Four conjectures on the primes p^2+18m and q^2-18n between the squares p^2, q^2 of a pair of twin primes

Abstract. In this paper I make the following four conjectures: (I) there exist always a prime of the form p^2 + 18*m between the squares p^2 and q^2 of a pair of twin primes [p, q = p + 2], beside the pair [3, 5]; examples: for [p^2, q^2] = [11^2, 13^2] = [121, 169] there exist the primes 139 = 121 + 1*18) and 157 = 121 + 2*18; for [p^2, q^2] = [17^2, 19^2] = [289, 361] there exist the prime 307 = 289 + 1*18); (II) there exist always a prime of the form q^2 – 18*n between the squares p^2 and q^2 of a pair of twin primes [p, q = p + 2], beside the pair [3, 5]; examples: for [p^2, q^2] = [11^2, 13^2] = [121, 169] there exist the prime 151 = 169 – 1*18); for [p^2, q^2] = [17^2, 19^2] = [289, 361] there exist the prime 307 = 361 – 3*18); (III) there exist an infinity of r primes of the form p^2 + 18*m or q^2 – 18*n between the squares p^2 and q^2 of a pair of twin primes [p, q = p + 2] such that the number obtained concatenating p^2 to the right with r is prime; example: 121139 is prime; (IV) there exist an infinity of r primes of the form p^2 + 18*m or q^2 - 18*n between the squares p^2 and q^2 of a pair of twin primes [p, q = p + 2] such that the number obtained concatenating q^2 to the left with r is prime; example: 139169 is prime. Of course, the conjectures (III) and (IV) imply that there exist an infinity of pairs of twin primes.

Conjecture 1:

There exist always a prime of the form p^2 + 18*m between the squares p^2 and q^2 of a pair of twin primes [p, q = p + 2], beside the pair [3, 5].

Verifying the conjecture:
(for the first n pairs of twin primes beside [3, 5])

: for [p^2, q^2] = [5^2, 7^2] = [25, 49] there exist the prime 43 = 25 + 1*18;
: for [p^2, q^2] = [11^2, 13^2] = [121, 169] there exist the prime 139 = 121 + 1*18 and 157 = 121 + 2*18;
: for [p^2, q^2] = [17^2, 19^2] = [289, 361] there exist the prime 307 = 289 + 1*18;
: for [p^2, q^2] = [29^2, 31^2] = [841, 961] there exist the primes 859 = 841 + 1*18 and 877 = 841 + 2*18;

: for [p^2, q^2] = [41^2, 43^2] = [1681, 1849] there exist the primes 1699 = 1681 + 1*18 and 1753 = 1681 + 4*18 and 1789 = 1681 + 6*18;
: for [p^2, q^2] = [59^2, 61^2] = [3481, 3721] there exist the primes 3499 = 3481 + 1*18 and 3517 = 3481 + 2*18 and 3571 = 3481 + 5*18 and 3607 = 3481 + 7*18 and 3643 = 3481 + 9*18 and 3697 = 3481 + 12*18.

Conjecture 2:

There exist always a prime of the form q^2 – 18*n between the squares p^2 and q^2 of a pair of twin primes [p, q = p + 2], beside the pair [3, 5].

Verifying the conjecture:
(for the first n pairs of twin primes beside [3, 5])

: for [p^2, q^2] = [5^2, 7^2] = [25, 49] there exist the prime 31 = 49 – 1*18;
: for [p^2, q^2] = [11^2, 13^2] = [121, 169] there exist the prime 151 = 169 – 1*18;
: for [p^2, q^2] = [17^2, 19^2] = [289, 361] there exist the prime 307 = 361 – 3*18;
: for [p^2, q^2] = [29^2, 31^2] = [841, 961] there exist the primes 907 = 961 – 3*18 and 853 = 961 – 6*18;
: for [p^2, q^2] = [41^2, 43^2] = [1681, 1849] there exist the primes 1831 = 1849 – 1*18 and 1777 = 1849 – 4*18 and 1759 = 1849 – 5*18 and 1741 = 1849 – 6*18 and 1723 = 1849 – 7*18;
: for [p^2, q^2] = [59^2, 61^2] = [3481, 3721] there exist the primes 3631 = 3721 - 5*18 and 3613 = 3721 – 6*18 and 3599 = 3721 – 9*18 and 3541 = 3721 – 10*18.

Conjecture 3:

There exist an infinity of r primes of the form p^2 + 18*m or q^2 - 18*n between the squares p^2 and q^2 of a pair of twin primes [p, q = p + 2] such that the number obtained concatenating p^2 to the right with r is prime.

The sequence of such primes:

: 2531, obtained for [p^2, r, q^2] = [25, 43, 49];
: 2543, obtained for [p^2, r, q^2] = [25, 43, 49];
: 121139, obtained for [p^2, r, q^2] = [121, 139, 169];
: 121151, obtained for [p^2, r, q^2] = [121, 151, 169];

: 121157, obtained for [p^2, r, q^2] = [121, 157, 169];
: 841859, obtained for [p^2, r, q^2] = [841, 859, 961];
: 16811741, obtained for [p^2, r, q^2] = [1681, 1741, 1849];
: 16811831, obtained for [p^2, r, q^2] = [1681, 1831, 1849];
: 34813607, obtained for [p^2, r, q^2] = [3481, 3607, 3721];
: 34813613, obtained for [p^2, r, q^2] = [3481, 3613, 3721];
: 34813631, obtained for [p^2, r, q^2] = [3481, 3631, 3721];
(...)

Conjecture 4:

There exist an infinity of r primes of the form p^2 + 18*m or q^2 - 18*n between the squares p^2 and q^2 of a pair of twin primes [p, q = p + 2] such that the number obtained concatenating q^2 to the left with r is prime.

The sequence of such primes:

: 4349, obtained for [p^2, r, q^2] = [25, 43, 49];
: 139169, obtained for [p^2, r, q^2] = [121, 139, 169];
: 151169, obtained for [p^2, r, q^2] = [121, 151, 169];
: 307361, obtained for [p^2, r, q^2] = [289, 307, 361];
: 35713721, obtained for [p^2, r, q^2] = [3481, 3571, 3721];
(..)