The Relativistic Solenoid

Carles R. Paul[1,], Albert Serra[2], and Ricard Bosch[3]*

It is a mistake to believe that the primitive experiments, known as the origin of the physical sciences, have been sufficiently studied and therefore it is impossible to extract from them some new and important knowledge. This view has contributed to the perpetuation of some misconceptions. The study of such experiments from other points of view, and applying new techniques, makes it possible to expand their meaning and understanding. Einstein must have thought this way since in 1905 decided to study the Faraday disk and, by doing so, discovered the theory of relativity, according to which the magnetic field is a consequence of the relative motion of different signs electric charges. The verifications of the theory of general relativity by cosmological experiments have led to the belief that the special relativity theory is irrelevant in terrestrial dimensions and speed. Therefore, it is important to correct this error by simple laboratory experiments, whose explanation is only possible by using special relativity theory.

1 Introduction

The lack of basic relativistic electrodynamics experiments in the laboratory, makes it difficult to study and understand relativity and puts it more and more away from basic experimentation and hence from scientific knowledge.

The relativistic electrodynamics is commonly ignored in the teaching of electromagnetism and in practical applications. This view gets worse by the verifications of the theory of general relativity using astronomical experiments and at speeds in the order of magnitude of the speed of light. All of this has led to the view that relativity, both special and general, is irrelevant in everyday physics. Surely, today, it would be difficult to find applications of general relativity into everyday's life (except[1] for the GPS) but this is not the case of special relativity. We are immersed in a world dominated by electromagnetic technology and the basic scenario for electromagnetism is special relativity. Still, it remains away from basic experimentation, indispensable for understanding the relativistic behavior of nature.

Einstein, over time, was developing a passion for experimentation and in a letter to his friend Michele Besso[2] claimed:

"In my old age I am developing a passion for experimentation".

Following this Einstein's line of thought, a review of the first experiments in electromagnetism has been conducted from the point of view of special relativity. Such experimental review has consisted in the design and study of several basic experiments of electromagnetism, whose presentation starts with the experiment of this paper.

Einstein, in the early twentieth century, by studying the Faraday disk[3], discovers the special relativity theory. This theory shows that the magnetic forces are actually electrical forces arising when varying the linear density of opposite sign charges, due to the relativistic length contraction, and relative speed between charges. In his famous 1905 publication[4], he theoretically shows that the only real flux is that of the electrical charges and that is the reason for questioning ourselves about the physical significance of magnetic "flux".

To understand conceptually this extraordinary publication by Einstein, it seems appropriate to begin the experimental study and review, through an experiment demonstrating that the magnetic "flux" does not exist.

The interaction between two parallel conductors by which a stationary continuous current flows, generates an attractive or repulsive force between them, depending on whether the currents have the same direction or opposite direction. This force is directly proportional to the product of the currents intensity and the cables length and inversely proportional to the distance that separates them. This experiment was conducted in 1820 by Ampere[5] using a device known today as the Ampere balance. These

* E-mail: paul@tecnocampus.cat
1 Department of Mechatronics, ESUP, Mataro, Spain
2 Departmen of Physics,Faculty of Science, ULA, Merida, Venezuela
3 Department of Electrical Engineering, UPC, Barcelona, Spain

forces can be calculated by defining the magnetic field or by the relativistic effect.

However, there is a great conceptual and experimental difference. According to the electromagnetic theory, the interaction force between two parallel conductors is due to a magnetic flux which is perpendicular to the plane defined by the conductors. On the other hand, in the relativistic electrodynamics, the interaction force between the two parallel conductors is due to an electric field contained in the plane of the conductors and perpendicular to them. That is, the forces move in the direction of the electric field.

2 Rectilinear conductor and solenoid interaction

The experiment on which this publication is based, is a modification of Ampere's experiment, in which one of the rectilinear conductors has been replaced by a solenoid or coil, whose length is much greater than its diameter. According to the electromagnetic theory, a great magnetic flux density is generated inside the solenoid and a very weak flow outside and near the center of the solenoid. In this case, it is said that the magnetic flux is confined within the solenoid as it is in a toroidal coil.

