# Newton's Gravitational Law over Dark Matter

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**Abstract**   The shape of a gravity body can play a critical role in affecting the movement of an object moving in its vicinity of close range.   If the shape is disregarded while the gravity body is regarded as a point mass regardless of the distance between the two bodies, the concept of dark matter may find its chance to surface up in theoretical treatment concerning gravity.

**Key words**     on-axis effect, off-axis effect, dark matter, "normal" gravitational force, "normal" speed,  flat rotation

**Introduction**   Through studying several special cases on the relationship between shape and gravitation, we will explore how the materials at a certain distance from the center of the Milky Way galaxy show up with speeds higher than "normal".   The so called "normal" speed referred to in this article is the speed conventionally believed to be possessed by an object that is moving around a point mass in a large distance.   The speed so obtained is derived according to Newtonian gravitational law.

Since the situation involving such conventional treatment repeats many times in this article, the term "normal" speed or "normal" force will be used here with the inseparable quotation marks.  Almost all cases presented here are hypothetically assumed in geometry, but they sure would lead us to have a peek at how the shape of a gravity body can lever the movement of some objects that appear in its vicinity of close range.   Being so levered, though, all these movements cannot get away from the governing of Newton's gravitational law. Finally, we will see how theoretically impossible it is for the Magellanic Clouds to have ever possessed any satellite status about the Milky Way.   All this study is proposed without the involvement of dark matter.



Fig. 1

## Case 1.  Gravity on the Axis of a Bar

In **Fig. 1**, object $A$ of mass $m$ is on the axis of a homogeneous bar with a distance $D$ from one end of this bar.  The bar of mass $M$  has a length of $L(=2a)$ .   The gravitational force
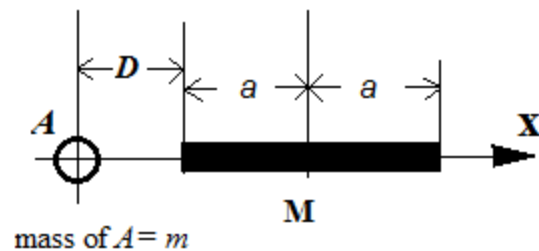
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between *A* and each differential mass element *dm* of the bar is

$$df = G\frac{m \cdot dM}{x^2} \qquad (Eq.\ 1)$$

where G is the universal gravitational constant.

Since $dM = \frac{M}{L}dx$, we get

$$df = G\frac{mM}{Lx^2}dx \qquad (Eq.\ 2)$$

Thus the total force $F_{1-1}$ between *A* and the bar is

$$F_{1-1} = \int_{D}^{D+2a} G\frac{mM}{Lx^2}dx$$

$$= G\frac{mM}{D(D+2a)}$$

$$= G\frac{mM}{D^2 + 2aD} \qquad (Eq.\ 3)$$

The tangential speed $V_{1-1}$ that is large enough for *A* to resist the bar's gravitational pull will lead to:

$$m\frac{v_{1-1}^2}{D+a} = G\frac{mM}{D^2+2aD} \qquad (Eq.\quad 4)$$

And therefore,

$$v_{1-1}^2 = \frac{GM(D+a)}{D^2+2aD} \qquad (Eq.\quad 5)$$

Had the bar become a point mass and stayed at where its original mass center is, the gravitational force $F_{1-2}$ between it and *A* should be

$$F_{1-2} = G\frac{mM}{(D+a)^2} \qquad (Eq.\quad 6)$$

The centrifugal force corresponding to $F_{1-2}$ for *A* to resist the pull of the point mass is

$$v_{1-2}^2 = \frac{GM}{D+a} \qquad (Eq.\quad 7)$$

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The comparison between $V_{1-1}$ and $V_{1-2}$ would lead to

$$\frac{v_{1-1}^2}{v_{2-2}^2} = \frac{D^2 + 2aD + a^2}{D^2 + 2Da} \qquad (Eq. \quad 8)$$

In order to make $V_{1-1} \approx V_{1-2}$, we need $D >> a$ so that length $a$ becomes trivial in Eq. 3 and the bar can then be regarded as a point mass. The smaller the distance $D$ is, the higher the magnitude $V_{1-1}$ needs to become if $A$ is to survive the gravitational pull of the bar. Once $A$ survives the pull at this point, it will retain this higher than "normal" momentum forever until something else brakes on it.

We use the term **on-axis effect** to name the effect that leads to $F_{1-1} > F_{1-2}$ and thus also leads to $v_{1-1} > v_{1-2}$, where $F_{1-2}$ is the "normal" force and $V_{1-2}$ is the "normal" speed.

**Case 2   Gravity off the Axis of a Bar, Situation 1**

**Step (a)**

The same two gravity bodies in **Fig. 1** are rearranged so that $A$ is located a distance of $h$ away directly below the end point J of the bar. (**Fig 2-a**)

In **Fig 2a**, $q$ is the distance between the two mass centers, and thus

$$q^2 = a^2 + h^2 \qquad (Eq. \quad 9)$$

Line $p$ represents the distance between the mass center of $A$ and the differential element $dx$ of the bar.  Therefore,

$$p^2 = x^2 + h^2 \qquad (Eq. \quad 10)$$

The gravitational force $df_1$ between $dx$ and $A$ is

$$df_1 = \frac{Gm}{p^2} dM$$
$$= \frac{Gm}{x^2 + h^2} dM$$
$$= \frac{Gm}{x^2 + h^2} \cdot \frac{M}{2a} dx \qquad (Eq. \quad 11)$$
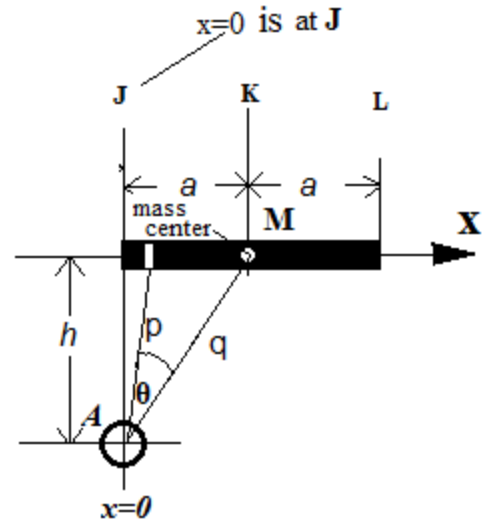


