Applied Mechanics and Materials
ISSN: 1662-7482, Vol. 841, pp 184-191
doi:10.4028/www.scientific.net/AMM.841.184
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Submitted: 2016-02-26
Revised: 2016-03-01
Accepted: 2016-03-07
Online: 2016-06-21

An Introduction to Bipolar Single Valued Neutrosophic Graph Theory

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Keywords: Single valued neutrosophic set, bipolar neutrosophic set, single valued neutrosophic graph, bipolar single valued neutrosophic graphs.

Abstract. In this paper, we first define the concept of bipolar single neutrosophic graphs as the generalization of bipolar fuzzy graphs, N-graphs, intuitionistic fuzzy graph, single valued neutrosophic graphs and bipolar intuitionistic fuzzy graphs.

1. Introduction

Zadeh [9] coined the term ‘degree of membership’ and defined the concept of fuzzy set in order to deal with uncertainty. Atanassov [8] incorporated the degree of non-membership in the concept of fuzzy set as an independent component and defined the concept of intuitionistic fuzzy set. Smarandache [2] grounded the term ‘degree of indeterminacy’ as an independent component and defined the concept of neutrosophic set from the philosophical point of view to deal with incomplete, indeterminate and inconsistent information in real world. The concept of neutrosophic set is a generalization of the theory of fuzzy set, intuitionistic fuzzy set. Each element of a neutrosophic set has three membership degrees including a truth membership degree, an indeterminacy membership degree, and a falsity membership degree which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. Therefore, if their range is restrained within the real standard unit interval [0, 1], the neutrosophic set is easily applied to engineering problems. For this purpose, Wang et al. [6] introduced the concept of the single-valued neutrosophic set (SVNS) as a subclass of the neutrosophic set. Recently, Deli et al. [7] defined the concept of bipolar neutrosophic, as a generalization of single valued neutrosophic set, and bipolar fuzzy graph, also studying some of their related properties. The neutrosophic set theory of and their extensions have been applied in various domains [22] (refer to the site http://fs.gallup.unm.edu/NSS/).

When the relations between nodes (or vertices) in problems are indeterminate, the concept of fuzzy graphs [15] and its extensions, such as intuitionistic fuzzy graphs [11, 16], N-graphs [13], bipolar fuzzy graphs [11, 12, 14], bipolar intuitionistic fuzzy graphs [1] are not suitable. For this purpose, Smarandache [3] defined four main categories of neutrosophic graphs, two based on literal indeterminacy (I), calling them I-edge neutrosophic graph and I-vertex neutrosophic graph; these concepts are deeply studied and gained popularity among some researchers [4, 5, 19, 20, 21] due to their applications in the real world problems. The two others graphs are based on (t, i, f) components, and are called: (t, i, f)-edge neutrosophic graph and (t, i, f)-vertex neutrosophic graph; but these new concepts are not developed at all yet. Later on, Broumi et al. [18] introduced a third neutrosophic graph model. The single valued neutrosophic graph is the generalization of fuzzy graph and intuitionstic fuzzy graph. Also, the same authors [17] introduced neighborhood degree of a vertex and closed neighborhood degree of a vertex in the single valued neutrosophic graph, as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph.

In this paper, motivated by the works of Deli et al. [7] and Broumi et al. [18], we introduced the concept of bipolar single valued neutrosophic graph and proved some propositions.

2. Preliminaries

In this section, we mainly recall some notions, which we are also going to use in the rest of the paper. The readers are referred to [6, 7, 10, 11, 13, 15, 18] for further details and background.

Definition 2.1 [6]

Let U be an universe of a discourse; then, the neutrosophic set A is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ U}, where the functions T, I, F: U→]$^-0,1^+$[define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ U to the set A with the condition: $^-0 \leq T_A(x)+ I_A(x)+ F_A(x) \leq 3^+$.

Definition 2.2 [7]

A bipolar neutrosophic set A in X is defined as an object of the form A={<x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x)>: x ∈ X}, where T^P, I^P, F^P:X→ [1, 0] and T^N, I^N, F^N: X→ [-1, 0]. The positive membership degree T^P(x), I^P(x), F^P(x) denotes the truth membership, indeterminate membership and false membership of an element ∈ X corresponding to a bipolar neutrosophic set A, and the negative membership degree T^N(x), I^N(x), F^N(x) denotes the truth membership, indeterminate membership and false membership of an element ∈ X to some implicit counter-property corresponding to a bipolar neutrosophic set A.

