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Soft neutrosophic semigroup and their generalization

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Abstract Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft neutrosophic semigroup,soft neutosophic bisemigroup, soft neutrosophic N-semigroup with the discuissionf of some of their characteristics. We also introduced a new type of soft neutrophic semigroup, the so called soft strong neutrosophic semigoup which is of pure neutrosophic character. This notion also foound in all the other corresponding notions of soft neutrosophic thoery. We also given some of their properties of this newly born soft structure related to the strong part of neutrosophic theory.

Keywords Neutrosophic semigroup, neutrosophic bisemigroup, neutrosophic N-semigroup, soft set, soft semigroup, soft neutrosophic semigroup, soft neutrosophic bisemigroup, soft neutrosophic N-semigroup.

§1. Introduction and preliminaries

Florentine Smarandache for the first time introduced the concept of neutrosophy in 1995, which is basically a new branch of philosophy which actually studies the origin, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interval valued fuzzy set. Neutrosophic logic is used to overcome the problems of impreciseness, indeterminate, and inconsistencies of date etc. The theory of neutrosophy is so applicable to every field of algebra. W. B. Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings,neutrosophic vector spaces,neutrosophic groups,neutrosophic bigroups and neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrosophic

N-semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N-loops, and so on. Mumtaz ali et al. introduced nuetrosophic LA-semigroups.

Molodtsov introduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in [2, 9, 10]. Some properties and algebra may be found in [1] . Feng et al. introduced soft semirings in [5]. By means of level soft sets an adjustable approach to fuzzy soft set can be seen in [6]. Some other concepts together with fuzzy set and rough set were shown in [7, 8].

This paper is about to introduced soft nuetrosophic semigroup, soft neutrosophic group, and soft neutrosophic N-semigroup and the related strong or pure part of neutrosophy with the notions of soft set theory. In the proceeding section, we define soft neutrosophic semigroup, soft neutrosophic strong semigroup, and some of their properties are discussed. In the next section, soft neutrosophic bisemigroup are presented with their strong neutrosophic part. Also in this section some of their characterization have been made. In the last section soft neutrosophic N-semigroup and their corresponding strong theory have been constructed with some of their properties.

§2. Definition and properties

Definition 2.1. Let S be a semigroup, the semigroup generated by S and I i.e. $S \cup I$ denoted by $\langle S \cup I \rangle$ is defined to be a neutrosophic semigroup where I is indeterminacy element and termed as neutrosophic element.

It is interesting to note that all neutrosophic semigroups contain a proper subset which is a semigroup.

Example 2.1. Let $Z = \{$the set of positive and negative integers with zero$\}$, Z is only a semigroup under multiplication. Let $N(S) = \{\langle Z \cup I \rangle\}$ be the neutrosophic semigroup under multiplication. Clearly $Z \subset N(S)$ is a semigroup.

Definition 2.2. Let $N(S)$ be a neutrosophic semigroup. A proper subset P of $N(S)$ is said to be a neutrosophic subsemigroup, if P is a neutrosophic semigroup under the operations of $N(S)$. A neutrosophic semigroup $N(S)$ is said to have a subsemigroup if $N(S)$ has a proper subset which is a semigroup under the operations of $N(S)$.

Theorem 2.1. Let $N(S)$ be a neutrosophic semigroup. Suppose P_1 and P_2 be any two neutrosophic subsemigroups of $N(S)$ then $P_1 \cup P_2$ (i.e. the union) the union of two neutrosophic subsemigroups in general need not be a neutrosophic subsemigroup.

Definition 2.3. A neutrosophic semigroup $N(S)$ which has an element e in $N(S)$ such that $e * s = s * e = s$ for all $s \in N(S)$, is called as a neutrosophic monoid.

Definition 2.4. Let $N(S)$ be a neutrosophic monoid under the binary operation $*$. Suppose e is the identity in $N(S)$, that is $s * e = e * s = s$ for all $s \in N(S)$. We call a proper subset P of $N(S)$ to be a neutrosophic submonoid if

1. P is a neutrosophic semigroup under $*$.

2. $e \in P$, i.e., P is a monoid under $*$.

Definition 2.5. Let $N(S)$ be a neutrosophic semigroup under a binary operation $*$. P be a proper subset of $N(S)$. P is said to be a neutrosophic ideal of $N(S)$ if the following conditions are satisfied.

1. P is a neutrosophic semigroup.

2. For all $p \in P$ and for all $s \in N(S)$ we have $p * s$ and $s * p$ are in P.

Definition 2.6. Let $N(S)$ be a neutrosophic semigroup. P be a neutrosophic ideal of $N(S)$, P is said to be a neutrosophic cyclic ideal or neutrosophic principal ideal if P can be generated by a single element.

Definition 2.7. Let $(BN(S), *, o)$ be a nonempty set with two binary operations $*$ and o. $(BN(S), *, o)$ is said to be a neutrosophic bisemigroup if $BN(S) = P1 \cup P2$ where atleast one of $(P1, *)$ or $(P2, o)$ is a neutrosophic semigroup and other is just a semigroup. $P1$ and $P2$ are proper subsets of $BN(S)$, i.e. $P1 \subsetneq P2$.

If both $(P1, *)$ and $(P2, o)$ in the above definition are neutrosophic semigroups then we call $(BN(S), *, o)$ a strong neutrosophic bisemigroup. All strong neutrosophic bisemigroups are trivially neutrosophic bisemigroups.

Example 2.2. Let $(BN(S), *, o) = \{0, 1, 2, 3, I, 2I, 3I, S(3), *, o\} = (P_1, *) \cup (P_2, o)$ where $(P_1, *) = \{0, 1, 2, 3, I, 2I, 3I, *\}$ and $(P_2, o) = (S(3), o)$. Clearly $(P_1, *)$ is a neutrosophic semigroup under multiplication modulo 4. (P_2, o) is just a semigroup. Thus $(BN(S), *, o)$ is a neutrosophic bisemigroup.

Definition 2.8. Let $(BN(S) = P1 \cup P2; o, *)$ be a neutrosophic bisemigroup. A proper subset $(T, o, *)$ is said to be a neutrosophic subbisemigroup of $BN(S)$ if

1. $T = T1 \cup T2$ where $T1 = P1 \cap T$ and $T2 = P2 \cap T$.

2. At least one of $(T1, o)$ or $(T2, *)$ is a neutrosophic semigroup.

Definition 2.9. Let $(BN(S) = P_1 \cup P_2, o, *)$ be a neutrosophic strong bisemigroup. A proper subset T of $BN(S)$ is called the strong neutrosophic subbisemigroup if $T = T_1 \cup T_2$ with $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and if both $(T_1, *)$ and (T_2, o) are neutrosophic subsemigroups of $(P_1, *)$ and (P_2, o) respectively. We call $T = T_1 \cup T_2$ to be a neutrosophic strong subbisemigroup, if atleast one of $(T_1, *)$ or (T_2, o) is a semigroup then $T = T_1 \cup T_2$ is only a neutrosophic subsemigroup.