Fig. 2a

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If $df_1$ is projected on q, we get

$$df'_1 = df_1\cos\theta$$

$$= \frac{Gm}{x^2 + h^2} \cdot \frac{M}{2a}dx \cdot \frac{p^2 + q^2 - (a - x)^2}{2pq}$$

$$= \frac{Gm}{x^2 + h^2} \cdot \frac{M}{2a}dx \cdot \frac{(x^2 + h^2) + (a^2 + h^2) - (a - x)^2}{2\sqrt{x^2 + h^2} \cdot \sqrt{a^2 + h^2}} \qquad (Eq. \quad 12)$$

The total force between $A$ and the segment JK of the bar is

$$F_{2-1} = \int_0^a df'_1$$

$$= \int_0^a \frac{Gm}{x^2 + h^2} \cdot \frac{M}{2a} \cdot \frac{(x^2 + h^2) + (a^2 + h^2) - (a - x)^2}{2\sqrt{x^2 + h^2} \cdot \sqrt{a^2 + h^2}} \cdot dx$$

$$= \frac{GmM}{2h\sqrt{a^2 + h^2}} \qquad (Eq. \quad 13)$$

**Step (b)**

Fig. 2b is a duplicate of Fig 2a but point x=0 is located at K for calculation convenience.

In Fig. 2b, $s$ represents the distance between the mass center of $A$ and the differential element $dx$ on the bar. Therefore,

$$s^2 = h^2 + (a + x)^2 \qquad (Eq. 14)$$

The gravitational force $df_2$ between $dx$ and $A$ is

$$df_2 = \frac{Gm}{s^2}dM$$

$$= \frac{Gm}{(a + x)^2 + h^2}dM$$

$$= \frac{Gm}{(a + x)^2 + h^2} \cdot \frac{M}{2a}dx \qquad (Eq. \quad 15)$$

When $df_2$ is projected on line q, we have

x=0 at K
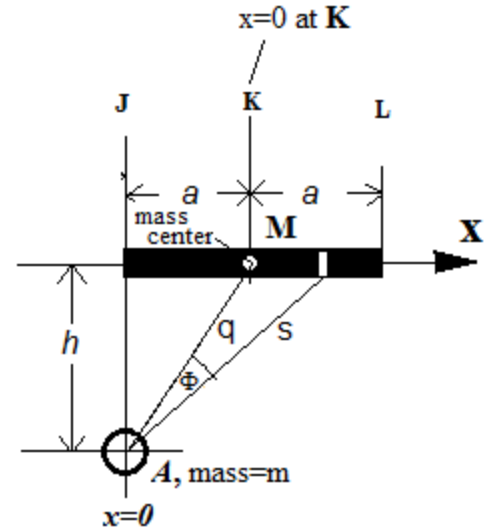
J      K      L

$\leftarrow a \quad \ast \quad a \rightarrow$

mass center  M       X

h       q   s

$A$, mass=m

x=0

Fig. 2 b

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$$df'_2 = df_2 cos\emptyset$$

$$= \frac{Gm}{(a+x)^2+h^2} \cdot \frac{M}{2a}dx \cdot \frac{s^2+q^2-x^2}{2sq}$$

$$= \frac{Gm}{(a+x)^2+h^2} \cdot \frac{M}{2a}dx \cdot \frac{h^2+(a+x)^2+(a^2+h^2)-x^2}{2\sqrt{h^2+(a+x)^2} \cdot \sqrt{a^2+h^2}} \qquad (Eq. \quad 16)$$

The total force between $A$ and the segment KL of the bar is

$$F_{2-2} = \int_0^a df'_2$$

$$= \int_0^a \frac{Gm}{(a+x)^2+h^2} \cdot \frac{M}{2a} \cdot \frac{h^2+(a+x)^2+(a^2+h^2)-x^2}{2\sqrt{h^2+(a+x)^2} \cdot \sqrt{a^2+h^2}} \cdot dx$$

$$= \frac{GmM}{2\sqrt{a^2+h^2}\sqrt{4a^2+h^2}} \qquad (Eq. \quad 17)$$

**Step (c)**

The total force between $A$ and the mass center of the bar is $F_{2-3}=F_{2-1}+F_{2-2}$ and thus

$$F_{2-3} = \frac{GmM}{2h\sqrt{a^2+h^2}} + \frac{GmM}{2\sqrt{a^2+h^2}\sqrt{4a^2+h^2}} \qquad (Eq. \quad 18)$$

If $A$ happens to move at speed $V_{2-3}$ in a direction perpendicular to line q, the centrifugal force thus needed to resist the bar's gravitational pull will lead to:

$$m\frac{v_{2-3}^2}{\sqrt{a^2+h^2}} = \frac{GmM}{2h\sqrt{a^2+h^2}} + \frac{GmM}{2\sqrt{a^2+h^2}\sqrt{4a^2+h^2}} \qquad (Eq. \quad 19)$$

Had the bar become a point mass and stayed at where its original mass center is, the "normal" gravitational force between $A$ and this point mass will be

$$F_{2-4} = G\frac{mM}{a^2+h^2} \qquad (Eq. \quad 20)$$

The "normal" centrifugal force for $A$ corresponding to $F_{2-4}$ would lead to

$$m\frac{v_{2-4}^2}{\sqrt{a^2+h^2}} = G\frac{mM}{a^2+h^2} \qquad (Eq. \quad 21)$$

Thus, we can have the comparison between $v_{2\text{-}3}$ and $v_{2\text{-}4}$ as

$$\frac{v_{2-3}^2}{v_{2-4}^2} = \frac{\sqrt{a^2 + h^2}}{2h} + \frac{\sqrt{a^2 + h^2}}{2\sqrt{4a^2 + h^2}} \qquad (Eq. \quad 22)$$

Let $h=na$, where $n \neq 0$, Eq. 22 leads to

$$\frac{v_{2-3}^2}{v_{2-4}^2} = \frac{\sqrt{1 + n^2}}{2n} + \frac{\sqrt{1 + n^2}}{2\sqrt{4 + n^2}} \qquad (Eq. \quad 23)$$

If $n=1$, for example, we have

$$\frac{v_{2-3}^2}{v_{2-4}^2} = 1.8 \qquad (EQ. \quad 24)$$

If $n=3$, however, we will have

$$\frac{v_{2-3}^2}{v_{2-4}^2} = 0.96 \qquad (Eq. \quad 25)$$

If we must introduce dark matter to explain the phenomenon brought up by Eq 24, how do we explain the phenomenon brought up by Eq. 25?

