Primes obtained concatenating two primes with the same digital root respectively digital sum

Abstract. In this paper I make the following three conjectures: (I) for any k having one of the values 1, 2, 4, 5, 7 or 8, there exist an infinity of primes obtained concatenating two primes that both have the digital root equal to k; (II) for any n positive integer, not divisible by 3, n ≥ 4, there exist primes obtained concatenating two primes that both have the digital sum equal to n; (III) there exist an infinity of values of n, positive integer, for which exist an infinity of primes obtained concatenating two primes that both have the digital sum equal to n.

Conjecture 1:

For any k having one of the values 1, 2, 4, 5, 7 or 8, there exist an infinity of primes p obtained concatenating two primes q1 and q2 that both have the digital root equal to k.

Note: the operator "\\" it will be used with the meaning "concatenated to".

The sequence of p when dr(q1) = dr(q2) = 1:
: 1973 (19\\73), 3719 (37\\19), 10937 (109\\37), 10973 (109\\73), 19163 (19\\163), 19181 (19\\181)...

The sequence of p when dr(q1) = dr(q2) = 2:
: 1129 (11\\29), 8311 (83\\11), 8329 (83\\29), 10111 (101\\11), 11173 (11\\173)...

The sequence of p when dr(q1) = dr(q2) = 4:
: 1367 (13\\67), 3167 (31\\67), 10313 (103\\13), 10331 (103\\31), 13103 (13\\103)...

The sequence of p when dr(q1) = dr(q2) = 5:
: 523 (5\\23), 541 (5\\41), 2341 (23\\41), 4159 (41\\59), 5113 (5\\113), 5167 (5\\167), 5923 (59\\23)...

The sequence of p when dr(q1) = dr(q2) = 7:
: 617 (61\\7), 743 (7\\43), 761 (7\\61), 797 (7\\97 or 79\\7)...

The sequence of p when dr(q1) = dr(q2) = 8:

: 1753 (17\\53), 1789 (17\\89), 8971 (89\\71), 10753 (107\\53), 10771 (107\\71), 10789 (107\\89), 17107 (17\\107)...

Conjecture 2:

For any n positive integer not divisible by 3, n ≥ 4, there exist primes p obtained concatenating two primes q1 and q2 that both have the digital sum equal to n.

The least prime p obtained for the following values of n:
: for n = 4, p = 10313 (103\\13) is prime;
: for n = 5, p = 523 (5\\23) is prime;
: for n = 7, p = 617 (61\\7) is prime;
: for n = 8, p = 1753 (17\\53) is prime;
: for n = 10, p = 3719 (37\\19) is prime;
: for n = 11, p = 4729 (47\\29) is prime;
: for n = 13, p = 19339 (193\\139) is prime;
: for n = 14, p = 59149 (59\\149) is prime;
: for n = 16, p = 27779 (277\\79) is prime;
: for n = 17, p = 89269 (89\\269) is prime;
: for n = 19, p = 199379 (199\\379) is prime;
: for n = 20, p = 389479 (389\\479) is prime;
: for n = 22, p = 499787 (499\\787) is prime;
: for n = 23, p = 797887 (797\\887) is prime.

Random primes p when s(q1) = s(q2) = 4:
: p = 1000003100000000003,
where q1 = 1000003 and q2 = 100000000003;
: p = 100000000000000003103,
where q1 = 100000000000000003 and q2 = 103;
: p = 1000000000000000031003,
where q1 = 1000000000000000003 and q2 = 1003.

Conjecture 3:

There exist an infinity of values of n, positive integer, for which exist an infinity of primes obtained concatenating two primes that both have the digital sum equal to n.