$
V\left(\left[\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right]\Rightarrow\mathbf{Q}\left\langle\mathbf{x}^{\mathrm{pc}}\right\rangle\right)\hat{=}V\left(\neg\bigwedge_{\mathbf{z}}\mathbf{Q}\langle\mathbf{z}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathbf{Q}\langle\emptyset\rangle\right)\hat{=}\,0.
\tag{3.39_3}
$$

Either of the two concurrent relation schemata (3.39) and (3.39_1) can be particularized (concretized) with respect to euautographic interpretands the APLOT's occurring in them, for instance, thus:

$$V\left(\left[\bigwedge_z \mathbf{Q}\langle z\rangle\right] \Rightarrow \mathbf{Q}\langle x\rangle\right) \hat{=} V\left(\neg \bigwedge_z \mathbf{Q}\langle z\rangle\right) \hat{\cdot} V\left(\mathbf{Q}\langle x\rangle\right) \hat{=} 0 \qquad (3.39_4)$$

or thus:

$$V\left(\left[\bigwedge_z \mathbf{Q}\langle z\rangle\right] \Rightarrow \mathbf{Q}\langle \emptyset\rangle\right) \hat{=} V\left(\neg \bigwedge_z \mathbf{Q}\langle z\rangle\right) \hat{\cdot} V\left(\mathbf{Q}\langle \emptyset\rangle\right) \hat{=} 0. \qquad (3.39_5)$$

At the same time, the relation schemata (3.39_4) and (3.39_5) are at the same time the pertinent particularizations of (3.39_2) and (3.39_3) with respect to euautographic interpretands of of '**z**', '$\mathbf{x}^{pv}$', and '$\mathbf{x}^{pc}$'. Like remarks apply, *mutatis mutandis*, to (3.40).

3) In connection with the above explanations, it may be remarked that many relations of A_1 and $\mathbf{A}_1$, which are relevant to Aristotelian logic and generally to other branches of formal logic, turn out to be trivial. Demonstrating some of these trivialities is one of the objects of this study, although I do not, as a rule, point to the trivialities of A_1 as "trivialities".•

***Th 3.6.**

$$\begin{gathered} V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right) \\ \hat{=} V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \wedge \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right) \\ \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right) \\ \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} \left[1 \hat{-} V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\right] \hat{=} 0. \end{gathered} \qquad (3.46)$$

$$\begin{gathered} V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{v},\mathbf{u}\rangle\right) \\ \hat{=} V\left(\left[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle \vee \mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right] \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \\ \hat{=} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right) \\ \hat{=} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} \left[1 \hat{-} V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\right] \hat{=} 0. \end{gathered} \qquad (3.47)$$

$$V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} 0. \qquad (3.48)$$

$$V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} 0. \qquad (3.49)$$

Proof: From (3.1) and from the variant of (3.6) with '**x**' in place of '**z**', it follows by the pertinent versions of the Fusion Law (II.4.29) that

$$\begin{gathered} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} \left[\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right] \\ \hat{\cdot} \left[\hat{\cdot}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right] \\ \hat{=} \hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right) \hat{\cdot} \left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right] \\ \hat{=} \hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right] \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right), \end{gathered} \qquad (3.46_1)$$

$$\begin{aligned} V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle) &\mathrel{\hat{=}}\left[\hat{\bullet}_{\mathbf{x}}\left[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\right] \\ &\quad\mathbin{\hat{\cdot}}\left[\hat{\bullet}_{\mathbf{z}}\left[1\mathbin{\hat{-}}V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\right]\right] \\ &\mathrel{\hat{=}}\hat{\bullet}_{\mathbf{x}}\left[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\right]\right] \\ &\mathrel{\hat{=}}\hat{\bullet}_{\mathbf{x}}\left[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\mathrel{\hat{=}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle). \end{aligned} \tag{3.46$_2$}$$

Hence,

$$\begin{aligned} V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle) &\mathrel{\hat{=}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle) \\ &\mathrel{\hat{=}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\right]\mathrel{\hat{=}}0, \end{aligned} \tag{3.46$_3$}$$

$$\begin{aligned} V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle) &\mathrel{\hat{=}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle) \\ &\mathrel{\hat{=}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle)\right]\mathrel{\hat{=}}0, \end{aligned} \tag{3.46$_4$}$$

$$\begin{aligned} &V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\wedge\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle]) \\ &\mathrel{\hat{=}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\wedge\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle) \\ &\mathrel{\hat{=}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle)\right]\mathrel{\hat{=}}0, \end{aligned} \tag{3.46$_5$}$$

which prove (3.46). At the same time, by (3.1), (3.2), and (3.6), it follows that

$$\begin{aligned} V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) &\mathrel{\hat{=}}V(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \\ \mathrel{\hat{=}}V(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle) &\mathrel{\hat{=}}V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\mathrel{\hat{=}}0, \end{aligned} \tag{3.47$_1$}$$

$$\begin{aligned} V(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) &\mathrel{\hat{=}}V(\neg\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle)\mathbin{\hat{\cdot}}V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \\ \mathrel{\hat{=}}V(\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle) &\mathrel{\hat{=}}V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle)\mathrel{\hat{=}}0, \end{aligned} \tag{3.47$_2$}$$

$$\begin{aligned} &V([\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\vee\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle]\Rightarrow\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \\ &\mathrel{\hat{=}}V(\neg[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\vee\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle])\mathbin{\hat{\cdot}}V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \\ &\mathrel{\hat{=}}\left[1\mathbin{\hat{-}}V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle)\right]\mathbin{\hat{\cdot}}V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \\ &\mathrel{\hat{=}}\left[1\mathbin{\hat{-}}V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle)\right]\mathbin{\hat{\cdot}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle) \\ &\mathrel{\hat{=}}V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\wedge\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle])\mathrel{\hat{=}}0. \end{aligned} \tag{3.47$_3$}$$

Hence, the train of identities (3.47) is just another form of (3.46). In analogy with (3.46_1), it follows by (3.4) and by the variant of (3.6) with '**x**' in place of '**z**' that

$$\begin{aligned} &V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\mathrel{\hat{=}}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle) \\ &\mathrel{\hat{=}}\left[\hat{\bullet}_{\mathbf{x}}\left[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\right]\mathbin{\hat{\cdot}}\left[\hat{\bullet}_{\mathbf{z}}\left[1\mathbin{\hat{-}}V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\right]\right] \\ &\mathrel{\hat{=}}\hat{\bullet}_{\mathbf{x}}\left[\left[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\right]\mathrel{\hat{=}}0, \end{aligned} \tag{3.48$_1$}$$

which proves (3.48). At the same time, by (3.4) and (3.6),

$$\begin{aligned} V(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) &\mathrel{\hat{=}}V(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \\ \mathrel{\hat{=}}V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\mathbin{\hat{\cdot}}V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle) &\mathrel{\hat{=}}V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\mathrel{\hat{=}}0, \end{aligned} \tag{3.49$_1$}$$

so that (3.49) is just another form of (3.48). QED.•

Cmt 3.10. 1) Identities (3.46)–(3.49) are equivalent to the implications:

$$\begin{gathered}\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle, \mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle,\\ \mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow [\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \wedge \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle],\end{gathered} \tag{3.46a}$$

$$\begin{gathered}\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle, \mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle,\\ [\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle \vee \mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle] \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle,\end{gathered} \tag{3.47a}$$

$$\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle, \tag{3.48a}$$

$$\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle. \tag{3.49a}$$

(cf. Cmts 3.3 and 3.5).

2) By (3.46) and (3.47), it follows that

$$\begin{gathered}V([\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle] \wedge [\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle])\\ \hat{=} 1 \hat{-} V(\neg[\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle]) \hat{\cdot} V(\neg[\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle])\\ \hat{=} 1 \hat{-} [1 \hat{-} V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle)] \hat{\cdot} [1 \hat{-} V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle)]\\ \hat{=} 1 \hat{-} 1 \hat{\cdot} 1 \hat{=} 0,\end{gathered} \tag{3.50}$$

$$\begin{gathered}V([\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle] \wedge [\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle])\\ \hat{=} 1 \hat{-} V(\neg[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle]) \hat{\cdot} V(\neg[\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle])\\ \hat{=} 1 \hat{-} [1 \hat{-} V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle)] \hat{\cdot} [1 \hat{-} V(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle)]\\ \hat{=} 1 \hat{-} 1 \hat{\cdot} 1 \hat{=} 0,\end{gathered} \tag{3.51}$$

or, equivalently,

$$[\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle] \wedge [\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle], \tag{3.50a}$$

$$[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle] \wedge [\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle]. \tag{3.51a}$$

3) The validity indices of the converses of the separate relations (3.46a)–(3.49a) can be calculated in the same way. Still, these calculations are redundant because it is clear without any calculations that, according to (3.16), (3.17), and (3.20)–(3.25), the antecedent and the consequent of any of those relations have different properties and therefore they cannot be equivalent.•

***Th 3.7.**

$$\begin{gathered}V(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\\ \hat{=} \hat{\curlyvee}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{=} V(\bigvee\nolimits_{\mathbf{x}} \mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle).\end{gathered} \tag{3.52}$$

$$\begin{gathered}V(\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle)\\ \hat{=} \hat{\curlyvee}_{\mathbf{y}} V(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle) \hat{=} V(\bigvee\nolimits_{\mathbf{y}} \mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle).\end{gathered} \tag{3.53}$$

$$
\begin{aligned}
&V\left(\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\wedge\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\vee\left[\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\right)\\
&\hat{=}V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\left[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\vee\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right)\\
&\hat{=}V\left(\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\vee\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right)\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\right]\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}V\left(\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right]\vee\left[\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\vee\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right).
\end{aligned}
\tag{3.54}
$$

$$
\begin{aligned}
&V\left(\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\vee\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\wedge\left[\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\right)\\
&\hat{=}V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\left[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\wedge\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right)\\
&\hat{=}V\left(\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\wedge\left[\mathsf{E}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right)\\
&\hat{=}1\hat{-}\left[1\hat{-}\dot{\bigwedge}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\dot{\wedge}\left[1\hat{-}\dot{\bigwedge}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\right]\\
&\hat{=}V\left(\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right]\wedge\left[\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right).
\end{aligned}
\tag{3.55}
$$

Proof: From (3.4) and from the variant of (3.2) with '**x**' in place of '**z**' (see (2.1)), it follows by the pertinent versions of the Fusion Law (II.4.29) that

$$
\begin{aligned}
&V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\dot{\wedge}\dot{\bigwedge}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\dot{\wedge}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\dot{\wedge}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{+}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\dot{\wedge}1\right]\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\hat{=}\dot{\bigwedge}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right),
\end{aligned}
\tag{3.52$_1$}
$$

Then, by (3.4), (3.2), and (3.52$_1$), it follows that

$$
\begin{aligned}
&V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right).
\end{aligned}
\tag{3.52$_2$}
$$

Thus, (3.52) is established. At the same time, by (3.16) with $m \vartriangleright 1$ and $n \vartriangleright 1$, the variants of (3.52$_1$) and (3.52$_2$) with '**u**' and '**v**' exchanged and with '**y**' in place of '**x**' become:

$$
\begin{aligned}
&V\left(\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathsf{I}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\hat{=}V\left(\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\\
&\hat{=}\dot{\bigwedge}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\hat{=}V\left(\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right),
\end{aligned}
\tag{3.53$_1$}
$$

$$
\begin{aligned}
&V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right) \hat{=} V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\\
&\hat{=} V\left(\neg\mathsf{E}_1\langle\mathbf{v},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right) \hat{=} V\left(\mathsf{E}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\\
&\hat{=} V\left(\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{v},\mathbf{u}\rangle\right),
\end{aligned}
\tag{3.53$_2$}
$$

which prove (3.53). Consequently, by (3.52_1)–(3.53_2),

$$
\begin{aligned}
&V\left(\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \wedge \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right] \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\neg\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \wedge \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right) \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} \left[1 \hat{-} \left[1 \hat{-} V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\right]\right] \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} \left[V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\right] \hat{\cdot} \left[V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\right]\\
&\hat{=} V\left(\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right] \vee \left[\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\right)\\
&\hat{=} \left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right] \hat{\cdot} \left[\hat{\boldsymbol{\cdot}}_{\mathbf{y}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right] \hat{=} \left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}} \left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=} V\left(\left[\bigvee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right] \vee \left[\bigvee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right) \hat{=} V\left(\bigvee_{\mathbf{x}} \left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \vee \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right]\right),
\end{aligned}
\tag{3.54$_1$}
$$

$$
\begin{aligned}
&V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \left[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle \vee \mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right)\\
&\hat{=} V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\\
&\hat{=} \left[V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\right] \hat{\cdot} \left[V\left(\neg\mathsf{E}_1\langle\mathbf{v},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\right]\\
&\hat{=} V\left(\left[\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right] \vee \left[\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right)\\
&\hat{=} V\left(\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right] \vee \left[\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\right),
\end{aligned}
\tag{3.54$_2$}
$$

$$
\begin{aligned}
&V\left(\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \vee \mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right] \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} \left[1 \hat{-} V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\right] \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} \left[V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{+} V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right) \hat{-} V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\right]\\
&\quad \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=} \hat{\boldsymbol{\cdot}}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{+} \hat{\boldsymbol{\cdot}}_{\mathbf{y}} V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right) \hat{-} \left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right] \hat{\cdot} \left[\hat{\boldsymbol{\cdot}}_{\mathbf{y}} V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\right]\\
&\hat{=} V\left(\bigvee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{+} V\left(\bigvee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right) \hat{-} V\left(\bigvee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\bigvee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\\
&\hat{=} V\left(\left[\bigvee_{\mathbf{x}} \mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right] \wedge \left[\bigvee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right)\\
&\hat{=} V\left(\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right] \wedge \left[\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle \Rightarrow \mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\right)\\
&\hat{=} 1 \hat{-} V\left(\neg\bigvee_{\mathbf{x}} \mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\bigvee_{\mathbf{y}} \mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\\
&\hat{=} 1 \hat{-} \left[1 \hat{-} \hat{\boldsymbol{\cdot}}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right] \hat{\cdot} \left[1 \hat{-} \hat{\boldsymbol{\cdot}}_{\mathbf{y}} V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\right],
\end{aligned}
\tag{3.55$_1$}
$$

$$
\begin{aligned}
&V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\left[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\wedge\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right)\\
&\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{+}V\left(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\hat{-}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{+}\hat{\bullet}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\hat{-}\left[\hat{\bullet}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\right]\\
&\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{+}V\left(\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\hat{-}\left[V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\hat{\cdot}\left[V\left(\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\right]\\
&\hat{=}V\left(\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right]\wedge\left[\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right)\\
&\hat{=}V\left(\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\wedge\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right)\\
&\hat{=}V\left(\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\wedge\left[\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\right),
\end{aligned}
\tag{3.55$_2$}
$$

because

$$
\begin{aligned}
&1\hat{-}V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\wedge\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{+}V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\hat{-}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right),
\end{aligned}
\tag{3.55$_3$}
$$

$$
\begin{aligned}
&V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{I}_1\langle\mathbf{v},\mathbf{u}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{E}\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{E}\langle\mathbf{v},\mathbf{u}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{E}\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{E}\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}\langle\mathbf{u},\mathbf{v}\rangle\right).
\end{aligned}
\tag{3.55$_4$}
$$

Hence, (3.54) and (3.55) are also established.•

Cmt 3.11. The trains of identities (3.52)–(3.55) are evidently equivalent to the following trains of equivalences:

$$
\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\Leftrightarrow\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\Leftrightarrow\bigvee_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle,
\tag{3.52a}
$$

$$
\left[\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\Leftrightarrow\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\Leftrightarrow\bigvee_{\mathbf{z}}\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle,
\tag{3.53a}
$$

$$
\begin{aligned}
&\left[\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\wedge\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\\
&\Leftrightarrow\left[\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\vee\left[\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\right]\\
&\Leftrightarrow\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\left[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\vee\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right]\\
&\Leftrightarrow\left[\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\vee\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right]\\
&\Leftrightarrow\left[\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right]\vee\left[\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right]\Leftrightarrow\bigvee_{\mathbf{x}}\left[\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\vee\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right],
\end{aligned}
\tag{3.54a}
$$

$$
\begin{aligned}
&\left[\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\vee\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\\
&\Leftrightarrow\left[\left[\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\wedge\left[\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\right]\\
&\Leftrightarrow\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\left[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\wedge\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right]\\
&\Leftrightarrow\left[\left[\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right]\wedge\left[\mathsf{E}_1\langle\mathbf{v},\mathbf{u}\rangle\Rightarrow\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle\right]\right]\\
&\Leftrightarrow\left[\left[\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right]\wedge\left[\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right].
\end{aligned}
\tag{3.55a}
$$

Depending on the mental attitude of the interpreter, the trains of identities (3.52)–(3.55) can be understood either as trains of unspecified valid (kyrological) identities of A_1 or as trains of valid identities of $\mathbf{A}_1$, i.e. of valid panlogographic schemata of euautographic identities of A_1. In this case, for instance, the kyrology (3.55) of A_1 signifies, – or, from either of the two other viewpoints, the panlogographic kyrology schema (3.55) of $\mathbf{A}_1$ of euautographic kyrologies of A_1 or the panlogographic kyrology (3.55) of $\mathbf{A}_1$ signifies, – that the relations

$$V([\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\vee\mathsf{A}_1\langle\mathbf{v},\mathbf{u}\rangle]\Rightarrow\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\text{ and }V(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\Rightarrow[\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\wedge\mathsf{O}_1\langle\mathbf{v},\mathbf{u}\rangle])$$

are (i) kyrologies if $\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle$ and hence $\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle$ is a kyrology, (ii) antikyrologies if $\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle$ and hence $\bigvee_{\mathbf{y}}\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle$ is an antikyrology, or else (iii) udeterologies if otherwise.

The previous statement itself expresses a *conditional property* of PSJ’s and it illustrates how some *categorical properties* of PSJ’s can be used as secondary rules of inference of A_1 (cf. statements i–xii of Cmt 3.7). Similar *conditional* statements apply, *mutatis mutandis*, to (3.53)–(3.55). Th 3.8 that is stated and proved in the next subsection is another example of using relations of $\mathbf{A}_1$ as conditions on relations of A_1.•

3.2. Conditional properties of the PSJ’s

Preliminary Remark 3.1. The final expressions (validity indices) in the trains of relations (3.1)–(3.6) are schemata, belonging to $\mathbf{A}_1$, of the validity integrons of euautographic relations belonging to A_1. Each of the latter can be reduced either to 0 or to 1, or else to an algebraic form in *molecular validity integrons*, which cannot be reduced further either to 0 or to 1, i.e. which are *NNI’s* (*non-numeral integrons*). Therefore, any concrete euautographic relation of the range of a PSJ is either a kyrology (valid relation) or an antikyrology (antivalid relation), or else an udeterology (vav-neutral relation). The decision in favor of exactly one of the three classes can be made only upon the relation placeholder ‘**P**’ is either specified properly or particularized. The following syntactic theorem illustrates the dependence of the validity indices of ESJ’s as given by (3.1)–(3.6) on the validity indices of ‘$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$’ and ‘$\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle$’ in the general hypothetical form. This theorem can be regarded as an introduction into Aristotelian logic of A_1 that will be developed in section 5 of this chapter.•

Th 3.8. 1) If $V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,0$ then

$$\begin{gathered} V(\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle) \\ \hat{=}\,V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,0, \\ V(\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle) \\ \hat{=}\,V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,1, \\ \text{for each } n\,\vec{\in}\,\{1,2,3\}. \end{gathered} \tag{3.56}$$

That is to say, if $\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$ is a kyrology then $\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle$, $\neg\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle$, $\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle$, and $\neg\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle$ at $n\,\vec{\in}\,\{1,2,3\}$, and also $\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$, and $\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$ are kyrologies, whereas $\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle$, $\neg\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle$, $\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle$, and $\neg\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle$ at $n\,\vec{\in}\,\{1,2,3\}$, and also $\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$ and $\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$ are antikyrologies.

2) If $V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,1$ then

$$\begin{gathered} V(\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle) \\ \hat{=}\,V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,0, \\ V(\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle) \\ \hat{=}\,V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\,\hat{=}\,1, \\ \text{for each } n\,\vec{\in}\,\{1,2,3\}. \end{gathered} \tag{3.57}$$

That is to say, if $\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$ is an antikyrology then $\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle$, $\neg\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle$, $\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle$, and $\neg\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle$ at $n\,\vec{\in}\,\{1,2,3\}$, and also $\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$ and $\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$ are kyrologies, whereas $\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle$, $\neg\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle$, $\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle$, and $\neg\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle$ at $n\,\vec{\in}\,\{1,2,3\}$, and also $\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$ and $\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$ are antikyrologies.

Proof: By the algebraic negation law (ANL) (II.7.1γ) with '$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$' in place of '$\mathbf{P}$', it follows that

$$\begin{gathered} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,0 \text{ if and only if } V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,1\,\hat{-}\,V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,1, \\ V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,1 \text{ if and only if } V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,1\,\hat{-}\,V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,0. \end{gathered} \tag{3.58}$$

At the same time, from the definitions of the terms "kyrology" ("valid relation") and "antikyrology" ("antivalid relation"), it follows that, under Ax 2.1,

$$\begin{gathered} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,0 \text{ if and only if } V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle)\,\hat{=}\,0, \\ V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,1 \text{ if and only if } V(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle)\,\hat{=}\,1, \end{gathered} \tag{3.59}$$

and similarly with any ordered pair of APLOT's ⟨'**z**','**v**'⟩, ⟨'**z**','**w**'⟩, ⟨'**z**','**x**'⟩, etc in place of ⟨'**x**','**y**'⟩. Indeed, the relation condition '$V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{=}\,0$', e.g., is supposed to hold for any APVOT's of A_1 being accidental denotata (instances, interpretands) of the APLOT's '$\mathbf{z}$' and '$\mathbf{u}$' of A_1. However, all APLOT's '$\mathbf{u}$' to '$\mathbf{z}$', '$\mathbf{u}_1$' to '$\mathbf{z}_1$', '$\mathbf{u}_2$' to '$\mathbf{z}_2$', etc have the same range. Therefore, particularly,

$$\begin{gathered} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \hat{=} 0 \text{ if and only if } V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle) \hat{=} 0, \\ V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \hat{=} 1 \text{ if and only if } V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle) \hat{=} 1, \end{gathered} \tag{3.60}$$

Hence, it follows from (3.1)–(3.6) that:

1) If $V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \hat{=} V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle) \hat{=} 0$ and if, hence, $V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \hat{=} V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle) \hat{=} 1$

then

$$\begin{gathered} V(\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} \hat{\vee}_{\mathbf{z}}[1 \hat{-} 1 \hat{\vee} 0] \hat{=} \hat{\vee}_{\mathbf{z}} 1 \hat{=} 1, \\ V(\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} \hat{\vee}_{\mathbf{z}}[1 \hat{-} 1 \hat{\vee} 1] \hat{=} \hat{\vee}_{\mathbf{z}} 0 \hat{=} 0, \\ \text{for each } n \,\bar{\in}\, \{1,2,3\}, \end{gathered} \tag{3.56$_1$}$$

$$V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} \hat{\vee}_{\mathbf{z}}[1 \hat{\vee} 1] \hat{=} \hat{\vee}_{\mathbf{z}} 1 \hat{=} 1. \tag{3.56$_2$}$$

2) If $V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \hat{=} V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle) \hat{=} 1$ and if, hence, $V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \hat{=} V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle) \hat{=} 0$

then

$$\begin{gathered} V(\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{A}_n\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} \hat{\vee}_{\mathbf{z}}[1 \hat{-} 0 \hat{\vee} 1] \hat{=} \hat{\vee}_{\mathbf{z}} 1 \hat{=} 1, \\ V(\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{E}_n\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} \hat{\vee}_{\mathbf{z}}[1 \hat{-} 0 \hat{\vee} 0] \hat{=} \hat{\vee}_{\mathbf{z}} 1 \hat{=} 1, \\ \text{for each } n \,\bar{\in}\, \{1,2,3\}, \end{gathered} \tag{3.57$_1$}$$

$$V(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle) \hat{=} \hat{\vee}_{\mathbf{z}}[0 \hat{\vee} 0] \hat{=} \hat{\vee}_{\mathbf{z}} 0 \hat{=} 0. \tag{3.57$_2$}$$

QED.•

Ex 3.1. Either of the substitutions

$$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \rhd [\mathbf{Q}\langle\mathbf{z},\mathbf{u}\rangle \vee \neg\mathbf{Q}\langle\mathbf{z},\mathbf{u}\rangle], \tag{3.61}$$

$$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \rhd \bigvee_{\mathbf{x}_1} \neg[\mathbf{Q}\langle\mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle\mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle] \tag{3.62}$$

satisfies the condition $V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \hat{=} 0$, whereas either of the substitutions

$$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \rhd [\mathbf{Q}\langle\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle\mathbf{z},\mathbf{u}\rangle], \tag{3.63}$$

$$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \rhd \bigwedge_{\mathbf{x}_1} [\mathbf{Q}\langle\mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle\mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle] \tag{3.64}$$

satisfies the condition $V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \hat{=} 1$. In this case,

$$\mathbf{Q}\langle\mathbf{z},\mathbf{u}\rangle \vee \neg\mathbf{Q}\langle\mathbf{z},\mathbf{u}\rangle \tag{3.61$_1$}$$

is a version of the conventional form of the law of excluded middle,

$$\neg[\mathbf{Q}\langle\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle\mathbf{z},\mathbf{u}\rangle] \tag{3.63$_1$}$$

is one of a great many forms of the same law, and

$$\bigvee_{\mathbf{x}_1} \neg\left[\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right], \tag{3.62$_1$}$$

$$\neg \bigwedge_{\mathbf{x}_1} \left[\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right] \tag{3.64$_1$}$$

are two versions of the General Law of Denial of Russell's Paradox in A_1 and $\mathbf{A}_1$, which has been established in Th II.8.10. For more clarity, here follow the pertinent AEADP's:

$$\begin{gathered} V\left(\mathbf{Q}\langle \mathbf{z},\mathbf{u}\rangle \vee \neg\mathbf{Q}\langle \mathbf{z},\mathbf{u}\rangle\right) \hat{=} V\left(\mathbf{Q}\langle \mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} \left[1 \hat{-} V\left(\mathbf{Q}\langle \mathbf{z},\mathbf{u}\rangle\right)\right] \\ \hat{=} V\left(\mathbf{Q}\langle \mathbf{z},\mathbf{u}\rangle\right) \hat{-} V\left(\mathbf{Q}\langle \mathbf{z},\mathbf{u}\rangle\right) \hat{=} 0, \end{gathered} \tag{3.61$_2$}$$

$$\begin{gathered} V\left(\bigvee_{\mathbf{x}_1} \neg\left[\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right]\right) \\ \hat{=} \widehat{\cdot}_{\mathbf{x}_1} \left[1 \hat{-} \left[\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right]\right] \\ \hat{=} \widehat{\cdot}_{\mathbf{x}_1} \left[V\left(\neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right)\right] \\ \hat{=} \widehat{\cdot}_{\mathbf{x}_1} \left[V\left(\neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right)\right] \\ \hat{=} \left[V\left(\neg\mathbf{Q}\langle \mathbf{z},\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathbf{Q}\langle \mathbf{z},\mathbf{z},\mathbf{u}\rangle\right)\right] \\ \hat{\cdot} \left[\widehat{\cdot}_{\mathbf{x}_1} \left[V\left(\neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right)\right]\right] \\ \hat{=} 0 \hat{\cdot} \left[\widehat{\cdot}_{\mathbf{x}_1} \left[V\left(\neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right)\right]\right] \hat{=} 0, \end{gathered} \tag{3.62$_2$}$$

$$V\left(\mathbf{Q}\langle \mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle \mathbf{z},\mathbf{u}\rangle\right) \hat{=} 1 \hat{-} V\left(\neg\mathbf{Q}\langle \mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathbf{Q}\langle \mathbf{z},\mathbf{u}\rangle\right) \hat{=} 1, \tag{3.63$_2$}$$

$$\begin{gathered} V\left(\bigwedge_{\mathbf{x}_1} \left[\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right]\right) \\ \hat{=} V\left(\neg\bigvee_{\mathbf{x}_1} \neg\left[\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right]\right) \\ \hat{=} 1 \hat{-} V\left(\bigvee_{\mathbf{x}_1} \neg\left[\mathbf{Q}\langle \mathbf{x}_1,\mathbf{z},\mathbf{u}\rangle \wedge \neg\mathbf{Q}\langle \mathbf{x}_1,\mathbf{x}_1,\mathbf{u}\rangle\right]\right) \hat{=} 1. \end{gathered} \tag{3.64$_2$}\bullet$$

Cmt 3.12. 1) If $\mathbf{P}\langle \mathbf{z},\mathbf{u}\rangle$ is an udeterology then $\mathbf{P}\langle \mathbf{x},\mathbf{y}\rangle$ (e.g.) and particularly $\mathbf{P}\langle \mathbf{z},\mathbf{v}\rangle$ are also udeterologies, and vice versa. In this case, it can, for instance, happen that

$$\begin{aligned} & V\left(\bigvee_{\mathbf{x}} \neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right) \hat{=} \widehat{\cdot}_{\mathbf{x}} V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right) \\ & \hat{=} \widehat{\cdot}_{\mathbf{y}} V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right) \hat{=} V\left(\bigvee_{\mathbf{y}} \neg\mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right) \hat{=} 0, \end{aligned} \tag{3.65}$$

so that, equivalently,

$$\begin{aligned} & V\left(\bigwedge_{\mathbf{x}} \mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right) \hat{=} 1 \hat{-} \widehat{\cdot}_{\mathbf{x}} V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right) \\ & \hat{=} 1 \hat{-} \widehat{\cdot}_{\mathbf{y}} V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right) \hat{=} V\left(\bigwedge_{\mathbf{y}} \neg\mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right) \hat{=} 1. \end{aligned} \tag{3.66}$$

That is to say, $\bigvee_{\mathbf{x}} \neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle$ and hence $\bigvee_{\mathbf{y}} \neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle$ are kyrologies while, equivalently, $\bigwedge_{\mathbf{x}} \mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle$ and hence $\bigwedge_{\mathbf{y}} \mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle$ are antikyrologies. In this case, it follows from (3.5) and (3.6) that

$$V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1, \tag{3.67}$$

$$V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}0, \tag{3.68}$$

i.e. $\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$ and $\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$ are kyrologies while $\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$ and $\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$ are antikyrologies. In general, if $\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$ and hence $\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle$ are udeterologies then the validity index of any concrete euautographic relation in the range of $\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle$ or $\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle$ is determined by that concrete **P**, and it should therefore be calculated concretely.

2) There are infinitely many arbitrary specifications of the relation placeholder '**P**' (as (3.61)–(3.64) and many others, not necessarily valid or antivalid ones), which are not immediately relevant to Aristotelian logic. In addition and above all, '$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$' and '$\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle$' can be specified single-mindedly by substitution (2.30) so as to allow interpretation some pertinent BESJ's of A_1 by verbal syllogistic judgments (VSJ's) of Aristotelian logic. •

4. Validity indices of the BStPSJ's and BESJ's

4.1. Validity indices of the BStPSJ's

Preliminary Remark 4.1. In this section, I shall specify the subject matter of section 3 under substitutions (2.30)–(2.32) subject to Df 2.4(2) and develop the results thus obtained further when possible. The proviso: «subject to Df 2.4(2)» is self-evident and therefore it will not be mentioned in the sequel. The pertinent straightforward instances of theorems of section 3 will be stated as corollaries and hence without any proof. Still, alternatively, all the corollaries can be proved by repeating the proofs of their source theorems under Df 2.4(3) instead of Df 2.1, because Df 2.4(3) is the instance of Df 2.1 under substitutions (2.30)–(2.32).

All specific relations of $\mathbf{A}_1$, which are obtained in either of the above pure mechanical ways, are visually similar to their general precursors of section 3. However, semantic properties of the specific relations are essentially different from those of the general relations. First of all, the generic panlogographic placeholder '$\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle$', e.g., is an udeterology of $\mathbf{A}_1$ that may assume euautographic relations of A_1 of all three kinds: valid, antivalid, and vav-neutral, i.e. kyrologies, antikyrologies, and udeterologies. By contrast, the specific panlogographic placeholder '$\mathbf{F}(\mathbf{z},\mathbf{u})$' is a structural (detailed) panlogographic schema of $\mathbf{A}_1$ of binary euautographic ordinary relations (BEOR's) of A_1. That is to say, '$\mathbf{F}(\mathbf{z},\mathbf{u})$' *is a panlogographic udeterology of* $\mathbf{A}_1$ *that may assume only udeterologies of* A_1 *as its euautographic instances* (*interpretands*). Consequently, neither of the assumptions: «If $V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\hat{=}0$» and «If $V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\hat{=}1$», which I have made in stating Th 3.8, can be satisfied by the substitution: $\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \triangleright \mathbf{F}(\mathbf{z},\mathbf{u})$. Therefore, Th 3.8, cannot be specified by substitutions (2.30)–(2.32) at all. Instead, I shall illustrate the corollaries from other theorems of section 3 by concrete euautographic relations of A_1.•

Crl 4.1:** ***The instance of Th 3.1 subject to Df 2.4(3).

$$\begin{gathered}V(\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \\ \hat{=} V\left(\bigwedge_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})]\right) \hat{=} V\left(\neg \bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \neg \mathbf{F}(\mathbf{z},\mathbf{v})]\right) \\ \hat{=} V\left(\neg \bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \neg[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})]]\right) \\ \hat{=} 1 \dot{-} \dot{\wedge}_{\mathbf{z}} V([\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \neg \mathbf{F}(\mathbf{z},\mathbf{v})]) \hat{=} 1 \dot{-} \dot{\wedge}_{\mathbf{z}}[1 \dot{-} V(\neg \mathbf{F}(\mathbf{z},\mathbf{u})) \dot{\wedge} V(\mathbf{F}(\mathbf{z},\mathbf{v}))] \\ \text{for each } n \vec{\in} \{1,2,3\}.\end{gathered} \tag{4.1}$$

$$\begin{gathered}V(\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg \mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \\ \hat{=} V\left(\neg \bigwedge_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})]\right) \hat{=} V\left(\bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \neg \mathbf{F}(\mathbf{z},\mathbf{v})]\right) \\ \hat{=} V\left(\bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \neg[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})]]\right) \\ \hat{=} \dot{\wedge}_{\mathbf{z}} V(\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \neg \mathbf{F}(\mathbf{z},\mathbf{v})) \hat{=} \dot{\wedge}_{\mathbf{z}}[1 \dot{-} V(\neg \mathbf{F}(\mathbf{z},\mathbf{u})) \dot{\wedge} V(\mathbf{F}(\mathbf{z},\mathbf{v}))] \\ \text{for each } n \vec{\in} \{1,2,3\}.\end{gathered} \tag{4.2}$$

$$\begin{gathered}V(\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \\ \hat{=} V\left(\bigwedge_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \neg \mathbf{F}(\mathbf{z},\mathbf{v})]\right) \hat{=} V\left(\neg \bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \mathbf{F}(\mathbf{z},\mathbf{v})]\right) \\ \hat{=} V\left(\neg \bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge [\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})]]\right) \\ \hat{=} 1 \dot{-} \dot{\wedge}_{\mathbf{z}} V(\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \mathbf{F}(\mathbf{z},\mathbf{v})) \hat{=} 1 \dot{-} \dot{\wedge}_{\mathbf{z}}[1 \dot{-} V(\neg \mathbf{F}(\mathbf{z},\mathbf{u})) \dot{\wedge} V(\neg \mathbf{F}(\mathbf{z},\mathbf{v}))] \\ \text{for each } n \vec{\in} \{1,2,3,4\}.\end{gathered} \tag{4.3}$$

$$\begin{gathered}V(\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg \mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \\ \hat{=} V\left(\neg \bigwedge_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \neg \mathbf{F}(\mathbf{z},\mathbf{v})]\right) \hat{=} V\left(\bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \mathbf{F}(\mathbf{z},\mathbf{v})]\right) \\ \hat{=} V\left(\bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge [[\mathbf{F}(\mathbf{z},\mathbf{u}) \Rightarrow \mathbf{F}(\mathbf{z},\mathbf{v})]]]\right) \\ \hat{=} \dot{\wedge}_{\mathbf{z}} V(\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \mathbf{F}(\mathbf{z},\mathbf{v})) \hat{=} \dot{\wedge}_{\mathbf{z}}[1 \dot{-} V(\neg \mathbf{F}(\mathbf{z},\mathbf{u})) \dot{\wedge} V(\neg \mathbf{F}(\mathbf{z},\mathbf{v}))] \\ \text{for each } n \vec{\in} \{1,2,3,4\}.\end{gathered} \tag{4.4}$$

$$\begin{gathered}V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg \mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V\left(\bigwedge_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \mathbf{F}(\mathbf{z},\mathbf{v})]\right) \\ \hat{=} 1 \dot{-} \dot{\wedge}_{\mathbf{z}}[1 \dot{-} V(\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \mathbf{F}(\mathbf{z},\mathbf{v}))] \hat{=} 1 \dot{-} \dot{\wedge}_{\mathbf{z}}[V(\neg \mathbf{F}(\mathbf{z},\mathbf{u})) \dot{\wedge} V(\neg \mathbf{F}(\mathbf{z},\mathbf{v}))].\end{gathered} \tag{4.5}$$

$$\begin{gathered}V(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg \mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V\left(\bigvee_{\mathbf{z}} \neg[\mathbf{F}(\mathbf{z},\mathbf{u}) \wedge \mathbf{F}(\mathbf{z},\mathbf{v})]\right) \\ \hat{=} \dot{\wedge}_{\mathbf{z}}[V(\neg \mathbf{F}(\mathbf{z},\mathbf{u})) \dot{\wedge} V(\neg \mathbf{F}(\mathbf{z},\mathbf{v}))] \hat{=} \left[\dot{\wedge}_{\mathbf{z}_1} V(\neg \mathbf{F}(\mathbf{z}_1,\mathbf{u}))\right] \dot{\wedge} \left[\dot{\wedge}_{\mathbf{z}_2} V(\neg \mathbf{F}(\mathbf{z}_2,\mathbf{v}))\right] \\ \hat{=} V\left(\bigvee_{\mathbf{z}_1} \neg \mathbf{F}(\mathbf{z}_1,\mathbf{u})\right) \dot{\wedge} V\left(\bigvee_{\mathbf{z}_2} \neg \mathbf{F}(\mathbf{z}_2,\mathbf{v})\right).\end{gathered} \tag{4.6}$$
•

Cmt 4.1: ***A specification of Cmt 3.1 subject to Df 2.4(3).*** Just as in the general case, owing to (4.1)–(4.6), the only pertinent syllogistic figures to be studied are those which are formed with the help of the BStPSB'es (2.37) at $n \triangleright 1$ (e.g.) and at $n \triangleright 4$. Henceforth, I shall therefore, consider the left hand sides of identities (4.1)–(4.4) only at $n \triangleright 1$.•

***Th 4.1.** 1)

$$V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} 1, V(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} 0$$
$$\text{for each } \mathbf{F} \vec{\in} \mathrm{K}_{\in}^{\mathrm{pc}} \to \{\in,\subseteq,=,\subset\}, \tag{4.7}$$

subject to (2.36ε), (2.47), and (4.6).

2) The validity integrons

$$V(\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})), V(\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})), V(\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})), V(\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}))$$
$$\text{for each } \mathbf{F} \vec{\in} \mathrm{K}_{\in}^{2} \text{ and each } n \vec{\in} \{1,2,3\}, \tag{4.8}$$

$$V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})), V(\mathsf{E}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})), V(\mathsf{I}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})), V(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))$$
$$\text{for each } \mathbf{F} \vec{\in} \kappa^{2\mathrm{pv}}, \tag{4.9}$$

subject to (2.33)–(2.36) (or (2.33ε)–(2.36ε), when applicable), (2.45), (2.46), and (4.1)–(4.6), are *validity indices*, i.e. none of them can be reduced neither to 0 nor to 1. Accordingly, the pertinent BStPSJ's of $\mathbf{A}_1$, being operata of the operator V, and all BESJ's of A_1, being their instances (interpretands), are *udeterologies* (*neutral relations*).

Proof: According to theorems (IV.1.49), (IV.2.7), (IV.2.19), and (IV.2.21),

$$V\left(\bigvee_{\mathbf{z}} \neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \hat{=} 0 \text{ for each } \mathbf{F} \vec{\in} \mathrm{K}_{\in}^{\mathrm{pc}} \to \{\in,\subseteq,\subset,=\}. \tag{4.10}$$

Consequently, (4.5) and (4.6) yield (4.7). Item 2 of the theorem follows from the fact that '$V(\mathbf{F}(\mathbf{z},\mathbf{u}))$' and '$V(\mathbf{F}(\mathbf{z},\mathbf{v}))$' are udetorologies of $\mathbf{A}_{1\in}$, whose ranges are the sets of udetorologies of $\mathsf{A}_{1\in}$ thus patterned. QED.•

Cnv 4.1. In view of Th 4.1(1), the quadruples:

$$(\mathsf{A}_{\mathbf{F}4}(u,v), \mathsf{E}_{\mathbf{F}4}(u,v), \mathsf{I}_{\mathbf{F}4}(u,v), \mathsf{O}_{\mathbf{F}4}(u,v)) \text{ at } \mathbf{F} \vec{\in} \mathrm{K}_{\in}^{\mathrm{pc}}, \tag{4.11}$$

subject to (2.47), will hereafter be disregarded as symmetric BStPSB's or BESB's.•

Crl 4.2:** ***The instance of Th 3.2 subject to Df 2.4(3). For each $n \vec{\in} \{1,2,3\}$:

$$V(\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{z}|V(\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{=}0} V(\neg\mathbf{F}(\mathbf{z},\mathbf{v})) \hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{z}|V(\mathbf{F}(\mathbf{z},\mathbf{v}))\hat{=}1} V(\mathbf{F}(\mathbf{z},\mathbf{u})), \tag{4.12}$$

$$V(\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} \hat{\cdot}_{\mathbf{z}|V(\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{=}0} V(\neg\mathbf{F}(\mathbf{z},\mathbf{v})) \hat{=} \hat{\cdot}_{\mathbf{z}|V(\mathbf{F}(\mathbf{z},\mathbf{v}))\hat{=}1} V(\mathbf{F}(\mathbf{z},\mathbf{u})). \tag{4.13}$$

$$V(\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{z}|V(\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{=}0} V(\mathbf{F}(\mathbf{z},\mathbf{v})) \hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{z}|V(\mathbf{F}(\mathbf{z},\mathbf{v}))\hat{=}0} V(\mathbf{F}(\mathbf{z},\mathbf{u})), \tag{4.14}$$

$$V(\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \hat{=} \hat{\cdot}_{\mathbf{z}|V(\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{=}0} V(\mathbf{F}(\mathbf{z},\mathbf{v})) \hat{=} \hat{\cdot}_{\mathbf{z}|V(\mathbf{F}(\mathbf{z},\mathbf{v}))\hat{=}0} V(\mathbf{F}(\mathbf{z},\mathbf{u})). \tag{4.15}$$ •

*__Crl 4.3:__ ***The instance of Th 3.3 subject to Df 2.4(3).***

$$V(\mathsf{E}_{\mathbf{F}m}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{I}_{\mathbf{F}m}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{I}_{\mathbf{F}n}(\mathbf{v},\mathbf{u})) \hat{=} V(\mathsf{E}_{\mathbf{F}n}(\mathbf{v},\mathbf{u})),$$
$$V(\mathsf{I}_{\mathbf{F}m}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{E}_{\mathbf{F}m}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{E}_{\mathbf{F}n}(\mathbf{v},\mathbf{u})) \hat{=} V(\mathsf{I}_{\mathbf{F}n}(\mathbf{v},\mathbf{u})) \tag{4.16}$$
$$\text{for each } m \vec{\in} \{1,2,3,4\} \text{ and each } n \vec{\in} \{1,2,3,4\};$$

$$V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{O}_{\mathbf{F}4}(\mathbf{v},\mathbf{u})) \hat{=} V(\mathsf{A}_{\mathbf{F}4}(\mathbf{v},\mathbf{u})),$$
$$V(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{v},\mathbf{u})) \hat{=} V(\mathsf{O}_{\mathbf{F}4}(\mathbf{v},\mathbf{u})); \tag{4.17}$$

– in agreement with Crl 4.1.•

Cmt 4.2: ***A specification of Cmt 3.3 subject to Df 2.4(3).*** The trains of equivalences (4.16) and (4.17) can equivalently be stated as the following trains of equivalences:

$$\mathsf{E}_{\mathbf{F}m}(\mathbf{u},\mathbf{v}) \Leftrightarrow \neg\mathsf{I}_{\mathbf{F}m}(\mathbf{u},\mathbf{v}) \Leftrightarrow \neg\mathsf{I}_{\mathbf{F}n}(\mathbf{v},\mathbf{u}) \Leftrightarrow \mathsf{E}_{\mathbf{F}n}(\mathbf{v},\mathbf{u}),$$
$$\mathsf{I}_{\mathbf{F}m}(\mathbf{u},\mathbf{v}) \Leftrightarrow \neg\mathsf{E}_{\mathbf{F}m}(\mathbf{u},\mathbf{v}) \Leftrightarrow \neg\mathsf{E}_{\mathbf{F}n}(\mathbf{v},\mathbf{u}) \Leftrightarrow \mathsf{I}_{\mathbf{F}n}(\mathbf{v},\mathbf{u}), \tag{4.16a}$$
$$\text{for each } m \vec{\in} \{1,2,3,4\} \text{ and each } n \vec{\in} \{1,2,3,4\};$$

$$\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \Leftrightarrow \neg\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \Leftrightarrow \neg\mathsf{O}_{\mathbf{F}4}(\mathbf{v},\mathbf{u}) \Leftrightarrow \mathsf{A}_{\mathbf{F}4}(\mathbf{v},\mathbf{u}),$$
$$\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \Leftrightarrow \neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \Leftrightarrow \neg\mathsf{A}_{\mathbf{F}4}(\mathbf{v},\mathbf{u}) \Leftrightarrow \mathsf{O}_{\mathbf{F}4}(\mathbf{v},\mathbf{u}). \tag{4.17a}$$

Df 4.1: ***A specification of Df 3.1 subject to Df 2.4(3).*** 1) The BStPSJ's:

$$\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}) \text{ and } \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}) \text{ at } n \vec{\in} \{1,2,3\} \tag{4.18}$$

are called *asymmetric ones* in the sense that they are not invariant under the permutation of '**u**' and '**v**'. By contrast, all other BStPSJ's selected out of the list (2.37), i.e.

$$\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}) \text{ and } \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}) \text{ at } n \vec{\in} \{1,2,3,4\}, \text{ and } \mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \text{ and } \mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}), \tag{4.19}$$

are called *symmetric BStPSJ's* in the sense that they are *invariant* under the permutation of '**u**' and '**v**', in accordance with Crl 4.3. Consequently, any of the three BStPSB's given by (2.37) at $n \vec{\in} \{1,2,3\}$ is called an *asymmetric* one, while the BStPSB with $n \triangleright 4$ is called a *symmetric* one.

2) Any BESJ in the range of a given BStPSJ is called an *asymmetric BESJ* if the BStPSJ is an asymmetric one and a *symmetric BESJ* if the BStPSJ is a symmetric one. Any BESB in the range of a given BStPSB is called an *asymmetric BESB* if the BStPSB is an asymmetric one and a *symmetric BESB* if the BStPSB is a symmetric one.•

*__Crl 4.4:__ ***The instance of Th 3.4 subject to Df 2.4(3).***

$$V\left(\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})\right) \hat{=} V\left(\neg\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})\right) \hat{=} 0 \text{ for each } n \vec{\in} \{1,2,3\}. \tag{4.20}$$

$$V\left(\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})\right) \hat{=} V\left(\neg\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})\right) \hat{=} 1 \text{ for each } n \vec{\in} \{1,2,3\}. \tag{4.21}$$

$$\begin{gathered} V\left(\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})\right) \hat{=} V\left(\neg\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})\right) \\ \hat{=} 1 \hat{-} \hat{\sum}_{\mathbf{z}} V(\mathbf{F}(\mathbf{z},\mathbf{u})) \hat{=} V\left(\neg\bigvee_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u})\right) \text{ for each } n \vec{\in} \{1,2,3,4\}. \end{gathered} \tag{4.22}$$

$$\begin{gathered} V\left(\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})\right) \hat{=} V\left(\neg\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})\right) \\ \hat{=} \hat{\sum}_{\mathbf{z}} V(\mathbf{F}(\mathbf{z},\mathbf{u})) \hat{=} V\left(\bigvee_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u})\right) \text{ for each } n \vec{\in} \{1,2,3,4\}. \end{gathered} \tag{4.23}$$

$$V\left(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{u})\right) \hat{=} V\left(\neg\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{u})\right) \hat{=} 1 \hat{-} \hat{\sum}_{\mathbf{z}} V(\neg\mathbf{F}(\mathbf{z},\mathbf{u})) \hat{=} V\left(\bigwedge_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u})\right). \tag{4.24}$$

$$V\left(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{u})\right) \hat{=} V\left(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{u})\right) \hat{=} \hat{\sum}_{\mathbf{z}} V(\neg\mathbf{F}(\mathbf{z},\mathbf{u})) \hat{=} V\left(\bigvee_{\mathbf{z}}\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right). \tag{4.25}$$ •

Cmt 4.3: ***A specification of Cmt 3.4 subject to Df 2.4(3).*** By (4.20), all euautographic relations

$$\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{u}) \text{ and } \neg\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{u}) \text{ at } n \vec{\in} \{1,2,3\} \tag{4.26}$$

are theorems and hence kyrologies (valid relations) of A_1, while by (4.21), all euautographic relations

$$\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{u}) \text{ and } \neg\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{u}) \text{ at } n \vec{\in} \{1,2,3\} \tag{4.27}$$

are antitheorems and hence antikyrologies. At the same time, the ultimate values of the validity integrons:

$$V\left(\neg\bigvee_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u})\right),\ V\left(\bigvee_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u})\right),\ V\left(\bigwedge_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u})\right),\ V\left(\bigvee_{\mathbf{z}}\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right), \tag{4.28}$$

occurring in (4.22)–(4.25) respectively, depend on a particular euautographic interpretand of ‘**F**’ as follows.

a) Since no specific (atypical) axiom have been imposed on elements of the set κ^{2pv}, defined by (2.46), in any phase of $\mathsf{A}_{1\in}$, therefore for each $\mathbf{F} \vec{\in} \kappa^{2pv}$ none of the four validity integrons relations (4.28) reduces either to 0 or to 1, so that every ESR (euautographic slave-relation) of any one of the four integrons is a vav-neutral one, i.e. a udeterology.

b) By the pertinent version of (II.8.2), it follows from (4.10) that

$$\begin{gathered} V\left(\bigwedge_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u})\right) \hat{=} V\left(\neg\bigvee_{\mathbf{z}}\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \hat{=} 1 \hat{-} V\left(\bigvee_{\mathbf{z}}\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \hat{=} 1 \\ \text{for each } \mathbf{F} \vec{\in} \mathrm{K}_{\in}^{\mathrm{pc}} \to \{\in,\subseteq,\subset,=\}. \end{gathered} \tag{4.28$_1$}$$

By (4.10) and (4.28$_1$), identities (4.24) and (4.25) turn into (4.7) with **v**=**u**, i.e. into

$$V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{u})) \hat{=} V(\neg\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{u})) \hat{=} 1, V(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{u})) \hat{=} V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{u})) \hat{=} 0$$
$$\text{for each } \mathbf{F} \vec{\in} \mathrm{K}^{\mathrm{pc}}_{\in} \to \{\in,\subseteq,=,\subset\}. \tag{4.7$_1$}$$

c) By (IV.1.45) and (IV.1.47), it follows that

$$V(\vee_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u})) \hat{=} 0, V(\neg\vee_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u})) \hat{=} 1 \hat{-} V(\vee_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u})) \hat{=} 1$$
$$\text{for each } \mathbf{F} \vec{\in} \{\subseteq,=\}, \tag{4.28$_2$}$$

so that (4.22) and (4.23) become

$$V(\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})) \hat{=} V(\neg\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})) \hat{=} 1, V(\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})) \hat{=} V(\neg\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{u})) \hat{=} 0$$
$$\text{for each } \mathbf{F} \vec{\in} \{\subseteq,=\} \text{ and each } n \vec{\in} \{1,2,3,4\}. \tag{4.22$_1$}$$

At the same time, for each $\mathbf{F} \vec{\in} \{\in,\subset\}$, the validity integrons $V(\neg\vee_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u}))$ and $V(\vee_{\mathbf{z}}\mathbf{F}(\mathbf{z},\mathbf{u}))$ do not reduce either to 0 or to 1, and hence so do the trains (4.22) and (4.23).•

Crl 4.5:** ***The instance of Th 3.5 subject to Df 2.4(3).

$$\begin{aligned} &V(\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \neg[\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \neg\mathbf{F}(\mathbf{x},\mathbf{v})]) \\ &\hat{=} V(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{\cdot} V(\neg[\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \neg\mathbf{F}(\mathbf{x},\mathbf{v})]) \\ &\hat{=} V(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{\cdot} [V(\neg\mathbf{F}(\mathbf{x},\mathbf{u})) \hat{\cdot} V(\mathbf{F}(\mathbf{x},\mathbf{v}))] \hat{=} 0. \end{aligned} \tag{4.29}$$

$$\begin{aligned} &V([\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \neg\mathbf{F}(\mathbf{x},\mathbf{v})] \Rightarrow \mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \\ &\hat{=} V(\neg[\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \neg\mathbf{F}(\mathbf{x},\mathbf{v})]) \hat{\cdot} V(\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \\ &\hat{=} V(\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{\cdot} [V(\neg\mathbf{F}(\mathbf{x},\mathbf{u})) \hat{\cdot} V(\mathbf{F}(\mathbf{x},\mathbf{v}))] \hat{=} 0. \end{aligned} \tag{4.30}$$

$$\begin{aligned} &V(\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \neg[\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \mathbf{F}(\mathbf{x},\mathbf{v})]) \\ &\hat{=} V(\neg\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{\cdot} V(\neg[\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \mathbf{F}(\mathbf{x},\mathbf{v})]) \\ &\hat{=} V(\neg\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{\cdot} [V(\neg\mathbf{F}(\mathbf{x},\mathbf{u})) \hat{\cdot} V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))] \hat{=} 0. \end{aligned} \tag{4.31}$$

$$\begin{aligned} &V([\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \mathbf{F}(\mathbf{x},\mathbf{v})] \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \\ &\hat{=} V(\neg[\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \mathbf{F}(\mathbf{x},\mathbf{v})]) \hat{\cdot} V(\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \\ &\hat{=} V(\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{\cdot} [V(\neg\mathbf{F}(\mathbf{x},\mathbf{u})) \hat{\cdot} V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))] \hat{=} 0. \end{aligned} \tag{4.32}$$

$$\begin{aligned} &V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \Rightarrow [\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \mathbf{F}(\mathbf{x},\mathbf{v})]) \\ &\hat{=} V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{\cdot} V(\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \mathbf{F}(\mathbf{x},\mathbf{v})) \\ &\hat{=} V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{\cdot} [1 \hat{-} V(\neg\mathbf{F}(\mathbf{x},\mathbf{u})) \hat{\cdot} V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))] \hat{=} 0. \end{aligned} \tag{4.33}$$

$$\begin{aligned} &V(\neg[\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \mathbf{F}(\mathbf{x},\mathbf{v})] \Rightarrow \mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \\ &\hat{=} V(\mathbf{F}(\mathbf{x},\mathbf{u}) \wedge \mathbf{F}(\mathbf{x},\mathbf{v})) \hat{\cdot} V(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \\ &\hat{=} V(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{\cdot} [1 \hat{-} V(\neg\mathbf{F}(\mathbf{x},\mathbf{u})) \hat{\cdot} V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))] \hat{=} 0. \end{aligned} \tag{4.34}$$

$$\begin{aligned} &V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \Rightarrow \mathbf{F}(\mathbf{x},\mathbf{u})) \hat{=} V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}) \Rightarrow \mathbf{F}(\mathbf{x},\mathbf{v})) \\ &\hat{=} V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{\cdot} V(\mathbf{F}(\mathbf{x},\mathbf{u})) \hat{=} V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{\cdot} V(\mathbf{F}(\mathbf{x},\mathbf{v})) \hat{=} 0. \end{aligned} \tag{4.35}$$

$$\begin{aligned}&\mathsf{V}(\neg\mathbf{F}(\mathbf{x},\mathbf{u})\Rightarrow \mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))\,\hat{=}\,\mathsf{V}(\neg\mathbf{F}(\mathbf{x},\mathbf{v})\Rightarrow \mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))\\&\hat{=}\,\mathsf{V}(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))\,\hat{\cdot}\,\mathsf{V}(\mathbf{F}(\mathbf{x},\mathbf{u}))\,\hat{=}\,\mathsf{V}(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))\,\hat{\cdot}\,\mathsf{V}(\mathbf{F}(\mathbf{x},\mathbf{v}))\,\hat{=}\,0.\end{aligned} \tag{4.36}$$
•

Cmt 4.4: ***A specification of Cmt 3.5 subject to Df 2.4(3).*** By the pertinent versions of (II.4.40a), identities (4.29)–(4.36) can be restated as the corresponding equivalences, e.g.:

$$\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\Rightarrow\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})], \tag{4.29a}$$

$$[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})]\Rightarrow\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}), \tag{4.30a}$$

ec, which are the pertinent specifications of equivalences (3.29a), (3.30a), etc.•

Cmt 4.5. Semantic properties of identities (4.29)–(4.36) are completely different from semantic properties of the original identities (3.29)–(3.36). Namely, according to Th 4.1 and Cnv 4.1, all euautographic instances of the relation schemata '$\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})$', to '$\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})$', '$\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})$', '$\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})$', '$\mathbf{F}(\mathbf{x},\mathbf{u})$', and '$\mathbf{F}(\mathbf{x},\mathbf{v})$' are udeterologies (neutral relations) of $\mathsf{A}_{1\in}$. Therefore, no implications similar to statements i–xii of Cmt 3.7 can be deduced from identities (4.29)–(4.36). In this connection, it is noteworthy that under (4.7) each of the kyrological (valid) identities (4.33)–(4.36) turns into the kyrology $0\,\hat{=}\,0$.•

Cmt 4.6: ***A specification of Cmt 3.8 subject to Df 2.4(3).*** By (4.1)–(4.6), identities (4.29)–(4.34) can be rewritten as:

$$\begin{aligned}&\mathsf{V}(\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\Rightarrow\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})])\\&\hat{=}\,\mathsf{V}(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}))\,\hat{\cdot}\,\mathsf{V}(\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})])\\&\hat{=}\,\mathsf{V}(\neg\textstyle\bigwedge_{\mathbf{z}}\neg[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{z},\mathbf{v})])\,\hat{\cdot}\,\mathsf{V}(\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})])\\&\hat{=}\,\mathsf{V}([\textstyle\bigwedge_{\mathbf{z}}\neg[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{z},\mathbf{v})]]\Rightarrow\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})])\,\hat{=}\,0,\end{aligned} \tag{4.29'}$$

$$\begin{aligned}&\mathsf{V}([\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})]\Rightarrow\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}))\\&\hat{=}\,\mathsf{V}(\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})])\,\hat{\cdot}\,\mathsf{V}(\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}))\\&\hat{=}\,\mathsf{V}(\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})])\,\hat{\cdot}\,\mathsf{V}(\textstyle\bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{z},\mathbf{v})])\\&\hat{=}\,\mathsf{V}([\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})]\Rightarrow\textstyle\bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{z},\mathbf{v})])\,\hat{=}\,0,\end{aligned} \tag{3.30'}$$

$$\begin{aligned}&\mathsf{V}(\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\Rightarrow\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})])\\&\hat{=}\,\mathsf{V}(\neg\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}))\,\hat{\cdot}\,\mathsf{V}(\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})])\\&\hat{=}\,\mathsf{V}(\neg\textstyle\bigwedge_{\mathbf{z}}\neg[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\mathbf{F}(\mathbf{z},\mathbf{v})])\,\hat{\cdot}\,\mathsf{V}(\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})])\\&\hat{=}\,\mathsf{V}([\textstyle\bigwedge_{\mathbf{z}}\neg[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\mathbf{F}(\mathbf{z},\mathbf{v})]]\Rightarrow\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})])\,\hat{=}\,0,\end{aligned} \tag{3.31'}$$

$$V([\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})]\Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}))$$
$$\hat{=}V(\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})])\hat{\cdot}V(\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}))$$
$$\hat{=}V(\neg\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v}))\hat{\cdot}V(\bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\mathbf{F}(\mathbf{z},\mathbf{v})]) \tag{3.32'}$$
$$\hat{=}V([\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})]\Rightarrow \bigvee_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\mathbf{F}(\mathbf{z},\mathbf{v})])\hat{=}0,$$

$$V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\Rightarrow[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})])$$
$$\hat{=}V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))\hat{\cdot}V(\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v}))$$
$$\hat{=}V(\neg\bigwedge_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\mathbf{F}(\mathbf{z},\mathbf{v})])\hat{\cdot}V(\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})) \tag{3.33'}$$
$$\hat{=}V([\bigwedge_{\mathbf{z}}[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\mathbf{F}(\mathbf{z},\mathbf{v})]]\Rightarrow[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})])\hat{=}0.$$

$$V(\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})]\Rightarrow \mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))$$
$$\hat{=}V(\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v}))\hat{\cdot}V(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))$$
$$\hat{=}V(\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v}))\hat{\cdot}V(\bigvee_{\mathbf{z}}\neg[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\mathbf{F}(\mathbf{z},\mathbf{v})]) \tag{3.34'}$$
$$\hat{=}V(\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})]\Rightarrow \bigvee_{\mathbf{z}}\neg[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\mathbf{F}(\mathbf{z},\mathbf{v})])\hat{=}0,$$

which are the pertinent instances of (3.29')-(3.34').