Of course, when $n \to \infty$, we no longer need to be concerned with dark matter, as Eq. 23 would give us a value very close to 1, fitting our conventional concept that the bar can be viewed as a point mass.

If $F_{2\text{-}3}$ is to be resolved on the line connecting A and J, we have $F_{2-5}$, where

$$F_{2-5} = F_{2-3} \cdot \frac{h}{\sqrt{a^2 + h^2}}$$
$$= \frac{GmM}{2(a^2 + h^2)} \cdot \left(1 + \frac{h}{\sqrt{4a^2 + h^2}}\right) \qquad (Eq. \ 26)$$

## Case 3a   Gravity off the Axis of a Bar, Situation 2

In **Fig. 3a,** we duplicate the bar in **Fig. 2a** or **Fig. 2b** and "weld" it with the original bar end to end and thus form a new bar.

On each side of the mass center of this longer homogeneous bar, the half bar has a length of *2a* (therefore the total length is *4a*).

The gravitational force $F_{3-1}$ between *A* and the full length new bar is two times of $F_{2-5}$ found in Eq.  26 and therefore

$$F_{3-1} = 2 \cdot \frac{GmM}{2(a^2 + h^2)} \cdot \left(1 + \frac{h}{\sqrt{4a^2 + h^2}}\right)$$
$$= \frac{GmM}{a^2 + h^2} \cdot \left(1 + \frac{h}{\sqrt{4a^2 + h^2}}\right) \qquad (Eq. \quad 27)$$

The tangential speed $v_{3-1}$ that is large enough for *A* to resist the bar's gravitational pull will lead to

$$m \frac{v_{3-1}^2}{h} = \frac{GmM}{a^2 + h^2} \cdot \left(1 + \frac{h}{\sqrt{4a^2 + h^2}}\right) \quad (Eq. \quad 28)$$

Had the bar become a point mass and stayed at where its mass center has been, the "normal" gravitational force $F_{3-2}$ between *A* and the bar will be

$$F_{3-2} = G \frac{m(2M)}{h^2} \qquad (Eq. \quad 29)$$

The "normal" centrifugal force corresponding to $F_{3-2}$ would lead to

$$m \frac{v_{3-2}^2}{h} = G \frac{m(2M)}{h^2} \qquad (Eq. \quad 30)$$

Thus, we can have the comparison between $v_{3-1}$ and $v_{3-2}$ as

$$\frac{v_{3-1}^2}{v_{3-2}^2} = \frac{h^2(h + \sqrt{4a^2 + h^2})}{2(a^2 + h^2)\sqrt{4a^2 + h^2}} \qquad (Eq. \quad 31)$$

Let *h=na*, where $n \neq 0$, we have



Locate x=0 at J
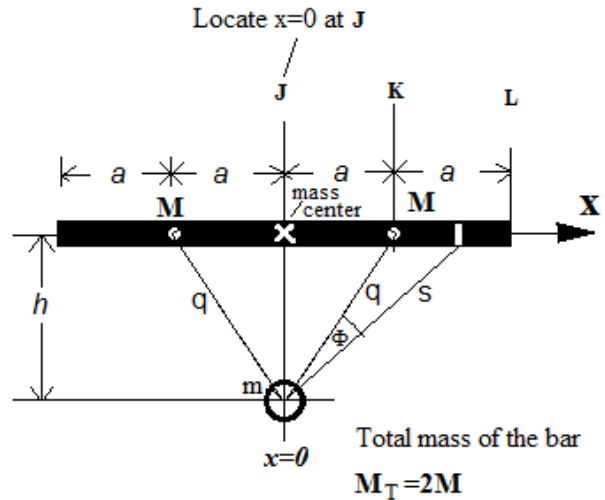
Total mass of the bar

$M_T = 2M$

**Fig.  3a**

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$$\frac{v_{3-1}^2}{v_{3-2}^2} = \frac{n^3}{2(1+n^2)\sqrt{4+n^2}} + \frac{n^2}{2(1+n^2)} \qquad \text{(Eq. \quad 32)}$$

Each term on the right side of Eq. 32 is smaller than 0.5. Therefore, $v_{3-1}$ is forever smaller than $v_{3-2}$ for any value of $n$. Dark matter must fail in explaining phenomenon brought up by Eq. 32.

**Case 3b   Off-axis Effect**

In **Fig. 3b,** we are going to compare the dynamic status of $A$ between location E and F.