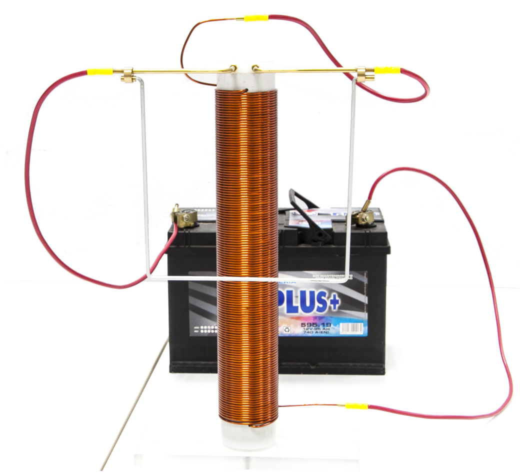

Figure 1 Image of a relativistic solenoid

The experiment consists on the interaction between the currents of the circular turns at the solenoid center with a diameter D = 6 cm, height H = 36.5 cm and 147 loops around a rectilinear conductor of 21 cm and 9.16g mass, contained in the turn plane and at a distance d = 2.3 cm to its border. Fig. 1 and 2.

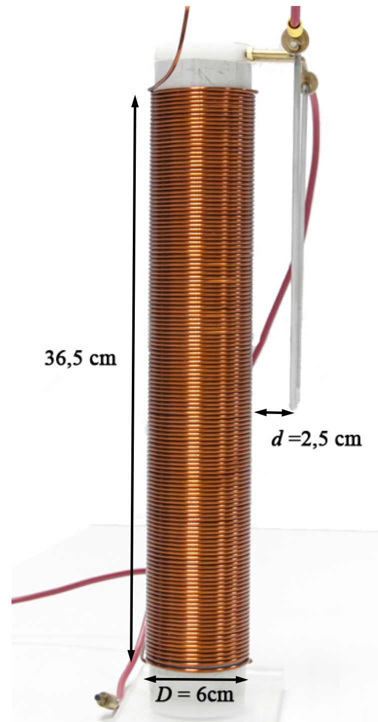

Figure 2 Relativistic solenoid dimensions

When connecting the circuit so that the current in the rectilinear conductor is in the same direction as the current near the coil, an attraction force appears, fig. 3. We found that a repulsive force appears if we connect the circuit so that the current in the rectilinear conductor is opposite direction to the current next to the coil.

The experimentally observed strong force is difficult to explain from the point of view of the magnetic field, since in the exterior, and close to the coil's center, the magnetic field can be considered null. The explanation of the experiment only makes sense considering the relativistic electrodynamics.

3 Scheme between the solenoid and the rectilinear current

Using as a starting point the relativistic argument used by Edward Purcell[6], the interaction between the rectilinear conductor and the solenoid is analyzed in the plane formed by the rectilinear conductor and the transversal

Figure 3 Attraction force on the rectilinear conductor

section of the solenoid closest to the rectilinear conductor in order to simplify the calculation.