Example 2.1

Let X = $\{x_1, x_2, x_3\}$;

A = $\left\{\begin{matrix} <x_1, 0.5, 0.3, 0.1, -0.6, -0.4, -0.05> \\ <x_2, 0.3, 0.2, 0.7, -0.02, -0.3, -0.02> \\ <x_3, 0.8, 0.05, 0.4, -0.6, -0.6, -0.03> \end{matrix}\right\}$ is a bipolar neutrosophic subset of X.

Definition 2.3 [7]

Let A_1 = {<x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x)>} and A_2 = {<x, T_2^P(x), I_2^P(x), F_2^P(x), T_2^N(x), I_2^N(x), F_2^N(x) >} be two bipolar neutrosophic sets. Then, $A_1 \subseteq A_2$ if and only if $T_1^P(x) \leq T_2^P(x)$, $I_1^P(x) \leq I_2^P(x)$, $F_1^P(x) \geq F_2^P(x)$ and $T_1^N(x) \geq T_2^N(x)$, $I_1^N(x) \geq I_2^N(x)$, $F_1^N(x) \leq F_2^N(x)$ for all x ∈ X.

Definition 2.4 [15]

A fuzzy graph with V as the underlying set is a pair G = (σ, μ), where σ: V → [0, 1] is a fuzzy subset and μ: V × V → [0, 1] is a fuzzy relation on σ such that μ(x, y) ≤ σ(x) ∧ σ(y) for all x, y ∈ V where ∧ stands for minimum.

Definition 2.5 [13]

By a *N*-graph G of a graph G^*, we mean a pair G= (μ_1, μ_2) where μ_1 is an *N*-function in V and μ_2 is an *N*-relation on E such that μ_2(u,v) ≥ max (μ_1(u), μ_1(v)) all u, v ∈ V.

Definition 2.6 [10]

An intuitionistic fuzzy graph is of the form G = (V, E), where

i. V = $\{v_1, v_2,\ldots, v_n\}$ such that μ_1: V→ [0,1] and γ_1: V → [0,1] denoting the degree of membership and non-membership of the element v_i ∈ V, respectively, and 0≤ $\mu_1(v_i)$ + $\gamma_1(v_i)$) ≤ 1 for every v_i ∈ V, (i = 1, 2, ……. n), (1)

ii. E ⊆ V x V where μ_2: VxV→[0,1] and γ_2: VxV→ [0,1] are such that $\mu_2(v_i, v_j) \leq$ min [$\mu_1(v_i)$, $\mu_1(v_j)$] and $\gamma_2(v_i, v_j) \geq$ max [$\gamma_1(v_i)$, $\gamma_1(v_j)$] and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every (v_i, v_j) ∈ E, (i, j = 1,2, ……. n). (2)

Definition 2.7 [11]

Let X be a non-empty set. A bipolar fuzzy set A in X is an object having the form A = {(x, μ_A^P(x), μ_A^N(x)) | x ∈ X}, where μ_A^P(x): X → [0, 1] and μ_A^N(x): X → [−1, 0] are mappings.

Definition 2.8 [11]

A bipolar fuzzy graph of a graph G^*= (V, E) is a pair G = (A,B), where A = (μ_A^P, μ_A^N) is a bipolar fuzzy set in V and B = (μ_B^P, μ_B^N) is a bipolar fuzzy set on E $\subseteq$ V x V such that μ_B^P(xy) $\leq$ min{μ_A^P(x), μ_A^P(y)} for all xy $\in E$, μ_B^N(xy) $\geq$ min{μ_A^N(x), μ_A^N(y)} for all xy $\in E$ and μ_B^P(xy) = μ_B^N(xy) = 0 for all xy $\in \tilde{V}^2$ –E. Here A is called bipolar fuzzy vertex set of V, and B - the bipolar fuzzy edge set of E.