Definition 2.10. Let $(BN(S) = P_1 \cup P_2 *, o)$ be any neutrosophic bisemigroup. Let J be a proper subset of $B(NS)$ such that $J_1 = J \cap P_1$ and $J_2 = J \cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic bi-ideal of $BN(S)$.

Definition 2.11. Let $(BN(S), *, o)$ be a strong neutrosophic bisemigroup where $BN(S) = P_1 \cup P_2$ with $(P_1, *)$ and (P_2, o) be any two neutrosophic semigroups. Let J be a proper subset of $BN(S)$ where $I = I_1 \cup I_2$ with $I_1 = J \cap P_1$ and $I_2 = J \cap P_2$ are neutrosophic ideals of the neutrosophic semigroups P_1 and P_2 respectively. Then I is called or defined as the strong neutrosophic bi-ideal of $B(N(S))$.

Union of any two neutrosophic bi-ideals in general is not a neutrosophic bi-ideal. This is true of neutrosophic strong bi-ideals.

Definition 2.12. Let $\{S(N), *_1, \ldots, *_N\}$ be a non empty set with N-binary operations defined on it. We call $S(N)$ a neutrosophic N-semigroup (N a positive integer) if the following conditions are satisfied.

1. $S(N) = S_1 \cup \ldots \cup S_N$ where each S_i is a proper subset of $S(N)$ i.e. $S_i \subsetneq S_j$ or $S_j \subsetneq S_i$ if $i \neq j$.

2. $(S_i, *_i)$ is either a neutrosophic semigroup or a semigroup for $i = 1, 2, \ldots, N$.

If all the N-semigroups $(Si, *i)$ are neutrosophic semigroups (i.e. for $i = 1, 2, \ldots, N$) then we call $S(N)$ to be a neutrosophic strong N-semigroup.

Example 2.3. Let $S(N) = \{S_1 \cup S_2 \cup S_3 \cup S_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic 4-semigroup where

$S_1 = \{Z_{12}$, semigroup under multiplication modulo 12$\}$.

$S_2 = \{0, 1, 2, 3, I, 2I, 3I$, semigroup under multiplication modulo 4$\}$, a neutrosophic semigroup.

$S_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} ; a, b, c, d \in \langle R \cup I \rangle \right\}$, neutrosophic semigroup under matrix multiplication and $S_4 = \langle Z \cup I \rangle$, neutrosophic semigroup under multiplication.

Definition 2.13. Let $S(N) = \{S_1 \cup S_2 \cup \ldots \cup S_N, *_1, \ldots, *_N\}$ be a neutrosophic N-semigroup. A proper subset $P = \{P_1 \cup P_2 \cup \ldots \cup P_N, *_1, *_2, \ldots, *_N\}$ of $S(N)$ is said to be a neutrosophic Nsubsemigroup if $P_i = P \cap S_i, i = 1, 2, \ldots, N$ are subsemigroups of S_i in which atleast some of the subsemigroups are neutrosophic subsemigroups.

Definition 2.14. Let $S(N) = \{S_1 \cup S_2 \cup \ldots \cup S_N, *_1, \ldots, *_N\}$ be a neutrosophic strong N-semigroup. A proper subset $T = \{T_1 \cup T_2 \cup \ldots \cup T_N, *_1, \ldots, *_N\}$ of $S(N)$ is said to be a neutrosophic strong sub N-semigroup if each $(T_i, *_i)$ is a neutrosophic subsemigroup of $(S_i, *_i)$ for $i = 1, 2, \ldots, N$ where $T_i = T \cap S_i$.

If only a few of the $(Ti, *i)$ in T are just subsemigroups of $(Si, *i)$ (i.e. $(Ti, *i)$ are not neutrosophic subsemigroups then we call T to be a sub N-semigroup of $S(N)$.

Definition 2.15. Let $S(N) = \{S_1 \cup S_2 \cup \ldots \cup S_N, *_1, \ldots, *_N\}$ be a neutrosophic N-semigroup. A proper subset $P = \{P_1 \cup P_2 \cup \ldots \cup P_N, *_1, \ldots, *_N\}$ of $S(N)$ is said to be a neutrosophic N-subsemigroup, if the following conditions are true,

i. P is a neutrosophic sub N-semigroup of $S(N)$.

ii. Each $P_i = P \cap S_i, i = 1, 2, \ldots, N$ is an ideal of S_i.

Then P is called or defined as the neutrosophic N-ideal of the neutrosophic N-semigroup $S(N)$.

Definition 2.16. Let $S(N) = \{S_1 \cup S_2 \cup \ldots \cup S_N, *_1, \ldots, *_N\}$ be a neutrosophic strong N-semigroup. A proper subset $J = \{I_1 \cup I_2 \cup \ldots \cup I_N\}$ where $I_t = J \cap S_t$ for $t = 1, 2, \ldots, N$ is said to be a neutrosophic strong N-ideal of $S(N)$ if the following conditions are satisfied.

1. Each is a neutrosophic subsemigroup of $S_t, t = 1, 2, \ldots, N$ i.e. It is a neutrosophic strong N-subsemigroup of $S(N)$.

2. Each is a two sided ideal of S_t for $t = 1, 2, \ldots, N$.

Similarly one can define neutrosophic strong N-left ideal or neutrosophic strong right ideal of $S(N)$.

A neutrosophic strong N-ideal is one which is both a neutrosophic strong N-left ideal and N-right ideal of $S(N)$.

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subset E$. Molodtsov [12] defined the soft set in the following manner:

Definition 2.17. A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \longrightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e-elements of the soft set (F, A), or as the set of e-approximate elements of the soft set.

Example 2.4. Suppose that U is the set of shops. E is the set of parameters and each parameter is a word or senctence. Let E={high rent, normal rent, in good condition, in bad condition}. Let us consider a soft set (F, A) which describes the attractiveness of shops that Mr. Z is taking on rent. Suppose that there are five houses in the universe $U = \{h_1, h_2, h_3, h_4, h_5\}$ under consideration, and that $A = \{e_1, e_2, e_3\}$ be the set of parameters where

e_1 stands for the parameter high rent.

e_2 stands for the parameter normal rent.

e_3 stands for the parameter in good condition.

Suppose that

$F(e_1) = \{h_1, h_4\}$.

$F(e_2) = \{h_2, h_5\}$.

$F(e_3) = \{h_3, h_4, h_5\}$.

The soft set (F, A) is an approximated family $\{F(e_i), i = 1, 2, 3\}$ of subsets of the set U which gives us a collection of approximate description of an object. Thus, we have the soft set (F, A) as a collection of approximations as below:

$(F, A) = \{$high rent $= \{h_1, h_4\}$, normal rent $= \{h_2, h_5\}$, in good condition $= \{h_3, h_4, h_5\}\}$.

Definition 2.18. For two soft sets (F, A) and (H, B) over U, (F, A) is called a soft subset of (H, B) if

1. $A \subseteq B$.

2. $F(e) \subseteq G(e)$, for all $e \in A$.

This relationship is denoted by $(F, A) \tilde{\subset} (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \tilde{\supset} (H, B)$.

Definition 2.19. Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).

Definition 2.20. (F, A) over U is called an absolute soft set if $F(e) = U$ for all $e \in A$ and we denote it by U.