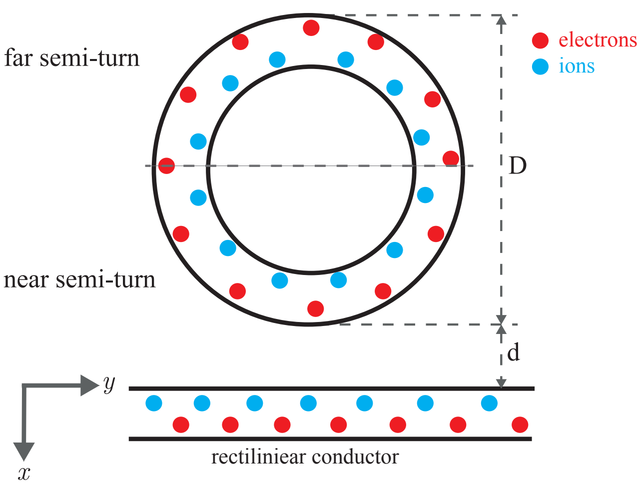

Figure 4 Scheme solenoid-rectilinear conductor

The force acting on the rectilinear conductor is due to the relativistic interaction between the charges of the solenoid and the conductor. It is calculated, first, the force exerted on the electrons and, later, on the ions of the rectilinear conductor on the side of the near and far semi-turns. On the grounds of the relativistic interaction between the rectilinear conductor and the semi-circumferences, there is no difference. That is, for the purposes of calculation, we consider equivalent to the interaction of two parallel rectilinear conductors.

4 Interaction on the rectilinear conductor electrons

4.1 Near semi-turn influence

It is considered, first, the scheme on the laboratory system, where the electrons of the rectilinear conductor move at a speed v_- and the electrons of the closest semi-turn move in the same direction and speed v_- . In the laboratory system the ions are at rest. By simplifying the scheme on an electron with q charge on the rectilinear conductor, it is obtained:

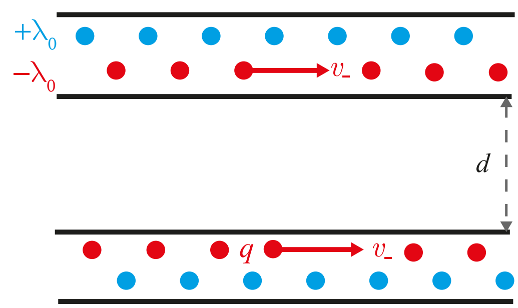

Figure 5 Electron laboratory system for near interaction

and the Lorentz factor is:

$$\beta_0 = \beta = \frac{v}{c} \quad \gamma_0 = \gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{1}$$

In the inertial Σ' system of q charge, the speed is zero. The relative speed of the electrons in the semi-turn is $v'_0 = 0$ and the relative speed of ions is $-v$

$$v'_- = \frac{v_- - v_-}{1 - \frac{v_- v_-}{c^2}} = 0 \tag{2}$$

$$\begin{aligned} \beta'_- &= \frac{v'_-}{c} = 0 \\ \gamma'_- &= 1 \end{aligned} \tag{3}$$

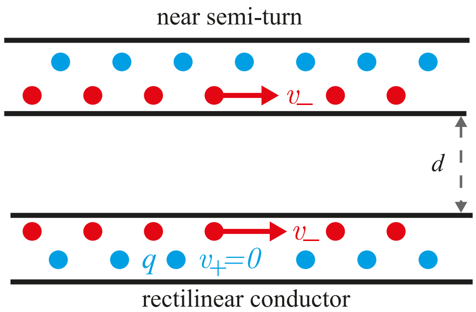

Figure 6 The Σ' system electrons for the near interaction

In the Σ' system the electron's speed is smaller than in the laboratory system and, therefore, its relative distance increases with respect to the laboratory system. Consequently, the linear density of negative charge decreases in a factor: $\frac{\lambda_0}{\gamma_0} = \frac{\lambda_0}{\gamma}$ in the referenceΣ'system. The negative charge density in Σ' is determined by:

$$\lambda'_- = \frac{\lambda_0}{\gamma}\gamma'_- = \frac{\lambda_0}{\gamma} \tag{4}$$

The ions gain a $-v\hat{y}$ speed and, thus, suffer a relativistic contraction in a $\sqrt{1-\beta^2} = \frac{1}{\gamma}$ factor, increasing the positive charge linear density in $\lambda'_+ = \lambda_0\gamma$

Then, the linear charge total density in the Σ' system is:

$$\lambda' = \lambda'_+ - \lambda'_- = \lambda_0\gamma\beta^2 \tag{5}$$

A positive charge total density appears that generates a $\mathbf{E}'_{x'}$ electric field, in the vecinity of the q charge, which we can approximate as:

$$\mathbf{E}'_{x'} = \frac{\lambda'}{2\pi\varepsilon_0 d}\hat{x} = \frac{\lambda_0\gamma\beta^2}{2\pi\varepsilon_0 d}\hat{x} \tag{6}$$