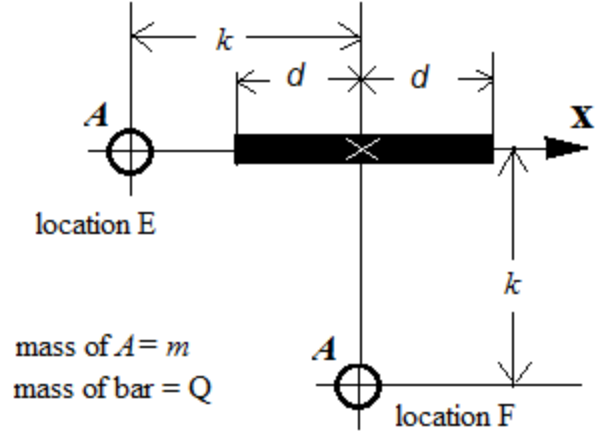At location E, Eq. 3 gives us the gravitational force received by $A$ as



**Fig. 3b**

$$F_{3b-1} = G\frac{mQ}{(k-d)^2 + 2a(k-d)} \qquad (Ea. \quad 33)$$

The speed $V_{3b-1}$ matching the corresponding centrifugal force for $A$ to survive the pull from the bar leads to

$$v_{3b-1}^2 = \frac{GQk}{(k-d)^2 + 2d(k-d)} \qquad (Eq. \quad 34)$$

At location F, Eq. 27 gives us the gravitational force received by $A$ as

$$F_{3b-2} = \frac{Gm(\frac{Q}{2})}{(\frac{d}{2})^2 + k^2} \cdot \left(1 + \frac{k}{\sqrt{4(\frac{d}{2})^2 + k^2}}\right) \qquad (Eq. \quad 35)$$

The speed $V_{3b\text{-}2}$ matching the corresponding centrifugal force for **A** to survive the pull from the bar at F leads to

$$v_{3b-2}^2 = \frac{G\left(\frac{Q}{2}\right)k}{(\frac{d}{2})^2 + k^2} \cdot \left(1 + \frac{k}{\sqrt{4(\frac{d}{2})^2 + k^2}}\right) \qquad (Eq. \quad 36)$$

Therefore we can further have

$$\frac{v_{3b-1}^2}{v_{3b-2}^2} = \frac{(d^2 + 4k^2)\sqrt{d^2 + k^2}}{2[k^2 - d^2](\sqrt{d^2 + k^2} + k)} \qquad (Eq. \quad 37)$$

Letting $k = nd$, where $n \neq 0$, we have

$$\frac{v_{3b-1}^2}{v_{3b-2}^2} = \frac{(1 + 4n^2)\sqrt{1 + n^2}}{2(n^2 - 1)(n + \sqrt{1 + n^2})} \qquad (Eq. \quad 38)$$

If $n + e > 1$, but with $e \to 0,$ Eq. 38, easily leads us to have high value for the ratio of the two speeds.

So, if we must regard the bar as a point mass in explaining the speed of **A,** then, at location E we must face inexplicable reason for A's higher than "normal" speed. When A moves to area near location F, we may perplex even more, because, carrying the momentum equipped at E, **A** now is encountered weaker and weaker than "normal" gravitational pull at F. Indeed, we can expect that A is going to fly away from the bar. For example, if $n=2$, the ratio in Eq. 38 is 1.49, or $v_{3b-1} = 1.22v_{3b-2}$. The bar definitely can no longer bind A with gravitation at F. From the behavior of A at location F, should we conclude that some apparent mass from the bar has lost its gravity?

We use the term *off-axis effect* to name the effect that leads **A** to receive weaker than "normal" gravitational force $F_{3b-2}$ at F. Eq. 38 tells us that the off-axis effect will diminish as $n \to \infty$ and the bar can be regarded as a point mass at remote distance.

## Case 4   Gravity in the Vicinity of a Cross

In **Fig 4**, two bars of length $2a$ and mass $M$ each are placed perpendicularly crossing each other at their dead centers. Body $A$ is a distance $a$ from each bar, and therefore it is a distance $q$ away from the mass center of the cross, where $q = \sqrt{2}a$. Taking advantage of the analysis shown with **Fig. 2a** and **2b**, replacing $h$ in **Eq. 18** with $a$, we can have the gravitational force $F_{4\text{-}1}$ between $A$ and the mass center of the cross as

mass of each bar
$= \mathbf{M}$

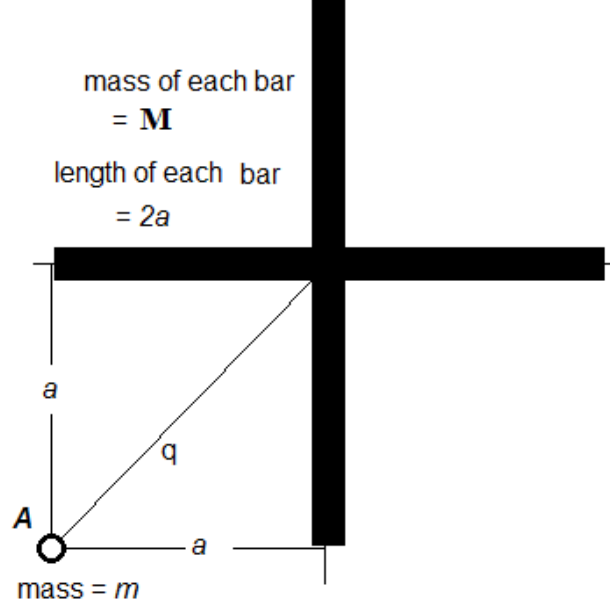length of each  bar
$= 2a$

$a$

$q$

$A$

$a$

mass $= m$

**Fig. 4**

$$F_{4-1} = 2\left(\frac{GmM}{2a\sqrt{a^2 + a^2}} + \frac{GmM}{2\sqrt{a^2 + a^2}\sqrt{4a^2 + a^2}}\right)$$

$$= GmM\left(\frac{\sqrt{5} + 1}{a^2\sqrt{10}}\right) \qquad\qquad (Eq. \quad 39)$$

The tangential speed $V_{4\text{-}1}$ that is large enough for $A$ to resist the cross's gravitational pull will lead to:

$$m\frac{v_{4-1}^2}{\sqrt{2}a} = GmM\left(\frac{\sqrt{5} + 1}{a^2\sqrt{10}}\right) \qquad\qquad (Eq. \quad 40 )$$

Had the cross become a point mass and stayed at where its mass center has been, the "normal" gravitational force $F_{4\text{-}2}$ between $A$ and the cross will be

$$F_{4-2} = G\frac{m(2M)}{(\sqrt{2}a)^2} = \frac{GmM}{a^2} \qquad (Eq. \quad 41)$$

The "normal" centrifugal force for $A$ corresponding to $F_{4\text{-}2}$ thus leads to a "normal" tangential speed $V_{4\text{-}2}$ as shown below

$$m\frac{v_{4-2}^2}{\sqrt{2}a} = G\frac{m(2M)}{2a^2} \qquad (Eq. \quad 42)$$

Thus, we can have the comparison between $V_{4\text{-}1}$ and $V_{4\text{-}2}$ as

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$$\frac{v_{4-1}^2}{v_{4-2}^2} = \frac{\dfrac{\sqrt{5}+1}{a^2\sqrt{10}}}{\dfrac{1}{a^2}} \approx 1.023 \qquad (Eq. \quad 43)$$

Eq. 43 thus shows that, at the location as shown in **Fig. 4**, the tangential velocity for $A$ to survive the pull will not change much whether the gravitational influence is from a cross or a point mass of the same mass.