Definition 2.21. Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by$(F, A) \cap_R (G, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap G(c)$ for all $c \in C = A \cap B$.

Definition 2.22. The extended intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $e \in C$, $H(e)$ is defined as

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cap G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \cap_\varepsilon (G, B) = (H, C)$.

Definition 2.23. The resticted union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $e \in C$, $H(e)$ is defined as the soft set $(H, C) = (F, A) \cup_R (G, B)$ where $C = A \cap B$ and $H(c) = F(c) \cup G(c)$ for all $c \in C$.

Definition 2.24. The extended union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $e \in C$, $H(e)$ is defined as

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \cup_\varepsilon (G, B) = (H, C)$.

Definition 2.25. A soft set (F, A) over S is called a soft semigroup over S if $(F, A) \overset{\curlywedge}{\circ} (F, A) \subseteq (F, A)$.

It is easy to see that a soft set (F, A) over S is a soft semigroup if and only if $\phi \neq F(a)$ is a subsemigroup of S.

Definition 2.26. A soft set (F, A) over a semigroup S is called a soft left (right) ideal over S, if $(S, E) \subseteq (F, A), ((F, A) \subseteq (S, E))$.

A soft set over S is a soft ideal if it is both a soft left and a soft right ideal over S.

Proposition 2.1. A soft set (F, A) over S is a soft ideal over S if and only if $\phi \neq F(a)$ is an ideal of S.

Definition 2.27. Let (G, B) be a soft subset of a soft semigroup (F, A) over S, then (G, B) is called a soft subsemigroup (ideal) of (F, A) if $G(b)$ is a subsemigroup (ideal) of $F(b)$ for all $b \in A$.

§3. Soft neutrosophic semigroup

Definition 3.1. Let $N(S)$ be a neutrosophic semigroup and (F, A) be a soft set over $N(S)$. Then (F, A) is called soft neutrosophic semigroup if and only if $F(e)$ is neutrosophic subsemigroup of $N(S)$, for all $e \in A$.

Equivalently (F, A) is a soft neutrosophic semigroup over $N(S)$ if $(F, A) \overset{\curlywedge}{\circ} (F, A) \subseteq (F, A)$, where $\tilde{N}_{(N(S),A)} \neq (F, A) \neq \underset{\sim}{\overset{\sim}{\phi}}$.

Example 3.1. Let $N(S) = \langle Z^+ \cup \{0\}^+ \cup \{I\}\rangle$ be a neutrosophic semigroup under $+$. Consider $P = \langle 2Z^+ \cup I\rangle$ and $R = \langle 3Z^+ \cup I\rangle$ are neutrosophic subsemigroup of $N(S)$. Then clearly for all $e \in A$, (F, A) is a soft neutrosophic semigroup over $N(S)$, where $F(x_1) = \{\langle 2Z^+ \cup I\rangle\}$, $F(x_2) = \{\langle 3Z^+ \cup I\rangle\}$.

Theorem 3.1. A soft neutrosophic semigroup over $N(S)$ always contain a soft semigroup over S.

Proof. The proof of this theorem is straight forward.

Theorem 3.2. Let (F, A) and (H, A) be two soft neutrosophic semigroups over $N(S)$. Then their intersection $(F, A) \cap (H, A)$ is again soft neutrosophic semigroup over $N(S)$.

Proof. The proof is staight forward.

Theorem 3.3. Let (F, A) and (H, B) be two soft neutrosophic semigroups over $N(S)$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic semigroup over $N(S)$.

Remark 3.1. The extended union of two soft neutrosophic semigroups (F, A) and (K, B) over $N(S)$ is not a soft neutrosophic semigroup over $N(S)$.

We take the following example for the proof of above remark.

Example 3.2. Let $N(S) = \langle Z^+ \cup I \rangle$ be the neutrosophic semigroup under $+$. Take $P_1 = \{\langle 2Z^+ \cup I \rangle\}$ and $P_2 = \{\langle 3Z^+ \cup I \rangle\}$ to be any two neutrosophic subsemigroups of $N(S)$. Then clearly for all $e \in A$, (F, A) is a soft neutrosophic semigroup over $N(S)$, where $F(x_1) = \{\langle 2Z^+ \cup I \rangle\}, F(x_2) = \{\langle 3Z^+ \cup I \rangle\}$.

Again Let $R_1 = \{\langle 5Z^+ \cup I \rangle\}$ and $R_2 = \{\langle 4Z^+ \cup I \rangle\}$ be another neutrosophic subsemigroups of $N(S)$ and (K, B) is another soft neutrosophic semigroup over $N(S)$, where $K(x_1) = \{\langle 5Z^+ \cup I \rangle\}, K(x_3) = \{\langle 4Z^+ \cup I \rangle\}$.

Let $C = A \cup B$. The extended union $(F, A) \cup_\varepsilon (K, B) = (H, C)$ where $x_1 \in C$, we have $H(x_1) = F(x_1) \cup K(x_1)$ is not neutrosophic subsemigroup as union of two neutrosophic subsemigroup is not neutrosophic subsemigroup.

Proposition 3.1. The extended intersection of two soft neutrosophic semigroups over $N(S)$ is soft neutrosophic semigruop over $N(S)$.

Remark 3.2. The restricted union of two soft neutrosophic semigroups (F, A) and (K, B) over $N(S)$ is not a soft neutrosophic semigroup over $N(S)$.

We can easily check it in above example.

Proposition 3.2. The restricted intersection of two soft neutrosophic semigroups over $N(S)$ is soft neutrosophic semigroup over $N(S)$.

Proposition 3.3. The AND operation of two soft neutrosophic semigroups over $N(S)$ is soft neutrosophic semigroup over $N(S)$.

Proposition 3.4. The OR operation of two soft neutosophic semigroup over $N(S)$ may not be a soft nuetrosophic semigroup over $N(S)$.

Definition 3.2. Let $N(S)$ be a neutrosophic monoid and (F, A) be a soft set over $N(S)$. Then (F, A) is called soft neutrosophic monoid if and only if $F(e)$ is neutrosophic submonoid of $N(S)$, for all $x \in A$.

Example 3.3. Let $N(S) = \langle Z \cup I \rangle$ be a neutrosophic monoid under $+$. Let $P = \langle 2Z \cup I \rangle$ and $Q = \langle 3Z \cup I \rangle$ are neutrosophic submonoids of $N(S)$. Then (F, A) is a soft neutrosophic monoid over $N(S)$, where $F(x_1) = \{\langle 2Z \cup I \rangle\}, F(x_2) = \{\langle 3Z \cup I \rangle\}$.

Theorem 3.4. Every soft neutrosophic monoid over $N(S)$ is a soft neutrosophic semigroup over $N(S)$ but the converse is not true in general.

Proof. The proof is straightforward.

Proposition 3.5. Let (F, A) and (K, B) be two soft neutrosophic monoids over $N(S)$. Then

1. Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $N(S)$ is not soft neutrosophic monoid over $N(S)$.

2. Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $N(S)$ is soft neutrosophic monoid over $N(S)$.

