Continuity in Choices

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1 Introduction

Consider a sketch about which the artist has no idea prior to the process of drawing, but develops on his way along stepwise. A list of distinct sketches from which he can choose for each step, right from start to the end, is given. Closing the loop marks the last step in the sketch.

The sketch, which are of two types is explained below.

Type 1: Here, anything but the first choice is used for each step. The idea is to make a sketch of close second choices. The process is repeated till the loop is closed.

Type 2: Here, anything but the first and second choice is used for each step. The loop is closed with all the steps made out of third choices.

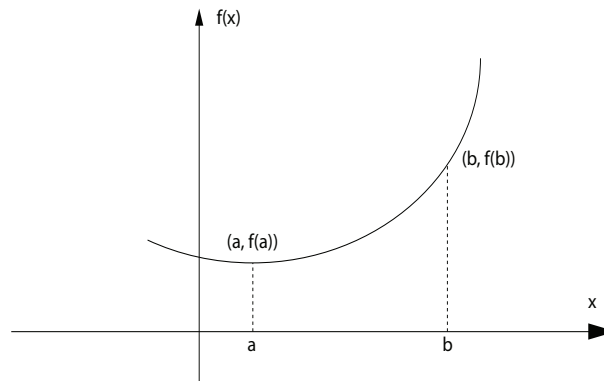## 2 Continuity in Choices for 2D Calculus

Figure 1: fig a

First derivative at a, $\dfrac{df(x)}{dx} = \lim\limits_{(b-a)\to 0} \dfrac{f(b) - f(a)}{b - a}$, and Second derivative at a, $\dfrac{d^2 f(x)}{dx^2} = d\left(\dfrac{df(x)}{dx}\right)$

'n' is the sketch count in the list and 't' and 'l' are the sketches dismissed.

Let the list of sketches be set $(x^1, ..., x^n)$.

$f^{'}(x)$ is the sketch drawn by choosing for each step, anything in the set, but x^t.

$f^{''}(x)$ is the sketch drawn by choosing for each step, anything in the set, but x^t and x^l.

We can redraw the diagram as below.

1. If $f^{''}(x) = 0$, then for some x, $f^{'}(x)$ is a constant. Consequently, f(x) is not a continuous function.

2. If $f^{''}(x) \neq 0$, then $f^{'}(x)$ is never a constant for any x. Consequently, f(x) is a continuous function.

The analogy in the Continuity of choices

1. Function f(x) is the traits specific to the person who selects it.

2. a and a+h are the symbols which we can choose from.

3. Here, output diagram is the connected sybmols within two specific symbols.

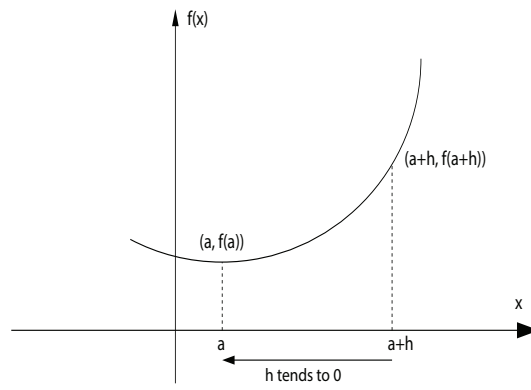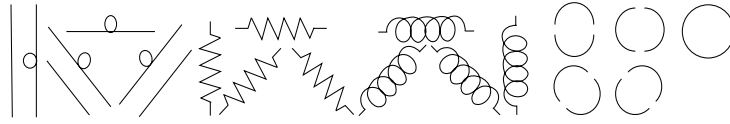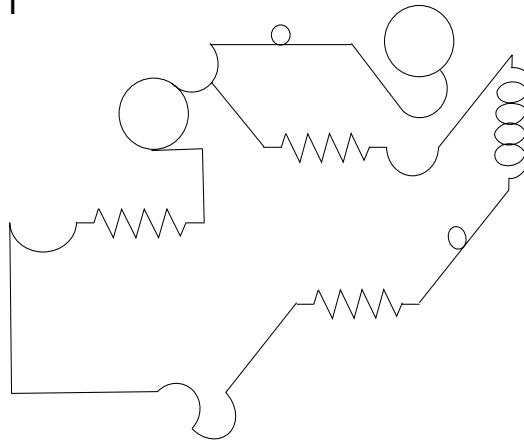*Orcid Id: 0000-0002-3356-3462 Email: vikipr@gmail.com

Figure 2: fig b

These two end symbols correspond to a and a+h respectively.