The Σ' force on q charge is determined by:

$$\mathbf{F}'_{x'} = q\mathbf{E}'_{x'} = \frac{q\lambda_0\gamma\beta^2}{2\pi\varepsilon_0 d}\hat{x} \tag{7}$$

The force observed in the laboratory system and considering that $q = -e$

$$\mathbf{F}_1 = \frac{1}{\gamma}\mathbf{F}'_{x'} = -\frac{e\lambda_0\beta^2}{2\pi\varepsilon_0 d}\hat{x} = -\frac{evI_0}{2\pi\varepsilon_0 c^2 d}\hat{x} \tag{8}$$

An attractive force on the rectilinear conductor electrons appears.

4.2 Far semi-turn influence

The laboratory system simplified scheme where the solenoid electrons move at the same speed than the rectilinear conductors electrons, but in the opposite sense, while the ions remain still. The inertial Σ'' system in q

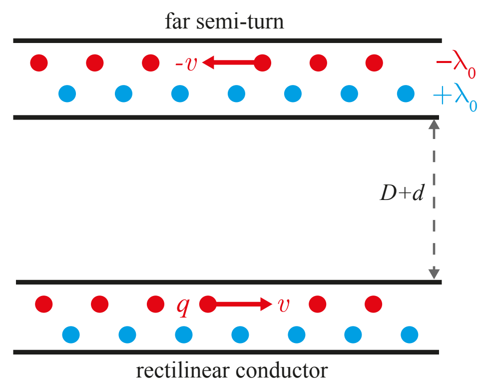

Figure 7 Electrons laboratory system with far interaction

charge, with respect to the charge movement in far semi-turns.

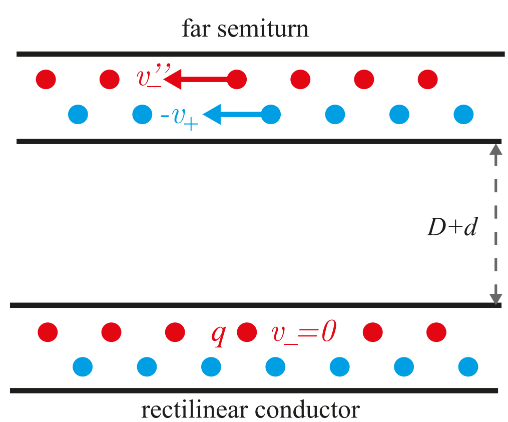

Figure 8 The Σ'' system

In Σ'' as in Σ' the ions undergo a $\sqrt{1-\beta^2}$ relativistic contraction, leading to an increase in the positive charge linear density $\lambda''_+ = \lambda_0\gamma$.

Instead the electrons in Σ'' move faster than in the laboratory system and, therefore, suffer greater relativistic contraction. The relative speed in Σ'' is determined by the sum of relativistic speeds.

$$v''_- = \frac{-v_- - v_-}{1 + \frac{v_-^2}{c^2}} = -\frac{2v_-}{1 + \frac{v_-^2}{c^2}} \tag{9}$$

and Lorentz factor γ''_- $\gamma''_- = \frac{1}{\sqrt{1-\beta''^2_-}}$ where $\beta''_- = \frac{2\beta}{1+\beta^2}$ being its value:

$$\gamma''_0 = \gamma^2 \left(1+\beta^2\right) \tag{10}$$

The negative charge linear density in Σ'' is, thus:

$$\lambda''_- = \frac{\lambda_0}{\gamma}\gamma''_- = \lambda_0\gamma\left(1+\beta^2\right) \tag{11}$$

and the total charge linear density in σ'' is

$$\lambda'' = \lambda''_+ - \lambda''_- = \lambda_0\gamma - \lambda_0\gamma\left(1+\beta^2\right) = -\lambda_0\gamma\beta^2 \tag{12}$$

