Primes obtained concatenating p-1 with q^2 where p and q are primes or Poulet numbers

Abstract. In this paper I make the following eight conjectures: (Ia) for any p prime, p > 3, there exist an infinity of primes q such that the number n obtained concatenating p – 1 to the right with q^2 is prime; (Ib) there exist an infinity of terms in any of the sequences above (for any p) such that r = (p – 1)*q^2 + 1 is prime; (IIa) for any q prime, q > 3, there exist an infinity of primes p such that the number n obtained concatenating q^2 to the left with p – 1 is prime; (IIb) there exist an infinity of terms in any of the sequences above (for any q) such that r = (p – 1)*q^2 + 1 is prime; (IIIa) for any Poulet number P, not divisible by 3, there exist an infinity of primes q such that the number n obtained concatenating P – 1 to the right with q^2 is prime; (IIIb) there exist an infinity of terms in any of the sequences above (for any P) such that r = (P – 1)*q^2 + 1 is prime; (IVa) for any Poulet number Q, not divisible by 3 or 5, there exist an infinity of primes p such that the number n obtained concatenating Q^2 to the left with p - 1 is prime; (IVb) there exist an infinity of terms in any of the sequences above (for any Q) such that r = (p – 1)*Q^2 + 1 is prime.

Conjecture 1a:

For any p prime, p > 3, there exist an infinity of primes q such that the number n obtained concatenating p – 1 to the right with q^2 is prime (example: for p = 5, the number n = 449 obtained for q = 7 is prime).

The sequence of primes n for p = 5:

: 449, 4289, 41681, 41849, 42209, 43481, 43721, 45329, 46889, 49409 (...) obtained for q = 7, 17, 41, 59, 61, 79, 83, 97 (...)

The sequence of primes n for p = 7:

: 6121, 6361, 6529, 6841, 6961, 61681, 66889 (...) obtained for q = 7, 19, 23, 29, 31, 41, 83 (...) [note the chain of four primes n (6361, 6529, 6841, 6961) obtained for four consecutive primes q (19, 23, 29, 31]

The sequence of primes n for p = 11:

: 1049,10169, 10289, 10529, 101681 (...) obtained for q = 7, 13, 17, 23, 41 (...)

The sequence of primes n for p = 13:

: 1249, 12289, 12841, 121369, 122209, 124489, 125329, 126241, 127921 (...) obtained for q = 7, 17, 29, 37, 47, 67, 73, 79, 89 (...)

The sequence of primes n for p = 17:

: 16361, 16529, 162209, 163481, 165041, 169409 (...) obtained for q = 19, 23, 47, 59, 71, 97 (...)

Conjecture 1b:

There exist an infinity of terms in any of the sequences above (for any p) such that r = (p – 1)*q^2 + 1 is prime.

The sequence of primes r for p = 5:

: 197 (= 4*49 + 1), 8837 (= 4*2209 + 1), 21317 (= 4*5329 + 1)...

The sequence of primes r for p = 7:

: 727 (= 6*121 + 1), 41047 (= 6*841 + 1)...

The sequence of primes r for p = 11:

: 491 (= 10*49 + 1), 16811 (= 10*1681 + 1)...

The sequence of primes r for p = 13:

: 3469 (= 12*289 + 1), 10093 (= 12*841 + 1), 63949 (= 12*5329 + 1)...

The sequence of primes r for p = 17:

: 55697 (= 16*3481 + 1), 80657 (= 16*5041 + 1)...

Conjecture 2a:

For any q prime, q > 3, there exist an infinity of primes p such that the number n obtained concatenating q^2 to the left with p - 1 is prime;

The sequence of primes n for q^2 = 7^2 = 49:

: 449, 1049, 1249, 3049, 4049, 4649, 5849, 8849, 9649 (...), obtained for p = 5, 11, 13, 31, 41, 47, 59, 89, 97 (...)

The sequence of primes n for q^2 = 11^2 = 121:

: 6121, 18121, 52121, 70121, 78121 (...), obtained for p = 7, 19, 53, 71, 79 (...)

