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Lorentz Velocity Transformation and Energy Mass and Momentum Transformation

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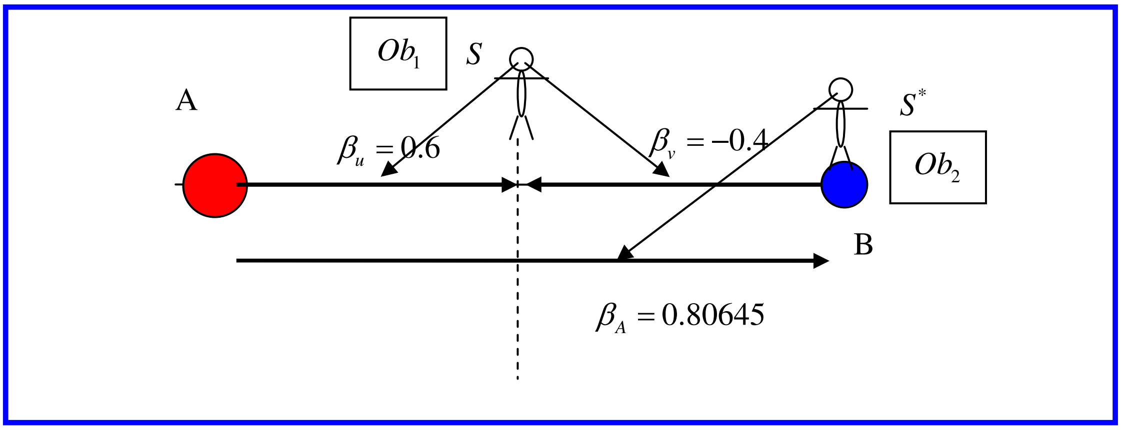
Introduction

Velocity transformation is used to change system coordinates in order to simplify the solution of problems that involve relativistic collisions of particles. In this paper I check how the energy of a particle A that moves with velocity β_A toward an observer S^* is seen by an observer S, under the condition that both observers sees the same energy. I found that although observer in S^* sees particle A with a real rest mass momentum and energy. Observer S sees two particles. If they move in opposite direction, One of those particles have an imaginary rest mass momentum and energy.
I could not decide if the case is only a numerical problem or it is also physical. I expect that those who will read this paper are more experienced then me and will find if the imaginary rest mass is physically true or false. And will suggest how to use it in practical situations.



Lorentz Velocity Transformation

Lorentz Velocity transformation enables us to change observer point of view on moving objects . Now we want to know what happens to the mass energy and momentum under this transformation.

Assume

β_v is the velocity of the B (S^*) measured in the S frame ($S \triangleq$ Earth)

β_u is the velocity of A measured in the S frame and $\beta_u \neq \beta_v$

And β_A is the velocity of A relative to B measured in the S^* frame

Given

$$\beta_u = 0.6$$
$$\beta_v = -0.4$$

β_A is found from the Lorentz velocity transformation

$$\beta_A = \frac{\beta_u - \beta_v}{1-\beta_u\beta_v} = \frac{0.6-(-0.4)}{1-0.6(-0.4)} = 0.80645$$

Let compute the square of the energy E^2 of A as measured by B. remember that A move relative to B with velocity β_A

The relativistic energy is given by

$$E = \frac{m_0c^2}{\sqrt{1-\beta^2}} = \frac{E_0}{\sqrt{1-\beta^2}}$$

And the energy squared is

$$E^2 = \left[\frac{E_0}{\sqrt{1-\beta^2}}\right]^2 = \frac{E_0{}^2}{1-\beta^2} = \frac{E_0{}^2}{2}\left(\frac{1}{(1-\beta)} + \frac{1}{(1+\beta)}\right)$$

The square of the energy is vary useful because the momentum and rest mass are hidden part of it

$$E^2 = p^2c^2 + E_0{}^2 = p^2c^2 + \left(m_0c^2\right)^2$$

Let define

$$\frac{1}{L_0}=\frac{E_0^{\ 2}}{2}=\frac{\left(m_0c^2\right)^2}{2} \qquad\qquad \frac{1}{L}=E^2=\left[\frac{E_0}{\sqrt{1-\beta^2}}\right]^2=\frac{E_0^{\ 2}}{1-\beta^2}$$

So

$$E^2=\frac{1}{L}=\frac{1}{L_0}\left(\frac{1}{1-\beta}+\frac{1}{1+\beta}\right)=\frac{E_0^{\ 2}}{2}\left[\frac{1}{1-\beta}+\frac{1}{1+\beta}\right]$$

Now, and this is the main target of this paper; I want to compute the energy square of A as measured by an observer on B.

