**Any two successive left factorials can represent only the first and second terms of an arithmetic progression of positive integers**

Prashanth R. Rao

Abstract: In this paper we show that any two successive left factorials can only represent the first and second terms and not any other pair of successive terms in any arithmetic progression of positive integers.

Results:

A left factorial of "n" is represented as !n and
!n=0!+1!+2!+……+(n-1)!

Therefore

!(n+1)= 0!+1!+2!+……+(n-1)!+n!

The question we have raised is whether !n and !(n+1), represent the $k^{th}$ and $(k+1)^{th}$ term of an arithmetic progression of the form a+mb where a and b are positive integers and m=0,1,2,3,………..
Let us say that for some non-negative integer k, !n and !(n+1) represent the $k^{th}$ and $(k+1)^{th}$ terms of such arithmetic progression.
Then !n=a+(k-1)b
!(n+1)=a+kb

Therefore !(n+1)-!n=a+kb-a-(k-1)b
or
(0!+1!+2!+……+(n-1)!)+n!-( 0!+1!+2!+……+(n-1)!)=b
or n!=b
Substituting
!n=a+(k-1)(n!)
For all n>2,
$!n<n!$, therefore for any positive integers a and (k-1) and n>2, !n=a+(k-1)(n!) is impossible.
Therefore (k-1) must be 0, i.e., k=1.
Substituting !n=a+0.(n!)=a

Therefore the two successive left factorials !n and !(n+1) are of the form a and a+b.
These are the first and second terms of the arithmetic progression.