Poulet numbers in Smarandache prime partial digital sequence and a possible infinite set of primes

Abstract. Though the well known Fermat's conjecture on the diophantine equation x^n + y^n = z^n is named "Fermat's big theorem", in fact probably much more important for number theory is what is called "Fermat's little theorem" which was the most important step up to that time in order to discover a primality criterion. This exceptional criterion of primality still has its exceptions: Fermat pseudoprimes, numbers which "behave" like primes though they are no primes; but they are still a class of numbers at least as interesting as the class of primes. Among Fermat pseudoprimes two classes of numbers are particularly distinguished: Poulet numbers (relative Fermat pseudoprimes) and Carmichael numbers (absolute Fermat pseudoprimes). The initial aim of this paper was only to see which Poulet numbers can be obtained concatenating primes (or, in other words, whichever admit a deconcatenation in prime numbers) but, inspired by a characteristic of a subset of Poulet numbers, I also made the following conjecture: there exist an infinity of primes p obtained concatenating to the right a prime q having the sum of the digits s(q) equal to a multiple of 5 with 3.

The *Smarandache prime-partial-digital sequence* (see A019549 in OEIS):
: 23, 37, 53, 73, 113, 137, 173, 193, 197, 211, 223, 227, 229, 233, 241, 257, 271, 277, 283, 293, 311, 313, 317, 331, 337, 347, 353, 359, 367, 373, 379, 383, 389, 397, 433, 523, 541, 547, 557, 571, 577, 593, 613, 617, 673, 677, 719, 727, 733, 743, 757, 761, 773, 797, 977 (...)

The *Fermat pseudoprimes to base two (Poulet numbers) sequence* (see A001567 in OEIS):
: 23, 37, 53, 73, 113, 137, 173, 193, 197, 211, 223, 227, 229, 233, 241, 257, 271, 277, 283, 293, 311, 313, 317, 331, 337, 347, 353, 359, 367, 373, 379, 383, 389, 397, 433, 523, 541, 547, 557, 571, 577, 593, 613, 617, 673, 677, 719, 727, 733, 743, 757, 761, 773, 797, 977 (...)

Conjecture:

There exist an infinity of Poulet numbers P which admit a deconcatenation in prime numbers.

The sequence of such Poulet numbers P:

: P = 341 (3 and 41 are primes);
: P = 561 (5 and 61 are primes);
: P = 1729 (17 and 29 are primes);
: P = 2701 (2 and 701 are primes);
: P = 2821 (2 and 821 are primes);
: P = 3277 (3 and 277 are primes, also 3 and 2 and 7 and 7 are primes);
: P = 4371 (43 and 71 are primes);
: P = 8911 (89 and 11 are primes);
: P = 13741 (137 and 41 are primes, also 13 and 7 and 41 are primes);
: P = 13747 (137 and 47 are primes, also 13 and 7 and 47 are primes);
: P = 23001 (2 and 3001 are primes);
: P = 25761 (257 and 61 are primes, also 2 and 5 and 761 are primes);
: P = 29341 (293 and 41 are primes, also 2 and 9341 are primes);
: P = 33153 (331 and 53 are primes);
: P = 35333 (3533 and 3 are primes, also 353 and 3 and 3 or 3 and 53 and 3 and 3 or 3 and 5 and 3 and 3 and 3);
: P = 49141 (491 and 41 are primes);
: P = 63973 (6397 and 3 are primes);
: p = 83333 (83 and 3 and 3 and 3 are primes);
: p = 88357 (883 and 5 and 7 are primes);
: P = 101101 (101 is prime);
: P = 137149 (137 and 149 are primes, also 13 and 7 and 149 are primes);
: P = 149281 (149 and 281 are primes);
: P = 157641 (157 and 641 are primes);
: P = 172081 (17 and 2081 are primes);
: P = 196093 (19609 and 3 are primes);
: P = 212421 (2 and 12421 are primes);
: P = 215749 (2 and 15749 are primes);
: P = 220729 (2207 and 29 are primes);
: P = 226801 (2 and 26801 are primes);
: p = 233017 (2 and 3301 and 7 are primes);
: p = 253241 (2 and 53 and 241 are primes);
: P = 264773 (2647 and 73 are primes, also 2647 and 7 and 3 are primes);
: P = 272251 (2 and 72251 are primes, also 2 and 7 and 2251 are primes, also 2 and 7 and 2 and 251);
: p = 276013 (2 and 7 and 601 and 3 are primes);
: p = 289941 (2 and 89 and 941 are primes);
: P = 294271 (29 and 4271 are primes);
: P = 294409 (29 and 4409 are primes);
: p = 314821 (3 and 14821 are primes);
: p = 318361 (31 and 83 and 61 are primes);