Remember that the velocity of A relative to B is

$$\beta_A=\frac{\beta_u-\beta_v}{1-\beta_u\beta_v}$$

The velocity is substituted in the energy

$$E_{\beta_A}^{\ \ 2}=\frac{1}{L_A}=\frac{1}{L_0}\left[\frac{1}{1-\beta_A}+\frac{1}{1+\beta_A}\right]$$

$$E_{\beta_A}^{\ \ 2}=\frac{1}{L_A}=\left(\frac{1}{L_0}\right)\frac{1}{1-\dfrac{\beta_u-\beta_v}{1-\beta_u\beta_v}}+\left(\frac{1}{L_0}\right)\frac{1}{1+\dfrac{\beta_u-\beta_v}{1-\beta_u\beta_v}}$$

And I want to see if the following equation is linear and obey

$$E_{\beta_A}^{\ \ 2}=\frac{1}{L_A}=E_{\beta_u}^{\ \ 2}\oplus E_{\beta_v}^{\ \ 2}$$

The full development of the target equation is given in Appendix A
It is found that as expected

$$E_{\beta_A}^{\ \ 2}=\frac{1}{L_A}=E_{\beta_u}^{\ \ 2}+E_{\beta_v}^{\ \ 2}$$

where

$$E_{\beta_u}^{\ \ 2}=\frac{1}{L_{\beta_u}}=\frac{1}{L_{u0}}\frac{1}{\left(1-\beta_u^{\ 2}\right)}$$

$$E_{\beta_v}^{\ \ 2}=\frac{1}{L_{\beta_v}}=\frac{1}{L_{v0}}\frac{1}{\left(1-\beta_v^{\ 2}\right)}$$

Or a bit more elegant

$$\frac{1}{L_A}=\frac{1}{L_{\beta_u}}+\frac{1}{L_{\beta_v}}$$

Where

$$\frac{1}{L_{u0}}=\left[\frac{2\beta_u\left(1-\beta_u\beta_v\right)}{\left(\beta_v+\beta_u\right)}\right]\frac{1}{L_0}=\left[\frac{2\beta_u\left(1-\beta_u\beta_v\right)}{\left(\beta_v+\beta_u\right)}\right]\frac{E_0^{\ 2}}{2}$$

$$\frac{1}{L_{v0}}=\left[\frac{2\beta_v\left(1-\beta_u\beta_v\right)}{\left(\beta_v+\beta_u\right)}\right]\frac{1}{L_0}=\left[\frac{2\beta_v\left(1-\beta_u\beta_v\right)}{\left(\beta_v+\beta_u\right)}\right]\frac{E_0^{\ 2}}{2}$$

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Pay attention to the following limitation

$$\beta_v + \beta_u \neq 0$$

In many problems it is given that $u = -v$

A numeric example

$$\beta_v = -0.4$$
$$\beta_u = 0.6$$
$$\beta_A = 0.80645$$

If we assume

$$E_0 = 1$$

Then

$$\frac{1}{L_{u0}} = \left[\frac{2\beta_u\left(1-\beta_u\beta_v\right)}{\left(\beta_v+\beta_u\right)}\frac{E_0^{\ 2}}{2}\right] = \left[\frac{2\cdot 0.6\left(1-0.6(-0.4)\right)}{\left((-0.4)+0.6\right)}\frac{E_0^{\ 2}}{2}\right] = 3.72$$

$$\frac{1}{L_{v0}} = \left[\frac{2\beta_v\left(1-\beta_u\beta_v\right)}{\left(\beta_v+\beta_u\right)}\frac{E_0^{\ 2}}{2}\right] = \left[\frac{2\cdot(-0.4)\left(1-0.6(-0.4)\right)}{\left((-0.4)+0.6\right)}\frac{E_0^{\ 2}}{2}\right] = -2.48$$

pay attention to the negative result in the previous equation

$$E_{\beta_A}^{\ 2} = \frac{1}{L_A} = \frac{1}{L_u} + \frac{1}{L_v} = \frac{1}{L_{u0}}\frac{1}{\left(1-\beta_u^{\ 2}\right)} + \frac{1}{L_{v0}}\frac{1}{\left(1-\beta_v^{\ 2}\right)}$$

$$E_{\beta_A}^{\ 2} = \frac{1}{L_A} = \frac{3.72}{\left(1-0.6^2\right)} + \frac{-2.48}{\left(1-(-0.4)^2\right)} = 5.8125 - 2.952 = 2.86$$

