Heisenberg's Potentia in Quantum Mechanics and Discrete Subgroups of Lie Groups

Syed Afsar Abbas
Jafar Sadiq Research Institute
Azim Green Home, New Sir Syed Nagar, Aligarh - 202002, India
(e-mail : drafsarabbas@gmail.com)

Abstract

The concept of "potentia" as proposed by Heisenberg to understand the structure of quantum mechanics, has just remained a fanciful speculation as of now. In this paper we provide a physically consistent and a mathematically justified ontology of this model based on a fundamental role played by the discrete subgroups of the relevant Lie groups. We show that as such, the space of "potentia" arises as a coexisting dual space to the real three dimensional space, while these two sit piggyback on each other, such that the collapse of wave function can be understood in a natural manner. Quantum nonlocality and quantum jumps arise as a natural consequence of this model.

Keywords : quantum mechanics, nonlocality, quantum jumps, wave-function collapse, potentia, discrete subgroups of Lie groups

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Quantum nonlocality, quantum jumps, wave-function collapse; essentially all puzzling aspects of quantum mechanics, are believed to occur in the hypothetical "potentia" as proposed by Heisenberg [1]. However as of now it has just remained a fanciful conjecture. In this paper we develop a consistent physical model with appropriate mathematical structure to map this "potentia" faithfully. We find that the disrete subgroups of the relevant Lie groups, play a basic role to provide a realistic interpretation of the concept of potentia.

The two electron wave function is [2] ($\uparrow \equiv \chi^{1/2}_{1/2}$ and $\downarrow \equiv \chi^{-1/2}_{1/2}$),

$$\psi = \frac{1}{\sqrt{2}}[\uparrow (1) \downarrow (2) - \downarrow (1) \uparrow (2)] \tag{1}$$

This is antisymmetric under the exchange of state labels $\uparrow \leftrightarrow \downarrow$ while the number labels are fixed in the sequence (12). Should we associate these numbers with particle numbers 1 and 2 for the two particles ? However the whole wave function of this two electron system has been built with the explicit aim of ensuring that the particle number has no physical significance [2, Ch. XIV]. So what do we these numbers represent?

Let us take the space in which the two electrons reside as,

$$SU(2)_S \otimes SO(3)_l \tag{2}$$

Here $SO(3)_l$ specifies the three-dimensional x-, y- and z-space. Now for a single electron the particle position in the $SO(3)_l$ space, as the expectation value $< x >$, is a well defined quantity. But no such values $< x_1 >$ and $< x_2 >$ for the position occurs for this composite system of two electrons. So what does it mean that the exact numbers 1 and 2 sit in the function (eqn. 1) with no manifestation of any uncertainty whatsoever.

Here we propose that this sequential number (12) in the above eqn. (1) does not exist in the ordianry $SO(3)_l$ space. It actually exists in the space of "potentia" as proposed by Heisenberg [1]. But wherefrom does this potentia pop up?

Note that the particles are sitting in group space as given in eqn. (2). Here the group structure $SU(2)$ has a centre of Z_2 (addition modulo 2 with elements [0,1]). Then the factor group,

$$\frac{SU(2)_S}{Z_2} \cong SO(3)_p \tag{3}$$

Here given the group structure in eqn. (2), there is no justification in associating the above orthogonal group with the group $SO(3)_l$. We should treat it as another independent SO(3) group and is thus labelled with another subscript "p". Now we identify it with the word "potentia" assuming that this space is defining the space of potentia.

Note that Z_2 is a discrete subgroup which is internal to the group $SU(2)_S$. Hence we label the fundamental representation of SU(2) group with its Z_2 centre elements [0,1] as,

$$\begin{pmatrix} \uparrow (0) \\ \downarrow (1) \end{pmatrix} \tag{4}$$

The justification for the above is as follows.

We know that in the symmetric group for three particles S_3 we have an antisymmmteric state as given by the Young diagram,

$$\begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array} \tag{5}$$

But this is zero for electrons as these are representations of the group $SU(2)_S$. This is normally stated as a manifestation of the fact for the group SU(2) totally antisummetric function occurs for a Young diangram with two boxes in a column. Thus the above Young digram is zero for SU(2) representation states. Thus an extra physical constraint is invoked to ensure the vanishing of the represetation in eqn. (5).