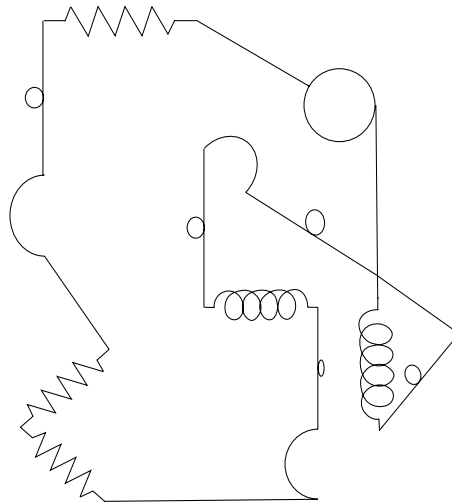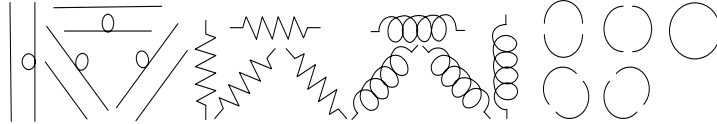Some examples of this idea are added in the following pages.
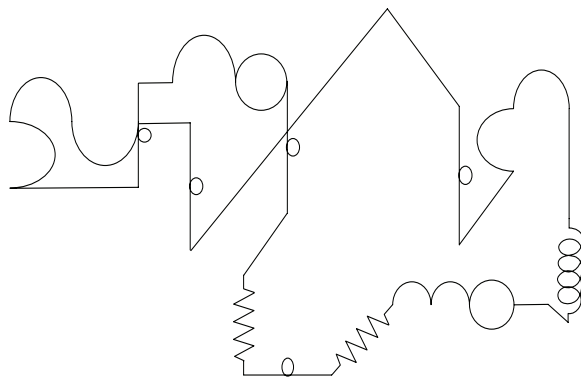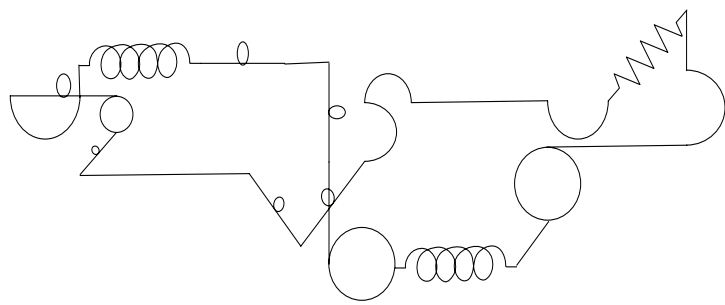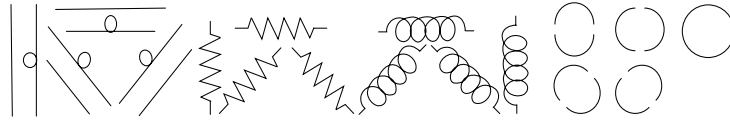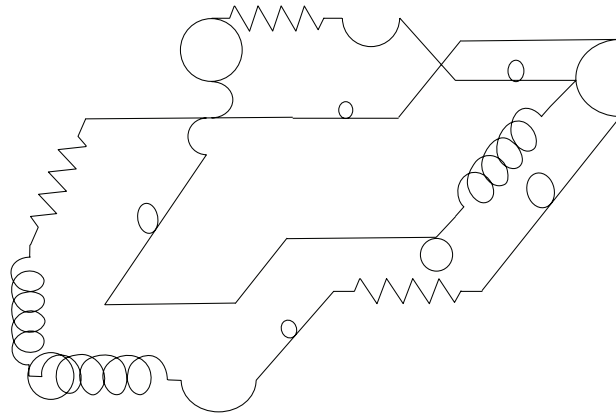
Type 1

