Conjecture on the pairs of primes (p,q=p+k) involving concatenation

Abstract. In this paper I make the following conjecture: there exist an infinity of pairs of primes (p, q), where q – p = k, for any even number k, such that the number obtained concatenating p with k then with q is prime. Note that is not necessary, as is stipulated in the Polignac's Conjecture, for the primes p and q to be consecutive (though, for the particular cases k = 2 and k = 4, of course that p and q are consecutive, which means that the conjecture above can be regarded as well as a stronger statement than the Twin primes Conjecture).

Conjecture:

There exist an infinity of pairs of primes (p, q), where q – p = k, for any even number k, such that the number r obtained concatenating p with k then with q is prime. Note that is not necessary, as is stipulated in the Polignac's Conjecture, for the primes p and q to be consecutive (though, for the particular cases k = 2 and k = 4, of course that p and q are consecutive, which means that the conjecture above can be regarded as well as a stronger statement than the Twin primes Conjecture).

The sequence of primes r for k = 2:

: 11213, 29231, 41243, 1012103, 1372139, 1912193, 3112313, 3472349, 4312433, 6172619, 6412643 (...)

obtained for [p, q] = [11, 13], [29, 31], [41, 43], [101, 103], [137, 139], [191, 193], [311, 313], [347, 349], [431, 433], [617, 619], [641, 643]...

(see A001359 in OEIS for the pairs of "twin primes" [p, q = p + 2])

The sequence of primes r for k = 4:

: 7411, 19423, 37441, 1634167, 2234227, 4574461, 6134617, 6434647, 7574761, 8594863, 9074911 (...)

obtained for [p, q] = [7, 11], [19, 23], [37, 41], [163, 167], [223, 227], [457, 461], [613, 617], [643, 647], [757, 761], [859, 863], [907, 911]...

(see A046132 in OEIS for the pairs of "cousin primes" [p, q = p + 4])

The sequence of primes r for k = 6:

: 17623, 23629, 37643, 41647, 47653, 61667, 73679, 83689, 976103, 1036109, 1076113, 1516157, 1676173, 1736179, 2276233(...)

obtained for [p, q] = [11, 17], [13, 19], [17, 23], [23, 29], [37, 43], [41, 47], [47, 53], [61, 67], [73, 79], [83, 89], [97, 103], [103, 109], [107, 113], [151, 157], [167, 173], [173, 179], [227, 233]...

(see A023201 in OEIS for the pairs of "sexy primes" [p, q = p + 6])

The sequence of primes r for k = 8:

: 5813, 23831, 29837, 53861, 71879, 89897, 1018109, 1318139, 2638271, 2698277, 5698577, 7018709 (...)

obtained for [p, q] = [5, 13], [23, 31], [29, 37], [53, 61], [71, 79], [89, 97], [101, 109], [131, 139], [263, 271], [269, 277], [569, 577], [701, 709]...

(see A023202 in OEIS for the pairs of primes [p, q = p + 8])

The sequence of primes r for k = 10:

: 131023, 311041, 611071, 1213101223, 1471101481, 1489191499, 1867101877 (...)

obtained for [p, q] = [13, 23], [31, 41], [61, 71], [1213, 1223], [1471, 1481], [1489, 1499], [1867, 1877]...

(see A023203 in OEIS for the pairs of primes [p, q = p + 10])

The sequence of primes r for k = 12:

: 51217, 191231, 411253, 471259, 591271, 1021121033, 1091121103, 1117121129 (...)

obtained for [p, q] = [5, 17], [19, 31], [41, 53], [47, 59], [59,71], [1021, 1031], [1091, 1103], [1117, 1129]...

(see A046133 in OEIS for the pairs of primes [p, q = p + 12])

The sequence of primes r for k = 14:

: 51419, 291443, 1019141033, 1187141201, 1223141237, 1283141297, 1367141381 (...)

obtained for [p, q] = [5, 19], [29, 43], [1019, 1033], [[1187, 1201], [1223, 1237], [1283, 1297], [1367, 1381]...

(see A153417 in OEIS for the pairs of primes [p, q = p + 14])

The sequence of primes r for k = 16:

: 431659, 1291161307, 1693161709, 2671162687, 2713162729, 2887162903 (...)

obtained for [p, q] = [43, 59], [1291, 1307], [1693, 1709], [2671, 2687], [2713, 2729], [2887, 2903]...

(see A049488 in OEIS for the pairs of primes [p, q = p + 16])

The sequence of primes r for k = 18:

: 111829, 191837, 231841, 531871, 611879, 711889, 791897, 1013181031, 1021181039, 1051181069 (...)

obtained for [p, q] = [11, 29], [19, 37], [23, 41], [53, 71], [61, 79], [71, 89], [79, 97], [1013, 1031], [1021, 1039], [1051, 1069]...

(see A153418 in OEIS for the pairs of primes [p, q = p +)

Note: the examples above, i.e. k = 2, 4, 6, 8, 10, 12, 14, 16, 18, covers any possible digital root of k: 1, 2, 3, 4, 5, 6, 7, 8 or 9.