**Case 5a   Gravity at the Edge of a Cross**

In **Fig. 5**, the gravitational force $F_{5\text{-}1}$ between $A$ and the vertical bar can be calculated according to Eq. 3.  In so doing, D in Eq. 3 is replaced with $D = q - a$ .  Therefore, we have

$$\begin{aligned} F_{5-1} &= G\frac{mM}{(q-a)^2 + 2a(q-a)} \\ &= \frac{GmM}{q^2 - a^2} \qquad (Eq.\ 44) \end{aligned}$$



mass of each bar = **M**

length of each  bar = 2a

Fig. 5

The gravitational force $F_{5\text{-}2}$ between $A$ and the horizontal bar can be calculated according to Eq. 27.  In doing so, $M$ in Eq. 27 is replaced with $M/2$, $a$ is replaced with $a/2$, $h$ is replaced with $q$.  Then,
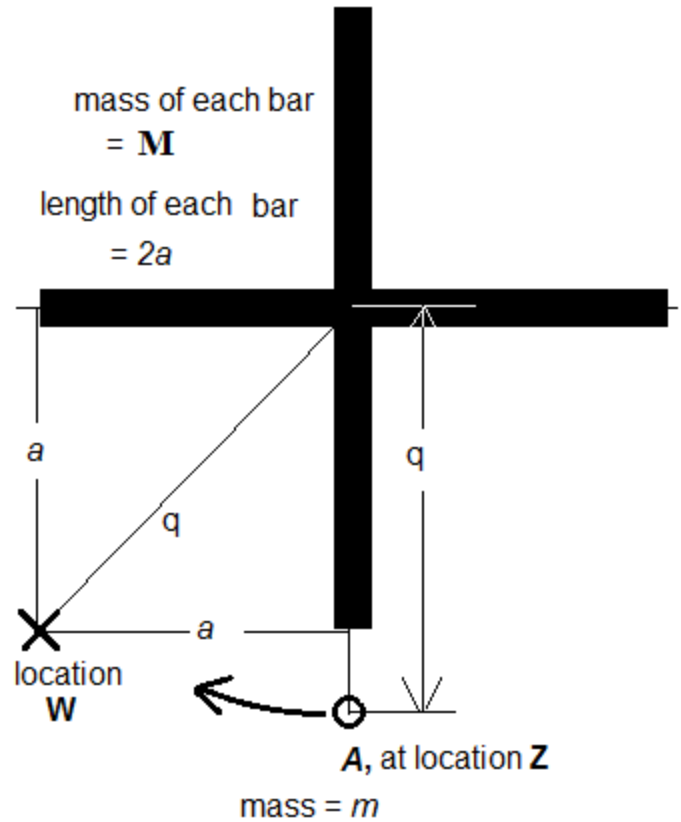
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$$F_{5-2} = \frac{Gm(\frac{M}{2})}{(\frac{a}{2})^2 + (\sqrt{2}a)^2} \cdot \left( 1 + \frac{\sqrt{2}a}{\sqrt{4(\frac{a}{2})^2 + (\sqrt{2}a)^2}} \right) \cong 0.403 \frac{GmM}{a^2} \qquad (Eq. \quad 45)$$

The total gravitational force $F_{5\text{-}3}$ between $A$ and both bars together is then

$$F_{5-3} = F_{5-1} + F_{5-2} = 1.403 \left( \frac{GmM}{a^2} \right) = 1.403 \, F_{4-2} \qquad (Eq. \quad 46)$$

where $F_{4-2}$ is the gravitational force that $A$ would have received if the cross had been a point mass at the mass center of the cross (See Eq. 41).

The tangential speed $v_{5\text{-}3}$ that can equip $A$ with enough centrifugal force against the cross's gravitational pull will lead to:

$$m\frac{v_{5-3}^2}{\sqrt{2}a} = 1.403 \left( \frac{GmM}{a^2} \right) \qquad (Eq. \quad 47)$$

Applying Eq. 41 in comparing the centrifugal force displayed in Eq. 47, we have

$$\frac{v_{5-3}^2}{v_{4-2}^2} = 1.403, \quad or$$
$$v_{5-3} = 1.184 v_{4-2} \qquad (Eq. \quad 48)$$

Eq. 48 means that, if a point mass becomes a cross shown in our picture, body $A$ needs its tangential speed to be 0.184 times higher than $v_{4-2}$, the "normal" speed. If not so, $A$ will be gravitationally sucked toward the cross. However, as $A$ leaves $Z$ but moves toward location $W$, the speed that $A$ carries will enable it to fly with extra momentum. Bound by a weaker gravitational force now, body $A$ may tend to fly away from the cross. However soon the on-axis effect of the next arm will come in to arrest it and stabilize its orbit about the cross again.

In **Fig. 6**, let's imagine that the tangential momentum of each of objects $A$, $B$, and $C$ has enabled them to survive the gravitational pull of the cross.