3. Their restricted union $(F, A) \cup_{R} (K, B)$ over $N(S)$ is not soft neutrosophic monoid over $N(S)$.

4. Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $N(S)$ is soft neutrosophic monoid over $N(S)$.

Proposition 3.6. Let (F, A) and (H, B) be two soft neutrosophic monoid over $N(S)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic monoid over $N(S)$.

2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic monoid over $N(S)$.

Definition 3.3. Let (F, A) be a soft neutrosophic semigroup over $N(S)$, then (F, A) is called Full-soft neutrosophic semigroup over $N(S)$ if $F(x) = N(S)$, for all $x \in A$. We denote it by $N(S)$.

Theorem 3.5. Every Full-soft neutrosophic semigroup over $N(S)$ always contain absolute soft semigroup over S.

Proof. The proof of this theorem is straight forward.

Definition 3.4. Let (F, A) and (H, B) be two soft neutrosophic semigroups over $N(S)$. Then (H, B) is a soft neutrosophic subsemigroup of (F, A), if

1. $B \subset A$.

2. $H(a)$ is neutrosophic subsemigroup of $F(a)$, for all $a \in B$.

Example 3.4. Let $N(S) = \langle Z \cup I \rangle$ be a neutrosophic semigroup under $+$. Then (F, A) is a soft neutrosophic semigroup over $N(S)$, where $F(x_1) = \{\langle 2Z \cup I \rangle\}, F(x_2) = \{\langle 3Z \cup I \rangle\}, F(x_3) = \{\langle 5Z \cup I \rangle\}$.

Let $B = \{x_1, x_2\} \subset A$. Then (H, B) is soft neutrosophic subsemigroup of (F, A) over $N(S)$, where $H(x_1) = \{\langle 4Z \cup I \rangle\}, H(x_2) = \{\langle 6Z \cup I \rangle\}$.

Theorem 3.6. A soft neutrosophic semigroup over $N(S)$ have soft neutrosophic subsemigroups as well as soft subsemigroups over $N(S)$.

Proof. Obvious.

Theorem 3.7. Every soft semigroup over S is always soft neutrosophic subsemigroup of soft neutrosophic semigroup over $N(S)$.

Proof. The proof is obvious.

Theorem 3.8. Let (F, A) be a soft neutrosophic semigroup over $N(S)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic subsemigroups of (F, A) then

1. $\cap_{i \in I} (H_i, B_i)$ is a soft neutrosophic subsemigroup of (F, A).

2. $\wedge_{i \in I} (H_i, B_i)$ is a soft neutrosophic subsemigroup of $\wedge_{i \in I} (F, A)$.

3. $\cup_{i \in I} (H_i, B_i)$ is a soft neutrosophic subsemigroup of (F, A) if $B_i \cap B_j = \phi$, for all $i \neq j$.

Proof. Straightforward.

Definition 3.5. A soft set (F, A) over $N(S)$ is called soft neutrosophic left (right) ideal over $N(S)$ if $N(S) \overset{\wedge}{\circ} (F, A) \subseteq (F, A)$, where $\tilde{N}_{(N(S), A)} \neq (F, A) \neq \widetilde{\phi}$ and $N(S)$ is Full-soft neutrosophic semigroup over $N(S)$.

A soft set over $N(S)$ is a soft neutrosophic ideal if it is both a soft neutrosophic left and a soft neutrosophic right ideal over $N(S)$.

Example 3.5. Let $N(S) = \langle Z \cup I \rangle$ be the neutrosophic semigroup under multiplication. Let $P = \langle 2Z \cup I \rangle$ and $Q = \langle 4Z \cup I \rangle$ are neutrosophic ideals of $N(S)$. Then clearly (F, A) is a soft neutrosophic ideal over $N(S)$, where $F(x_1) = \{\langle 2Z \cup I \rangle\}, F(x_2) = \{\langle 4Z \cup I \rangle\}$.

Proposition 3.7. (F, A) is soft neutrosophic ideal if and only if $F(x)$ is a neutrosophic ideal of $N(S)$, for all $x \in A$.

Theorem 3.9. Every soft neutrosophic ideal (F, A) over $N(S)$ is a soft neutrosophic semigroup but the converse is not true.

Proposition 3.8. Let (F, A) and (K, B) be two soft neutrosophic ideals over $N(S)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $N(S)$ is soft neutrosophic ideal over $N(S)$.
2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $N(S)$ is soft neutrosophic ideal over $N(S)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $N(S)$ is soft neutrosophic ideal over $N(S)$.
4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $N(S)$ is soft neutrosophic ideal over $N(S)$.

Proposition 3.9.

1. Let (F, A) and (H, B) be two soft neutrosophic ideal over $N(S)$.
2. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic ideal over $N(S)$.
3. Their OR operation $(F, A) \vee (H, B)$ is soft neutrosophic ideal over $N(S)$.

Theorem 3.10. Let (F, A) and (G, B) be two soft semigroups (ideals) over S and T respectively. Then $(F, A) \times (G, B)$ is also a soft semigroup (ideal) over $S \times T$.

Proof. The proof is straight forward.

Theorem 3.11. Let (F, A) be a soft neutrosophic semigroup over $N(S)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic ideals of (F, A) then

1. $\cap_{i \in I}(H_i, B_i)$ is a soft neutrosophic ideal of (F, A).
2. $\wedge_{i \in I}(H_i, B_i)$ is a soft neutrosophic ideal of $\wedge_{i \in I}(F, A)$.
3. $\cup_{i \in I}(H_i, B_i)$ is a soft neutrosophic ideal of (F, A).
4. $\vee_{i \in I}(H_i, B_i)$ is a soft neutrosophic ideal of $\vee_{i \in I}(F, A)$.

Definition 3.6. A soft set (F, A) over $N(S)$ is called soft neutrosophic principal ideal or soft neutrosophic cyclic ideal if and only if $F(x)$ is a principal or cyclic neutrosophic ideal of $N(S)$, for all $x \in A$.

Proposition 3.10. Let (F, A) and (K, B) be two soft neutrosophic principal ideals over $N(S)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $N(S)$ is not soft neutrosophic principal ideal over $N(S)$.
2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $N(S)$ is soft neutrosophic principal ideal over $N(S)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $N(S)$ is not soft neutrosophic principal ideal over $N(S)$.

4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $N(S)$ is soft neutrosophic principal ideal over $N(S)$.

Proposition 3.11. Let (F, A) and (H, B) be two soft neutrosophic principal ideals over $N(S)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic principal ideal over $N(S)$.
2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic principal ideal over $N(S)$.

§3. Soft neutrosophic bisemigroup

Definition 3.1. Let $\{BN(S), *_1, *_2\}$ be a neutrosophic bisemigroup and let (F, A) be a soft set over $BN(S)$. Then (F, A) is said to be soft neutrosophic bisemigroup over $BN(G)$ if and only if $F(x)$ is neutrosophic subbisemigroup of $BN(G)$ for all $x \in A$.