2) Any one of the six relations (4.29')-(4.34') is an instance of a certain one of the four relations (3.39), (3.40), (3.39a), and (3.40a) under a certain one of the four pairs of substitutions:

$$\mathbf{Q}\langle\mathbf{z}\rangle\rhd\neg[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{z},\mathbf{v})],\mathbf{Q}\langle\mathbf{x}\rangle\rhd\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})], \tag{4.37$_1$}$$

$$\mathbf{Q}\langle\mathbf{z}\rangle\rhd[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{z},\mathbf{v})],\mathbf{Q}\langle\mathbf{x}\rangle\rhd[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{x},\mathbf{v})], \tag{4.37$_2$}$$

$$\mathbf{Q}\langle\mathbf{z}\rangle\rhd\neg[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\mathbf{F}(\mathbf{z},\mathbf{v})],\mathbf{Q}\langle\mathbf{x}\rangle\rhd\neg[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})], \tag{4.37$_3$}$$

$$\mathbf{Q}\langle\mathbf{z}\rangle\rhd[\mathbf{F}(\mathbf{z},\mathbf{u})\wedge\mathbf{F}(\mathbf{z},\mathbf{v})],\mathbf{Q}\langle\mathbf{x}\rangle\rhd[\mathbf{F}(\mathbf{x},\mathbf{u})\wedge\mathbf{F}(\mathbf{x},\mathbf{v})], \tag{4.37$_4$}$$

being the pertinent instances of substitutions (3.41$_1$)–(3.41$_4$) respectively. To be specific, relations (4.29'), (4.31'), and (4.33') are instances of (3.39) subject to (4.37$_1$), (4.37$_3$), and (4.37$_4$) respectively, while relations (4.30'), (4.32'), and (4.34') are instances of (3.40) subject to (4.37$_2$), (4.37$_4$), and (4.37$_3$) respectively.•

Crl 4.6:** ***The instance of Th 3.6 subject to Df 2.4(3).

$$V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\Rightarrow \mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}))\hat{=}V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\Rightarrow \mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u}))$$
$$\hat{=}V(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\Rightarrow[\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\wedge\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})])$$
$$\hat{=}V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))\hat{\cdot}V(\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}))\hat{=}V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))\hat{\cdot}V(\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})) \tag{4.38}$$
$$\hat{=}V(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v}))\hat{\cdot}[1\hat{-}V(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}))\hat{\cdot}V(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u}))]\hat{=}0,$$

$$
\begin{aligned}
V\left(\mathsf{O}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) &\mathrel{\hat{=}} V\left(\mathsf{O}_{\mathbf{F1}}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{v},\mathbf{u})\right) \\
&\mathrel{\hat{=}} V\left(\left[\mathsf{O}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \vee \mathsf{O}_{\mathbf{F1}}(\mathbf{v},\mathbf{u})\right] \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\mathrel{\hat{=}} V\left(\mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \hat{\cdot} V\left(\neg\mathsf{O}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right) \mathrel{\hat{=}} V\left(\mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \hat{\cdot} V\left(\neg\mathsf{O}_{\mathbf{F1}}(\mathbf{v},\mathbf{u})\right) \\
&\mathrel{\hat{=}} V\left(\mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \hat{\cdot} \left[1 \hat{-} V\left(\mathsf{O}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right) \hat{\cdot} V\left(\mathsf{O}_{\mathbf{F1}}(\mathbf{v},\mathbf{u})\right)\right] \mathrel{\hat{=}} 0,
\end{aligned}
\tag{4.39}
$$

$$
V\left(\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{I}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right) \mathrel{\hat{=}} V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \hat{\cdot} V\left(\mathsf{I}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right) \mathrel{\hat{=}} 0\,, \tag{4.40}
$$

$$
V\left(\mathsf{E}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \mathrel{\hat{=}} V\left(\mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \hat{\cdot} V\left(\neg\mathsf{E}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right) \mathrel{\hat{=}} 0\,, \tag{4.41}
$$

which are the pertinent instances of identities (3.46)–(3.49) respectively.•

Cmt 4.7: ***A specification of Cmt 3.10 subject to Df 2.4(3).*** 1) Identities (4.38)–(4.41) are equivalent to the equivalences:

$$
\begin{gathered}
\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{A}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}),\, \mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{A}_{\mathbf{F1}}(\mathbf{v},\mathbf{u}), \\
\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \left[\mathsf{A}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \wedge \mathsf{A}_{\mathbf{F1}}(\mathbf{v},\mathbf{u})\right],
\end{gathered}
\tag{4.38a}
$$

$$
\begin{gathered}
\mathsf{O}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}),\, \mathsf{O}_{\mathbf{F1}}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}), \\
\left[\mathsf{O}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \vee \mathsf{O}_{\mathbf{F1}}(\mathbf{v},\mathbf{u})\right] \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}),
\end{gathered}
\tag{4.39a}
$$

$$
\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{I}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}), \tag{4.40a}
$$

$$
\mathsf{E}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}), \tag{4.41a}
$$

which are the pertinent specifications of equivalences (3.46a)–(3.49a).

2) By (4.38) and (4.39), it follows that

$$
\begin{aligned}
&V\left(\left[\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{A}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right] \wedge \left[\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{A}_{\mathbf{F1}}(\mathbf{v},\mathbf{u})\right]\right) \\
&\mathrel{\hat{=}} 1 \hat{-} V\left(\neg\left[\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{A}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right]\right) \hat{\cdot} V\left(\neg\left[\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{A}_{\mathbf{F1}}(\mathbf{v},\mathbf{u})\right]\right) \\
&\mathrel{\hat{=}} 1 \hat{-} \left[1 \hat{-} V\left(\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{A}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right)\right] \hat{\cdot} \left[1 \hat{-} V\left(\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{A}_{\mathbf{F1}}(\mathbf{v},\mathbf{u})\right)\right] \\
&\mathrel{\hat{=}} 1 \hat{-} 1 \hat{\cdot} 1 \mathrel{\hat{=}} 0,
\end{aligned}
\tag{4.42}
$$

$$
\begin{aligned}
&V\left(\left[\mathsf{O}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right] \wedge \left[\mathsf{O}_{\mathbf{F1}}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{v},\mathbf{u})\right]\right) \\
&\mathrel{\hat{=}} 1 \hat{-} V\left(\neg\left[\mathsf{O}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right]\right) \hat{\cdot} V\left(\neg\left[\mathsf{O}_{\mathbf{F1}}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{v},\mathbf{u})\right]\right) \\
&\mathrel{\hat{=}} 1 \hat{-} \left[1 \hat{-} V\left(\mathsf{O}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right)\right] \hat{\cdot} \left[1 \hat{-} V\left(\mathsf{O}_{\mathbf{F1}}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{v},\mathbf{u})\right)\right] \\
&\mathrel{\hat{=}} 1 \hat{-} 1 \hat{\cdot} 1 \mathrel{\hat{=}} 0,
\end{aligned}
\tag{4.43}
$$

or, equivalently,

$$
\left[\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{A}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right] \wedge \left[\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{A}_{\mathbf{F1}}(\mathbf{v},\mathbf{u})\right], \tag{4.42a}
$$

$$
\left[\mathsf{O}_{\mathbf{F1}}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right] \wedge \left[\mathsf{O}_{\mathbf{F1}}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F4}}(\mathbf{v},\mathbf{u})\right]; \tag{4.43a}
$$

(4.42), (4.43), (4.42a), and (4.43a) are the pertinent instances of (3.50), (3.51), (3.50a), and (3.51a) respectively.

3) The validity indices of the converses of the separate relations (4.38a)–(4.41a) can be calculated in the same way. Still, these calculations are redundant

because it is clear without any calculations that, according to (4.16), (4.17), and (4.20)–(4.25), the antecedent and the consequent of any of those relations have different properties and therefore they cannot be equivalent.•

Crl 4.7:**The instance of Th 3.7 subject to Df 2.4(3)*****.**

$$\begin{aligned} &V\left(\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right) \hat{=} V\left(\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right) \\ &\hat{=} \overset{\frown}{\cdot}_{\mathbf{x}} V(\mathbf{F}(\mathbf{x},\mathbf{u})) \hat{=} V\left(\bigvee_{\mathbf{x}} \mathbf{F}(\mathbf{x},\mathbf{u})\right), \end{aligned} \tag{4.44}$$

$$\begin{aligned} &V\left(\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right) \hat{=} V\left(\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right) \\ &\hat{=} \overset{\frown}{\cdot}_{\mathbf{y}} V(\mathbf{F}(\mathbf{y},\mathbf{v})) \hat{=} V\left(\bigvee_{\mathbf{y}} \mathbf{F}(\mathbf{y},\mathbf{v})\right), \end{aligned} \tag{4.45}$$

$$\begin{aligned} &V\left(\left[\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \wedge \mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right] \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right) \\ &\hat{=} V\left(\left[\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right] \vee \left[\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right]\right) \\ &\hat{=} V\left(\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \left[\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \vee \mathsf{O}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right]\right) \\ &\hat{=} V\left(\left[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right] \vee \left[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right]\right) \\ &\hat{=} \left[\overset{\frown}{\cdot}_{\mathbf{x}} V(\mathbf{F}(\mathbf{x},\mathbf{u}))\right] \hat{\cdot} \left[\overset{\frown}{\cdot}_{\mathbf{y}} V(\mathbf{F}(\mathbf{y},\mathbf{v}))\right] \hat{=} \left[\overset{\frown}{\cdot}_{\mathbf{x}} \left[V(\mathbf{F}(\mathbf{x},\mathbf{u})) \hat{\cdot} V(\mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right] \\ &\hat{=} V\left(\left[\bigvee_{\mathbf{x}} \mathbf{F}(\mathbf{x},\mathbf{u})\right] \vee \left[\bigvee_{\mathbf{y}} \mathbf{F}(\mathbf{y},\mathbf{v})\right]\right) \hat{=} V\left(\bigvee_{\mathbf{x}} \left[\mathbf{F}(\mathbf{x},\mathbf{u}) \vee \mathbf{F}(\mathbf{x},\mathbf{v})\right]\right), \end{aligned} \tag{4.46}$$

$$\begin{aligned} &V\left(\left[\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \vee \mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right] \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right) \\ &\hat{=} V\left(\left[\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right] \wedge \left[\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right]\right) \\ &\hat{=} V\left(\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \left[\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \wedge \mathsf{O}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right]\right) \\ &\hat{=} V\left(\left[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right] \wedge \left[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right]\right) \\ &\hat{=} 1 \hat{-} \left[1 \hat{-} \overset{\frown}{\cdot}_{\mathbf{x}} V(\mathbf{F}(\mathbf{x},\mathbf{u}))\right] \hat{\cdot} \left[1 \hat{-} \overset{\frown}{\cdot}_{\mathbf{y}} V(\mathbf{F}(\mathbf{y},\mathbf{v}))\right] \\ &\hat{=} V\left(\left[\bigvee_{\mathbf{x}} \mathbf{F}(\mathbf{x},\mathbf{u})\right] \wedge \left[\bigvee_{\mathbf{y}} \mathbf{F}(\mathbf{y},\mathbf{v})\right]\right), \end{aligned} \tag{4.47}$$

which are the pertinent instances of identities (3.52)–(3.55) respectively.•

Cmt 4.8: ***A specification of Cmt 3.11 subject to Df 2.4(3)*****.** Equivalences (3.52a)–(3.55a) reduce to:

$$\left[\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right] \Leftrightarrow \left[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right] \Leftrightarrow \bigvee_{\mathbf{z}} \mathbf{F}(\mathbf{z},\mathbf{u}), \tag{4.44a}$$

$$\left[\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right] \Leftrightarrow \left[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right] \Leftrightarrow \bigvee_{\mathbf{z}} \mathbf{F}(\mathbf{z},\mathbf{v}), \tag{4.45a}$$

$$\begin{aligned} &\left[\left[\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \wedge \mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right] \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right] \\ &\Leftrightarrow \left[\left[\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right] \vee \left[\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u}) \Rightarrow \mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right]\right] \\ &\Leftrightarrow \left[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \left[\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \vee \mathsf{O}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right]\right] \\ &\Leftrightarrow \left[\left[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right] \vee \left[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v}) \Rightarrow \mathsf{O}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\right]\right] \\ &\Leftrightarrow \left[\left[\bigvee_{\mathbf{x}} \mathbf{F}(\mathbf{x},\mathbf{u})\right] \vee \left[\bigvee_{\mathbf{y}} \mathbf{F}(\mathbf{y},\mathbf{v})\right]\right] \Leftrightarrow \bigvee_{\mathbf{x}} \left[\mathbf{F}(\mathbf{x},\mathbf{u}) \vee \mathbf{F}(\mathbf{x},\mathbf{v})\right], \end{aligned} \tag{4.46a}$$

$$\begin{aligned}&[[\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\vee\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})]\Rightarrow\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})]\\ &\Leftrightarrow[[\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\Rightarrow\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})]\wedge[\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})\Rightarrow\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})]]\\ &\Leftrightarrow[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\Rightarrow[\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\wedge\mathsf{O}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})]]\\ &\Leftrightarrow[[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\Rightarrow\mathsf{O}_{1}\langle\mathbf{u},\mathbf{v}\rangle]\wedge[\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\Rightarrow\mathsf{O}_{\mathbf{F}1}(\mathbf{v},\mathbf{u})]]\\ &\Leftrightarrow[[\bigvee_{\mathbf{x}}\mathbf{F}(\mathbf{x},\mathbf{u})]\wedge[\bigvee_{\mathbf{y}}\mathbf{F}(\mathbf{y},\mathbf{v})]]\end{aligned}\qquad(4.47\text{a})\bullet$$

4.2. Validity indices of the BESJ's

°**Crl 4.8:** ***Particularization of PLMT's (4.1)–(4.6) for any*** $\mathbf{F}\triangleright\mathbf{F}^{\mathrm{pv}}\mathrel{\vec\in}\kappa^{2\mathrm{pv}}$**.** The following identities are the results of the simultaneous substitution $\mathbf{F}\triangleright f^2$ and (2.52) throughout (4.1)–(4.6):

$$\begin{aligned}&V(\mathsf{A}_{f^2n}(u,v))\mathrel{\hat=}V(\neg\mathsf{O}_{f^2n}(u,v))\\ &\mathrel{\hat=}V(\bigwedge_z[f^2(z,u)\Rightarrow f^2(z,v)])\mathrel{\hat=}V(\bigwedge_z\neg[f^2(z,u)\wedge\neg f^2(z,v)])\\ &\mathrel{\hat=}1\mathrel{\hat-}\hat{\cdot}_z[1\mathrel{\hat-}V(\neg f^2(z,u))\mathrel{\hat\cdot}V(f^2(z,u))]\text{ for each } n\mathrel{\vec\in}\{1,2,3\},\end{aligned}\qquad(4.1\mu_0)$$

$$\begin{aligned}&V(\mathsf{O}_{f^2n}(u,v))\mathrel{\hat=}V(\neg\mathsf{A}_{f^2n}(u,v))\\ &\mathrel{\hat=}V(\neg\bigwedge_z[f^2(z,u)\Rightarrow f^2(z,v)])\mathrel{\hat=}V(\bigvee_z[f^2(z,u)\wedge\neg f^2(z,v)])\\ &\mathrel{\hat=}\hat{\cdot}_z[1\mathrel{\hat-}V(\neg f^2(z,u))\mathrel{\hat\cdot}V(f^2(z,v))]\text{ for each } n\mathrel{\vec\in}\{1,2,3\},\end{aligned}\qquad(4.2\mu_0)$$

$$\begin{aligned}&V(\mathsf{E}_{f^2n}(u,v))\mathrel{\hat=}V(\neg\mathsf{I}_{f^2n}(u,v))\mathrel{\hat=}V(\bigwedge_z\neg[f^2(z,u)\wedge f^2(z,v)])\\ &\mathrel{\hat=}1\mathrel{\hat-}\hat{\cdot}_z[1\mathrel{\hat-}V(\neg f^2(z,u))\mathrel{\hat\cdot}V(\neg f^2(z,v))]\text{ for each } n\mathrel{\vec\in}\{1,2,3,4\},\end{aligned}\qquad(4.3\mu_0)$$

$$\begin{aligned}&V(\mathsf{I}_{f^2n}(u,v))\mathrel{\hat=}V(\neg\mathsf{E}_{f^2n}(u,v))\mathrel{\hat=}V(\bigvee_z[f^2(z,u)\wedge f^2(z,v)])\\ &\mathrel{\hat=}\hat{\cdot}_z[1\mathrel{\hat-}V(\neg f^2(z,u))\mathrel{\hat\cdot}V(\neg f^2(z,v))]\text{ for each } n\mathrel{\vec\in}\{1,2,3,4\},\end{aligned}\qquad(4.4\mu_0)$$

$$\begin{aligned}&V(\mathsf{A}_{f^24}(u,v))\mathrel{\hat=}V(\neg\mathsf{O}_{f^24}(u,v))\mathrel{\hat=}V(\bigwedge_{\mathbf{z}}[f^2(z,u)\wedge f^2(z,u)])\\ &\mathrel{\hat=}1\mathrel{\hat-}\hat{\cdot}_z[V(\neg f^2(z,u))\mathrel{\hat\cdot}V(\neg f^2(z,u))],\end{aligned}\qquad(4.5\mu_0)$$

$$\begin{aligned}&V(\mathsf{O}_{f^24}(u,v))\mathrel{\hat=}V(\neg\mathsf{I}_{f^24}(u,v))\mathrel{\hat=}V(\bigvee_z\neg[f^2(z,u)\wedge f^2(z,u)])\\ &\mathrel{\hat=}\hat{\cdot}_z[V(\neg f^2(z,u))\mathrel{\hat\cdot}V(\neg f^2(z,u))]\\ &\mathrel{\hat=}[\hat{\cdot}_{z_1}V(\neg f^2(z_1,u))]\mathrel{\hat\cdot}[\hat{\cdot}_{z_2}V(\neg f^2(z_2,v))]\\ &\mathrel{\hat=}V(\bigvee_{z_1}\neg f^2(z_1,u))\mathrel{\hat\cdot}V(\bigvee_{z_2}\neg f^2(z_2,v)),\end{aligned}\qquad(4.6\mu_0)$$

and similarly with any $\mathbf{F}\mathrel{\vec\in}\kappa^{2\mathrm{pv}}$ subject to (2.46) in place of f^2. Like any one of the trains of identities (4.1)–(4.6), the respective one of the trains (4.1μ₀)–(4.6μ₀) and its any variant with $\mathbf{F}\mathrel{\vec\in}\kappa^{2\mathrm{pv}}$ are not reducible either to 0 or to 1.•

°**Crl 4.9:** ***Particularization of PLMT's (4.1)–(4.4) for each*** $\mathbf{F}\,\vec{\in}\,\{\in,\subseteq,=,\subset\}$**.**

1) In accordance with (2.50) and (2.51), PLMT's (4.1)–(4.4) become for each $n\,\vec{\in}\,\{1,2,3\}$:

$$\begin{gathered} V(\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}))\hat{=}V(\neg\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \\ \hat{=}V(\bigwedge\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\Rightarrow[\mathbf{zFv}]])\hat{=}V(\neg\bigvee\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\wedge\neg[\mathbf{zFv}]]) \\ \hat{=}V(\neg\bigvee\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\wedge\neg[[\mathbf{zFu}]\Rightarrow[\mathbf{zFv}]]]) \\ \hat{=}1\hat{-}\hat{\vee}_{\mathbf{z}}V([\mathbf{zFu}]\wedge\neg[\mathbf{zFv}])\hat{=}1\hat{-}\hat{\vee}_{\mathbf{z}}[1\hat{-}V(\neg[\mathbf{zFu}])\hat{\cdot}V(\mathbf{zFv})], \end{gathered} \tag{4.1ε}$$

$$\begin{gathered} V(\mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}))\hat{=}V(\neg\mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \\ \hat{=}V(\neg\bigwedge\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\Rightarrow[\mathbf{zFv}]])\hat{=}V(\bigvee\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\wedge\neg[\mathbf{zFv}]]) \\ \hat{=}V(\bigvee\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\wedge\neg[[\mathbf{zFu}]\Rightarrow[\mathbf{zFv}]]]) \\ \hat{=}\hat{\vee}_{\mathbf{z}}V([\mathbf{zFu}]\wedge\neg[\mathbf{zFv}])\hat{=}\hat{\vee}_{\mathbf{z}}[1\hat{-}V(\neg[\mathbf{zFu}])\hat{\cdot}V(\mathbf{zFv})], \end{gathered} \tag{4.2ε}$$

$$\begin{gathered} V(\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}))\hat{=}V(\neg\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \\ \hat{=}V(\bigwedge\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\Rightarrow\neg[\mathbf{zFv}]])\hat{=}V(\neg\bigvee\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\wedge[\mathbf{zFv}]]) \\ \hat{=}V(\neg\bigvee\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\wedge[[\mathbf{zFu}]\Rightarrow[\mathbf{zFv}]]]) \\ \hat{=}1\hat{-}\hat{\vee}_{\mathbf{z}}V([\mathbf{zFu}]\wedge[\mathbf{zFv}])\hat{=}1\hat{-}\hat{\vee}_{\mathbf{z}}[1\hat{-}V(\neg[\mathbf{zFu}])\hat{\cdot}V(\neg[\mathbf{zFv}])], \end{gathered} \tag{4.3ε}$$

$$\begin{gathered} V(\mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v}))\hat{=}V(\neg\mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})) \\ \hat{=}V(\neg\bigwedge\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\Rightarrow\neg[\mathbf{zFv}]])\hat{=}V(\bigvee\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\wedge[\mathbf{zFv}]]) \\ \hat{=}V(\bigvee\nolimits_{\mathbf{z}}[[\mathbf{zFu}]\wedge[[[\mathbf{zFu}]\Rightarrow[\mathbf{zFv}]]]]) \\ \hat{=}\hat{\vee}_{\mathbf{z}}V([\mathbf{zFu}]\wedge[\mathbf{zFv}])\hat{=}\hat{\vee}_{\mathbf{z}}[1\hat{-}V(\neg[\mathbf{zFu}])\hat{\cdot}V(\neg[\mathbf{zFv}])]. \end{gathered} \tag{4.4ε}$$

2) In accordance with Th 4.1 and Cnv 4.1, the case of n=1 is disregarded.•

°**Crl 4.10:** ***Particularization of PLMT's (4.1ε)–(4.4ε) for any*** $\mathbf{F}\triangleright\mathbf{F}^{\mathrm{pc}}\,\vec{\in}\,\mathrm{K}_{\in}^{\mathrm{pc}}$**.** For each $n\,\vec{\in}\,\{1,2,3\}$, the following identities are the results of the simultaneous substitution $\mathbf{F}\triangleright\in$ and (2.52) throughout (4.1ε)–(4.4ε):

$$\begin{gathered} V(\mathsf{A}_{\in n}(u,v))\hat{=}V(\neg\mathsf{O}_{\in n}(u,v)) \\ \hat{=}V(\bigwedge\nolimits_{z}[[z\in u]\Rightarrow[z\in v]])\hat{=}V(\neg\bigvee\nolimits_{z}[[z\in u]\wedge\neg[z\in v]]) \\ \hat{=}V(\neg\bigvee\nolimits_{z}[[z\in u]\wedge\neg[[z\in u]\Rightarrow[z\in v]]]) \\ \hat{=}1\hat{-}\hat{\vee}_{z}V([z\in u]\wedge\neg[z\in v])\hat{=}1\hat{-}\hat{\vee}_{z}[1\hat{-}V(\neg[z\in u])\hat{\cdot}V(z\in v)], \end{gathered} \tag{4.1μ₁}$$

$$\begin{gathered} V(\mathsf{O}_{\in n}(u,v))\hat{=}V(\neg\mathsf{A}_{\in n}(u,v)) \\ \hat{=}V(\neg\bigwedge\nolimits_{z}[[z\in u]\Rightarrow[z\in v]])\hat{=}V(\bigvee\nolimits_{\mathbf{z}}[[z\in u]\wedge\neg[z\in v]]) \\ \hat{=}V(\bigvee\nolimits_{z}[[z\in u]\wedge\neg[[z\in u]\Rightarrow[z\in v]]]) \\ \hat{=}\hat{\vee}_{z}V([z\in u]\wedge\neg[z\in v])\hat{=}\hat{\vee}_{z}[1\hat{-}V(\neg[z\in u])\hat{\cdot}V(z\in v)], \end{gathered} \tag{4.2μ₁}$$

$$
\begin{gathered}
V\left(\mathsf{E}_{\in n}(u,v)\right)\hat{=}V\left(\neg\mathsf{I}_{\in n}(u,v)\right)\\
\hat{=}V\left(\bigwedge_z\left[[z\in u]\Rightarrow\neg[z\in v]\right]\right)\hat{=}V\left(\neg\bigvee_z\left[[z\in u]\wedge[z\in v]\right]\right)\\
\hat{=}V\left(\neg\bigvee_z\left[[z\in u]\wedge\left[[z\in u]\Rightarrow[z\in v]\right]\right]\right)\\
\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_z V\left([z\in u]\wedge[z\in v]\right)\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_z\left[1\hat{-}V\left(\neg[z\in u]\right)\hat{\cdot}V\left(\neg[z\in v]\right)\right],
\end{gathered}
\tag{4.3μ1}
$$

$$
\begin{gathered}
V\left(\mathsf{I}_{\in n}(u,v)\right)\hat{=}V\left(\neg\mathsf{E}_{\in n}(u,v)\right)\\
\hat{=}V\left(\neg\bigwedge_z\left[[z\in u]\Rightarrow\neg[z\in v]\right]\right)\hat{=}V\left(\bigvee_z\left[[z\in u]\wedge[z\in v]\right]\right)\\
\hat{=}V\left(\bigvee_z\left[[z\in u]\wedge\left[[z\in u]\Rightarrow[z\in v]\right]\right]\right)\\
\hat{=}\hat{\boldsymbol{\cdot}}_z V\left([z\in u]\wedge[z\in v]\right)\hat{=}\hat{\boldsymbol{\cdot}}_z\left[1\hat{-}V\left(\neg[z\in u]\right)\hat{\cdot}V\left(\neg[z\in v]\right)\right];
\end{gathered}
\tag{4.4μ1}
$$

and similarly with any of the three predicate-signs $\subseteq$, $=$, and $\subset$ in place of $\in$. Like any one of the trains of identities (4.1ε)–(4.4ε), the respective one of the trains ($4.1\mu_1$)–($4.6\mu_1$) and its any variant with $\mathbf{F}\,\vec{\in}\,\{\subseteq,=,\subset\}$ are not reducible either to 0 or to 1.•

Cmt 4.9. By definition (IV.1.3), it follows from EMT's ($4.1\mu_1$) and ($4.2\mu_1$) that

$$V\left(\mathsf{A}_{\in n}(u,v)\right)\hat{=}V(u\subseteq v),\tag{4.1μ1+}$$

$$V\left(\mathsf{O}_{\in n}(u,v)\right)\hat{=}V\left(\neg\mathsf{A}_{\in n}(u,v)\right)\hat{=}1\hat{-}V(u\subseteq v),\tag{4.2μ1+}$$

in agreement with (2.64).•

°**Th 4.2.** For each $n\,\vec{\in}\,\{1,2,3\}$:

$$
\begin{gathered}
V\left(\mathsf{A}_{\in n}(\emptyset,v)\right)\hat{=}V\left(\neg\mathsf{O}_{\in n}(\emptyset,v)\right)\hat{=}V(\emptyset\subseteq v)\\
\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_z\left[1\hat{-}V\left(\neg[z\in\emptyset]\right)\hat{\cdot}V(z\in v)\right]\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_z\left[1\hat{-}0\hat{\cdot}V(z\in v)\right]\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_z 1\hat{=}0,
\end{gathered}
\tag{4.1μ2}
$$

$$
\begin{gathered}
V\left(\mathsf{O}_{\in n}(\emptyset,v)\right)\hat{=}V\left(\neg\mathsf{A}_{\in n}(\emptyset,v)\right)\hat{=}V\left(\neg[\emptyset\subseteq v]\right)\\
\hat{=}1\hat{-}V\left(\mathsf{A}_{\in n}(\emptyset,v)\right)\hat{=}1\hat{-}0\hat{=}1,
\end{gathered}
\tag{4.2μ2}
$$

$$
\begin{gathered}
V\left(\mathsf{E}_{\in n}(\emptyset,v)\right)\hat{=}V\left(\neg\mathsf{I}_{\in n}(\emptyset,v)\right)\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_z\left[1\hat{-}V\left(\neg[z\in\emptyset]\right)\hat{\cdot}V\left(\neg[z\in v]\right)\right]\\
\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_z\left[1\hat{-}0\hat{\cdot}V\left(\neg[z\in v]\right)\right]\hat{=}1\hat{-}\hat{\boldsymbol{\cdot}}_z 1\hat{=}0,
\end{gathered}
\tag{4.3μ2}
$$

$$V\left(\mathsf{I}_{\in n}(\emptyset,v)\right)\hat{=}V\left(\neg\mathsf{E}_{\in n}(\emptyset,v)\right)\hat{=}1\hat{-}V\left(\mathsf{E}_{\in n}(\emptyset,v)\right)\hat{=}1\hat{-}0\hat{=}1,\tag{4.4μ2}$$

and similarly with $\subset$ in place of $\in$.

Proof: In developing the final results in ($4.1\mu_2$) and ($4.3\mu_2$), use of the identity

$$V\left(\neg[z\in\emptyset]\right)\hat{=}0,\tag{4.48}$$

being the pertinent instance of identity (IV.3.8), has been made. At the same time, the identity

$$V(z\sqsupset\emptyset)\hat{=}V\left(\neg[z\subset\emptyset]\right)\hat{=}0\tag{4.49}$$

is an analogous instance of identity (IV.3.10). Therefore, the variants of identities with are also identities (valid equalities). By contrast, neither of the validity integrons $V(\neg[z \subseteq \emptyset])$ and $V(\neg[z = \emptyset])$ does not reduce either to 0 or to 1, so that both ER's $\neg[z \subseteq \emptyset]$ and $\neg[z = \emptyset]$ are vav-neutral (udeterologous). Consequently, the variants of identities ($4.1\mu_2$)–($4.4\mu_2$) with $\subseteq$ or with = in place of $\in$ are invalid (not valid).•

°**Th 4.3.** 1) For each $n \,\vec{\in}\, \{1,2,3\}$:

$$V(\mathsf{A}_{\in n}(u,\emptyset)) \,\hat{=}\, V(\neg \mathsf{O}_{\in n}(u,\emptyset)) \,\hat{=}\, V(u \in \emptyset) \,\hat{=}\, 1 \,\hat{-}\, \hat{\cdot}_z [1 \,\hat{-}\, V(\neg[z \in u]) \,\hat{\cdot}\, V(z \in \emptyset)]$$
$$\hat{=}\, 1 \,\hat{-}\, \hat{\cdot}_z [1 \,\hat{-}\, V(\neg[z \in u]) \,\hat{\cdot}\, 1] \,\hat{=}\, 1 \,\hat{-}\, \hat{\cdot}_z V(z \in u), \tag{4.1μ_3}$$

$$V(\mathsf{O}_{\in n}(u,\emptyset)) \,\hat{=}\, V(\neg \mathsf{A}_{\in n}(u,\emptyset)) \,\hat{=}\, V(\neg[u \subseteq \emptyset])$$
$$\hat{=}\, 1 \,\hat{-}\, V(\mathsf{A}_{\in n}(u,\emptyset)) \,\hat{=}\, \hat{\cdot}_z V(z \in u), \tag{4.2μ_3}$$

$$V(\mathsf{E}_{\in n}(u,\emptyset)) \,\hat{=}\, V(\neg \mathsf{I}_{\in n}(u,\emptyset)) \,\hat{=}\, 1 \,\hat{-}\, \hat{\cdot}_z [1 \,\hat{-}\, V(\neg[z \in u]) \,\hat{\cdot}\, V(\neg[z \in \emptyset])]$$
$$\hat{=}\, 1 \,\hat{-}\, \hat{\cdot}_z [1 \,\hat{-}\, V(\neg[z \in u]) \,\hat{\cdot}\, 0] \,\hat{=}\, 1 \,\hat{-}\, \hat{\cdot}_z 1 \,\hat{=}\, 0, \tag{4.3μ_3}$$

$$V(\mathsf{I}_{\in n}(u,\emptyset)) \,\hat{=}\, V(\neg \mathsf{E}_{\in n}(u,\emptyset)) \,\hat{=}\, 1 \,\hat{-}\, V(\mathsf{E}_{\in n}(u,\emptyset)) \,\hat{=}\, 1 \,\hat{-}\, 0 \,\hat{=}\, 1. \tag{4.4μ_3}$$

2) The variants of identities ($4.1\mu_3$) and ($4.2\mu_3$) with any one of the predicate-signs $\subseteq$, =, and $\subset$ in place of $\in$, and the variants of identities ($4.3\mu_3$) and ($4.4\mu_3$) with any one of the predicate-sign $\subset$ (but not with $\subseteq$ or with =) in place of $\in$ are also identities.

Proof: None of the validity integrons $V(z \in \emptyset)$, $V(z \subseteq \emptyset)$, $V(z = \emptyset)$, and $V(z \subset \emptyset)$ reduces either to 0 or to 1, so that all ER's $z \in \emptyset$, $z \subseteq \emptyset$, $z = \emptyset$, and $z \subset \emptyset$ are vav-neutral (udeterologous). Hence, the final results in ($4.1\mu_3$) and ($4.2\mu_3$) are irreducible. The train of identities ($4.3\mu_3$) and its variant with $\subset$ in place of $\in$ are proved with the help of (4.48) and (4.49) respectively.•

5. Aristotelian logic of $A_{1\in}$

5.1. Underlying nomenclature

Preliminary Remark 5.1. 1) In accordance with Df 1.1, a *formal categorical syllogism* (*FCS*) is *one of 19 three-judgment three-term formal* (*schematic*) *rules of deductive inference of a categorical judgment*, called the *conclusion*, *from two known categorical judgments*, called the *premises*. The traditional form of an FCS, in which the premises and conclusion are stated as three separate *judgment forms* (*JF's*), each of which ends with a full stop, is called a *verbal staccato form* (*VSF*) *of the FCS*. Alternatively, an FCS can be asserted in the form of a *hypothetical statement schema*, in which the antecedent is the conjunction of two premises and the consequent is the conclusion. This form of an FCS is called the *logographic legato form* (*LLF*) *of the FCS* and also a *formal hypothetico-categorical syllogism* (*FHCS*), *formal quantified transitive law* (*FQTL*), or *formal syllogistic implication* (*FSI)*. It is essential that in passage from the staccato form of an FCS to its legato form, the premises and conclusion do not alter and hence they remain *categorical* (*unconditional*), i.e. *neither hypothetical nor disjunctive*. Therefore, both forms are equivalent, while the legato form is preferable because it is naturally incorporated into logistic systems.

2) There are four standard *syllogistic JF's* (*SJF's*), which are called *universal affirmative*, *universal negative*, *particular affirmative*, and *particular negative* ones, and which are distinguished by the four capital or small vowel letters "A", "E", "I", "O" or "a", "e", "i", "o" in that order as *conventional* (*traditional*) *code* (*catch*) *letters* that were derived from the two Latin words "***a**ff**i**rmo*" and "*n**e**g**o***". The 19 FCS's are conventionally divided into 4 *syllogistic figures*, defined by (1.12), and 10 *syllogistic moods*, defined by (1.13), so that each concrete FCS is uniquely determined by both its figure and its mood. Accordingly, each concrete FCS is distinguished by its proper three-syllable mnemonic code name (catchword), in which the sequence of three vowels, selected out of the four code letters "a", "e", "i", "o", indicates the mood of the FCS, while the consonants are tacitly and unsystematically associated with the figure of the FCS. For convenience in description and study, I have also divided all FCS's into 6 mood-related groups, defined by (1.14). Each mood-related group contains FCS's of all pertinent moods independent of their figures. At the same time,

the consonant letters occurring in the catchword of an FCS are irrelevant to the mood of the FCS. However, it turns out that *all catchwords of the FCS's of each one of the six mood-related groups of FCS's begin with the same consonant letter*, provided that the conventional catchword "*Darapti*" is replaced with "*Barapti*". Consequently, the catchword "Barapti" of my own is used in this treatise instead of the conventional catchword "Darapti".

3) A syllogistic judgment (SJ), being a concrete instance of an SJF, is a *veracious* (*accidentally true*) syllogistic proposition (extended declarative two-term sentence), the understanding being that an instance of either of the two *premise forms* of an FCS is *veracious* either *by assumption* or *by the previous knowledge*, while the instance of the *conclusion form* of the FCS is *veracious by inference* (*deduction*) *of the FCS itself*. At the same time, in order to serve as a *categorical*, i.e. *unconditional*, *rule of inference*, an FCS should be *valid* in the sense that it should be a *conformal catxenographic* (briefly *CFCX*, not necessarily *catlogographic*) *interpretand* of an appropriate *valid ER* (*euautographic relation*) as its *interpretans*. If an ER that serves as an interpretans of a given FCS is *vav-neutral* and if the CFCX interpretand of the ER turns into the FCS in the result of adopting a certain CFCX axiom or certain CFCX axioms then the FCS is *veracious*, and *not valid*, and hence it is in fact a *formal conditional syllogism* (*FCdS*). That is to say, in this case the common name "*formal categorical syllogism*" ("*FCS*") of the syllogism in question becomes a *misnomer*.

4) In contrast to FCS's, which can be presented both in the staccato form and in the legato form, the ER's of $\mathsf{A}_{1\in}$, which are interpretantia of the FCS's, and the PLR's of $\boldsymbol{\mathsf{A}}_{1\in}$, which are interpretantia of the above ER's, can be written only in the legato form, i.e. as inseparable single whole PLR's and ER's respectively. Also, I shall need, as belonging to the IML (inclusive metalanguage) of $\mathsf{A}_{1\in}$, a system of verbal names (taxonyms) describing the PLR's and ER's in question through a genus and the differentia. For convenience, I shall use abbreviations of those names. A system of forming these names and their abbreviations and also the names and the abbreviations as such are described in the following two definitions.•

Df 5.1. 1) Unless stated otherwise, the noun "syllogism" will be used as an abbreviation of any of the synonymous count names: "*syllogistic implication*" ("*SJ*"), "*hypothetico-categorical syllogism*" ("*HCS*"), and "*quantified transitive law*"

("*QTL*"). Accordingly, the adjective "*syllogistic*" means: «*of or relating to a syllogism*».

2) Let for each $n \in \{1,2,3,4\}$ any of the four metalogographs '$\mathsf{U}_n\langle \mathbf{u},\mathbf{v}\rangle$', '$\mathsf{V}_n\langle \mathbf{u},\mathbf{v}\rangle$', and '$\mathsf{W}_n\langle \mathbf{u},\mathbf{v}\rangle$' be a *metalogographic placeholder* (*MLPH*), whose range is the set of four PSJ's: $\{\mathsf{A}_n\langle \mathbf{u},\mathbf{v}\rangle, \mathsf{E}_n\langle \mathbf{u},\mathbf{v}\rangle, \mathsf{I}_n\langle \mathbf{u},\mathbf{v}\rangle, \mathsf{O}_n\langle \mathbf{u},\mathbf{v}\rangle\}$; and similarly with any other ordered pair of letters, which is selected out of the three letters '**u**', '**v**', '**w**' instead of ⟨'**u**','**v**'⟩. Let

$$\begin{aligned}
&(1\mathsf{UVW})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{U}_n\langle \mathbf{w},\mathbf{v}\rangle \wedge \mathsf{V}_n\langle \mathbf{u},\mathbf{w}\rangle \Rightarrow \mathsf{W}_n\langle \mathbf{u},\mathbf{v}\rangle],\\
&(2\mathsf{UVW})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{U}_n\langle \mathbf{v},\mathbf{w}\rangle \wedge \mathsf{V}_n\langle \mathbf{u},\mathbf{w}\rangle \Rightarrow \mathsf{W}_n\langle \mathbf{u},\mathbf{v}\rangle],\\
&(3\mathsf{UVW})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{U}_n\langle \mathbf{w},\mathbf{v}\rangle \wedge \mathsf{V}_n\langle \mathbf{w},\mathbf{u}\rangle \Rightarrow \mathsf{W}_n\langle \mathbf{u},\mathbf{v}\rangle],\\
&(4\mathsf{UVW})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{U}_n\langle \mathbf{v},\mathbf{w}\rangle \wedge \mathsf{V}_n\langle \mathbf{w},\mathbf{u}\rangle \Rightarrow \mathsf{W}_n\langle \mathbf{u},\mathbf{v}\rangle],
\end{aligned} \tag{5.1}$$

in analogy with (1.11). The four meta-syntactic relation schemata serving as definientia of definition (5.1), in which the substituends of 'U_n', 'V_n', and 'W_n' are supposed to be selected and arranged after the manner of any of the 19 FCS's, are called the *panlogographic syllogistic figures* (*PSFs*) in reference to the specific mutual arrangements of the placeholders '**u**', '**v**', and '**w**' relative to one another. Hence, the strings $(1\mathsf{UVW})_n$, $(2\mathsf{UVW})_n$, $(3\mathsf{UVW})_n$, and $(4\mathsf{UVW})_n$ are MLPH's, which have, in analogy with (1.12), the following ranges:

$$\begin{aligned}
&(1\mathsf{UVW})_n \in \{(1\mathsf{AAA})_n, (1\mathsf{EAE})_n, (1\mathsf{AII})_n, (1\mathsf{EIO})_n\},\\
&(2\mathsf{UVW})_n \in \{(2\mathsf{AEE})_n, (2\mathsf{EAE})_n, (2\mathsf{EIO})_n, (2\mathsf{AOO})_n\},\\
&(3\mathsf{UVW})_n \in \{(3\mathsf{AAI})_n, (3\mathsf{AII})_n, (3\mathsf{IAI})_n, (3\mathsf{EAO})_n, (3\mathsf{EIO})_n, (3\mathsf{OAO})_n\},\\
&(4\mathsf{UVW})_n \in \{(4\mathsf{AAI})_n, (4\mathsf{AEE})_n, (4\mathsf{IAI})_n, (4\mathsf{EAO})_n, (4\mathsf{EIO})_n\}.
\end{aligned} \tag{5.2}$$

A PLR, being an instance of any one of the four definientia of (5.1) subject to (5.2), i.e. an instance $[\mathsf{U}_n \wedge \mathsf{V}_n] \Rightarrow \mathsf{W}_n$ of the schematic *metalogographic implication*: '$[\mathsf{U}_n \wedge \mathsf{V}_n] \Rightarrow \mathsf{W}_n$', subject to (5.1) and (5.2), is called a *panlogographic syllogistic implication* (*PSI*). Any concrete string in the range of each one of the four MLPH's is called the *panlogographic logical predicate* (*PLLP*) of the corresponding PSI, the understanding being that the latter is uniquely determined by the former. In this case, the given number from 1 to 4 occurring in the PLLP of a PSI indicates the figure of the PSI, whereas the sequence of three code letters selected out of the four ones 'A', 'E', 'I', and 'O' indicates the *sequence of qualities of the major premise, minor premise, and conclusion of the PSI* (in that order) – the sequence, which is called the *mood of the PSI* or less explicitly a *panlogographic syllogistic mood*. In accordance with (5.2), there is the following *ten* syllogistic moods, corresponding to those of FCS's, (1.13):

$$\mathsf{AAA, AAI, AII, IAI, AEE, EAE, EAO, EIO, AOO, OAO}. \qquad (5.3)$$

3) In accordance with the APLADM (advanced panlogographic algebraic decision method) $\mathbf{D}_1$ of $\mathbf{A}_1$, a PSI $[\mathsf{U}_n \wedge \mathsf{V}_n] \Rightarrow \mathsf{W}_n$ is a *panlogographic slave relation* (*PLSR*), which should satisfy a certain *panlogographic master*, or *decision, theorem* (*PLMT* or *PLDT*), the form of which allows unambiguously classifying it as one of the following three kinds (classes): (a) a *valid*, or *kyrological*, *PSI* (*VPSI* or *KPSI*); (b) an *antivalid*, or *antikyrological*, *PSI* (*AVPSI* or *AKPSI*); (c) a *vav-neutral*, or *udeterological*, *PSI* (*vav-NPSI* or *UPSI*). The PLMT (PLDT) of a PSI is called a *syllogistic one* (*SPLMT* or *SPLDT*).

4) An ER of A_1, being an instance (interpretand) of a PSI, is called a *euautographic syllogistic implication* (*ESI*). In this case, the pertinent instance (interpretand) of the PLLP of the PSI is called a *euautographic logical predicate* (*ELP*) *of the ESI*, the understanding being the ESP is uniquely determined by its ELP. In accordance with the AEADM (advanced euautographic algebraic decision method) D_1 of A_1, an ESI is a *euautographic slave relation* (*ESR*), which should satisfy a certain *euautographic master*, or *decision*, *theorem* (*EMT* or *EDT*), the form of which allows unambiguously classifying it as one of the following three kinds (classes): (a) a *valid*, or *kyrological*, *ESI* (*VESI* or *KESI*); (b) an *antivalid*, or *antikyrological*, *ESI* (*AVESI* or *AKESI*); (c) a *vav-neutral*, or *udeterological*, *ESI* (*vav-NESI* or *UESI*). The EMT (EDT) of an ESI is called a *syllogistic one* (*SEMT* or *SEDT*).

5) In this treatise, most definitions and most theorems concerning ER's in general and those concerning ESI's in particular are condensed into the appropriate definition and theorem schemata belonging to $\mathbf{A}_1$. Accordingly, every ESI of the range of a VPSI (KPSI) is a VESI (KESI).

6) In accordance with (5.1) and (5.2), each one of the 19 PSI's at each $n \in \{1,2,3,4\}$ is identified by its LLP (logographic logical predicate) or VLP (verbal logical predicate), which are, in contrast to the LLP's and VLP's of FCS's, set in the

bold-faced upright Gothic (sans serif) font, called Bold-Faced Roman Arial Narrow Font, and are suffixed with the subscript placeholder '$_n$' that assumes four values: '$_1$', '$_2$', '$_3$', '$_4$', corresponding to the four types of PSB's.

7) For convenience in calculating the validity indices of the separate PSI's of each 19-member set, I divide the latter's into six *mood-related groups* corresponding to the six mood-related groups (1.14) of the FCS's, namely:

$$\mathsf{AAA\&AAI,\ AII\&IAI,\ AEE\&EAE,\ EAO,\ EIO,\ AOO\&OAO}. \tag{5.4}$$

Just as in the case of FCS's, in connection with this division, I use the catchword "Barapti" instead of the conventional catchword "Darapti" for the reason that has been explained in item 2 of Preliminary Remark 5.1. In the following definition, the PSI's are given for each $n \in \{1,2,3,4\}$ in accordance with their mood-related groups.•

Df 5.2.

1°) Group AAA&AAI

1) $\mathsf{Barbara}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{1AAA})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{A}_n\langle \mathbf{w},\mathbf{v}\rangle \wedge \mathsf{A}_n\langle \mathbf{u},\mathbf{w}\rangle \Rightarrow \mathsf{A}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

2) $\mathsf{Barapti}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{3AAI})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{A}_n\langle \mathbf{w},\mathbf{v}\rangle \wedge \mathsf{A}_n\langle \mathbf{w},\mathbf{u}\rangle \Rightarrow \mathsf{I}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

3) $\mathsf{Bamalip}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{4AAI})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{A}_n\langle \mathbf{v},\mathbf{w}\rangle \wedge \mathsf{A}_n\langle \mathbf{w},\mathbf{u}\rangle \Rightarrow \mathsf{I}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

2°) Group AII&IAI

4) $\mathsf{Darii}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{1AII})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{A}_n\langle \mathbf{w},\mathbf{v}\rangle \wedge \mathsf{I}_n\langle \mathbf{u},\mathbf{w}\rangle \Rightarrow \mathsf{I}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

5) $\mathsf{Datisi}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{3AII})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{A}_n\langle \mathbf{w},\mathbf{v}\rangle \wedge \mathsf{I}_n\langle \mathbf{w},\mathbf{u}\rangle \Rightarrow \mathsf{I}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

6) $\mathsf{Disamis}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{3IAI})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{I}_n\langle \mathbf{w},\mathbf{v}\rangle \wedge \mathsf{A}_n\langle \mathbf{w},\mathbf{u}\rangle \Rightarrow \mathsf{I}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

7) $\mathsf{Dimatis}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{4IAI})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{I}_n\langle \mathbf{v},\mathbf{w}\rangle \wedge \mathsf{A}_n\langle \mathbf{w},\mathbf{u}\rangle \Rightarrow \mathsf{I}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

3°) Group EAE&AEE

8) $\mathsf{Celarent}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{1EAE})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{E}_n\langle \mathbf{w},\mathbf{v}\rangle \wedge \mathsf{A}_n\langle \mathbf{u},\mathbf{w}\rangle \Rightarrow \mathsf{E}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

9) $\mathsf{Camestres}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{2AEE})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{A}_n\langle \mathbf{v},\mathbf{w}\rangle \wedge \mathsf{E}_n\langle \mathbf{u},\mathbf{w}\rangle \Rightarrow \mathsf{E}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

10) $\mathsf{Cesare}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{2EAE})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{E}_n\langle \mathbf{v},\mathbf{w}\rangle \wedge \mathsf{A}_n\langle \mathbf{u},\mathbf{w}\rangle \Rightarrow \mathsf{E}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

11) $\mathsf{Calemes}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{4AEE})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{A}_n\langle \mathbf{v},\mathbf{w}\rangle \wedge \mathsf{E}_n\langle \mathbf{w},\mathbf{u}\rangle \Rightarrow \mathsf{E}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

4°) Group EAO

12) $\mathsf{Felapton}_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to (\mathsf{3EAO})_n\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle \to [\mathsf{E}_n\langle \mathbf{w},\mathbf{v}\rangle \wedge \mathsf{A}_n\langle \mathbf{w},\mathbf{u}\rangle \Rightarrow \mathsf{O}_n\langle \mathbf{u},\mathbf{v}\rangle]$.

13) $\mathsf{Fesapo}_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to(\mathsf{4EAO})_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to[\mathsf{E}_n\langle\mathbf{v},\mathbf{w}\rangle\wedge\mathsf{A}_n\langle\mathbf{w},\mathbf{u}\rangle\Rightarrow\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle]$.

5°) Group EIO

14) $\mathsf{Ferio}_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to(\mathsf{1EIO})_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to[\mathsf{E}_n\langle\mathbf{w},\mathbf{v}\rangle\wedge\mathsf{I}_n\langle\mathbf{u},\mathbf{w}\rangle\Rightarrow\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle]$.

15) $\mathsf{Festino}_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to(\mathsf{2EIO})_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to[\mathsf{E}_n\langle\mathbf{v},\mathbf{w}\rangle\wedge\mathsf{I}_n\langle\mathbf{u},\mathbf{w}\rangle\Rightarrow\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle]$.

16) $\mathsf{Feriso}_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to(\mathsf{3EIO})_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to[\mathsf{E}_n\langle\mathbf{w},\mathbf{v}\rangle\wedge\mathsf{I}_n\langle\mathbf{w},\mathbf{u}\rangle\Rightarrow\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle]$.

17) $\mathsf{Fresison}_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to(\mathsf{4EIO})_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to[\mathsf{E}_n\langle\mathbf{v},\mathbf{w}\rangle\wedge\mathsf{I}_n\langle\mathbf{w},\mathbf{u}\rangle\Rightarrow\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle]$.

6°) Group AOO&OAO

18) $\mathsf{Baroco}_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to(\mathsf{2AOO})_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to[\mathsf{A}_n\langle\mathbf{v},\mathbf{w}\rangle\wedge\mathsf{O}_n\langle\mathbf{u},\mathbf{w}\rangle\Rightarrow\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle]$.

19) $\mathsf{Bocardo}_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to(\mathsf{3OAO})_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\to[\mathsf{O}_n\langle\mathbf{w},\mathbf{v}\rangle\wedge\mathsf{A}_n\langle\mathbf{w},\mathbf{u}\rangle\Rightarrow\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle]$.•

Cmt 5.1. 1) By Th 3.1 and Cmt 3.1, it follows that, in each one of 19 items of Df 5.2, three PSI's having the subscripts '$_1$', '$_2$', '$_3$' are equivalent. Therefore, I shall calculate the validity indices only for those PSI's whose PLLP's and PhLP's (catchwords) carry either one of the subscripts '$_1$' and '$_4$'.

2) At the same time, in accordance with Df 3.1, a PSI is said to be *asymmetric* if at least one of its three constituent PSJ's is asymmetric and *symmetric* if its all constituent PSJ's are symmetric. Hence, a PSI is asymmetric its PLLP and PhLP (catchword) carry any one of the three subscripts '$_1$', '$_2$', '$_3$' and symmetric if its its PLLP and PhLP carry the subscripts '$_4$'. It will be shown in due course later on that asymmetric PSI's and particularly those carrying the subscript '$_1$' can be interpreted by the traditional FCS's.

3) The results of calculations of the validity indices of the 19 PSI's at $n \rhd 1$ or $n \rhd 4$ will be summarized in two articles under the heads "Th 5.1" or "Th 5.2" respeectively. The calculations themselves forming proofs of the separate items of either theorem are APLADP's (advance panlogographic algebraic decision procedures) for the respective slave PSI's.

4) Calculations of the validity index of any given PSI will begin with the pertinent instance of the *valid* relation schema:

$$\begin{aligned}V([\mathsf{U}_n\wedge\mathsf{V}_n]\Rightarrow\mathsf{W}_n)&\hat{=}V(\neg[\mathsf{U}_n\wedge\mathsf{V}_n])\,\hat{\cdot}\,V(\mathsf{W}_n)\hat{=}[1\hat{-}V(\mathsf{U}_n\wedge\mathsf{V}_n)]\,\hat{\cdot}\,V(\mathsf{W}_n)\\&\hat{=}V(\neg\mathsf{U}_n)\,\hat{\cdot}\,V(\neg\mathsf{V}_n)\,\hat{\cdot}\,V(\mathsf{W}_n)\end{aligned}\tag{5.5}$$

subject to Df 5.1.

5) All calculations will be performed by means of the following identity schemata and their pertinent variants:

$$V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} 1 \hat{-} \hat{\vdots}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right], \tag{5.6}$$

$$V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} \hat{\vdots}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right], \tag{5.7}$$

$$\begin{gathered} V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \\ \hat{=} 1 \hat{-} \hat{\vdots}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right], \end{gathered} \tag{5.8}$$

$$\begin{gathered} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \\ \hat{=} \hat{\vdots}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right], \end{gathered} \tag{5.9}$$

$$V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} 1 \hat{-} \hat{\vdots}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right], \tag{5.10}$$

$$V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} \hat{\vdots}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right] \tag{5.11}$$

All these relation schemata are comprised in Th 3.1 and they are given here for convenience in cross-references. I shall sometimes make use of the relation schemata (3.16) at $m \ddot{\in} \{1,4\}$ and $n \ddot{\in} \{1,4\}$ and (3.17), and also of the appropriate variants of these relations in no connection with the final expressions in (5.8)–(5.11), although the former immediately follow from the latter.

6) In calculating almost all validity indices, which turn out to be 0, I shall make use the following self-evident lemmas:

$$V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{=} V(\mathbf{Q}) \text{ if and only if } V(\mathbf{Q}) \hat{\cdot} V(\mathbf{R}) \hat{=} V(\mathbf{Q}) \hat{\cdot} \left[1 \hat{-} V(\neg\mathbf{R})\right] \hat{=} 0\,. \tag{5.12}$$

$$V(\mathbf{Q}) \hat{\cdot} V(\mathbf{R}) \hat{=} V(\mathbf{Q}) \text{ if and only if } V(\mathbf{Q}) \hat{\cdot} V(\neg\mathbf{R}) \hat{=} V(\mathbf{Q}) \hat{\cdot} \left[1 \hat{-} V(\mathbf{R})\right] \hat{=} 0\,. \tag{5.12a}$$

These two relations are variants of each other with ‘**R**’ and ‘¬**R**’ exchanged. I shall not mention either of the relations when I use it, but the reader will readily recognize all instances of (5.12) and (5.12a) which occur in various APLADP’s.