Definition 2.9 [18]

A single valued neutrosophic graph (SVNG) of a graph G^*= (V, E) is a pair G = (A, B), where:

i. V = {v_1, v_2,…, v_n} such that T_A:V→[0, 1], I_A:V→[0, 1] and F_A:V→[0, 1] denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element v_i $\in$ V, respectively, and 0$\leq$ $T_A(v_i)$ + $I_A(v_i)$ +$F_A(v_i)$ $\leq$3 for every v_i $\in$ V (i=1, 2, …, n). (3)

ii. E $\subseteq$ V x V, where T_B:V x V →[0, 1], I_B:V x V →[0, 1] and F_B:V x V →[0, 1] are such that $T_B(v_i,v_j)$ $\leq$ min [$T_A(v_i)$, $T_A(v_j)$], $I_B(v_i,v_j)$ $\geq$ max [$I_A(v_i)$, $I_A(v_j)$] and $F_B(v_i,v_j)$ $\geq$ max [$F_A(v_i)$, $F_A(v_j)$] and 0$\leq$ $T_B(v_i,v_j)$ + $I_B(v_i,v_j)$+ $F_B(v_i,v_j)$ $\leq$3, for every (v_i,v_j) $\in$ E (i, j = 1, 2,…, n). (4)

3. Bipolar Single Valued Neutrosophic Graphs

In this section, we firstly define the concept of a bipolar single valued neutrosophic relation.

Definition 3.1

Let X be a non-empty set. Then we call a mapping A = (x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x)):X × X → [−1, 0] × [0, 1] a bipolar single valued neutrosophic relation on X such that T_A^P(x, y) $\in$ [0, 1], I_A^P(x, y) $\in$ [0, 1], F_A^P(x, y) $\in$ [0, 1], and T_A^N(x, y) $\in$ [−1, 0], I_A^N(x, y) $\in$ [−1, 0], F_A^N(x, y) $\in$ [−1, 0].

Definition 3.2

Let A = (T_A^P, I_A^P, F_A^P, T_A^N, I_A^N, F_A^N) and B = (T_B^P, I_B^P, F_B^P, T_B^N, I_B^N, F_B^N) be a bipolar single valued neutrosophic graph on a set X. If B = (T_B^P, I_B^P, F_B^P, T_B^N, I_B^N, F_B^N) is a bipolar single valued neutrosophic relation on A = (T_A^P, I_A^P, F_A^P, T_A^N, I_A^N, F_A^N) then:

$T_B^P(x, y) \leq \min(T_A^P(x), T_A^P(y))$, $T_B^N(x, y) \geq \max(T_A^N(x), T_A^N(y))$ (5)

$I_B^P(x, y) \geq \max(I_A^P(x), I_A^P(y))$, $I_B^N(x, y) \leq \min(I_A^N(x), I_A^N(y))$ (6)

$F_B^P(x, y) \geq \max(F_A^P(x), F_A^P(y))$, $F_B^N(x, y) \leq \min(F_A^N(x), F_A^N(y))$, for all $x, y \in X$. (7)

A bipolar single valued neutrosophic relation B on X is called symmetric if $T_B^P(x, y) = T_B^P(y, x)$, $I_B^P(x, y) = I_B^P(y, x)$, $F_B^P(x, y) = F_B^P(y, x)$ and $T_B^N(x, y) = T_B^N(y, x)$, $I_B^N(x, y) = I_B^N(y, x)$, $F_B^N(x, y) = F_B^N(y, x)$, for all $x, y \in X$.

Definition 3.3

A bipolar single valued neutrosophic graph of a graph G^* = (V, E) is a pair G = (A, B), where A = (T_A^P, I_A^P, F_A^P, T_A^N, I_A^N, F_A^N) is a bipolar single valued neutrosophic set in V, and B = (T_B^P, I_B^P, F_B^P, T_B^N, I_B^N, F_B^N) is a bipolar single valued neutrosophic set in $\tilde{V}^2$, such that

$T_B^P(x, y) \leq \min(T_A^P(\boldsymbol{v_i}), T_A^P(\boldsymbol{v_j}))$, $T_B^N(x, y) \geq \max(T_A^N(\boldsymbol{v_i}), T_A^N(\boldsymbol{v_j}))$ (8)

$I_B^P(x, y) \geq \max(I_A^P(\boldsymbol{v_i}), I_A^P(\boldsymbol{v_j}))$, $I_B^N(x, y) \leq \min(I_A^N(\boldsymbol{v_i}), I_A^N(\boldsymbol{v_j}))$, and (9)

$F_B^P(x, y) \geq \max(F_A^P(\boldsymbol{v_i}), F_A^P(\boldsymbol{v_j}))$, $F_B^N(x, y) \leq \min(F_A^N(\boldsymbol{v_i}), F_A^N(\boldsymbol{v_j}))$, for all xy $\in \tilde{V}^2$. (10)

Notation

An edge of BSVNG is denoted by e_{ij} $\in$ E or $\boldsymbol{v_i v_j}$ $\in$ E.