Example 3.1. Let $BN(S) = \{0, 1, 2, I, 2I, \langle Z \cup I \rangle, \times, +\}$ be a neutosophic bisemigroup. Let $T = \{0, I, 2I, \langle 2Z \cup I \rangle, \times, +\}, P = \{0, 1, 2, \langle 5Z \cup I \rangle, \times, +\}$ and $L = \{0, 1, 2, Z, \times, +\}$ are neutrosophic subbisemigroup of $BN(S)$. The (F, A) is clearly soft neutrosophic bisemigroup over $BN(S)$, where $F(x_1) = \{0, I, 2I, \langle 2Z \cup I \rangle, \times, +\}, F(x_2) = \{0, 1, 2, \langle 5Z \cup I \rangle, \times, +\}, F(x_3) = \{0, 1, 2, Z, \times, +\}$.

Theorem 3.1. Let (F, A) and (H, A) be two soft neutrosophic bisemigroup over $BN(S)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic bisemigroup over $BN(S)$.

Proof. Straightforward.

Theorem 3.2. Let (F, A) and (H, B) be two soft neutrosophic bisemigroups over $BN(S)$ such that $A \cap B = \phi$, then their union is soft neutrosophic bisemigroup over $BN(S)$.

Proof. Straightforward.

Proposition 3.1. Let (F, A) and (K, B) be two soft neutrosophic bisemigroups over $BN(S)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $BN(S)$ is not soft neutrosophic bisemigroup over $BN(S)$.
2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $BN(S)$ is soft neutrosophic bisemigroup over $BN(S)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $BN(S)$ is not soft neutrosophic bisemigroup over $BN(S)$.
4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $BN(S)$ is soft neutrosophic bisemigroup over $BN(S)$.

Proposition 3.2. Let (F, A) and (K, B) be two soft neutrosophic bisemigroups over $BN(S)$. Then

1. Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic bisemigroup over $BN(S)$.
2. Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic bisemigroup over $BN(S)$.

Definition 3.2. Let (F, A) be a soft neutrosophic bisemigroup over $BN(S)$, then (F, A) is called Full-soft neutrosophic bisemigroup over $BN(S)$ if $F(x) = BN(S)$, for all $x \in A$. We denote it by $BN(S)$.

Definition 3.3. Let (F, A) and (H, B) be two soft neutrosophic bisemigroups over $BN(S)$. Then (H, B) is a soft neutrosophic subbisemigroup of (F, A), if

1. $B \subset A$.

2. $H(x)$ is neutrosophic subbisemigroup of $F(x)$, for all $x \in B$.

Example 3.2. Let $BN(S) = \{0, 1, 2, I, 2I, \langle Z \cup I \rangle, \times, +\}$ be a neutosophic bisemigroup. Let $T = \{0, I, 2I, \langle 2Z \cup I \rangle, \times, +\}, P = \{0, 1, 2, \langle 5Z \cup I \rangle, \times, +\}$ and $L = \{0, 1, 2, Z, \times, +\}$ are neutrosophic subbisemigroup of $BN\,(S)$. The (F, A) is clearly soft neutrosophic bisemigroup over $BN\,(S)$, where $F\,(x_1) = \{0, I, 2I, \langle 2Z \cup I \rangle, \times, +\}, F\,(x_2) = \{0, 1, 2, \langle 5Z \cup I \rangle, \times, +\}, F\,(x_3) = \{0, 1, 2, Z, \times, +\}$.

Then (H, B) is a soft neutrosophic subbisemigroup of (F, A), where $H\,(x_1) = \{0, I, \langle 4Z \cup I \rangle, \times, +\}, H\,(x_3) = \{0, 1, 4Z, \times, +\}$.

Theorem 3.3. Let (F, A) be a soft neutrosophic bisemigroup over $BN\,(S)$ and $\{(H_i, B_i)\,;\, i \in I\}$ be a non-empty family of soft neutrosophic subbisemigroups of (F, A) then

1. $\cap_{i \in I}\,(H_i, B_i)$ is a soft neutrosophic subbisemigroup of (F, A).

2. $\wedge_{i \in I}\,(H_i, B_i)$ is a soft neutrosophic subbisemigroup of $\wedge_{i \in I}\,(F, A)$.

3. $\cup_{i \in I}\,(H_i, B_i)$ is a soft neutrosophic subbisemigroup of (F, A) if $B_i \cap B_j = \phi$, for all $i \neq j$.

Proof. Straightforward.

Theorem 3.4. (F, A) is called soft neutrosophic biideal over $BN\,(S)$ if $F(x)$ is neutrosophic biideal of $BN(S)$, for all $x \in A$.

Example 3.3. Let $BN(S) = (\{\langle Z \cup I \rangle, 0, 1, 2, I, 2I, +, \times\}(\times$ under multiplication modulo 3)). Let $T = \{\langle 2Z \cup I \rangle, 0, I, 1, 2I, +, \times\}$ and $J = \{\langle 8Z \cup I \rangle, \{0, 1, I, 2I\}, +\times\}$ are ideals of $BN\,(S)$. Then (F, A) is soft neutrosophic biideal over $BN\,(S)$, where $F\,(x_1) = \{\langle 2Z \cup I \rangle, 0, I, 1, 2I, +, \times\}, F\,(x_2) = \{\langle 8Z \cup I \rangle, \{0, 1, I, 2I\}, +\times\}$.

Theorem 3.5. Every soft neutrosophic biideal (F, A) over $BS\,(N)$ is a soft neutrosophic bisemigroup but the converse is not true.

Proposition 3.3. Let (F, A) and (K, B) be two soft neutrosophic biideals over $BN(S)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $BN(S)$ is not soft neutrosophic biideal over $BN(S)$.

2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $BN(S)$ is soft neutrosophic biideal over $BN(S)$.

3. Their restricted union $(F, A) \cup_R (K, B)$ over $BN(S)$ is not soft neutrosophic biideal over $BN(S)$.

4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $BN(S)$ is soft neutrosophic biideal over $BN(S)$.

Proposition 3.4. Let (F, A) and (H, B) be two soft neutrosophic biideal over $BN(S)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic biideal over $BN(S)$.

2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic biideal over $BN(S)$.

Theorem 3.6.

Let (F, A) be a soft neutrosophic bisemigroup over $BN\,(S)$ and $\{(H_i, B_i)\,;\, i \in I\}$ is a non empty family of soft neutrosophic biideals of (F, A) then

1. $\cap_{i \in I}\,(H_i, B_i)$ is a soft neutrosophic biideal of (F, A).

2. $\wedge_{i \in I}\,(H_i, B_i)$ is a soft neutrosophic biideal of $\wedge_{i \in I}\,(F, A)$.

§4. Soft neutrosophic strong bisemigroup

Definition 4.1. Let (F, A) be a soft set over a neutrosophic bisemigroup $BN(S)$. Then (F, A) is said to be soft strong neutrosophic bisemigroup over $BN(G)$ if and only if $F(x)$ is neutrosophic strong subbisemigroup of $BN(G)$ for all $x \in A$.