7) The original relations (5.6)–(5.11) as they are will be used for representing the validity integron of the consequent of a PSI. When I write the variants of relations (5.6)–(5.11) for representing the validity indices of the first and second conjuncts of the antecedent of the PSI serving as the definiens of each given item of Df 5.2, I shall respectively employ ‘**x**’ and ‘**y**’ as placeholders for the bound (dummy) APVOT’s.•

5.2. The master (decision) theorems and validity indices of the PSI's

Th 5.1: ***The PLMT's and validity indices of asymmetric PSI's.***

1°) Group AAA&AAI

$$\begin{aligned}
&V\left(\mathrm{Barbara}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((1\mathrm{AAA})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{A}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{A}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathrm{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\widehat{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\hat{\cdot}\left[\widehat{\cdot}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[1\hat{-}\widehat{\cdot}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}\tag{5.13}$$

$$\begin{aligned}
&V\left(\mathrm{Barapti}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((3\mathrm{AAI})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{A}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathrm{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\widehat{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\hat{\cdot}\left[\widehat{\cdot}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[\widehat{\cdot}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\widehat{\cdot}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\left[\widehat{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\hat{\cdot}\left[\widehat{\cdot}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{y}}\left[\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right)\hat{\hat{=}}\mathrm{J}^1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle.
\end{aligned}\tag{5.14}$$

$$\begin{aligned}
&V\left(\mathrm{Bamalip}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((4\mathrm{AAI})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathrm{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\widehat{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\hat{\cdot}\left[\widehat{\cdot}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[\widehat{\cdot}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\widehat{\cdot}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\left[\widehat{\cdot}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{\cdot}\left[\widehat{\cdot}_{\mathbf{y}}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{y}}\left[\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right]\right)\hat{\hat{=}}\mathrm{J}^2\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle.
\end{aligned}\tag{5.15}$$

2°) Group AII&IAI

$$\begin{aligned}
&V\left(\mathrm{Datisi}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\mathrm{Darii}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left((3\mathrm{AII})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((1\mathrm{AII})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{A}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{I}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathrm{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{A}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{I}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathrm{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\widehat{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[1\hat{-}\widehat{\cdot}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[\widehat{\cdot}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}\tag{5.16}$$

$$
\begin{aligned}
&V\left(\mathrm{Dimatis}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\mathrm{Disamis}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left((4\mathrm{IAI})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((3\mathrm{IAI})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{I}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathrm{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{I}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathrm{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[1\hat{-}\hat{\prod}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}
\tag{5.17}
$$

3°) Group EAE&AEE

$$
\begin{aligned}
&V\left(\mathrm{Cesare}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\mathrm{Celarent}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left((2\mathrm{EAE})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((1\mathrm{EAE})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{E}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{A}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathrm{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{E}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{A}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathrm{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\prod}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\prod}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[1\hat{-}\hat{\prod}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}
\tag{5.18}
$$

$$
\begin{aligned}
&V\left(\mathrm{Calemes}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\mathrm{Camestres}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left((4\mathrm{AEE})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((2\mathrm{AEE})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{E}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathrm{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{E}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathrm{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\prod}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\prod}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[1\hat{-}\hat{\prod}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}
\tag{5.19}
$$

4°) Group EAO

$$
\begin{aligned}
&V\left(\mathrm{Fesapo}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\mathrm{Felapton}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left((4\mathrm{EAO})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((3\mathrm{EAO})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{E}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathrm{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathrm{E}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathrm{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathrm{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\prod}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\prod}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\prod}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\left[\hat{\prod}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\hat{\cdot}\left[\hat{\prod}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{y}}\left[\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right)\hat{\equiv}\mathrm{J}^3\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle.
\end{aligned}
\tag{5.20}
$$

5°) Group EIO

$$
\begin{aligned}
&V\left(\mathsf{Fresison}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{Feriso}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\mathsf{Festino}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{Ferio}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left((4\mathsf{EIO})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((3\mathsf{EIO})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left((2\mathsf{EIO})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((1\mathsf{EIO})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{I}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{I}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{I}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{I}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\dot{\wedge}\left[1\dot{-}\dot{\bigwedge}_{\mathbf{y}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{z}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}
\tag{5.21}
$$

6°) Group AOO&OAO

$$
\begin{aligned}
&V\left(\mathsf{Baroco}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((2\mathsf{AOO})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\dot{\wedge}\left[1\dot{-}\dot{\bigwedge}_{\mathbf{y}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{z}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}
\tag{5.22}
$$

$$
\begin{aligned}
&V\left(\mathsf{Bocardo}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((3\mathsf{AOO})_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{O}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[1\dot{-}\dot{\bigwedge}_{\mathbf{x}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{y}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right]\\
&\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{z}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}
\tag{5.23}
$$

Proof:

1) ***An APLADP for (5.13):*** By the variant of (5.7) with ‘**w**’ in place of ‘**u**’ and by that with ‘**w**’ in place of ‘**v**’, it follows through the FFL (Fusion and Fission Law) (II.4.29) that

$$
\begin{aligned}
&V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{y}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\dot{\wedge}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[1\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\dot{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right].
\end{aligned}
\tag{5.13$_1$}
$$

By (5.13_1) and (5.7), another application of the FFL yields:

$$
\begin{aligned}
&\left[V\left(\neg \mathsf{A}_1\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{A}_1\langle \mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}V\left(\neg \mathsf{A}_1\langle \mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\quad\hat{\cdot}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\right.\\
&\quad\left.\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\mathbf{i}\langle \mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,
\end{aligned}
\tag{5.13$_2$}
$$

where

$$
\begin{aligned}
\mathbf{i}\langle \mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle &\stackrel{\rightarrow}{\hat{=}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\quad\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\\
&\quad\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\\
&\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right).
\end{aligned}
\tag{5.13$_3$}
$$

By (5.13$_1$) and (5.13$_3$), relation (5.13$_2$) reduces to

$$
\begin{aligned}
&\left[V\left(\neg \mathsf{A}_1\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{A}_1\langle \mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}V\left(\neg \mathsf{A}_1\langle \mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{=}V\left(\neg \mathsf{A}_1\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{A}_1\langle \mathbf{u},\mathbf{w}\rangle\right).
\end{aligned}
\tag{5.13$_4$}
$$

Hence,

$$
\begin{aligned}
V\left(\mathsf{Barbara}_1\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)&\hat{=}\left[V\left(\neg \mathsf{A}_1\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{A}_1\langle \mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}V\left(\mathsf{A}_1\langle \mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[V\left(\neg \mathsf{A}_1\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{A}_1\langle \mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}V\left(\neg \mathsf{A}_1\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\hat{=}0.
\end{aligned}
\tag{5.13$_5$}
$$

2) ***An APLADP for (5.14):*** By the variant of (5.7) with '**w**' in place of '**u**' and by that with '**w**' and '**u**' in place of '**u**' and '**v**' respectively, it follows through the FFL that

$$
\begin{aligned}
&V\left(\neg \mathsf{A}_1(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg \mathsf{A}_1(\mathbf{w},\mathbf{u})\right)\\
&\hat{=}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\right]\right]\\
&\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\mathbf{i}\langle \mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,
\end{aligned}
\tag{5.14$_1$}
$$

where

$$
\begin{aligned}
&\mathbf{i}\langle \mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle\\
&\stackrel{\rightarrow}{\hat{=}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\right]\\
&\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[V\left(\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{+}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right).
\end{aligned}
\tag{5.14$_2$}
$$

In developing the final result in (5.14_2), use of the pertinent version of (II.5.10) has been made. At the same time, by (5.14_2), relation (5.9) can be rewritten as

$$
\begin{aligned}
V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle) \mathrel{\hat{=}} V(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle) \mathrel{\hat{=}} V(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle) \mathrel{\hat{=}} V(\neg\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle)\\
\mathrel{\hat{=}} \hat{\bullet}_{\mathbf{z}} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)
\end{aligned}
\tag{5.14$_3$}
$$

(cf. (12.4)). Hence, by (5.14_1)–(5.14_3), more applications of the FFL yield:

$$
\begin{aligned}
&V(\mathsf{Barapti}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle) \mathrel{\hat{=}} [V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{v}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle)] \mathbin{\hat{\cdot}} V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\\
&\mathrel{\hat{=}} [\hat{\bullet}_{\mathbf{x}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]] \mathbin{\hat{\cdot}} [\hat{\bullet}_{\mathbf{z}} V(\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)]\\
&\mathrel{\hat{=}} \hat{\bullet}_{\mathbf{x}} [[1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)] \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]\\
&\mathrel{\hat{=}} \hat{\bullet}_{\mathbf{x}} [[1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)] \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]\\
&\mathrel{\hat{=}} \hat{\bullet}_{\mathbf{x}} [V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]\\
&\mathrel{\vec{\hat{=}}} \hat{\bullet}_{\mathbf{x}} [V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]]\\
&\mathrel{\hat{=}} [\hat{\bullet}_{\mathbf{x}} V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)] \mathbin{\hat{\cdot}} [\hat{\bullet}_{\mathbf{y}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle)]]\\
&\mathrel{\hat{=}} V({\textstyle\bigvee_{\mathbf{x}}} \mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V({\textstyle\bigvee_{\mathbf{y}}} [\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle \wedge \mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle]).
\end{aligned}
\tag{5.14$_5$}
$$

3) ***APLADP for (5.15):*** By two pertinent variants of (5.7), it follows through the FFL that

$$
\begin{aligned}
&V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle)\\
&\mathrel{\hat{=}} [\hat{\bullet}_{\mathbf{x}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)]] \mathbin{\hat{\cdot}} [\hat{\bullet}_{\mathbf{y}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)]]\\
&\mathrel{\hat{=}} \hat{\bullet}_{\mathbf{x}} [[1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)] \mathbin{\hat{\cdot}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)]]\\
&\mathrel{\hat{=}} \hat{\bullet}_{\mathbf{x}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)].
\end{aligned}
\tag{5.15$_1$}
$$

Hence, by (5.15_1) and by (5.9), another application of the FFL yields:

$$
\begin{aligned}
&V(\mathsf{Bamalip}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle) \mathrel{\hat{=}} [V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle)] \mathbin{\hat{\cdot}} V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\\
&\mathrel{\hat{=}} [\hat{\bullet}_{\mathbf{x}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)]]\\
&\mathbin{\hat{\cdot}} [\hat{\bullet}_{\mathbf{z}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)]] \mathrel{\hat{=}} \hat{\bullet}_{\mathbf{x}} \mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,
\end{aligned}
\tag{5.15$_2$}
$$

where

$$
\begin{aligned}
&\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle\\
&\mathrel{\vec{\hat{=}}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)]\\
&\mathbin{\hat{\cdot}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]\\
&\mathrel{\hat{=}} 1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\\
&\mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} [1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)]]\\
&\mathrel{\hat{=}} 1 \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \mathbin{\dot{-}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\\
&\mathbin{\dot{-}} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \mathbin{\hat{\cdot}} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \mathbin{\hat{\cdot}} V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)]
\end{aligned}
\tag{5.15$_3$}
$$

By (5.15_3) and by the relation $V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{=}1\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)$, the final expression in (5.15_3) can be developed further thus:

$$
\begin{aligned}
\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle &\hat{=} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{+}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\\
&\hat{-}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\\
&\hat{=} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\\
&\hat{+}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{=} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\\
&\hat{=} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right).
\end{aligned}
\qquad (5.15_4)
$$

Hence, by more applications of the FFL, (5.15_2) becomes

$$
\begin{aligned}
V\left(\mathsf{Bamalip}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) &\hat{=} \hat{\bullet}_{\mathbf{x}}\,\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle\\
&\hat{=} \hat{\bullet}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=} \left[\hat{\bullet}_{\mathbf{x}}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=} V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{y}}\left[\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right]\right).
\end{aligned}
\qquad (5.15_5)
$$

4) *APLADP for (5.16):* By the variant of (5.7) with '**w**' in place of '**v**' and by (5.9), it follows through the FFL that

$$
\begin{aligned}
&V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right].
\end{aligned}
\qquad (5.16_1)
$$

By (5.16_1) and by the variant of (5.9) with '**w**' in place of '**v**', another application of the FFL yields:

$$
\begin{aligned}
&\left[V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\right]\hat{\cdot}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\\
&\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\hat{=}\hat{\bullet}_{\mathbf{x}}\,\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,
\end{aligned}
\qquad (5.16_2)
$$

where

$$
\begin{aligned}
\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle \;\vec{\hat{=}}\; & \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right] \\
& \hat{\cdot} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right] \\
\vec{\hat{=}}\; & 1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \\
& \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \hat{-} V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right] \\
\hat{=}\; & 1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle).
\end{aligned}
\qquad (5.16_3)
$$

By (5.16_1) and (5.16_3), relation (5.16_2) reduces to

$$
\begin{aligned}
& \left[V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{v}\rangle) \hat{\cdot} V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\right] \hat{\cdot} V(\mathsf{I}_1\langle\mathbf{u},\mathbf{w}\rangle) \\
& \hat{=} \widehat{\cdot}_{\mathbf{x}} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right] \\
& \hat{=} V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{v}\rangle) \hat{\cdot} V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle).
\end{aligned}
\qquad (5.16_4)
$$

Hence,

$$
\begin{aligned}
V(\mathsf{Darii}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle) & \hat{=} \left[V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{v}\rangle) \hat{\cdot} V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\right] \hat{\cdot} V(\neg\mathsf{I}_1\langle\mathbf{u},\mathbf{w}\rangle) \\
& \hat{=} \left[V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{v}\rangle) \hat{\cdot} V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\right] \hat{\cdot} \left[1 \hat{-} V(\mathsf{I}_1\langle\mathbf{u},\mathbf{w}\rangle)\right] \hat{=} 0.
\end{aligned}
\qquad (5.16_5)
$$

5) ***APLADP for (5.17):*** By the variant of (5.7) with '**w**' and '**u**' in place of '**u**' and '**v**' respectively and by (5.9), it follows through the FFL that

$$
\begin{aligned}
& V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle) \hat{\cdot} V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle) \\
& \hat{=} \left[\widehat{\cdot}_{\mathbf{y}} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\right]\right] \hat{\cdot} \left[\widehat{\cdot}_{\mathbf{z}} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\right]\right] \\
& \hat{=} \widehat{\cdot}_{\mathbf{y}} \left[\left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\right] \hat{\cdot} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle)\right]\right] \\
& \hat{=} \widehat{\cdot}_{\mathbf{y}} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle) \hat{-} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\right].
\end{aligned}
\qquad (5.17_1)
$$

By (5.17_1) and by the variant of (5.9) with '**w**' in place of '**u**', another application of the FFL yields:

$$
\begin{aligned}
& V(\mathsf{I}_1\langle\mathbf{w},\mathbf{v}\rangle) \hat{\cdot} \left[V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle) \hat{\cdot} V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\right] \\
& \hat{=} \left[\widehat{\cdot}_{\mathbf{x}} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\right] \\
& \hat{\cdot} \left[\widehat{\cdot}_{\mathbf{y}} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle) \hat{-} V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\right]\right] \\
& \hat{=} \widehat{\cdot}_{\mathbf{x}} \mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,
\end{aligned}
\qquad (5.17_2)
$$

where

$$
\begin{aligned}
\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle \;\vec{\hat{=}}\; & \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right] \\
& \hat{\cdot} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\right] \\
\hat{=}\; & 1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \\
& \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \hat{\cdot} \left[1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{-} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\right] \\
\hat{=}\; & 1 \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle) \hat{\cdot} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle) \hat{-} V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle) \hat{\cdot} V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle).
\end{aligned}
\qquad (5.17_3)
$$

By (5.17$_1$) and (5.17$_3$), relation (5.17$_2$) reduces to

$$
\begin{aligned}
&V(\mathsf{I}_1\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}\left[V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle)\hat{\cdot}V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\right]\\
&\hat{=}V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle)\hat{\cdot}V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle).
\end{aligned}
\tag{5.17$_4$}
$$

Hence,

$$
\begin{aligned}
&V(\mathsf{Disamis}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle)\hat{=}V(\neg\mathsf{I}_1\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}\left[V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle)\hat{\cdot}V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\right]\\
&\hat{=}\left[1\hat{-}V(\mathsf{I}_1\langle\mathbf{w},\mathbf{v}\rangle)\right]\hat{\cdot}\left[V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle)\hat{\cdot}V(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle)\right]\hat{=}0.
\end{aligned}
\tag{5.17$_5$}
$$

6) ***APLADP for (5.18):*** By the variant of (5.9) with ‘**w**’ in place of ‘**u**’ and by the variant of (5.7) with ‘**w**’ in place of ‘**v**’, it follows through the FFL that

$$
\begin{aligned}
&V(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{w}\rangle)\\
&\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\hat{\cdot}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right],
\end{aligned}
\tag{5.18$_1$}
$$

By (5.18$_1$) and (5.9), another application of the FFL yields:

$$
\begin{aligned}
&\left[V(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{w}\rangle)\right]\hat{\cdot}V(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\right]\\
&\hat{\cdot}\left[\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\right]\right]\hat{=}\hat{\bullet}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,
\end{aligned}
\tag{5.18$_2$}
$$

where

$$
\begin{aligned}
&\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle\\
&\overset{\rightarrow}{\hat{=}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\\
&\hat{\cdot}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\\
&\hat{=}1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\\
&\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}\left[1\hat{-}V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\\
&\hat{=}1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle).
\end{aligned}
\tag{5.18$_3$}
$$

By (5.18$_1$) and (5.18$_3$), relation (5.18$_2$) reduces to

$$
\begin{aligned}
&\left[V(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{w}\rangle)\right]\hat{\cdot}V(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\\
&\hat{=}V(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{w}\rangle).
\end{aligned}
\tag{5.18$_4$}
$$

Hence,

$$V\left(\mathsf{Celarent}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}\left[V\left(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)$$
$$\hat{=}\left[V\left(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\right]\hat{=}0. \quad (5.18_5)$$

7) ***APLADP for (5.19):*** By the variant of (5.7) with ‘**v**’ and ‘**w**’ in place of ‘**u**’ and ‘**v**’ respectively and by the variant of (5.9) ‘**w**’ in place of ‘**v**’, it follows through the FFL that

$$V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{w}\rangle\right)$$
$$\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]$$
$$\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right] \quad (5.19_1)$$
$$\hat{=}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right],$$

By (5.19_1) and (5.9),

$$\left[V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)$$
$$\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right] \quad (5.19_2)$$
$$\hat{\cdot}\left[\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}\hat{\bullet}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,$$

In developing the final two equalities in (5.19_2), use of the following train of equalities has been made:

$$\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle$$
$$\overset{\rightarrow}{\hat{=}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]$$
$$\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]$$
$$\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right) \quad (5.19_3)$$
$$\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]$$
$$\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right).$$

By (5.19_1) and (5.19_3), relation (5.19_2) reduces to

$$\left[V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)$$
$$\hat{=}\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right] \quad (5.19_4)$$
$$\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{w}\rangle\right).$$

Hence,

$$V\left(\mathsf{Camestres}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}\left[V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)$$
$$\hat{=}\left[V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}V\left(\neg V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\right)\right]\hat{=}0. \quad (5.19_5)$$

8) ***APLADP for (5.20):*** By the variant of (5.7) with ‘**w**’ and ‘**u**’ in place of ‘**u**’ and ‘**v**’ respectively and also by (5.7), it follows through the FFL that

$$
\begin{aligned}
&V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{y}}\left[\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right].
\end{aligned}
\tag{5.20$_1$}
$$

By (5.20$_1$) and by the variant of (5.9) with '**w**' in place of '**u**', another application of the FFL yields:

$$
\begin{aligned}
&V\left(\mathsf{Felapton}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\right]\\
&\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,
\end{aligned}
\tag{5.20$_2$}
$$

where

$$
\begin{aligned}
&\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle\overset{\rightarrow}{\hat{=}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\\
&\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\\
&\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\\
&\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\\
&\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right).
\end{aligned}
\tag{5.20$_3$}
$$

Upon setting $V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{=}1\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)$ in the last term in the final expression in (5.20$_3$), that expression can be developed further thus:

$$
\begin{aligned}
&\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle\\
&\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\\
&\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{+}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\\
&\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\\
&\hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\\
&\hat{=}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\\
&\hat{=}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right).
\end{aligned}
\tag{5.20$_4$}
$$

Hence, in analogy with (5.14$_5$) and (5.15$_5$), relation (5.20$_2$) becomes:

$$
\begin{aligned}
V(\mathsf{Felapton}_1\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle) &\hat{=} \dot{\bigwedge}_{\mathbf{x}}\left[V(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\,\hat{\wedge}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\right]\right]\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}V(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\right]\hat{\wedge}\left[\dot{\bigwedge}_{\mathbf{y}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle)\right]\right]\\
&\hat{=} V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\wedge}\left[V\left(\bigvee_{\mathbf{y}}\left[\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right]\right)\right]
\end{aligned}
\qquad (5.20_5)
$$

9) ***APLADP for (5.21):*** By the variant of (5.9) with '**w**' in place of '**u**' and by (5.7), it follows through the FFL that

$$
\begin{aligned}
&V(\neg\mathsf{E}_1\langle \mathbf{w},\mathbf{v}\rangle)\,\hat{\wedge}\,V(\mathsf{O}_1\langle \mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\right]\right]\hat{\wedge}\left[\dot{\bigwedge}_{\mathbf{z}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{z},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\mathbf{P}\langle \mathbf{z},\mathbf{v}\rangle)\right]\right]\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\right]\hat{\wedge}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\right]\right]\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\right].
\end{aligned}
\qquad (5.21_1)
$$

By (5.21_1) and by the variant of (5.9) with '**w**' in place of '**v**', another application of the FFL yields:

$$
\begin{aligned}
&\left[V(\neg\mathsf{E}_1\langle \mathbf{w},\mathbf{v}\rangle)\,\hat{\wedge}\,V(\mathsf{O}_1\langle \mathbf{u},\mathbf{v}\rangle)\right]\hat{\wedge}\,V(\mathsf{I}_1\langle \mathbf{u},\mathbf{w}\rangle)\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\right]\right]\\
&\quad\hat{\wedge}\left[\dot{\bigwedge}_{\mathbf{y}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle)\right]\right]\hat{=}\dot{\bigwedge}_{\mathbf{x}}\mathbf{i}\langle \mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,
\end{aligned}
\qquad (5.21_2)
$$

where

$$
\begin{aligned}
&\mathbf{i}\langle \mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle\\
&\vec{\hat{=}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\right]\\
&\qquad\hat{\wedge}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\right]\\
&\vec{\hat{=}}\,1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\\
&\quad\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\,\hat{\wedge}\left[1\,\hat{-}\,V(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\right]\\
&\hat{=}\,1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle).
\end{aligned}
\qquad (5.21_3)
$$

By (5.21_1) and (5.21_3), relation (5.21_2) reduces to

$$
\begin{aligned}
&\left[V(\neg\mathsf{E}_1\langle \mathbf{w},\mathbf{v}\rangle)\,\hat{\wedge}\,V(\mathsf{O}_1\langle \mathbf{u},\mathbf{v}\rangle)\right]\hat{\wedge}\,V(\mathsf{I}_1\langle \mathbf{u},\mathbf{w}\rangle)\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle)\,\hat{\wedge}\,V(\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle)\,\hat{\wedge}\,V(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle)\right]\\
&\hat{=}V(\neg\mathsf{E}_1\langle \mathbf{w},\mathbf{v}\rangle)\,\hat{\wedge}\,V(\mathsf{O}_1\langle \mathbf{u},\mathbf{v}\rangle).
\end{aligned}
\qquad (5.21_4)
$$

Hence,

$$
\begin{aligned}
V(\mathsf{Ferio}_1\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle)&\hat{=}\left[V(\neg\mathsf{E}_1\langle \mathbf{w},\mathbf{v}\rangle)\,\hat{\wedge}\,V(\mathsf{O}_1\langle \mathbf{u},\mathbf{v}\rangle)\right]\hat{\wedge}\,V(\neg\mathsf{I}_1\langle \mathbf{u},\mathbf{w}\rangle)\\
&\hat{=}\left[V(\neg\mathsf{E}_1\langle \mathbf{w},\mathbf{v}\rangle)\,\hat{\wedge}\,V(\mathsf{O}_1\langle \mathbf{u},\mathbf{v}\rangle)\right]\hat{\wedge}\left[1\,\hat{-}\,V(\mathsf{I}_1\langle \mathbf{u},\mathbf{w}\rangle)\right]\hat{=}\,0.
\end{aligned}
\qquad (5.21_5)
$$

10) ***APLADP for (5.22):*** By the variant of (5.7) with '**v**' and '**w**' in place of '**u**' and '**v**' respectively and by (5.7), it follows that

$$
\begin{aligned}
&V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle)\,\hat{\cdot}\,V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}\left[\hat{:}_{\mathbf{x}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\right]\hat{\cdot}\left[\hat{:}_{\mathbf{z}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\right]\right]\\
&\hat{=}\,\hat{:}_{\mathbf{x}}\left[\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\hat{\cdot}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\right]\\
&\hat{=}\,\hat{:}_{\mathbf{x}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right].
\end{aligned}
\tag{5.22$_1$}
$$

By (5.22$_1$) and by the variant of (5.7) with '**w**' in place of '**v**', another application of the FFL yields:

$$
\begin{aligned}
&\left[V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle)\,\hat{\cdot}\,V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\right]\hat{\cdot}\,V(\mathsf{O}_1\langle\mathbf{u},\mathbf{w}\rangle)\\
&\hat{=}\left[\hat{:}_{\mathbf{x}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\right]\\
&\hat{\cdot}\left[\hat{:}_{\mathbf{y}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle)\right]\right]\hat{=}\,\hat{:}_{\mathbf{x}}\,\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,
\end{aligned}
\tag{5.22$_2$}
$$

where

$$
\begin{aligned}
&\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle\\
&\overset{\to}{\hat{=}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\\
&\hat{\cdot}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\\
&\hat{=}\,1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\\
&\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\,\hat{\cdot}\left[1\,\hat{-}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\\
&\hat{=}\,1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle).
\end{aligned}
\tag{5.22$_3$}
$$

By (5.22$_1$) and (5.22$_3$), relation (5.22$_2$) reduces to

$$
\begin{aligned}
&\left[V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle)\,\hat{\cdot}\,V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\right]\hat{\cdot}\,V(\mathsf{O}_1\langle\mathbf{u},\mathbf{w}\rangle)\\
&\hat{=}\,\hat{:}_{\mathbf{x}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\\
&\hat{=}\,V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle)\,\hat{\cdot}\,V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle).
\end{aligned}
\tag{5.22$_4$}
$$

Hence,

$$
\begin{aligned}
&V(\mathsf{Baroco}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle)\,\hat{=}\left[V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle)\,\hat{\cdot}\,V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\right]\hat{\cdot}\,V(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{w}\rangle)\\
&\hat{=}\left[V(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle)\,\hat{\cdot}\,V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\right]\hat{\cdot}\left[1\,\hat{-}\,V(\mathsf{O}_1\langle\mathbf{u},\mathbf{w}\rangle)\right]\hat{=}\,0.
\end{aligned}
\tag{5.22$_5$}
$$

11) ***APLADP for (5.23):*** By the variant of (5.7) with '**w**' and '**u**' in place of '**u**' and '**v**' respectively and by (5.7), it follows through the FFL that

$$
\begin{aligned}
&V(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}\left[\hat{:}_{\mathbf{y}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\right]\right]\hat{\cdot}\left[\hat{:}_{\mathbf{z}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\right]\right]\\
&\hat{=}\,\hat{:}_{\mathbf{y}}\left[\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\right]\hat{\cdot}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle)\right]\right]\\
&\hat{=}\,\hat{:}_{\mathbf{y}}\left[1\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle)\,\hat{-}\,V(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle)\,\hat{\cdot}\,V(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\right].
\end{aligned}
\tag{5.23$_1$}
$$

By (5.23_1) and by the variant of (5.7) with '**w**' in place of '**u**', another application of the FFL yields:

$$\begin{gathered}V\left(\mathsf{O}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\right]\\ \hat{=}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\ \hat{\cdot}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right]\\ \hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle,\end{gathered}\tag{5.23$_2$}$$

where

$$\begin{gathered}\mathbf{i}\langle\mathbf{x},\mathbf{u},\mathbf{v},\mathbf{w}\rangle\overset{\rightarrow}{\hat{=}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\\ \hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\\ \hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\\ \hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{-}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\\ \hat{=}1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right).\end{gathered}\tag{5.23$_3$}$$

By (5.23_1) and (5.23_3), relation (5.23_2) reduces to

$$\begin{gathered}V\left(\mathsf{O}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\right]\\ \hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\\ \hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right).\end{gathered}\tag{5.23$_4$}$$

Hence,

$$\begin{gathered}V\left(\mathsf{Bocardo}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\right]\\ \hat{=}\left[1\hat{-}V\left(\mathsf{O}_1\langle\mathbf{w},\mathbf{v}\rangle\right)\right]\hat{\cdot}\left[V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\right]\hat{=}0.\end{gathered}\tag{5.23$_4$}\bullet$$

Cmt 5.2. 1) In formulating and proving the separate items of Th 5.1, I utilize the fact that the initial validity integrons of certain PSI's turn out to be equal owing to the symmetry relations (3.16) with $m \triangleright 1$ and $n \triangleright 1$. Also, the initial validity integrons in (5.16) and (5.17), or (5.18) and (5.19), and their entire APLADP's are variants of each other with '**u**' and '**x**' exchanged with '**v**' and '**y**' respectively. Making use of this additional symmetry, the number of independent APLADP's could be reduced by two. Still, I have decided to present all the APLADP's straightforwardly for more clarity and also for the purpose of verification of the calculations. In this connection, the following general remark regarding decision procedures may be in order.

2) Given a relation of A_1 or of $\boldsymbol{\mathsf{A}}_1$, there are, as a rule, several decision procedures for the relation, which differ in order of calculations. All the procedures are accomplished in accordance with the same rules, so that they must result in the

same validity index of the relation. Still, some of the procedures turn out to be shorter and simpler than some others. For instance, in calculating the validity-integron $V(\text{Bocardo}_1\langle \mathbf{u}, \mathbf{w}, \mathbf{v}\rangle)$, I might, have calculated $V(\neg \text{O}_1\langle \mathbf{w}, \mathbf{v}\rangle) \hat{\cdot} V(\neg \text{A}_1\langle \mathbf{w}, \mathbf{u}\rangle)$ first, and then

$$[V(\neg \text{O}_1\langle \mathbf{w}, \mathbf{v}\rangle) \hat{\cdot} V(\neg \text{A}_1\langle \mathbf{w}, \mathbf{u}\rangle)] \hat{\cdot} V(\text{O}_1\langle \mathbf{u}, \mathbf{v}\rangle).$$

However, the order of calculations, which I have actually adopted, allows obtaining the final result simpler and faster. A like remark applies, *mutatis mutandis*, to some other individual decision procedures comprised in the proof of the theorem. Thus, in spite of the fact that any AEADP or APLADP is mechanical, choice of the optimal ADP is a kind of art that is acquired by experience.•

Cmt 5.3. According to Th 5.1, the validity indices of the asymmetric PSI's:

$$\begin{array}{l} \text{Barapti}_1\langle \mathbf{u}, \mathbf{w}, \mathbf{v}\rangle, \text{Bamalip}_1\langle \mathbf{u}, \mathbf{w}, \mathbf{v}\rangle, \\ \text{Fesapo}_1\langle \mathbf{u}, \mathbf{w}, \mathbf{v}\rangle, \text{Felapton}_1\langle \mathbf{u}, \mathbf{w}, \mathbf{v}\rangle \end{array} \tag{5.24}$$

turn out to identically equal neither to 0 nor to 1, so that these four PSI's are udeterological (neutral) schemata or, using the pertinent terminology of Df 5.1(3,4), they are vav-neutral, or udeterological, PSI's (NPSI's or UPSI's). This means that the range of any of the four UPSI's contains ER's (euautographic relations) of A_1 of all the three v*alidity classes* (*validity values*): *kyrologies* (*valid relations*), *antikyrologies* (*antivalid relations*), and *udeterologies* (*vav-neutral relations*). The fact that the range of any of the four UPSI's on the list (5.24) contains udeterologies of A_1 will be made explicit in subsection 6.2. The fact that the above mentioned range contains kyrologies and antikyrologies of A_1 will be demonstrated in Th 5.3 that is stated and proved below this comment. By contrast, the validity indices of all remaining asymmetric PSI's at $n \triangleright 1$, 15 PSI's altogether, turn out to be 0, so that these are valid, or kyrological, PSI's (VPSI's or KPSI's). The range of any KPSI contains only valid, or kyrilogical, ESI's (VESI's or KESI's) of A_1.

In connection with the above result of Th 5.1, it will be recalled that Hilbert and Ackermann [1950, pp. 48–54, 53ff] have shown that 15 FCS's other than Darapti(u, w, v) (latter Barapti(u, w, v)), Bamalip(u, w, v), Felapton(u, w, v), and Fesapo(u, w, v) are deducible from Boolean algebra, whereas the above four are not. It is clear that the peculiar character of the above four FCS's is relevant to the fact that they can be traced back to the UPSI's on the list (5.24) or, more specifically, to certain vav-neutral, or udeterological, ESI's (NESI's or UESI's) in the ranges the UPSI's, while all concrete instances of the remaining 15 PSI's are kyrologies (valid euautographic relations) of A_1. The semantic (catlogographic) conditions (1.49)–(1.51) of deducibility of the four peculiar FCS's from $\mathbf{A}_1$ via A_1 will be established rigorously in the next section. •

Th 5.2: ***The PLMT's and validity indices of symmetric PSI's.***

1°) Group AAA&AAI

$$\begin{gathered} V\left(\mathsf{Barbara}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left((1\mathsf{AAA})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \\ \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \\ \hat{=} \left[\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right] \\ \hat{\cdot} \left[1 \hat{-} \hat{\cdot}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right] \hat{=} 0. \end{gathered} \tag{5.25}$$

$$\begin{gathered} V\left(\mathsf{Bamalip}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{Barapti}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \\ \hat{=} V\left((4\mathsf{AAI})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left((3\mathsf{AAI})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \\ \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{v},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \\ \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \\ \hat{=} \left[\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right] \\ \hat{\cdot} \left[\hat{\cdot}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right] \hat{=} 0. \end{gathered} \tag{5.26}$$

2°) Group AII&IAI

$$\begin{gathered} V\left(\mathsf{Datisi}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{Darii}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \\ \hat{=} V\left((3\mathsf{AII})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left((1\mathsf{AII})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \\ \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{I}_4\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \\ \hat{=} V\left(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{I}_4\langle\mathbf{u},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \\ \hat{=} \left[\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right] \hat{\cdot} \left[1 \hat{-} \hat{\cdot}_{\mathbf{y}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right] \\ \hat{\cdot} \left[\hat{\cdot}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right] \hat{=} 0. \end{gathered} \tag{5.27}$$

$$\begin{gathered} V\left(\mathsf{Dimatis}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{Disamis}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \\ \hat{=} V\left((4\mathsf{IAI})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left((3\mathsf{IAI})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \\ \hat{=} V\left(\neg\mathsf{I}_4\langle\mathbf{v},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \\ \hat{=} V\left(\neg\mathsf{I}_4\langle\mathbf{w},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right) \\ \hat{=} \left[1 \hat{-} \hat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right] \\ \hat{\cdot} \left[\hat{\cdot}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right] \hat{=} 0. \end{gathered} \tag{5.28}$$

3°) Group EAE&AEE

$$\begin{aligned}
&V\left(\mathrm{Cesare}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\mathrel{\hat{=}}V\left(\mathrm{Celarent}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}V\left((2\mathrm{EAE})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\mathrel{\hat{=}}V\left((1\mathrm{EAE})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}V\left(\neg\mathrm{E}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathrm{A}_4\langle\mathbf{u},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathrm{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}V\left(\neg\mathrm{E}_4\langle\mathbf{w},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathrm{A}_4\langle\mathbf{u},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathrm{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}\left[\hat{\bullet}_{\mathbf{x}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\mathbin{\hat{\cdot}}\left[\hat{\bullet}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}\hat{\bullet}_{\mathbf{z}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\mathrel{\hat{=}}0.
\end{aligned}\tag{5.29}$$

$$\begin{aligned}
&V\left(\mathrm{Calemes}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\mathrel{\hat{=}}V\left(\mathrm{Camestres}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}V\left((4\mathrm{AEE})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\mathrel{\hat{=}}V\left((2\mathrm{AEE})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}V\left(\neg\mathrm{A}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathrm{E}_4\langle\mathbf{w},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathrm{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}V\left(\neg\mathrm{A}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathrm{E}_4\langle\mathbf{u},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathrm{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}\left[\hat{\bullet}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\mathbin{\hat{\cdot}}\left[\hat{\bullet}_{\mathbf{y}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}\hat{\bullet}_{\mathbf{z}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\mathrel{\hat{=}}0.
\end{aligned}\tag{5.30}$$

4°) Group EAO

$$\begin{aligned}
&V\left(\mathrm{Fesapo}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\mathrel{\hat{=}}V\left(\mathrm{Felapton}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}V\left((4\mathrm{EAO})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\mathrel{\hat{=}}V\left((3\mathrm{EAO})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}V\left(\neg\mathrm{E}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathrm{A}_4\langle\mathbf{w},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathrm{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}V\left(\neg\mathrm{E}_4\langle\mathbf{w},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathrm{A}_4\langle\mathbf{w},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathrm{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}\left[\hat{\bullet}_{\mathbf{x}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\mathbin{\hat{\cdot}}\left[\hat{\bullet}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right]\\
&\mathbin{\hat{\cdot}}\left[\hat{\bullet}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\mathrel{\hat{=}}0.
\end{aligned}\tag{5.31}$$

5°) Group EIO

$$
\begin{aligned}
&V\left(\mathsf{Fresison}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{Feriso}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\mathsf{Festino}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{Ferio}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left((\mathsf{4EIO})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((\mathsf{3EIO})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left((\mathsf{2EIO})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((\mathsf{1EIO})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{E}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{I}_4\langle\mathbf{w},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{E}_4\langle\mathbf{w},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{I}_4\langle\mathbf{w},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{E}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{I}_4\langle\mathbf{u},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{E}_4\langle\mathbf{w},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{I}_4\langle\mathbf{u},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\dot{\wedge}\left[1\hat{-}\dot{\bigwedge}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}
\tag{5.32}
$$

6°) Group AOO&OAO

$$
\begin{aligned}
&V\left(\mathsf{Baroco}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((\mathsf{2AOO})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{A}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\dot{\wedge}\left[1\hat{-}\dot{\bigwedge}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}
\tag{5.33}
$$

$$
\begin{aligned}
&V\left(\mathsf{Bocardo}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left((\mathsf{3AOO})_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{w},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[1\hat{-}\dot{\bigwedge}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right)\right]\right]\\
&\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}
\tag{5.34}
$$

Proof:

1) ***APLADP for (5.25):*** By the variants of (5.11) with ‘**w**’ in place of ‘**u**’ and by that with ‘**w**’ in place of ‘**v**’, it follows through the FFL (Fusion and Fission Law) (II.4.29) that

$$
\begin{aligned}
&V\left(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle\right)\dot{\wedge}V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle\right)\\
&\hat{=}\left[\dot{\bigwedge}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}(\mathbf{x},\mathbf{w})\right)\dot{\wedge}V\left(\neg\mathbf{P}(\mathbf{x},\mathbf{v})\right)\right]\right]\dot{\wedge}\left[\dot{\bigwedge}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[\left[V\left(\neg\mathbf{P}(\mathbf{x},\mathbf{w})\right)\dot{\wedge}V\left(\neg\mathbf{P}(\mathbf{x},\mathbf{v})\right)\right]\dot{\wedge}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\dot{\bigwedge}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\dot{\wedge}V\left(\neg\mathbf{P}(\mathbf{x},\mathbf{v})\right)\dot{\wedge}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right].
\end{aligned}
\tag{5.25_1}
$$

By (5.25_1) and (5.11), another application of the FFL yields:

$$
\begin{aligned}
&[V(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle)]\hat{\cdot}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}[\hat{\bullet}_{\mathbf{x}}[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}(\mathbf{x},\mathbf{v}))\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)]]\\
&\hat{\cdot}[\hat{\bullet}_{\mathbf{z}}[V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)]]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}[[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}(\mathbf{x},\mathbf{v}))\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)]\\
&\hat{\cdot}[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}(\mathbf{x},\mathbf{v}))\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)]\\
&\hat{=}V(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle).
\end{aligned}
\tag{5.25$_2$}
$$

Hence, by (5.25$_2$) and (5.10),

$$
\begin{aligned}
&V(\mathsf{Barbara}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle)\hat{=}[V(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle)]\hat{\cdot}V(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}[V(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle)]\hat{\cdot}[1\hat{-}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle)]\hat{=}0.
\end{aligned}
\tag{5.25$_3$}
$$

2) ***APLADP for (5.26):*** By the pertinent variants of (5.11) and by (5.25$_1$),

$$
\begin{aligned}
&V(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{u}\rangle)\hat{=}V(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle)\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)].
\end{aligned}
\tag{5.26$_1$}
$$

Hence, by (5.26$_1$) and (5.9), it follows through the FFL that

$$
\begin{aligned}
&V(\mathsf{Barapti}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle)\hat{=}[V(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{u}\rangle)]\hat{\cdot}V(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}[\hat{\bullet}_{\mathbf{x}}[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)]]\\
&\hat{\cdot}[\hat{\bullet}_{\mathbf{z}}[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)]]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}[[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)]\\
&\hat{\cdot}[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\\
&\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{=}0.
\end{aligned}
\tag{5.26$_2$}
$$

3) ***APLADP for (5.27):*** By the variant of (5.11) with '**w**' in place of '**u**' and by (5.9), it follows through the FFL that

$$
\begin{aligned}
&V(\neg\mathsf{A}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}V(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}[\hat{\bullet}_{\mathbf{x}}[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]]\hat{\cdot}[\hat{\bullet}_{\mathbf{z}}[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)]]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}[[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]\hat{\cdot}[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)]]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}[1\hat{-}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)]]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}[V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}V(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)].
\end{aligned}
\tag{5.27$_1$}
$$

By (5.27$_1$) and by the variant of (5.9) with '**w**' in place of '**u**', another application of the FFL yields:

$$
\begin{aligned}
&\left[V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\mathbin{\hat{\cdot}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{w}\rangle\right)\\
&\mathrel{\hat{=}}\left[\hat{\bullet}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\quad\mathbin{\hat{\cdot}}\left[\hat{\bullet}_{\mathbf{y}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\mathrel{\hat{=}}\hat{\bullet}_{\mathbf{x}}\left[\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right.\\
&\quad\left.\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\mathrel{\hat{=}}\hat{\bullet}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\\
&\mathrel{\hat{=}}V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{v}\rangle\right).
\end{aligned}
\tag{5.27$_2$}
$$

Hence, by (5.27$_2$),

$$
\begin{aligned}
V\left(\mathsf{Darii}_4\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)&\mathrel{\hat{=}}\left[V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\mathbin{\hat{\cdot}}V\left(\neg\mathsf{I}_4\langle \mathbf{u},\mathbf{w}\rangle\right)\\
&\mathrel{\hat{=}}\left[\left[V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\right]\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{w}\rangle\right)\right]\mathrel{\hat{=}}0.
\end{aligned}
\tag{5.27$_3$}
$$

4) ***APLADP for (5.28):*** By the variant of (5.11) with '**w**' in place of '**u**' and '**u**' in place of '**v**', and also by (5.9), it follows through the FFL that

$$
\begin{aligned}
&V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\\
&\mathrel{\hat{=}}\left[\hat{\bullet}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\right]\right]\mathbin{\hat{\cdot}}\left[\hat{\bullet}_{\mathbf{z}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle \mathbf{z},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\mathrel{\hat{=}}\hat{\bullet}_{\mathbf{y}}\left[\left[V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\right]\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right)\right]\right]\\
&\mathrel{\hat{=}}\hat{\bullet}_{\mathbf{y}}\left[\left[V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\right]\mathbin{\hat{\cdot}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right)\right]\right]\\
&\mathrel{\hat{=}}\hat{\bullet}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\right].
\end{aligned}
\tag{5.28$_1$}
$$

By (5.28$_1$) and by the variant of (5.9) with '**w**' in place of '**u**', another application of FFL yields:

$$
\begin{aligned}
&V\left(\mathsf{I}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}\left[V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\\
&\mathrel{\hat{=}}\left[\hat{\bullet}_{\mathbf{x}}\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\quad\mathbin{\hat{\cdot}}\left[\hat{\bullet}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\mathrel{\hat{=}}\hat{\bullet}_{\mathbf{x}}\left[\left[1\mathbin{\hat{-}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\right.\\
&\quad\left.\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\\
&\mathrel{\hat{=}}\hat{\bullet}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\\
&\mathrel{\hat{=}}V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{v}\rangle\right).
\end{aligned}
\tag{5.28$_2$}
$$

Hence, by (5.28$_2$),

$$
\begin{aligned}
V\left(\mathsf{Disamis}_4\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)&\mathrel{\hat{=}}V\left(\neg\mathsf{I}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\mathbin{\hat{\cdot}}\left[V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\\
&\mathrel{\hat{=}}\left[1\mathbin{\hat{-}}V\left(\mathsf{I}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\right]\mathbin{\hat{\cdot}}\left[V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{u}\rangle\right)\mathbin{\hat{\cdot}}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\mathrel{\hat{=}}0.
\end{aligned}
\tag{5.28$_3$}
$$

5) *APLADP for (5.29):* By the variant of (5.9) with ‘**w**’ in place of ‘**u**’ and by the variant of (5.11) ‘**w**’ in place of ‘**v**’, and also by (5.9), it follows through the FFL that

$$
\begin{aligned}
&\mathsf{V}(\neg\mathsf{E}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle)\\
&\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[1\dot{-}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}\left[\mathsf{V}(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\left[1\dot{-}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\hat{\cdot}\left[\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}\left[1\dot{-}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right].
\end{aligned}
\tag{5.29$_1$}
$$

By (5.29$_1$) and (5.9), another application of the FFL yields:

$$
\begin{aligned}
&\left[\mathsf{V}(\neg\mathsf{E}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle)\right]\hat{\cdot}\mathsf{V}(\neg\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\right]\\
&\quad\hat{\cdot}\left[\hat{\bullet}_{\mathbf{z}}\left[1\dot{-}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\left[\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\right.\\
&\quad\left.\hat{\cdot}\left[1\dot{-}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\\
&\hat{=}\mathsf{V}(\neg\mathsf{E}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle).
\end{aligned}
\tag{5.29$_2$}
$$

Hence, by (5.29$_2$),

$$
\begin{aligned}
&\mathsf{V}(\mathsf{Celarent}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle)\hat{=}\left[\mathsf{V}(\neg\mathsf{E}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle)\right]\hat{\cdot}\mathsf{V}(\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle)\\
&\hat{=}\left[\mathsf{V}(\neg\mathsf{E}_4\langle\mathbf{w},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{w}\rangle)\right]\hat{\cdot}\left[1\dot{-}\mathsf{V}(\neg\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle)\right]\hat{=}0.
\end{aligned}
\tag{5.29$_3$}
$$

6) *APLADP for (5.30):* By the variant of (5.11) with ‘**v**’ and ‘**w**’ in place of ‘**u**’ and ‘**v**’ respectively and by the variant of (5.9) with ‘**w**’ in place of ‘**v**’, it follows through the FFL that

$$
\begin{aligned}
&\mathsf{V}(\neg\mathsf{A}_4\langle\mathbf{v},\mathbf{w}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathsf{E}_4\langle\mathbf{u},\mathbf{w}\rangle)\\
&\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}\left[1\dot{-}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\left[\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\hat{\cdot}\left[1\dot{-}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\hat{\cdot}\left[1\dot{-}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\right]\right]\\
&\hat{=}\hat{\bullet}_{\mathbf{x}}\left[\mathsf{V}(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle)\hat{\cdot}\mathsf{V}(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle)\right].
\end{aligned}
\tag{5.30$_1$}
$$

By (5.30$_1$) and (5.9), another application of the FFL yields:

$$
\begin{aligned}
&\left[V\left(\neg \mathsf{A}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{E}_4\langle\mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}V\left(\neg \mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\quad\hat{\cdot}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{z}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right.\\
&\quad\left.\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\\
&\hat{=}V\left(\neg \mathsf{A}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{E}_4\langle\mathbf{u},\mathbf{w}\rangle\right).
\end{aligned}
\tag{5.30$_2$}
$$

Hence, by (5.30$_2$),

$$
\begin{aligned}
&V\left(\mathsf{Camestres}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}\left[V\left(\neg \mathsf{A}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{E}_4\langle\mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}V\left(\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\quad\hat{=}\left[V\left(\neg \mathsf{A}_4\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{E}_4\langle\mathbf{u},\mathbf{w}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}V\left(\neg \mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\right]\hat{=}0.
\end{aligned}
\tag{5.30$_3$}
$$

7) ***APLADP for (5.31):*** By the variant of (5.9) with '**w**' in place of '**u**' and by the variant of (5.11) with '**w**' and '**u**' in place of '**u**' and '**w**' respectively, it follows through the FFL that

$$
\begin{aligned}
&V\left(\neg \mathsf{E}_4\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{A}_4\langle\mathbf{w},\mathbf{u}\rangle\right)\\
&\hat{=}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{y}}\left[V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\right)\right]\right]\\
&\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\hat{\cdot}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\right]\right]\\
&\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right].
\end{aligned}
\tag{5.31$_1$}
$$

By (5.31$_1$) and (5.11), another application of the FFL yields:

$$
\begin{aligned}
&V\left(\mathsf{Felapton}_4\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\\
&\hat{=}\left[V\left(\neg \mathsf{E}_4\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathsf{A}_4\langle\mathbf{w},\mathbf{u}\rangle\right)\right]\hat{\cdot}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\quad\hat{\cdot}\left[\hat{\boldsymbol{\cdot}}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\boldsymbol{\cdot}}_{\mathbf{x}}\left[\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right.\\
&\quad\left.\hat{\cdot}\left[V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\hat{=}0.
\end{aligned}
\tag{5.31$_2$}
$$

8) ***APLADP for (5.32):*** By the variant of (5.9) with '**w**' in place of '**u**' and by (5.8), it follows through the FFL that

$$
\begin{aligned}
&V\left(\neg \mathsf{E}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\cdot}_{\mathbf{x}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\cdot}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle \mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\hat{\cdot}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right].
\end{aligned}
\qquad (5.32_1)
$$

By (5.32_1) and by the variant of (5.9) with ‘**w**’ in place of ‘**v**’, another application of the FFL yields:

$$
\begin{aligned}
&\left[V\left(\neg \mathsf{E}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\hat{\cdot}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{w}\rangle\right)\\
&\hat{=}\left[\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{\cdot}\left[\hat{\cdot}_{\mathbf{y}}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right.\\
&\left.\hat{\cdot}\left[1\hat{-}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\\
&V\left(\neg \mathsf{E}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right).
\end{aligned}
\qquad (5.32_2)
$$

Hence, by (5.32_2),

$$
\begin{aligned}
&V\left(\mathsf{Ferio}_4\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}\left[V\left(\neg \mathsf{E}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\hat{\cdot}V\left(\neg \mathsf{I}_4\langle \mathbf{u},\mathbf{w}\rangle\right)\\
&\hat{=}\left[V\left(\neg \mathsf{E}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}V\left(\mathsf{I}_4\langle \mathbf{u},\mathbf{w}\rangle\right)\right]\hat{=}0.
\end{aligned}
\qquad (5.32_3)
$$

9) ***APLADP for (5.33):*** By the variant of (5.11) with ‘**v**’ and ‘**w**’ in place of ‘**u**’ and ‘**v**’ respectively and also by (5.8), it follows through the FFL that

$$
\begin{aligned}
&V\left(\neg \mathsf{A}_4\langle \mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\cdot}_{\mathbf{z}}\left[V\left(\neg\mathbf{P}\langle \mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\hat{\cdot}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right].
\end{aligned}
\qquad (5.33_1)
$$

By (5.33_1) and by the variant of (5.11) with ‘**w**’ in place of ‘**v**’, another application of the FFL yields:

$$
\begin{aligned}
&\left[V\left(\neg \mathsf{A}_4\langle \mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{w}\rangle\right)\\
&\hat{=}\left[\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\quad\hat{\cdot}\left[\hat{\cdot}_{\mathbf{y}}\left[V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[\left[V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right.\\
&\quad\left.\hat{\cdot}\left[V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\\
&\hat{=}V\left(\neg \mathsf{A}_4\langle \mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right).
\end{aligned}
\tag{5.33$_2$}
$$

Hence, by (5.33$_2$),

$$
\begin{aligned}
V\left(\mathsf{Baroco}_4\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)&\hat{=}\left[V\left(\neg \mathsf{A}_4\langle \mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\hat{\cdot}V\left(\neg \mathsf{O}_4\langle \mathbf{u},\mathbf{w}\rangle\right)\\
&\hat{=}\left[V\left(\neg \mathsf{A}_4\langle \mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\hat{\cdot}\left[1\hat{-}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{w}\rangle\right)\right]\hat{=}0.
\end{aligned}
\tag{5.33$_3$}
$$

10) ***APLADP for (5.34):*** By the variant of (5.11) with ‘**w**’ and ‘**u**’ in place of ‘**u**’ and ‘**v**’ respectively and also by (5.8), it follows through the FFL that

$$
\begin{aligned}
&V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\\
&\hat{=}\left[\hat{\cdot}_{\mathbf{y}}\left[V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right)\right]\right]\hat{\cdot}\left[\hat{\cdot}_{\mathbf{z}}\left[V\left(\neg \mathbf{P}\langle \mathbf{z},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{z},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{y}}\left[\left[V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right)\right]\hat{\cdot}\left[V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{y}}\left[V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\right].
\end{aligned}
\tag{5.34$_1$}
$$

By (5.34$_1$) and by the variant of (5.11) with ‘**w**’ in place of ‘**u**’, another application of the FFL yields:

$$
\begin{aligned}
&V\left(\mathsf{O}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\\
&\hat{=}\left[\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\
&\quad\hat{\cdot}\left[\hat{\cdot}_{\mathbf{y}}\left[V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{y},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[\left[V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\right]\right.\\
&\quad\left.\hat{\cdot}\left[V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\
&\hat{=}\hat{\cdot}_{\mathbf{x}}\left[V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg \mathbf{P}\langle \mathbf{x},\mathbf{w}\rangle\right)\right]\\
&\hat{=}V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right).
\end{aligned}
\tag{5.34$_2$}
$$

Hence, by (5.34$_2$),

$$
\begin{aligned}
V\left(\mathsf{Bocardo}_4\langle \mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)&\hat{=}V\left(\neg \mathsf{O}_4\langle \mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}\left[V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\\
&\hat{=}\left[1\hat{-}\mathsf{O}_4\langle \mathbf{w},\mathbf{v}\rangle\right]\hat{\cdot}\left[V\left(\neg \mathsf{A}_4\langle \mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_4\langle \mathbf{u},\mathbf{v}\rangle\right)\right]\hat{=}0.
\end{aligned}
\tag{5.34$_3$}\bullet
$$

Cmt 5.4 (*mutatis mutandis*, the same as Cmt 5.3(1)). In formulating and proving Th 5.2, I utilize the fact that the initial validity integrons of certain PSI’s turn out to be equal owing to the symmetry relations (3.16) with $m \triangleright 4$ and $n \triangleright 4$ or owing to the symmetry relations (3.17). Also, the initial validity integrons in (5.27) and (5.28), or (5.29) and (5.30), and their entire APLADP's are variants of each other with '**u**' and '**x**' exchanged with '**v**' and '**y**' respectively.•

Th 5.3. a) If

$$V\left(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{=}0, \tag{5.35}$$

i.e. if $\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$ is a kyrology, then

$$\begin{aligned}&V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\&\hat{=}V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}0,\end{aligned} \tag{5.36}$$

$$\begin{aligned}&V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\&\hat{=}V\left(\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1,\end{aligned} \tag{5.36'}$$

$$\mathsf{J}^1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\hat{=}\mathsf{J}^2\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\hat{=}\mathsf{J}^3\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\hat{=}0. \tag{5.37}$$

b) If

$$V\left(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right)\hat{=}1, \tag{5.38}$$

i.e. if $\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$ is an antikyrology, then

$$\begin{aligned}&V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\&\hat{=}V\left(\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}0,\end{aligned} \tag{5.39}$$

$$\begin{aligned}&V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\\&\hat{=}V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1\end{aligned} \tag{5.39'}$$

$$\mathsf{J}^1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\hat{=}\mathsf{J}^2\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\hat{=}\mathsf{J}^3\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\hat{=}1. \tag{5.40}$$

It is understood that all the above relations are invariant under replacement of '**x**', '**y**', '**u**', '**v**', and '**w**' with any other mutually different PLOT's subject to Ax 2.1 and Cnv 2.1.

Proof: With allowance for the last reservation regarding '**x**' and '**y**', identities (5.6)–(5.11) and also the trains of identities (5.14), (5.15), and (5.20), each starting from the third expression from end, can be developed further thus:

$$V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1\mathbin{\hat{-}}\boldsymbol{\hat{\cdot}}_{\mathbf{z}}\left[1\mathbin{\hat{-}}1\mathbin{\hat{\cdot}}0\right]\hat{=}0, \tag{5.6a}$$

$$V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1, \tag{5.7a}$$

$$V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1\mathbin{\hat{-}}\boldsymbol{\hat{\cdot}}_{\mathbf{z}}\left[1\mathbin{\hat{-}}1\mathbin{\hat{\cdot}}1\right]\hat{=}1, \tag{5.8a}$$

$$V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}0\,, \tag{5.9a}$$

$$V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1\,\hat{-}\,\hat{\bullet}_{\mathbf{z}}\left[1\,\hat{\cdot}\,1\right]\hat{=}0\,, \tag{5.10a}$$

$$V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1, \tag{5.11a}$$

$$\begin{aligned}\mathsf{J}^1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle &\vec{\hat{=}}\,\hat{\bullet}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[1\,\hat{-}\,V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}\,V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\ &\hat{=}\,\hat{\bullet}_{\mathbf{x}}\left[0\,\hat{\cdot}\left[1\,\hat{-}\,1\,\hat{\cdot}\,1\right]\right]\hat{=}0,\end{aligned} \tag{5.14a}$$

$$\begin{aligned}\mathsf{J}^2\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle &\hat{=}\,\hat{\bullet}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\,\hat{-}\,V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}\,V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\ &\hat{=}\,\hat{\bullet}_{\mathbf{x}}\left[0\,\hat{\cdot}\left[1\,\hat{-}\,0\,\hat{\cdot}\,1\right]\right]\hat{=}0,\end{aligned} \tag{5.15a}$$

$$\begin{aligned}\mathsf{J}^3\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle &\hat{=}\,\hat{\bullet}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[1\,\hat{-}\,V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}\,V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\ &\hat{=}\,\hat{\bullet}_{\mathbf{x}}\left[0\,\hat{\cdot}\left[1\,\hat{-}\,1\,\hat{\cdot}\,0\right]\right]\hat{=}0\end{aligned} \tag{5.20a}$$

if (5.35) holds, or thus:

$$V\left(\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1\,\hat{-}\,\hat{\bullet}_{\mathbf{z}}\left[1\,\hat{-}\,0\,\hat{\cdot}\,1\right]\hat{=}0\,, \tag{5.6b}$$

$$V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1, \tag{5.7b}$$

$$V\left(\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1\,\hat{-}\,\hat{\bullet}_{\mathbf{z}}\left[1\,\hat{-}\,0\,\hat{\cdot}\,0\right]\hat{=}0\,, \tag{5.8b}$$

$$V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_1\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\mathsf{I}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{E}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1, \tag{5.9b}$$

$$V\left(\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}1\,\hat{-}\,\hat{\bullet}_{\mathbf{z}}\left[0\,\hat{\cdot}\,0\right]\hat{=}1, \tag{5.10b}$$

$$V\left(\mathsf{O}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_4\langle\mathbf{u},\mathbf{v}\rangle\right)\hat{=}0\,, \tag{5.11b}$$

$$\begin{aligned}\mathsf{J}^1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle &\vec{\hat{=}}\,\hat{\bullet}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[1\,\hat{-}\,V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}\,V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\ &\hat{=}\,\hat{\bullet}_{\mathbf{x}}\left[1\,\hat{\cdot}\left[1\,\hat{-}\,0\,\hat{\cdot}\,0\right]\right]\hat{=}1,\end{aligned} \tag{5.14b}$$

$$\begin{aligned}\mathsf{J}^2\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle &\hat{=}\,\hat{\bullet}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}\left[1\,\hat{-}\,V\left(\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}\,V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\right]\right]\\ &\hat{=}\,\hat{\bullet}_{\mathbf{x}}\left[1\,\hat{\cdot}\left[1\,\hat{-}\,1\,\hat{\cdot}\,0\right]\right]\hat{=}1,\end{aligned} \tag{5.15b}$$

$$\begin{aligned}\mathsf{J}^3\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle &\hat{=}\,\hat{\bullet}_{\mathbf{x}}\left[V\left(\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}\left[1\,\hat{-}\,V\left(\neg\mathbf{P}\langle\mathbf{x},\mathbf{u}\rangle\right)\hat{\cdot}\,V\left(\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\right]\right]\\ &\hat{=}\,\hat{\bullet}_{\mathbf{x}}\left[1\,\hat{\cdot}\left[1\,\hat{-}\,0\,\hat{\cdot}\,1\right]\right]\hat{=}1\end{aligned} \tag{5.20b}$$

if (5.38) holds. QED.•

Cmt 5.5. 1) Under either condition (5.35) or (5.38), all items of Ths 5.1 and 5.2 remain valid, as must be. Particularly, by (5.7a)–(5.9a) or by (5.7b)–(5.9b), it immediately follows from the initial, factorized expressions in (5.14), (5.15), and (5.20) respectively that

$$V\left(\mathsf{Barapti}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} 1 \hat{\cdot} 1 \hat{\cdot} 0 \hat{=} 0, \tag{5.14a'}$$

$$V\left(\mathsf{Bamalip}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} 1 \hat{\cdot} 1 \hat{\cdot} 0 \hat{=} 0, \tag{5.15a'}$$

$$V\left(\mathsf{Fesapo}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{Felapton}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} 0 \hat{\cdot} 1 \hat{\cdot} 1 \hat{=} 0, \tag{5.20a'}$$

if (5.35) holds, or that

$$V\left(\mathsf{Barapti}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} 1 \hat{\cdot} 1 \hat{\cdot} 1 \hat{=} 1, \tag{5.14b'}$$

$$V\left(\mathsf{Bamalip}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{A}_1\langle\mathbf{v},\mathbf{w}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{I}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} 1 \hat{\cdot} 1 \hat{\cdot} 1 \hat{=} 1, \tag{5.15b'}$$

$$V\left(\mathsf{Fesapo}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\mathsf{Felapton}_1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right) \hat{=} V\left(\neg\mathsf{E}_1\langle\mathbf{w},\mathbf{v}\rangle\right) \hat{\cdot} V\left(\neg\mathsf{A}_1\langle\mathbf{w},\mathbf{u}\rangle\right) \hat{\cdot} V\left(\mathsf{O}_1\langle\mathbf{u},\mathbf{v}\rangle\right) \hat{=} 1 \hat{\cdot} 1 \hat{\cdot} 1 \hat{=} 1, \tag{5.20b'}$$

if (5.38) holds. Identities (5.14a')–(5.20a') are in agreement with (5.14a)–(5.20a), whereas (5.14b')–(5.20b') are in agreement with (5.14b)–(5.20b).