If we compute directly using $\beta_A = 0.806$ we get the same result

$$E_{\beta_A}^{\ 2} = \left[\frac{E_0}{\sqrt{1-\beta_{u^*}^{*\ 2}}}\right]^2 = \frac{E_0^{\ 2}}{1-\beta_{u^*}^{*\ 2}}$$

$$E_{\beta_A}^{\ 2} = \frac{1}{1-0.806^2} = 2.86$$

Let now compute the momentum from

$$E^2 = p^2c^2 + E_0^{\ 2}$$

for A

$$E_u^{\ 2} = \frac{E_{0u}^{\ 2}}{1-\beta_u^{\ 2}} = \frac{3.72}{\left(1-0.6^2\right)} = 5.8125$$

$$p_u^{\ 2}c^2 = E_u^{\ 2} - E_{0u}^{\ 2} = 5.8125 - 3.72 = 2.0925$$

and

$$p_u c = \sqrt{2.0925} = \pm 1.446$$

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for B

$$E_v{}^2 = \frac{E_{0v}{}^2}{1-\beta_v{}^2} = \frac{-2.48}{\left(1-0.4^2\right)} = -2.952$$

$$p_v{}^2c^2 = E_v{}^2 - E_{0v}{}^2 = -2.952 - \left(-2.48\right) = -0.472$$

$$E_{0v}{}^2 = \left(m_{0v}c^2\right)^2 = -2.48$$

$$E_{0v} = m_{0v}c^2 = \sqrt{-2.48} = \pm j1.574$$

$$p_v c = \pm j0.687 \qquad j = \sqrt{-1}$$

In that case it follows that the rest mass and momentum are imaginary numbers

$$E_{\beta_A}{}^2 = \frac{E_{0\beta_A}{}^2}{1-\beta_{\beta_A}{}^2} = \frac{1}{1-0.806^2} = 2.86$$

$$p_{\beta_A}{}^2c^2 = E_{\beta_A}{}^2 - E_{0\beta_A}{}^2 = 2.86 - 1 = 1.86$$

$$p_{\beta_A}c = \sqrt{1.86} = 1.36$$

the momentum of the two particles is not conserved

$$E_{\beta_A}{}^2 = p_{\beta_A}{}^2c^2 + E_{0\beta_A}{}^2$$

$$E_{\beta_A}{}^2 = p_u{}^2c^2 + E_{0u}{}^2 + p_v{}^2c^2 + E_{0v}{}^2$$

$$p_{\beta_A}{}^2c^2 + E_{\beta_A 0}{}^2 = p_u{}^2c^2 + E_{0u}{}^2 + p_v{}^2c^2 + E_{0v}{}^2 =$$

$$p_{\beta_A}{}^2c^2 - p_u{}^2c^2 - p_v{}^2c^2 = E_{0u}{}^2 + E_{0v}{}^2 - E_{\beta_A 0}{}^2$$

$$1.86 - 2.0925 - \left(-0.472\right) = 3.72 - 2.48 - 1 = 0.24$$

Summery

Velocity transformation is used in many cases.
I don't know what are the physical conclusions of energy mass and momentum transformation?. What is the meaning of the singularity? $\beta_v + \beta_u = 0$
And why we get imaginary mass and momentum

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Lorentz Velocity Transformation

Problem 1

Two rockets approach each other and, each moving with the same speed as measured by a stationary observer on the Earth. Their relative speed is $u_{rel} = 0.7c$