To any object in a situation similar to that of **A, B,** and **C**, the general expression for the gravitational force $F_{5\text{-}4}$ it receives from the cross can be written as (Refer to Eq. 27 and 34, with proper replacement of corresponding quantities)

$F_{5-4} = Force\ from\ vertical\ bar + force\ from\ horizontal\ bar$

$$= \frac{GmM}{q^2 - a^2} + \frac{Gm\left(\frac{M}{2}\right)}{\left(\frac{a}{2}\right)^2 + q^2} \cdot \left(1 + \frac{q}{\sqrt{4\left(\frac{a}{2}\right)^2 + q^2}}\right)$$

$$= GmM\left[\frac{1}{q^2 - a^2} + \frac{2\sqrt{q^2 + a^2} + 2q}{(4q^2 + a^2)\sqrt{q^2 + a^2}}\right] \qquad (Eq. \quad 49)$$
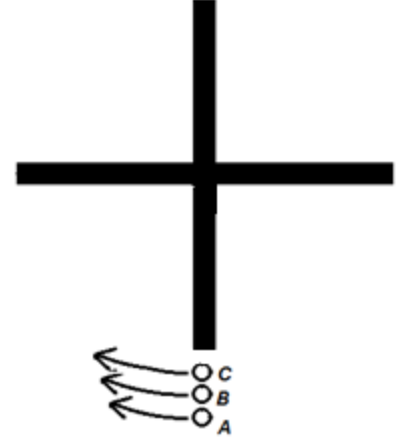


Fig. 6

The tangential speed $v_{5-4}$ corresponding to $F_{5-4}$ would show

$$v_{5-4}^2 = GMq\left[\frac{1}{q^2 - a^2} + \frac{2\sqrt{q^2 + a^2} + 2q}{(4q^2 + a^2)\sqrt{q^2 + a^2}}\right] \qquad (Eq. \quad 50)$$

Let $q = na$, where $n \neq 0$, correspondingly, Eq. 49 and Eq. 50 will become

$$F_{5-4} = \frac{GmM}{a^2}\left[\frac{1}{n^2 - 1} + \frac{2\sqrt{n^2 + 1} + 2n}{(4n^2 + 1)\sqrt{n^2 + 1}}\right] \qquad (Eq. \quad 51)$$

and

$$v_{5-4}^2 = GMa \cdot n\left[\frac{1}{n^2 - 1} + \frac{2\sqrt{n^2 + 1} + 2n}{(4n^2 + 1)\sqrt{n^2 + 1}}\right] \qquad (Eq. \quad 50)$$

Below is a chart showing how $F_{5-4}$ and the ratio $v_{5-4}/v_{4-2}$ change in accordance with $n=1.05$, $n=1.1$, $n=1.2$, $n=1.3$, $n=1.4$, $n=2$, and $n=5$.

Note 1: The so called $F_{"normal"}$ in the chart is the gravitational force that a moving object receives from the cross but the cross has been shrunk into a point mass of the same mass quantity at its mass center.

Note 2: $a$ is the arm length of the cross, $q$ is the distance between the moving object and the mass center of the cross.

| $n$ | a | q | $F_{5-4}$, set as S $value \times \dfrac{GmM}{a^2}$ | $F_{"normal"}$, set as T $value \times \dfrac{Gm(2M)}{a^2}$ | S/T | $v_{5-4}/v_{4-2}$ |
|---|---|---|---|---|---|---|
| 1.05 | 1a | 1.05a | 10.39 | 1.81 | 5.74 | 2.4 |
| 1.1 | 1a | 1.1a | 5.39 | 1.65 | 3.26 | 1.81 |
| 1.2 | 1a | 1.2a | 2.8 | 1.39 | 2.02 | 1.42 |
| 1.3 | 1a | 1.3a | 1.91 | 1.18 | 1.61 | 1.27 |
| 1.4 | 1a | 1.4a | 1.45 | 1.02 | 1.42 | 1.19 |
| 2 | 1a | 2a | 0.556 | 0.556 | 1.12 | 1.06 |
| 5 | 1a | 5a | 0.81 | 0.08 | 1.01 | 1 |

Data from the chart suggest that moving objects at the edge of the cross can have speed that is 240% as high as its "normal" speed, which is acquired without the consideration of the shape of the cross, but instead, only the cross's total mass as a point mass is considered. Subsequently, this chart will lead us to visualize that, with respect to the center of the cross, object $C$ in **Fig. 6** will move faster than $B$, which in turn moves faster than $A$.

**Case 5b  On the Gravity of a Softened Cross and on the Rotation Arms of the Milky Way.**

If the lower arm of the cross is: (1) a rotating body with respect to the mass center of the entire cross and (2) composed of loose materials, all the materials in this arm must display the same movement pattern as what $A$, $B$, and $C$ are showing in **Fig. 7a.**

The same reasoning must equally apply to other arms of the cross, if all other arms also possess the same nature as that of the lower arm. **(Fig. 7b)**
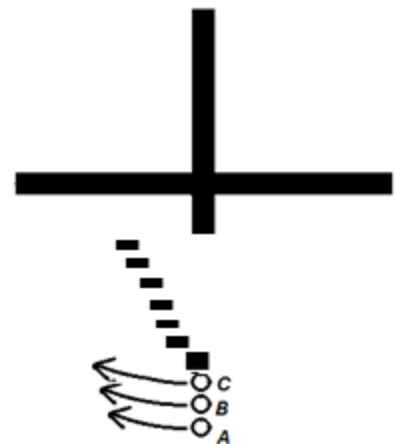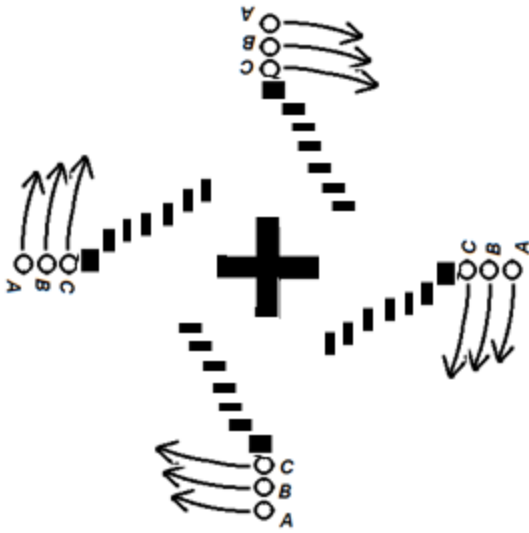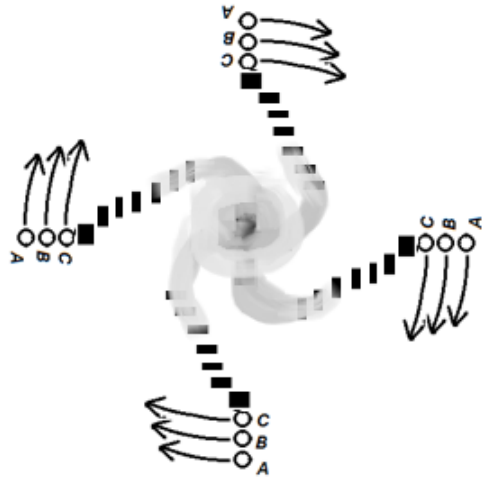