2) It follows from (5.14), (5.15), and (5.20) that identities (5.40) also hold if

$$V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right) \hat{=} 0, \tag{5.41}$$

i.e. if $\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle$ is a kyrology. At the same time, if (5.35) holds then

$$V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right) \hat{=} \hat{\cdot}_{\mathbf{x}} V\left(\mathbf{P}\langle\mathbf{x},\mathbf{y}\rangle\right) \hat{=} \hat{\cdot}_{\mathbf{x}} 0 \hat{=} 0, \tag{5.42}$$

i.e. (5.35) implies (5.41), but not vice versa.

3) If (5.35) holds then one of the three pertinent multipliers $V(\neg\mathsf{U}_n)$, $V(\neg\mathsf{V}_n)$, and $V(\neg\mathsf{W}_n)$ occurring in any item of Th 5.1 or 5.2 equals 0 owing to (5.36). If (5.38) holds then one of the three pertinent multipliers $V(\neg\mathsf{U}_n)$, $V(\neg\mathsf{V}_n)$, and $V(\neg\mathsf{W}_n)$ occurring in any item of Th 5.1 or 5.2, except (5.14), (5.15), and (5.20), equals 0 owing to (5.39) and (5.39'), whereas all the three pertinent multipliers $V(\neg\mathsf{U}_n)$, $V(\neg\mathsf{V}_n)$, and $V(\neg\mathsf{W}_n)$ occurring in any one of the items (5.14), (5.15), and (5.20) of Th 5.1 equal 1 owing to (5.39), as was demonstrated above in item 1 of this Comment.

4) In proving each separate item (train of identities) of Ths 5.1 and 5.2, I proceed from the respective instance of the schema (5.5), whereas the pertinent

product $V(\neg \mathsf{U}_n) \hat{\cdot} V(\neg \mathsf{V}_n) \hat{\cdot} V(\mathsf{W}_n)$ is calculated with the help of the APLADM (advance panlogographic algebraic decision method) of $\mathbf{A}_1$ with allowance for the specific structure of each multiplier and without making any additional assumptions. In accordance with Ax 2.1 and Cnv 2.1, '$\mathbf{P}\langle \mathbf{x}, \mathbf{y} \rangle$' (e.g.) is a PLR of $\mathbf{A}_1$, whose range is a class of ER's of A_1, each of which contains at least two *different* EOT's (euautographic ordinary atomic terms) of the ranges of the PLOT's '**x**' and '**y**'. At the same time, the PLR '$\mathbf{P}\langle \mathbf{x}, \mathbf{y} \rangle$' is vav-neutral (kyrological), so that it may assume, as its concrete instances (interpretands, accidental denotata), ER's of A_1 of all three kinds: *valid*, *antivalid*, and *vav-neutral* ones, i.e. *kyrologies*, *antikyrologies*, and *udeterologies*. Consequently, the equality $V(\mathbf{P}\langle \mathbf{x}, \mathbf{y} \rangle) \hat{=} 0$ is a *condition* that '$\mathbf{P}\langle \mathbf{x}, \mathbf{y} \rangle$' may take only on kyrologies of A_1 as its instances, whereas $V(\mathbf{P}\langle \mathbf{x}, \mathbf{y} \rangle) \hat{=} 1$ is an alternative *condition* that '$\mathbf{P}\langle \mathbf{x}, \mathbf{y} \rangle$' may take only on antikyrologies of A_1 as its instances. By proving Th 5.3, I have just illustrated that in either of the above two cases each item of Ths 5.1 and 5.2 is valid as it must be. However, these two cases of Th 5.1 and the whole of Th 5.2 are irrelevant to Aristotelian logic proper. In the next subsection, these theorems will be specified in accordance with Df 2.4, so that '$\mathbf{P}\langle \mathbf{x}, \mathbf{y} \rangle$' will be replaced with the *vav-neutral* (*udeterological*) *structural PLR* (*StPLR*) '$\mathbf{F}(\mathbf{x}, \mathbf{y})$', which may assume only vav-neutral (udeterological) ER's of A_1 as its concrete instances (interpretands). This instance of Th 5.1 (and of its of its equivalent counterparts corresponding to $n \vec{\in} \{2,3\}$ instead of $n \vec{\hat{=}} 1$) has direct relevance to Aristotelian logic.•

5.3. A summary

1) Each displayed train of identities of Ths 5.1 and 5.2 is the *panlogographic master*, or *decision, theorem* (*PLMT* or *PLDT*) for the respective *slave PSI* (*panlogographic syllogistic implication*) or *PSI's*. At the same time, in accordance with Th 3.1 and Cmts 3.1 and 5.1, every one of 11 PLMT's (PLDT's) (5.13)–(5.23) of Th 5.1 applies also with either subscript '$_2$' or '$_3$' in place of '$_1$'. Therefore, 11 PLMT's (5.13)–(5.23) comprised in Th 5.1 and 10 PLMT's (5.25)–(5.34) comprised in Th 5.2 can be summarized and generalized as follows:

$$V\left(\left(\mathsf{1UVW}\right)_n\left(\mathbf{u},\mathbf{w},\mathbf{v}\right)\right)\hat{=}V\left(\neg\mathsf{U}_n\left(\mathbf{w},\mathbf{v}\right)\right)\hat{\cdot}V\left(\neg\mathsf{V}_n\left(\mathbf{u},\mathbf{w}\right)\right)\hat{\cdot}V\left(\mathsf{W}_n\left(\mathbf{u},\mathbf{v}\right)\right)\hat{=}0$$
$$\text{for each }\left(\mathsf{1UVW}\right)_n\in\left\{\left(\mathsf{1AAA}\right)_n,\left(\mathsf{1EAE}\right)_n,\left(\mathsf{1AII}\right)_n,\left(\mathsf{1EIO}\right)_n\right\}\tag{5.43}$$
$$\text{and for each }n\in\{1,2,3,4\},$$

$$V\left(\left(\mathsf{2UVW}\right)_n\left(\mathbf{u},\mathbf{w},\mathbf{v}\right)\right)\hat{=}V\left(\neg\mathsf{U}_n\left(\mathbf{v},\mathbf{w}\right)\right)\hat{\cdot}V\left(\neg\mathsf{V}_n\left(\mathbf{u},\mathbf{w}\right)\right)\hat{\cdot}V\left(\mathsf{W}_n\left(\mathbf{u},\mathbf{v}\right)\right)\hat{=}0$$
$$\text{for each }\left(\mathsf{2UVW}\right)_n\in\left\{\left(\mathsf{2AEE}\right)_n,\left(\mathsf{2EAE}\right)_n,\left(\mathsf{2EIO}\right)_n,\left(\mathsf{2AOO}\right)_n\right\}\tag{5.44}$$
$$\text{and for each }n\in\{1,2,3,4\},$$

$$V\left(\left(\mathsf{3AAI}\right)_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_n\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{A}_n\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle\right)$$
$$\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{y}}\left[\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right)\hat{\bar{=}}\,\mathsf{J}^1\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\tag{5.45}$$
$$\text{for each }n\in\{1,2,3\},$$

$$V\left(\left(\mathsf{4AAI}\right)_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\neg\mathsf{A}_n\langle\mathbf{v},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{A}_n\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{I}_n\langle\mathbf{u},\mathbf{v}\rangle\right)$$
$$\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{y}}\left[\neg\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\wedge\mathbf{P}\langle\mathbf{y},\mathbf{w}\rangle\right]\right)\hat{\bar{=}}\,\mathsf{J}^2\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\tag{5.46}$$
$$\text{for each }n\in\{1,2,3\},$$

$$V\left(\left(\mathsf{4EAO}\right)_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)\hat{=}V\left(\left(\mathsf{3EAO}\right)_n\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle\right)$$
$$\hat{=}V\left(\neg\mathsf{E}_n\langle\mathbf{w},\mathbf{v}\rangle\right)\hat{\cdot}V\left(\neg\mathsf{A}_n\langle\mathbf{w},\mathbf{u}\rangle\right)\hat{\cdot}V\left(\mathsf{O}_n\langle\mathbf{u},\mathbf{v}\rangle\right)\tag{5.47}$$
$$\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{P}\langle\mathbf{x},\mathbf{w}\rangle\right)\hat{\cdot}V\left(\bigvee_{\mathbf{y}}\left[\mathbf{P}\langle\mathbf{y},\mathbf{u}\rangle\wedge\neg\mathbf{P}\langle\mathbf{y},\mathbf{v}\rangle\right]\right)\hat{\bar{=}}\,\mathsf{J}^3\langle\mathbf{u},\mathbf{w},\mathbf{v}\rangle$$
$$\text{for each }n\in\{1,2,3\},$$

$$V\left(\left(\mathsf{3UVW}\right)_n\left(\mathbf{u},\mathbf{w},\mathbf{v}\right)\right)\hat{=}V\left(\neg\mathsf{U}_n\left(\mathbf{w},\mathbf{v}\right)\right)\hat{\cdot}V\left(\neg\mathsf{V}_n\left(\mathbf{w},\mathbf{u}\right)\right)\hat{\cdot}V\left(\mathsf{W}_n\left(\mathbf{u},\mathbf{v}\right)\right)\hat{=}0$$
$$\text{for each }\left(\mathsf{3UVW}\right)_n\in\left\{\left(\mathsf{3AII}\right)_n,\left(\mathsf{3IAI}\right)_n,\left(\mathsf{3EIO}\right)_n,\left(\mathsf{3OAO}\right)_n\right\}\tag{5.48}$$
$$\text{and for each }n\in\{1,2,3\},$$

$$V\left(\left(\mathsf{4UVW}\right)_n\left(\mathbf{u},\mathbf{w},\mathbf{v}\right)\right)\hat{=}V\left(\neg\mathsf{U}_n\left(\mathbf{v},\mathbf{w}\right)\right)\hat{\cdot}V\left(\neg\mathsf{V}_n\left(\mathbf{w},\mathbf{u}\right)\right)\hat{\cdot}V\left(\mathsf{W}_n\left(\mathbf{u},\mathbf{v}\right)\right)\hat{=}0$$
$$\text{for each }\left(\mathsf{4UVW}\right)_n\in\left\{\left(\mathsf{4AEE}\right)_n,\left(\mathsf{4IAI}\right)_n,\left(\mathsf{4EIO}\right)_n\right\}\tag{5.49}$$
$$\text{and for each }n\in\{1,2,3\},$$

$$V\left(\left(\mathsf{3UVW}\right)_4\left(\mathbf{u},\mathbf{w},\mathbf{v}\right)\right)\hat{=}V\left(\neg\mathsf{U}_4\left(\mathbf{w},\mathbf{v}\right)\right)\hat{\cdot}V\left(\neg\mathsf{V}_4\left(\mathbf{w},\mathbf{u}\right)\right)\hat{\cdot}V\left(\mathsf{W}_4\left(\mathbf{u},\mathbf{v}\right)\right)\hat{=}0$$
$$\text{for each }\left(\mathsf{3UVW}\right)_4\in\left\{\left(\mathsf{3AAI}\right)_4,\left(\mathsf{3AII}\right)_4,\left(\mathsf{3IAI}\right)_4,\left(\mathsf{3EAO}\right)_4,\left(\mathsf{3EIO}\right)_4,\left(\mathsf{3OAO}\right)_4\right\},\tag{5.50}$$

$$V\left(\left(\mathsf{4UVW}\right)_4\left(\mathbf{u},\mathbf{w},\mathbf{v}\right)\right)\hat{=}V\left(\neg\mathsf{U}_4\left(\mathbf{v},\mathbf{w}\right)\right)\hat{\cdot}V\left(\neg\mathsf{V}_4\left(\mathbf{w},\mathbf{u}\right)\right)\hat{\cdot}V\left(\mathsf{W}_4\left(\mathbf{u},\mathbf{v}\right)\right)\hat{=}0$$
$$\text{for each }\left(\mathsf{4UVW}\right)_4\in\left\{\left(\mathsf{4AAI}\right)_4,\left(\mathsf{4AEE}\right)_4,\left(\mathsf{4IAI}\right)_4,\left(\mathsf{4EAO}\right)_4,\left(\mathsf{4EIO}\right)_4\right\},\tag{5.51}$$

The PLMT (PLDT) for a PSI (panlogographic syllogistic implication) being its PLSR (panlogographic slave relation) is alternatively called a *panlogographic syllogistic master*, or *decision, theorem* (*PLSMT* or *PLSDT*). The 12 PLMT's (PLDT's) (5.45)–(5.47) are said to be *inhomogeneous ones* (*IHPLSMT's* or *IHPLSDT's*), whereas the rest of PLMT's, (5.43), (5.44), and (5.48)–(5.51), are said to be *homogeneous ones*

(*HPLSMT's* or *HPLSDT's*). An HPLSMT is the PLMT (PLDT) of a certain KPSI, whereas an IHPLSMT is the PLMT (PLDT) of a certain UPSI.

2) Thus, according to Ths 5.1 and 5.2, there are, using the pertinent terminology introduced in Cmt 5.1(2), 15 asymmetric KPSI's and 4 asymmetric UPSI at $n \triangleright 1$, and 19 symmetric KPSI's at $n \triangleright 4$. At the same time, according to (5.43)–(5.49), there are in fact 45 asymmetric KPSI's and 12 UPSI's corresponding to $n \vec{\in} \{1,2,3\}$. Every KPSI belongs to $\mathbf{A}_1$ and is a schema of an infinite number of KESI's of A_1. Every UPSI also belongs to $\mathbf{A}_1$ and is a schema of an infinite number of ESI's of A_1 of all three classes: KESI's, AKESI's, and UEKSI's.

3) The HPLSMT's (5.43), (5.44), and (5.48)–(5.51) can be condensed into the *metalogographic*, i.e. *metalinguistic logographic*, *schema*:

$$V([\mathsf{U}_n \wedge \mathsf{V}_n] \Rightarrow \mathsf{W}_n) \hat{=} V(\neg\mathsf{U}_n) \hat{\cdot} V(\neg\mathsf{V}_n) \hat{\cdot} V(\mathsf{W}_n) \hat{=} 0, \tag{5.52}$$

while the *valid slave relation of the schema*:

$$[\mathsf{U}_n \wedge \mathsf{V}_n] \Rightarrow \mathsf{W}_n \tag{5.53}$$

condenses valid slave PLI's of the separate HPLSMT's. Both schemata (5.52) and (5.53) belong to the IML (inclusive metalanguage) of A_1, i.e. of both A_1 and $\mathbf{A}_1$. In this case, 'U_n', 'V_n', and 'W_n' are *metalogographic*, i.e. *metalinguistic logographic*, *placeholders* (*MLPH's*) that are used for mentioning the respective PSJ's, i.e. the two premises (two conjuncts of the antecedent) and the conclusion (consequent), of a certain KPSI (valid PSI) of the range of schema (5.53) – a KPSI, which belongs to $\mathbf{A}_1$ and which is in turn a placeholder of KESI's (valid ESI's) belonging to A_1.

4) Any HPLSMT in the range of the schema (5.52), i.e. any one of the HPLSMT's (5.43), (5.44), and (5.48)–(5.51), *can be used* as *a secondary objective* (*algebraic*) *rule of decision of* A_1, while *the slave PSI of the HPLSMT* in the range of the schema (5.53) *becomes a secondary subjective* (*logical*) *rule of inference of* A_1; both rules belong to $\mathbf{A}_1$ and they are therefore called a *panlogographic objective syllogistic decision rule* (*POSDR*) *of* A_1 and a *panlogographic subjective syllogistic decision rule* (*PSSDR*) *of* A_1 in that order. For (5.52) to hold, it is sufficient (but not necessary) that at least one of the three validity integrons $V(\neg\mathsf{U}_n)$ or $V(\neg\mathsf{V}_n)$ or $V(\mathsf{W}_n)$ should equal 0. Hence, for (5.53) to hold, it is sufficient (but not necessary) that at least one of the three slave relations $\neg\mathsf{U}_n$, $\neg\mathsf{V}_n$, and W_n should be valid. Therefore, a

panlogographic instance of the schema (5.52) that is used as a POSDR of A_1 or a panlogographic instance of the schema (5.53) that is used as a PSSDR of A_1 has a few aspects, which can, most generally, be expressed respectively thus:

a) If any two of the three validity integrons $V(\neg \mathsf{U}_n)$ or $V(\neg \mathsf{V}_n)$ or $V(\mathsf{W}_n)$ equal 1 then the remaining one equals 0.

b) If any two of the three slave relations U_n, V_n, and W_n are valid then the third one is also valid.

Particularly, both a POSDR of A_1 and a PSIDR of A_1 can be used *after the manner of FCS's* as follows:

$$\text{If } V(\mathsf{U}_n) \hat{=} V(\mathsf{V}_n) \hat{=} 0 \text{ then } V(\mathsf{W}_n) \hat{=} 0. \qquad (5.54)$$

If U_n and V_n are kyrologies then W_n is a kyrology. (5.55)

Still, in order to satisfy (5.52), PLR's U_n, V_n, and W_n should not necessarily satisfy the above condition a) or particularly (5.54). Each of them can be vav-neutral (udeterological) as was in fact assumed in proving Ths 5.1. and 5.2. At the same time, a ER in the range of a vav-neutral PLR can either be valid or antivalid or vav-neutral, but it cannot be either true or antitrue (false) just because it cannot be semantically interpreted psychically (mentally, not physically, not by substitution).

5) The totality of HPLSMT's of $\mathbf{A}_{1\in}$ and of the KPSI's being their PLSR's (panlogographic slave relations) is denoted by '$\mathbf{A}_{1\mathrm{A}}$' and is called *Aristotelian phase*, or *Aristotelian logic*, *of* $\mathbf{A}_{1\in}$, whereas euautographic instances of the above PLR's (panlogographic relations) belonging to $\mathsf{A}_{1\in}$ is denoted by '$\mathsf{A}_{1\mathrm{A}}$' and is called *Aristotelian phase*, or *Aristotelian logic*, *of* $\mathsf{A}_{1\in}$. Accordingly, the pair of interrelated calculi $\mathbf{A}_{1\mathrm{A}}$ and $\mathsf{A}_{1\mathrm{A}}$ is denoted by '$\mathrm{A}_{1\mathrm{A}}$' and is called is called *Aristotelian phase*, or *Aristotelian logic*, *of* $\mathrm{A}_{1\in}$. Since $\mathbf{A}_{1\in}$ has a simple universal ADM (algebraic decision method) $\mathbf{D}_{1\in}$, therefore any secondary inference rules are unimportant for $\mathbf{A}_{1\in}$, except for the most fundamental and easily memorable ones, which are useful. At the same time, the KPSI's in the capacity of secondary formation rules of $\mathbf{A}_{1\in}$ are complicated and are therefore difficult for remembering. Still, the very fact that $\mathbf{A}_{1\mathrm{A}}$, which is much more extensive than the original verbal Aristotelian logic, can be constructed within $\mathbf{A}_{1\in}$ and also the fact that at the same time the CFCL (conformal

catlogographic) interpretands of various versions of $\mathsf{A}_{1\mathsf{A}}$ turn out to be interpretnds of the verbal Aristotelian logic are amazing.

6) Unlike the KPSI's, the four UPSI's on the list (5.24) and two equivalent quadruples of UPSI's with the subscripts $_2$ and $_3$ in place of $_1$ satisfy IHPLSMT's (5.45)–(5.47). The latter cannot be used as any *secondary rules of decision of* A_1, and therefore none of the UPSI's can be used as a *secondary rule of inference of* A_1.

7) In accordance with Df 2.2, the whole of the previous portion of this section apply under the following substitutions:

$$\tilde{\mathsf{A}} \mapsto \mathsf{A},\ \tilde{\mathsf{E}} \mapsto \mathsf{E},\ \tilde{\mathsf{I}} \mapsto \mathsf{I},\ \tilde{\mathsf{O}} \mapsto \mathsf{O} \tag{5.56}$$

(the same as (2.7)) in the PLLP's (panlogographic logical predicates) of PSJ's and PSI's;

$$\tilde{\mathsf{a}} \mapsto \mathsf{a},\ \tilde{\mathsf{e}} \mapsto \mathsf{e},\ \tilde{\mathsf{i}} \mapsto \mathsf{i},\ \tilde{\mathsf{o}} \mapsto \mathsf{o} \tag{5.57}$$

in the VLP's (panlogographic logical predicates, catch words) of PSI's;

$$\langle \mathbf{w}, \mathbf{x} \rangle \mapsto \langle \mathbf{x}, \mathbf{w} \rangle, \langle \mathbf{v}, \mathbf{x} \rangle \mapsto \langle \mathbf{x}, \mathbf{v} \rangle \tag{5.58}$$

in the major premise (first conjunct of the antecedent) of a PSI;

$$\langle \mathbf{u}, \mathbf{y} \rangle \mapsto \langle \mathbf{y}, \mathbf{u} \rangle, \langle \mathbf{w}, \mathbf{y} \rangle \mapsto \langle \mathbf{y}, \mathbf{w} \rangle \tag{5.59}$$

in the minor premise (second conjunct of the antecedent) of a PSI;

$$\langle \mathbf{u}, \mathbf{z} \rangle \mapsto \langle \mathbf{z}, \mathbf{u} \rangle,\ \langle \mathbf{v}, \mathbf{z} \rangle \mapsto \langle \mathbf{z}, \mathbf{v} \rangle. \tag{5.60}$$

(the same as (2.8) in the conclusion (antecedent) of a PSI. In addition, the substitution

$$\tilde{\mathsf{J}} \mapsto \mathsf{J} \tag{5.61}$$

should be made in the very last expression (definiendum) of each one of the identities (5.44)–(5.46) accompanied with the pertinent ones of the substitutions (5.58) and (5.59) in the next to last expression. Substitutions (5.56) imply the substitutions:

$$\tilde{\mathsf{U}} \mapsto \mathsf{U},\ \tilde{\mathsf{V}} \mapsto \mathsf{V},\ \tilde{\mathsf{W}} \mapsto \mathsf{W}, \tag{5.62}$$

the understanding being that each one of the three letters '$\tilde{\mathsf{U}}$', '$\tilde{\mathsf{V}}$', and '$\tilde{\mathsf{W}}$' is a placeholder whose range is the set of four letters: '$\tilde{\mathsf{A}}$', '$\tilde{\mathsf{E}}$', '$\tilde{\mathsf{I}}$', and '$\tilde{\mathsf{O}}$'. At the same time, in the result of substitutions (5.56), all occurrences of a three-letter string such as 'EAO' or 'EIO' should be replaced with occurrences of the respective string such as '$\tilde{\mathsf{E}}\tilde{\mathsf{A}}\tilde{\mathsf{O}}$' or '$\tilde{\mathsf{E}}\tilde{\mathsf{I}}\tilde{\mathsf{O}}$'. Likewise, in the result of substitutions (5.57), all occurrences of a catchword such as '$\mathsf{Felapton}$' or 'Ferio' should be replaced with the respective catch word such as ‘$\tilde{\mathsf{F}}\tilde{\mathsf{e}}\tilde{\mathsf{l}}\tilde{\mathsf{a}}\mathsf{pt}\tilde{\mathsf{o}}\mathsf{n}$’ or ‘$\tilde{\mathsf{F}}\tilde{\mathsf{e}}\mathsf{r}\tilde{\mathsf{i}}\tilde{\mathsf{o}}$’. In accordance with Df 2.2(3), the variant of any expression subject to substitutions (5.56)–(5.62) will be qualified *transposed* and it can, when convenient, be referred to by the same double position numeral (if it has one) attached with the letter “t”.•

6. Aristotelian logic of $\mathsf{A}_{1\in}$ (*continued*)

6.1. Binary structural panlogographic syllogistic implications (BStPSI’s)

Df 6.1. 1) The whole of previous section in the exclusion of item 6 of subsection 5.3 applies under the substitutions indicated in Df 2.4, which should be supplemented by the following ones:

$$\begin{gathered}\mathsf{Barbara}_n \triangleright \mathsf{Barbara}_{\mathbf{F}n}, \ldots, \mathsf{Bocardo}_n \triangleright \mathsf{Bocardo}_{\mathbf{F}n},\\ (1\mathsf{AAA})_n \triangleright (1\mathsf{AAA})_{\mathbf{F}n}, \ldots, (4\mathsf{EIO})_n \triangleright (4\mathsf{EIO})_{\mathbf{F}n} \text{ for each } n \in \{1,2,3,4\};\end{gathered} \tag{6.1}$$

$$\mathsf{J}^1 \triangleright \mathsf{J}^{\mathbf{1}}_{\mathbf{F}}, \mathsf{J}^2 \triangleright \mathsf{J}^{\mathbf{2}}_{\mathbf{F}}, \mathsf{J}^3 \triangleright \mathsf{J}^{\mathbf{3}}_{\mathbf{F}}. \tag{6.2}$$

2) The specific instance of a *PSI* (*panlogographic syllogistic implication*) that results by a certain conjunction of interconnected substitutions that are indicated in Df 2.4 and in the previous item of this definition is called a *binary structural*, or *detailed*, *PSI* (*BStPSI*). All pertinent BStPSI’s are given in the following Crl 6.1 of Df 5.2•

Cmt 6.1. Just as Df 5.2, Crl 6.1 applies at each $n \vec{\in} \{1,2,3,4\}$. However, it follows from Crl 4.1 that, just as PSI’s in the general case, the only unredundant BStPSI’s are those corresponding to $n \triangleright 1$ (e.g.) and $n \triangleright 4$. The identities, which are the pertinent instances of identities (5.6)–(5.11) and which are selected out of identities (4.1)–(4.6) to determine the validity integrons of the pertinent unredundant BStPSJ’s, are given in Crl 6.2. Two other corollaries, Crls 6.3 and 6.4, are the pertinent instances of Ths 5.1.and 5.2 respectively.•

Crl 6.1: *The BStPSI’s*

1°) Group AAA&AAI

1) $\mathsf{Barbara}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (1\mathsf{AAA})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{v}) \wedge \mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{w}) \Rightarrow \mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

2) $\mathsf{Barapti}_{\mathbf{F}n} \to (3\mathsf{AAI})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{v}) \wedge \mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{u})(w,u) \Rightarrow \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

3) $\mathsf{Bamalip}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (4\mathsf{AAI})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{A}_{\mathbf{F}n}(\mathbf{v},\mathbf{w}) \wedge \mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{u}) \Rightarrow \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

2°) Group AII&IAI

4) $\mathsf{Darii}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{1AII})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{v}) \wedge \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{w}) \Rightarrow \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

5) $\mathsf{Datisi}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{3AII})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{v}) \wedge \mathsf{I}_{\mathbf{F}n}(\mathbf{w},\mathbf{u}) \Rightarrow \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

6) $\mathsf{Disamis}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{3IAI})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{I}_{\mathbf{F}n}(\mathbf{w},\mathbf{v}) \wedge \mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{u}) \Rightarrow \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

7) $\mathsf{Dimatis}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{4IAI})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{I}_{\mathbf{F}n}(\mathbf{v},\mathbf{w}) \wedge \mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{u}) \Rightarrow \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

3°) Group EAE&AEE

8) $\mathsf{Celarent}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{1EAE})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{E}_{\mathbf{F}n}(\mathbf{w},\mathbf{v}) \wedge \mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{w}) \Rightarrow \mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

9) $\mathsf{Camestres}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{2AEE})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{A}_{\mathbf{F}n}(\mathbf{v},\mathbf{w}) \wedge \mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{w}) \Rightarrow \mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

10) $\mathsf{Cesare}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{2EAE})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{E}_{\mathbf{F}n}(\mathbf{v},\mathbf{w}) \wedge \mathsf{A}_{\mathbf{F}n}(\mathbf{u},\mathbf{w}) \Rightarrow \mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

11) $\mathsf{Calemes}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{4AEE})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{A}_{\mathbf{F}n}(\mathbf{v},\mathbf{w}) \wedge \mathsf{E}_{\mathbf{F}n}(\mathbf{w},\mathbf{u}) \Rightarrow \mathsf{E}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

4°) Group EAO

12) $\mathsf{Felapton}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{3EAO})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{E}_{\mathbf{F}n}(\mathbf{w},\mathbf{v}) \wedge \mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

13) $\mathsf{Fesapo}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{4EAO})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{E}_{\mathbf{F}n}(\mathbf{v},\mathbf{w}) \wedge \mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

5°) Group EIO

14) $\mathsf{Ferio}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{1EIO})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{E}_{\mathbf{F}n}(\mathbf{w},\mathbf{v}) \wedge \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{w}) \Rightarrow \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

15) $\mathsf{Festino}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{2EIO})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{E}_{\mathbf{F}n}(\mathbf{v},\mathbf{w}) \wedge \mathsf{I}_{\mathbf{F}n}(\mathbf{u},\mathbf{w}) \Rightarrow \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$

16) $\mathsf{Feriso}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{3EIO})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to [\mathsf{E}_{\mathbf{F}n}(\mathbf{w},\mathbf{v}) \wedge \mathsf{I}_{\mathbf{F}n}(\mathbf{w},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

17) $\mathsf{Fresison}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{4EIO})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v})$

$$\to [\mathsf{E}_{\mathbf{F}n}(\mathbf{v},\mathbf{w})(v,w) \wedge \mathsf{I}_{\mathbf{F}n}(\mathbf{w},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})].$$

6°) Group AOO&OAO

18) $\mathsf{Baroco}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{2AOO})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \leftrightarrow [\mathsf{A}_{\mathbf{F}n}(\mathbf{v},\mathbf{w}) \wedge \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{w}) \Rightarrow \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})]$.

19) $\mathsf{Bocardo}_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v}) \to (\mathsf{3OAO})_{\mathbf{F}n}(\mathbf{u},\mathbf{w},\mathbf{v})$

$$\to [\mathsf{O}_{\mathbf{F}n}(\mathbf{w},\mathbf{v}) \wedge \mathsf{A}_{\mathbf{F}n}(\mathbf{w},\mathbf{u}) \Rightarrow \mathsf{O}_{\mathbf{F}n}(\mathbf{u},\mathbf{v})].\bullet$$

Crl 6.2:** ***The PLMT's (PLDT's) of the unredundant PStPSJ's.

$$V(\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{z}}[1 \hat{-} V(\neg\mathbf{F}(\mathbf{z},\mathbf{u})) \hat{\cdot} V(\mathbf{F}(\mathbf{z},\mathbf{v}))], \tag{6.3}$$

$$V(\mathsf{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{=} \hat{\cdot}_{\mathbf{z}}[1 \hat{-} V(\neg\mathbf{F}(\mathbf{z},\mathbf{u})) \hat{\cdot} V(\mathbf{F}(\mathbf{z},\mathbf{v}))], \tag{6.4}$$

$$\begin{aligned} V(\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) &\hat{=} V(\neg\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})) \hat{=} V(\mathsf{E}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \hat{=} V(\neg\mathsf{I}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})) \\ &\hat{=} 1 \hat{-} \hat{\cdot}_{\mathbf{z}}[1 \hat{-} V(\neg\mathbf{F}(\mathbf{z},\mathbf{u})) \hat{\cdot} V(\neg\mathbf{F}(\mathbf{z},\mathbf{v}))], \end{aligned} \tag{6.5}$$

$$
\begin{aligned}
V\left(\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right) \,\hat{=}\, V\left(\neg\mathsf{E}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right) \,\hat{=}\, V\left(\mathsf{I}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\right) \,\hat{=}\, V\left(\neg\mathsf{E}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\right)& \\
\,\hat{=}\, \hat{\bullet}_{\mathbf{z}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{v})\right)\right]&,
\end{aligned}
\tag{6.6}
$$

$$
V\left(\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\right) \,\hat{=}\, V\left(\neg\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\right) \,\hat{=}\, 1 \,\hat{-}\, \hat{\bullet}_{\mathbf{z}}\left[V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{v})\right)\right], \tag{6.7}
$$

$$
V\left(\mathsf{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\right) \,\hat{=}\, V\left(\neg\mathsf{A}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\right) \,\hat{=}\, \hat{\bullet}_{\mathbf{z}}\left[V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{v})\right)\right], \tag{6.8}
$$

subject to (2.33) and (2.36) (or (2.33ε) and (2.36ε), when applicable, –see Th 4.1 and Cnv 4.1).•

Crl 6.3:** ***The PLMT's (PLDT's) of the unredundant asymmetric BStPSI's.

1°) Group AAA&AAI

$$
\begin{aligned}
&V\left(\mathsf{Barbara}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \,\hat{=}\, V\left((\mathsf{1AAA})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\,\hat{=}\, V\left(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{w},\mathbf{v})\right) \,\hat{\cdot}\, V\left(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{w})\right) \,\hat{\cdot}\, V\left(\mathsf{A}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right) \\
&\,\hat{=}\, \left[\hat{\bullet}_{\mathbf{x}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{w})\right) \,\hat{\cdot}\, V\left(\mathbf{F}(\mathbf{x},\mathbf{v})\right)\right]\right] \,\hat{\cdot}\, \left[\hat{\bullet}_{\mathbf{y}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\mathbf{F}(\mathbf{y},\mathbf{w})\right)\right]\right] \\
&\,\hat{\cdot}\, \left[1 \,\hat{-}\, \hat{\bullet}_{\mathbf{z}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\mathbf{F}(\mathbf{z},\mathbf{v})\right)\right]\right] \,\hat{=}\, 0.
\end{aligned}
\tag{6.9}
$$

$$
\begin{aligned}
&V\left(\mathsf{Barapti}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \,\hat{=}\, V\left((\mathsf{3AAI})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\,\hat{=}\, V\left(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{w},\mathbf{v})\right) \,\hat{\cdot}\, V\left(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{w},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right) \\
&\,\hat{=}\, \left[\hat{\bullet}_{\mathbf{x}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{w})\right) \,\hat{\cdot}\, V\left(\mathbf{F}(\mathbf{x},\mathbf{v})\right)\right]\right] \,\hat{\cdot}\, \left[\hat{\bullet}_{\mathbf{y}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{w})\right) \,\hat{\cdot}\, V\left(\mathbf{F}(\mathbf{y},\mathbf{u})\right)\right]\right] \\
&\,\hat{\cdot}\, \left[\hat{\bullet}_{\mathbf{z}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{v})\right)\right]\right] \\
&\,\vec{\hat{=}}\, \hat{\bullet}_{\mathbf{x}}\left[V\left(\mathbf{F}(\mathbf{x},\mathbf{w})\right) \,\hat{\cdot}\, \left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{v})\right)\right]\right] \\
&\,\hat{=}\, \left[\hat{\bullet}_{\mathbf{x}} V\left(\mathbf{F}(\mathbf{x},\mathbf{w})\right)\right] \,\hat{\cdot}\, \left[\hat{\bullet}_{\mathbf{y}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{v})\right)\right]\right] \\
&\,\hat{=}\, V\left(\bigvee\nolimits_{\mathbf{x}}\mathbf{F}(\mathbf{x},\mathbf{w})\right) \,\hat{\cdot}\, V\left(\bigvee\nolimits_{\mathbf{y}}\mathbf{F}(\mathbf{y},\mathbf{u}) \wedge \mathbf{F}(\mathbf{y},\mathbf{v})\right) \,\overleftarrow{\hat{=}}\, \mathsf{J}^{1}_{\mathbf{F}}(\mathbf{u},\mathbf{w},\mathbf{v}).
\end{aligned}
\tag{6.10}
$$

$$
\begin{aligned}
&V\left(\mathsf{Bamalip}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \,\hat{=}\, V\left((\mathsf{4AAI})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\,\hat{=}\, V\left(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{w})\right) \,\hat{\cdot}\, V\left(\neg\mathsf{A}_{\mathbf{F}1}(\mathbf{w},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\mathsf{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right) \\
&\,\hat{=}\, \left[\hat{\bullet}_{\mathbf{x}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{v})\right) \,\hat{\cdot}\, V\left(\mathbf{F}(\mathbf{x},\mathbf{w})\right)\right]\right] \,\hat{\cdot}\, \left[\hat{\bullet}_{\mathbf{y}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{w})\right) \,\hat{\cdot}\, V\left(\mathbf{F}(\mathbf{y},\mathbf{u})\right)\right]\right] \\
&\,\hat{\cdot}\, \left[\hat{\bullet}_{\mathbf{z}}\left[1 \,\hat{-}\, V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{v})\right)\right]\right] \\
&\,\hat{=}\, \hat{\bullet}_{\mathbf{x}}\left[V\left(\mathbf{F}(\mathbf{x},\mathbf{v})\right) \,\hat{\cdot}\, \left[1 \,\hat{-}\, V\left(\mathbf{F}(\mathbf{x},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{w})\right)\right]\right] \\
&\,\hat{=}\, \left[\hat{\bullet}_{\mathbf{x}} V\left(\mathbf{F}(\mathbf{x},\mathbf{v})\right)\right] \,\hat{\cdot}\, \left[\hat{\bullet}_{\mathbf{y}}\left[1 \,\hat{-}\, V\left(\mathbf{F}(\mathbf{y},\mathbf{u})\right) \,\hat{\cdot}\, V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{w})\right)\right]\right] \\
&\,\hat{=}\, V\left(\bigvee\nolimits_{\mathbf{x}}\mathbf{F}(\mathbf{x},\mathbf{v})\right) \,\hat{\cdot}\, V\left(\bigvee\nolimits_{\mathbf{y}}\left[\neg\mathbf{F}(\mathbf{y},\mathbf{u}) \wedge \mathbf{F}(\mathbf{y},\mathbf{w})\right]\right) \,\overleftarrow{\hat{=}}\, \mathsf{J}^{2}_{\mathbf{F}}(\mathbf{u},\mathbf{w},\mathbf{v}).
\end{aligned}
\tag{6.11}
$$

2°) Group AII&IAI

$$
\begin{aligned}
&V\left(\text{Datisi}_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left(\text{Darii}_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=} V\left((3\text{AII})_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left((1\text{AII})_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=} V\left(\neg\text{A}_{\mathbf{F1}}(\mathbf{w},\mathbf{v})\right) \hat{\cdot} V\left(\neg\text{I}_{\mathbf{F1}}(\mathbf{w},\mathbf{u})\right) \hat{\cdot} V\left(\text{I}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=} V\left(\neg\text{A}_{\mathbf{F1}}(\mathbf{w},\mathbf{v})\right) \hat{\cdot} V\left(\neg\text{I}_{\mathbf{F1}}(\mathbf{u},\mathbf{w})\right) \hat{\cdot} V\left(\text{I}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{w})\right) \hat{\cdot} V\left(\mathbf{F}(\mathbf{x},\mathbf{v})\right)\right]\right]\\
&\hat{\cdot} \left[1 \hat{-} \widehat{\cdot}_{\mathbf{y}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{u})\right) \hat{\cdot} V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{w})\right)\right]\right]\\
&\hat{\cdot} \left[\widehat{\cdot}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \hat{\cdot} V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{v})\right)\right]\right] \hat{=} 0.
\end{aligned}
\tag{6.12}
$$

$$
\begin{aligned}
&V\left(\text{Dimatis}_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left(\text{Disamis}_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=} V\left((4\text{IAI})_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left((3\text{IAI})_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=} V\left(\neg\text{I}_{\mathbf{F1}}(\mathbf{v},\mathbf{w})\right) \hat{\cdot} V\left(\neg\text{A}_{\mathbf{F1}}(\mathbf{w},\mathbf{u})\right) \hat{\cdot} V\left(\text{I}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right)\\
&= V\left(\neg\text{I}_{\mathbf{F1}}(\mathbf{w},\mathbf{v})\right) \hat{\cdot} V\left(\neg\text{A}_{\mathbf{F1}}(\mathbf{w},\mathbf{u})\right) \hat{\cdot} V\left(\text{I}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=} \left[1 \hat{-} \widehat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{w})\right) \hat{\cdot} V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{v})\right)\right]\right]\\
&\hat{\cdot} \left[\widehat{\cdot}_{\mathbf{y}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{w})\right) \hat{\cdot} V\left(\mathbf{F}(\mathbf{y},\mathbf{u})\right)\right]\right]\\
&\hat{\cdot} \left[\widehat{\cdot}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \hat{\cdot} V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{v})\right)\right]\right] \hat{=} 0.
\end{aligned}
\tag{6.13}
$$

3°) Group EAE&AEE

$$
\begin{aligned}
&V\left(\text{Cesare}_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left(\text{Celarent}_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=} V\left((2\text{EAE})_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left((1\text{EAE})_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=} V\left(\neg\text{E}_{\mathbf{F1}}(\mathbf{v},\mathbf{w})\right) \hat{\cdot} V\left(\neg\text{A}_{\mathbf{F1}}(\mathbf{u},\mathbf{w})\right) \hat{\cdot} V\left(\text{E}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=} V\left(\neg\text{E}_{\mathbf{F1}}(\mathbf{w},\mathbf{v})\right) \hat{\cdot} V\left(\neg\text{A}_{\mathbf{F1}}(\mathbf{u},\mathbf{w})\right) \hat{\cdot} V\left(\text{E}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{w})\right) \hat{\cdot} V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{v})\right)\right]\right] \hat{\cdot} \left[\widehat{\cdot}_{\mathbf{y}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{v})\right) \hat{\cdot} V\left(\mathbf{F}(\mathbf{y},\mathbf{w})\right)\right]\right]\\
&\hat{\cdot} \left[1 \hat{-} \widehat{\cdot}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \hat{\cdot} V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{v})\right)\right]\right] \hat{=} 0.
\end{aligned}
\tag{6.14}
$$

$$
\begin{aligned}
&V\left(\text{Calemes}_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left(\text{Camestres}_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=} V\left((4\text{AEE})_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left((2\text{AEE})_{\mathbf{F1}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=} V\left(\neg\text{A}_{\mathbf{F1}}(\mathbf{v},\mathbf{w})\right) \hat{\cdot} V\left(\neg\text{E}_{\mathbf{F1}}(\mathbf{w},\mathbf{u})\right) \hat{\cdot} V\left(\text{E}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=} V\left(\neg\text{A}_{\mathbf{F1}}(\mathbf{v},\mathbf{w})\right) \hat{\cdot} V\left(\neg\text{E}_{\mathbf{F1}}(\mathbf{u},\mathbf{w})\right) \hat{\cdot} V\left(\text{E}_{\mathbf{F1}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=} \left[\widehat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{x},\mathbf{v})\right) \hat{\cdot} V\left(\mathbf{F}(\mathbf{x},\mathbf{w})\right)\right]\right] \hat{\cdot} \left[\widehat{\cdot}_{\mathbf{y}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{u})\right) \hat{\cdot} V\left(\neg\mathbf{F}(\mathbf{y},\mathbf{w})\right)\right]\right]\\
&\hat{\cdot} \left[1 \hat{-} \widehat{\cdot}_{\mathbf{z}}\left[1 \hat{-} V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{u})\right) \hat{\cdot} V\left(\neg\mathbf{F}(\mathbf{z},\mathbf{v})\right)\right]\right] \hat{=} 0.
\end{aligned}
\tag{6.15}
$$

4°) Group EAO

$$\begin{aligned}
&V\left(\mathrm{Fesapo}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left(\mathrm{Felapton}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left((4\mathrm{EAO})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((3\mathrm{EAO})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{\mathbf{F}1}(\mathbf{v},\mathbf{w})\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\mathbf{F}1}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathrm{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{\mathbf{F}1}(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\mathbf{F}1}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathrm{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}\left[\hat{\prod}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{x},\mathbf{w}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right]\hat{\cdot}\left[\hat{\prod}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{y},\mathbf{w}))\hat{\cdot}V(\mathbf{F}(\mathbf{y},\mathbf{u}))\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_{\mathbf{z}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{\cdot}V(\mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right]\\
&\hat{=}\hat{\prod}_{\mathbf{x}}\left[V(\mathbf{F}(\mathbf{x},\mathbf{w}))\hat{\cdot}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{x},\mathbf{u}))\hat{\cdot}V(\mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right]\\
&\hat{=}\left[\hat{\prod}_{\mathbf{x}}V(\mathbf{F}(\mathbf{x},\mathbf{w}))\right]\hat{\cdot}\left[\hat{\prod}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{y},\mathbf{u}))\hat{\cdot}V(\mathbf{F}(\mathbf{y},\mathbf{v}))\right]\right]\\
&\hat{=}V\left(\bigvee_{\mathbf{x}}\mathbf{F}(\mathbf{x},\mathbf{w})\right)\hat{\cdot}V\left(\bigvee_{\mathbf{y}}\left[\mathbf{F}(\mathbf{y},\mathbf{u})\wedge\neg\mathbf{F}(\mathbf{y},\mathbf{v})\right]\right)\hat{\bar{=}}\mathrm{J}^3_{\mathbf{F}}(\mathbf{u},\mathbf{w},\mathbf{v}).
\end{aligned}\tag{6.16}$$

5°) Group EIO

$$\begin{aligned}
&V\left(\mathrm{Fresison}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left(\mathrm{Feriso}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\mathrm{Festino}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left(\mathrm{Ferio}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left((4\mathrm{EIO})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((3\mathrm{EIO})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left((2\mathrm{EIO})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((1\mathrm{EIO})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{\mathbf{F}1}(\mathbf{v},\mathbf{w})\right)\hat{\cdot}V\left(\neg\mathrm{I}_{\mathbf{F}1}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathrm{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{\mathbf{F}1}(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg\mathrm{I}_{\mathbf{F}1}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathrm{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{\mathbf{F}1}(\mathbf{v},\mathbf{w})\right)\hat{\cdot}V\left(\neg\mathrm{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{w})\right)\hat{\cdot}V\left(\mathrm{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{\mathbf{F}1}(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg\mathrm{I}_{\mathbf{F}1}(\mathbf{u},\mathbf{w})\right)\hat{\cdot}V\left(\mathrm{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}\left[\hat{\prod}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{x},\mathbf{w}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right]\\
&\hat{\cdot}\left[1\hat{-}\hat{\prod}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{y},\mathbf{u}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{y},\mathbf{w}))\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_{\mathbf{z}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{\cdot}V(\mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right]\hat{=}0.
\end{aligned}\tag{6.17}$$

6°) Group AOO&OAO

$$\begin{aligned}
&V\left(\mathrm{Baroco}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((2\mathrm{AOO})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathrm{A}_{\mathbf{F}1}(\mathbf{v},\mathbf{w})\right)\hat{\cdot}V\left(\neg\mathrm{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{w})\right)\hat{\cdot}V\left(\mathrm{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}\left[\hat{\prod}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))\hat{\cdot}V(\mathbf{F}(\mathbf{x},\mathbf{w}))\right]\right]\hat{\cdot}\left[1\hat{-}\hat{\prod}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{y},\mathbf{u}))\hat{\cdot}V(\mathbf{F}(\mathbf{y},\mathbf{w}))\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_{\mathbf{z}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{\cdot}V(\mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right]\hat{=}0.
\end{aligned}\tag{6.18}$$

$$\begin{aligned}
&V\left(\mathrm{Bocardo}_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((3\mathrm{AOO})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathrm{O}_{\mathbf{F}1}(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\mathbf{F}1}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathrm{O}_{\mathbf{F}1}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}\left[1\hat{-}\hat{\prod}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{x},\mathbf{w}))\hat{\cdot}V(\mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right]\hat{\cdot}\left[\hat{\prod}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{y},\mathbf{w}))\hat{\cdot}V(\mathbf{F}(\mathbf{y},\mathbf{u}))\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_{\mathbf{z}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{\cdot}V(\mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right]\hat{=}0.
\end{aligned}\tag{6.19}$$
•

Crl 6.4: ***The PLMT's (PLDT's) of the unredundant symmetric BStPSI's.***

1°) Group AAA&AAI

$$
\begin{aligned}
&V\left(\mathsf{Barbara}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((\mathsf{1AAA})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{w})\right)\hat{\cdot}V\left(\mathsf{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}\left[\mathop{\hat{\cdot}}_{\mathbf{x}}\left[V(\neg\mathbf{F}(\mathbf{x},\mathbf{w}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right]\hat{\cdot}\left[\mathop{\hat{\cdot}}_{\mathbf{y}}\left[V(\neg\mathbf{F}(\mathbf{y},\mathbf{u}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{y},\mathbf{w}))\right]\right]\\
&\hat{\cdot}\left[1\hat{-}\mathop{\hat{\cdot}}_{\mathbf{z}}\left[V(\neg\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right]\hat{=}0.
\end{aligned}
\tag{6.20}
$$

$$
\begin{aligned}
&V\left(\mathsf{Bamalip}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left(\mathsf{Barapti}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left((\mathsf{4AAI})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((\mathsf{3AAI})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{v},\mathbf{w})\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathsf{I}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathsf{I}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}\left[\mathop{\hat{\cdot}}_{\mathbf{x}}\left[V(\neg\mathbf{F}(\mathbf{x},\mathbf{w}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right]\hat{\cdot}\left[\mathop{\hat{\cdot}}_{\mathbf{y}}\left[V(\neg\mathbf{F}(\mathbf{y},\mathbf{w}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{y},\mathbf{u}))\right]\right]\\
&\hat{\cdot}\left[\mathop{\hat{\cdot}}_{\mathbf{z}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right]\hat{=}0.
\end{aligned}
\tag{6.21}
$$

2°) Group AII&IAI

$$
\begin{aligned}
&V\left(\mathsf{Datisi}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left(\mathsf{Darii}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left((\mathsf{3AII})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((\mathsf{1AII})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg\mathsf{I}_{\mathbf{F4}}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathsf{I}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg\mathsf{I}_{\mathbf{F4}}(\mathbf{u},\mathbf{w})\right)\hat{\cdot}V\left(\mathsf{I}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}\left[\mathop{\hat{\cdot}}_{\mathbf{x}}\left[V(\neg\mathbf{F}(\mathbf{x},\mathbf{w}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right]\\
&\hat{\cdot}\left[1\hat{-}\mathop{\hat{\cdot}}_{\mathbf{y}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{y},\mathbf{u}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{y},\mathbf{w}))\right]\right]\\
&\hat{\cdot}\left[\mathop{\hat{\cdot}}_{\mathbf{z}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{y},\mathbf{v}))\right]\right]\hat{=}0.
\end{aligned}
\tag{6.22}
$$

$$
\begin{aligned}
&V\left(\mathsf{Dimatis}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left(\mathsf{Disamis}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left((\mathsf{4IAI})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((\mathsf{3IAI})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathsf{I}_{\mathbf{F4}}(\mathbf{v},\mathbf{w})\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathsf{I}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathsf{I}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg\mathsf{A}_{\mathbf{F4}}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathsf{I}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}\left[1\hat{-}\mathop{\hat{\cdot}}_{\mathbf{x}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{x},\mathbf{w}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right]\\
&\hat{\cdot}\left[\mathop{\hat{\cdot}}_{\mathbf{y}}\left[V(\neg\mathbf{F}(\mathbf{y},\mathbf{w}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{y},\mathbf{u}))\right]\right]\\
&\hat{\cdot}\left[\mathop{\hat{\cdot}}_{\mathbf{z}}\left[1\hat{-}V(\neg\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right]\hat{=}0.
\end{aligned}
\tag{6.23}
$$

3°) Group EAE&AEE

$$\begin{aligned}
&V\left(\mathrm{Cesare}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left(\mathrm{Celarent}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\hat{=} V\left((2\mathrm{EAE})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left((1\mathrm{EAE})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\hat{=} V\left(\neg \mathrm{E}_{\mathbf{F4}}(\mathbf{v},\mathbf{w})\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{w})\right) \hat{\cdot} V\left(\mathrm{E}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\hat{=} V\left(\neg \mathrm{E}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\mathbf{F4}}(\mathbf{u},\mathbf{w})\right) \hat{\cdot} V\left(\mathrm{E}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\hat{=} \left[\hat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V(\neg \mathbf{F}(\mathbf{x},\mathbf{w})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{y}}\left[V(\neg \mathbf{F}(\mathbf{y},\mathbf{v})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{y},\mathbf{w}))\right]\right] \\
&\hat{\cdot} \left[1 \hat{-} \hat{\cdot}_{\mathbf{z}}\left[1 \hat{-} V(\neg \mathbf{F}(\mathbf{z},\mathbf{u})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right] \hat{=} 0.
\end{aligned} \tag{6.24}$$

$$\begin{aligned}
&V\left(\mathrm{Calemes}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left(\mathrm{Camestres}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\hat{=} V\left((4\mathrm{AEE})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left((2\mathrm{AEE})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\hat{=} V\left(\neg \mathrm{A}_{\mathbf{F4}}(\mathbf{v},\mathbf{w})\right) \hat{\cdot} V\left(\neg \mathrm{E}_{\mathbf{F4}}(\mathbf{w},\mathbf{u})\right) \hat{\cdot} V\left(\mathrm{E}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\hat{=} V\left(\neg \mathrm{A}_{\mathbf{F4}}(\mathbf{v},\mathbf{w})\right) \hat{\cdot} V\left(\neg \mathrm{E}_{\mathbf{F4}}(\mathbf{u},\mathbf{w})\right) \hat{\cdot} V\left(\mathrm{E}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\hat{=} \left[\hat{\cdot}_{\mathbf{x}}\left[V(\neg \mathbf{F}(\mathbf{x},\mathbf{v})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{x},\mathbf{w}))\right]\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{y}}\left[1 \hat{-} V(\neg \mathbf{F}(\mathbf{y},\mathbf{u})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{y},\mathbf{w}))\right]\right] \\
&\hat{\cdot} \left[1 \hat{-} \hat{\cdot}_{\mathbf{z}}\left[1 \hat{-} V(\neg \mathbf{F}(\mathbf{z},\mathbf{u})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right] \hat{=} 0.
\end{aligned} \tag{6.25}$$

4°) Group EAO

$$\begin{aligned}
&V\left(\mathrm{Fesapo}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left(\mathrm{Felapton}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\hat{=} V\left((4\mathrm{EAO})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left((3\mathrm{EAO})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\hat{=} V\left(\neg \mathrm{E}_{\mathbf{F4}}(\mathbf{v},\mathbf{w})\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\mathbf{F4}}(\mathbf{w},\mathbf{u})\right) \hat{\cdot} V\left(\mathrm{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\hat{=} V\left(\neg \mathrm{E}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\mathbf{F4}}(\mathbf{w},\mathbf{u})\right) \hat{\cdot} V\left(\mathrm{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\hat{=} \left[\hat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V(\neg \mathbf{F}(\mathbf{x},\mathbf{w})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right] \hat{\cdot} \left[\hat{\cdot}_{\mathbf{y}}\left[V(\neg \mathbf{F}(\mathbf{y},\mathbf{w})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{y},\mathbf{u}))\right]\right] \\
&\hat{\cdot} \left[\hat{\cdot}_{\mathbf{z}}\left[V(\neg \mathbf{F}(\mathbf{z},\mathbf{u})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right] \hat{=} 0.
\end{aligned} \tag{6.26}$$

5°) Group EIO

$$\begin{aligned}
&V\left(\mathrm{Fresison}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left(\mathrm{Feriso}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\hat{=} V\left(\mathrm{Festino}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left(\mathrm{Ferio}_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\hat{=} V\left((4\mathrm{EIO})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left((3\mathrm{EIO})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\hat{=} V\left((2\mathrm{EIO})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \hat{=} V\left((1\mathrm{EIO})_{\mathbf{F4}}(\mathbf{u},\mathbf{w},\mathbf{v})\right) \\
&\hat{=} V\left(\neg \mathrm{E}_{\mathbf{F4}}(\mathbf{v},\mathbf{w})\right) \hat{\cdot} V\left(\neg \mathrm{I}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right) \hat{\cdot} V\left(\mathrm{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\hat{=} V\left(\neg \mathrm{E}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right) \hat{\cdot} V\left(\neg \mathrm{I}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right) \hat{\cdot} V\left(\mathrm{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\hat{=} V\left(\neg \mathrm{E}_{\mathbf{F4}}(\mathbf{v},\mathbf{w})\right) \hat{\cdot} V\left(\neg \mathrm{I}_{\mathbf{F4}}(\mathbf{u},\mathbf{w})\right) \hat{\cdot} V\left(\mathrm{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\hat{=} V\left(\neg \mathrm{E}_{\mathbf{F4}}(\mathbf{w},\mathbf{v})\right) \hat{\cdot} V\left(\neg \mathrm{I}_{\mathbf{F4}}(\mathbf{u},\mathbf{w})\right) \hat{\cdot} V\left(\mathrm{O}_{\mathbf{F4}}(\mathbf{u},\mathbf{v})\right) \\
&\hat{=} \left[\hat{\cdot}_{\mathbf{x}}\left[1 \hat{-} V(\neg \mathbf{F}(\mathbf{x},\mathbf{w})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right] \\
&\hat{\cdot} \left[1 \hat{-} \hat{\cdot}_{\mathbf{y}}\left[1 \hat{-} V(\neg \mathbf{F}(\mathbf{y},\mathbf{u})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{y},\mathbf{w}))\right]\right] \\
&\hat{\cdot} \left[\hat{\cdot}_{\mathbf{z}}\left[V(\neg \mathbf{F}(\mathbf{z},\mathbf{u})) \hat{\cdot} V(\neg \mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right] \hat{=} 0.
\end{aligned} \tag{6.27}$$

6°) Group AOO&OAO

$$
\begin{aligned}
&V\left(\mathrm{Baroco}_{\mathbf{F}4}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((2\mathrm{AOO})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathrm{A}_{\mathbf{F}4}(\mathbf{v},\mathbf{w})\right)\hat{\cdot}V\left(\neg\mathrm{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{w})\right)\hat{\cdot}V\left(\mathrm{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}\left[\hat{\bullet}_{\mathbf{x}}\left[V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{x},\mathbf{w}))\right]\right]\hat{\cdot}\left[1\hat{-}\hat{\bullet}_{\mathbf{y}}\left[V(\neg\mathbf{F}(\mathbf{y},\mathbf{u}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{y},\mathbf{w}))\right]\right]\\
&\hat{\cdot}\left[\hat{\bullet}_{\mathbf{z}}\left[V(\neg\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right]\hat{=}0.
\end{aligned}
\tag{6.28}
$$

$$
\begin{aligned}
&V\left(\mathrm{Bocardo}_{\mathbf{F}4}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\hat{=}V\left((3\mathrm{AOO})_{\mathbf{F}1}(\mathbf{u},\mathbf{w},\mathbf{v})\right)\\
&\hat{=}V\left(\neg\mathrm{O}_{\mathbf{F}4}(\mathbf{w},\mathbf{v})\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\mathbf{F}4}(\mathbf{w},\mathbf{u})\right)\hat{\cdot}V\left(\mathrm{O}_{\mathbf{F}4}(\mathbf{u},\mathbf{v})\right)\\
&\hat{=}\left[1\hat{-}\hat{\bullet}_{\mathbf{x}}\left[V(\neg\mathbf{F}(\mathbf{x},\mathbf{w}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{x},\mathbf{v}))\right]\right]\hat{\cdot}\left[\hat{\bullet}_{\mathbf{y}}\left[V(\neg\mathbf{F}(\mathbf{y},\mathbf{w}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{y},\mathbf{u}))\right]\right]\\
&\hat{\cdot}\left[\hat{\bullet}_{\mathbf{z}}\left[V(\neg\mathbf{F}(\mathbf{z},\mathbf{u}))\hat{\cdot}V(\neg\mathbf{F}(\mathbf{z},\mathbf{v}))\right]\right]\hat{=}0.
\end{aligned}
\tag{6.29}\bullet
$$

6.2. Binary euautogographic syllogistic implications (BESI's)

Df 6.2. 1) Each one of the PLSMT's (PLSDT's) that are given in Crls 6.3 and 6.4 has an infinite number of concrete euautographic instances (interpretands, corollaries), any one of which can be written immediately (without any proof) in accordance with the rules of substitution summarized in Preliminary Remark 2.1. The same is true of the slave PSI of a PLSMT. A euautographic instance (interpretand) of a slave PSI is called:

a) a *routine binary euautographic syllogistic implication* (*RBESI*) if it is resulted by replacing all occurrences of '**F**' throughout the PLSMT, or correspondingly throughout the slave PSI, with occurrences of a certain APVOPS (atomic pseudo-variable predicate-sign) of the set κ^{2pv}, defined by (2.46), i.e. if $\mathbf{F}\triangleright\mathbf{F}^{pv}\vec{\in}\kappa^{2pv}$;

b) a *distinguished binary euautographic syllogistic implication* (*DBESI*) if it is resulted by replacing all occurrences of '**F**' throughout the PLSMT, or correspondingly throughout the slave PSI, with occurrences of a certain APCOPS (atomic pseudo-constant predicate-sign) of the set $\mathrm{K}_{\in}^{pc}$, defined by (2.47), i.e. if $\mathbf{F}\triangleright\mathbf{F}^{pc}\vec{\in}\vec{\mathrm{K}}_{\in}^{pc}$.