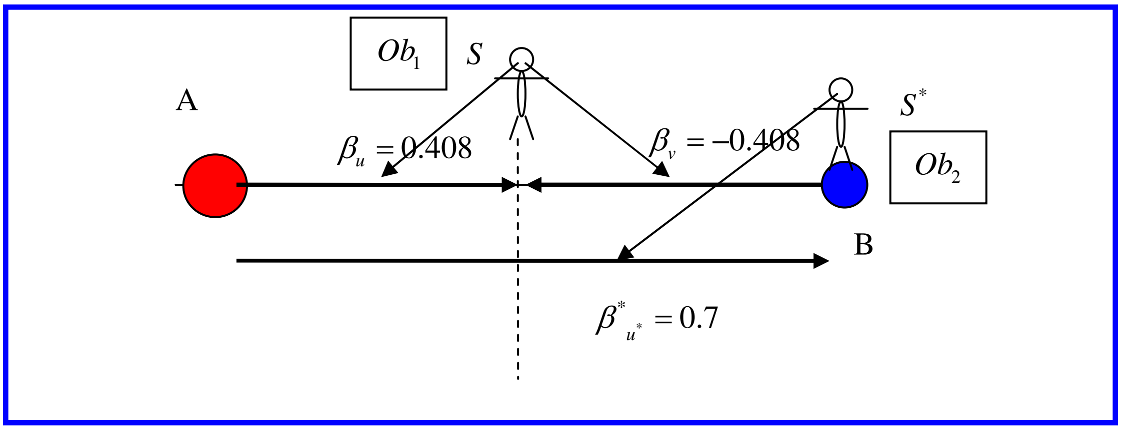
Determine the velocities of each rocket as measured by the stationary observer on Earth (S frame)

Solution

Assume a Galilean velocity transformation. Observer on rocket B will see rocket A moving toward his rocket at a speed $0.7c$ Another observer in between the two rockets will see rocket A moving to the right and rocket B moving to the left . Since both rockets have the same speed the speed of one rocket is expected to be

$$3.5c = \frac{0.7c}{2}$$

In a Lorentz velocity transformation, the velocity direction of each rocket is the same as in a Galilean velocity transformation, but the magnitude is different.



Galilean transformation

$$v = -0.35c$$

$$u = 0.35c$$

$$u^* = u - v = 0.35c - (-0.35c) = 0.7c$$

Assume the rockets and the frames move along the x direction

Lorentz velocity transformation is given by:

$$u^* = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\beta^*_{u^*} = \frac{u^*}{c} = \frac{\frac{u}{c} - \frac{v}{c}}{1 - \frac{uv}{c^2}} = \frac{\beta_u - \beta_v}{1 - \beta_u \beta_v}$$

v , $\beta_v = ?$ is the velocity of the S^* (B) frame relative to frame S (Earth)
u , β_u is the velocity of A measured in the S frame and $\beta_u = -\beta_v$
And u^* , $\beta_{u^*} = 0.7c$ is the velocity of A relative to B measured in the S^* frame
S^* frame is attached to B

$$u^* = \frac{u-(-u)}{1-\frac{u(-u)}{c^2}}$$

$$\beta^*_{u^*} = \frac{u^*}{c} = \frac{\frac{u}{c}-\frac{-u}{c}}{1-\frac{u(-u)}{c^2}} = \frac{\beta_u-(-\beta_u)}{1-\beta_u(-\beta_u)} = \frac{2\beta_u}{1+\beta_u^{\;2}}$$

$$0.7 = \frac{2\beta_u}{1+\beta_u^{\;2}} \quad \rightarrow \quad \beta_u = 0.408 \quad \beta_v = -0.408$$

