A true Kurepa conjecture implies Dirichlet-Kurepa primes: Two new classes of infinite primes within arithmetic progressions

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Abstract: The Kurepa conjecture states that the gcd (!n,n!) is equal to 2 for all n=2,3,….Although this conjecture has not been proven, in this paper we study the implication of a true Kurepa conjecture. We use (!n)/2 and (n!)/2 as components of two distinct arithmetic progressions and propose that both should have infinitely many primes as known from Dirichlet's theorem of Arithmetic progressions and these are named as Dirichlet-Kurepa primes Type 1 and Type 2 in their honor.

Results:

According to Kurepa Conjecture, the greatest common divisor of left factorial of n (denoted by !n) and the factorial of n (denoted by n!) is 2 for all positive integers "n" greater than 1. The left factorial for n is defined as
!n=0!+1!+2!+…….+(n-1)!
and n!=1.2.3……..n

The Kurepa conjecture is unverified for all n. However if true, then for all n>1,

gcd(!n,n!)=2 therefore
gcd((!n)/2, (n!)/2)=1 for all n>1

This means positive integers (!n)/2 and (n!)/2 are coprime with each other.

Consider the Arithmetic progression ((!n)/2)+m((n!)/2), this sequence contains infinitely many primes according to Dirichlet's Theorem of Arithmetic progressions (where m=0,1,2,3,……………..). We will call the primes in this sequence as Dirichlet-Kurepa primes Type 1.

Similarly the other arithmetic progression represented by ((n!)/2)+k((!n)/2) would also contain infinite primes for k=1,2,3,…………) according to Dirichlet's theorem. We will call the primes in this sequence as Dirichlet-Kurepa primes Type 2.