Here, the sextuple (v_i, T_A^P, I_A^P, F_A^P, T_A^N, I_A^N, F_A^N) denotes the positive degree of truth-membership, the positive degree of indeterminacy-membership, the positive degree of falsity-membership, the negative degree of truth-membership, the negative degree of indeterminacy-membership, the negative degree of falsity- membership of the vertex v_i.

The sextuple (e_{ij} , T_B^P, I_B^P, F_B^P, T_B^N, I_B^N, F_B^N) denotes the positive degree of truth-membership, the positive degree of indeterminacy-membership, the positive degree of falsity-membership, the negative degree of truth-membership, the negative degree of indeterminacy-membership, the negative degree of falsity- membership of the edge relation $e_{ij} = (v_i, v_j)$ on V× V.

Notes

i. When $T_A^P = I_A^P = F_A^P = 0$ and $T_A^N = I_A^N = F_A^N = 0$ for some i and j, then there is no edge between v_i and v_j . Otherwise there exists an edge between v_i and v_j .

ii. If one of the inequalities is not satisfied, then G is not a BSVNG.

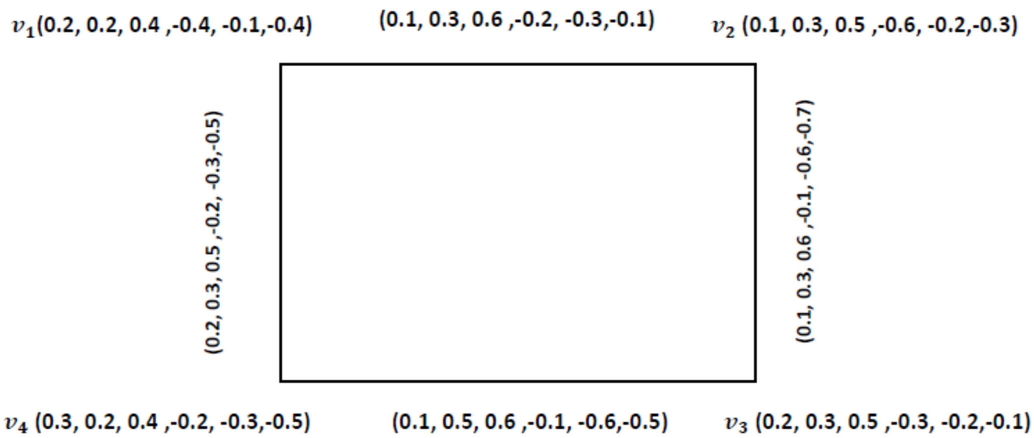

Fig.1: Bipolar single valued neutrosophic graph.

Proposition 3.1

A bipolar single valued neutrosophic graph is the generalization of the fuzzy graph.

Proof

Suppose G = (A, B) is a bipolar single valued neutrosophic graph. Then, by setting the positive indeterminacy-membership, positive falsity-membership and negative truth-membership, negative indeterminacy-membership, negative falsity-membership values of vertex set and edge set equals to zero, it reduces the bipolar single valued neutrosophic graph to a fuzzy graph.

Example 3.1

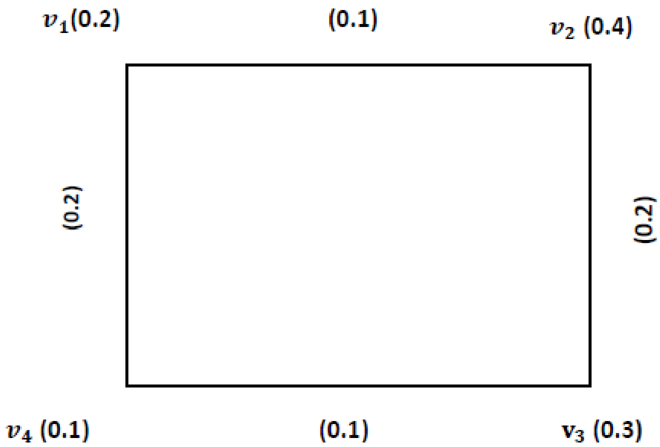

Fig. 2: Fuzzy graph

Proposition 3.2

A bipolar single valued neutrosophic graph is the generalization of the bipolar intuitionstic fuzzy graph.

Proof

Suppose G = (A, B) is a bipolar single valued neutrosophic graph. Then, by setting the positive indeterminacy-membership, negative indeterminacy-membership values of vertex set and edge set equals to zero, it reduces the bipolar single valued neutrosophic graph to a bipolar intuitionistic fuzzy graph.