Let check the results,

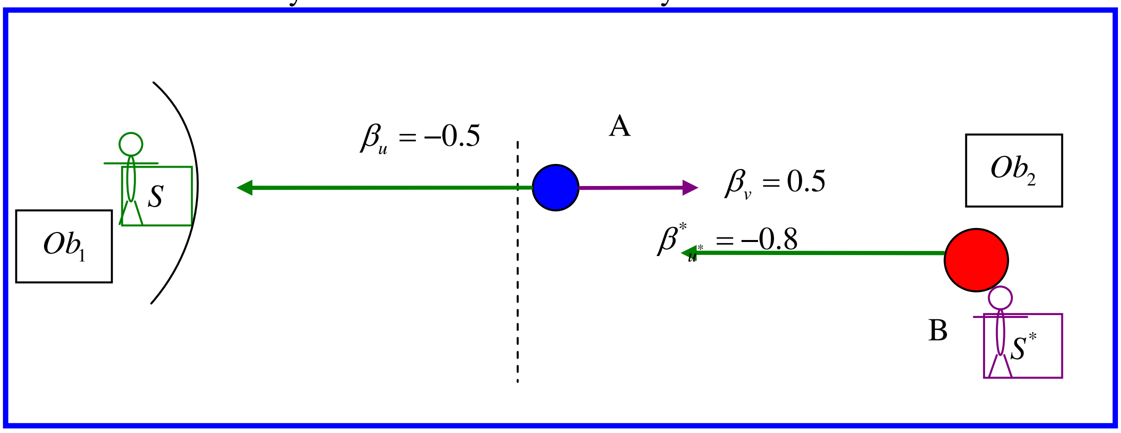
$$\beta^*_{u^*} = \frac{\beta_u - \beta_v}{1-\beta_u\beta_v} = \frac{0.408-(-0.408)}{1-0.408(-0.408)} = 0.7$$

$$\beta_v = -0.408$$

$$\beta_u = 0.408$$

Problem 2

A stationary observer on Earth observes rockets A and B moving in the same direction toward the Earth.
Rocket A has velocity 0.5c and rockets B has velocity 0.80c.
Determine the velocity of rockets A as measured by an observer at rest in rockets B.



Solution

Assume a Galilean velocity transformation. In a Galilean velocity transformation B is quicker then A. Observer on rocket B will see rocket A moving toward his rocket at a speed $0.8c - 0.5c = 0.3c$
In a Lorentz velocity transformation, the velocity direction of each rocket is the same, as in a Galilean velocity transformation, but the magnitude is different.

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We take the S frame to be attached to the Earth and the S^* frame to be attached to rocket B moving with velocity $u^* = -0.8c$ along the x axis.
rocket A has velocity $u = -0.5c$ in S.
It follows from

$$u^* = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\beta^*_{u^*} = \frac{\beta_u - \beta_v}{1 - \beta_u \beta_v} = \frac{-0.5 - \beta_v}{1 - (-0.5)\beta_v} = -0.8$$

$$\beta_v = 0.5$$

$$\beta_u = -0.5$$

$$\beta^*_{u^*} = -0.8$$

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Appendix A

$$\beta_A = \frac{\beta_u - \beta_v}{1-\beta_u\beta_v}$$

$$\frac{1}{L_A} = \frac{1}{(1-\beta_A)L_0} + \frac{1}{(1+\beta_A)L_0}$$

$$\frac{1}{L_A} = \frac{1}{\left(1-\frac{\beta_u-\beta_v}{1-\beta_u\beta_v}\right)L_0} + \frac{1}{\left(1+\frac{\beta_u-\beta_v}{1-\beta_u\beta_v}\right)L_0}$$

$$\frac{1}{L_A} = \frac{1}{(1-\beta_u-\beta_u\beta_v+\beta_v)}\left[\frac{1-\beta_u\beta_v}{L_0}\right] + \frac{1}{(1-\beta_v+\beta_u-\beta_u\beta_v)}\left[\frac{1-\beta_u\beta_v}{L_0}\right]$$

$$\frac{1}{L_A} = \frac{1}{(1-\beta_u)(1+\beta_v)}\left[\frac{1-\beta_u\beta_v}{L_0}\right] + \frac{1}{(1-\beta_v)(1+\beta_u)}\left[\frac{1-\beta_u\beta_v}{L_0}\right]$$

$$\frac{1}{(1-\beta_u)(1+\beta_v)} = \left(\frac{1}{\beta_v+\beta_u}\right)\left(\frac{1}{1-\beta_u} - \frac{1}{1+\beta_v}\right)$$

$$\frac{1}{(1+\beta_u)(1-\beta_v)} = \left(\frac{-1}{\beta_v+\beta_u}\right)\left(\frac{1}{1+\beta_u} - \frac{1}{1-\beta_v}\right)$$

$$\frac{1}{L_A} = \left[\frac{1-\beta_u\beta_v}{L_0}\right]\left\{\left(\frac{1}{\beta_v+\beta_u}\right)\left(\frac{1}{1-\beta_u} - \frac{1}{1+\beta_v}\right) - \left(\frac{1}{\beta_v+\beta_u}\right)\left(\frac{1}{1+\beta_u} - \frac{1}{1-\beta_v}\right)\right\}$$