Type 2
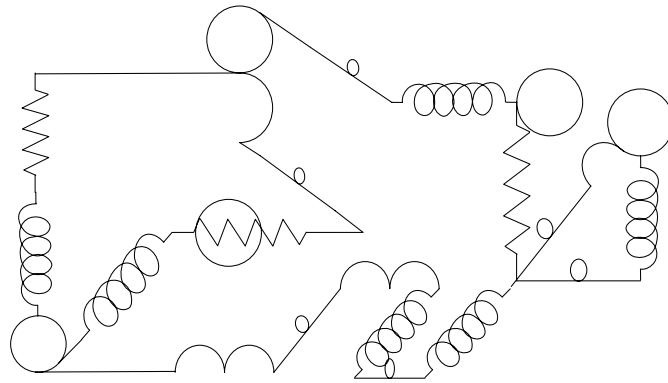
Figure 3: fig f

Type 1

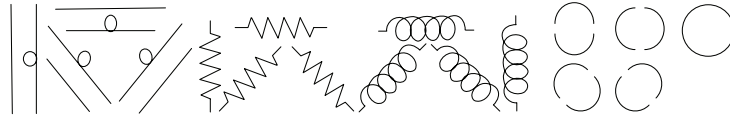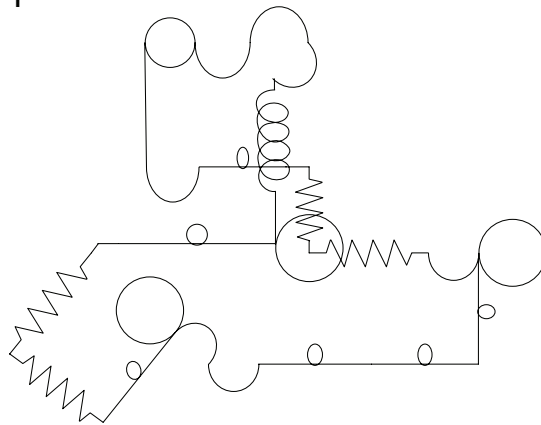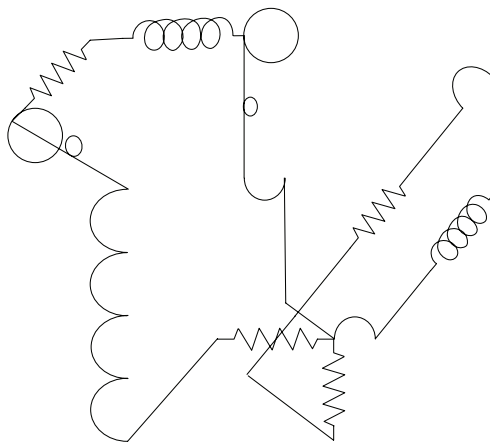Type 2

Figure 4: fig f

Type1

Type2

Figure 5: fig f

Type 1

Type 2

Figure 6: fig f

3   Differentiation in Basic Terms

$$\lambda : R \to R \ \text{(Linear Transformation)}$$
$$\lambda(h) = f^{'}(a) \times h$$
$$\lambda(h) = f(a+h) - f(a)$$

As we can see, $\lambda(h)$ is just the difference between the function of point $(a+h)$ and function of point a

$$\lim_{h \to 0} \frac{f(a+h) - f(a) - \lambda(h)}{h} = 0$$

How are these comparable to Continuity in Choices?

In math, it is actually a leap of thought from transformation within R^n coordinate system to transformation from R^n to R^m coordinate system.