Example 3.2

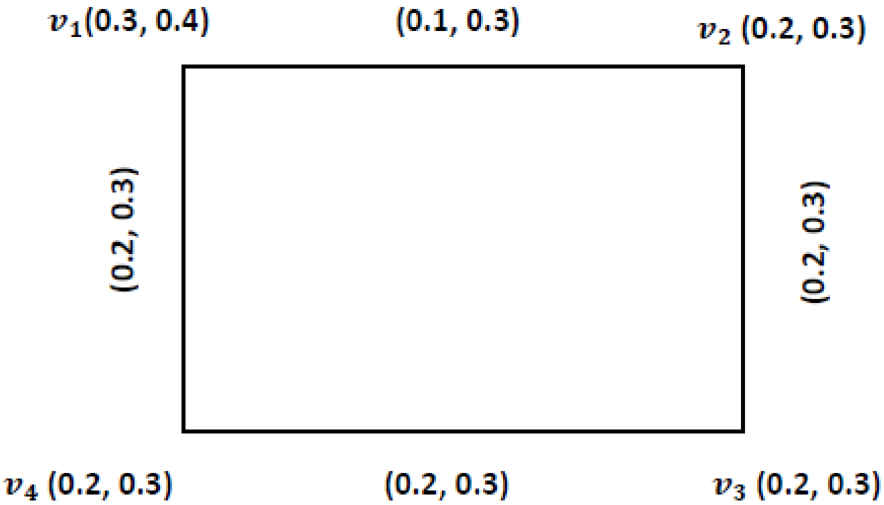

Fig.3: Intuitionistic fuzzy graph.

Proposition 3.3

A bipolar single valued neutrosophic graph is the generalization of the single valued neutrosophic graph.

Proof

Suppose G = (A, B) is a bipolar single valued neutrosophic graph. Then, by setting the negative truth-membership, negative indeterminacy-membership, negative falsity-membership values of vertex set and edge set equals to zero, it reduces the bipolar single valued neutrosophic graph to a single valued neutrosophic graph.

Example 3.3

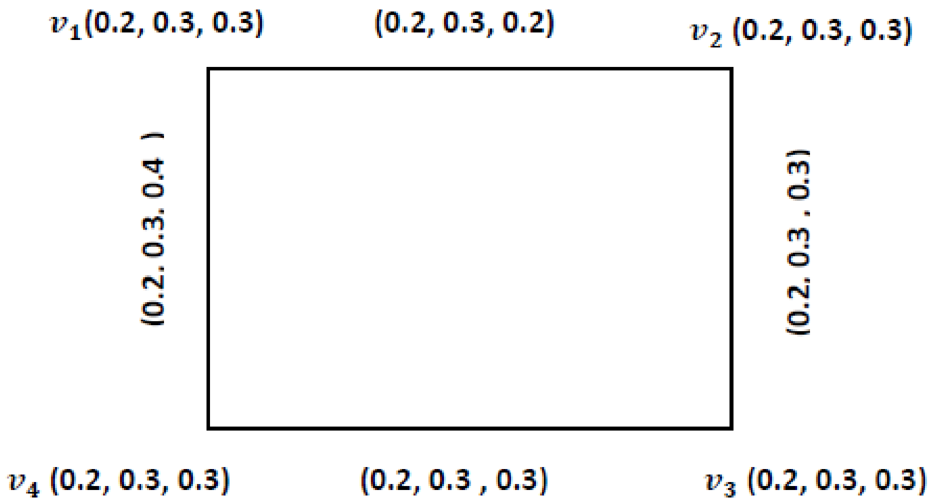

Fig. 4: Single valued neutrosophic graph.

Proposition 3.4

A bipolar single valued neutrosophic graph is the generalization of the bipolar intuitionstic fuzzy graph.

Proof

Suppose G = (A, B) is a bipolar single valued neutrosophic graph. Then, by setting the positive indeterminacy-membership, negative indeterminacy-membership values of vertex set and edge set equals to zero, it reduces the bipolar single valued neutrosophic graph to a bipolar intuitionistic fuzzy graph.