**Fig. 7a**

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Fig. 7b



Fig. 7c

However, as our inspection moves closer and closer to the center of the cross, we must notice that the arm length of the cross is getting shorter and shorter. The ever shortened arms of the cross must lead two things to happen: (1) The contrast between the on-axis effect and off-axis effect gradually diminishes; (2) movement of the objects about the mass center should show more and more obviously a pattern that is gravitationally governed by a point mass. When this happening is in progress, we cannot ignore one fact, which is that the angular velocity of the moving objects near the center is higher than that of those farther away from the center. The higher and higher angular velocities of the materials toward the center gradually blur out any distinctive feature of a cross. Instead, they just come together and present a rapidly spinning cloud. (**Fig. 7c**)

The problem is that, unless the cloud is absolutely homogeneous, given enough time, the spinning cloud will sooner or later evolve into a rotating bar. The reason for the appearance of such a bar, ironically, is exactly because the gravity in this range is more and more dominantly governed by a point mass. This point mass must be an extremely compacted and massive one if it is to stabilize the movement of so many objects traveling in orbits of short radius around it.

Let's suppose that some objects of more prominent mass inside a spinning and inhomogeneous cloud happen to have concentrated along a certain radial direction with respect to the mass center of the cloud, such as those shown along line OJ and OK in **Fig. 7d.** Having so joined by a random chance, these groups would act together like a bar shown in **Fig. 1** to a certain extend. So the newly formed bar, although a broken one, would exert their gravitational influence through the on–axis effect onto those materials flying near the end of such the bar. Highly potentially, the flying objects are recruited by the bar. Once so recruited, the newly joining material would contribute to beef up the gravity strength of the materials gathering of the bar and



Fig. 7d

further escalated the bar's on-axis gravitational strength. Those material groups like **L** and **R,** they are located at the area that the off-axis effect of the bar is more obvious. Depending on the angular velocity they already possess, they may slowly drift (with rotation movement about the cloud's center) either toward the center or away from the center. To those drifting toward the center, their ever shortened rotation radius may accelerate them to plunge into the bar. To those drifting away from the center, their ever lengthening rotating radius and thus decreasing angular velocity may just make them sooner or later be arrested by the bar's sweeping. Either way, the bar is an unstoppable gravitational predator once so formed. As to the bars OJ and OK, once they stabilize their predator position, the centrifugal force and their own on-axis effect exerted on each other will line them up on one straight line across the cloud's center.
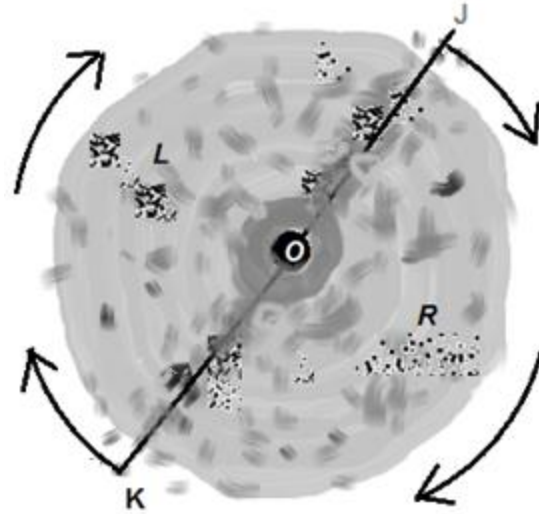
When materials of a huge quantity were tossed together in the remotely old days, no one can ever expect that a solid gravity body with a shape of high regularity could have formed itself like what is presented as the cross in this article. When the materials of various sizes were so randomly thrown at each other, the momentum between them is impossible to be exactly canceling each other out. The vector sum of all the off-center residual momentum contributed by each material chunk then forces the entire gathering to rotate about the center of the overall material formation.
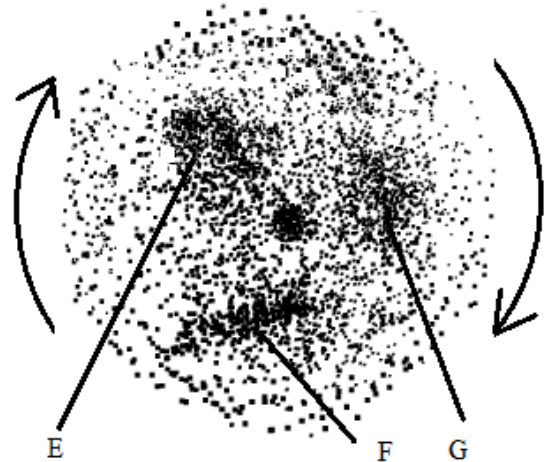


Fig. 8

The same randomness must also prevent the appearance of absolute homogeneity of material distribution across the entire formation. At areas where more materials have come together, the seed of a future rotation arm is planted. As shown in **Fig. 8**, blobs E, F, and G can all lure the formation of some rotation arms inside the big rotating formation.