4   Idea of a Differentiable in Manifolds

4.1   Differentiable in 2-Dim

A function $f(a) : R \to R$ is differentiable at $a \in R$, if there is a linear transformation $\lambda(h) : R \to R$ such that

$$\lim_{h \to 0} \frac{f(a+h) - f(a) - \lambda(h)}{h} = 0$$

4.2   Generalizing to Higher Dimensions

A function $f(a) : R^n \to R^m$ is differentiable at $a \in R^n$, if there is a linear transformation $\lambda(h) : R^n \to R^m$ such that

$$\lim_{h \to 0} \frac{\|f(a+h) - f(a) - \lambda(h)\|}{\|h\|} = 0$$

Here, h is a point of R^n. $f(a+h) - f(a) - \lambda(h)$ is a point of R^m. Hence, norm signs are essential.

Note for λ: λ is denoted by $Df(a)$. λ is also called as derivative of f at a or called the linear transformation λ.

Note for h: In a transformation within R^n, h is $(a+h) - a$. h is a distance so small, that it can be considered a point. So, it is also a point in manifolds.

5   Leap of Space

Introduction of λ helps in the leap of space from transformation within a coordinate system to transformation between two coordinate systems.

$$f^{'}(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\lambda(h)}{h}$$

We can see from the above equation that $\lambda(h)$ is equal to $f(a+h) - f(a)$.

$$\lambda(h) = f^{'}(a) \times h$$

In continuity in choices, this means $\lambda(h) = $ (Select first, reject, select another) $\times$ (difference in symbol choices)

6   Pictorial Representation

Here $f'(x)$ refers to type 1. Select one of the symbols. Reject the selected symbol and select another.
Here $f''(x)$ refers to type 2. Select one of the symbols. Reject the selected symbol and select another.
Again, reject the selected symbol and select another.

First Person	Third Person	Second Person
$f'(x)$: Select one for first person. Reject and select another.	Both these selections for first and second person should be same.	$f'(x)$: Select one for second person. Reject and select another.
$f''(x)$: Select one for first person. Reject and select another. Again, Reject and select another.	Both these selections for first and second person should be same.	$f''(x)$: Select one for second person. Reject and select another. Again, Reject and select another.

Table 1: Function takes a point from R^n into R^m

As we can see from the table, third person should select the same symbol for both first and second person. He should be able to predict the selection for both of them, for $f'(x)$ and $f''(x)$.
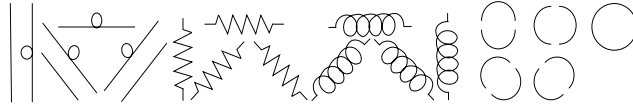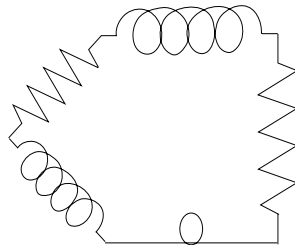

Type 1
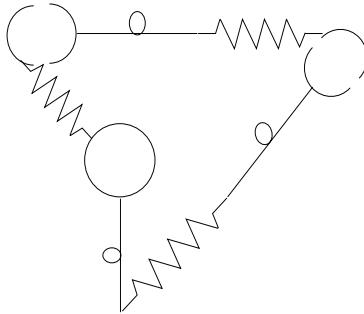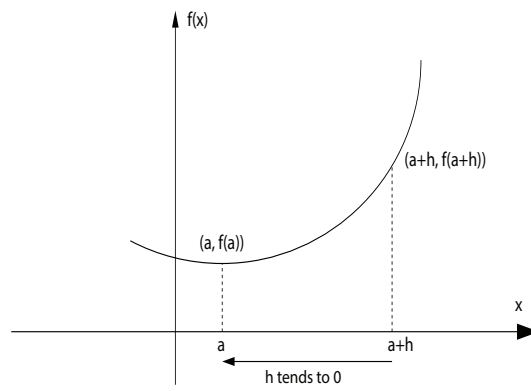

Type 2

Figure 7: fig f

7 $\lambda(h) + f(a)$

Figure 8: fig f

$\lambda(h) + f(a)$ is a good approximation at a.
$\lambda(h) + f(a)$ is equal to the function of the highlighted region in the figure. .