Conjecture on the consecutive concatenation of the terms of an arithmetic progression

Abstract. In this paper I make the following conjecture: for any arithmetic progression a + b*k, where at least one of a and b is different than 1, that also satisfies the conditions imposed by the Dirichlet's Theorem (a and b are positive coprime integers) is true that the sequence obtained by the consecutive concatenation of the terms a + b*k has an infinity of prime terms. Example: for [a, b] = [7, 11], the sequence obtained by consecutive concatenation of 7, 18, 29, 40, 51, 62, 73 (...) has the prime terms 718294051, 7182940516273 (...). If this conjecture were true, the fact that the *Smarandache consecutive numbers sequence* 1, 12, 123, 1234, 12345 (...) could have not any prime term (thus far there is no prime number known in this sequence, though there have been checked the first about 40 thousand terms) would be even more amazing.

Conjecture:

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Note:

If this conjecture were true, the fact that the *Smarandache consecutive numbers sequence* 1, 12, 123, 1234, 12345 (...) could have not any prime term (thus far there is no prime number known in this sequence, though there have been checked the first about 40 thousand terms) would be even more amazing.

The sequence of primes for [a, b] = [2, 1]:

: 2, 23, 23456789,
  2345678910111213141516171819202122324252627 (...)

The sequence of primes for [a, b] = [3, 1]:

:   3, 3456789101112131415161718 19 (...)

The sequence of primes for [a, b] = [4, 1]:

:   4567, 45678910111213 (...)

The sequence of primes for [a, b] = [5, 1]:

:   5, 567891011121314151617 (...)

The sequence of primes for [a, b] = [6, 1]:

:   67, 678910111213 (...)

The sequence of primes for [a, b] = [25, 1]:

:   25262728293031323334353637383940414243444546474849
(...)
Note that 25 and 49 are both squares of primes
(twins); question: are there any other pairs of
squares of primes $[p^2, q^2]$ having the property
that concatenating the numbers p^2, $p^2 + 1$, ...,
q^2 is obtained a prime?

The sequence of primes for [a, b] = [1, 2]:

:   13, 135791113151719, 1357911131517192123 2527 2931
(...)
This sequence is known as *Smarandache concatenated
odd sequence* and Florentin Smarandache conjectured
that there exist an infinity of prime terms of this
sequence. The terms of this sequence are primes for
the following values of n: 2, 10, 16, 34, 49, 2570
(the term corresponding to n = 2570 is a number with
9725 digits); there is no other prime term known
though where checked the first about 26 thousand
terms of this sequence.

The sequence of primes for [a, b] = [1, 3]:

:   14710131619, 147101316192225283 13437 (...)

In my previous paper "Conjecture on the consecutive
concatenation of the numbers nk+1 where k multiple
of 3" I already conjectured that this sequence has
an infinity of prime terms.

The sequence of primes for [a, b] = [1, 4]:

: 159131721252933373145495357616569737781 (...)

The sequence of primes for [a, b] = [1, 20]:

: 121416181101121141161181 (...)

The sequence of primes for [a, b] = [3, 2]:

: 3579111315171921232527 29 (...)

The sequence of primes for [a, b] = [5, 2]:

: 57911131517, 5791113151719212325272931 3335373941
(...)

The sequence of primes for [a, b] = [17, 16]:

: 1733496581, 173349658197113,
1733496581971131291451611 77,
1733496581971131291451 611 77193209 (...)

The sequence of primes for [a, b] = [19, 30]:

: 1949, 19497910913916919 9229259289319 (...)

The sequence of primes for [a, b] = [23, 30]:

: 23, 2353831131431 73203233263293323,
235383113143173203233263293323353383 (...)

The sequence of primes for [a, b] = [61, 30]:

: 61, 6191121151181, 6191121151181211241271301331
(...)

The sequence of primes for [a, b] = [1729, 30]:

: 17291759, 172917591789181918 4918791909 (...)

The sequence of primes for [a, b] = [1729, 60]:

: 1729178918491909 (...)

The sequence of primes for [a, b] = [1729, 90]:

: 17291819, 1729181919091999 (...)

The sequence of primes for [a, b] = [341, 90]:

:   341431521611, 341431521611701 (...)

The sequence of primes for [a, b] = [341, 340]:

:   341681 (...)

The sequence of primes for [a, b] = [561, 560]:

:   5611121 (...)

The sequence of primes for [a, b] = [1729, 1728]:

:   17293457 (...)

Note that from a Poulet number P (see above 341, 561, 1729) is obtained often a prime Q when is concatenated with 2*P – 1 (it is also the case for P = 1387 (Q = 13872773), P = 3277 (Q = 32776553) etc.