Example 4.1. Let $BN(S) = \{0, 1, 2, I, 2I, \langle Z \cup I\rangle, \times, +\}$ be a neutrosophic bisemigroup. Let $T = \{0, I, 2I, \langle 2Z \cup I\rangle, \times, +\}$ and $R = \{0, 1, I, \langle 4Z \cup I\rangle, \times, +\}$ are neutrosophic strong subbisemigroups of $BN(S)$. Then (F, A) is soft neutrosophic strong bisemigroup over $BN(S)$, where $F(x_1) = \{0, I, 2I, \langle 2Z \cup I\rangle, \times, +\}, F(x_2) = \{0, I, 1, \langle 4Z \cup I\rangle, \times, +\}$.

Theorem 4.1. Every soft neutrosophic strong bisemigroup is a soft neutrosophic bisemigroup but the converse is not true.

Proposition 4.1. Let (F, A) and (K, B) be two soft neutrosophic strong bisemigroups over $BN(S)$. Then

1. Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $BN(S)$ is not soft neutrosophic strong bisemigroup over $BN(S)$.

2. Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $BN(S)$ is soft neutrosophic stong bisemigroup over $BN(S)$.

3. Their restricted union $(F, A) \cup_{R} (K, B)$ over $BN(S)$ is not soft neutrosophic stong bisemigroup over $BN(S)$.

4. Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $BN(S)$ is soft neutrosophic strong bisemigroup over $BN(S)$.

Proposition 4.2. Let (F, A) and (K, B) be two soft neutrosophic strong bisemigroups over $BN(S)$. Then

1. Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic strong bisemigroup over $BN(S)$.

2. Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic strong bisemigroup over $BN(S)$.

Definition 4.2. Let (F, A) and (H, B) be two soft neutrosophic strong bisemigroups over $BN(S)$. Then (H, B) is a soft neutrosophic strong subbisemigroup of (F, A), if

1. $B \subset A$.

2. $H(x)$ is neutrosophic strong subbisemigroup of $F(x)$, for all $x \in B$.

Example 4.2. Let $BN(S) = \{0, 1, 2, I, 2I, \langle Z \cup I\rangle, \times, +\}$ be a neutrosophic bisemigroup. Let $T = \{0, I, 2I, \langle 2Z \cup I\rangle, \times, +\}$ and $R = \{0, 1, I, \langle 4Z \cup I\rangle, \times, +\}$ are neutrosophic strong subbisemigroups of $BN(S)$. Then (F, A) is soft neutrosophic strong bisemigroup over $BN(S)$, where $F(x_1) = \{0, I, 2I, \langle 2Z \cup I\rangle, \times, +\}, F(x_2) = \{0, I, \langle 4Z \cup I\rangle, \times, +\}$.

Then (H, B) is a soft neutrosophic strong subbisemigroup of (F, A), where $H(x_1) = \{0, I, \langle 4Z \cup I\rangle, \times, +\}$.

Theorem 4.2. Let (F, A) be a soft neutrosophic strong bisemigroup over $BN(S)$ and $\{(H_i, B_i); i \in I\}$ be a non empty family of soft neutrosophic strong subbisemigroups of (F, A) then

1. $\cap_{i \in I} (H_i, B_i)$ is a soft neutrosophic strong subbisemigroup of (F, A).

2. $\wedge_{i \in I} (H_i, B_i)$ is a soft neutrosophic strong subbisemigroup of $\wedge_{i \in I} (F, A)$.

3. $\cup_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong subbisemigroup of (F, A) if $B_i \cap B_j = \phi$, for all $i \neq j$.

Proof. Straightforward.

Definition 4.3. (F, A) over $BN(S)$ is called soft neutrosophic strong biideal if $F(x)$ is neutosophic strong biideal of $BN(S)$, for all $x \in A$.

Example 4.3. Let $BN(S) = (\{\langle Z \cup I\rangle, 0, 1, 2, I, 2I\}, +, \times(\times$ under multiplication modulo 3)). Let $T = \{\langle 2Z \cup I\rangle, 0, I, 1, 2I, +, \times\}$ and $J = \{\langle 8Z \cup I\rangle, \{0, 1, I, 2I\}, +\times\}$ are neutrosophic strong ideals of $BN(S)$. Then (F, A) is soft neutrosophic strong biideal over $BN(S)$, where $F(x_1) = \{\langle 2Z \cup I\rangle, 0, I, 1, 2I, +, \times\}, F(x_2) = \{\langle 8Z \cup I\rangle, \{0, 1, I, 2I\}, +\times\}$.

Theorem 4.3. Every soft neutrosophic strong biideal (F, A) over $BS(N)$ is a soft neutrosophic bisemigroup but the converse is not true.

Theorem 4.4. Every soft neutrosophic strong biideal (F, A) over $BS(N)$ is a soft neutrosophic strong bisemigroup but the converse is not true.

Proposition 4.3. Let (F, A) and (K, B) be two soft neutrosophic strong biideals over $BN(S)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $BN(S)$ is not soft neutrosophic strong biideal over $BN(S)$.

2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $BN(S)$ is soft neutrosophic strong biideal over $BN(S)$.

3. Their restricted union $(F, A) \cup_R (K, B)$ over $BN(S)$ is not soft neutrosophic strong biideal over $BN(S)$.

4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $BN(S)$ is soft neutrosophic stong biideal over $BN(S)$.

Proposition 4.4. Let (F, A) and (H, B) be two soft neutrosophic strong biideal over $BN(S)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic strong biideal over $BN(S)$.

2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic strong biideal over $BN(S)$.

Theorem 4.5. Let (F, A) be a soft neutrosophic strong bisemigroup over $BN(S)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic strong biideals of (F, A) then

1. $\cap_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong biideal of (F, A).

2. $\wedge_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong biideal of $\wedge_{i\in I}(F, A)$.

§5. Soft neutrosophic N-semigroup

Definition 5.1. Let $\{S(N), *_1, \ldots, *_N\}$ be a neutrosophic N-semigroup and (F, A) be a soft set over $\{S(N), *_1, \ldots, *_N\}$. Then (F, A) is termed as soft neutrosophic N-semigroup if and only if $F(x)$ is neutrosophic sub N-semigroup, for all $x \in A$.

Example 5.1. Let $S(N) = \{S_1 \cup S_2 \cup S_3 \cup S_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic 4-semigroup where

$S_1 = \{Z_{12}$, semigroup under multiplication modulo 12$\}$.

$S_2 = \{0, 1, 2, 3, I, 2I, 3I$, semigroup under multiplication modulo 4$\}$, a neutrosophic semigroup.

$S_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a,b,c,d \in \langle R \cup I \rangle \right\}$, neutrosophic semigroup under matrix multiplication.