An equal charge density than in σ' appears, but opposite direction, the $\mathbf{E}''_{x''}$ electric field is:

$$\mathbf{E}''_{x''} = \frac{\lambda''}{2\pi\varepsilon_0(D+d)}\hat{x}'' = -\frac{\lambda_0\gamma\beta^2}{2\pi\varepsilon_0(D+d)}\hat{x}'' \tag{13}$$

The force on q is:

$$\mathbf{F}''_{x''} = q\mathbf{E}''_{x''} = -\frac{q\lambda_0\gamma\beta^2}{2\pi\varepsilon_0(D+d)}\hat{x}'' \tag{14}$$

In the laboratory system and considering $q = -e$ is finally obtained:

$$\mathbf{F}_2 = \frac{1}{\gamma}\mathbf{F}''_{x''} = \frac{e\lambda_0\beta^2}{2\pi\varepsilon_0(D+d)}\hat{x} \tag{15}$$

which we observe is a repulsive force.

4.3 Total force on the rectilinear conductor electrons

The total force produced by the solenoid charges on an electron located in the rectilinear conductor is determined by $\mathbf{F}_e = \mathbf{F}_1 + \mathbf{F}_2$ an obtained

$$\mathbf{F}_e = \frac{e\lambda_0\beta^2}{2\pi\varepsilon_0}\left[\frac{D}{(D+d)\,d}\right]\hat{x} \tag{16}$$

Where a resulting attractive force is observed. In the case of $D \gg d$, it is simplified to:

$$\mathbf{F}_e \simeq -\frac{e\lambda_0\beta^2}{2\pi\varepsilon_0}\frac{1}{d}\hat{x} \tag{17}$$

The attraction force increases as we approach to the solenoid. In contrast to the established version by the classical electromagnetic theory that predicts a very weak external magnetic field and decreases as it approaches to the solenoid.

5 Interaction on the rectilinear conductor electrons

Below, it is considered the solenoid charges interaction on the rectilinear conductor ions. For simplicity, it is calculated, first, the near solenoid semi-turn force, and subsequently the far solenoid semi-turn force.

5.1 Near semi-turn influence on the ions

Interaction of the ions with the near semi-turn of the solenoid in the laboratory system. In the laboratory system, ions are idle and observe solenoid electrons move at a speed $\mathbf{v} = v_-\hat{y}$ These contract at a $\frac{1}{\gamma}$ factor and their negative charge density increases at $\lambda_0\gamma$. The positive charge density does not change for ions, thus the total charge linear density observed in this case is:

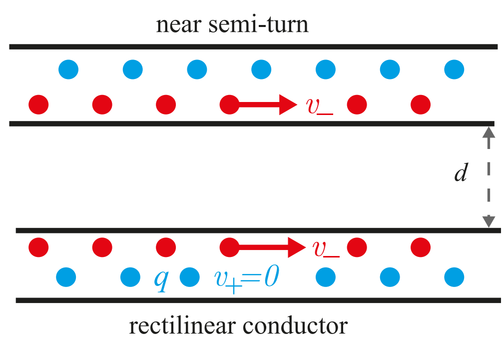

Figure 9 Ions system laboratory with near interaction

$$\lambda_{p_1} = \lambda_0 - \lambda_0\gamma = \lambda_0\left(1-\gamma\right) \tag{18}$$

which is a negative charge linear density. The electric field associated with this charge density is:

$$\mathbf{E}_{p_1} = \frac{\lambda_p}{2\pi\varepsilon_0 d}\hat{x} = \frac{\lambda_0(1-\gamma)}{2\pi\varepsilon_0 d}\hat{x} \tag{19}$$

and the ion force on the charge $q = e$,

$$\mathbf{F}_{p_1} = \frac{e\lambda_0(1-\gamma)}{2\pi\varepsilon_0 d}\hat{x} = -\frac{e\lambda_0(\gamma-1)}{2\pi\varepsilon_0 d}\hat{x} \tag{20}$$

which is an attractive force by being $\gamma > 1$

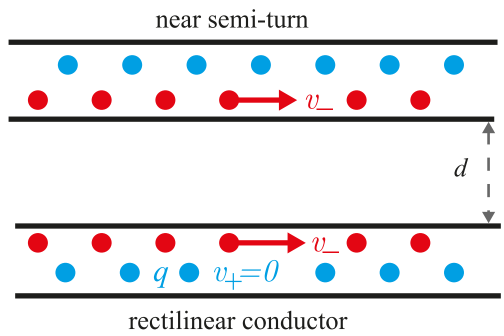

Figure 10 Ions laboratory system with far interaction

5.2 Far semi-turn influence on ions

In the scheme in the laboratory system:

We can see that the situation is symmetrical in relation to the near semi-turn, but the distance is now D+d. The force is then:

$$\mathbf{F}_{p_2} = -\frac{e\lambda_0(\gamma-1)}{2\pi\varepsilon_0(D+d)}\hat{x} \tag{21}$$

5.3 Total force on the ions

The total force on the rectilinear conductor ions is attractive and at a value of:

$$\mathbf{F}_p = \mathbf{F}_{p_1} + \mathbf{F}_{p_2} = -\frac{e\lambda_0(\gamma-1)}{2\pi\varepsilon_0}\left[\frac{1}{D+d}+\frac{1}{d}\right]\hat{x} \tag{22}$$

The presence of an attractive total force on the rectilinear conductor ions at rest in the laboratory system cannot be explained by classic electromagnetics theory. Lorentz force acts only when a charge is moving inside a magnetic field. In this case, the ions gain no speed, but a force acts due to the relative speed between them and the solenoid electrons.

6 Total interaction on the rectilinear conductor

The total interaction between the solenoid and the rectilinear conductor is a consequence of the movement relativity of different sign charges. In this approximate resolution it is obtained that $\mathbf{F}_T = \mathbf{F}_e + \mathbf{F}_p$ and simplifying, the total attractive force result per unit of length on the rectilinear conductor is obtained:

$$\mathbf{F}_T = -\frac{e\lambda_0}{2\pi\varepsilon_0}\left\{\beta^2\left[\frac{1}{d}-\frac{1}{D+d}\right]+(\gamma-1)\left[\frac{1}{d}+\frac{1}{D+d}\right]\right\}\hat{x} \tag{23}$$

Considering a solenoid diameter much greater than the distance to the rectilinear conductor $D \gg d$, an inverse proportional relationship to d is obtained:

$$\mathbf{F}_T = -\frac{e\lambda_0}{2\pi\varepsilon_0}\frac{1}{d}\left\{\beta^2+\gamma-1\right\}\hat{x} \tag{24}$$

7 Currents interaction interpretation

As it has been proven, the interaction between a circle and a rectilinear current, can be interpreted as the current of two opposite current semi-turns interacting with the rectilinear current.

If the d distance between the rectilinear current on the border of the circular turn is very large in relation to the diameter of the turn, the interaction force tends to be canceled. Conversely, if the d distance is approximately equal to the D diameter of the turn, the opposite forces exerted by the two rectilinear semi-turns on the rectilinear currents do not cancel.

If the direction of the near and rectilinear semi-turn currents is the same, then, an attractive force results. Conversely, if the direction of the near and rectilinear semi-turn currents is opposed, the resulting force is repulsive. These attraction or repulsion forces increase when decreasing the d distance.

These forces are detectable in the center of a compact, length-indefinite solenoid. They grow by increasing the diameter of the solenoid and its current intensity and also by reducing the distance to the border of the solenoid. These forces cannot be attributed to the solenoid magnetic flux for the following reasons: It is assumed that the supposed magnetic flux is confined within the solenoid. And in the exterior, the flux density decreases with distance and as it approaches its center.

It is possible to see that the attraction or repulsion force that the solenoid produces on the rectilinear current, transforming it into a pendulum or modified ampere balance, increases when approaching the solenoid central turns, contrary to the idea of decreasing and annulling itself if it were due to magnetic flux.