The sequence of primes n for q^2 = 13^2 = 169:

: 10169, 18169, 30169, 40169, 42169, 58169, 60169, 66169, 72169, 88169 (...), obtained for p = 11, 19, 31, 41, 43, 59, 61, 67, 73, 89 (...)

The sequence of n for q^2 = 104729^2 = 10968163441:

: 1810968163441, 4010968163441, 5210968163441, 7810968163441, 8810968163441 (...), obtained for p = 19, 41, 53, 79, 89 (...)

Conjecture 2b:

There exist an infinity of terms in any of the sequences above (for any q) such that r = (p – 1)*q^2 + 1 is prime.

The sequence of primes r for q^2 = 7^2 = 49:

: 197, 491, 1471 (= 30*49 + 1), 2843 (= 58*49 + 1)...

The sequence of primes r for q^2 = 11^2 = 121:

: 727, 2179 (= 18*121 + 1), 9439 (= 70*121 + 1)...

The sequence of primes r for q^2 = 13^2 = 169:

: 6761 (= 40*169 + 1), 9803 (= 58*169 + 1), 10141 (= 60*169 + 1)...

Conjecture 3a:

For any Poulet number P, not divisible by 3, there exist an infinity of primes q such that the number n obtained concatenating P – 1 to the right with q^2 is prime.

The sequence of primes n for P = 341:

: 34049, 340169, 3409409 (...), obtained for q = 11, 13, 97 (...)

The sequence of primes n for P = 1105:

: 1104289, 11041369, 11042209, 11043481, 11044489, 11046241, 11047921 (...), obtained for q = 17, 37, 47, 59, 67, 79, 89 (...)

The sequence of primes n for P = 1387:

: 1386361, 13861369, 13862809, 13867921 (...), obtained for q = 19, 37, 53, 89 (...)

The sequence of primes n for P = 1729:

: 172849, 1728121, 1728361, 17281681, 17283481, 17286889, 17289409 (...), obtained for q = 7, 11, 19, 41, 59, 83, 97 (...)

Conjecture 3b:

There exist an infinity of terms in any of the sequences above (for any P) such that r = (P – 1)*q^2 + 1 is prime.

The sequence of primes r for P = 341:

: 16661 (= 340*49 + 1), 3199061 (= 340*9409 + 1)...

The sequence of primes r for P = 1105:

: 319057 (= 1104*289 + 1)...

The sequence of primes r for P = 1387:

: 14871781 (= 1104*7921 + 1)...

The sequence of primes r for P = 1729:

: 84673 (= 1728*49 + 1), 209089 (= 1728*121 + 1), 6015169 (= 1728*3481 + 1)...

Conjecture 4a:

For any Poulet number Q, not divisible by 3 or 5, there exist an infinity of primes p such that the number n obtained concatenating Q^2 to the left with p - 1 is prime.

The sequence of primes n for Q^2 = 341^2 = 116281:

: 6116281, 18116281, 40116281, 42116281, 58116281, 60116281, 72116281, 78116281 (...), obtained for p = 7, 19, 41, 43, 59, 61, 73, 79 (...)

The sequence of primes n for Q^2 = 1387^2 = 1923769:

: 181923769, 281923769, 881923769 (...), obtained for p = 19, 23, 89 (...)

The sequence of primes n for Q^2 = 1729^2 = 2989441:

: 62989441, 162989441, 222989441, 722989441, 822989441 (...), obtained for p = 7, 17, 23, 72, 82 (...)

Conjecture 4b:

There exist an infinity of terms in any of the sequences above (for any Q) such that r = (p – 1)*Q^2 + 1 is prime.

The sequence of primes r for Q^2 = 341^2 = 116281:

: 697687 (= 6*116281 + 1), 6744299 (= 58*116281 + 1), 6976861 (= 6*116281 + 1), 8372233 (= 72*116281 + 1)...

The sequence of primes r for Q^2 = 1387^2 = 1923769:

: 169291673 (= 88*1923769 + 1)...

The sequence of primes r for Q^2 = 1729^2 = 2989441:

: 65767703 (= 22*2989441 + 1)...