It goes without saying that, in either of the above two acts of euautographic interpretation of the panlogographic interpretans, i.e. of the PLSMT or of the slave PSI, all occurrences of the six APLOT's '**u**' to '**z**' in the interpretans should be replaced with occurrences of certain distinct AEOT's (atomic euautographic ordinary terms) of the set τ, defined by (2.48) and (2.49), as indicated in Preliminary Remark

2.1. An RBESI or a DBESI is indiscriminately called a *BESI* (*binary euautographic syllogistic implication*).

2) In agreement with item 6 of subsection 1.5, a euautographic instance (interpretand) of a PLSMT (PLSDT) is called a *euautographic syllogistic master*, or *decision theorem* (*ESMT* or *ESDT*), the understanding being that its *euautographic slave relation* (*ESR*) is a BESI being a euautographic instance (interpretand) of the slave BStPSI of the PLSMT (PLSDT). Also, I shall, as before, use the abbreviations: "*HESMT*" (or "*HEMDT*") for "*homogeneous* ESMT" (or "*homogeneous* ESDT") and "*IHESMT*" (or "*IHEMDT*") for "*inhomogeneous* ESMT" (or "*inhomogeneous* ESDT").

3) A BESI is said to be an *asymmetric* one if it is an interpretand (instance) of an asymmetric BStPSI and a *symmetric* one if it is an interpretand (instance) of a symmetric BStPSI. In accordance with Th 4.1(1), the *symmetric DBESI's* are *degenerate* and therefore they are disregarded.●

°**Crl 6.5:** ***The EMT's (EDT's) of asymmetric and symmetric RBESI's.***

1) The following trains of identities (6.10μ_0), (6.11μ_0), and (6.16μ_0) are results of the simultaneous substitutions $\mathbf{F} \triangleright f^2$ and (2.55) throughout identities (6.10), (6.11), and (6.16) respectively:

$$\begin{aligned}
&V\left(\mathrm{Barapti}_{f^2 1}(u,w,v)\right) \hat{=} V\left((\mathrm{3AAI})_{f^2 1}(u,w,v)\right) \\
&\hat{=} V\left(\neg \mathrm{A}_{f^2 1}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{f^2 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{I}_{f^2 1}(u,v)\right) \\
&\hat{=} \left[\hat{\bullet}_x \left[1 \hat{-} V\left(\neg f^2(x,w)\right) \hat{\cdot} V\left(f^2(x,v)\right)\right]\right] \hat{\cdot} \left[\hat{\bullet}_y \left[1 \hat{-} V\left(\neg f^2(y,w)\right) \hat{\cdot} V\left(f^2(y,u)\right)\right]\right] \\
&\quad \hat{\cdot} \left[\hat{\bullet}_z \left[1 \hat{-} V\left(\neg f^2(z,u)\right) \hat{\cdot} V\left(\neg f^2(z,v)\right)\right]\right] \qquad (6.10\mu_0) \\
&\vec{\hat{=}}\, \hat{\bullet}_x \left[V\left(f^2(x,w)\right) \hat{\cdot} \left[1 \hat{-} V\left(\neg f^2(x,u)\right) \hat{\cdot} V\left(\neg f^2(x,v)\right)\right]\right] \\
&\hat{=} \left[\hat{\bullet}_x V\left(f^2(x,w)\right)\right] \hat{\cdot} \left[\hat{\bullet}_y \left[1 \hat{-} V\left(\neg f^2(y,u)\right) \hat{\cdot} V\left(\neg f^2(y,v)\right)\right]\right] \\
&\hat{=} V\left(\bigvee_x f^2(x,w)\right) \hat{\cdot} V\left(\bigvee_y f^2(y,u) \wedge f^2(y,v)\right) \overleftarrow{\hat{=}}\, \mathrm{J}^1_{f^2}(u,w,v).
\end{aligned}$$

$$
\begin{aligned}
&V\left(\mathrm{Bamalip}_{f^2 1}(u,w,v)\right)\hat{=}V\left((\mathrm{4AAI})_{f^2 1}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathrm{A}_{f^2 1}(v,w)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{f^2 1}(w,u)\right)\hat{\cdot}V\left(\mathrm{I}_{f^2 1}(u,v)\right)\\
&\hat{=}\left[\widehat{\bullet}_x\left[1\hat{-}V\left(\neg f^2(x,v)\right)\hat{\cdot}V\left(f^2(x,w)\right)\right]\right]\hat{\cdot}\left[\widehat{\bullet}_y\left[1\hat{-}V\left(\neg f^2(y,w)\right)\hat{\cdot}V\left(f^2(y,u)\right)\right]\right]\\
&\hat{\cdot}\,\widehat{\bullet}_z\left[1\hat{-}V\left(\neg f^2(z,u)\right)\hat{\cdot}V\left(\neg f^2(z,v)\right)\right] \qquad (6.11\mu_0)\\
&\hat{=}\,\widehat{\bullet}_x\left[V\left(f^2(x,v)\right)\hat{\cdot}\left[1\hat{-}V\left(f^2(x,u)\right)\hat{\cdot}V\left(\neg f^2(x,w)\right)\right]\right]\\
&\hat{=}\left[\widehat{\bullet}_x V\left(f^2(x,v)\right)\right]\hat{\cdot}\left[\widehat{\bullet}_{\mathbf{y}}\left[1\hat{-}V\left(f^2(y,u)\right)\hat{\cdot}V\left(\neg f^2(y,w)\right)\right]\right]\\
&\hat{=}V\left(\bigvee_x f^2(x,v)\right)\hat{\cdot}V\left(\bigvee_y\left[\neg f^2(y,u)\wedge f^2(y,w)\right]\right)\overleftarrow{\hat{=}}\,\mathrm{J}^2_{f^2}(u,w,v).
\end{aligned}
$$

$$
\begin{aligned}
&V\left(\mathrm{Fesapo}_{f^2 1}(u,w,v)\right)\hat{=}V\left(\mathrm{Felapton}_{f^2 1}(u,w,v)\right)\\
&\hat{=}V\left((\mathrm{4EAO})_{f^2 1}(u,w,v)\right)\hat{=}V\left((\mathrm{3EAO})_{f^2 1}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{f^2 1}(w,v)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{f^2 1}(w,u)\right)\hat{\cdot}V\left(\mathrm{O}_{f^2 1}(u,v)\right)\\
&\hat{=}\left[\widehat{\bullet}_x\left[1\hat{-}V\left(\neg f^2(x,w)\right)\hat{\cdot}V\left(\neg f^2(x,v)\right)\right]\right]\\
&\hat{\cdot}\left[\widehat{\bullet}_y\left[1\hat{-}V\left(\neg f^2(y,w)\right)\hat{\cdot}V\left(f^2(y,u)\right)\right]\right] \qquad (6.16\mu_0)\\
&\hat{\cdot}\left[\widehat{\bullet}_z\left[1\hat{-}V\left(\neg f^2(z,u)\right)\hat{\cdot}V\left(f^2(z,v)\right)\right]\right]\\
&\hat{=}\,\widehat{\bullet}_x\left[V\left(f^2(x,w)\right)\hat{\cdot}\left[1\hat{-}V\left(\neg f^2(x,u)\right)\hat{\cdot}V\left(f^2(x,v)\right)\right]\right]\\
&\hat{=}\left[\widehat{\bullet}_x V\left(f^2(x,w)\right)\right]\hat{\cdot}\left[\widehat{\bullet}_y\left[1\hat{-}V\left(\neg f^2(y,u)\right)\hat{\cdot}V\left(f^2(y,v)\right)\right]\right]\\
&\hat{=}V\left(\bigvee_x f^2(x,w)\right)\hat{\cdot}\left[V\left(\bigvee_y\left[f^2(y,u)\wedge\neg f^2(y,v)\right]\right)\right]\overleftarrow{\hat{=}}\,\mathrm{J}^3_{f^2}(u,w,v).
\end{aligned}
$$

Like any one of the trains (6.10), (6.11), and (6.16), the respective one of the trains (6.10μ_0), 6.11μ_0), and (6.16μ_0) is irreducible either to 0 or to 1.

2) The analogous results of the simultaneous substitutions $\mathbf{F}\triangleright f^2$ and (2.55) throughout the rest of the PLMT's comprised in Crl 6.3, – namely throughout identities (6.9), (6.12)–(6.15), and (6.17)–(6.19), – and also throughout all PLMT's (6.20)–(6.29) comprised in Crl 6.4 can be written down straightforwardly likewise. These instances (interpretands) of the PLMT's will be obviously understood and be referred to as (6.9μ_0), (6.12μ_0)–(6.15μ_0), (6.17μ_0)–(6.19μ_0), and (6.20μ_0)–(6.29μ_0) respectively. Also, the EMT's of asymmetric and symmetric RBESI's in question can be summarized in analogy with (5.43), (5.44), and (5.48)–(5.51) as follows:

$$
\begin{aligned}
&V\left((\mathrm{1UVW})_{f^2 n}(u,w,v)\right)\hat{=}V\left(\neg\mathrm{U}_{f^2 n}(w,v)\right)\hat{\cdot}V\left(\neg\mathrm{V}_{f^2 n}(u,w)\right)\hat{\cdot}V\left(\mathrm{W}_{f^2 n}(u,v)\right)\hat{=}0\\
&\text{for each }(\mathrm{1UVW})_{f^2 n}\,\vec{\in}\left\{(\mathrm{1AAA})_{f^2 n},(\mathrm{1EAE})_{f^2 n},(\mathrm{1AII})_{f^2 n},(\mathrm{1EIO})_{f^2 n}\right\} \qquad (6.30)\\
&\text{and for each } n\,\vec{\in}\,\{1,2,3,4\},
\end{aligned}
$$

$$V\left((2\mathrm{UVW})_{f^2 n}(u,w,v)\right) \hat{=} V\left(\neg \mathrm{U}_{f^2 n}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{V}_{f^2 n}(u,w)\right) \hat{\cdot} V\left(\mathrm{W}_{f^2 n}(u,v)\right) \hat{=} 0$$
$$\text{for each } (2\mathrm{UVW})_{f^2 n} \vec{\in} \left\{(2\mathrm{AEE})_{f^2 n}, (2\mathrm{EAE})_{f^2 n}, (2\mathrm{EIO})_{f^2 n}, (2\mathrm{AOO})_{f^2 n}\right\} \quad (6.31)$$
$$\text{and for each } n \vec{\in} \{1,2,3,4\},$$

$$V\left((3\mathrm{UVW})_{f^2 n}(u,w,v)\right) \hat{=} V\left(\neg \mathrm{U}_{f^2 n}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{V}_{f^2 n}(w,u)\right) \hat{\cdot} V\left(\mathrm{W}_{f^2 n}(u,v)\right) \hat{=} 0$$
$$\text{for each } (3\mathrm{UVW})_{f^2 n} \vec{\in} \left\{(3\mathrm{AII})_{f^2 n}, (3\mathrm{IAI})_{f^2 n}, (3\mathrm{EIO})_{f^2 n}, (3\mathrm{OAO})_{f^2 n}\right\} \quad (6.32)$$
$$\text{and for each } n \vec{\in} \{1,2,3\},$$

$$V\left((4\mathrm{UVW})_{f^2 n}(u,w,v)\right) \hat{=} V\left(\neg \mathrm{U}_{f^2 n}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{V}_{f^2 n}(w,u)\right) \hat{\cdot} V\left(\mathrm{W}_{f^2 n}(u,v)\right) \hat{=} 0$$
$$\text{for each } (4\mathrm{UVW})_{f^2 n} \vec{\in} \left\{(4\mathrm{AEE})_{f^2 n}, (4\mathrm{IAI})_{f^2 n}, (4\mathrm{EIO})_{f^2 n}\right\} \quad (6.33)$$
$$\text{and for each } n \vec{\in} \{1,2,3\},$$

$$V\left((3\mathrm{UVW})_{f^2 4}(u,w,v)\right) \hat{=} V\left(\neg \mathrm{U}_{f^2 4}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{V}_{f^2 4}(w,u)\right) \hat{\cdot} V\left(\mathrm{W}_{f^2 4}(u,v)\right) \hat{=} 0$$
$$\text{for each } (3\mathrm{UVW})_{f^2 4} \vec{\in} \left\{(3\mathrm{AAI})_{f^2 4}, (3\mathrm{AII})_{f^2 4}, (3\mathrm{IAI})_{f^2 4}, \right. \quad (6.34)$$
$$\left. (3\mathrm{EAO})_{f^2 4}, (3\mathrm{EIO})_{f^2 4}, (3\mathrm{OAO})_{f^2 4}\right\},$$

$$V\left((4\mathrm{UVW})_{f^2 4}(u,w,v)\right) \hat{=} V\left(\neg \mathrm{U}_{f^2 4}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{V}_{f^2 4}(w,u)\right) \hat{\cdot} V\left(\mathrm{W}_{f^2 4}(u,v)\right) \hat{=} 0$$
$$\text{for each } (4\mathrm{UVW})_{f^2 4} \vec{\in} \left\{(4\mathrm{AAI})_{f^2 4}, (4\mathrm{AEE})_{f^2 4}, (4\mathrm{IAI})_{f^2 4}, (4\mathrm{EAO})_{f^2 4}, (4\mathrm{EIO})_{f^2 4}\right\}. \quad (6.35)$$

In this case, the catchwords of all pertinent RBESI's can be recovered with the help of Crls 6.3 and 6.4.

3) The above items 1 and 2 apply with any $\mathbf{F} \vec{\in} \kappa^{2\mathrm{pv}}$ subject to (2.46) in place of f^2.•

Th 6.1: ***The EMT's (EDT's) of asymmetric DBESI's.***

A) $\mathbf{F} \triangleright \in$

1°) Group AAA&AAI

$$\begin{aligned} V\left(\mathrm{Barbara}_{\in 1}(u,w,v)\right) &\hat{=} V\left((1\mathrm{AAA})_{\in 1}(u,w,v)\right) \\ &\hat{=} V\left(\neg \mathrm{A}_{\in 1}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\in 1}(u,w)\right) \hat{\cdot} V\left(\mathrm{A}_{\in 1}(u,v)\right) \\ &\hat{=} \left[\overset{\frown}{\cdot}_x \left[1 \hat{-} V(\neg[x \in w]) \hat{\cdot} V(x \in v)\right]\right] \hat{\cdot} \left[\overset{\frown}{\cdot}_y \left[1 \hat{-} V(\neg[y \in u]) \hat{\cdot} V(y \in w)\right]\right] \\ &\quad \hat{\cdot} \left[1 \hat{-} \overset{\frown}{\cdot}_z \left[1 \hat{-} V(\neg[z \in u]) \hat{\cdot} V(z \in v)\right]\right] \hat{=} 0. \end{aligned} \quad (6.9\mu_1)$$

$$\begin{aligned} V\left(\mathrm{Barapti}_{\in 1}(u,w,v)\right) &\hat{=} V\left((3\mathrm{AAI})_{\in 1}(u,w,v)\right) \\ &\hat{=} V\left(\neg \mathrm{A}_{\in 1}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\in 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{I}_{\in 1}(u,v)\right) \\ &\hat{=} \left[\overset{\frown}{\cdot}_x \left[1 \hat{-} V(\neg[x \in w]) \hat{\cdot} V(x \in v)\right]\right] \hat{\cdot} \left[\overset{\frown}{\cdot}_y \left[1 \hat{-} V(\neg[y \in w]) \hat{\cdot} V(y \in u)\right]\right] \\ &\quad \hat{\cdot} \left[\overset{\frown}{\cdot}_z \left[1 \hat{-} V(\neg[z \in u]) \hat{\cdot} V(\neg[z \in v])\right]\right] \\ &\hat{=} \overset{\frown}{\cdot}_x \left[V(x \in w) \hat{\cdot} \left[1 \hat{-} V(\neg[x \in u]) \hat{\cdot} V(\neg[x \in v])\right]\right] \\ &\hat{=} \left[\overset{\frown}{\cdot}_x V(x \in w)\right] \hat{\cdot} \left[\overset{\frown}{\cdot}_y \left[1 \hat{-} V(\neg[y \in u]) \hat{\cdot} V(\neg[y \in v])\right]\right] \\ &\hat{=} V\left(\bigvee_x [x \in w]\right) \hat{\cdot} V\left(\bigvee_y \left[[y \in u] \wedge [y \in v]\right]\right) \hat{\bar{=}} \mathrm{J}^1_{\in}(u,w,v). \end{aligned} \quad (6.10\mu_1)$$

$$\begin{aligned}
&V\left(\mathrm{Bamalip}_{\in 1}(u,w,v)\right)\hat{=}V\left((4\mathrm{AAI})_{\in 1}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathrm{A}_{\in 1}(v,w)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\in 1}(w,u)\right)\hat{\cdot}V\left(\mathrm{I}_{\in 1}(u,v)\right)\\
&\hat{=}\left[\hat{\prod}_x\left[1\hat{-}V(\neg[x\in v])\hat{\cdot}V(x\in w)\right]\right]\hat{\cdot}\left[\hat{\prod}_y\left[1\hat{-}V(\neg[y\in w])\hat{\cdot}V(y\in u)\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_z\left[1\hat{-}V(\neg[z\in u])\hat{\cdot}V(\neg[z\in v])\right]\right]\\
&\hat{=}\hat{\prod}_x\left[V(x\in v)\hat{\cdot}\left[1\hat{-}V(x\in u)\hat{\cdot}V(\neg[x\in w])\right]\right]\\
&\hat{=}\left[\hat{\prod}_x V(x\in v)\right]\hat{\cdot}\left[\hat{\prod}_y\left[1\hat{-}V(y\in u)\hat{\cdot}V(\neg[y\in w])\right]\right]\\
&\hat{=}V\left(\bigvee_x[x\in v]\right)\hat{\cdot}V\left(\bigvee_y\left[\neg[y\in u]\wedge[y\in w]\right]\right)\hat{\overline{=}}\,\mathrm{J}_{\in}^{2}(u,w,v).
\end{aligned}\qquad(6.11\mu_1)$$

2°) Group AII&IAI

$$\begin{aligned}
&V\left(\mathrm{Datisi}_{\in 1}(u,w,v)\right)\hat{=}V\left(\mathrm{Darii}_{\in 1}(u,w,v)\right)\\
&\hat{=}V\left((3\mathrm{AII})_{\in 1}(u,w,v)\right)\hat{=}V\left((1\mathrm{AII})_{\in 1}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathrm{A}_{\in 1}(w,v)\right)\hat{\cdot}V\left(\neg\mathrm{I}_{\in 1}(w,u)\right)\hat{\cdot}V\left(\mathrm{I}_{\in 1}(u,v)\right)\\
&\hat{=}V\left(\neg\mathrm{A}_{\in 1}(w,v)\right)\hat{\cdot}V\left(\neg\mathrm{I}_{\in 1}(u,w)\right)\hat{\cdot}V\left(\mathrm{I}_{\in 1}(u,v)\right)\\
&\hat{=}\left[\hat{\prod}_x\left[1\hat{-}V(\neg[x\in w])\hat{\cdot}V(x\in v)\right]\right]\\
&\hat{\cdot}\left[1\hat{-}\hat{\prod}_y\left[1\hat{-}V(\neg[y\in u])\hat{\cdot}V(\neg[y\in w])\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_z\left[1\hat{-}V(\neg[z\in u])\hat{\cdot}V(\neg[z\in v])\right]\right]\hat{=}0.
\end{aligned}\qquad(6.12\mu_1)$$

$$\begin{aligned}
&V\left(\mathrm{Dimatis}_{\in 1}(u,w,v)\right)\hat{=}V\left(\mathrm{Disamis}_{\in 1}(u,w,v)\right)\\
&\hat{=}V\left((4\mathrm{IAI})_{\in 1}(u,w,v)\right)\hat{=}V\left((3\mathrm{IAI})_{\in 1}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathrm{I}_{\in 1}(v,w)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\in 1}(w,u)\right)\hat{\cdot}V\left(\mathrm{I}_{\in 1}(u,v)\right)\\
&\hat{=}V\left(\neg\mathrm{I}_{\in 1}(w,v)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\in 1}(w,u)\right)\hat{\cdot}V\left(\mathrm{I}_{\in 1}(u,v)\right)\\
&\hat{=}\left[1\hat{-}\hat{\prod}_x\left[1\hat{-}V(\neg[x\in w])\hat{\cdot}V(\neg[x\in v])\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_y\left[1\hat{-}V(\neg[y\in w])\hat{\cdot}V(y\in u)\right]\right]\\
&\hat{\cdot}\left[\hat{\prod}_z\left[1\hat{-}V(\neg[z\in u])\hat{\cdot}V(\neg[z\in v])\right]\right]\hat{=}0.
\end{aligned}\qquad(6.13\mu_1)$$

3°) Group EAE&AEE

$$\begin{aligned}
&V\left(\mathrm{Cesare}_{\in 1}(u,w,v)\right)\hat{=}V\left(\mathrm{Celarent}_{\in 1}(u,w,v)\right)\\
&\hat{=}V\left((2\mathrm{EAE})_{\in 1}(u,w,v)\right)\hat{=}V\left((1\mathrm{EAE})_{\in 1}(u,w,v)\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{\in 1}(v,w)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\in 1}(u,w)\right)\hat{\cdot}V\left(\mathrm{E}_{\in 1}(u,v)\right)\\
&\hat{=}V\left(\neg\mathrm{E}_{\in 1}(w,v)\right)\hat{\cdot}V\left(\neg\mathrm{A}_{\in 1}(u,w)\right)\hat{\cdot}V\left(\mathrm{E}_{\in 1}(u,v)\right)\\
&\hat{=}\left[\hat{\prod}_x\left[1\hat{-}V(\neg[x\in w])\hat{\cdot}V(\neg[x\in v])\right]\right]\hat{\cdot}\left[\hat{\prod}_y\left[1\hat{-}V(\neg[y\in v])\hat{\cdot}V(y\in w)\right]\right]\\
&\hat{\cdot}\left[1\hat{-}\hat{\prod}_z\left[1\hat{-}V(\neg[z\in u])\hat{\cdot}V(\neg[z\in v])\right]\right]\hat{=}0.
\end{aligned}\qquad(6.14\mu_1)$$

$$
\begin{aligned}
&V\left(\mathrm{Calemes}_{\in 1}(u,w,v)\right) \hat{=} V\left(\mathrm{Camestres}_{\in 1}(u,w,v)\right)\\
&\hat{=} V\left((4\mathrm{AEE})_{\in 1}(u,w,v)\right) \hat{=} V\left((2\mathrm{AEE})_{\in 1}(u,w,v)\right)\\
&\hat{=} V\left(\neg \mathrm{A}_{\in 1}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{E}_{\in 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{E}_{\in 1}(u,v)\right)\\
&\hat{=} V\left(\neg \mathrm{A}_{\in 1}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{E}_{\in 1}(u,w)\right) \hat{\cdot} V\left(\mathrm{E}_{\in 1}(u,v)\right)\\
&\hat{=} \left[\hat{\boldsymbol{\cdot}}_x \left[1 \hat{-} V\left(\neg[x \in v]\right) \hat{\cdot} V(x \in w)\right]\right] \hat{\cdot} \left[\hat{\boldsymbol{\cdot}}_y \left[1 \hat{-} V\left(\neg[y \in u]\right) \hat{\cdot} V\left(\neg[y \in w]\right)\right]\right]\\
&\hat{\cdot} \left[1 \hat{-} \hat{\boldsymbol{\cdot}}_z \left[1 \hat{-} V\left(\neg[z \in u]\right) \hat{\cdot} V\left(\neg[z \in v]\right)\right]\right] \hat{=} 0.
\end{aligned}
\tag{6.15μ_1}
$$

4°) Group EAO

$$
\begin{aligned}
&V\left(\mathrm{Fesapo}_{\in 1}(u,w,v)\right) \hat{=} V\left(\mathrm{Felapton}_{\in 1}(u,w,v)\right)\\
&\hat{=} V\left((4\mathrm{EAO})_{\in 1}(u,w,v)\right) \hat{=} V\left((3\mathrm{EAO})_{\in 1}(u,w,v)\right)\\
&\hat{=} V\left(\neg \mathrm{E}_{\in 1}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\in 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{O}_{\in 1}(u,v)\right)\\
&\hat{=} V\left(\neg \mathrm{E}_{\in 1}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\in 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{O}_{\in 1}(u,v)\right)\\
&\hat{=} \left[\hat{\boldsymbol{\cdot}}_x \left[1 \hat{-} V\left(\neg[x \in w]\right) \hat{\cdot} V\left(\neg[x \in v]\right)\right]\right]\\
&\hat{\cdot} \left[\hat{\boldsymbol{\cdot}}_y \left[1 \hat{-} V\left(\neg[y \in w]\right) \hat{\cdot} V(y \in u)\right]\right]\\
&\hat{\cdot} \left[\hat{\boldsymbol{\cdot}}_z \left[1 \hat{-} V\left(\neg[z \in u]\right) \hat{\cdot} V(z \in v)\right]\right]\\
&\hat{=} \hat{\boldsymbol{\cdot}}_x \left[V(x \in w) \hat{\cdot} \left[1 \hat{-} V\left(\neg[x \in u]\right) \hat{\cdot} V(x \in v)\right]\right]\\
&\hat{=} \left[\hat{\boldsymbol{\cdot}}_x V(x \in w)\right] \hat{\cdot} \left[\hat{\boldsymbol{\cdot}}_y \left[1 \hat{-} V\left(\neg[y \in u]\right) \hat{\cdot} V(y \in v)\right]\right]\\
&\hat{=} V\left(\bigvee_x [x \in w]\right) \hat{\cdot} \left[V\left(\bigvee_y \left[[y \in u] \wedge \neg[y \in v]\right]\right)\right] \hat{\bar{=}} \mathrm{J}^3_{\in}(u,w,v).
\end{aligned}
\tag{6.16μ_1}
$$

5°) Group EIO

$$
\begin{aligned}
&V\left(\mathrm{Fresison}_{\in 1}(u,w,v)\right) \hat{=} V\left(\mathrm{Feriso}_{\in 1}(u,w,v)\right)\\
&\hat{=} V\left(\mathrm{Festino}_{\in 1}(u,w,v)\right) \hat{=} V\left(\mathrm{Ferio}_{\in 1}(u,w,v)\right)\\
&\hat{=} V\left((4\mathrm{EIO})_{\in 1}(u,w,v)\right) \hat{=} V\left((3\mathrm{EIO})_{\in 1}(u,w,v)\right)\\
&\hat{=} V\left((2\mathrm{EIO})_{\in 1}(u,w,v)\right) \hat{=} V\left((1\mathrm{EIO})_{\in 1}(u,w,v)\right)\\
&\hat{=} V\left(\neg \mathrm{E}_{\in 1}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{I}_{\in 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{O}_{\in 1}(u,v)\right)\\
&\hat{=} V\left(\neg \mathrm{E}_{\in 1}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{I}_{\in 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{O}_{\in 1}(u,v)\right)\\
&\hat{=} V\left(\neg \mathrm{E}_{\in 1}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{I}_{\in 1}(u,w)\right) \hat{\cdot} V\left(\mathrm{O}_{\in 1}(u,v)\right)\\
&\hat{=} V\left(\neg \mathrm{E}_{\in 1}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{I}_{\in 1}(u,w)\right) \hat{\cdot} V\left(\mathrm{O}_{\in 1}(u,v)\right)\\
&\hat{=} \left[\hat{\boldsymbol{\cdot}}_x \left[1 \hat{-} V\left(\neg[x \in w]\right) \hat{\cdot} V\left(\neg[x \in v]\right)\right]\right]\\
&\hat{\cdot} \left[1 \hat{-} \hat{\boldsymbol{\cdot}}_y \left[1 \hat{-} V\left(\neg[y \in u]\right) \hat{\cdot} V\left(\neg[y \in w]\right)\right]\right]\\
&\hat{\cdot} \left[\hat{\boldsymbol{\cdot}}_z \left[1 \hat{-} V\left(\neg[z \in u]\right) \hat{\cdot} V(z \in v)\right]\right] \hat{=} 0.
\end{aligned}
\tag{6.17μ_1}
$$

6°) Group AOO&OAO

$$
\begin{aligned}
&V\left(\text{Baroco}_{\in 1}(u,w,v)\right) \hat{=} V\left((2\text{AOO})_{\in 1}(u,w,v)\right) \\
&\hat{=} V\left(\neg \text{A}_{\in 1}(v,w)\right) \hat{\cdot} V\left(\neg \text{O}_{\in 1}(u,w)\right) \hat{\cdot} V\left(\text{O}_{\in 1}(u,v)\right) \\
&\hat{=} \left[\hat{\bullet}_x \left[1 \hat{-} V(\neg[x \in v]) \hat{\cdot} V(x \in w)\right]\right] \hat{\cdot} \left[1 \hat{-} \hat{\bullet}_y \left[1 \hat{-} V(\neg[y \in u]) \hat{\cdot} V(y \in w)\right]\right] \\
&\hat{\cdot} \left[\hat{\bullet}_z \left[1 \hat{-} V(\neg[z \in u]) \hat{\cdot} V(z \in v)\right]\right] \hat{=} 0.
\end{aligned}
\tag{6.18μ_1}
$$

$$
\begin{aligned}
&V\left(\text{Bocardo}_{\in 1}(u,w,v)\right) \hat{=} V\left((3\text{AOO})_{\in 1}(u,w,v)\right) \\
&\hat{=} V\left(\neg \text{O}_{\in 1}(w,v)\right) \hat{\cdot} V\left(\neg \text{A}_{\in 1}(w,u)\right) \hat{\cdot} V\left(\text{O}_{\in 1}(u,v)\right) \\
&\hat{=} \left[1 \hat{-} \hat{\bullet}_x \left[1 \hat{-} V(\neg[x \in w]) \hat{\cdot} V(x \in v)\right]\right] \hat{\cdot} \left[\hat{\bullet}_y \left[1 \hat{-} V(\neg[y \in w]) \hat{\cdot} V(y \in u)\right]\right] \\
&\hat{\cdot} \left[\hat{\bullet}_z \left[1 \hat{-} V(\neg[z \in u]) \hat{\cdot} V(z \in v)\right]\right] \hat{=} 0.
\end{aligned}
\tag{6.19μ_1}
$$

The EMT's (6.9μ_1)–(6.19μ_1) are results of the simultaneous substitutions $\mathbf{F} \triangleright \in$ and (2.55) throughout the PLMT's (6.9)–(6.19). In this case, like any one of the trains (6.10), (6.11), and (6.16), and also like their instances at any $\mathbf{F} \vec{\in} \kappa^{2pv}$ subject to (2.46) (see Crl 6.5)), the respective one of the trains (6.10μ_1), (6.11μ_1), and (6.16μ_1) is irreducible either to 0 or to 1. As can be seen from the next item B, some euautographic instances of (6.10), (6.11), and (6.16) do not share this property.

B) $\mathbf{F} \vec{\in} \{\subseteq, =, \subset\}$

The variants of (6.9μ_1)–(6.19μ_1) with any one of the three predicate-sign $\subseteq$, =, and $\subset$ in place of $\in$ are the instances of (6.9)–(6.19) subject to the substitution of that predicate-sign for '**F**', which is accompanied by the simultaneous substitutions (2.55). Therefore, all those variants are valid. At the same time, according to the pertinent instances of theorems (IV.1.45) and (IV.1.47),

$$V\left(\bigvee_x [x \subseteq w]\right) \hat{=} 0,\; V\left(\bigvee_x [x \subseteq v]\right) \hat{=} 0, \tag{6.36}$$

$$V\left(\bigvee_x [x = w]\right) \hat{=} 0,\; V\left(\bigvee_x [x = v]\right) \hat{=} 0, \tag{6.37}$$

while no such euautographic theorems exist with $\in$ or $\subset$ in place of $\subseteq$ or =. By (6.36) or (6.37), it follows from the variants of (6.10μ_1), (6.11μ_1), and (6.16μ_1) with $\subseteq$ or = in place of $\in$ that

$$\text{J}^1_{\subseteq}(u,w,v) \hat{=} \text{J}^2_{\subseteq}(u,w,v) \hat{=} \text{J}^3_{\subseteq}(u,w,v) \hat{=} 0, \tag{6.38}$$

$$\text{J}^1_{=}(u,w,v) \hat{=} \text{J}^2_{=}(u,w,v) \hat{=} \text{J}^3_{=}(u,w,v) \hat{=} 0. \tag{6.39}$$

Hence, the above variants of (6.10μ_1), (6.11μ_1), and (6.16μ_1) are *homogeneous*:

$$
\begin{aligned}
&V\left(\mathrm{Barapti}_{\subseteq 1}(u,w,v)\right) \,\hat{=}\, V\left((\mathrm{3AAI})_{\subseteq 1}(u,w,v)\right)\\
&\hat{=}\, V\left(\neg \mathrm{A}_{\subseteq 1}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\subseteq 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{I}_{\subseteq 1}(u,v)\right)\\
&\hat{=}\, \left[\widehat{\bullet}_x \left[1 \hat{-} V\left(\neg [x \subseteq w]\right) \hat{\cdot} V(x \subseteq v)\right]\right] \hat{\cdot} \left[\widehat{\bullet}_y \left[1 \hat{-} V\left(\neg [y \subseteq w]\right) \hat{\cdot} V(y \subseteq u)\right]\right]\\
&\hat{\cdot} \left[\widehat{\bullet}_z \left[1 \hat{-} V\left(\neg [z \subseteq u]\right) \hat{\cdot} V\left(\neg [z \subseteq v]\right)\right]\right] \,\hat{=}\, 0,
\end{aligned}
\tag{6.10μ₂}
$$

$$
\begin{aligned}
&V\left(\mathrm{Bamalip}_{\subseteq 1}(u,w,v)\right) \,\hat{=}\, V\left((\mathrm{4AAI})_{\subseteq 1}(u,w,v)\right)\\
&\hat{=}\, V\left(\neg \mathrm{A}_{\subseteq 1}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\subseteq 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{I}_{\subseteq 1}(u,v)\right)\\
&\hat{=}\, \left[\widehat{\bullet}_x \left[1 \hat{-} V\left(\neg [x \subseteq v]\right) \hat{\cdot} V(x \subseteq w)\right]\right] \hat{\cdot} \left[\widehat{\bullet}_y \left[1 \hat{-} V\left(\neg [y \subseteq w]\right) \hat{\cdot} V(y \subseteq u)\right]\right]\\
&\hat{\cdot} \left[\widehat{\bullet}_z \left[1 \hat{-} V\left(\neg [z \subseteq u]\right) \hat{\cdot} V\left(\neg [z \subseteq v]\right)\right]\right] \,\hat{=}\, 0,
\end{aligned}
\tag{6.11μ₂}
$$

$$
\begin{aligned}
&V\left(\mathrm{Fesapo}_{\subseteq 1}(u,w,v)\right) \,\hat{=}\, V\left(\mathrm{Felapton}_{\subseteq 1}(u,w,v)\right)\\
&\hat{=}\, V\left((\mathrm{4EAO})_{\subseteq 1}(u,w,v)\right) \,\hat{=}\, V\left((\mathrm{3EAO})_{\subseteq 1}(u,w,v)\right)\\
&\hat{=}\, V\left(\neg \mathrm{E}_{\subseteq 1}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\subseteq 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{O}_{\subseteq 1}(u,v)\right)\\
&\hat{=}\, V\left(\neg \mathrm{E}_{\subseteq 1}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{\subseteq 1}(w,u)\right) \hat{\cdot} V\left(\mathrm{O}_{\subseteq 1}(u,v)\right)\\
&\hat{=}\, \left[\widehat{\bullet}_x \left[1 \hat{-} V\left(\neg [x \subseteq w]\right) \hat{\cdot} V\left(\neg [x \subseteq v]\right)\right]\right]\\
&\hat{\cdot} \left[\widehat{\bullet}_y \left[1 \hat{-} V\left(\neg [y \subseteq w]\right) \hat{\cdot} V(y \subseteq u)\right]\right]\\
&\hat{\cdot} \left[\widehat{\bullet}_z \left[1 \hat{-} V\left(\neg [z \in u]\right) \hat{\cdot} V(z \in v)\right]\right] \,\hat{=}\, 0.
\end{aligned}
\tag{6.16μ₂}
$$

$$
\begin{aligned}
&V\left(\mathrm{Barapti}_{=1}(u,w,v)\right) \,\hat{=}\, V\left((\mathrm{3AAI})_{=1}(u,w,v)\right)\\
&\hat{=}\, V\left(\neg \mathrm{A}_{=1}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{=1}(w,u)\right) \hat{\cdot} V\left(\mathrm{I}_{=1}(u,v)\right)\\
&\hat{=}\, \left[\widehat{\bullet}_x \left[1 \hat{-} V\left(\neg [x = w]\right) \hat{\cdot} V(x = v)\right]\right] \hat{\cdot} \left[\widehat{\bullet}_y \left[1 \hat{-} V\left(\neg [y = w]\right) \hat{\cdot} V(y = u)\right]\right]\\
&\hat{\cdot} \left[\widehat{\bullet}_z \left[1 \hat{-} V\left(\neg [z = u]\right) \hat{\cdot} V\left(\neg [z = v]\right)\right]\right] \,\hat{=}\, 0,
\end{aligned}
\tag{6.10μ₃}
$$

$$
\begin{aligned}
&V\left(\mathrm{Bamalip}_{=1}(u,w,v)\right) \,\hat{=}\, V\left((\mathrm{4AAI})_{=1}(u,w,v)\right)\\
&\hat{=}\, V\left(\neg \mathrm{A}_{=1}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{=1}(w,u)\right) \hat{\cdot} V\left(\mathrm{I}_{=1}(u,v)\right)\\
&\hat{=}\, \left[\widehat{\bullet}_x \left[1 \hat{-} V\left(\neg [x = v]\right) \hat{\cdot} V(x = w)\right]\right] \hat{\cdot} \left[\widehat{\bullet}_y \left[1 \hat{-} V\left(\neg [y = w]\right) \hat{\cdot} V(y = u)\right]\right]\\
&\hat{\cdot} \left[\widehat{\bullet}_z \left[1 \hat{-} V\left(\neg [z = u]\right) \hat{\cdot} V\left(\neg [z = v]\right)\right]\right] \,\hat{=}\, 0,
\end{aligned}
\tag{6.11μ₃}
$$

$$
\begin{aligned}
&V\left(\mathrm{Fesapo}_{=1}(u,w,v)\right) \,\hat{=}\, V\left(\mathrm{Felapton}_{=1}(u,w,v)\right)\\
&\hat{=}\, V\left((\mathrm{4EAO})_{=1}(u,w,v)\right) \,\hat{=}\, V\left((\mathrm{3EAO})_{=1}(u,w,v)\right)\\
&\hat{=}\, V\left(\neg \mathrm{E}_{=1}(v,w)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{=1}(w,u)\right) \hat{\cdot} V\left(\mathrm{O}_{=1}(u,v)\right)\\
&\hat{=}\, V\left(\neg \mathrm{E}_{=1}(w,v)\right) \hat{\cdot} V\left(\neg \mathrm{A}_{=1}(w,u)\right) \hat{\cdot} V\left(\mathrm{O}_{=1}(u,v)\right)\\
&\hat{=}\, \left[\widehat{\bullet}_x \left[1 \hat{-} V\left(\neg [x = w]\right) \hat{\cdot} V\left(\neg [x = v]\right)\right]\right]\\
&\hat{\cdot} \left[\widehat{\bullet}_y \left[1 \hat{-} V\left(\neg [y = w]\right) \hat{\cdot} V(y = u)\right]\right]\\
&\hat{\cdot} \left[\widehat{\bullet}_z \left[1 \hat{-} V\left(\neg [z = u]\right) \hat{\cdot} V(z = v)\right]\right] \,\hat{=}\, 0.
\end{aligned}
\tag{6.16μ₃}
$$

At the same time, the variants of (6.10μ_1), (6.11μ_1), and (6.16μ_1) with $\subset$ in place of $\in$ remain *inhomogeneous*:

$$
\begin{aligned}
&V\left(\mathsf{Barapti}_{\subset 1}(u,w,v)\right) \hat{=} V\left((\mathsf{3AAI})_{\subset 1}(u,w,v)\right) \\
&\hat{=} V\left(\neg\mathsf{A}_{\subset 1}(w,v)\right) \hat{\cdot} V\left(\neg\mathsf{A}_{\subset 1}(w,u)\right) \hat{\cdot} V\left(\mathsf{I}_{\subset 1}(u,v)\right) \\
&\hat{=} \left[\hat{\cdot}_x\left[1 \hat{-} V(\neg[x \subset w]) \hat{\cdot} V(x \subset v)\right]\right] \hat{\cdot} \left[\hat{\cdot}_y\left[1 \hat{-} V(\neg[y \subset w]) \hat{\cdot} V(y \subset u)\right]\right] \\
&\hat{\cdot} \left[\hat{\cdot}_z\left[1 \hat{-} V(\neg[z \subset u]) \hat{\cdot} V(\neg[z \subset v])\right]\right] \\
&\hat{=} \hat{\cdot}_x\left[V(x \subset w) \hat{\cdot} \left[1 \hat{-} V(\neg[x \subset u]) \hat{\cdot} V(\neg[x \subset v])\right]\right] \\
&\hat{=} \left[\hat{\cdot}_x V(x \subset w)\right] \hat{\cdot} \left[\hat{\cdot}_y\left[1 \hat{-} V(\neg[y \subset u]) \hat{\cdot} V(\neg[y \subset v])\right]\right] \\
&\hat{=} V\left(\bigvee_x [x \subset w]\right) \hat{\cdot} V\left(\bigvee_y [[y \subset u] \wedge [y \subset v]]\right) \hat{\bar{=}} \mathsf{J}^1_{\subset}(u,w,v).
\end{aligned}
\qquad (6.10\mu_4
$$

$$
\begin{aligned}
&V\left(\mathsf{Bamalip}_{\subset 1}(u,w,v)\right) \hat{=} V\left((\mathsf{4AAI})_{\subset 1}(u,w,v)\right) \\
&\hat{=} V\left(\neg\mathsf{A}_{\subset 1}(v,w)\right) \hat{\cdot} V\left(\neg\mathsf{A}_{\subset 1}(w,u)\right) \hat{\cdot} V\left(\mathsf{I}_{\subset 1}(u,v)\right) \\
&\hat{=} \left[\hat{\cdot}_x\left[1 \hat{-} V(\neg[x \subset v]) \hat{\cdot} V(x \subset w)\right]\right] \hat{\cdot} \left[\hat{\cdot}_y\left[1 \hat{-} V(\neg[y \subset w]) \hat{\cdot} V(y \subset u)\right]\right] \\
&\hat{\cdot} \left[\hat{\cdot}_z\left[1 \hat{-} V(\neg[z \subset u]) \hat{\cdot} V(\neg[z \subset v])\right]\right] \\
&\hat{=} \hat{\cdot}_x\left[V(x \subset v) \hat{\cdot} \left[1 \hat{-} V(x \subset u) \hat{\cdot} V(\neg[x \subset w])\right]\right] \\
&\hat{=} \left[\hat{\cdot}_x V(x \subset v)\right] \hat{\cdot} \left[\hat{\cdot}_y\left[1 \hat{-} V(y \subset u) \hat{\cdot} V(\neg[y \subset w])\right]\right] \\
&\hat{=} V\left(\bigvee_x [x \subset v]\right) \hat{\cdot} V\left(\bigvee_y [\neg[y \subset u] \wedge [y \subset w]]\right) \hat{\bar{=}} \mathsf{J}^2_{\subset}(u,w,v).
\end{aligned}
\qquad (6.11\mu_4)
$$

$$
\begin{aligned}
&V\left(\mathsf{Fesapo}_{\subset 1}(u,w,v)\right) \hat{=} V\left(\mathsf{Felapton}_{\subset 1}(u,w,v)\right) \\
&\hat{=} V\left((\mathsf{4EAO})_{\subset 1}(u,w,v)\right) \hat{=} V\left((\mathsf{3EAO})_{\subset 1}(u,w,v)\right) \\
&\hat{=} V\left(\neg\mathsf{E}_{\subset 1}(v,w)\right) \hat{\cdot} V\left(\neg\mathsf{A}_{\subset 1}(w,u)\right) \hat{\cdot} V\left(\mathsf{O}_{\subset 1}(u,v)\right) \\
&\hat{=} V\left(\neg\mathsf{E}_{\subset 1}(w,v)\right) \hat{\cdot} V\left(\neg\mathsf{A}_{\subset 1}(w,u)\right) \hat{\cdot} V\left(\mathsf{O}_{\subset 1}(u,v)\right) \\
&\hat{=} \left[\hat{\cdot}_x\left[1 \hat{-} V(\neg[x \subset w]) \hat{\cdot} V(\neg[x \subset v])\right]\right] \\
&\hat{\cdot} \left[\hat{\cdot}_y\left[1 \hat{-} V(\neg[y \subset w]) \hat{\cdot} V(y \subset u)\right]\right] \\
&\hat{\cdot} \left[\hat{\cdot}_z\left[1 \hat{-} V(\neg[z \subset u]) \hat{\cdot} V(z \subset v)\right]\right] \\
&\hat{=} \hat{\cdot}_x\left[V(x \subset w) \hat{\cdot} \left[1 \hat{-} V(\neg[x \subset u]) \hat{\cdot} V(x \subset v)\right]\right] \\
&\hat{=} \left[\hat{\cdot}_x V(x \subset w)\right] \hat{\cdot} \left[\hat{\cdot}_y\left[1 \hat{-} V(\neg[y \subset u]) \hat{\cdot} V(y \subset v)\right]\right] \\
&\hat{=} V\left(\bigvee_x [x \subset w]\right) \hat{\cdot} \left[V\left(\bigvee_y [[y \subset u] \wedge \neg[y \subset v]]\right)\right] \hat{\bar{=}} \mathsf{J}^3_{\subset}(u,w,v).
\end{aligned}
\qquad (6.16\mu_4)\bullet
$$

Cmt 6.2. I recall that in accordance with Crl 4.1 the validity indices of asymmetric BStPSI's and BESI's at $n \vec{\in} \{2,3\}$ are the same as those at $n \rhd 1$. That is to say, all trains of identities, which are included under any one of the logical titles Crl 6.2, Crl 6.3, Crl 6.5, and Th 6.1, and which involve the LLP's or VLP's with the subscript '$_1$', remain valid if that subscript is replaced with '$_n$' subject to $n \vec{\in} \{1,2,3\}$, while the rest of a train remain unaltered. Particularly, in analogy with (6.30)–(6.35), the HESMT's for BESTI's with $\mathbf{F}\mathrel{\vec{\in}}\{\in,\subseteq,=,\subset\}$ and $n\mathrel{\vec{\in}}\{1,2,3\}$ can be summarized thus:

$$\begin{gathered}V\left((1\mathrm{UVW})_{\mathbf{F}n}(u,w,v)\right)\mathrel{\hat{=}}V\left(\neg\mathrm{U}_{\mathbf{F}n}(w,v)\right)\mathrel{\hat{\cdot}}V\left(\neg\mathrm{V}_{\mathbf{F}n}(u,w)\right)\mathrel{\hat{\cdot}}V\left(\mathrm{W}_{\mathbf{F}n}(u,v)\right)\mathrel{\hat{=}}0\\ \text{for each } (1\mathrm{UVW})_{\mathbf{F}n}\mathrel{\vec{\in}}\left\{(1\mathrm{AAA})_{\mathbf{F}n},(1\mathrm{EAE})_{\mathbf{F}n},(1\mathrm{AII})_{\mathbf{F}n},(1\mathrm{EIO})_{\mathbf{F}n}\right\}\\ \text{and each } \mathbf{F}\mathrel{\vec{\in}}\{\in,\subseteq,=,\subset\},\end{gathered}\tag{6.40}$$

$$\begin{gathered}V\left((2\mathrm{UVW})_{\mathbf{F}n}(u,w,v)\right)\mathrel{\hat{=}}V\left(\neg\mathrm{U}_{\mathbf{F}n}(v,w)\right)\mathrel{\hat{\cdot}}V\left(\neg\mathrm{V}_{\mathbf{F}n}(u,w)\right)\mathrel{\hat{\cdot}}V\left(\mathrm{W}_{\mathbf{F}n}(u,v)\right)\mathrel{\hat{=}}0\\ \text{for each } (2\mathrm{UVW})_{\mathbf{F}n}\mathrel{\vec{\in}}\left\{(2\mathrm{AEE})_{\mathbf{F}n},(2\mathrm{EAE})_{\mathbf{F}n},(2\mathrm{EIO})_{\mathbf{F}n},(2\mathrm{AOO})_{\mathbf{F}n}\right\}\\ \text{and each } \mathbf{F}\mathrel{\vec{\in}}\{\in,\subseteq,=,\subset\},\end{gathered}\tag{6.41}$$

$$\begin{gathered}V\left((3\mathrm{UVW})_{\mathbf{F}n}(u,w,v)\right)\mathrel{\hat{=}}V\left(\neg\mathrm{U}_{\mathbf{F}n}(w,v)\right)\mathrel{\hat{\cdot}}V\left(\neg\mathrm{V}_{\mathbf{F}n}(w,u)\right)\mathrel{\hat{\cdot}}V\left(\mathrm{W}_{\mathbf{F}n}(u,v)\right)\mathrel{\hat{=}}0\\ \text{for each } (3\mathrm{UVW})_{\mathbf{F}n}\mathrel{\vec{\in}}\left\{(3\mathrm{AAI})_{\mathbf{F}n},(3\mathrm{AII})_{\mathbf{F}n},(3\mathrm{IAI})_{\mathbf{F}n},(3\mathrm{EAO})_{\mathbf{F}n},(3\mathrm{EIO})_{\mathbf{F}n},\right.\\ \left.(3\mathrm{OAO})_{\mathbf{F}n}\right\}\text{and each } \mathbf{F}\mathrel{\vec{\in}}\{\subseteq,=\},\end{gathered}\tag{6.42}$$

$$\begin{gathered}V\left((4\mathrm{UVW})_{\mathbf{F}n}(u,w,v)\right)\mathrel{\hat{=}}V\left(\neg\mathrm{U}_{\mathbf{F}n}(v,w)\right)\mathrel{\hat{\cdot}}V\left(\neg\mathrm{V}_{\mathbf{F}n}(w,u)\right)\mathrel{\hat{\cdot}}V\left(\mathrm{W}_{\mathbf{F}n}(u,v)\right)\mathrel{\hat{=}}0\\ \text{for each } (4\mathrm{UVW})_{\mathbf{F}n}\mathrel{\vec{\in}}\left\{(4\mathrm{AAI})_{\mathbf{F}n},(4\mathrm{AEE})_{\mathbf{F}n},(4\mathrm{IAI})_{\mathbf{F}n},(4\mathrm{EAO})_{\mathbf{F}n},(4\mathrm{EIO})_{\mathbf{F}n}\right\}\\ \text{and each } \mathbf{F}\mathrel{\vec{\in}}\{\subseteq,=\},\end{gathered}\tag{6.43}$$

$$\begin{gathered}V\left((3\mathrm{UVW})_{\mathbf{F}n}(u,w,v)\right)\mathrel{\hat{=}}V\left(\neg\mathrm{U}_{\mathbf{F}n}(w,v)\right)\mathrel{\hat{\cdot}}V\left(\neg\mathrm{V}_{\mathbf{F}n}(w,u)\right)\mathrel{\hat{\cdot}}V\left(\mathrm{W}_{\mathbf{F}n}(u,v)\right)\mathrel{\hat{=}}0\\ \text{for each } (3\mathrm{UVW})_{\mathbf{F}n}\mathrel{\vec{\in}}\left\{(3\mathrm{AII})_{\mathbf{F}n},(3\mathrm{IAI})_{\mathbf{F}n},(3\mathrm{EIO})_{\mathbf{F}n},(3\mathrm{OAO})_{\mathbf{F}n}\right\}\\ \text{and each } \mathbf{F}\mathrel{\vec{\in}}\{\in,\subset\},\end{gathered}\tag{6.44}$$

$$\begin{gathered}V\left((4\mathrm{UVW})_{\mathbf{F}n}(u,w,v)\right)\mathrel{\hat{=}}V\left(\neg\mathrm{U}_{\mathbf{F}n}(v,w)\right)\mathrel{\hat{\cdot}}V\left(\neg\mathrm{V}_{\mathbf{F}n}(w,u)\right)\mathrel{\hat{\cdot}}V\left(\mathrm{W}_{\mathbf{F}n}(u,v)\right)\mathrel{\hat{=}}0\\ \text{for each } (4\mathrm{UVW})_{\mathbf{F}n}\mathrel{\vec{\in}}\left\{(4\mathrm{AEE})_{\mathbf{F}n},(4\mathrm{IAI})_{\mathbf{F}n},(4\mathrm{EIO})_{\mathbf{F}n}\right\}\\ \text{and each } \mathbf{F}\mathrel{\vec{\in}}\{\in,\subset\}.\end{gathered}\tag{6.45}$$ •

6.3. Binary conformal catlogographic syllogistic implication schemata (BCFCLSIS'ta)

Df 6.3. 1) In accordance with Ax I.8.1(2), Df I.8.4(2), and Cmt II.7.5(7), and also in agreement with the item 7 of subsection 1.5, the *analo-catlographic* (and hence *analo-homolographic*) substitutions (I.8.18) and (II.7.32a), i.e.

$$u\mapsto u,\ v\mapsto v,\ w\mapsto w,\ x\mapsto x,\ y\mapsto y,\ z\mapsto z,\tag{6.46}$$

$$V\mapsto V,\tag{6.47}$$

without any quotation marks, throughout any separate *euautographic master*, or *decision*, *theorem* (*EMT* or *EDT*) and hence throughout its *euautographic slave relation* (*ESR*), such as e.g. as an ESJ (euautographic syllogistic judgment) or an ESI (euautographic syllogistic implication), result in the *catlogographic relations* (*CLR's*), which are respectively called the *conformal catlogographic* (*CFCL*) *interpretand of the EMT* (*EDT*) and the *CFCL of the ESR* or, more generally, a *CFCL master*, or *decision*, *theorem* (*CFCLMT* or *CFCLDT*) and a *catlogographic slave relation* (*CLSR*). Consequently, the CFCL interpretand of a BESJ is called a *binary catlogographic syllogistic judgment schema* (*BCLSJS*, pl. *BCLSJS'ta*), or *form* (*BCLSJF*), and similarly the CFCL interpretand of a BESI is called a *binary catlogographic syllogistic implication schema* (*BCLSIS*, pl. *BCLSIS'ta*), or *form* (*BCLSIF*). A CFCLMT or its CLSR will indiscriminately be called a *CFCL relation* (*CFCLR*).

2) In accordance with the above said, a BCFCLSIS has the same LLP (logographic logical predicate) and the same VLP (verbal logical predicate) as those of the BESI being its interpretans. For instance, for each $n \vec{\in} \{1,2,3,4\}$, $\mathsf{Barapti}_{f^2 n}(u,w,v)$ (or $(\mathsf{3AAI})_{f^2 n}(u,w,v)$), is the CFCL interpretand of $\mathsf{Barapti}_{f^2 n}(u,w,v)$ (or $(\mathsf{3AAI})_{f^2 n}(u,w,v)$), and similarly with any $\mathbf{F} \vec{\in} \kappa^{2\mathrm{pv}}$ subject to (2.46) in place of f^2, and also similarly, for each $n \vec{\in} \{1,2,3\}$, with any $\mathbf{F} \vec{\in} \vec{\mathrm{K}}^{\mathrm{pc}}_{\in}$ subject to (2.47) in place of f^2. At the same time, the CFCL interpretands of the RBESI's, whose EMT's (EDT's) have been explicated in Clr 6.5, are irrelevant to Aristotelian logic as interpretands of FCS's and in general they are impractical until some $\mathbf{F} \vec{\in} \kappa^{2\mathrm{pv}}$ are distinguished from the others by logographic or verbal axioms and are utilized in practice of logical reasoning. By contrast, the CFCL interpretands of the DBESI's, whose EMT's (EDT's) are comprised in Th 6.1, turn out to be natural interpretands of the respective FCS's of Aristotelian logic. Therefore in what follows, I shall discuss the CFCL interpretands of the DBESI's in greater detail. When necessary, I shall refer to the CFCL interpretand of an EMT comprised in Th 6.1 by using the variant of the bookmark of the EMT with the letter 'κ' in place of 'μ', although the CFCL interpretand is not actually written down. Likewise, the CFCL interpretands of the EMT's (6.40)–(6.45), i.e. the variants of the latter subject to the substitutions (6.46) and (6.47), will be referred to as (6.40κ)–(6.45κ) respectively. In this way, I shall avoid writing down a CFCLMT, which is obviously understood if the EMT being its interpretans is already written down somewhere in the treatise.•

Df 6.4. 1) In accordance with Ax 8.1**(**6), a CLR, i.e. the CFCL interpretand of a vavn-decided ER, preserves the validity-value of the ER and acquires the respective *tautologousness-value* that is compatible with its validity-value so that the CLR is said to be *tautologous* (*universally true*) or *antitautologous* (*universally antitrue*, *universally false*, *contradictory*) or else *ttatt-neutral* (*neutral with respect to tautologousness and antitautologousness, neither tautologous nor antitatologous*) if and only if it is *valid* (*kyrologous*) or *antivalid* (*antikyrologous*) or *vav-neutral neutral with respect to validity and antivalidity* (*neither valid nor antivalid, udeterologous*) respectively.

2) A *ttatt-neutral* CLR, i.e. the CFCL interpretand of a vav-neutral ER, is called a *transformative CLR* (*TCLR*) and also the *transformative CFCL* (*TCFCL*) *interpretand of the ER* in either one of the following two cases:

a) The CLR is *assumed* (*postulated, taken for granted*) to be *veracious,* i.e. *accidentally true,* in the sense that it is *conformable to a certain fact of interrelation of classes*.

b) The CLR is a CFCLMT (CFCLDT), which is developed further with allowance for the *catlogographic postulates* (see the previous item) so as to result in the *transformed*, or *transformative*, *CFCLMT* (*TCFCLMT*), according to which its CLSR is unambiguously decided to be one of the following three kinds: *veracious* (*accidentally true*), *antiveracious* (*accidentally antitrue*, *accidentally false*), or *vravr-neutral* (*vravr-indeterminate*, *neither veracious nor antiveracious*).

A CLR, i.e. the CFCL interpretand of an ER, is said to be a *conservative CLR* (*CCLR*) and also the *conservative CFCL* (*TCFCL*) *interpretand of the ER* if it is not transformative. Particularly, a CLSR is said to be a conservative one if and only if it is a *ttatt-determinate* (*ttatt-unnutral*, *tautologous* or *antitautologous*) *CLR*, i.e. the CFCL interpretand of a *vav-determinate* (*vav-unnutral*, *valid* or *antivalid*) *ER*, or else a *suspended ttatt-neutral* (*ttatt-indeterminate*) *CLR*. Likewise, a CFCLMT (CFCLMT) is called a *conservative one* (*CCFCLMT* or *CCFCLMT*) if it is not transformative.

3) The *tautologousness-value tautologousness* (*universal truth*) or the *veracity-value veracity* (*accidental truth*) is indiscriminately called the *truth-value truth*; whereas the *tautologousness-value antitautologousness* (*universal antitruth, universal falsity*, *contradictoriness*) or the *veracity-value antiveracity* (*accidental antitruth, accidental falsity*) is indiscriminately called the *truth-value antitruth*

(*falsity*); the veracity-value vravr-neutrality is alternatively called the *truth-value truth-antitruth neutrality* (*tat-neutrality*) and vice versa. Hence, the tautologousness-values tautologousness and antitautologousness are at the same time the truth-values truth and antitruth respectively, but not vice versa.