This simple experiment confirms the relativistic electrodynamics and denies the existence of magnetic flux. The existence of this relativistic macroscopic electric field can explain the Aharanov-Bohn[7] quantum effect without resorting to the mathematical artifice of the potential vector.

The presence of relativistic macroscopic electric field must necessarily act on the charges of the Aharanov-Bohm experiment, explaining its effect without the existence of the potential vector outside the solenoid. Edward Purcell mentioned in his book[8] on electromagnetism:

“If we had to analyze every system of moving charges by transforming back and forth among various coordinate systems, our task would grow both tedious and confusing. There is a better way. The overall effect of one current on another, or of a current on a moving charge, can be described completely and concisely by working with the magnetic field”

The recognition of the magnetic field as a mathematical artifice to facilitate the calculation of the interaction forces between currents is really important to understand that the widespread belief of the physical existence of a perpendicular field to the forces, which we call magnetic, is a misconception that hinders the understanding of physics. So must have thought Einstein because, when discussing an article by Planck[9], he said:

"The essential thing is the content, not mathematics. With mathematics anything can be proven”

Therefore, Faraday’s disk and law of induction, which gave rise to the magnetic field flux, are not understandable[10]. Not even because the disk is an exception to Faraday’s law of induction[11],[12]. Because of this exception that cannot be explained and questions the law of induction, Faraday disk has been removed from the literature on electromagnetism, despite its scientific and historical importance. In his incomprehension, Einstein discovered relativity.

In future articles, it will be shown, by using simple experiments, other exceptions and paradoxes due to the magnetic flux.

8 Conclusions

Through elementary experiments on electromagnetism, it is possible to demonstrate its relativistic nature. Space contraction, due to the speed, varies the linear density of different sign charges in its relative movement inside the conductors, breaking its neutrality. This neutrality lost generates a perpendicular electric field to the conductors, generating forces in the direction of the electric field.

Surprisingly, the relativistic contraction due to the small drift speed of mm/s charges, seems to be the source of electromagnetic forces. However, when we consider the large amount of charges simultaneously in motion and in the same direction, in the order of 10^{20} charges, agree with experimental results.

This simple experiment demonstrates that due to the absence of a magnetic field, the quantum effect attributed to the vector potential of this field, is actually a relativistic electrodynamics macroscopic effect. And we directly observe that the interaction forces between currents are perpendicular to the conductors and parallel to the relativistic electric field. Hence, without the need of assuming the existence of a magnetic field perpendicular to the forces direction.

To be able to see and interpret correctly the relativistic forces, small circuits and high current intensity are required. The presence of magnets or big coils, it make us to believe that these forces are due to a magnetic field perpendicular to them.

Acknowledgements. We thank MSc. Jhony M. Jaimes for the discussion and contribution to translate this paper.

Key words. Special Relativity, Aharonov-Bohm Effect, electrodynamics, magnetic flux,

References

[1] Haustein, Mario, Chemnitz University of Technology (2009).
[2] Calaprice, Alice, The Einstein Almanach (The Johns Hopkins University Press, 2005), p. 45.
[3] Faraday, Michael and Philosophical Transactions R.Soc.Lond. **122**, 125 (1832).
[4] Einstein, Albert, Annalen der Physik **17**, 891 (1905).
[5] Ampere, A.M, Annales de chimie et de physique **15**, 59 (1820).
[6] Edward M.Purcell, David J. Morin, and C. Thirdauthor, Electricity and Magnetism (Cambridge University Press,2013)
[7] Aharonov, Y., Bohm, D. Phys.Rev **115**, 485 (1959)
[8] Edward M.Purcell, David J. Morin, and Electricity and Magnetism (Cambridge University Press, City, 2013), p. 267.
[9] Denis Brian, Einstein (Acento Ediciones, 2004)
[10] Serra Valls, A., Gago Bousquet, C.. AJP **38**, 1273 (1970)
[11] R.P. Feynman, R.B. Leighton, and M. Sans, The Feynman Lectures on Physics (Addison-Wesley, New York, 1964), Vol. II
[12] Serra Valls, A., ArXiv Physics **0012009**,(2000)