: p = 323713 (32371 and 3 are primes, also 3 and 2 and 3 and 7 and 13);
: p = 334153 (3 and 3 and 41 and 53 are primes)
: P = 387731 (3877 and 31 are primes);
: P = 401401 (401 is prime);
(note that 101101 is also a Poulet number, and 101 is also prime);
: p = 423793 (42379 and 3 are primes);
: P = 443719 (443 and 719 are primes, also 443 and 7 and 19);
: P = 476971 (47 and 6971 are primes);
: P = 481573 (48157 and 3 are primes);
: P = 486737 (48673 and 7 are primes);
: P = 493697 (49369 and 7 are primes);
(...)

Note:

The Poulet number 1729 (Hardy-Ramanujan number), can be deconcatenated in 17 and 29, primes, and also the Poulet number 137149 can be deconcatenated in two primes: 137 = 17 + 120 and 149 = 29 + 120; the numbers obtained concatenating the primes of the form 17 + 120*k with primes with the form 29 + 120*k might be interesting to study; one other Poulet number which can be deconcatenated in two numbers x and y such that x – y = 12 is 253241 but this time 253 is not prime but a semiprime; also 1729 = 7*13*19 and 137149 = 67*23*89 and the numbers of the form (60*k + 7)*(10*k + 13)*(70*k + 19) might be interesting to study.

Note:

An other interesting thing is that the Poulet numbers 63973, 196093 and 481573, beside the fact that are obtained from primes of the form 6*k + 1 concatenated to the right with 3, have in common the sum of their digits, *i.e.* 28.

Conjecture:

There exist an infinity of primes p obtained concatenating to the right a prime q having the sum of the digits s(q) equal to a multiple of 5 with 3.

The sequence of such primes p for s(q) = 5:

: P = 53 (q = 5 prime);
: p = 233 (q = 23 prime and 2 + 3 = 5);
: P = 4013 (q = 401 prime and 4 + 0 + 1 = 5);
: P = 10133 (q = 1013 prime and 1 + 0 + 1 + 3 = 5);

: P = 10313 (q = 1031 prime and 1 + 0 + 3 + 1 = 5);
(...)

The sequence of such primes p for s(q) = 10:

: P = 193 (q = 19 prime and 1 + 9 = 10);
: P = 373 (q = 37 prime and 3 + 7 = 10);
: P = 733 (q = 73 prime and 7 + 3 = 10);
: P = 1093 (q = 109 prime and 1 + 0 + 9 = 10);
: P = 2713 (q = 271 prime and 2 + 7 + 1 = 10);
: P = 5233 (q = 523 prime and 5 + 2 + 3 = 10);
: P = 5413 (q = 541 prime and 5 + 2 + 3 = 10);
: P = 6133 (q = 613 prime and 6 + 1 + 3 = 10);
: P = 10093 (q = 1009 prime and 1 + 0 + 0 + 9 = 10);
: P = 11173 (q = 1117 prime and 1 + 1 + 1 + 7 = 10);
(...)
: P = 1043113 (q = 104311 prime s(q) = 10);
(...)

The sequence of such primes p for s(q) = 20:

: P = 4793 (q = 479 prime and 4 + 7 + 9 = 20);
: P = 5693 (q = 569 prime and 5 + 6 + 9 = 20);
: P = 9293 (q = 929 prime and 9 + 2 + 9 = 20);
: P = 9473 (q = 947 prime and 9 + 4 + 7 = 20);
: P = 9833 (q = 983 prime and 9 + 8 + 3 = 20);
: P = 12893 (q = 1289 prime and 1 + 2 + 8 + 9 = 20);
(...)
: P = 1047173 (q = 104717 prime s(q) = 20);
(...)

The sequence of such primes p for s(q) = 25:

: P = 9973 (q = 997 prime and 9 + 9 + 7 = 25);
: P = 16993 (q = 1699 prime and 1 + 6 + 9 + 9 = 25);
: P = 18793 (q = 1879 prime and 1 + 8 + 7 + 9 = 25);
: P = 19873 (q = 1987 prime and 1 + 9 + 8 + 7 = 25);
(...)
: P = 1036873 (q = 103687 prime s(q) = 25);
(...)