Here we invoke an internal mathematical condition from eqn. (4) to ensure the vanishing of the above state. The above is consistently explained by putting the Z_2 labels in the Young diagram for the SU(2) fundamental representation for two SU(2) particles as,

$$\begin{array}{|c|} \hline 0 \\ \hline 1 \\ \hline \end{array} \tag{6}$$

And thus for three particles the relevant non-zero Young diagram is,

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 1 & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array} \tag{7}$$

Note that the labels $[0, 1] \cong [1, 2]$ and thus in eqn. (1) the sequential labels (12) correspond to the center Z_2 lables. Next how is the subscipt "p" on the orbital group is justified ? It is justified as the centre being a discrete group, the exchange over this space is a jump between 1 and 2 with infinite speed This is instantaneous exchange in this space with $c \rightarrow \infty$, that is infinte velocity. So this exchange occurs over this potentia given by the orbital space $SO(3)_p$. Now due to the instantaneous exchange it is justified to use it to map 'potentia' of Heisenberg.

Clearly the particles which are represented by ($\uparrow$, $\downarrow$) exist simultaneously at corrresponding locations in the ordinary orbital space $SO(3)_l$ and which should be dual to the potentia space $SO(3)_p$. The points at which the particles are defined in both the spaces is what makes these two spaces to sit piggyback on each other. When measuremt is preformed in our $SO(3)_l$ space then the wave function collapse (or better reduction) occurs and nonlocality (as per the state given in eqn.(1)) is manifested in our $SO(3)_l$ space.

It turns out that we have been able to identify the potentia for a two electron system quite easily. How about a single particle state as defined by Schroedinger equation? Now the spherical harmonics $Y^l_m(\theta, \phi)$ are rotation group basis states in our $SO(3)_l$ space [3]. Note that the $SO(3)_l$ transformations are defined by three parameters. So why are the spherical harmonics defined only by two parameters (θ, ϕ) ? Classically to describe a rigid body three parameters are needed. In quantum mechanics one has only point particles and no rigid bodies. Only two parameters are needed to give their angular coordinates. The third coordinate though hidden, still should manifest itself [3, p. 327]. This shows up as the phase of the quantum mechanical wave function $e^{i\phi}\ \psi$. This gives the group $U(1)$ for the phase part of the state. Now given the additive group of the real number line $\mathcal{R}$ and the infinite set of integers $\mathcal{Z}$ then the factor group,

$$\frac{\mathcal{R}}{\mathcal{Z}} \cong U(1) \sim SO(2)_p \tag{8}$$

Now $SO(2)_l$ is a subgroup of the orbital space $SO(3)_l$. However we identify the above $SO(2)_p$ as an independent and different orbital space which is labelled by the set $\mathcal{Z}$. We have taken the cue from the above set Z_2 for the two particle system. Hence we suggest that this potentia space of $SO(2)_p$ which labels the particle in that space by the discrete set $\mathcal{Z}$. Let us propose that the spaces $SO(2)_l$ and $SO(2)_p$ are simultaneous and dual to each other and sitting piggyback on each other. Thus particle travels in this potentia by jumping over the numbers 1,2,3.. instantaneously while it travels continuously and with velocity $v \leq c$ in our orbital space.

When observation is made in the space $SO(2)_l$ then the wave function collapses in such a manner that in the potentia space with jumps in $\mathcal{Z}$ from $1 \to 2 \to 3 \to 4 \to \cdots$ occur. Clearly for a bound state these jumps would correspond to instantaeous quantum jumps in the potentia space. So quantum jumps do not occur in real $SO(2)_l$ space but in the $SO(2)_p$ potentia space with infinite velocity.

We see that there is indeed an absolute space with $c \to \infty$ coexisting with the real space with a finite velocity c. But in quantum mechnanics it manifest itself in the above unique fashion as a potentia for a single particle quantum mechanics and for a two fermion system. This provides a physically consistent and mathematically justified ontology to Heisenberg's potentia to understand how quantum mechanics functions.

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