$S_4 = \langle Z \cup I \rangle$, neutrosophic semigroup under multiplication. Let $T = \{T_1 \cup T_2 \cup T_3 \cup T_4, *_1, *_2, *_3, *_4\}$ is a neutosophic sub 4-semigroup of $S(4)$, where $T_1 = \{0,2,4,6,8,10\} \subseteq Z_{12}$, $T_2 = \{0, I, 2I, 3I\} \subset S_2$, $T_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a,b,c,d \in \langle Q \cup I \rangle \right\} \subset S_3$, $T4 = \{\langle 5Z \cup I \rangle\} \subset S_4$, the neutrosophic semigroup under multiplication. Also let $P = \{P_1 \cup P_2 \cup P_3 \cup P_4, *_1, *_2, *_3, *_4\}$ be another neutrosophic sub 4-semigroup of $S(4)$, where $P_1 = \{0,6\} \subseteq Z_{12}$, $P_2 = \{0,1,I\} \subset S_2$, $P_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a,b,c,d \in \langle Z \cup I \rangle \right\} \subset S_3$, $P_4 = \{\langle 2Z \cup I \rangle\} \subset S_4$. Then (F, A) is soft neutrosophic 4-semigroup over $S(4)$, where

$$\begin{aligned} F(x_1) &= \{0,2,4,6,8,10\} \cup \{0, I, 2I, 3I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a,b,c,d \in \langle Q \cup I \rangle \right\} \cup \{\langle 5Z \cup I \rangle\}, \\ F(x_2) &= \{0,6\} \cup \{0,1,I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a,b,c,d \in \langle Z \cup I \rangle \right\} \cup \{\langle 2Z \cup I \rangle\}. \end{aligned}$$

Theorem 5.1. Let (F, A) and (H, A) be two soft neutrosophic N-semigroup over $S(N)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic N-semigroup over $S(N)$.

Proof. Straightforward.

Theorem 5.2. Let (F, A) and (H, B) be two soft neutrosophic N-semigroups over $S(N)$ such that $A \cap B = \phi$, then their union is soft neutrosophic N-semigroup over $S(N)$.

Proof. Straightforward.

Proposition 5.1. Let (F, A) and (K, B) be two soft neutrosophic N-semigroups over $S(N)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $S(N)$ is not soft neutrosophic N-semigroup over $S(N)$.
2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $S(N)$ is soft neutrosophic N-semigroup over $S(N)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $S(N)$ is not soft neutrosophic N-semigroup over $S(N)$.
4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $S(N)$ is soft neutrosophic N-semigroup over $S(N)$.

Proposition 5.2. Let (F, A) and (K, B) be two soft neutrosophic N-semigroups over $S(N)$. Then

1. Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic N-semigroup over $S(N)$.
2. Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic N-semigroup over $S(N)$.

Definition 5.2. Let (F, A) be a soft neutrosophic N-semigroup over $S(N)$, then (F, A) is called Full-soft neutrosophic N-semigroup over $S(N)$ if $F(x) = S(N)$, for all $x \in A$. We denote it by $S(N)$.

Definition 5.3. Let (F, A) and (H, B) be two soft neutrosophic N-semigroups over $S(N)$. Then (H, B) is a soft neutrosophic sub N-semigroup of (F, A), if

1. $B \subset A$.
2. $H(x)$ is neutrosophic sub N-semigroup of $F(x)$, for all $x \in B$.

Example 5.2. Let $S(N) = \{S_1 \cup S_2 \cup S_3 \cup S_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic 4-semigroup where

$S_1 = \{Z_{12}$, semigroup under multiplication modulo 12$\}$.

$S_2 = \{0, 1, 2, 3, I, 2I, 3I$, semigroup under multiplication modulo 4$\}$, a neutrosophic semigroup.

$S_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle R \cup I \rangle \right\}$, neutrosophic semigroup under matrix multiplication.

$S_4 = \langle Z \cup I \rangle$, neutrosophic semigroup under multiplication. Let $T = \{T_1 \cup T_2 \cup T_3 \cup T_4, *_1, *_2, *_3, *_4\}$ is a neutosophic sub 4-semigroup of $S(4)$, where $T_1 = \{0, 2, 4, 6, 8, 10\} \subseteq Z_{12}$, $T_2 = \{0, I, 2I, 3I\} \subset S_2$, $T_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Q \cup I \rangle \right\} \subset S_3$, $T_4 = \{\langle 5Z \cup I \rangle\} \subset S_4$, the neutrosophic semigroup under multiplication. Also let $P = \{P_1 \cup P_2 \cup P_3 \cup P_4, *_1, *_2, *_3, *_4\}$ be another neutrosophic sub 4-semigroup of $S(4)$, where $P_1 = \{0, 6\} \subseteq Z_{12}$, $P_2 = \{0, 1, I\} \subset S_2$, $P_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Z \cup I \rangle \right\} \subset S_3$, $P_4 = \{\langle 2Z \cup I \rangle\} \subset S_4$. Also let $R = \{R_1 \cup R_2 \cup R_3 \cup R_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic sub 4-semigroup os $S(4)$ where $R_1 = \{0, 3, 6, 9\}, R_2 = \{0, I, 2I\}, R_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle 2Z \cup I \rangle \right\}, R_4 = \{\langle 3Z \cup I \rangle\}$.
Then (F, A) is soft neutrosophic 4-semigroup over $S(4)$, where

$$F(x_1) = \{0, 2, 4, 6, 8, 10\} \cup \{0, I, 2I, 3I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Q \cup I \rangle \right\} \cup \{\langle 5Z \cup I \rangle\},$$

$$F(x_2) = \{0, 6\} \cup \{0, 1, I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Z \cup I \rangle \right\} \cup \{\langle 2Z \cup I \rangle\},$$

$$F(x_3) = \{0, 3, 6, 9\} \cup \{0, I, 2I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle 2Z \cup I \rangle \right\} \cup \{\langle 3Z \cup I \rangle\}.$$

Clearly (H, B) is a soft neutrosophic sub N-semigroup of (F, A), where

$$H(x_1) = \{0, 4, 8\} \cup \{0, I, 2I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Z \cup I \rangle \right\} \cup \{\langle 10Z \cup I \rangle\},$$

$$H(x_3) = \{0, 6\} \cup \{0, I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle 4Z \cup I \rangle \right\} \cup \{\langle 6Z \cup I \rangle\}.$$

Theorem 5.3. Let (F, A) be a soft neutrosophic N-semigroup over $S(N)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic sub N-semigroups of (F, A) then

1. $\cap_{i\in I}(H_i, B_i)$ is a soft neutrosophic sub N-semigroup of (F, A).

2. $\wedge_{i\in I}(H_i, B_i)$ is a soft neutrosophic sub N-semigroup of $\wedge_{i\in I}(F, A)$.

3. $\cup_{i\in I}(H_i, B_i)$ is a soft neutrosophic sub N-semigroup of (F, A) if $B_i \cap B_j = \phi$, for all $i \neq j$.

Proof. Straightforward.

Definition 5.4. (F, A) over $S(N)$ is called soft neutrosophic N-ideal if $F(x)$ is neutosophic N-ideal of $S(N)$, for all $x \in A$.

Theorem 5.4. Every soft neutrosophic N-ideal (F, A) over $S(N)$ is a soft neutrosophic N-semigroup but the converse is not true.

Proposition 5.3. Let (F, A) and (K, B) be two soft neutrosophic N-ideals over $S(N)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $S(N)$ is not soft neutrosophic N-ideal over $S(N)$.

2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $S(N)$ is soft neutrosophic N-ideal over $S(N)$.

3. Their restricted union $(F, A) \cup_R (K, B)$ over $S(N)$ is not soft neutrosophic N-ideal over $S(N)$.

4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $S(N)$ is soft neutrosophic N-ideal over $S(N)$.