Example 3.4

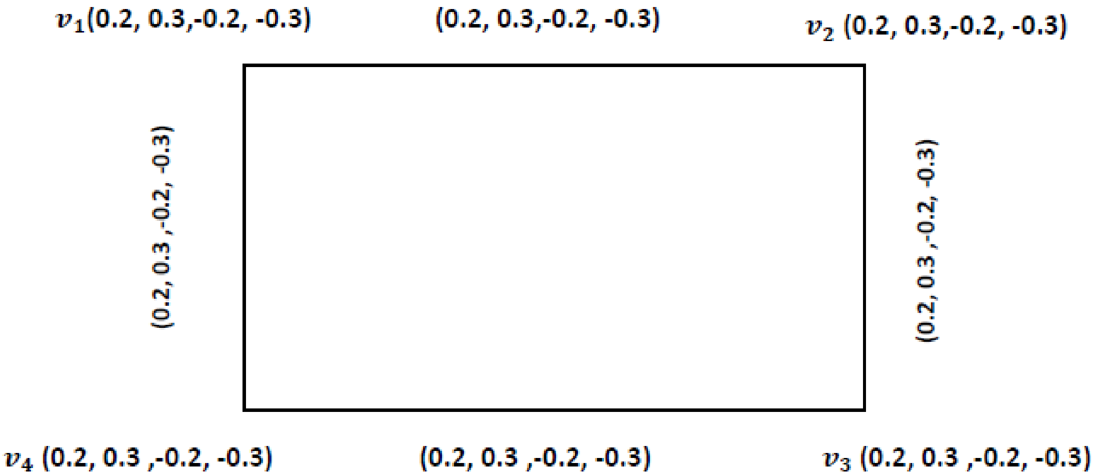

Fig.5: Bipolar intuitionistic fuzzy graph.

Proposition 3.5

A bipolar single valued neutrosophic graph is the generalization of the *N*-graph.

Proof

Suppose G = (A, B) is a bipolar single valued neutrosophic graph. Then, by setting the positive degree membership such truth-membership, indeterminacy-membership, falsity-membership and negative indeterminacy-membership, negative falsity-membership values of vertex set and edge set equals to zero, it reduces the single valued neutrosophic graph to a *N*-graph.

Example 3.5

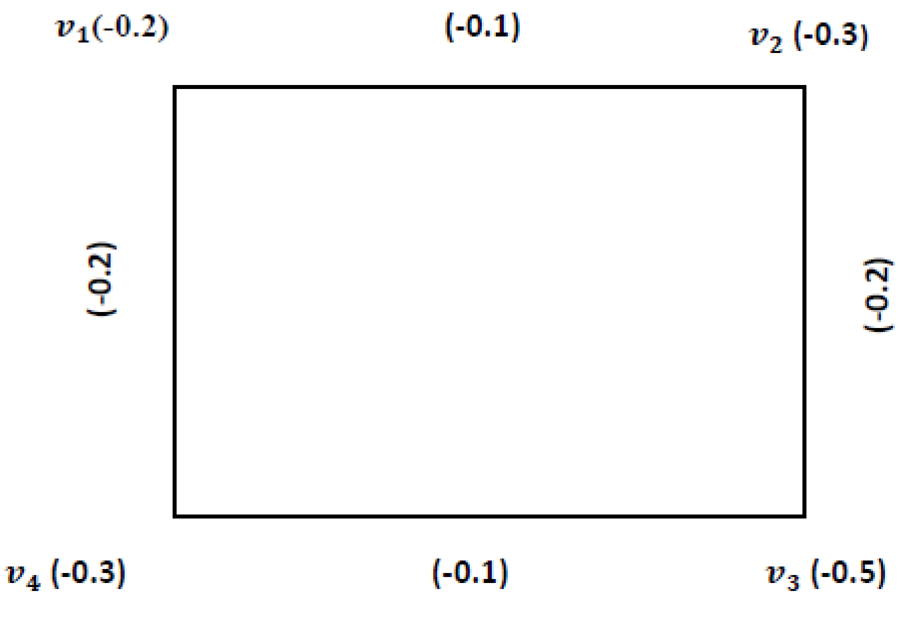

Fig. 6: *N*-graph.

4. Conclusion

In this paper, we have introduced the concept of bipolar single valued neutrosophic graphs and also proved that the most widely used extensions of fuzzy graphs are particular cases of bipolar single valued neutrosophic graphs. So our future work will focus on: (1) The study of certains types of bipolar single valued neutrosophic graphs such as, complete bipolar single valued neutrosophic graphs, strong bipolar single valued neutrosophic graphs, regular bipolar single valued neutrosophic graphs. (2) The concept of energy of bipolar single valued neutrosophic graphs. (3) The study about applications, especially in traffic light problem.

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