4) The above-said applies particularly to the CCFCL interpretand of a DBESI and to the CCFCL interpretands of the constituent ER's of the DBESI. In order to explicate the difference between a *valid* DBESI and its CCFCL interpretand, I shall, by way of example, use the DBESI $\mathsf{Barbara}_{\in 1}(\mathsf{u},\mathsf{w},\mathsf{v})$, i.e. $(\mathsf{1AAA})_{\in 1}(\mathsf{u},\mathsf{w},\mathsf{v})$, whose EMT (EDT) is given as the train of identities (6.9μ_1) of Th 6.1, and I shall also use its CCFCL interpretand $\mathsf{Barbara}_{\in 1}(u,w,v)$, i.e. $(\mathsf{1AAA})_{\in 1}(u,w,v)$, whose CCFCLMT (CCFCLDT) is the CCFCL interpretand of the EMT (6.9μ_1), i.e.

$$\begin{gathered} V(\mathsf{Barbara}_{\in 1}(u,w,v)) \hat{=} V((\mathsf{1AAA})_{\in 1}(u,w,v)) \\ \hat{=} V(\neg\mathsf{A}_{\in 1}(w,v)) \hat{\cdot} V(\neg\mathsf{A}_{\in 1}(u,w)) \hat{\cdot} V(\mathsf{A}_{\in 1}(u,v)) \hat{=} 0. \end{gathered} \tag{6.9κ_1}$$

5) In accordance with Crl 4.10, each one of the three multipliers $\mathsf{A}_{\in 1}(\mathsf{w},\mathsf{v})$, $\mathsf{A}_{\in 1}(\mathsf{u},\mathsf{w})$, and $\mathsf{A}_{\in 1}(\mathsf{u},\mathsf{v})$, occurring in (6.9$\mu_1$), is a *vav-neutral* ER of $\mathsf{A}_{1\mathsf{A}}$ Therefore, I *may neither deduce* from (6.9μ_1) and *assert* that $V(\neg\mathsf{A}_{\in 1}(\mathsf{w},\mathsf{v})) \hat{=} 0$ or $V(\neg\mathsf{A}_{\in 1}(\mathsf{u},\mathsf{w})) \hat{=} 0$ or $V(\mathsf{A}_{\in 1}(\mathsf{u},\mathsf{v})) \hat{=} 0$ *nor may I assume* that $V(\neg\mathsf{A}_{\in 1}(\mathsf{w},\mathsf{v})) \hat{=} 1$ and $V(\neg\mathsf{A}_{\in 1}(\mathsf{u},\mathsf{w})) \hat{=} 1$ to *conclude* from (6.9μ_1) that $V(\mathsf{A}_{\in 1}(\mathsf{u},\mathsf{v})) \hat{=} 0$, – just as I may not assume, e.g., that $V(\mathsf{p}) \hat{=} 0$ or that $V(\mathsf{q}) \hat{=} 1$. All the above *unassertive euautographic equalities are vav-neutral ER's* (cf. Cmt IV.1.8).

6) In contrast to u, v, and w, being *euautographic pseudo-variables*, *u*, *v*, and *w* are *catlogographic variables* that may assume (take on) some *classes* as their *accidental denotata*, – classes that may stand in relations with one another. In this case, the *CLR* (*catlogographic relation*) $\mathsf{A}_{\in 1}(w,v)$, e.g., is said to be:

a) *veracious* or *accidentally true*, which is expressed formally as

$$V(\mathsf{A}_{\in 1}(w,v)) \hat{=} 0, \tag{6.48a}$$

if it is conformable to the relation between the classes *w* and *v*;

b) *antiveracious* or *accidentally antitrue* (*accidentally false*), which is expressed formally as

$$V(\mathsf{A}_{\in 1}(w,v)) \hat{=} 1 \text{ or } V(\neg\mathsf{A}_{\in 1}(w,v)) \hat{=} 0, \tag{6.48b}$$

if it is not conformable to the relation between the classes *w* and *v*, but $\neg\mathsf{A}_{\in 1}(w,v)$ is;

c) *veracity-antiveracity neutral* (*vravr-neutral*) or *veracity-antiveracity indeterminate* (*vravr-indeterminate*), which is expressed formally as

$$V\left(\mathsf{A}_{\in 1}(w,v)\right) \mathrel{\hat{=}} i_{\sim}\left|\mathsf{A}_{\in 1}(w,v)\right\rangle \qquad (6.48c)$$

subject to the idempotent law:

$$i_{\sim}\left|\mathsf{A}_{\in 1}(w,v)\right\rangle \mathbin{\hat{\wedge}} i_{\sim}\left|\mathsf{A}_{\in 1}(w,v)\right\rangle \mathrel{\hat{=}} i_{\sim}\left|\mathsf{A}_{\in 1}(w,v)\right\rangle, \qquad (6.48c_{+})$$

if it is *neither veracious nor antiveracious*.

A like definition applies with $\mathsf{A}_{\in 1}(u,w)$ or $\mathsf{A}_{\in 1}(u,v)$ in place of $\mathsf{A}_{\in 1}(w,v)$. Once the CCFCL interpretand, as $\mathsf{A}_{\in 1}(u,w)$, $\mathsf{A}_{\in 1}(u,v)$, or $\mathsf{A}_{\in 1}(w,v)$, of a vav-neutral ER, as $\mathsf{A}_{\in 1}(\mathsf{w},\mathsf{v})$, $\mathsf{A}_{\in 1}(\mathsf{u},\mathsf{w})$, and $\mathsf{A}_{\in 1}(\mathsf{u},\mathsf{v})$ respectively, is assigned with a certain one of the above three veracity-values either by assumption, i.e. by taking it for granted, or by inference (see item 7 below), the former becomes a *transformative conformal catlogographic* (*TCFCL*) *interpretand* of the latter, although its appearance remains unaltered.

7) As has already been indicated in Cmt II.7.5(7), the domain of definition of the kernel-sign (operator) V is an extension of domain of definition of the kernel-sign V from the PLR's and ER's onto the CCFCLR's, being the CCFCL interpretands of the ER's. In contrast to the kernel-sign V, which is called the *validity-sign* or, when regarded as an abbreviation of $\mathsf{V}(\ )$, the *validity-operator*, the kernel-sign V, is called the *truth-sign* or, when regarded as an abbreviation of $V(\ )$, the *truth-operator*. By extension, V satisfies the same rules of inference and decision as V. Therefore, as far as there is no danger of confusion, it is convenient to use V equivocally for denoting and mentioning both itself and its CFCL extension, V. However, the most essential semantic properties of the CCFCL and TCFCL interpretands of BESI's turn out to be unexpressible if V is used equivocally. Therefore, in this discussion, I systematically distinguish between V and V.

8) In accordance with the terminology introduced in Ax I.8.1(6), the *binary euautographic syllogistic implication* (*BESI*) $\mathsf{Barbara}_{\in 1}(\mathsf{u},\mathsf{w},\mathsf{v})$, or $(\mathsf{1AAA})_{\in 1}(\mathsf{u},\mathsf{w},\mathsf{v})$, satisfying the EMT (6.9μ_1) is called a *valid*, or *kyrologous*, *ER* (*euautographic relation*) and also a *euautographic kyrology*, whereas the *binary catlogographic*

syllogistic implication (*BCLSIS*) $\mathsf{Barbara}_{\in 1}(u,w,v)$, or $(\mathsf{1AAA})_{\in 1}(u,w,v)$, satisfying the CCFCLMT (CCFCLDT) (6.9κ_1), is called a *tautologous*, or *universally true*, *CLR* (*catlogographic relation*) and also a *catlogographic tautology*. By Df 6.2(2), the EMT (EDT) (6.9μ_1) is alternatively called a *euautographic syllogistic master*, or *decision*, *theorem* (*ESMT* or *ESDT*) or, more specifically, a *homogeneous one* (*HESMT* or *HESDT*) – in contrast to *inhomogeneous ones* (*IHESMT's* or *IHESDT's*). Accordingly, the CCFCLMT (CCFCLDT) (6.9κ_1) will be called a *conservative conformal catlogographic syllogistic master*, or *decision*, *theorem* (*CCFCLSMT* or *CCFCLSDT*) or, more specifically, a *homogeneous one* (*HCCFCLSMT* or *HCCFCLSDT*) – in contrast to *inhomogeneous ones* (*IHCCFCLSMT's* or *IHCCFCLSDT's*). I shall use the abbreviations: "*HESMT*" (or "*HEMDT*") for "*homogeneous* ESMT" (or "*homogeneous* ESDT"), "*IHESMT*" (or "*IHEMDT*") for "*inhomogeneous* ESMT" (or "*inhomogeneous* ESDT"), "*HCFCLSMT*" (or "*HCFCLDT*") for "*homogeneous* CFCLSMT" (or "*homogeneous* CFCLSDT"), and "*IHCFCLSMT*" (or "*IHCFCLSDT*") for "*inhomogeneous* CFCLSMT" (or "*inhomogeneous* CFCLSDT"). In the sequel, I may also use the self-explanatory variants of some of the above abbreviations with "*T*CFCL" ("*transformative* CFCL") in place of "*C*CFCL" ("*conservative* CFCL").

9) According to item 5 of this definition, the HESMT (6.9μ_1) is not an inference rule. By contrast, from items 6–8 of this definition, it follows that the HCCFCLDT (6.9κ_1) is a *catlogographic inference rule*, according to which

$$\text{if } V(\mathsf{A}_{\in 1}(w,v)) \hat{=} 0 \text{ and } V(\mathsf{A}_{\in 1}(u,w)) \hat{=} 0 \text{ then } V(\mathsf{A}_{\in 1}(u,v)) \hat{=} 0, \tag{6.49}$$

i.e.

$$\text{if '}\mathsf{A}_{\in 1}(w,v)\text{' and '}\mathsf{A}_{\in 1}(u,w)\text{' are veracious then '}\mathsf{A}_{\in 1}(u,v)\text{' is veracious} \tag{6.43'}$$

or simply

$$\text{if } \mathsf{A}_{\in 1}(w,v) \text{ and } \mathsf{A}_{\in 1}(u,w) \text{ then } \mathsf{A}_{\in 1}(u,v), \tag{6.49''}$$

which is which is a semi-verbal form of $\mathsf{Barbara}_{\in 1}(u,w,v)$, i.e. of $(\mathsf{1AAA})_{\in 1}(u,w,v)$ (cf. (1.44)–(1.44'')). Consequently, (6.49'') can be used as an interpretand of the FCS $\text{Barbara}(u,w,v)$, i.e. $(1\text{AAA})(u,w,v)$, in accordance with the formal definition:

$$\begin{gathered}\mathrm{Barbara}(u,w,v)\leftrightarrow(1\mathrm{AAA})(u,w,v)\\ \rightarrow[\mathsf{A}(w,v)\wedge\mathsf{A}(u,w)\Rightarrow\mathsf{A}(u,v)]\rightarrow[\mathsf{A}_{\in1}(w,v)\wedge\mathsf{A}_{\in1}(u,w)\Rightarrow\mathsf{A}_{\in1}(u,v)]\\ \rightarrow(1\mathsf{AAA})_{\in1}(u,w,v)\leftrightarrow\mathsf{Barbara}_{\in1}(u,w,v)\end{gathered}\tag{6.50}$$

(cf. (1.45)). It is understood that definition (6.50) applies also with each $\mathbf{F}\,\vec{\in}\,\{\subseteq,\subset,=\}$ in place of $\in$ and that each one of the four definitions applies with $_2$ or $_3$, – or, putting it differently, with $_n$ subject to $n\,\vec{\in}\,\{1,2,3\}$, – in place of $_1$.

10) The above items 5–9 apply, *mutatis mutandis*, with each one of the HEMT's (6.12μ_1)–(6.15μ_1) and (6.17μ_1)–(6.19μ_1) and with the respective one of their CCFCL interpretands (6.12κ_1)–(6.15κ_1) and (6.17κ_1)–(6.19κ_1) in place of the HEMT (6.9μ_1) and its CCFCL interpretand (6.9κ_1) respectively.

11) In addition, the above items 5–9 apply, *mutatis mutandis*, with each one of the HEMT's (6.10μ_2), (6.11μ_2), (6.16μ_2)¸ (6.10μ_3), (6.11μ_3), (6.16μ_3) and with the respective one of their CCFCL interpretands (6.10κ_2), (6.11κ_2), (6.16κ_2)¸ (6.10κ_3), (6.11κ_3), (6.16κ_3) in place of the HEMT (6.9μ_1) and its CCFCL interpretand (6.9κ_1) respectively. By contrast, the EMT's (6.10μ_1), (6.11μ_1), (6.16μ_1)¸ (6.10μ_4), (6.11μ_4), (6.16μ_4) and hence their CCFCL interpretands (6.10κ_1), (6.11κ_1), (6.16κ_2)¸ (6.10κ_4), (6.11κ_4), (6.16κ_4) are inhomogeneous. Hence, the BCFCLSIS'ta (binary conformal catlogographic syllogistic implication schemata) being CLSR's (catlogographic slave relations) of the latter IHCCFCLSMT's are ttatt-neutral so that they are not catlogographic inference rules. Therefore, they cannot serve as interpretands of any FCS's. At the same time, I can to adopt the catlogographic postulates (1.49)–(1.51) and the like postulates with $\subset$ in place of $\in$ thus turning the above *IHC*CFCLSMT's into the respective *veracious ttatt-neutral HT*CFCLSMT's (see (1.49b)–(1.51b) and their variants with $\subset$ in place of $\in$). The CLSR's of these HTCFCLSMT's are *veracious ttstt-neutral* BCFCLSI's which can be used as interpretands of of the respective FCS's (see (1.45$_1$)–(1.48$_1$) and their variants with $\subset$ in place of $\in$).

Appendices: Metalinguistic themes

A1. Anglicized morphological constructions of Greek and Latin origin

I have already mention in section 1, that most terms used that are in this treatise are ones of my own which I form by combining Anglicized morphemes of Greek or Latin origin that have the appropriate etymological sense. Some of the morphemes are established ones that can be found, e.g., in WTNID or PED either as separate vocabulary entries or as constituent parts of longer vocabulary entries. The other morphemes are ones of my own which I form in accordance with *The Oxford Dictionary of Modern Greek* (*Greek–English* and *English–Greek*) by Pring [1982] and *Cassell's Latin Dictionary* (*Latin–English* and *English–Latin*) by Simpson [1959]. I also use these dictionaries for selecting and explaining the appropriate etymological senses of established morphemes, when desired. In citing or paraphrasing a dictionary definition, I preserve the style and particularly most of the abbreviations that are used in the dictionary from which the definition is taken.

Simpson [1959] is a bilingual *Classic* (*Old*) *Latin*-English and English-*Classic Latin* dictionary, and therefore its authenticity as an etymological source of the *established* Anglicized Classic Latin words is unquestionable. By contrast, Pring [1982] is a bilingual *Modern Greek*-English and English-*Modern Greek* dictionary. Modern Greek is an inseparable mixture of two parts: *Katharevusa* and *Demotic*. Katharevusa, – from the Greek noun "καθαρεύουσα" \kaθarévusa\ meaning *purified*, or *formal, language*, – is the part of Modern Greek conforming to vocabulary and grammar (phonetics, morphology, and syntax) of *Classical* (*Attic*) *Greek* and tending to reject loanwords. *Demotic*, – from the Greek adjective "δημόσιος" \ðimósios\ meaning *public*), – is the part of Modern Greek that includes Greek words in modern demotic (colloquial) usage and is characterized by free acceptance of loanwords and simplification of inflections. Katharevusa and Demotic are interwoven in the Modern Greek so tightly that it is more correct to treat of Katharevusa and Demotic as *features* of the language rather than as its parts. Here follow some most general characteristics of Modern Greek and of his dictionary by Pring himself.

«When Greece won independence in 1830, Athens and the Peloponnese became the political core of the new kingdom, and it is their dialects which form the basis of the standard spoken Greek of today. But the official language of a modern state could not be wrought out of the folklore of the medieval peasantry. Efforts were made to produce a purified form of Greek (‘katharevusa’), suitable for modern needs. But they became too deeply influenced by the spirit of Atticism; and the problem of finding a natural prose medium supple enough to provide expression in both formal and colloquial terms was not solved to anybody’s satisfaction. Now, after more than a century of independence, Greeks are still frustrated by the ‘language question’. But with the spread of education and the growth of journalism and broadcasting, the question begins to solve itself. Demotic and katharevusa cannot be kept apart, and a form of Greek is already emerging which combines features of both.» (Preface to the Greek–English part of Pring [1982], p. viii).

«The nucleus of the modern vocabulary has been handed down from ancient Greek. It is supplemented by several strata of loanwords, of which the chief are: Latin from the Hellenistic and Byzantine periods); Italian (from Venetian and Genoese occupation of Greek lands after 1200), Turkish (from the period 1453–1830), French, and, to the lesser degree, English (during the last 1300 years). The abundant resources of derivation and composition which Greek possesses have made easy the creation of new words, especially scientific terms, out of the native stock. These include translation-words, such as *σιδηρόδρομος* (chemin-de-fer) ⟨meaning *railway*; etymologically, the noun “*σίδηρο*” means *iron*, and the noun “*δρόμος*” *way* or *road* – Ya. I.⟩, and the reborrowing from the Hellenic period coinage already current elsewhere, as *ἀεροπλάνον* (aeroplane)» (*ibdem*, p. xi).

«The Greek of daily use is based broadly on *demotic* (or ‘common Greek’) rather than the more formal *katharevusa* (or ‘puristic Greek’), which is by tradition the language of Law, the Church, the official world, and the domain of science and technology. But the terms demotic and katharevusa (D and K hereinafter) have another, separate, meaning. Besides indicating degrees of formality in the manner of expression, they also designate more precise differences arising in morphology and syntax.» (Preface to the English-Greek part of Pring [1982], p. v).

«… the terms D and K have disparate ranges of meaning, and may not always prove adequate signposts for those who explore the paths of Greek...

This being so, too much concern with ideas of a 'demotic versus katharevusa' conflict may bring more confusion than clarity to the matter. The dictionary's function is to reflect usage without being committed to either one or other of these philosophical watchwords; and any preemptive application of D or K labels to headwords would be too suggestive of a dichotomy that belongs rather to the 'language question' than to the language, and too likely to obscure the interplay of those variegated strands of usage from which the fabric of Greek is woven.» (*ibdem*, pp. vi, vii).

In accordance with the above-said, in explaining the etymological senses of established English morphemes or in forming new Anglicized morphemes or morphological constructions of Greek origin, differences in the D and K inflections or diacritics of the pertinent Greek words are immaterial. In forming a new Anglicized morpheme or morphological construction, the main criteria of its acceptability is its congruency from the standpoint of the English grammar and lexicon and not the fact whether its Greek etymon is a K one or a D, particularly loaned, one. That is to say, in this case it does not matter whether or not the Greek etymon existed in the Plato and Aristotle time and, if it did, how the word was spelled or pronounced then. Still, the etymology of Classical Latin words that is given in Simpson's dictionary in the supposedly original Greek spelling or the etymology of English words that is given in authoritative explanatory dictionaries of the English language, particularly in WTNID, in the transliterated spelling allows in most cases establishing whether a given Greek word is an ancient one or a loaned one.

The spoken Modern Greek language is a *polytonic* one (not to be confused with a tone language as any of a great many of Chinese dialects). Accordingly, the Modern Greek written language, which is an *alphabetic* (*lettered*) language, has a great many (more than twenty) single or combined *diacritics* (*diacritical marks*) which are placed over or under individual vowel letters for indicating variations in phonetic values or tones of the corresponding speech sounds. An attempt to introduce all of them into a typeset would be a nightmare for the typesetter (cf. http://www.pauthun.org). Therefore, only a part of the diacritics is usually written down. Particularly, Pring makes use at least of the following ten diacritics: the *acute accent* (*accent mark*) of the form ´; the *grave accent* of the form `; the *circumflex accent*, or briefly the *circumflex*, of the form ⁀ for indicating a raising-falling tone of long vowel sounds; the *diaeresis* or *dieresis* (from the Greek verb “διαρῶ” \ðiaro\ meaning «divide», pl. “*diaereses*” or “*diereses*”) of the form ¨; the *Greek dialytika tonos* of the form ΅ (from the Greek etymons: the adjective “διαλυτικόσ” \ðialitikos\ meaning «dissolvent», the kindred plural noun “διαλυτικά” \ðialitika\ meaning «diaeresis», and the noun “τόνος” \tonos\ meaning «tone» or, in the grammar, «accent»); the two simple diacritics of the forms ’ and ‛, each taken alone or in combination with the following ´; the combined diacritic consisting of ’ and ⁀ over it (see, e.g., the vocabulary entries “ευ” and “ευγε”). The circumflex accent ⁀ is sometimes used interchangeably with ~ (cf., e.g., the definiendum “τυχαῖος” in the Greek-English part of Pring’s dictionary and its token “τυχαῖος” that is used as the definiens of the definiendum “accidental” in the English-Greek part of that dictionary). According to Pring (*ibidem*, Preface to the Greek–English dictionary p. xiv), some Modern Greek vowel-digraphs are pronounced as a single vowel sound unless the first letter of a digraph bears an acute accent or the second a diaeresis; a diaeresis can be used simultaneously with an acute accent in the form of a dialytika tonus (as in ΐ or ΰ). In these two cases each vowel letter has a separate phonetic value. Either of the words “divider” and “separator” is perhaps an appropriate English equivalent for “diaeresis”. In connection with the Greek accentuation, Pring (*ibidem*, p. xvi) says: «The difference between acute, grave, and circumflex accent does not affect pronunciation. In printed Greek the grave accent of final syllables is sometimes replaced by an acute. Fluctuations of usage in this matter is reflected in the dictionary.» (As an illustration of the above-said, cf. the vocabulary entry “εγώ” and its token with ` in place of ´ in the entry under the headword “ίδιος”.) Most of the above-mentioned diacritics are also used in the Greek etymons cited in the pertinent Latin vocabulary entries of Simpson’s dictionary.

Phonetic nuances of Greek words are irrelevant to my use of the words for forming terms of this treatise. Indeed, in spite of the fact that I do not speak Greek, I am able to form new Greek-based English terms with the help of Pring's dictionary. Therefore, for the sake of simplicity, in citing Greek entries of Pring's dictionary, I omit all diacritics except ´, ¨, and ΅, and also except ⌒ in case when this is the only accent of a word. The interested reader can recover the omitted diacritics of any cited Greek word with the help of Pring's dictionary. I indicate approximate modern native pronunciation of a cited Greek word after the word between back-slash virgules with the help of the phonetic symbols that are listed and described on pp. xiv-xvi of Pring's Preface to the Greek–English dictionary with the following provisos: the English letter "o" is used as a phonetic symbol for either of separately read Greek letters "ο" and "ω"; the English digraph "ch" is used interchangeably with the phonetic symbol "x"; the English digraph "th" is used interchangeably with the phonetic symbol "θ" indistinguishable from the Greek letter "θ"; the Greek letter "γ" is equivocally used as the pertinent phonetic symbol unless the letter occurs before "ι", "ε", [another] "γ", "ξ", "χ", or "κ". Thus, in the exclusion of "γ", "ç", and "ŋ", all remaining phonetic symbols are homonyms of the appropriate English letters or clusters of English letters. Therefore, the phonetic transcription of a cited Greek word is in fact a *transliteratum* (pl. *transliterata*") of the word in English alphabet, which is, when applicable, made with allowance for the above-mentioned peculiarity of Modern Greek vowel-digraphs. At the same time, one should remember that an Anglicized version of a Greek etymon (or, generally, of any foreign etymon) should not necessarily coincide with an English transliteratum of the latter, although the two sometimes coincide either partly or completely. In forming new Anglicized words of Greek etymons, certain tacit rules of replacing Greek letters or clusters of letters by the appropriate English ones (as done in WTNID and in most other explanatory dictionaries of the English language) are more important than both the modern native pronunciations of the etymons and their English transliterata.

In explaining the etymological sense of a new English term, I utilize the pertinent interrelated entries of both parts of Pring's dictionary, the Greek-English one and English-Greek one, if those entries are not conversable in regard to some meanings of their headwords (vocabulary entries). A like remark applies, *mutatis mutandis*, to my use of Simpson's dictionary.

In citing Latin headwords (vocabulary entries) of Simpson's dictionary, I preserve the *diacritics* ˇ and ¯ which are placed over a vowel letter (as in "ă" and "ā") to indicate that the vowel speech sound represented by the letter is short or long respectively. Although the diacritics are irrelevant to the established or new English terms which are formed of the cited Latin etymons, they serve for visually distinguishing between a Latin etymon and the counterpart English word; otherwise the two would have been homographs. Incidentally, in an alphabetic language, the sign of the form ˇ or ˘ that is placed over a vowel letter to indicate that the vowel speech sound represented by the letter is short is called a *breve* (cognate to "brief"), whereas the bar ¯ that is placed over a vowel letter to indicate that the vowel speech sound represented by the letter is long is called a *macron* (from the Greek adjective μακρός \makrós\ meaning «long»).

WTNID defines the terms "affix" and "combining form" and also the terms "infix" , "prefix", and suffix" as subterms of "affix" in this manner:

«^{2}**affix** … *n* –ES … **1 a :** a sound or sequence of sounds or, in writing, a letter or sequence of letters occurring as a bound form linguistic form attached to the beginning or end of a word, base, or phrase or inserted within a word or base and serving to produce a derivative word (as *un-* in *unite*, *-ate* in *chlorate*, *-ish* in *morning-after-ish*) or an inflectional form (as *-s* in *cats*) or the basis of part or all of a paradigm …– compare ^{2}INFIX, ^{2}PREFIX, ^{1}SUFFIX …

combining form *n* **:** a linguistic form that occurs only in compounds or derivatives and can be distinguished descriptively from an affix by its ability to occur as one immediate constituent of a form whose only other immediate constituent is an affix (as *cerpal-* in *cerpalic*) or by its being an allomorph of a morpheme that has another allomorph that may occur alone (as *electro-* representing *electric* in *electromagnetic* or *resini-* representing *resin* in *resiniferous, forma-* representing *formaldehyde* in *formalith*, *para-* representing *parachute* in *paratrooper*) or can be distinguished historically from an affix by the fact that it is borrowed from another language in which it is descriptively a word (as French *mal* giving English *mal-* in *malodorous*) or a combining form used (as Greek *kako-*, compound form of *kakos*, giving English *caco-* in *cacography*)

[2]**infix** … *n* –ES **:** a derivational of inflectional affix appearing in the body of a word or base rather than at its beginning or end (… English *stand* as contrasted with *stood*) – compare PREFIX

[2]**prefix** … *n* –ES … **1 :** a sound or sequence of sounds or, in writing, a letter or sequence of letters occurring as a bound form linguistic form attached to the beginning or end of a word, base, or phrase and serving to produce a derivative word or an inflectional form – compare AFFIX, INFIX, SUFFIX …

[1]**suffix** … *n* –ES … **1 :** a affix occurring at the end of a word, base, or phrase – compare PREFIX …»

In classifying English morphemes and particularly the new Anglicized ones of Greek origin, I shall follow the above definitions, but I disregard the part of Webster's definition of "combining form", according to which this term is declared to be incompatible with any of the four other terms, i.e. according to which a combining form is not an affix and vice versa. This feature is not sanctioned by using the words "prefix" and "suffix", both as nouns and as verbs, in practice and therefore it is impracticable. Consequently, I adopt the following definition.

Df A1.1. A *combining form* as described in Webster's definition of this term is called a prefix, suffix of infix depending on a position that it occupies in a complex word. Consequently, a combining form is an affix, i.e. a prefix, suffix, or infix, but not necessarily vice versa. That is to say, some affixes are combining forms, while the others are not. It is desirable to have a concise term for an *affix not being a combining form*, but unfortunately I am unable to suggest any.•

Dict A1.1 that is given below is an *etymological* English-Greek dictionary of most new and established morphemes that are used in the treatise either in combinations with the noun "nym" by one, two, or more for forming new complex monomials in analogy with "antonym", "homonym", etc., – e.g. "graphonym", "xenonym", "autographonym", cenautographonym", etc., – or in combinations with one another for forming abbreviations of some monomials thus obtained, – e.g. “autograph”, “cenautograph”, etc. The dictionary also includes a few words that will be used in no connection with the root “nym”. I have compiled Dict A1.1 mainly with the help of the Greek–English part of Pring’s dictionary, but in some cases I have also used the English–Greek part. It should be emphasized that the entries of the dictionary explain *the senses of the Greek etymons*, i.e. *the etymological senses*, of the Anglicized morphemes (or morphological constructions), and not the senses that will be attached to these morphemes by subsequent definitions of the technical terms in which the morphemes are be utilized.

Dict A1.1: *English–Greek Etymological Dictionary*

“*a*”- or “*an*”-, prefix, from either the allomorphic privative prefixes “α”- \a\ and “αν”- \an\ having the same sense as “il”-, “im”-, “in”-, “non”-, “un”-, “-less”.

“*ad*”- or “*ado*”-, comb. form, from the verb “άδω” \áðo\ meaning as *to sing*.

“*agno*”-, prefix , from the adj. “αγνός” \aγnós\ meaning *pure* (*clean*, *unmixed*), *chaste* – opposed to “*mict*”- or “*micto*”-.

“*agraph*”- or “*agrapho*”-, comb. form, from the adj. “άγραφος” \áγrafos\ meaning *unwritte*n.

“*aiti*”- or “*aitio*”-, prefix, denoting *cause*, from the noun “αιτία” \aitía\ meaning *cause* and also *reason*, but in fact contrasted with “λόγος” that means *reason* properly.

“*all*”- or “*allo*”-, prefix, from the pronoun and adjective “άλλος” \állos\ meaning *other*, *else*, *rest*; *next*; *different*; *more* (to be used as an antonym of “idio”-).

“*alythio*”-, prefix, from the adj. noun “αληθής” \alithís\ meaning *true* and from the noun “αλήθειας” \alíthias\ meaning *truth*.

“*analo*”-, comb form, or “*analogical*” and also “*analogous*”, adj., from the noun “αναδογία” \analogía\ meaning *a relation, proportion, ratio*.

“*ant*”- or “*anti*”-, prefix, from either of the allomorphic prefixes “άντ”- \ant\ and “άντι”- \ánti\ denoting *opposition, opposite situation,* or *negation*.

“*aphon*”- or “*aphono*”-, comb. form, from the adj. “άφωνος” \áfonos\ meaning *mute*; *without a* (singing) *voice*.

“*apl*”- or “*aplo*”-, prefix, from the adj. “απλός” \aplós\ meaning *simple*; *single*.

“*apt*”- or “*apto*”-, prefix, from the adj. “απτός” \aptós\ meaning as *tangible*.

“*arch*”- or “*arche*”- or “*archi*”-, prefix, from the following etymons: (I) the noun “αρχή” \arxí, archí\ having the same sense as “beginning”, “origin”; “principle”; “authority”; (II) the homonymous comb. form “αρχι”- \arçi, arhi\ having the same senses as “first”, “chief”, “master” ⟨“αρχέτυπον” \arçétipon\ means *an archetype*⟩.

“*atomo*”-, comb. form, from the noun “άτομον” \átomon\ meaning *an atom, person, individual*.

“*aut*”- or “*auto*”-, comb. form, from the comb. form “αυτ”- \aut\ denoting *self* or *same*.

“*bebe*”- or “*bebeo*”-, prefix, from the adj. “βέβαιος” \bébeos\ meaning *sure, certain*.

“*bio*”-, comb. form, from the noun “βίος” \bíos\ meaning *life*.

“*boob*”- or “*boobo*”-, prefix, from the adj. “βουβός” \bubós\ meaning *dumb*.

“*cal*”- or “*calo*”-, prefix, from the trans. verb “καλῶ” \kaló\ meaning *to call, name, term*; *summon*; *invite*.

“*cat*”- or “*cato*”- and also “*kat*”- or “*kato*”-, prefix, from the adv. and prep. “κάτω” \káto\ meaning *down, below, beneath, under*.

“*cen*”- or “*ceno*”- or “*coeno*”- and also “*caen*”- or “*caeno*” “*con*”-- or “*cono*”-, prefix, from the adj. “κοινός” \kinós\ meaning *common* or [*held*] *in common* (cf. the perfective, associative, and collective prefixes of L. origin: “*co*”-, “*col*”-, “*com*”-, “*con*”-, and “*cor*”-).

“*clas*”- or “*claso*”- and also “*class*”- or “*classo*”-, comb. form, from the loaned Gk. noun “κλάσις” \clássis\ meaning *a class* or *an age-group* and from the L. noun “classis” having the same meaning. Simpson [1959] explains the etymology of this noun thus: «**classis -**is, f. (connected with *κλέω* and calo, *to summon*), *a summoning*; hence *a group as summoned, a division, class*». There is likely an error or, perhaps, a misprint in the above parenthesis. Namely, the same dictionary says: «**cǎlo** (**kǎlo**) -are**-** (*καλῶ*), *to call, summon*…», which is in agreement with Pring [1982]. Nevertheless, the above Simpson’s proposition is correct in essence.

“*code*”, noun, from the noun “κῶδιξ” \kóðiks\ having the same sense as “*code*” (or as “*codex*”).

“*dactylo*”- from the noun “δάκτυλος” \ðáktilos\ meaning *a finger*.

“*dem*”- or “*demo*”- from the adj. **“**δημόσ|ιος**”** \ðimós|ios\ meaning *public*; ~“ία” \ía\ or ~“ίως” \íos\ is an adverb meaning *in public*. The name **“**τὸ ~ιον**”** \tò ~ion\ means *the state* or *the public*. The noun “δῆμος” \ðímos\ has the same meaning as “municipality” or “borough” (cf. “democracy”, “demagogue” or “demagog”, “demography”, etc.).

“*di*”- or “*dy*”- or “*dyo*”-, comb form, from the comb. form “δι”- \di\ and cardinal numeral “δύο” \dío\ or “δυό” \dió\, meaning *two* each, or from the L cardinal numeral “dŭŏ” (-“ae”, -“ŏ”) loaned from Gk. and meaning *two* as well.

“*dicto*”- from the noun “δείκτης” \ðíktis\ meaning *a forefinger*; *indicator*; *pointer, index, hand* – opposite to both “*icono*”- and “*ideo*”-.

“*dys*”- from the prefix “δυς”- \ðis\ meaning *difficult* or *bad*.

“*ec*”-, prefix, from the second of the synonymous comb forms. “εξ”- \eks\, “εκ”- \ek\, and “ξε”- \kse-\. meaning *out*, *off*.

“*echo*”-, comb. form., from the noun “ήχος” \íxos, íchos\ having the same meaning as “sound”.

“*end*”- or “*endo*”-, comb. form, from the adv. and prep. “ένδον” \énðon, énthon\ meaning *within*.

“*enneo*” or “*ennio*”- from the noun “έννοια” \énnia\ meaning *an idea*, *concept*; *meaning*, *sense* (not to be confused with the established combining form “*ennia*”- from the numeral “εννιά” \enniá\ meaning *nine*).

“*entity*”, noun, from the noun “οντότης” \ontótis\ meaning *an entity*, *being*; *individuality*.

“*epi*”-, prefix, from the prep. “έπι” \épi\ meaning (with acc.) *towards*; (with gen.) *on*.

“*epo*”-, prefix, from the intransitive verb “έπομαι” \épome\ meaning *follow*.

“*eu*”- from the Gk. adv. and comb. form “ευ”, \ev\ before voiced sounds or \ef\ otherwise, having the same meaning as “well” each, and from the L. interjection “eu!” associated with the above Gk. and having, according to Simpson [1959], the same meaning as “good!” or “well done!”.

“*exo*”-, comb. form, from the first of the synonymous preps. “εξ” \eks\ and “εκ” \ek\ meaning (origin) *from* and from and from the adv. & prep. “έξω” \éxo\ meaning *out, outside*; *abroad*.

“*geg*”- or “*gego*”- from the noun “γεγόνος” \jeγónos\ meaning *an event, fact*.

“*gen*”- or “*geno*”- from the noun “γένος” \jénos\ meaning *a race, tribe*; *genus*; *gender*.

“*gloss*”- or “*glosso*”- from the noun “γλῶσσα” \γlóssa\ meaning «*tongue*» or «*language*» and from its adjective derivative “γλοσσικός” \glossikós\ meaning «*lingual*» or «*linguistic*».

“*grammo*”- or -“*gram*” or -“*gramme*” from the noun “γράμμα” \γrámma\ having the same meaning as “letter” both in the sense of “primitive symbol” and in the sense of “message”; the latter meaning is disregarded.

“*grapho*”- or -“*graph*” from the noun “γραφή” \γrafí\ having the same sense as “*writing*”, and also from its kindred verb “γράφω” \γráfo\ having the same meaning as “*to write*”, and from its adjective derivative “γραφικός” \γrafikós\ having the same meaning as “*written*” or “*drawn*”.

“*gust*”- or “*gusto*”- from the noun “γεῦσισ” \γéfsis\ meaning «*tatse*», «*flavour*», or «*tasting*».

“*gymn*”- or “*gymno*”- from the adjective “γύμνος” \jímnos\ meaning *naked, nude, bare*.

“*hom*”- or “*homo*”- from the combining form “όμο”- \ómo-\ having the same meaning as “*same*” or “*together*”, or όμοιο”- \ómio-\ having the same meaning as “*similar*” or “*same*”, or from the adjective όμοιος” \ómios\ having the same meaning as “*similar*”, “*alike*”, “*like*”, or “*same*”.

“*homolo*”- , comb. form, or “*homological*” and also “*homologous*”, adj., from the noun “ομοδογία” \omología\ meaning *confession, admission*.

“*hyper*”- from the preposition “υπέρ” \ipér\, assuming the same meaning as (*with acc.*) “*over*” or “*above* ” or as (*with gen.*) “for” or “on behalf of”, or from the homographic combining form “υπέρ”-, assuming the same meaning as “*over*” or “*very much*” or as “on behalf of”; the Latin preposition and adverb “*sŭper*” from the same Greek etymons.

“*hypo*”- from the preposition “υπό” \ipó\ that assumes the same meanings as (*with acc.*) “*below*” or “*beneath*” or as (*with gen.*) “by” (*agent*) or from the combining form “υπ(ο)”- \ip(o)-\ assuming the same meanings as “*under*”, “*secretly*”, or “*slightly*”; the Latin preposition and adverb “*sŭb*” from the same Greek etymons.

“*icon*”- or “*icono*”- and also “*eicon*”- or “*eicono*”- or “*ikon*”- or “*ikono*”- , comb, form, from the synonymous nouns “εικών” \ikón\ or “εικόνα” \ikóna\ having the same meaning as “*picture*” or “*image*” – opposite to both “*dicto*”- and “*ideo*”-.

“*ideo*”-, comb, form, from the noun “ιδέα” \idéa\ having the same sense as “*idea*” or “*opinion*” – opposite to both “*dicto*”- and “*icono*”- (“*picto*”-).

“*idio*”- prefix, either from the adj. **“**ίδι|ος**”** \íðios\ having the same meaning as “[*of*] *one's own*”, “[*one*-]*self*” (as “myself”, “himself”, etc.), “*same* [*as*]”, or from the adv. **“**~α**”** having the same meaning as “*same as*” or “*like*”.

“*id*”- or “*ido*”-, prefix, from the noun “είδος” \íðos\ meaning *a sort*, *kind*, *species*.

“*is*”- or “*iso*”- prefix, from the adjective “ίσος” \ísos \ having the same meaning as “*equal (to)*” or “*the same (as)*”.

“*kin*”- or “*kine*”- or “*kino*”- or “*cin*”- or “*cino*”- and also “*kuno*”-, comb form, from the following etymons: the noun “κίνεμα” \kínema\ meaning *movement* (action); the noun “κίνεσις” \kínezis\ meaning *movement*, *motion* (action or abstract); trans. and intrans. verb “κουνῶ” \kunó\ meaning *to move*, *rock*, *shake*, or *swing*. Also, according to the English-Greek part of Pring's dictionary, the English trans. verb “to wag” is translated into Greek by a token of the above Greek verb ~ in place of ⌒ (cf. the pertinent remark regarding these diacritics in the discussion preceding this dictionary).

“*kuph*”- or “*kupho*”- from the adjective “κουφός” \kufós\ meaning *deaf* (not to be confused with the adj. “κοῦφος” \kúfos\ meaning *lightweight*, *not serious*; *empty* (hope, etc.).

“*kyr*”- or “*kyro*”- from the noun “κῦρος” \kíros\ meaning *validity* and also *weight* or *authority* (not to be confused with “κύριος”, – see below).

"*kyri*"- or "*kyrio*"- from the adjective "κύρι|ος" \kíri|os\ meaning *main*, *principal*, *chief* 〈**"**~ον όνομα**"** means *a proper name*〉 and from the homonymous noun meaning *a lord*, *master*, and also *a gentleman*, *mister*.

"*lexi*"- from the singular noun "λε̑ξις" \léksis\ having the same meaning as "word" and used for mentioning a *word singly*, as contrasted to the plural noun "λόγια" \lójia\ (sing. "λόγος" \lóɣos\) having the same meaning as "words" and used for mentioning *words in connected speech*.

"*log*"- or "*logo*"- from the following etymons: (I) the noun "λόγος" \lóɣos\ assuming the same meaning as: 1. "*speech*" (faculty); "*speech*", "*address*"; "*talk*", "*mention*", "*question*" (*of*); "*saying"*; "*word*" (in connected speech: pl. "τά λόγια" \tá lójia\ means «*the words*», – cf. "λε̑ξις" in the previous entry); 2. "*reason*", "*ground*"; "*account*", "*reckoning*"; (II) the noun "λογική" \lojikí, loyikí\ having the same meaning as "*logic*" or "*way of thinking*"; (III) the noun "λογικ|ό" \lojikó\, "~ά" \~á\ (*pl.*) having the same meaning as "*senses*", "*reason*" (sanity); (IV) the adjective "λογικός" \lojikos\ having the same meaning as "*rational*"; "*logical*", "*reasonable*".

"*mer*"- or "*mero*"- from the noun "μέρος" \méros\ meaning *a part.*

maz"- or "*mazo*"- from the noun "μάζα" \máza\ meaning *mass*.

"*melo*"- from the noun "μέλος" \mélos\ meaning *a member* (and also *a limb* or *a melody*).

"*meta*"-, comb. form, from the adv. "μετά" \metá\ meaning «*afterwards*», or from the homographic preposition meaning (with an accusative noun) «*after*».

"*mict*"- or "*micto*"-, prefix, from the adj. μικτός \mictós\ meaning *mixed* – opposed to "*agn*"- or "*agno*"-.

"*mnemon*", noun, from the nouns "μνήμη" \mními\, meaning *memory* or *remembrance,* and "όν" \ón\, explained below in this dictionary (see -"on"), and also from the adjective "μνήων" \mnímon\ meaning *mindful*, i.e. *bearing or keeping in mind*.

"*mono*"-, comb. form, from: the noun "μονάς" \monás\ the same meaning a *unit*, *monad*; the comb. form "μονο"- \mono\ denoting *single*; the adjective "μονός" \monós\ meaning *simple* (*not compound*) or *single* (*not double*).

"*morio*"- from the noun "μόριον" \mórion\ meaning *a molecule*.

"*morph*"- or "*morpho*"- or -"*morph*", comb. form, from the noun "μορφή" \morfí\ meaning *a form, shape*; *appearance*; *aspect*; *phase*.

"*myl*"- or "*mylo*"- or "*omyl*"- or "*omylo*"-, comb. form, from the synonymous verbs "μιλῶ" \miló\ and "ομιλῶ" \omiló\ having the same meaning as "to speak" each, and also from the Hebrew noun "מילה" \mila\ having the same meaning "word".

"*neo*"-, comb. form, from the adj. "νέος" \néos\ meaning «*new*», «*modern*»; «*young*».

"*nym*" or "*onym*" from the noun "όνομα" \ónoma\ having the same senses as the nouns "*name*" and (gram.) "*noun*".

"*ogc*"- or "*ogco*"-, prefrix, from the noun "όγκος" \ógkos\ meaning *mass, bulk, volume.*

"*omylitic*"- or "*omylitico*"-, comb. form, from the adj. ομιλητικός" \omilitiós\ meaning *communicative*.

-"*on*" or -"*n*", suffix, from the noun "όν" \ón\ meaning *a being* or *creature*.

"*opt*"- or "*opto*"-, prefix, from the adjective "οπτικός" \optikós\ having the same meaning as "visual" and also as "optical" or "optic"; or from the noun "οπτική" \optikí\ meaning *optics*.

"*organon*" from the Greek noun "όργανον" \órganon\) having the same meaning as "organ", "instrument", or "agent"; the Latin noun "orgănum" meaning «instrument» or «instrument», esp. «musical instrument», originates from the same Greek etymon.

"*or*"- or "*oro*"- from the noun "όρος" \óros\ meaning *a term* (word, expression).

"*orism*"- or "*orismo*"- from the noun "ορισμός" \orismós\ meaning *definition*.

"*orismen*"- or "*orismeno*"- from the adjective "ωρισμένος" \orisménos\ meaning *determinate, determined, fixed*, or *certain* (unspecified).

"*pale*"- or "*paleo*"-, comb. form, from the adj. "παλαιός" \paleós\ meaning *old* (not new) or *former*, and from the comb. form "παλιο"- \palio\ meaning *old* or being used as a *pejorative epithet.*

[1]"*pan*"- or "*pano*"- from the adv. "πάνω" \páno\ being a synonym of "απάνω" \apáno\ and having the same sense as "*up*", "*above*", "*over*", or "*on top*".

[2]"*pan*"- from the comb. form "παν"- \pan\ or "παμ"- \pam\ having the same sense as "*all*" and from the noun "πᾶν" \pán\ being a synonym of "πᾶς" \pás\ and having the same sense as "*everything*" or "*the whole world*".

"*par*"- or "*para*"- from the combining form "παρα"- \para\ having the same meaning as "*near*", "*beyond*", "*contrary*", "*excess*", etc.

"*pas*"- or "*pasi*"- from the adj. "πᾶν" \pás\ having the same sense as "*all*" or "*every*".

"*per*"- or "*peri*"-, prefix, from (I) the preposition "περί" \perí\ having the same meaning as: 1) (with gen.) "about" or "concerning", or 2) (with acc.) "*round*", "*near*"; "*round about*", "*approximately*"; (*concerned*) "with"; (II) the combining form "περι"- having the same meaning as 1) "*around*" or 2) "*very*".[7]

"*perigraph*"- or "*perigrapo*"-, comb form, from the noun "περιγραφή" \perigrafí\ meaning *description* and the kindred verb "περιγράφω" \perigráfo\ meaning *to describe*.

"*phon*"- or "*phono*"- or "*-phon*" from the noun "φωνή" \foní\ assuming the same senses as the nouns "*voice*", "*cry*", and "*shout*".

"*phot*"- or "*photo*"- from the noun "φῶς" \fós\ meaning *light*" or figuratively (faculty of) *sight*.

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[7]The school that Aristotle established in the fifty-third year of his age was the walk along the athletic field, on which he strolled up and down together with his scholars when teaching them. The athletic field was a part of the grounds of the temple of Apollo Luceus – the protector of flocks against wolves (from "λύκος" \lukos\ meaning «wolf»). The walk was called "Peripatos" (from "περίπατος" meaning «walk», «ride», «drive», «trip»). Aristotle's school took the Latinized name "the Luceum" from the name "Apollo Luceus", and the name "Peripatetic School" from "Peripatos". Accordingly, the scholars and later followers of Aristotle are called "Peripatetics".

“*physical*”, adj., or “*physico*”-, comb. form, from either of the nouns “φυσικά” \fisiká\ (neut. pl.) and “φυσική” \fisikí\ (fem. sing.) meaning *physics* and from the adjective “φυσικός” \fisikós\ meaning *natural*, *physical***.**

“*piez*”- or “*piezo*”-, comb. form, from the trans. verb “πίεζω” \píezo\ meaning *to press, squeeze*, *compress; constrain* and from the noun “πίεσις” \píesis\ meaning *pressure*

“*phys*”- or “*physo*”-, comb. form, from the noun “φύσις” \físis\ meaning *nature*; “φύσει” \físi\ means *by nature***.**

“*pleo*”- or “*pleio*”- or “*plio*”- from the adverbs: “πλάϊ” \plai\ having the same meaning as “at the side” (“πλάϊ-πλάϊ” has the same meaning as “side by side” or “in juxtaposition”) and “πλέον” \pléon\ having the same meaning as “more” (cognate of “plus” also originating from the above etymon).

“*poly*”-, comb. form, from the comb. form “πολυ”- \poli\ denoting *much*, *very* and from the adjectives. “πολλοί” \polli\ having the same meaning as “*many*” and “πολύς” \polís\ having the same meaning as “*much*”, “*many*”, “*great*”.

“*pragma*”- from the noun “πρᾶγμα” \práγma\ meaning *a thing* or *matter.*

“*pro*”-, prefix, means «*earlier than*», «*prior to*», or «*before*»; from the Greek prep. and comb. form “προ” \pro\ or from the Latin adv. and prep. “pro”, having the same meaning as “*before*” each.

“*prota*”- or “*proto*”- or “*prot*”-, comb. form, means «*earliest in time or lowest in organization*, *status*, *or in a series*»; from the following Greek etymons: “πρῶτα” \próta\, adv., having the same meaning as “(*at*) *first*” or “*before*”; “πρωτο”- \proto\, comb. form, denoting *first*; “πρώτος \prótos\, adj., meaning «*first*», «*foremost*»; “προτοῦ” \protú\, conj. and adv., having the same meaning as “*before*”.

“*protas*”- or “*protaso*”-, comb. form, from the following Greek noun: “πρότασις” \prótasis\, meaning *a proposal*, *proposition*, *motion*; (gram.) *a sentence, clause.*.

“*psychical*” or “*psychic*”, adj., and “*psycho*”- or “*psychico*”-, comb. form, from the noun “ψυχή” \psixí, psichí\ meaning *soul*; *heart*; *energy*, *spirit*, *courage* and from the adj. derivative “ψυχικός” \psiçikós, psihíkós\ meaning *psychical*; *psychic*.

"*semasi*"- or "*semasio*"-, prefix, from the noun "σεμασία" \semasía\ having the same sense as "*meaning*" and from the kindred verb "σεμασίνω" \semasíno\ meaning *to import*, *to mean*, or *to signify*.

"*shes*"- or "*sheso*"-, prefix, from the noun "σχέσις" \sçésis, shésis\ meaning *relation*, *connection*.

"*syllab*"- or "*syllabo*"-, prefix, from the noun "συλλαβή" \sillabí\ meaning *a syllable*.

"*syn*"- or "*syno*"- (cf. Cmt 4.1) or "*sym*"- from the combining form "συν"- \sin\ meaning *together* or *with* (the homonymous preposition "σύν" has the same meaning as "*with*" or, in mathematics, as "*plus*").

"*syndet*"- or "*syndeto*"- from the adjective "συνδετκός" \sindetikós\ meaning *connecting*.

"*taut*"- or "*tauto*"-, prefix, from the transitive verb "ταυτίζω" \taftízo\ meaning «to treat as identical» or «to identify»; the pl. neuter pronoun "ταῦτα" \táfta\ meaning «these»; the noun "ταυτότης" \taftótis\ means «identity (card)».

"*tax*"- or "*taxo*"-, comb. form, from the noun "τάξις" \táxis\ having the same meaning as "*order*" (the quality or state of being ordered or tidy) or as "*class*" or "*grade*".

"*techn*"- or "*techno*"-, comb. form, from the noun "τεχνητός" meaning *artificial*.

"*therm*"- or "*thermo*"-, comb. form, from the adj. "θερμός" \θermós\ meaning *hot*, *warm* and from the noun "θερμότης" \θermótis\ meaning *heat*, *warmth*.

"*tri*"-, comb. form, from the adj. "τρεῖς" \trís\ and the cardinal numeral "τρία" \tría\, meaning *three* each, and also from the L. adj. and cardinal numeral "trēs", "trĭa" loaned from Gk. and meaning *three* as well.

"*tych*"- from the following etymons: the noun "τύχη" \tíxi, tíhi\ meaning «*chance*», «*fortune*», «*fate*», or «*good luck*»; the adjective **"**τυχαῖ|ος**"** \tixéos, tihéos\ meaning «*fortunate*» or «*lucky*»; the homophonic adverb **"**~ως**"** meaning «*by chance*». Also, according to the English-Greek part of Pring's dictionary, the English vocabulary entry "*accidental*" is translated into Greek by a token of the above Greek adjective with ˜ in place of ⁀.

“*udeter*”- or “*udetero*”- from the adj. “ουδέτερος” \uðéteros, uthéteros\ meaning *neutral* or (gram.) *neuter* and from the homonymous pronoun meaning *neither*.

“*xeno*”- from the adj. “ξενικός” \ksenikós\ having the same meaning as “*foreign*” (to be used as an antonym of “auto”-).•

Cmt A1.1. 1) In the exclusion of “ido”-, “kuno”-, and “prota”-, all Anglicized prefixes and combining forms given in the above dictionary occur in the English lexicon, although some of them are used in association with the different etymological senses. For instance, the combining form “melo”- as described in Dict A1.1 is a homonym of the well-established English combining form (see WTNID), whereas the combining form “mylo”- is a homonym of the well-established English combining form meaning *molar*. Instead of “mylo”-, I might have employed the combining form “lalla”- or “lallo”-, which originates from the Greek singular noun “λαλιά” \laliá\, meaning «speech» or «talk», and from the kindred verb “λαλῶ” \lalo\, meaning «to speak» or «to talk». However, apart from being longer than “mylo”-, “lalla”- is used in the English noun “lallation” (see WTNID), and it has therefore undesired association.

2) The new English combining form “prota” is the transliteratum of the Greek adverb “πρῶτα” which has the same meaning as the conjunction and adverb “προτοῦ”. At the same time, “prota” is consonant with both “meta” and “para”. It is therefore convenient to use the morpheme “prota” interchangeably with or instead of “proto” as a *complimentary antonym of* “meta”.

3) It is also noteworthy that in all cases when the combining form “logo”- will be used alone or together with some other combining forms for qualifying the morpheme “nym” or some other morphemes, it will, as a rule, descriptive of both groups of meanings of the etymon “λόγος”, defined in items 1 and 2 of the vocabulary entry “logo”- (cf. “logograph”).•

Dict A1.2 that is given below is an *etymological* English-Latin dictionary of some Anglicized morphemes and words of Latin origin having counterparts among those of Greek origin. This dictionary has been compiled mainly with the help of the Latin–English part of Simpson’s dictionary in analogy with Dict A1, but it is irrelevant to nymology.

Dict A1.2. English-Latin Etymological Dictionary

"*bi*"-, comb. form, from the L. adv. "bĭs" meaning *twice*, *in two ways*.

"*certum*", adj., pl. "*certa*", from the L. neuter adj. "certum", pl. "certa" (masc.: sing. " certus", pl. "certi"; fem.: sing. "certa", pl. "certae") meaning *settled*, *resolved*, *decided*, *definite*, *certain*, *fixed*, *sure*, *undoubted*.

"*cis*"-, comb. form, from the L. prep. with acc. "*cis*" meaning *on this side of*.

"*denote*", trans. verb, and "*denotatum*", noun, from the L. Prsent Inf. verb "dēnŏtare" (Present Tense: "dēnŏto) meaning (1) *to mark out for another*, *designate precisely*; (2) *to take note of* (for ones own purposes).

"*extra*"-, comb. form, from the L. synonymous masc. adjs. "exter" and "extĕrus". (fem. -"a", neuter -"um") meaning *outward*, *foreign*, *strange*; compar.: "extĕrĭor", fem. and neut. -"ĭus", meaning *outer*; superl.: "extrēmus" or "extĭmus", fem. -"a", neuter -"um", meaning *outermost*; and from the cognate adjective "externus" (-"a", -"um") meaning *that is outside*, *external*.

"*graphic*", adj. from the L. masc. adj. "găphĭcus" (fem. -"a", and neut. -"um" in place of -"us") meaning *concerned with painting*.

"*incertum*", adj., pl. "*incerta*", from the L. neuter adj. "incertum", pl. "incerta" (masc.: sing. " incertus", pl. "incerti"; fem.: sing. "incerta", pl. "incertae") meaning *uncertain*, *doubtful*, *not sure*, *not known*, *obscure*.

"*intra*"-, comb. form, from the L. adv. and prep. "intrā" meaning *within*, *inside*.

-"*ject*" and -"*jection*", comb. form, from the L. verb "iăcĭo" (first person singular present indefinite), "iăcĕre" (present infinitive), "iēci" (first person singular present perfect), "iactum" (singular nominative masculine past participle) meaning [*I*] *lay*, *to lay*, [*I*] *have laid*, [*it is*] *laid*.

"*mental*", adj., from the L. noun "mens" (genitive "mentis") meaning *the mind*, *understanding*, *reason*, *intellect*, *judgment*.

"*multi*"-, comb. form, from the L. adj. "multus", -"a", -"um"; compar. "plūs", "plūris"; superl. "plūrĭmus", -"a", -"um"; meaning *much*, *many*.

"*noci*"-, comb. form, from the L. verb "nŏcĕo", -"ĕre", meaning *to hurt*, *injure*, or *harm*.

"*nomen*", noun, pl. "*nomina*", and -"*nomial*", comb. form, from the L. noun "nōmen", pl. "nōmĭna", meaning *a name*.

“*nudum*”, adj., pl. “*nuda*”, from the L. neuter adj. “nūdum”, pl. “nūda” (masc.: sing. “nūdus”, pl. “nūdi”; fem.: sing. “nūda”, pl. “nūdae”) meaning *naked, unclothed, bare*.

“*picture*”, noun, and “*picto*”-, comb. form, from the L. noun “pictūra”, pl. -“ae”, meaning *painting, the art of painting, a painting* or *picture, embroidery,* (of) *mosaic work*, and also *a word-picture* or *word-description*, so that its meaning differs from the meaning of its Greek etymon “γραφικός”.

“*trans*”-, comb. form, from the L. prep. with acc. “*trans*” meaning *over, across, on* or *to the other side of*.

“*vetum*”, adj., pl. “*veta*”, from the L. neuter adj. “vĕtum”, pl. “vĕta” (masc. sing. “vĕtus”, pl. “vĕti”; fem.: sing. “vĕta”, pl. “vĕtae”), meaning *old, ancient*.•

Cmt A1.2. 1) Diacritics (as ˇ, ¯, ˜, etc.) should be distinguished from the similar marks that are placed as distinguishing labels over letters or some other case characters in forming unit aphonic euautographs or logographs. In regard to such labels, I shall use the following terminology.

Df A1.2. When the marks ˇ, ˘, and ¯ are used as constituent parts of *aphonic* euautograpic or logographic characters, they will be called an *angle caron*, a *round caron*, and an *overbar* respectively. In a like use, the marks ˆ, ⌢, and ˜ are called an *angle cap*, a *round cap*, and an *overtilde* respectively; the word “*cap*” can be used interchangeably (synonymously) with “*overcaret*. Also, for the sake of brevity, I use the following verbs: “*to overbar*” (“*overbarred*’) meaning «to provide with an overbar» or «to provide with a bar over», “*to overtilde*” (“*overtilded*’) meaning «to provide with an overtilde» or «to provide with a tilde over», and “*to overcaret*” (“*overcareted*” or “*overcaretted*”) meaning «to provide with an overcarret» or «to provide with a caret over».•

A2. Trichotomies of classes and the hierarchy of privative prefixes

Df A2.1. The primary (original) division of all vavn-decided relations of A_1 or $\mathbf{A}_1$ into three classes: *valid, antivalid,* and *vav-neutral* (*vav-indeterminate*) is called the *basic decisional trichotomy* (*trisection, trifurcation*) *of the vavn-decided* *euautographic or panlogographic relations* respectively. The three secondary (defined) divisions of all vavn-decided euautographic into two complementary classes each, namely: (a) *valid* and *invalid*, (b) *antivalid* and *non-antivalid*, (c) *vav-neutral* (*vav-inteterminate*) and *vav-unneutral* (*vav-determinate*), are called the *subsidiary dichotomies* (*bisections*, *bifurcations*) *of the vavn-decided euautographic or panlogographic relations*. •

Cmt A2.1. In forming the above taxonomies and many other similar ones to be established in the treatise, I tacitly adopt a certain hierarchy of English privative prefixes, which is made explicit below in this section.•

Df A2.2. 1) Let a given class that is *ad hoc* called a *superclass* or *hypertaxon* or *genus* be provided with a proper name that is *ad hoc*, called a *hypertaxonym* or *generic taxonym*. Let the superclass be either divided into or, on the contrary, be composed by uniting *two or more disjoint complementary subclasses*, called also *hypotaxa* (*hypotaxons*), or *specific classes*, or *species*, *of the hypertaxon* (*genus*), – the subclasses, which are identified by their *taxonyms* (*taxonomic names)* that are ad hoc called *hypotaxonyms* or *specific taxonyms*. If the superclass consists of *two* disjoint subclasses then their hypotaxonyms are said to be the *mutually complementary*, or *antithetic* (*antithetical*), or *antipodal* (*antipodean*), *hypotaxonyms*, and also *complementary* (*antithetic*, *antipodal*), or *absolute*, *antonyms*, *with respect to the hypertaxonym*. If the superclass consists of *three* or *more* disjoint subclasses then their hypotaxonyms are said to be *non-complementary* (*non-antithetic*, *non-antipodal*), or *relative*, *antonyms*.