$$\frac{1}{L_A} = \left[\frac{1-\beta_u\beta_v}{(\beta_v+\beta_u)L_0}\right]\left\{\left(\frac{1}{1-\beta_u} - \frac{1}{1+\beta_v}\right) - \left(\frac{1}{1+\beta_u} - \frac{1}{1-\beta_v}\right)\right\}$$

$$\frac{1}{L_A} = \left[\frac{1-\beta_u\beta_v}{(\beta_v+\beta_u)L_0}\right]\left(\frac{1}{1-\beta_u} - \frac{1}{1+\beta_u} + \frac{1}{1-\beta_v} - \frac{1}{1+\beta_v}\right)$$

$$\frac{1}{L_A} = \left[\frac{1-\beta_u\beta_v}{(\beta_v+\beta_u)L_0}\right]\left(\frac{1+\beta_u-(1-\beta_u)}{1-\beta_u^{\ 2}} + \frac{1+\beta_v-(1-\beta_v)}{1-\beta_v^{\ 2}}\right)$$

$$\frac{1}{L_A} = \left[\frac{1-\beta_u\beta_v}{(\beta_v+\beta_u)L_0}\right]\left(\frac{2\beta_u}{1-\beta_u^{\ 2}} + \frac{2\beta_v}{1-\beta_v^{\ 2}}\right)$$

$$\frac{1}{L_A} = \left[\frac{1-\beta_u\beta_v}{(\beta_v+\beta_u)L_0}\right]\left(\frac{2\beta_u}{1-\beta_u^{\ 2}} + \frac{2\beta_v}{1-\beta_v^{\ 2}}\right)$$

$$\frac{1}{L_A} = \left[\frac{1-\beta_u\beta_v}{(\beta_v+\beta_u)L_0}\right]\left[\frac{2\beta_u}{1-\beta_u^{\ 2}}\right] + \left[\frac{1-\beta_u\beta_v}{(\beta_v+\beta_u)L_0}\right]\left[\frac{2\beta_v}{1-\beta_v^{\ 2}}\right]$$

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$$\frac{1}{L_u}=\frac{1}{L_0}\left[\frac{2\beta_u\left(1-\beta_u\beta_v\right)}{\left(\beta_v+\beta_u\right)}\right]\left(\frac{1}{1-\beta_u{}^2}\right)$$

$$\frac{1}{L_v}=\frac{1}{L_0}\left[\frac{2\beta_v\left(1-\beta_u\beta_v\right)}{\left(\beta_v+\beta_u\right)}\right]\left(\frac{1}{1-\beta_v{}^2}\right)$$

$$\frac{1}{L_{u0}}=\left[\frac{2\beta_u\left(1-\beta_u\beta_v\right)}{\left(\beta_v+\beta_u\right)}\right]\frac{1}{L_0}=\alpha_u\frac{1}{L_0}$$

$$\frac{1}{L_{v0}}=\left[\frac{2\beta_v\left(1-\beta_u\beta_v\right)}{\left(\beta_v+\beta_u\right)}\right]\frac{1}{L_0}=\alpha_v\frac{1}{L_0}$$

$$\frac{1}{L_A}=\frac{1}{L_u}+\frac{1}{L_v}=\frac{1}{L_{u0}}\frac{1}{\left(1-\beta_u{}^2\right)}+\frac{1}{L_{v0}}\frac{1}{\left(1-\beta_v{}^2\right)}=\alpha_u\frac{1}{L_0}\frac{1}{\left(1-\beta_u{}^2\right)}+\alpha_v\frac{1}{L_0}\frac{1}{\left(1-\beta_v{}^2\right)}$$

$$\frac{1}{L_A}=\alpha_u\frac{1}{L_0}\left(\frac{1}{1-\beta_u}+\frac{1}{1-\beta_u}\right)+\alpha_v\frac{1}{L_0}\left(\frac{1}{1-\beta_v}+\frac{1}{1+\beta_v}\right)$$

$$\frac{1}{L_A}=\frac{1}{L_{u0}}\left(\frac{1}{1-\beta_u}+\frac{1}{1-\beta_u}\right)+\frac{1}{L_{v0}}\left(\frac{1}{1-\beta_v}+\frac{1}{1+\beta_v}\right)$$

END

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