Proposition 5.4. Let (F, A) and (H, B) be two soft neutrosophic N-ideal over $S(N)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic N-ideal over $S(N)$.

2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic N-ideal over $S(N)$.

Theorem 5.5. Let (F, A) be a soft neutrosophic N-semigroup over $S(N)$ and $\{(H_i, B_i);\ i \in I\}$ is a non empty family of soft neutrosophic N-ideals of (F, A) then

1. $\cap_{i\in I}(H_i, B_i)$ is a soft neutrosophic N-ideal of (F, A).

2. $\wedge_{i\in I}(H_i, B_i)$ is a soft neutrosophic N-ideal of $\wedge_{i\in I}(F, A)$.

§6. Soft neutrosophic strong N-semigroup

Definition 6.1. Let $\{S(N), *_1, \ldots, *_N\}$ be a neutrosophic N-semigroup and (F, A) be a soft set over $\{S(N), *_1, \ldots, *_N\}$. Then (F, A) is called soft neutrosophic strong N-semigroup if and only if $F(x)$ is neutrosophic strong sub N-semigroup, for all $x \in A$.

Example 6.1. Let $S(N) = \{S_1 \cup S_2 \cup S_3 \cup S_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic 4-semigroup where

$S_1 = \langle Z_6 \cup I\rangle$, a neutrosophic semigroup.

$S_2 = \{0, 1, 2, 3, I, 2I, 3I$, semigroup under multiplication modulo 4$\}$, a neutrosophic semigroup.

$S_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle R \cup I\rangle \right\}$, neutrosophic semigroup under matrix multiplication.

$S_4 = \langle Z \cup I \rangle$, neutrosophic semigroup under multiplication. Let $T = \{T_1 \cup T_2 \cup T_3 \cup T_4, *_1, *_2, *_3, *_4\}$ is a neutosophic strong sub 4-semigroup of $S(4)$, where $T_1 = \{0, 3, 3I\} \subseteq \langle Z_6 \cup I \rangle$, $T_2 = \{0, I, 2I, 3I\} \subset S_2$, $T_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Q \cup I \rangle \right\} \subset S_3$, $T_4 = \{\langle 5Z \cup I \rangle\} \subset S_4$, the neutrosophic semigroup under multiplication. Also let $P = \{P_1 \cup P_2 \cup P_3 \cup P_4, *_1, *_2, *_3, *_4\}$ be another neutrosophic strong sub 4-semigroup of $S(4)$, where $P_1 = \{0, 2I, 4I\} \subseteq \langle Z_6 \cup I \rangle$, $P_2 = \{0, 1, I\} \subset S_2$, $P_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Z \cup I \rangle \right\} \subset S_3$, $P_4 = \{\langle 2Z \cup I \rangle\} \subset S_4$. Then (F, A) is soft neutrosophic strong 4-semigroup over $S(4)$, whereThen (F, A) is soft neutrosophic 4-semigroup over $S(4)$, where

$$\begin{aligned} F(x_1) &= \{0, 3, 3I\} \cup \{0, I, 2I, 3I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Q \cup I \rangle \right\} \cup \{\langle 5Z \cup I \rangle\}, \\ F(x_2) &= \{0, 2I, 4I\} \cup \{0, 1, I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Z \cup I \rangle \right\} \cup \{\langle 2Z \cup I \rangle\}. \end{aligned}$$

Theorem 6.1. Every soft neutrosophic strong N-semigroup is trivially a soft neutrosophic N-semigroup but the converse is not true.

Proposition 6.1. Let (F, A) and (K, B) be two soft neutrosophic strong N-semigroups over $S(N)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $S(N)$ is not soft neutrosophic strong N-semigroup over $S(N)$.
2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $S(N)$ is soft neutrosophic strong N-semigroup over $S(N)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $S(N)$ is not soft neutrosophic strong N-semigroup over $S(N)$.
4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $S(N)$ is soft neutrosophic strong N-semigroup over $S(N)$.

Proposition 6.2. Let (F, A) and (K, B) be two soft neutrosophic strong N-semigroups over $S(N)$. Then

1. Their *AND* operation $(F, A) \wedge (K, B)$ is soft neutrosophic strong N-semigroup over $S(N)$.
2. Their *OR* operation $(F, A) \vee (K, B)$ is not soft neutrosophic strong N-semigroup over $S(N)$.

Definition 6.2. Let (F, A) and (H, B) be two soft neutrosophic strong N-semigroups over $S(N)$. Then (H, B) is a soft neutrosophic strong sub N-semigroup of (F, A), if

1. $B \subset A$.
2. $H(x)$ is neutrosophic strong sub N-semigroup of $F(x)$, for all $x \in B$.

Theorem 6.2.

1. Let (F, A) be a soft neutrosophic strong N-semigroup over $S(N)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic stong sub N-semigroups of (F, A) then
2. $\cap_{i \in I} (H_i, B_i)$ is a soft neutrosophic strong sub N-semigroup of (F, A).

3. $\wedge_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong sub N-semigroup of $\wedge_{i\in I}(F, A)$.

4. $\cup_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong sub N-semigroup of (F, A) if $B_i \cap B_j = \phi$, for all $i \neq j$.

Proof. Straightforward.

Definition 6.3. (F, A) over $S(N)$ is called soft neutrosophic strong N-ideal if $F(x)$ is neutosophic strong N-ideal of $S(N)$, for all $x \in A$.

Theorem 6.3. Every soft neutrosophic strong N-ideal (F, A) over $S(N)$ is a soft neutrosophic strong N-semigroup but the converse is not true.

Theorem 6.4. Every soft neutrosophic strong N-ideal (F, A) over $S(N)$ is a soft neutrosophic N-semigroup but the converse is not true.

Proposition 6.3. Let (F, A) and (K, B) be two soft neutrosophic strong N-ideals over $S(N)$. Then

1. Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $S(N)$ is not soft neutrosophic strong N-ideal over $S(N)$. 2. Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $S(N)$ is soft neutrosophic strong N-ideal over $S(N)$.

3. Their restricted union $(F, A) \cup_R (K, B)$ over $S(N)$ is not soft neutrosophic strong N-ideal over $S(N)$.

4. Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $S(N)$ is soft neutrosophic strong N-ideal over $S(N)$.

Proposition 6.4. Let (F, A) and (H, B) be two soft neutrosophic strong N-ideal over $S(N)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic strong N-ideal over $S(N)$.

2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic strong N-ideal over $S(N)$.

Theorem 6.5. Let (F, A) be a soft neutrosophic strong N-semigroup over $S(N)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic strong N-ideals of (F, A) then

1. $\cap_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong N-ideal of (F, A).

2. $\wedge_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong N-ideal of $\wedge_{i\in I}(F, A)$.

Conclusion

This paper is an extension of neutrosphic semigroup to soft semigroup. We also extend neutrosophic bisemigroup, neutrosophic N-semigroup to soft neutrosophic bisemigroup, and soft neutrosophic N-semigroup. Their related properties and results are explained with many illustrative examples, the notions related with strong part of neutrosophy also established within soft semigroup.

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