2) Let each *specific* taxonym of a given *furcated* taxonomy be a *descriptio per genus et differentiam*, i.e. a *description* (*descriptive name*) *that defines* (*describes*) *the species* (specific class, subclass), which it denotes, *through the genus* (generic class, superclass), denoted by the pertinent *generic taxonym* (*head word*), *and through the differentia* (*difference*), denoted by the respective *epithet* (*qualifier*), single or compound (conjunctive). Then the terminology introduced in the items 1 and 2 applies also to the epithets occurring in the specific hypotaxonyms, e.g., “valid” and “antivalid” (or “kyro”- and “antikyro”-), “valid” and “invalid” (or “kyro”- and “anantikyro”-), “neutral” and “unneutral” (or “indeterminate” and “determinate”, or “udetero”- and “anudetero”-, or “anorsmeno”- and “orismeno”), etc. For instance, the

substantives “valid relation” and “antivalid relation” are antithetic antonyms with respect to the substantive“unneutral relation”, while the adjectives “valid” and “antivalid” are antithetic antonyms with respect to the adjective “unneutral” provided that all the three adjectives qualify the noun “relation”; the substantives “valid relation” and “inivalid relation” are antithetic antonyms with respect to the substantive “relation”, while the adjectives “valid” and “invalid” are antithetic antonyms with respect to *the zero qualifier*, i.e. with respect to the absence of any qualifier, to the noun “relation”; etc.•

Df A2.3. There are in English several privative prefixes: “a”- or “an”-, “ant”- or “anti”-, “il”-, “im”-, “in”-, “non”-, and “un”-. The description through a genus and the differentia is said to be *positive*, or *affirmative*, if the epithet denoting the differentia does not contain any privative prefix and *negative* if otherwise. If two negative descriptions differ from each other only in the privative prefixes occurring in their epithets then the prefix occurring in the description denoting the *narrower* species is said to be *stronger* than the prefix occurring in the other description denoting the *broader* species, and accordingly the latter prefix is said to be *weaker* than the former. Two different privative prefixes are said to be *incomparable* if they are attached to two different qualifiers.•

Cmt A2.2. The phenomenon of hierarchy of privative prefixes is well-known in English and in other native languages. For instance, Allen [2003] defines one group of meanings of the prefix “un-” thus:

«**un-**[1] *prefix* forming adjectives, nouns, and adverbs with the meanings: **1** not; lack of something: *unskilled*; *unbelief*. **2** opposite of or contrary to something: *ungrateful*; *unrest*. [Old English]

Usage note

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un- *and* **non-** Both these prefixes are used to produce negative forms of words. In cases when they both can be attached to the same root, the resulting *un-* word is generally stronger than the *non-* word. A *non-*professional tutor is one who is not qualified; *unprofessional behaviour* contravenes professional ethics. If somebody’s methods are described as

> *unscientific*, a criticism is usually implied (the methods do not come up to the standards required by science); if they are described as *non-scientific*, the effect is usually more neutral (the methods come from some other field than science).»

In accordance with Df A2.2, ***Usage note*** means that, given a specific positive taxonym consisting of a generic taxonym (head word) and a positive epithet to it, if there are in use two negative antonyms of the specific taxonym, one with the prefix "un"- and the other with the prefix "non"-, then the prefix "un"- is stronger than the prefix "non"-. Still, the prefix "anti"-, which I have employed in Dfs 1.1(24), 1.2, and A2.1, has the same property with respect to the prefix "non"- as "un"-. For instance, the well-established word "antiscientific" can be used interchangeably with "unscientific", whereas the new self-explanatory word "antiprofessional" can be used interchangeably with "unprofessional". Also, the prefix "anti"- thus used is by definition stronger than "in"-, because the class of invalid relations is broader than the class of antivalid relations. At the same time, in forming a negative *neonym* (*new name*), any privative prefix can be used, provided that its use is congruous with the English grammar. In this case, "non"- (especially when hyphenated) is the most universal prefix associated with the adverb "not".•

Rule A2.1: ***Principles of a primary trichotomy and of the three associated secondary dichotomies of the class.*** Let a given genus (general class, superclass), denoted by the appropriate generic taxonym (hypertaxonym, head name) be divided into three species (specific classes, subclasses) which are denoted by the appropriate specific taxonyms in the form of descriptions through a genus and the pertinent differentiae (differences) denoted by the appropriate epithets (qualifiers) to the generic taxonym. This trichotomy can be described as two successive dichotomies in three different ways. At the first step, the entire initial class is divided into one of its *trichotomal subclasses*, – which will *ad hoc* be referred as *the distinguished trichotomal subclass*, – and the union of two other subclasses, which is provided with an appropriate taxonym. At the second step, the union is divided into the two initial trichotomal subclasses of which it has been formed. A distinguished subclass can be selected out of the three given subclasses in three ways. In order to express the complementary character of the union of two trichotomal subclasses with respect to the distinguished one, it is natural to form the taxonym of the union in one of the following ways.

1) If the epithet occurring in the taxonym of the distinguished trichotomal subclass has no privative prefix then the taxonym of the union of two other trichotomal subclasses can be formed by attaching the epithet with a certain privative prefix provided that the word thus obtained is not used as an epithet in any specific taxonym of the trichotomy. For instance, "invalid" is a complementary antonym of "valid" in the class of relations of A_1 or $\mathbf{A}_1$, whereas "unneutral" is a complementary antonym of "neutral" in the same class. In this case, "antinvalid" is an antipodal antonym of "valid" in the class of determinate relations of A_1 or $\mathbf{A}_1$. Since the class of determinate (unneutral) relations includes (is a superclass of) antivalid relations, the prefix "anti"- is stronger than the prefix "in"-.

2) If the qualifier occurring in the taxonym of the distinguished trichotomal subclass has a privative prefix then the taxonym of the union of two other trichotomal subclasses can be formed by omitting the prefix provided that the remaining word is not used as an epithet in any specific taxonym of the trichotomy. For instance, "determinate" is a complementary antonym of "indeterminate" in the class of relations of A_1 or $\mathbf{A}_1$, the understanding being that the former is a synonym of "unneutral", while the latter is a synonym of "neutral".

3) If the qualifier occurring in the taxonym of the distinguished trichotomal subclass has a privative prefix that cannot be omitted because its omission would result in the epithet that is already used in one of the specific taxonyms of the trichotomy, then the complementary antonymous epithet of the negative epithet can be formed by adhering the latter with another, different privative prefix. For instance, it is impossible for the obvious reason to employ the name "valid relation" as a taxonym of the union of the class of valid relations and the class of neutral relations. Therefore, I have employed the epithet "non-antivalid" as a complementary antonym of "antivalid" and the substantive "anantikyrology" as a complementary antonym of "antikyrology".•

Cmt A2.3. Usually, juxtapositions of two privative prefixes are not utilized in the English lexicon. Therefore, the qualifier "non-antivalid" is a grammatical and lexical incongruity (barbarism), but it is justifiable from the standpoint of logical analysis. I shall not have many occasions to use this and similar qualifiers in the sequel. But such barbaric linguistic forms seem to be indispensable in stating the complete set of derivative secondary dichotomies of a class that has been trisected. In order to legitimize use of doubly negated qualifiers in case if I need some, I shall adopt the following *grammatical rule of this ML*.•

Rule A2.2. A privative prefix, hyphenated or not, can be applied to a word which has already a privative prefix, either a different one or the same one. Still, in accordance with the law of double negation, the juxtaposition of two tokens of the same privative prefix can be omitted from a word containing it without altering the sense of the word.•

Cmt A2.4. By Rule A2.2, it follows, e.g., that both "anti-antivalid" and "in-invalid" are synonyms of "valid", "un-unneutral" is a synonym of "neutral", "in-indeterminate" is a synonym of "determinate", and "non-non-antivalid" is a synonym of "antivalid". By contrast, "non-antivalid", e.g., is not a synonym of "valid". Still, it is noteworthy that two descriptions that differ from each other only in privative prefixes of their negative epithets can, depending on the generic name that they have in common, denote the same specific class. For instance, it will be shown in due course that there are in A_0 no pseudo-constant relations that could be qualified as neutral (indeterminate); i.e. a neutral relation of A_0 is necessarily a pseudo-variable one. Therefore, the taxonym (count name) "invalid pseudo-constant relation of A_0" denotes the same class as "antivalid pseudo-constant relation of A_0", whereas the taxonym "non-antivalid pseudo-constant relation of A_0" denotes the same class as "valid pseudo-constant relation of A_0".•

Cmt A2.3. 1) In connection with Rule A2.2 and Cmts A2.2 and A2.3, the following example demonstrates that there is no categorical ban on use double privative prefixes in NL's (native languages). According to various Greek-English (GE) and English-Greek (EG) dictionaries, including the GE and EG one by Pring [1982] (briefly, GEP and EGP), the Greek words "αναλυτός" and "ανάλυσις" \análusis\ and of their English parasynonyms "analytics" and "analysis" can be regarded as derivatives from, i.e. to be *analysed* into, the following Greek *basic* etymons:

"α"- \a\ or "αν"- \an\ (*privative prefix*) un-, in-, -less.

“λύω” \lúo\ *v.* solve.

“λύσις” \lúsis\ *s.f.* termination, dissolution (of *partnership, etc*); solving, settling; solution, answer; dénouement, ending.

“λυτός” \lutós\ *a.* solvable.

Consequently, the following morphological constructions are derivational:

“ανα”- \ana\ (*double privative prefix* –Ya. I.) *idenotes* up, back, again, intensification.

“άλυσις” \álusis\ *s.f.* chain, sequence, succession.

“άλυτος,” \álutos\ *a.* not united; unsolved; unsolvable; (*fig.*) indissoluable

“ανάλυσις” \análusis\ *s.f.* analysis.

“αναλυτικός” \analutikós\ *a.* analytic(al); detailed.

The peculiarity of the prefix “ανα”- is that it is the sequence of the privative prefix “α”- and its allomorph “αν”, so that it is an *intensifying positive* one. Therefore, just as “λυτός”, the adjective “αναλυτός” means *solvable* but apparently in the more specified sense of the expression “*solvable in detail*”. The Greek noun, or absolute adjective, “αναλυτός” and hence the derived English noun “analytics” should be understood respectively.

2) Aristotle is commonly and undisputedly called *the founder of logic* primarily owing to the collection of his six treatises that is known as «*Organon*». However, Aristotle himself did not give any indications that he considered the six treatises, compiled under the title “Organon” by the later Peripathetics, as a single whole study and he did not employ ither term “Organon” or “logic” either as the title or in the title of any of his treatises or their parts. He employed the term “logic” (“λογική” \lojikí, loyikí\ *s.f.*, pl. “λογικαί” \lojiké, loyiké\, as a close synonym of “*dialectics*” (“διαλεκτική” \ðialectikí\ *s.f.*, pl. διαλεκτικαί \ðialectiké\) meant *induction or deduction or both*. At the same time, Aristotle used the generic name “analytics” in the titles of his two Organon’s treatises, namely «*Prior Analytics*» and «*Posterior Analytics*», which he thus regarded as one work. The subject matter of the former is Aristotle’s theory of categorical syllogisms. In the light of the above etymological sense of the word “analytics”, Aristotle’s conjectural view on what he

did in the above two treatises can briefly be described thus: *detailed solution* of problems of dialectic (inductive or deductive) reasoning. The word "λογική" as a name of the science founded by «*Organon*» was reputedly used in writings of the Stoics. However, the Latin version of this name, "*logika*", from which the English noun "logic" is derived, was coined by Marcus Tullius Cicero (106–43 BC), a Roman statesman, orator, and author. Here follows the entry of Simpson [1959] confirming this fact:

> «**lŏgĭcus**, -a, -um (λογικός), *logical*; n. pl. as subst. **lŏgĭca**, - ōrum, *logic*: Cic.».•

A3. Pairs of antonymous polysemantic qualifiers

Preliminary Remark A3.1. In this treatise, in forming a description (descriptive name) of the species (specific class, subclass) of graphonyms (including statements) through the genus (general class, superclass) and the differentia (differences), I widely use various polysemantic (equivocal, ambiguous) adjectives as qualifiers (epithets) to the generic name (name of the genus, head name of the description) for denoting the differentia – such adjectives, e.g., as (in the alphabetic order): "advanced", atomic", "basic", "combined", "complex", "composite", "compound", "elemental", "fundamental", "molecular", "primary", "primitive", "secondary", etc. All these adjectives are *epistemologically relativistic* (*ad hoc*) ones, whose senses depend, in general outline, on the context, in which they occur, or, more specifically, on a generic name of graphonyms, to which they apply as qualifiers for denoting the pertinent differentia. There are no universal criteria for using the same qualifier in the names of graphonyms of different classes. Therefore, a graphonym is, after all, called by a given specific descriptive name, which involves a certain generic name and certain qualifiers to it, *if and only if is it is defined as one being so called*. Still, the above adjectives are usually used in pairs of *epistemologically relativistic antonyms*, complementary or not (see Cmt 5.1(3)). Consequently, it is possible to elaborate some general principles of forming such antonymous pairs. These principles, which allow the reader not to be lost in the extensive and extremely ramified unconventional terminology of this treatise, are explicated below in this appendix.•

Df A3.1. 1) A pasigraph (as a sign or formula) of A_1 or of any other logistic system of the treatise is said to be:

a) *atomic* if and only if it is, firstly, designed to be employed as a *unit* (*single whole*, *functionally indivisible*) *pasigraph* and if, secondly, it cannot be dissected (cleaved, analyzed) into two or more mutually disconnected figures *each of which* is classified as an atomic pasigraph;

b) *combined* if otherwise, i.e. if and only if the pasigraph is a *combination* of two or more atomic pasigraphs;

c) *molecular* if and only if it is a combined pasigraph that is designed to be employed as a *unit pasigraph* – just as an atomic one, but subject to a certain effective categorical (not conditional) criterion of its *integrity* other than the criterion of an atomic pasigraph stated in the item a);

d) *primitive* or *elemental* if and only if it is *either atomic or molecular*;

e) *complex* or *compound* if otherwise, i.e. if and only if it is not primitive.

2) A combination of two or more atomic pasigraphs is said to be a *juxtaposition*, and also a *linear*, or *juxtapositional*, *combination*, *of the pasigraphs* if it is a *linear sequence of the pasigraphs in the direction from left to right* and a *nonlinear* one if otherwise.

3) In accordance with Preliminary Remark A3.1, the adjectives "*primary*" and "*secondary*" are *epistemologically relativistic complementary antonyms*, whose senses depend on a generic name of graphonyms, to which they apply as qualifiers for denoting the pertinent differentia. Most often, the adjective "primary" is and will be used either in a broad sense as a synonym of some one of the adjectives or adjective equivalents "*initial*", "*original*", "*first and foremost*", "*of top priority*", "*introduced or defined in the first place*", etc, or in a narrow sense as a synonym of either of the adjectives "*undefined*" and "*postulated*" ("*taken for granted*"), while "secondary" is and will be used as an antonym of "primary" and hence as a synonym of one of the adjective equivalents: "*of less than first order or importance*", "*introduced or defined afterwards*", "*immediately or mediately defined in terms of or derived (inferred) from the respective primary*", etc, so that a secondary entity can be *immediately* defined in terms of be *immediately* derived (inferred) from *another secondary entity* of the same class.•

Cmt A3.1. 1) It will be recalled that *atomic logograph* is called a *lexigraph* and that, accordingly, an *atomic panlogograph* is called a *panlexigraph*. In accordance with item Df A3.1(1a), the qualifier "atomic" of a pasigraph (i.e. of a euautograph or logograph) is not descriptive of any *universal distinguishing topographic* (*topological*) properties of the pasigraph, i.e. of its *figure* (*form*, *inscription*). Particularly, this qualifier does not mean that the pasigraph should necessarily be connected. In other words, "atomic" should be understood as an abbreviation of the expression "*functionally indivisible*" or of the wordier expression "*functionally independent and self-subsistent*", and not as an abbreviation of either expression "*topographically connected*" or "*topographically indivisible*". An atomic pasigraph can consist of two or more disconnected figures, provided that none of them is classified as an atomic pasigraph. On the other hand, a topographically (topologically) connected pasigraph can be combined. Therefore, the criterion of *atomicity* of pasigraphs, which is indicated in Df A3.1(1a), can be called a *topographico-functional*, or *topologico-functional*, *one*. Here follow some examples of application of that criterion.

a) Any of the PAE's on the list (5.1) except the first six and any of those on the list (5.2) except the first four consists of at least two disconnected parts, namely, a base letter and a numeral subscript on it; the latter can itself be disconnected, e.g. $_{10}$, $_{11}$, etc. Similar subscripts and similar superscripts occur on the base letters of the PAE's on the lists (5.3^1)–(5.3^3), etc. In this case, it is not accidental that, in forming the *indexed* PAE's, the subscript or superscript on a base letter is set in the current Roman type and not in the Roman Arial Narrow Type. Indeed, in accordance with item 13 of Ax 5.1, the Arabic digits 0 and 1 in this Light-Faced Roman Arial Narrow Type are the *primary special* (*unordinary*) *atomic terms* (*formula-terms*, *term-formulas*), called also *primary atomic integrons*, both of A_n (both A_1 and A_0). Accordingly, the eight Arabic digits 2, 3, …, 9 in the same type will be employed as *secondary atomic integrons* of A_n. Thus, all the ten digits are atomic integrons of A_n. At the same time, the six letters *u*, *v*, *w*, *x*, *y*, *z* (without subscripts) on the list (5.1) in this Italic Arial Narrow type are also PAE's of A_1, while the four letters *p*, *q*, *r*, *s* (without subscripts) on the list (5.2) in the same font are atomic euautographs of both A_1 and A_0. Consequently, in accordance with Df A3.1(1a), the graphonym which is formed by furnishing any of the above-mentioned ten letters with a numeral subscript composed of the digits 0, 1, 2, …, 9 instead of 0, 1, 2, …, 9 could not be classified as an atomic one. Therefore, for avoidance of any artificial taxonomic incongruities, all digital indices which are employed in this treatise are set in the current Roman type.

b) Any of the special PAE's introduced in Ax 5.1(11) consists of two disconnected parts, one of which is the caret. Still, a caret alone is not used as an atomic euautograph of A_n. Therefore, in agreement with Df A3.1(1a), the above-mentioned careted algebraic signs are called *primary atomic* euaurographs (PAE's), whereas the sign $\hat{-}$, defined as $\hat{-} \rightarrow \hat{+}\,\hat{\div}$, is called a *secondary atomic euaurograph* (*SAE*).

c) Analogously, among the SAE on the list (5.5), $\subseteq$ consists of two disconnected parts: the figure $\subset$, which is used as another SAE, and the figure _, which is not used as any SAE of A_1. Therefore, the sign $\subseteq$ is qualified atomic in agreement with Df A3.1(1a). By contrast, either of the signs $\overline{\subseteq}$ and $\overline{\subset}$ can, in accordance with Cmt 5.3(2), be dissected into two disconnected parts, namely $\subseteq$ or $\subset$ respectively, which is used as an SAE of A_1, and ¯, being another SAE of both A_1 and A_0 (cf. $\overline{\vee}$, $\overline{\wedge}$, $\overline{\Rightarrow}$, etc), a synonym *of* $\neg$. Accordingly, both signs $\overline{\subseteq}$ and $\overline{\subset}$ are qualified molecular ad hence combined, and not atomic. At the same time, any of the conventional mathematical sign $\notin$, $\not\subseteq$, $\neq$, and $\not\subset$, being synonyms of $\overline{\in}$, $\overline{\subseteq}$, $\overline{=}$, and $\overline{\subset}$, is topographically connected, but at the same time it is a combination of the respective atomic sign $\in$, $\subseteq$, $=$, or $\subset$, and the atomic sign /, which is, like ¯, a synonym *of* $\neg$.

2) In accordance, with Df A3.1(1c), a molecular pasigraph is, like an atomic one, functionally indivisible, but it consists of two or more atomic pasigraphs. In this case, however, there is no universal criterion of *molecularity* of pasigraphs to allow distinguishing a molecular pasigraph from a complex one. The simplest combined relations and their validity-integrons are called respectively *molecular relations* and *molecular idempotent integrons* owing to their property of *integrity and ultimacy in ADP's* (*algebraic decision procedures*). All *molecular euautographic relations* (*MER's*) and *all molecular euautographic idempotent integrons* (*MEII's*) are *pseudo-variables*, while all *molecular panlogographic relations* (*MPLR's*) and *all molecular idempotent panolgographic integrons* (*MIPLI's*) are *place-holding variables.* In

treating of ADP's, dividing the combined relations of A_1 and their validity-integrons into molecular ones and complex ones and introducing the pertinent taxonyms (taxonomic names) is indispensable. By contrast, dividing some other classes of combined pasigraphs (as kernel-signs or the combined secondary decimal or binary digital integrons) into molecular ones and complex ones is done for convenience in description and is therefore dispensable.

3) In accordance with Df A3.1(1e), the qualifiers "complex" and compound" are synonyms. However, I shall use "compound" rather than "complex" when it is desirable to avoid undesirable associations of the latter with the meaning, which it has in the metaterm "complex number".

4) The complementary antonym of "*elementary*" is "*advanced*". Therefore, the classes denoted by the adjectives "*elemental*" ("*primitive*") and "*elementary*" are incomparable. Still, the adjective "basic" is sometimes used an antonym of "advance" and hence as a synonym of "elementary". At the same time, "basic" is also used as synonym of 'fundamental" and hence as a synonym of "elemental". This is just one of many unavoidable double or, in general, multiple uses of words, with which we live. It is hoped that this ambiguity of the word "basic" or of any other polysemantic word will be solved by the context, in which the word occurs. •

Df A3.2. 1) Two classes, particularly two *taxa* (*taxons*, *taxonomic classes*), are said to be *comparable* if and only if one of them is a *subclass* (*hypotaxon*, *part*) of the other or, equivalently, if and only if one of them is a *superclass* (*hypertaxon*, *whole*) of the other. Two classes are said to be *incomparable* if and only if they are *not comparable*. Two classes are said to be *compatible* or *conjoint* if and only if they *intersect*, and *incompatible* or *disjoint* if otherwise. *Comparable classes are compatible, but not necessarily vice versa*.

2) Two *taxonyms* (*taxonomic names*) are said to be *comparable*, *incomparable*, *compatible*, or *incompatible* and also *disjoint* if so are the taxa denoted by the taxonyms.

3) If two taxa are comparable then the taxonym of the subclass (hypotaxon) and the taxonym of the superclass (hypertaxon) are called a *hypotaxonym* and a *hypertaxonym*, or more generally a *subterm* and a *superterm*, *of* or *with respect to each other*.•

Cmt A3.2. 1) In accordance with Df A3.1, the class (or, more precisely, class-concept) that is denoted by either qualifier of any *pair of antonyms*:

a) “atomic” and “combined”, “primitive” and “complex” (“compound”), “molecular” and “complex”, and “atomic” and “molecular”

is *incomparable*, i.e. is *not standing in a subclass–superclass relation*, with the class denoted by either qualifier of the pair of antonyms:

b) “primary” and “secondary”.

Accordingly, in agreement with Df A3.2, I shall say that any pair of anonyms of the list a) is incomparable with the pair b). Likewise, in accordance with the pertinent earlier definitions, the pairs of antonymous qualifiers:

c) “ordinary” and “special,”

d) “euautographic” and “panlogographic”, “categorematic” (“formulary”) and “syncategorematic”, “main” (“principal”) and “auxiliary” (“subsidiary”), “structural” and “analytical”, etc

are mutually incomparable and any of them is incomparable tible with any of the pairs of the previous items a) and b).

2) Given a grammatically and lexically congruent hypothetical descriptive *specific* name of pasigraphs through an appropriate *generic name*, – e.g. “pasigraph” or any of its subterms (hypotaxonyms) such as:

“euautograph”, “panlogograph”, “categorem” (“formula”), “syncategorem”, “term”, “relation”, ‘integron”, “kernel-sign”, “logical connective”, “contractor”, “pseudo-quantifier”, “placeholder”, “schema”, etc,

– and through the conjunction of appropriate incomparable qualifiers, selected one from each of some incomparable pairs of antonyms given in the above items a)–d), the descriptive specific name is *meaningful* (*nonempty*, *has a denotatum*) if, in accordance with the atomic basis and formation rules of A_1, the generic name and the qualifiers are *compatible*, i.e. if the species of pasigraphs resulted by intersection of the genus denoted by the generic name and of the differentia denoted by the qualifiers is not empty, and the descriptive specific name is *meaningless* (*empty*, *has no denotatum*) and should therefore be disregarded if otherwise. Here follow a few examples illustrating the above-said.

a) An atomic, or combined, euautograph can be either primary or secondary, and conversely a primary, or secondary, euautograph can be either atomic or combined. That is to say, the descriptive specific names: “primary atomic euautograph”, “primary combined euautograph”, “secondary atomic euautograph”, and “secondary combined euautograph”, and also their synonyms with “primary” or “secondary” exchanged with “atomic” or “combined” are meaningful.

b) In accordance with the atomic basis and formation rules of A_1, there are in A_1 *no atomic pseudo-variable special terms*, *no combined euautographic ordinary terms*, *no atomic pseudo-constant relations*, either ordinary or special, and *no atomic pseudo-variable special relations*. That is to say, the classes that are designated by the count names, which have been used but not mentioned above, are *empty*.

c) In accordance with Cmt 5.3(1), the count names “secondary atomic punctuation mark” and “secondary atomic pseudo-variable ordinary predicate-sign” (“secondary APVOPS”) are empty (meaningless).

d) Some meaningful names of euautographs of A_1 become meaningless when they apply to euautographs of A_0. To be specific, *there are neither ordinary terms* (*OT’s*) *nor ordinary predicate-signs* (*OPS*’s) *in* A_0. That is to say, while the count names “ordinary term of A_1” and “ordinary predicate-sign of A_1” are nonempty, the count names “ordinary term of A_0” and “ordinary predicate-sign of A_0” are empty’

3) Any two incomparable pairs of antonymous qualifiers are mutually *independent*. However, owing to the *collective incompatibility* of a certain generic name of pasigraphs of A_1 and of the conjunction of some qualifiers to it, which results by the atomic basis and formation rules of A_1 and which has been described in the previous item, the conjunction of the same qualifiers to another, *compatible*, generic name may turn out to be redundant, so that one or more conjunct can be omitted without altering the denotatum of the description. Here follow a few examples illustrating this property.

a) In accordance with the item 2b, the count names of each one of the following pairs and their abbreviations are synonyms: (i) “*atomic pseudo-variable ordinary term*” (“*APVOT*”) and “*pseudo-variable ordinary term*” (“*PVOT*”); (ii) “*atomic pseudo-constant ordinary term*” (“*APCOT*”) and “*pseudo-constant ordinary term*” (“*PCOT*”); (iii) “*atomic euautographic ordinary term*” (“*AEOT*”) and "*euautographic ordinary term*" ("*EOT*"); (iv) "*atomic pseudo-variable ordinary relation*" ("*APVOR*") and "*atomic euautographic relation*" ("*AER*"). In this case, an EOT (AEOT) is either a PVOT (APVOT) or a PCOT (APCOT).

b) In accordance with the item 2c, the count names: "primary atomic punctuation mark" and "atomic punctuation mark", or "primary APVOPS" and "APVOPS" are synonyms (cf. Cmt 5.3(1)).

c) In accordance with the item 2d, the count names: "term of A_0 ", "special term of A_0", and "integron of A_0" are synonyms, whereas "the predicate-sign of A_0" and "the special predicate-sign of A_0" are synonymous proper names of $\hat{=}$.

4) Besides prepositive qualifiers, as those occurring on the lists a)c) and e) of the item 1 of this comment, a descriptive name of a species of pasigraphs may contain some postpositive qualifiers, If, for instance, the name "euautographic relation" is followed by either of the postpositive qualifiers "*of* A_1" and "*of* A_0" then the entire descriptive name may, without altering its denotatum, be abbreviated by omission of the prepositive qualifier "euautographic", because the latter denotes the same class as that denoted by "of A_1" and a strict superclass (a whole) of the class denoted by "of A_0". A like remark applies with "term", "kernel-sign", "predicate-sign", "formula", etc in place of "relation" Also, the whole of the above-said applies with "panlogographic" and '$\mathbf{A}$' in place of "euautographic" and 'A' respectively.•

Cmt A3.3. 1) All elements of B_{1OM} and B_{1Sp} and also all *selected* elements of B_{1OS} are called the *primary atomic* euautographs (*PA*E's) of the pertinent branch of A_1, while all *atomic* euautographs, which are defined in terms of some PAE's, are called *secondarsy atomic* euautographs (*SA*E's) of that branch. As was indicated in Cmt 5.3(2,3):, there are, particularly, the following options: (a) $\in$ is selected as a primary atomic predicate-signs, while $\subseteq$ and = are defined in terms of $\in$ and are therefore secondary atomic predicate-signs; (b) $\in$ is disregarded, $\subseteq$ is selected as a primary atomic predicate-sign, while = is defined in terms of $\subseteq$ and is therefore a secondary atomic predicate-sign; (c) both $\in$ and $\subseteq$ are is disregarded, while = is selected as a primary atomic predicate-sign. This example illustrates that the qualifier "atomic" is independent of either of the qualifiers "primary" and "secondary" and primary" are independent. In addition, it shows that, besides the fact that the pair of antonyms "primary" and "secondary" is epistemologically relativistic, these antonyms are *relative* in the sense that a graphonym that is qualified primary can be metamorphosed to become a secondary one and vice versa.

2) Like atomic euautographs, *formation rule*s and hence *terms* and *relations*, indiscriminately called *formulas* or *categoremata*, of A_1 can be divided into primary ones and secondary ones. The concrete definitions of the qualifiers "primary" and "secondary" to the generic names "formation rule" and "formula", and also to "term" and "relation", being subterms (hypotaxons) of "formula", will be made in the next subsection. Meanwhile, here follow two other important instances of using these qualifiers.

a) A *meta-relation*, i.e. a relation belonging to the *IML* is said to be

i) a *primary true meta-relation* or a *meta-axiom* if and only if it is taken for granted to be true and is laid down as being so;

ii) a *secondary true meta-relation* if and only if it is either a *definition* being a supplement to a meta-axiom or meta-axioms, or a *meta-theorem* that *is derived* (*inferred*, *proved*) *from other true meta-relations*, primary or secondary, mainly by means of some informal self-evident substitutions.

b) A relation of A_1 is said to be

i) a *primary valid relation*, or an *axiom*, *of* A_1 if and only if it is taken for granted to be valid and is laid down as being so;

ii) a *secondary valid relation*, or a *theorem*, *of* A_1 if and only if it *is derived* (*inferred*, *proved*) *from other valid relations*, primary or secondary, by means of D_1.

3) In agreement with Df A3.2(3), the narrow senses, which the qualifiers "primary" and "secondary" have in the previous two items, can, be expressed by the following descriptive adjective equivalents: "*taken for granted as being effective, true*, *or valid*" and "*being effective, true, or valid according to a subsequent definition or theorem*", respectively. Therefore, the setup of A_n can be changed in such a way that some atomic euautographs, some formulas, or some valid relations of A_n, which are qualified secondary, will become primary ones, while some other atomic euautographs, some other formulas, or some other valid relations of A_n, which are

qualified primary, will become secondary ones. Examples of such *primary–secondary duality* (*relativity*), which are analogous to that of either predicate-sign $\subseteq$ or =, will be given in due course. Particularly, some of the various *duality laws*, which will be stated in the sequel, can be interpreted as the respective relations of primary–secondary duality.

4) It has been pointed out in Cmt A3.2(3) that incomparable pairs of antonymous qualifiers such as “ordinary” and “special”, “atomic” and “combined”, “primitive” (“elemental”) and “complex”, and “primary” and “secondary” are independent from one another. Therefore, for instance, an ordinary or special euautograph can be either atomic or combined and at the same time it can be either primary or secondary. In this case, if a description (descriptive name) contains two or more epithets (qualifiers) to the same head name (generic name) then a permutation of the epithets does not change the sense of the description. For instance, the complex qualifiers “special primary atomic”, “primary atomic special”, “atomic special primary”, etc. to any of the noun “euautograph”, “panlogograph”, pasigraph”, etc are synonyms. However, in accordance, e.g., with the item 2b of this comment, the count noun “*endosemasiographic axiom*” is a synonym of the descriptive count name “*primary valid endosemasiographic relation*”, whereas the count noun “*endosemasiographic theorem*” is a synonym of the count descriptive count name “*secondary valid endosemasiographic relation*”. The two descriptive names are in turn synonyms of the descriptions “*endosemasiographic relation that is taken for granted to be valid*” and “*endosemasiographic relation that is deduced to be valid from some other valid relations*” in this order. Accordingly, either of the adjectives “primary” and “secondary” is a qualifier to the head name “*valid endosemasiographic relation*”, – or, from a somewhat different viewpoint, it is an adherent qualifier to the adjective “*valid*”, – and not a qualifier to the name “*endosemasiographic relation*”. Therefore, “*primary valid* endosemasiographic relation” is not a synonym of “*valid primary* endosemasiographic relation”, while “*secondary valid* endosemasiographic relation” is not a synonym of “*valid secondary* endosemasiographic relation”. That is to say, the qualifier “primary” or “secondary” is not commutable with the qualifier “valid”.•

A4. Individuals versus universals

A4.1. Domains

Df A4.1. 1) I shall use the word “*domain*” for denoting any particular *system of interrelated entities* (*beings*) that is intelligibly, but not necessarily consistently or even rationally from my viewpoint, expressed by a certain part of one of the *written native languages* (*WNL’s*), in which I have command. A *domain sensu stricto*, i.e. a domain in the narrow (restricted) sense, in which the *class-name* (*count noun*) “domain” will be used in the sequel, is a *domain of knowledge*, i.e. a *scientific domain* or, in other words, a field of *scientific* study and discourse (as logic, mathematics, physics, biology, etc) or a concrete self-contained treatise (discourse) in the field. A scientific domain is alternatively called a *theory* – to put forward the discursive (conceptual) aspect of the domain. Therefore, in the sequel, the noun “domain” will, unless stated otherwise, be used synecdochically instead of or interchangeably with the noun “theory”. However, in a broad sense, a work of fiction or a heroic or religious legend is also a domain. If a domain is a scientific (theoretic) one then its WNL can be and usually is augmented, with the help of that same WNL, by some defined or undefined graphonyms (graphic expressions), not belonging to the common lexicon of the WNL. In two different domains, the same graphonym (as a root, word, or logograph) may denote two completely different entities. Particularly, if both domains are scientific, in which the noun “class” is used as a technical term, then the denotatum of that noun may be different in the two domains. For instance, in a *biological taxonomy of bionts* (*BTB*), the noun “class” is used in a narrow sense, i.e. as “class” sensu stricto, for denoting a taxon (taxonomic class [sensu lato]) ranking between the *orders* and *divisions* of either kingdom **Plantae** or **Fungi** or between the orders and phyla of the kingdom **Animalia**. Also, a word that is admissible and meaningful in one of the domains can be inadmissible and hence meaningless in the other one.

2) The sense of the term “domain” as defined above and the sense of either term “*domain of definition of*” or “*scope of the definition*” are distinct. Still, a domain is a domain of definition of any term, whose sense is intelligibly and univocally defined (described) in the domain and which is used throughout the domain in

accordance with its definition. In this case, the domain is also the scope of the definition in question.•

Cmt A4.1. Here follow a few examples illustrating relationship between a class and its domain, being at the same time the pertinent domain of definition of the class and the scope of that definition. Some formal examples of that relationship will be given as I go along.

1) In [the domain of] Greek mythology, the following statement of Guirand [1959, p. 161] can be regarded as an axiom so that it is *formally* (*but not materially*) *true*:

> «In addition to Satyrs and the Sileni, another kind of monstrous creatures formed part of the cortege of Dionysus: the Centaurs. Their torso and head were those of a man; the rest of their body belonged to a horse.»

Therefore in this domain, the class *centaur* that is designated by the count name "centaur", exists and is not empty. Just as any other class (as *man*, *tree*, etc), designated by the respective count name, the class *centaur*, along with its name, can be used *xenonymously* in a *projective* (*polarized*, *extensional*, *connotative*) *mental node*, indicated in English by the indefinite article, for mentioning an as if extramental (exopsychical) *common* (*general*) *member* of that class, namely *a* centaur, which is denoted by its *common individual name* (*limited common name*) "a centaur" and which is another hypostasis of that same class. Accordingly in this domain, the name "a centaur" is *senseful*, – just as the *proper class-name* (*unlimited common name*) "centaur". At the same time, the count noun "centaur" is not a biological term, i.e. biology and particularly a BTB (biological taxonomy of bionts) is not a domain of definition of that noun. That is to say, in biology, both the unlimited common name "centaur" and the limited (indefinitely-articled) common name "a centaur" are *inadmissible* (*purposeless*, *functionless*) and hence *senseless* (*absurd*) *graphonyms*, i.e. *mere autographs*, which have no xenonymous denotata and which are not designed to be attached with any. Any grammatically congruent linguistic construction that contains a senseless graphonym as its constituent part is also senseless. For instance, from the standpoint of semantic analysis, the *quasi-sentence*: "A centaur is a mammal", its negation: "A centaur is not a mammal", and the inclusive disjunction of the above two: "A centaur is a mammal or a centaur is not a mammal" are senseless, i.e. they *have no denotata*. (Regarding the term "quasi-sentence", see Df 2.16 below in this section.) Consequently, all the three quasi-sentences are *paradoxical* in the sense that they are *neither true nor antitrue* (*false*) and that the notions of truth and antitruth (falsity) are not applicable to them, although the last quasi-sentence is *valid* from the standpoint of syntactic analysis. However, the graphonym "A centaur does not exist" is a *true* (*veracious) sentence*, because it is just a loose abbreviation of the true (veracious) sentence "The graphonym "a centaur" has no denotatum in the natural universe", in which the graphonym (graphic name sensu lato) "a centaur" is used autonymously. An inadmissible (purposeless, functionless) and hence senseless (absurd) and *indefinable extrinsic graphonym*, which is incompatible with the terminology of a given domain, as "centaur" in biology, will be called a *paralogy* (from the Greek adjective "παράλογος" \parálογos\ meaning *unreasonable* or *absurd*) – in contrast to a *naked* but *definable*, or more precisely *redefinable*, *intrinsic name*, which is conventionally called in Latin a *nomen nudum* (pl. "*nomina nuda*") and which can alternatively be called a *gymnonym* (from the Greek adjective "γυμνός" \γimnós\ meaning *naked*) or more specifically a *gymnograh* (*gymnograhonym*). Like remarks apply, *mutatis mutandis*, with the proper name "Pegasus", e.g., in place of both common names "centaur" and "a centaur".

2) A Merriam-Webster® [1981] (abbreviated as "WTNID" – "Webster's Third New International Dictionary of the English Language") defines one of the meanings of the noun "nothing" in this manner:

«[3]**nothing** … *n* -s …**1 a :** no thing at all **:** something that does not exist …»

This definition is a *persuasive one* and *not a real one*, i.e. *not* a traditional *definition through a genus and the difference* (*definitio per genus et differentiam*). Also, the second part of this definition is a *contradiction in adjecto*, and hence the noun "nothing" thus defined is a *nomen nudum* (*mere name*, *naked name*). In fact, the *sense* of the noun "nothing" depends on its domain and is therefore an *epistemologically relativistic* one. Most generally, given a domain, *nothing* is an abstract entity (being) serving as the *empty ground*, or *background*, of any *nonempty entity* of the domain – an entity that is regarded as being *something*, to which the nothing is opposed. That is to say, nothing is always the background of something, but not of everything. For instance, I may look at the cloudless blue sky and to say that I see nothing. Likewise,

a homogeneous *blank surface*, on which *figures* (*faces*) of graphonyms are depicted, is the pertinent empty ground, i.e. the pertinent nothing, of the graphonyms. In this case, a part of the ground of a standard shape and of standard linear sizes is called a *blank*, or *empty space*; e.g. an "n"-*space*, or *En Space*, is an empty space. The difference between graphic figures and their groun
d is relative. Escher's mosaic drawings in the *Alhambra style* are striking illustrations of this fact (see e.g. Hofstadter [1980, pp. 64–70] or Ernst [1985, pp. 35–41]). Various kinds of a nothing versus the respective something and dependence of a class on the domain (theory, system), to which it belongs, are illustrated bellow by some mathematical examples

3) Let 'S' denote any given full-scale *axiomatic set theory* (*AST*) – a *one-individual* one (*OIAST*) or a *many-individual* one (*MIAST*), to be denoted by 'S^{OI}' or 'S^{MI}' respectively. Let '$\varnothing$' denote the *empty set*, called also the *empty*, or *null*, *individual*, and let '$\mathcal{N}$', '$\mathcal{I}$', '$\mathcal{Q}$', '$\mathcal{R}$', and '$\mathcal{C}$' denote the sets of *natural*, *integer* (*integral*), *rational*, *real,* and *complex numbers* in that order, the understanding being that these sets are or can be defined in the framework of (derived from) S. Let the *indexed zeros* '0_N', '0_I', '0_Q', '0_R', and '0_C' denote the *null elements*, i.e. *null numbers*, of the sets $\mathcal{N}$, $\mathcal{I}$, $\mathcal{Q}$, $\mathcal{R}$, and $\mathcal{C}$ respectively. In this case, $0_N=\varnothing$, while 0_I, 0_Q, 0_R, and 0_C turn out to be mutually different *nonempty sets*, and hence they are *not individuals*, *either empty or nonempty* (see, e.g., Feferman [1964] or Burrill [1967]). The set 0_N, 0_I, 0_Q, 0_R, or 0_C can be called the *natural*, *integer*, *rational*, *real*, or *complex null-set* respectively ("*null*" can be not hyphenated). Consequently, any one of the sets 0_I, 0_Q, 0_R, and 0_C can indiscriminately be called a *nonempty null-set*, while 0_N, i.e. $\varnothing$, should, by way of emphatic comparison with the above term, be called *the empty null-set*. Thus, a null-set can be either empty or nonempty. That is to say, the term "null-set" alone, without the appropriate one of the prepositive qualifiers "empty" and "nonempty", is not a synonym of the terms "empty set", "empty individual" and "null individual". In accordance with the above-said, besides the three synonymous proper names "the empty set", "the empty individual", and "the null individual", $\varnothing$ can alternatively be called the *universal,* or *absolute*, *null*, and also the *void*, *of any set* or *class theory*, whereas the *null element* (*null number*) 0_I, 0_Q, 0_R, or 0_C can alternatively called the *relative null of* $\mathcal{I}$, $\mathcal{Q}$, $\mathcal{R}$, or $\mathcal{C}$ respectively, and only of

that set. A like terminology applies to the null element of any given algebraic system – such an element, e.g., the null element $\widehat{0}$ of an additive group or the null-vector $\widehat{0}_{(n)}$ of an *n*-dimensional linear (vector) space over a certain field. The occurrence of the word "null" in either of the terms "absolute null" ("universal null) and "relative null" can be used interchangeably with either one of the nouns "*nil*" and "*nothing*" without altering the meaning of a term. As an exception to the above general terminology, the *relative null* (*nil*, *nothing*) *of* $\mathcal{N}$ is the absolute null (nil, nothing) of any underlying set or class theory. Accordingly, the term "void" is not applicable to the relative null (nil, nothing) of any set except $\mathcal{N}$. Thus, the count names "null-class", "null", and "nothing" (in the above sense) are synonymous *proper names* of a certain multitudinous (many-member) class, i.e. *null-class*, *null*, or *nothing* (without any modifier), so that any of the names can be used in a *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, which is indicated in English by the indefinite article, for mentioning a *common* (*general*) *member* of that class, namely *a* null-class, *a* null, or *a* nothing, which is another hypostasis of that class. There is an indefinite number of distinct null-classes (nulls, nothings), although some of them are equivocally (homographically, homonymously) denoted by the same graphonym.

4) The most fundamental semantic axiom of S is the following conventional *axiom* [*schema*] *of comprehension* (or *of specification* or *of separation* or *of subsets)*, called also in German *Aussonderungaxiom* (*axiom of selection*): Given a condition $P_e\langle x\rangle$ that is imposed on the elements ("e") of the range of the variable '*x*', to every set *u* there is a set *v* whose elements (members) are exactly all those elements of *u*, for which $P_e\langle x\rangle$ is *veracious*, i.e. logographically

$$\bigwedge_u \bigvee_v \bigwedge_x \left[\left[x\in v\right] \Leftrightarrow \left[\left[x\in u\right]\wedge P_e\langle x\rangle\right]\right], \tag{A4.1}$$

where '$\wedge$' and '$\Leftrightarrow$' are read as "and" and "if and only if" respectively, while '$\bigwedge_u$' and '$\bigvee_v$' (e.g.) are synonyms of the conventional quantifiers '$(\forall u)$' and '$(\exists v)$', which are read as "for all *u*:" or "for every *u*:" and as "for some *v*:" or "for at least one *v*:" or "there exists at least one *v* such that", respectively. Consequently,

$$\begin{aligned} &\bigwedge_u \bigvee_v \left[\left[v=\left\{z\middle|\left[z\in u\right]\wedge P_{\mathrm{e}}(z)\right\}\right]\right. \\ &\left.\to \bigwedge_x \left[\left[x\in v\right]\Leftrightarrow\left[\left[x\in u\right]\wedge P_{\mathrm{e}}(x)\right]\right]\right] \end{aligned} \tag{A4.2}$$

is a contextual definition of the logographic term '$\{z|[z \in u] \wedge P_e(z)\}$' of S, which is read as: "the set of all z such that $z \in u$ and $P_e\langle z\rangle$". The above statements apply with "class" and 'C' in place of "set" and 'S' respectively, when the full-scale semantic one-individual theory in question includes both sets (regular, or small, classes) and irregular (proper) classes (see subsection I.9.3). The pertinent set-builders are used below for illustrating dependence of a set on a domain (theory), to which it belongs.

a) In the *set of real numbers*, $\mathcal{R}$, as *the given domain*, the condition $\sin z = 2$ as $P_e\langle z\rangle$ is *not satisfiable*, i.e. is *antiveracious*, for every value z of the variable 'z'. Therefore,

$$\{z|\sin z = 2 \text{ and } z \in \mathcal{R}\} = \varnothing, \tag{A4.3}$$

where $\varnothing$ is, as before, the *empty set*.

b) By contrast, the *homonymous* condition $\sin z = 2$ is *satisfiable*, i.e. is *veracious*, for some values of the *homonymous* variable 'z' of the set of complex numbers, $\mathcal{C}$, as another given domain and is *not satisfiable*, i.e. is *antiveracious*, for the other values of that variable in the same set. Therefore, the set S_2, defined as

$$S_2 \doteq \{z|\sin z = 2 \text{ and } z \in \mathcal{C}\}, \tag{A4.4}$$

is not empty and it can be calculated as follows. The condition $\sin z = 2$ occurring in (A4.4), can be rewritten as: $(1/2i)(e^{iz} - e^{-iz}) = 2$, which reduces to the quadratic equation: $e^{2iz} - 4ie^{iz} - 1 = 0$ with respect to e^{iz}. Therefore,

$$e^{iz} = 2i \pm \sqrt{-4+1} = (2 \pm \sqrt{3})i, \tag{A4.5}$$

whence

$$\begin{aligned} z &= -i\ln\left[(2 \pm \sqrt{3})i\right] = -i\left[\ln(2 \pm \sqrt{3}) + \ln i\right] \\ &= -i\ln(2 \pm \sqrt{3}) + (2n + \tfrac{1}{2})\pi \text{ for each } n \in I_{\mathrm{R}}, \end{aligned} \tag{A4.6}$$

because

$$\ln i = \ln\exp\left[(2n + \tfrac{1}{2})\pi i\right] = (2n + \tfrac{1}{2})\pi i \text{ for each } n \in I_{\mathrm{R}}; \tag{A4.7}$$

I_{R} is the set of real integers – strictly positive, strictly negative, and null. Consequently, (A4.4) becomes:

$$S_2 \doteq \{z|z = -i\ln(2 \pm \sqrt{3}) + (2n + \tfrac{1}{2})\pi \text{ and } n \in I_{\mathrm{R}}\}. \tag{A4.8}$$

c) A *broadest domain* of the sets ∅ and S_2 is S or C, provided that it includes a theory of real and complex numbers. Such a domain is at the same time a *domain of definition of* '∅', but it is not necessarily a domain of definition of 'S_2', because the latter logograph is an *ad hoc* one. •

Cnv A4.1. Unless stated otherwise, by "a domain", I shall henceforth mean a *scientific domain* (*domain of knowledge*) as defined in Df A4.1(1), i.e. either a field of scientific study and discourse as logic, mathematics, physics, biology, etc or a branch of the field or else a concrete study or discourse in the field or in the branch. When I put forward the discursive (theoretical) aspect of a scientific domain, I shall use the noun "domain" synecdochically instead of or interchangeably with the noun "theory". A self-contained logical calculus or a self-contained class, or set, or mass, theory, which has a well-defined nomenclature, i.e. phonographic (verbal) terminology and pasigraphic (logographic or euautographic or both) notation, will be called a *logical domain*. The *species* (*specific class*, *kind*) *of classes* that are called *sets* is specified in subsection I.9.3.•

A5. Nominalistic class and mass theories

A5.1. Nomenclature of $\mathsf{C}_1^{\mathsf{OI}}$ and M_1

Df A5.1. 1) In order to develop either a full-scale *one-individual nominalistic axiomatic class theory* (*OINACT*), denoted by '$\mathsf{C}_1^{\mathsf{OI}}$', or a full-scale *nominalistic axiomatic mass theory* (*NAMT*), which is unavoidably a *one-individual* theory as well and which will denoted by 'M_1', the AVCLOT's (atomic variable catlogographic ordinary terms) defined by Df 7.1(1) can be used either as catlogographic *class-valued variables* (*CVV's*) or as catlogographic *mass-valued variables* (*MVV's*) respectively, but not both simultaneously, whereas the ACCLOT (atomic constant catlogographic ordinary term) '∅' defined by Df 7.1(2) can be used either as a catlogographic *empty-class-valued constant* (*ECVC*) and as a catlogographic *empty-mass-valued constant* (*EMVC*), i.e. as an *empty-individual-valued constant* (*EIVC*), in both theories. That is to say, each of the six light-faced italic minuscule letters '*u*', '*v*', '*w*', '*x*', '*y*', and '*z*', taken alone or furnished either with any of the upright Arabic numeral subscripts '$_1$', '$_2$', etc or with any number of primes or with both labels simultaneously, is a *variable*, which can denote (assume as its accidental denotatum):

a) a *class*, a *nonempty* one or the *empty* one, i.e. the *empty individual*, denoted by '∅', the understanding being that a nonempty class can be a *member* (*element*) or a *subclass* (*part*) or both of another nonempty class and that the empty class is subclass both of itself and of every nonempty class, – if the variable belongs to $\mathsf{C}_1^{\mathsf{OI}}$;

b) *mass*, *nonempty* one or the *empty* one, i.e. the *empty individual*, denoted by '∅', the understanding being that nonempty mass can be *submass* (a *part*) of another nonempty mass, whereas the empty mass is a submass both of itself and of every nonempty mass, – if the variable belongs to M_1.

A class being a member of the range of any CVV is more specifically called a *particular class* and also an *element*. Analogously, mass being a value of an MVV is more specifically called *particular mass*, but it is not called "element" for the latter noun is customarily used as a contrary term of "mass".

2) At any place, $\mathsf{C}_1^{\mathsf{OI}}$ can be augmented by one or more *class-valued constants* (*CVC's*), accidental (circumstantial, ad hoc) ones or essential ones, each of which denotes a certain *universal class*, say *U* generally or $\omega_\varnothing$, or $\omega'_\varnothing$, specifically, that cannot be a member of any particular class and that satisfies the three semantic postulates, accidental ones (hypotheses) or essential ones (axioms), $\neg[U\in U]$, $u\in U$, and $\neg[U\in u]$, and similarly with any congeneric CVV in place of '*u*'; the above postulates are the CFCL interpretands of the respective euautographic axioms $\neg[\mathsf{U}\in\mathsf{U}]$, $\mathsf{u}\in\mathsf{U}$, and $\neg[\mathsf{U}\in\mathsf{u}]$ of $\bar{\mathsf{A}}_{1\in}$ (see Cmt 7.6(2)). Likewise, at any place, M_1 can be augmented by one or more *mass-valued constants* (*MVC's*), accidental (circumstantial, ad hoc) ones or essential ones, each of which denotes a certain *universal mass*, say *U* again, that cannot be a part of any particular mass and that satisfies the three semantic postulates, accidental ones (hypotheses) or essential ones (axioms), $U\subseteq U$, $u\subseteq U$, and $\neg[U\subseteq u]$, and similarly with any congeneric MVV in place of '*u*'; the above postulates are the CFCL interpretands of the respective euautographic axioms $\mathsf{U}\subseteq\mathsf{U}$, $\mathsf{u}\subseteq\mathsf{U}$, and $\neg[\mathsf{U}\subseteq\mathsf{u}]$ of $\bar{\mathsf{A}}_{1\subseteq}$ (see *ibid*.).

3) When the operators '$\bigvee_u$' and '$\bigwedge_u$', e.g., occur in to $\mathsf{C}_1^{\mathsf{OI}}$, they are synonyms of the conventional *quantifiers* '$(\exists u)$' and '$(\forall u)$', which are read as "*for some u*:" or "*for at least one u*:" or "*there exists at least one u such that*" and as "*for*

all u:” or “*for every u*:” respectively and similarly with any congeneric AVCLOT of the list (8.4) in the capacity of a CVV in place of ‘*u*’ (cf. item 5 of subsection 9.2). When, however, the operators ‘$\bigvee_u$’ and ‘$\bigwedge_u$’, e.g., occur in M_1, they are *qualifiers* (and *not quantifiers)*, which are rendered into ordinary English as “*for some u*:” (but neither as “*for at least one u*:” nor as “*there exists at least one u such that*”) and “*for every u*:” (but not as “*for all u*:”) respectively; and similarly with any congeneric AVCLOT of the list (8.4) in the capacity of an MVV in place of ‘*u*’. It is understood that any quantification of a CVV and any qualification of an MVV *is effective in the range of that variable*.

4) The logographic logical connectives and their verbal (phonographic) synonyms, which are employed in $\mathsf{C}_1^{\mathsf{OI}}$ and M_1, have been given in the item 6 of subsection 9.2 and they are cited below for convenience and for completeness of this discussion:

“not” for ‘$\neg$’,

“or” or “ior” for ‘$\vee$’,

“and” or “&” for ‘$\wedge$’,

“if … then –” or “… only if –” for ‘…$\Rightarrow$–’,

“if” for ‘$\Leftarrow$’,

‘if and only if” or “iff” for ‘$\Leftrightarrow$’,

“neither … nor –” for ‘$\not\vee$’ or ‘$\overline{\vee}$’,

“not both … and –” for ‘$\not\wedge$’ or ‘$\overline{\wedge}$’,

“but not” for ‘$\overline{\Rightarrow}$’¸“not … but –” for ‘$\overline{\Leftarrow}$’,

“either … or – but not both” or “xor” for ‘$\overline{\Leftrightarrow}$’,

“for some $*$:” or “for at least one $*$:” or “there exists at least one $*$ such that” for ‘$\bigvee_*$’,

“for all $*$” or “for every $*$” for ‘$\bigwedge_*$’,

the understanding being that in any one of the above definitions alike ellipses should be replaced alike by the appropriate relations or relation-valued variables.•

A5.2. Foundations of $\mathsf{C}_1^{\mathsf{OI}}$

Df A5.2: ***The interpretation of*** $\in$ in $\mathsf{C}_1^{\mathsf{OI}}$.

$$[x \text{ is a class-member of the class } u] \rightarrow [x \in u]. \qquad \text{(A5.1)}\bullet$$

Summary A5.1: ***Basic tautologous catlogographic relations (axioms, theorems, and definitions) of*** $\mathsf{C}_1^{\mathsf{OI}}$. The basic tautologous *catlogographic relations* (*CLR's*), which are given below in this summary and which are discriminately called *axioms*, *theorems*, or *definitions*, are the CFCL interpretands of the pertinent *axioms* and *theorems*, i.e. *valid* (*kyrologous*) *euautographic relations* (*ER's*) *of* $\mathsf{A}_{1\in}$, and of the pertinent *definitions of* $\mathsf{A}_{1\in}$, respectively.

1) *The axioms for* $\in$.

$$\neg[[x \in y] \wedge [y \in x]]. \qquad (\textit{The antisymmetry law}) \quad \text{(A5.2)}$$

$$\bigvee_u [x \in u]. \qquad (\textit{The incidence law for a nonempty class}) \quad \text{(A5.3)}$$

2) *Basic theorems for* $\in$.

$$\neg[x \in x]. \qquad (\textit{The antireflexivity law}) \quad \text{(A5.4)}$$

$$\bigvee_x \neg[x \in u]. \qquad (\textit{The incidence law for a nonmember of the class}) \quad \text{(A5.5)}$$

3) *The definition of* $\subseteq$.

$$[u \text{ is a subclass of } v] \rightarrow [u \subseteq v] \rightarrow \bigwedge_x [[x \in u] \Rightarrow [x \in v]]. \qquad \text{(A5.6)}$$

4) *Basic theorems for* $\subseteq$.

$$u \subseteq u. \qquad (\textit{The reflexivity law}) \quad \text{(A5.7)}$$

$$[u \subseteq v] \wedge [v \subseteq w] \Rightarrow [u \subseteq w]. \qquad (\textit{The transitivity law}) \quad \text{(A5.8)}$$

$$\bigvee_u [u \subseteq v]. \qquad (\textit{The first incidence law for } \subseteq) \quad \text{(A5.9)}$$

$$\bigvee_v [u \subseteq v]. \qquad (\textit{The second incidence law for } \subseteq) \quad \text{(A5.10)}$$

$$\bigvee_u \neg[u \subseteq v]. \qquad (\textit{The first incidence law for } \overline{\subseteq}) \quad \text{(A5.11)}$$

$$\left[\bigvee_v \neg[u \subseteq v]\right] \Leftrightarrow \left[\bigvee_x [x \in u]\right]. \qquad (\textit{The second incidence law for } \overline{\subseteq}) \quad \text{(A5.12)}$$

5) *The definition* (*law*) *of extensionality for classes:*

$$[u = v] \rightarrow [[u \subseteq v] \wedge [v \subseteq u]]. \qquad \text{(A5.13)}$$

6) *Basic theorems* (*laws*) *of equality for classes.*

$$u = u. \qquad (\textit{The reflexivity law}) \quad \text{(A5.14)}$$

$[u = v] \Leftrightarrow [v = u]$. (*The symmetry law*) (A5.15)

$[u = v] \wedge [v = w] \Rightarrow [u = w]$. (*The transitivity law*) (A5.16)

$\bigvee_u \neg[u = v]$. (*The incidence law for antiequalities*) (A5.17)

$\bigvee_u [u = v]$. (*The incidence law for equalities*) (A5.18)

7) *The definition of* $\subset$: *the first law of a strict subclass.*

$$[u \text{ is a strict subclass of } v] \to [u \subset v] \to [[u \subseteq v] \wedge \neg[v \subseteq u]]. \qquad \text{(A5.19)}$$

8) *Basic theorems for* $\subset$.

$[u \subset v] \Leftrightarrow [[u \subseteq v] \wedge \neg[u = v]]$.

(*The second law of a strict subclass*) (A5.20)

$\neg[u \subset u]$. (*The antireflexivity law for* $\subset$) (A5.21)

$[u \subset v] \wedge [v \subset w] \Rightarrow [u \subset w]$. (*The transitivity law*) (A5.22)

$\bigvee_u \neg[u \subset v]$. (*The first incidence law for* $\overline{\subset}$) (A5.23)

$\bigvee_v \neg[u \subset v]$. (*The second incidence law for* $\overline{\subset}$) (A5.24)

9) *The axiom* (*primary law*) *of an empty class*.

$$\neg[x \in \varnothing]. \qquad \text{(A5.25)}$$

10) *The theorem* (*law*) *of uniqueness of the empty class*.

$$[[\neg[x \in \varnothing]] \wedge [\neg[x' \in \varnothing']]] \Leftrightarrow [\varnothing = \varnothing']. \qquad \text{(A5.26)}$$

11) *Basic theorems* (*secondary laws*) *for the empty class*:

$\varnothing \subseteq v$. (*The first law*) (A5.27)

$\neg[v \subset \varnothing]$. (*The second law*) (A5.28)

All *catlogographic relations* (*CLR's*), which are classified above as axioms or as theorems are *tautologous* ones, for they are CFCL interpreteands of the respective analographic *valid* euautographic relations, i.e. *euautographic axioms* or *euautographic theorems* respectively, of $\mathsf{A}_{1\in}$; the euautographic theorems are established to be so by the appropriate EADP's (euautographic algebraic decision procedures).•

Summary A5.2: ***Basic veracious catlogographic relations (axioms, theorems, and definitions) of*** $\mathsf{C}_1^{\mathsf{OI}}$. The basic *veracious catlogographic relations* (*CLR's*), which are given below in this summary and which are discriminately called *axioms*, *theorems*, or *definitions*, are *pure semantic true*, *but not tautologically true*, *relations*, which are irrelevant to any euautographic relations (ER's) and to any definitions of $\mathsf{A}_{1\in}$.

1) *The axiom* [*schema*] *of comprehension* (or *of specification* or *of separation*) or (in German) *Aussonderungaxiom* (*axiom of selection*), *for classes*. Given a condition $P_{\mathrm{c}}\langle x\rangle$ that is imposed on the class-members ("c") of the range of the CVV '*x*', to every class *u* of classes there is a class *v* of classes whose members are exactly all those class-members of *u*, for which $P_{\mathrm{c}}\langle x\rangle$ is *veracious*, i.e. logographically

$$\bigwedge_u \bigvee_v \bigwedge_x [[x \in v] \Leftrightarrow [[x \in u] \wedge P_{\mathrm{c}}\langle x\rangle]], \tag{A5.29}$$

where '$\wedge$' and '$\Leftrightarrow$' are read as "and" and "if and only if" respectively, while '$\bigwedge_u$' and '$\bigvee_v$' (e.g.) are synonyms of the conventional quantifiers '$(\forall u)$' and '$(\exists v)$', which are read as "for all *u*:" or "for every *u*:" and as "for some *v*:" or "for at least one *v*:" or "there exists at least one *v* such that" respectively. Consequently,

$$\begin{aligned}&\bigwedge_u \bigvee_v [[v = \{z|[z \in u] \wedge P_{\mathrm{c}}\langle z\rangle\}] \\ &\rightarrow \bigwedge_x [[x \in v] \Leftrightarrow [[x \in u] \wedge P_{\mathrm{c}}\langle x\rangle]]],\end{aligned} \tag{A5.30}$$

is a contextual *definition* of the logographic term '$\{z|[z \in u] \wedge P_{\mathrm{c}}\langle z\rangle\}$' of $\mathsf{C}_1^{\mathsf{OI}}$, which is read as: "the class of all *z* such that $z\in u$ and $P_{\mathrm{c}}\langle z\rangle$". The operator '$\{\,|\,\}$' thus defined is called an *abstraction operator from a relation* (as $[z \in u] \wedge P_{\mathrm{c}}\langle z\rangle$) *to a class* (as $\{z|[z \in u] \wedge P_{\mathrm{c}}\langle z\rangle\}$) and also an *abstract class-builder*. The logographic term '$\{z|[z \in u] \wedge P_{\mathrm{c}}\langle z\rangle\}$' can be regarded as a logographic *description* through the *class-genus* (*generic class*) *u* and the *differentia* (*difference*) P_{c} of the *class-species* (*specific class*) $\{z|[z \in u] \wedge P_{\mathrm{c}}\langle z\rangle\}$, which it denotes.

2) *The axiom of pairing of classes.*

$$\bigvee_u \bigwedge_x [[x \in u] \Leftrightarrow [[x = x_1] \vee [x = x_2]]]. \tag{A5.31}$$

Consequently,

$$\begin{aligned}&\bigvee_u [[u = \{z|[z = x_1] \vee [z = x_2]\}] \\ &\rightarrow \bigwedge_x [[x \in u] \Leftrightarrow [[x = x_1] \vee [x = x_2]]]]\end{aligned} \tag{A5.32}$$

is a contextual *definition* of the logographic term '$\{z|[z = x_1] \vee [z = x_2]\}$' of $\mathsf{C}_1^{\mathsf{OI}}$, which is abbreviated further and rendered into ordinary language as follows:

$$[\text{The class of classes } x_1 \text{ and } x_2]$$
$$\rightarrow[\text{The unordered pair of classes } x_1 \text{ and } x_2]$$
$$\rightarrow \{x_1, x_2\} \rightarrow \{x|[x = x_1] \vee [x = x_2]\}. \quad \text{(A5.33)}$$

3) *The definition of the intersection of two classes.* By the pertinent instance of the axiom of comprehension (A5.29),

$$\bigvee_w \bigwedge_x [[x \in w] \Leftrightarrow [[x \in u] \wedge [x \in v]]]. \quad \text{(A5.34)}$$

Consequently,

$$\begin{gathered}\bigvee_w [[w = \{z|[z \in u] \wedge [z \in v]\}] \\ \rightarrow \bigwedge_x [[x \in w] \Leftrightarrow [[x \in u] \wedge [x \in v]]]]\end{gathered} \quad \text{(A5.35)}$$

is a contextual *definition* of the logographic term '$\{z|[z \in u] \wedge [z \in v]\}$' of $\mathsf{C}_1^{\mathsf{OI}}$, which is abbreviated further and rendered into ordinary language as follows:

$$[\text{The intersection of classes } u \text{ and } v]$$
$$\rightarrow [u \cap v] \rightarrow \{z|[z \in u] \wedge [z \in v]\}. \quad \text{(A5.36)}$$

4) *The axiom of the intersection of the class-members of a class.*

$$\bigvee_w \bigwedge_y [[y \in w] \Leftrightarrow \bigwedge_x [[x \in u] \wedge [y \in x]]]. \quad \text{(A5.37)}$$

Consequently,

$$\begin{gathered}\bigvee_w [[w = \{z| \bigwedge_x [[x \in u] \wedge [z \in x]]\}] \\ \rightarrow \bigwedge_y [[y \in w] \Leftrightarrow \bigwedge_x [[x \in u] \wedge [y \in x]]]]\end{gathered} \quad \text{(A5.38)}$$

is a contextual *definition* of the logographic term '$\{z| \bigwedge_x [[x \in u] \wedge [z \in x]]\}$' of $\mathsf{C}_1^{\mathsf{OI}}$, which is abbreviated further and rendered into ordinary language as follows:

$$[\text{The intersection of } \textit{the class-members of a class } u]$$
$$\rightarrow [\bigcap u] \leftrightarrow \bigcap_{x \in u} x \rightarrow \{z| \bigwedge_x [[x \in u] \wedge [z \in x]]\}. \quad \text{(A5.39)}$$

Note. The relation operatum $[[y \in w] \Leftrightarrow \bigwedge_x [[x \in u] \wedge [y \in x]]]$, occurring in (A5.37) does not have the form of the operatum of the axiom of comprehension (A5.29). Therefore, in contrast to the relation (A5.34), which underlies the definition (A5.36), I regard the relation (A5.37) as an axiom.

5) *The definition of the difference between two classes of classes.*

[The difference between the classes *u* and *v*]

↔[The complement of the class *v* in the class *u*]

$$\to [u - v] \to \{x | [x \in u] \wedge \neg [x \in v]\}. \quad \text{(A5.40)}$$

6) *The axiom of the union of two classes of classes.*

$$\bigvee_w \bigwedge_x [[x \in w] \Leftrightarrow [[x \in u] \vee [x \in v]]]. \quad \text{(A5.41)}$$

Consequently,

$$\bigvee_w [[w = \{z | [z \in u] \vee [z \in v]\}] \to \bigwedge_x [[x \in w] \Leftrightarrow [[x \in u] \vee [x \in v]]]] \quad \text{(A5.42)}$$

is a contextual *definition* of the logographic term '$\{z | [z \in u] \vee [z \in v]\}$' of $\mathsf{C}_1^{\mathsf{OI}}$, which is abbreviated further and rendered into ordinary language as follows:

[The union of classes *u* and *v*] $\to [u \cup v] \to \{z | [z \in u] \vee [z \in v]\}$]. (A5.43)

7) *The axiom of the union of the class-members of a class.*

$$\bigvee_w \bigwedge_y [[y \in w] \Leftrightarrow \bigvee_x [[x \in u] \vee [y \in x]]]. \quad \text{(A5.44)}$$

Consequently,

$$\bigvee_w [[w = \{z | \bigvee_x [[x \in u] \wedge [z \in x]]\}] \to \bigwedge_y [[y \in w] \Leftrightarrow \bigvee_x [[x \in u] \wedge [y \in x]]]] \quad \text{(A5.45)}$$

is a contextual *definition* of the logographic term '$\{z | \bigvee_x [[x \in u] \wedge [z \in x]]\}$' of $\mathsf{C}_1^{\mathsf{OI}}$, which is abbreviated further and rendered into ordinary language as follows:

[The union of *the class-members of a class u*]

$$\to [\bigcup u] \leftrightarrow \bigcup_{x \in u} x \to \{z | \bigvee_x [[x \in u] \wedge [z \in x]]\}. \quad \text{(A5.46)}$$

8) *The definitions of singletons and unordered multiples of classes.*

[The singleton of a class x_1] $\to \{x_1\} \to \{x_1, x_1\}$. (A5.47)

[The unordered triple of classes x_1, x_2, x_3]

→[The class of classes x_1, x_2, x_3]

$$\to \{x_1, x_2, x_3\} \to \{x_1, x_2\} \cup \{x_3\}. \quad \text{(A5.48)}$$

[The unordered *n*-tuple of classes $x_1, x_2, ..., x_n$]

$\rightarrow$[The class of classes $x_1, x_2, ..., x_n$]

$$\rightarrow \{x_1, x_2, ..., x_n\} \rightarrow \{x_1, x_2, ..., x_{n-1}\} \cup \{x_n\}. \tag{A5.49}$$

In contrast to an abstract class-builder '$\{\,|\,\}$', being an abstraction operator that produces a unique class from a given relation (see the item 1 of this Summary), the operator of aggregation

$$\underbrace{\{-,-,...,-\}}_{n}$$

produces a unique class from *n* given classes as its elements, and it is therefore called a *concrete class-builder*.

9) *The definition of ordered pairs of classes of classes*.

[The ordered pair of classes x_1 and x_2]

$\rightarrow$[The ordered pair of classes

with first coordinate x_1 and second coordinate x_2]

$$\rightarrow (x_1, x_2) \leftrightarrow \langle x_1, x_2 \rangle \rightarrow \{\{x_1\}, \{x_1, x_2\}\}. \tag{A5.50}$$

Note. If x_1 and x_2 are real numbers then the logograph '(x_1, x_2)' is ambiguous, for it may stand, not only for the ordered pair x_1 and x_2, but also for the open interval (x_1, x_2). Whenever there is a danger of confusion, I shall denote an ordered pair with the help of angle brackets.

By (A5.13) subject to (A5.6), it follows from (A5.50) as a *theorem* that

$$[(x_1, x_2) = (y_1, y_2)] \Leftrightarrow [[x_1 = y_1] \wedge [x_2 = y_2]]. \tag{A5.51}$$

10) *The definitions of ordered singles and ordered multiples of classes.*

$$\text{[The ordered single of a class } x_1\text{]} \rightarrow \{\{x_1\}\}. \tag{A5.52}$$

[The ordered *n*-tuple of classes $x_1, x_2, ..., x_n$]

$$\rightarrow \bar{x}_{[1,n]} \rightarrow (x_1, x_2, ..., x_n) \rightarrow (((...((x_1, x_2), x_3), ...), x_{n-1}), x_n). \tag{A5.53}$$

11) *The definitions of direct product of classes of classes.*

[The direct product of the classes u_1 and u_2]

$$\rightarrow [u_1 \times u_2] \rightarrow \{(x_1, x_2) | [x_1 \in u_1] \wedge [x_2 \in u_2]\}. \tag{A5.54}$$

[The direct product of the classes $u_1, u_2, ..., u_n$]

$$\to [u_1 \times u_2 \times ... \times u_n] \to [[[...[[u_1 \times u_2] \times u_3] \times ...] \times u_{n-1}] \times u_n]$$
$$\to \{(((...((x_1, x_2), x_3), ...), x_{n-1}), x_n) |$$
$$[[...[[[x_1 \in u_1] \wedge [x_2 \in u_2]] \wedge [x_3 \in u_3]] \wedge ...] \wedge [x_{n-1} \in u_{n-1}]] \wedge [x_n \in u_n]]\} \quad \text{(A5.55)}$$
$$\leftarrow \{(x_1, x_2, ..., x_n) | [x_1 \in u_1] \wedge [x_2 \in u_2] \wedge ... \wedge [x_n \in u_n]\}.$$

[The repeated (n–1)-fold direct product of the class u by itself]

$$\to u^{n\times} \to \underbrace{[[...[[u \times u] \times u] \times ...] \times u]}_{n}. \quad \text{(A5.56)}$$

12) *The first axiom of infinity for classes.* There exists a *unique universal class* $\omega_\varnothing$, denoted also by 'ω_0', such that

$$[\varnothing \in \omega_\varnothing] \wedge \bigwedge\nolimits_x [[[x \in \omega_\varnothing] \Rightarrow [[x \cup \{x\}] \in \omega_\varnothing]] \wedge [x \in \omega_\varnothing]]. \quad \text{(A5.57)}$$

Note. An implication ***P***⇒***Q*** has the truth value truth if the antecedent ***P*** is antitrue (false) Therefore, in (A5.57) and in similar relations in the sequel, I employ the pertinent *modus ponendo ponens* [***P***⇒***Q***]∧***P***, and not just its major premise ***P***⇒***Q*** as commonly done in stating the various axioms of infinity in the literature on set theory (see, e.g., Fraenkel et al [1973, pp. 46–49]. In this way, I may assert uniqueness of $\omega_\varnothing$ and of any analogous infinite recursive universal class from the very beginning and to avoid applying the *imaginary* and actually *circular* procedure of intersecting an indefinite number of undefined congeneric infinite recursive classes for obtaining a unique one (see, e.g., *ibid*, p. 47).

The class $[x \cup \{x\}]$ is called the *immediate successor* of the class x in the class $\omega_\varnothing$, whereas x is called the *immediate predecessor* of $[x \cup \{x\}]$ in the class $\omega_\varnothing$.

13) *The second axiom of infinity for classes.* There exists a *unique universal class* $\omega'_\varnothing$, denoted also by 'ω'_0', such that

$$[\varnothing \in \omega'_\varnothing] \wedge \bigwedge\nolimits_x [[[x \in \omega'_\varnothing] \Rightarrow [\{x\} \in \omega'_\varnothing]] \wedge [x \in \omega'_\varnothing]]. \quad \text{(A5.58)}$$

The singleton $\{x\}$ is called the *immediate successor* of the class x in the class $\omega'_\varnothing$, whereas x is called the *immediate predecessor* of $\{x\}$ in the class $\omega'_\varnothing$.

Note. It is possible to state an indefinite number of various axioms of infinity for classes, of which (A5.57) and (A5.58) are just most common ones due to Neumann [1923] and Zermello [1908] respectively.•

Cmt A5.1. 1) At x=v=∅, (A5.25), (A5.27), and (A5.28) become ¬[∅∈∅], ∅⊆∅, and ¬[∅⊂∅] respectively. Consequently, (A5.13) at u=v=∅ yields ∅=∅.

2) By (A5.40), it follows from the variant of (A5.36) with '[u-v]' and 'u' in place of 'u' and 'v' respectively that

a) [u-v]∩v=∅,

b) u-v=∅ if [u∩v]=∅,

c) [u-v]⊆u and [u-v]∪v=u if v⊆u.•

Cmt A5.2. $\mathsf{C}_1^{\mathsf{OI}}$ can be developed further into a *one-individual nominalistic axiomatic set theory* (*OINAST*) $\mathsf{S}_1^{\mathsf{OI}}$ as indicated in Cmt 9.1(3). In this case, every occurrence of the noun "class" throughout $\mathsf{C}_1^{\mathsf{OI}}$ can be replaced with an occurrence of the noun "set".•

A5.3. Foundations of M_1

Df A5.3: ***The interpretation of*** ⊆ in M_1.

$$[u \text{ is a submass (mass-part) of } v]\to[u\subseteq v]. \qquad \text{(A5.59)}\bullet$$

Summary A5.3: ***Basic tautologous catlogographic relations (axioms, theorems, and definitions) of*** M_1. The basic tautologous *catlogographic relations* (*CLR's*), which are given below in this summary and which are discriminately called *axioms*, *theorems*, or *definitions*, are the CFCL interpretands of the pertinent *axioms* and *theorems*, i.e. *valid* (*kyrologous*) *euautographic relations* (*ER's*) *of* $\mathsf{A}_{1\subseteq}$, and of the pertinent *definitions of* $\mathsf{A}_{1\subseteq}$, respectively.

1) *The axioms for* ⊆.

$$u \subseteq u\,. \qquad (\textit{The reflexivity law}) \quad \text{(A5.60)}$$

$$[u \subseteq v]\wedge[v \subseteq w]\Rightarrow[u \subseteq w]. \qquad (\textit{The transitivity law}) \quad \text{(A5.61)}$$

$$\vee_u \neg[u \subseteq v]. \qquad (\textit{The incidence law for antiinclusions}) \quad \text{(A5.62)}$$

2) *The incidence theorems* (*laws*) *for inclusions.*

$$\vee_u [u \subseteq v]. \qquad (\textit{The first incidence law for} \subseteq) \quad \text{(A5.63)}$$

$\bigvee_v [u \subseteq v]$. (*The second incidence law for* $\subseteq$) (A5.64)

3) *The definition* (*law*) *of extensionality for masses:*

$$[u = v] \to [[u \subseteq v] \wedge [v \subseteq u]]. \qquad \text{(A5.65)}$$

4) *Basic theorems* (*laws*) *of equality for masses.*

$u = u$. (*The reflexivity law*) (A5.66)

$[u = v] \Leftrightarrow [v = u]$. (*The symmetry law*) (A5.67)

$[u = v] \wedge [v = w] \Rightarrow [u = w]$. (*The transitivity law*) (A5.68)

$\bigvee_u \neg[u = v]$. (*The incidence law for antiequalities*) (A5.69)

$\bigvee_u [u = v]$. (*The incidence law for equalities*) (A5.70)

5) *The definition of* $\subset$: *the first law of strict submass.*

$$[u \text{ is a strict subclass of } v] \to [u \subset v] \to [[u \subseteq v] \wedge \neg[v \subseteq u]]. \qquad \text{(A5.71)}$$

6) *Basic theorems for* $\subset$.

$[u \subset v] \Leftrightarrow [[u \subseteq v] \wedge \neg[u = v]]$.

(*The second law of a strict subclass*) (A5.72)

$\neg[u \subset u]$. (*The antireflexivity law for* $\subset$) (A5.73)

$[u \subset v] \wedge [v \subset w] \Rightarrow [u \subset w]$. (*The transitivity law*) (A5.74)

$\bigvee_u \neg[u \subset v]$. (*The first incidence law for* $\overline{\subset}$) (A5.75)

$\bigvee_v \neg[u \subset v]$. (*The second incidence law for* $\overline{\subset}$) (A5.76)

7) *The axiom* (*primary law*) *of an empty mass*:

$$\varnothing \subseteq v. \qquad \text{(A5.77)}$$

8) *The theorem* (*law*) *of uniqueness of the empty mass*:

$$[[\varnothing \subseteq u] \wedge [\varnothing' \subseteq u']] \Leftrightarrow [\varnothing = \varnothing']. \qquad \text{(A5.78)}$$

9) *The theorem* (*secondary law*) *of the empty mass*:

$$\neg[v \subset \varnothing]. \qquad \text{(A5.79)}$$

Just as the CLR's given in Summary A5.1, the CLR's, which are classified above as axioms or as theorems are *tautologous* ones, for they are CFCL interpreteands of the respective analographic *valid* euautographic relations, i.e. *euautographic axioms* or *euautographic theorems* respectively, of $\mathsf{A}_{1\subseteq}$; the euautographic theorems are established to be so by the appropriate EADP's (euautographic algebraic decision procedures).•

Summary A5.4: ***Basic veracious catlogographic relations (axioms, theorems, and definitions) of*** M_1. The basic *veracious catlogographic relations* (*CLR's*), which are given below in this summary and which are discriminately called *axioms*, *theorems*, or *definitions*, are *pure semantic true*, *but not tautologically true*, *relations*, which are irrelevant to any euautographic relations (ER's) and to any definitions of $\mathsf{A}_{1\subseteq}$.

1) *The axiom* [*schema*] *of comprehension* (or *of specification* or *of separation*) or (in German) *Aussonderungaxiom* (*axiom of selection*), *for masses*. Given a condition $P_{\mathrm{m}}\langle x\rangle$ that is imposed on the masses ("m") of the range of the MVV '*x*', to every mass *u* there is a mass *v* whose parts (submasses) of *u*, for which $P_{\mathrm{m}}\langle x\rangle$ is *veracious*, i.e. logographically

$$\bigwedge_u \bigvee_v \bigwedge_x \left[\left[x \subseteq v\right] \Leftrightarrow \left[\left[x \subseteq u\right] \wedge P_{\mathrm{m}}\langle x\rangle\right]\right], \tag{A5.80}$$

where '$\wedge$' and '$\Leftrightarrow$' are read as "and" and "if and only if" respectively, while '$\bigwedge_u$' and '$\bigvee_v$' (e.g.) are qualifiers, which are read as "for every *u*:" (but not "for all *u*:") and as "for some *v*:" (but neither as "for at least one *v*:" nor "there exists at least one *v* such that") respectively. Consequently,

$$\begin{aligned}&\bigwedge_u \bigvee_v \left[\left[v = \left\{z \middle| \left[z \subseteq u\right] \wedge P_{\mathrm{m}}\langle z\rangle\right\}\right]\right.\\ &\left.\to \bigwedge_x \left[\left[x \subseteq v\right] \Leftrightarrow \left[\left[x \subseteq u\right] \wedge P_{\mathrm{m}}\langle x\rangle\right]\right]\right],\end{aligned} \tag{A5.81}$$

is a contextual *definition* of the logographic term '$\left\{z \middle| \left[z \subseteq u\right] \wedge P_{\mathrm{m}}\langle z\rangle\right\}$' of M_1, which is read as: "the mass of all *z* such that $z \subseteq u$ and $P_{\mathrm{m}}\langle z\rangle$". The operator '$\{\,|\,\}$' thus defined is called an *abstraction operator from a relation* (as $\left[\boldsymbol{z} \subseteq \boldsymbol{u}\right] \wedge \boldsymbol{P}_{\mathrm{m}}\langle \boldsymbol{z}\rangle$) *to a mass* (as $\left\{z \middle| \left[z \subseteq u\right] \wedge P_{\mathrm{m}}\langle z\rangle\right\}$) and also a *mass-builder*. Like '$\left\{z \middle| \left[z \in u\right] \wedge P_{\mathrm{e}}\langle z\rangle\right\}$' (see Df A5.3(1)), the logographic term '$\left\{z \middle| \left[z \subseteq u\right] \wedge P_{\mathrm{m}}\langle z\rangle\right\}$' can be regarded as a logographic *description* through the *mass-genus* (*generic mass*) *u* and the *differentia* (*difference*) P_{m} of the *mass-species* (*specific mass*) $\left\{z \middle| \left[z \subseteq u\right] \wedge P_{\mathrm{m}}\langle z\rangle\right\}$, which it denotes.

2) *The definition of the intersection of two masses.*

[The intersection of masses *u* and *v*]

$$\to[u\cap v]\to\{z|[z\subseteq u]\wedge[z\subseteq v]\}. \tag{A5.82}$$

3) *The definition of the difference between two masses.*

[The difference between the masses *u* and *v*]

↔[The complement of the mass *v* in the mass *u*]

$$\to[u - v]\to[u\cap v]\to\{x|[x\subseteq u]\wedge\neg[x\subseteq v]\}. \tag{A5.83}$$

4) *The axiom of union of two masses.*

$$\bigvee_w\bigwedge_x[[x\subseteq w]\Leftrightarrow[[x\subseteq u]\vee[x\subseteq u]]]. \tag{A5.84}$$

Consequently,

$$\begin{gathered}\bigvee_w[[w=\{z|[z\subseteq u]\vee[z\subseteq v]\}]\\ \to\bigwedge_x[[x\subseteq w]\Leftrightarrow[[x\subseteq u]\vee[x\subseteq v]]]]\end{gathered} \tag{A5.85}$$

is a contextual *definition* of the logographic term '$\{z|[z\subseteq u]\vee[z\subseteq v]\}$' of M_1, which is abbreviated further and rendered into ordinary language as follows:

[The union of masses *u* and *v*]

$$\to[u\cup v]\to\{z|[z\subseteq u]\vee[z\subseteq v]\}]. \tag{A5.86}$$•

Cmt A5.3. 1) At *v*=∅, (A5.77) and (A5.79) become ∅⊆∅ and ¬[∅⊂∅] respectively. Consequently, (A5.50) at *u*=*v*=∅ yields ∅=∅ (cf. the item 1 of Cmt A5.1).

2) By (A5.83), it follows from the variant of (A5.82) with '[*u*-*v*]' and '*u*' in place of '*u*' and '*v*' respectively that relations a)–c) of the item 2 of Cmt A5.1 retain.•

A5.4. Foundations of $\mathsf{C}_1^{\mathsf{MI}}$

Df A5.4: *Nomenclature of* $\mathsf{C}_1^{\mathsf{MI}}$. 1) In order to develop a *many-individual nominalistic axiomatic class theory* (*MINACT*), denoted by '$\mathsf{C}_1^{\mathsf{MI}}$', three sets of *element-valued variables* of different kinds are needed, – for instance, the following three, of which the first one is that mentioned above.

a) Each of the six light-faced italic minuscule (small) letters of the current type '*u*', '*v*', '*w*', '*x*', '*y*', and '*z*', i.e. a token of the respective AVCLOT, is a catlogographic CVV, i.e. a catlogographic variable that can denote (assume as its accidental denotatum) a *nonempty class* or the *empty class*, i.e. the *empty individual*, denoted by '∅', but which *cannot denote any nonempty individual.*

b) Each of the three *dotted* light-faced italic minuscule (small) letters of the current type '$\dot{u}$', '$\dot{v}$', and '$\dot{w}$' is a catlogographic variable whose range is *the class of all nonempty individuals of* $\mathsf{C}_1^{\mathsf{MI}}$.

c) Each of the three Greek letters '*ξ*', '*η*', and '*ζ*' is a catlogographic *element-valued variable* (*EVV*), i.e. a variable that that can denote any *element*, namely either a *nonempty class* or the *empty class* (*empty individual*) or else a *nonempty individual* (*NEI*).

d) Each of the three Greek letters '*α*', '*β*', and '*γ*' is a catlogographic *nonempty-individual-valued variable* (*NEIVV*), i.e. a variable that that can denote only an NEI.

Any of the above twelve letters can, when necessary, be furnished either with any of the upright Arabic numeral subscripts '$_1$', '$_2$', etc or with any number of primes or with both labels simultaneously, thus becoming another catlogographic variable with same range and hence of the same nomenclature. All variables having the same range are called *congeneric variables* or synecdochically *congeners*.

2) At any place, $\mathsf{C}_1^{\mathsf{MI}}$ can, like $\mathsf{C}_1^{\mathsf{OI}}$, be augmented by one or more *class-valued constants* (*CVC's*), accidental (circumstantial, ad hoc) ones or essential ones, each of which denotes a certain *universal class,* say *U* generally or $\omega_\varnothing$, $\omega'_\varnothing$, ω_α, or ω'_α (at a given *α*), that cannot be a member of any particular class and that satisfies the three semantic postulates, accidental ones (hypotheses) or essential ones (axioms), $\neg[U\in U]$, $u\in U$, and $\neg[U\in u]$, and similarly with any congeneric CVV in place of '*u*'; the above postulates are the CFCL interpretands of the respective euautographic axioms $\neg[\mathsf{U}\in\mathsf{U}]$, $\mathsf{u}\in\mathsf{U}$, and $\neg[\mathsf{U}\in\mathsf{u}]$ of $\bar{\mathsf{A}}_{1\in}$ (see Cmt 7.6(2)).

3) In accordance with the pertinent nomenclature of $\mathsf{A}_{1\in}$ (see also item 5 of subsection 9.2 and Df A5.1(2)), '$\bigvee_u$' and '$\bigwedge_u$', e.g., are synonyms of the conventional *quantifiers* '(∃*u*)' and '(∀*u*)', which are read as "*for some u*:" or "*for at least one u*:" or "*there exists at least one u such that*" and as "*for all u*:" or "*for every u*:" respectively. Similar nomenclature applies with any congener of '*u*' and also with

‘ξ’ or ‘α’ and with their congeners in place of ‘*u*’, the understanding being that any quantification of a CVV, EVV, or NEIVV *is effective in the range of that variable*.

4) The logographic logical connectives and their verbal (phonographic) synonyms, which are employed in $\mathsf{C}_1^{\mathsf{MI}}$, are given in the item 6 of subsection 9.2 and also in the item 4 of Df A5.1, i.e. they are the same as those employed in $\mathsf{C}_1^{\mathsf{OI}}$ and M_1.•

Df A5.5: ***The interpretation of*** $\in$ in $\mathsf{C}_1^{\mathsf{MI}}$. In order to pass from $\mathsf{C}_1^{\mathsf{OI}}$ to $\mathsf{C}_1^{\mathsf{MI}}$, the interpretation of $\in$ that given by definition (A5.1) should be *generalized* (*extended*) as an *interpretation* for *element-members* of the class, i.e. for both *class-members* and *NEI-members* simultaneously, e.g., thus:

$$[\xi \text{ is an element-member, or element, of the class } u]\to[\xi\in u], \qquad (\text{A5.1}_{\text{e}})$$

in accordance with the items 1a and 1c of Df A5.4. This interpretation can then pe *particularized* (*restricted*), e.g., thus:

$$[\alpha \text{ is an NEI-member of the class } \dot{u}\,]\to[\alpha\in \dot{u}\,], \qquad (\text{A5.1}_{\text{i}})$$

in accordance with the items 1b and 1d of Df A5.4. Here, and generally in what follows, the numeral bookmark (logical name) of a relation is provided with the subscript ‘$_{\text{e}}$’ (being the first letter of the word “element”) if the relation involves EVV’s and with the subscript ‘$_{\text{i}}$’ (being the first letter of the word “individual”) if the relation involves NEIVV’s. In this case, the CVV ‘*u*’ occurring in (A5.1_{e}), and generally in what follows, is freed of its initial range and it *connotes* (*designates*) *the class of elements*, i.e. *the class of both classes and NEI’s*.•

Summary A5.5: ***Incorporation of nonempty individuals into tautologous CLR’s of Summary A5.1.*** 1) When I indicate in Summary A5.1 that a certain relation of $\mathsf{C}_1^{\mathsf{OI}}$ is an axiom, theorem, or definition, I mean that that relation is the CFCL interpretand of the respective euautographic relation of $\mathsf{A}_{1\in}$. That is to say, in accordance with Ax 8.1(2), the euautographic interptetantia of the catlogographic relations (A5.2)–(A5.28) are analographic alphabetic variants of the latter with u, v, w, x, y, $\mathsf{\emptyset}$, $\mathsf{\emptyset}'$ in place of *u*, *v*, *w*, *x*, *y*, ∅, ∅′ respectively.

2) Particularly, the tautologous catlogographic axioms (A5.2), (A5.3), and (A5.25) are the CFCL interpretands of the euautographic axioms of $\mathsf{A}_{1\in}$:

$$\neg[[\mathsf{x}\in\mathsf{y}]\wedge[\mathsf{y}\in\mathsf{x}]], \qquad (\text{A5.2}_0)$$

$$\bigvee_u [x \in u], \tag{A5.3$_0$}$$

$$\neg[x \in \emptyset]. \tag{A5.25$_0$}$$

so that

$$V(\neg[[x \in y] \wedge [y \in x]]) \hat{=} 1 \hat{-} V([x \in y] \wedge [y \in x])$$
$$\hat{=} V(\neg[x \in y]) \hat{\cdot} V(\neg[y \in x]) \hat{=} [1 \hat{-} V(x \in y)] \hat{\cdot} [1 \hat{-} V(y \in x)] \hat{=} 0, \tag{A5.2$_1$}$$

$$V(\bigvee_u [x \in u]) \hat{=} \hat{\prod}_u V(x \in u) \hat{=} 0, \tag{A5.3$_1$}$$

$$V(\neg[x \in \emptyset]) \hat{=} 1 \hat{-} V(x \in \emptyset) \hat{=} 0, \tag{A5.25$_1$}$$

which are zeroed because the ER's (A5.2$_0$) and (A5.3$_0$) are axioms. The instance of (A5.2$_1$) with x in place of y becomes

$$V(\neg[x \in x]) \hat{=} 1 \hat{-} V(x \in x) \hat{=} 0. \tag{A5.4$_1$}$$

Hence,

$$V(\bigvee_u \neg[x \in u]) \hat{=} \hat{\prod}_u V(\neg[x \in u])$$
$$\hat{=} V(\neg[x \in x]) \hat{\cdot} \hat{\prod}_u V(\neg[x \in u]) \hat{=} 0 \hat{\cdot} \hat{\prod}_u V(\neg[x \in u]) \hat{=} 0, \tag{A5.5$_1$}$$

where use of the so-called *emission and absorption law* **(***EAL***)** or *advanced idempotent law 1* **(***AIL1***)** has been made. It immediately follows from EADP's (A5.4$_1$) and (A5.5$_1$) that

$$\neg[x \in x], \tag{A5.4$_0$}$$

$$\bigvee_u \neg[x \in u], \tag{A5.5$_0$}$$

which are the euautographic intrpretantia (anti-intrpretands) of the tautologous catlogographic theorems (A5.4) and (A5.5).

3) At the same time, none of the NEIVV's that are introduced in Df A5.5(1d) and hence none of the EVV's that are introduced in Df A5.5(1c) can stand to *the right of [a token of] the sign* $\in$. Therefore, the tautologous catlogographic theorem (A5.4) just retains in C_1^{MI}, because this can neither be *extended for NEI's* as '$\neg[\alpha \in \alpha]$' nor be *generalized for elements*, i.e. *for classes and NRI's simultaneously*, as '$\neg[\xi \in \xi]$'. The *tautologous* catlogographic theorem (A5.5) also holds *for classes only*, so that its *variant* (*substituend*)

$$\bigvee_\alpha \neg[\alpha \in u] \tag{A5.5$_i$}$$

(e.g), which is obtained by substitution of the NEIVV 'α', introduced in Df A5.5(1d), for 'x', introduced in Df A5.5(1a), is irrelevant to C_1^{OI} and hence it is *not a* *tautologous catlogographic theorem*. However, (A5.5$_i$) can be and *is* taken for granted as a *veracious catlogographic axiom of* $\mathrm{C}_1^{\mathrm{MI}}$. The *tautologous catlogographic theorem (A5.5) for classes* and the *veracious catlogographic axiom (A5.5$_i$), being its extension* (*continuation*) *for NEI's*, can then be generalized as a *single true catlogographic theorem* (*corollary*) *for elements*, i.e. for both *classes* and *NEI's*:

$$\bigvee_{\xi}\neg[\xi \in u] \tag{A5.5$_e$}$$

(e.g.), which is obtained by substitution of the EVV 'ξ', introduced in Df A5.5(1c), for '*x*' in (A5.5) or for '*α*' in (A5.5$_i$).

4) Likewise, the *tautologous catlogographic axioms* (A5.3) and (A5.25) can be extended for NEI's as the *veracious catlogographic axioms*:

$$\bigvee_{u}[\alpha \in u], \tag{A5.3$_i$}$$

$$\neg[\alpha \in \varnothing] \tag{A5.25$_i$}$$

and be then generalized for elements, i.e. for both classes and NEI's, as single *true catlogographic axiom*s:

$$\bigvee_{u}[\xi \in u]. \tag{A5.3$_e$}$$

$$\neg[\xi \in \varnothing]. \tag{A5.25$_e$}$$

5) Under the definition (A5.1$_e$), the definition (A5.6) is generalized thus:

$$[u \text{ is a subclass of } v] \to [u \subseteq v] \to \bigwedge_{\xi}[[\xi \in u] \Rightarrow [\xi \in v]], \tag{A5.6$_e$}$$

whereas the pertinent generalization of the definition (A5.19) is just the pertinent *homograph* of the latter.

6) Like (A5.5), the tautologous catlogographic theorems (A5.11) and (A5.12) are the CFCL interpretands of the euautographic theorems:

$$\bigvee_{u}\neg[u \subseteq v], \tag{A5.11$_0$}$$

$$[\bigvee_{v}\neg[u \subseteq v]] \Leftrightarrow [\bigvee_{x}[x \in u]]. \tag{A5.12$_0$}$$

These are proved with the help of the appropriate EADP's, which result in the EDT's (euautographic decision theorems):

$$V(\bigvee_{u}\neg[u \subseteq v]) \hat{=} \curlywedge_{u} V(\neg[u \subseteq v]) \hat{=} 0, \tag{A5.11$_1$}$$

$$V(\bigvee_{v}\neg[u \subseteq v]) \hat{=} \curlywedge_{v} V(\neg[u \subseteq v]) \hat{=} \curlywedge_{x} V(x \in u) \hat{=} V(\bigvee_{x}[x \in u]), \tag{A5.12$_1$}$$

respectively. Just as in EADP (A5.5$_1$), in developing (A5.11$_1$) or (A5.12$_1$), use of the relation (A5.4$_1$) or of its appropriate alphabetic variant has been made. Therefore, if the CVV ‘*u*’, now freed of its initial range, connotes the class of elements, i.e. of both classes and NEI’s, then for the reason indicated above in the item 3 both the relation (A5.11) and the relation:

$$\lfloor \bigvee_{v} \neg[u \subseteq v] \rfloor \Leftrightarrow \lfloor \bigvee_{\xi} [\xi \in u] \rfloor, \qquad (A5.12_e)$$

being the pertinent variant of the relation (A5.12), are *not tautologous catlogographic theorems*. However, both the above *homograph* of relation (A5.11) and the relation ($A5.12_e$) can be and *are* taken for granted as a *veracious catlogographic axioms of* C_1^{MI}.

7) In proving the euautographic theorems being interpretantia of all other *tautologous catlogographic theorems* given in Summary A5.1, namely (A5.7)–(A5.10), (A5.13)–(A5.18), (A5.20)–(A5.24), and (A5.26)–(A5.28), neither the relation ($A5.4_1$) nor any of its alphabetic variants is utilized. However, in order to incorporate the above-mentioned tautologous catlogographic theorems into C_1^{MI}, the CVV’s ‘*u*’ and ‘*v*’, occurring in those theorems, should be freed of their initial ranges and be assigned to the new ranges that comprise elements, i.e. both the classes and the NEI’s, – as has already been done in the catlogographic definitions and in the other pertinent tautologous catlogograpic relations (axioms and theorems). That is to say, the former CVV’s should be replaced with their *homographs* of the above broader ranges. The catlogographic relations thus obtained are not tautologous, because they have no proofs in $\mathsf{A}_{1\in}$. Therefore, in order to utilize these relations as groundwork of C_1^{MI}, they are taken for granted as *veracious catlogographic axioms of* C_1^{MI} – just as (A5.11).

8) In addition, the occurrences of the CVV’s ‘*u*’ and ‘*v*’ throughout the item 6 of Summary A5.1 should be replaced with occurrences of the EVV’s ‘ξ’ and ‘η’ (e.g.) respectively and the resulting relations are taken for granted as the following *basic veracious axioms* (*laws*) *of equality for elements*:

$$\xi = \xi. \qquad (\textit{The reflexivity law}) \quad (A5.14_e)$$

$$[\xi = \eta] \Leftrightarrow [\eta = \xi]. \qquad (\textit{The symmetry law}) \quad (A5.15_e)$$

$$[\xi = \eta] \wedge [\eta = \zeta] \Rightarrow [\xi = \zeta]. \qquad (\textit{The transitivity law}) \quad (A5.16_e)$$

$$\bigvee_{\xi} \neg[\xi = \eta]. \qquad (\textit{The incidence law for antiequalities}) \quad (A5.17_e)$$

$$\vee_{\xi}[\xi = \eta].\qquad\qquad (\textit{The incidence law for equalities})\quad (A5.18_e)$$

In connection with use of the sign = in the above axioms, the following remarks will be in order.

9) As has already been indicated in the item 3 of this summary, none of the NEIVV's that are introduced in Df A5.5(1d) and hence none of the EVV's that are introduced in Df A5.5(1c) can stand to *the right of [a token of] the sign* ∈. That is to say, the string such as

$$\text{'}\alpha\in\alpha\text{', '}\neg[\alpha\in\alpha]\text{', '}\alpha\in\beta\text{', '}\neg[\alpha\in\beta]\text{', '}\alpha\in\xi\text{', '}\neg[\alpha\in\xi]\text{',}$$
$$\text{'}\xi\in\alpha\text{', '}\neg\,[\xi\in\alpha]\text{', '}\xi\in\eta\text{', '}\neg[\xi\in\eta]\text{'}\qquad (A5.87)$$

are *inadmissible* – just as inadmissible are the sentences: "A centaur is a mammal" and "A centaur is not a mammal" in the domain of biology. Consequently, the strings

$$\text{'}\alpha\subseteq\alpha\text{', '}\neg[\alpha\subseteq\alpha]\text{', '}\alpha\subseteq\beta\text{', '}\neg[\alpha\subseteq\beta]\text{', '}\alpha\subseteq\xi\text{', '}\neg[\alpha\subseteq\xi]\text{',}$$
$$\text{'}\xi\subseteq\alpha\text{', '}\neg\,[\xi\subseteq\alpha]\text{', '}\xi\subseteq\eta\text{', '}\neg[\xi\subseteq\eta]\text{',}\qquad (A5.88)$$

being the variants of the above strings with '⊆' in place of '∈', are also inadmissible owing to the definition (A5.6), whereas the strings

$$\text{'}\alpha\subset\alpha\text{', '}\neg[\alpha\subset\alpha]\text{', '}\alpha\subset\beta\text{', '}\neg[\alpha\subset\beta]\text{', '}\alpha\subset\xi\text{', '}\neg[\alpha\subset\xi]\text{',}$$
$$\text{'}\xi\subset\alpha\text{', '}\neg\,[\xi\subset\alpha]\text{', '}\xi\subset\eta\text{', '}\neg[\xi\subset\eta]\text{',}\qquad (A5.89)$$

being the variants of the strings (A5.87) or (A5.88) with '⊂' in place of '∈' or '⊆' respectively, are inadmissible, owing to the definition (A5.19). At first glance, it seems that the strings

$$\text{'}\alpha=\alpha\text{', '}\neg[\alpha=\alpha]\text{', '}\alpha=\beta\text{', '}\neg[\alpha=\beta]\text{', '}\alpha=\xi\text{', '}\neg[\alpha=\xi]\text{',}$$
$$\text{'}\xi=\alpha\text{', '}\neg\,[\xi=\alpha]\text{', '}\xi=\eta\text{', '}\neg[\xi=\eta]\text{',}\qquad (A5.90)$$

which are the variants of the strings (A5.87), (A5.88), or (A5.89) with '=' in place of '∈', '⊆', or '⊂' respectively, should be disregarded as well owing to the definition (A5.13). However, this is not the case for the following reason. I have already indicated above in this Summary, particularly in the item 2, that the tautologous catlogographic axioms and theorems as given in Summary A5.1 are the CFCL interpretands of certain euautographic axioms and theorems of $\mathsf{A}_{1\in}$. In this case, in accordance with the items 3–5 of Df 7.1, the equality signs that are introduced in the organons $\mathsf{A}_{1=}$, $\mathsf{A}_{1\subseteq}$, and $\mathsf{A}_{1\in}$ are three different *homographs*. Accordingly, the sign = occurring in the first four strings on the list (A5.90) can be regarded as that introduced in $\mathsf{A}_{1=}$, whereas the sign = occurring in the remaining six strings on that list and also occurring in the relations (A5.14$_e$)–(A5.18$_e$) can be regarded as a generalization (synthesis) of the two homographic signs = introduced in $\mathsf{A}_{1=}$, and in $\mathsf{A}_{1\in}$ respectively.•

Summary A5.5: ***Incorporation of nonempty individuals into the veracious CLR's of Summary A5.2.*** All CLR's indicted in Summary A5.2 are *veracious*, i.e. *true but not tautologically true*. That is to say, they have, either syntactically or semantically, no direct relevance to any EADP's of ER's and to any definitions of $\mathsf{A}_{1\in}$. Consequently, in the exclusion of the CLR's that are given in the items 4, 7, 12, and 13 of Summary A5.2, all other veracious CLR's that are given in the items 1–3, 5, 6, and 8–11 of Summary A5.2 and that will ad hoc be referred to as *the selected ones* can be and *are* straightforwardly restated with allowance for Dfs A5.4 and A5.5 and Summary A5.4 so as to become *veracious CLR's of* $\mathsf{C}_1^{\mathsf{MI}}$. In order to facilitate passage from the above selected veracious CLR's of $\mathsf{C}_1^{\mathsf{OI}}$ to the respective *general* CLR's of $\mathsf{C}_1^{\mathsf{MI}}$ that are given below, I have providently stated the former in such a form to allow obtaining the latter CLR's by replacing occurrences of certain CVV's in the former with occurrences of the appropriate EVV's. To be specific, to *generalize* the selected veracious CLR's of $\mathsf{C}_1^{\mathsf{OI}}$, given in the items 1–3, 5, 6, and 8–11 of Summary A5.2, all occurrences of the letters '*x*', '*y*', and '*z*' throughout those items and all relevant occurrences of the word "*class*" should be replaced with occurrences of the letters 'ξ', 'η', and 'ζ' and with occurrences of the word "*element*" respectively. For the obvious reason, the veracious CLR's that are given in the items 4, 7, 12, and 13 of Summary A5.2 cannot be altered, so that these retain unaltered as ones of $\mathsf{C}_1^{\mathsf{MI}}$. At the same time, two additional axioms of infinity for NEI's are laid down after the manner of the items 12 and 13. For convenience in further references, the results of all pertinent changes that are done in Summary A5.2 are given below. The numbers of the following items correspond to the numbers of the respective modified items of Summary A5.2. Consequently, items 4 and 7 are absent below.

1) *The axiom* [*schema*] *of comprehension* (or *of specification* or *of separation*) or (in German) *Aussonderungaxiom* (*axiom of selection*), *for classes.* Given a condition $P_e\langle\xi\rangle$ that is imposed on the elements ("e") of the range of the EVV 'ξ', to

every class u there is a class v whose elements (members) are exactly all those elements of u, for which $P_{\mathrm{e}}\langle\xi\rangle$ is *veracious*, i.e. logographically

$$\bigwedge_u \bigvee_v \bigwedge_\xi \left[\left[\xi \in v\right] \Leftrightarrow \left[\left[\xi \in u\right] \wedge P_e\langle\xi\rangle\right]\right], \qquad \text{(A5.29}_{\mathrm{e}}\text{)}$$

where '$\wedge$' and '$\Leftrightarrow$' are read as "and" and "if and only if" respectively, while '$\bigwedge_u$' and '$\bigvee_v$' (e.g.) are synonyms of the conventional quantifiers '$(\forall u)$' and '$(\exists v)$', which are read as "for all *u*:" or "for every *u*:" and as "for some *v*:" or "for at least one *v*:" or "there exists at least one *v* such that" respectively. Consequently,

$$\begin{aligned}&\bigwedge_u \bigvee_v \left[\left[v = \left\{\zeta \middle| \left[\zeta \in u\right] \wedge P_e\langle\zeta\rangle\right\}\right]\right.\\ &\left.\rightarrow \bigwedge_\xi \left[\left[\xi \in v\right] \Leftrightarrow \left[\left[\xi \in u\right] \wedge P_e\langle\xi\rangle\right]\right]\right],\end{aligned} \qquad \text{(A5.30}_{\mathrm{e}}\text{)}$$

is a contextual *definition* of the logographic term '$\left\{\zeta\left[\zeta \in u\right] \wedge P_e\langle\zeta\rangle\right\}$' of $\mathsf{C}_1^{\mathsf{MI}}$, which is read as: "the class of all *z* such that $z \in u$ and $P_{\mathrm{e}}\langle\zeta\rangle$". The operator '$\{\,|\,\}$' thus defined is called an *abstraction operator from a relation* (as $\left[\zeta \in u\right] \wedge P_{\mathrm{e}}\langle\zeta\rangle$) *to a class* (as $\left\{\zeta \middle| \left[\zeta \in u\right] \wedge P_e\langle\zeta\rangle\right\}$) and also an *abstract class-builder*. The logographic term '$\left\{\zeta \middle| \left[\zeta \in u\right] \wedge P_e\langle\zeta\rangle\right\}$' can be regarded as a logographic *description* through the *class-genus* (*generic class*) u and the *differentia* (*difference*) P_{e} of the *class-species* (*specific class*) $\left\{\zeta \middle| \left[\zeta \in u\right] \wedge P_e\langle\zeta\rangle\right\}$, which it denotes.

2) *The axiom of pairing of elements.*

$$\bigvee_u \bigwedge_\xi \left[\left[\xi \in u\right] \Leftrightarrow \left[\left[\xi = \xi_1\right] \vee \left[\xi = \xi_2\right]\right]\right]. \qquad \text{(A5.31}_{\mathrm{e}}\text{)}$$

Consequently,

$$\begin{aligned}&\bigvee_u \left[\left[u = \left\{\zeta \middle| \left[\zeta = \xi_1\right] \vee \left[\zeta = \xi_2\right]\right\}\right]\right.\\ &\left.\rightarrow \bigwedge_\xi \left[\left[\xi \in u\right] \Leftrightarrow \left[\left[\xi = \xi_1\right] \vee \left[\xi = \xi_2\right]\right]\right]\right]\end{aligned} \qquad \text{(A5.32}_{\mathrm{e}}\text{)}$$

is a contextual *definition* of the logographic term '$\left\{\zeta \middle| \left[\zeta = \xi_1\right] \vee \left[\zeta = \xi_2\right]\right\}$' of $\mathsf{C}_1^{\mathsf{MI}}$, which is abbreviated further and rendered into ordinary language as follows:

[The class of elements ξ_1 and ξ_2]

$\rightarrow$[The unordered pair of elements ξ_1 and ξ_2]

$$\rightarrow \{\xi_1, \xi_2\} \rightarrow \left\{\xi \middle| \left[\xi = \xi_1\right] \vee \left[\xi = \xi_2\right]\right\}. \qquad \text{(A5.33}_{\mathrm{e}}\text{)}$$

3) *The definition of the intersection of two classes of elements.* By the pertinent instance of the axiom of comprehension (A5.29$_e$),

$$\bigvee_w \bigwedge_\xi [[\xi \in w] \Leftrightarrow [[\xi \in u] \wedge [\xi \in v]]]. \qquad \text{(A5.34}_e\text{)}$$

Consequently,

$$\begin{aligned} &\bigvee_w [[w = \{\zeta | [\zeta \in u] \vee [\zeta \in v]\}] \\ &\rightarrow \bigwedge_\xi [[\xi \in w] \Leftrightarrow [[\xi \in u] \vee [\xi \in v]]]] \end{aligned} \qquad \text{(A5.35}_e\text{)}$$

is a contextual *definition* of the logographic term '$\{\zeta | [\zeta \in u] \vee [\zeta \in v]\}$' of $\mathsf{C}_1^{\mathsf{OI}}$, which is abbreviated further and rendered into ordinary language as follows:

[The intersection of the classes *u* and *v* of elements]

$$\rightarrow [u \cap v] \rightarrow \{\zeta | [\zeta \in u] \wedge [\zeta \in v]\}. \qquad \text{(A5.36}_e\text{)}$$

5) *The definition of the difference between two classes of elements.*

[The difference between the classes *u* and *v* of elements]

↔[The complement of the class *v* in the class *u*]

$$\rightarrow [u - v] \rightarrow \{\xi | [\xi \in u] \ \wedge \neg [\xi \in v]\}. \qquad \text{(A5.40}_e\text{)}$$

6) *The axiom of union of two classes of elements.*

$$\bigvee_w \bigwedge_\xi [[\xi \in w] \Leftrightarrow [[\xi \in u] \vee [\xi \in v]]]. \qquad \text{(A5.41}_e\text{)}$$

Consequently,

$$\begin{aligned} &\bigvee_w [[w = \{\zeta | [\zeta \in u] \vee [\zeta \in v]\}] \\ &\rightarrow \bigwedge_\xi [[\xi \in w] \Leftrightarrow [[\xi \in u] \vee [\xi \in v]]]] \end{aligned} \qquad \text{(A5.42}_e\text{)}$$

is a contextual *definition* of the logographic term '$\{\zeta | [\zeta \in u] \vee [\zeta \in v]\}$' of $\mathsf{C}_1^{\mathsf{MI}}$, which is abbreviated further and rendered into ordinary language as follows:

[The union of the classes *u* and *v* of elements]

$$\rightarrow [u \cup v] \rightarrow \{\zeta | [\zeta \in u] \vee [\zeta \in v]\}]. \qquad \text{(A5.43}_e\text{)}$$

8) *The definitions of singletons and unordered multiples of elements.*

[The singleton of an element ξ_1]$\rightarrow \{\xi_1\} \rightarrow \{\xi_1, \xi_1\}$. (A5.47$_e$)

[The unordered triple of elements ξ_1, ξ_2, ξ_3]

→[The class of elements ξ_1, ξ_2, ξ_3]

$$\rightarrow \{\xi_1, \xi_2, \xi_3\} \rightarrow \{\xi_1, \xi_2\} \cup \{\xi_3\}. \qquad \text{(A5.48}_e\text{)}$$

[The unordered n-tuple of elements $\xi_1,\xi_2,...,\xi_n$]

$$\to\text{[The class of elements } \xi_1,\xi_2,...,\xi_n\text{]}$$

$$\to\{\xi_1,\xi_2,...,\xi_n\}\to\{\xi_1,\xi_2,...,\xi_{n-1}\}\cup\{\xi_n\}. \qquad (\text{A5.49}_e)$$

In contrast to an abstract class-builder '$\{\,|\,\}$', being an abstraction operator that produces a unique class of elements from a given relation (see the item 1 of this Summary), the operator of aggregation

$$\underbrace{\{-,-,...,-\}}_{n}$$

produces a unique class from n given elements as its members, and it is therefore called a *concrete class-builder*.

9) *The definition of ordered pairs of elements*.

[The ordered pair of elements ξ_1 and ξ_2]

→[The ordered pair of elements

with first coordinate ξ_1 and second coordinate ξ_2]

$$\to(\xi_1,\xi_2)\leftrightarrow\langle\xi_1,\xi_2\rangle\to\{\{\xi_1\},\{\xi_1,\xi_2\}\}. \qquad (\text{A5.50}_e)$$

Note. If ξ_1 and ξ_2 are real numbers then the logograph '(ξ_1,ξ_2)' is ambiguous, for it may stand, not only for the ordered pair ξ_1 and ξ_2, but also for the open interval (ξ_1,ξ_2). Whenever there is a danger of confusion, I shall denote an ordered pair with the help of angle brackets.

By the pertinent *homograph* of (A5.13) subject to (A5.6$_e$), it follows from (A5.50$_e$) as a *theorem* that

$$[(\xi_1,\xi_2)=(\eta_1,\eta_2)]\Leftrightarrow[[\xi_1=\eta_1]\wedge[\xi_2=\eta_2]]. \qquad (\text{A5.51}_e)$$

10) *The definitions of ordered singles and ordered multiples of elements.*

$$\text{[The ordered single of an element } \xi_1\text{]}\to\{\{\xi_1\}\}. \qquad (\text{A5.52}_e)$$

[The ordered n-tuple of classes $\xi_1,\xi_2,...,\xi_n$]

$$\to\overline{\xi}_{[1,n]}\to(\xi_1,\xi_2,...,\xi_n)\to(((...((\xi_1,\xi_2),\xi_3),...),\xi_{n-1}),\xi_n). \qquad (\text{A5.53}_e)$$

11) *The definitions of direct products of classes of elements.*

[The direct product of the classes u_1 and u_2]

$$\to [u_1 \times u_2] \to \{(\xi_1,\xi_2)|[\xi_1 \in u_1] \wedge [\xi_2 \in u_2]\}. \qquad \text{(A5.54}_\text{e}\text{)}$$

[The direct product of the classes $u_1, u_2, ..., u_n$]

$$\begin{gathered}\to [u_1 \times u_2 \times ... \times u_n] \to [[[...[[u_1 \times u_2] \times u_3] \times ...] \times u_{n-1}] \times u_n] \\ \to \{(((...((\xi_1,\xi_2),\xi_3),...),\xi_{n-1}),\xi_n)| \\ [[...[[[\xi_1 \in u_1] \wedge [\xi_2 \in u_2]] \wedge [\xi_3 \in u_3]] \wedge ...] \wedge [\xi_{n-1} \in u_{n-1}]] \wedge [\xi_n \in u_n]]\} \\ \leftarrow \{(\xi_1,\xi_2,...,\xi_n)|[\xi_1 \in u_1] \wedge [\xi_2 \in u_2] \wedge ... \wedge [\xi_n \in u_n]\}.\end{gathered} \qquad \text{(A5.55}_\text{e}\text{)}$$

[The repeated (n–1)-fold direct product of the class u by itself]

$$\to u^{n\times} \to \underbrace{[[...[[u \times u] \times u] \times ...] \times u]}_{n}. \qquad \text{(A5.56}_\text{e}\text{)}$$

12) *The first axiom of infinity for elements.* For each NEI α there exists a *unique universal class* ω_α such that

$$\begin{gathered}[[\alpha \in \omega_\alpha] \wedge [\{\alpha\} \in \omega_\alpha]] \\ \wedge \bigwedge_x [[[x \in \omega_\alpha] \Rightarrow [[x \cup \{x\}] \in \omega_\alpha]] \wedge [x \in \omega_\alpha]].\end{gathered} \qquad \text{(A5.57}_\text{e}\text{)}$$

The singleton {α} is called the *immediate successor* of the NEI α in the class ω_α, whereas α is called the *immediate predecessor* of {α}. The class $[x \cup \{x\}]$ is called the *immediate successor* of the class x in the class ω_α, whereas x is called the *immediate predecessor* of $[x \cup \{x\}]$ in the class ω_α.•

13) *The second axiom of infinity for elements.* For each NEI α there exists a *unique universal class* ω'_α such that

$$[\alpha \in \omega'_\alpha] \wedge \bigwedge_\xi [[[\xi \in \omega'_\alpha] \Rightarrow [\{\xi\} \in \omega'_\alpha]] \wedge [\xi \in \omega'_\alpha]]. \qquad \text{(A5.58}_\text{e}\text{)}$$

The singleton {ξ} is called the *immediate successor* of the NEI ξ in the class ω'_α, whereas ξ is called the *immediate predecessor* of {ξ} in the class ω'_α.•

Cmt A5.4. Cmt A5.1 applies with 'ξ=v=∅' and '(A5.25$_\text{e}$)' in place of 'x=v=∅' and '(A5.25)'.•

Summary A5.6: ***The taxonomy of objects of*** $\mathsf{C}_1^{\mathsf{MI}}$**.**

1) An object of $\mathsf{C}_1^{\mathsf{MI}}$ is called:

a) an *element* if it can be or is a *member* of class;

b) a *universal class* it cannot be or is not a *member* of class;

c) a *particular class* and also a *class-element* if it is an element, but not necessarily vice versa.

2) An element of $\mathsf{C}_1^{\mathsf{MI}}$ is called:

a) a *divisible element* and also a *nonempty class* if it has at least one element, which is predicated to be its *member*, i.e. which is a *predicative* of at least one element;

b) the *empty indivisible element* or the *empty individual* and also the *empty class* or the *empty set*, conventionally denoted by '∅', if it is *denied to be a predicative of any element including itself*;

c) a *nonempty indivisible element* or a *nonempty individual* if is *rejected* (*not allowed*, *prevented from*) being a predicative of any other element including itself, although it can be predicated both to be a member of a nonempty class and not to be a member of the empty class;

d) an *indivisible*, or *memberless*, *element* and also an *individual* if it is either the empty individual or a nonempty individual.

3) If $\mathsf{C}_1^{\mathsf{MI}}$ is freed of all nonempty individuals then it turns into $\mathsf{C}_1^{\mathsf{OI}}$. Every *element* of $\mathsf{C}_1^{\mathsf{OI}}$ is a particular class, a nonempty one or the empty one, so that the taxa (metaterms) "element", "class-element", and "particular class" become synonyms.•

Cmt A5.5 (*mutatis mutandis*, word for word the same as Cmt A5.2)**.** $\mathsf{C}_1^{\mathsf{MI}}$ can be developed further into a *many-individual nominalistic axiomatic set theory* (*MINAST*) $\mathsf{S}_1^{\mathsf{MI}}$ as indicated in Cmt 9.1**(**3). In this case, every occurrence of the noun "class" throughout $\mathsf{C}_1^{\mathsf{MI}}$ can be replaced with an occurrence of the noun "set".•

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