From a state shown in **Fig. 8** to a state shown in **Fig. 9a** for the nowadays galaxy of Milky Way, there is a long history of transition similar to what is illustrated in **Fig. 7c** (for the overall formation of the entire cloud of the future galaxy) and **Fig. 7d** (for the formation of a central bar that is part of the entire rotating formation). In a more strict sense, we can say that the Milky Way has two types of rotation arms:  the straight arms, such as what is shown as the Galactic Bar at the galaxy center, and the spiral arms, such as what are shown in areas outside where the Bar is sweeping.  It takes more time for the straight Bar to come into shape than the spiral arms.  Those get recruited as one of the members in the Bar may move with all kinds of orbit in different shape with respect to the dead center of the Bar, from lanky ellipses to near perfect circles.  Their orbital plane may even form any angle with the ecliptic, from lying perfectly within it to being perpendicular to it.
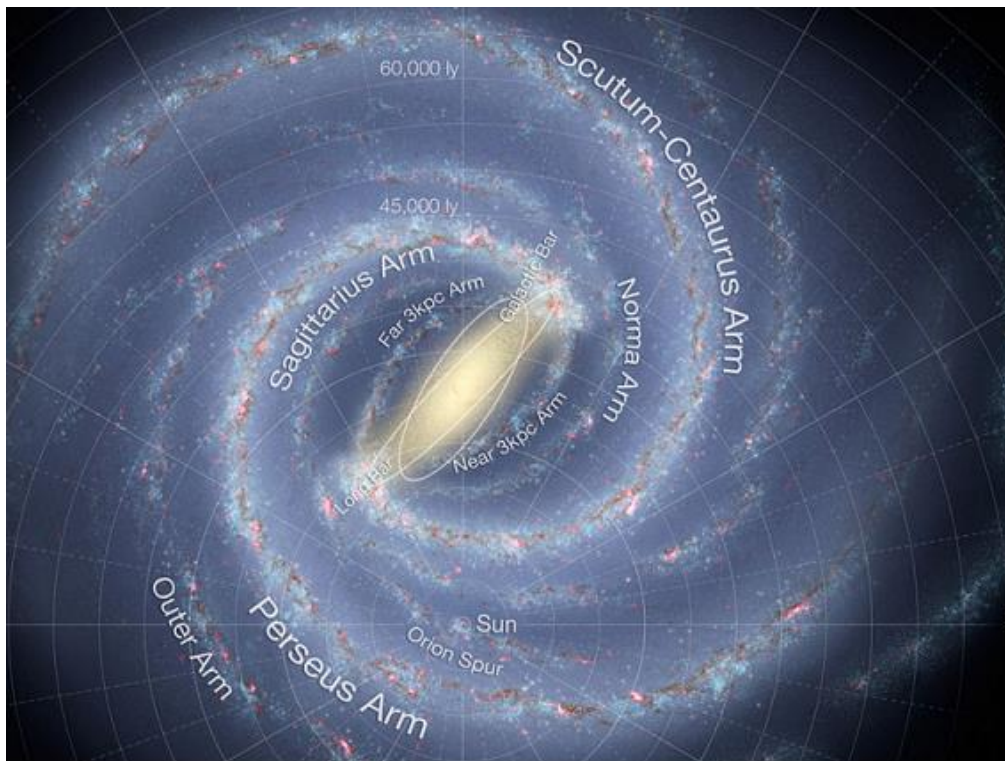


NASA/JPL-Caltech/ESO/R. Hurt - http://www.eso.org/public/images/eso1339e/

Fig. 9a

The analysis of **Fig. 5** would easily suggest to us that the spiral rotation arms may not be an unchanged establishment over time.   Somewhere there may be some material chunk that finds itself having entered a region with speed higher than necessary to balance the gravity field there and thus advanced to join the next arm.  On the contrary, some may find itself not with enough angular momentum to keep up with the peers around it and gradually lag behind and eventually fall into the arm that is coming after.  However, given the movement stability of the formation that has been established today, all these migrations can only happen in an extremely slow process.   It is this slow process that has introduced the formation of some minor spiral rotation arms in the Milky Way's rotation disk found in **Fig. 9a**.

**Fig. 9a**, shows two major spiral arms for the entire Milky Way, one flowing out from each end of the rotating bar.  Although two bars are identified in the photo, the close proximity between them allows us to consider them working as one.   It seems common among rotating galaxies that fundamentally two spiral arms are found for the entire galaxy, with one spiral to be dragged following each end of the rotation bar.  (**Fig. 9b, 9c, 9d, 9e**)



**Fig. 9 b**

Photo credit: NASA. Gov



**Fig. 9c**

Photo credit: NASA.gov

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**Fig. 9d**

Photo credit: content.time.com



Photo credit:

creciendocontunegocio.celeris.cl

**Fig. 9 e**

If we consider the on-axis effect, the phenomenon that one major spiral arm follows at each end of the rotation bar should appear highly natural. As the bar rotates, somewhere along its axis there must appear certain location at which materials chunks, such as object *A* in **Fig. 9f**, cannot have enough angular momentum to catch up with the bar's angular advancement. The centrifugal force disengages it a little from the bar. As this happens, its angular movement must somewhat lag behind the bar's. However, the strong "extra" gravitational force because of the on-axis effect must continue to bind object *A* in a "controllable" distance. In some sense, *A* taking its position is just as natural as some celestial body taking the Lagrangian point in some other gravitational system, although the cause is different. Staying away from the bar with the same reason like **A**'s, object B lags behind even more. The more being away from the region of the on-axis effect for *B* means the more for it to be in the region where off-axis effect is pronounced. However, the gravitational pull from A will not let go of *B* freely. Object *A* and *B* would also work together to drag *C* along while *C* has been even further away from the end of the rotation bar. This reaction continues so that a ribbon of materials are joining together to form a spiral formation following at the end of the bar. The same also happens at the other end of the bar. To the material chunks happening not at a close vicinity of the bar end, they would move away, waiting to be caught by the upcoming but extensively long spiral arms that is led by the other end of the bar, or just directly absorbed by the bar if its angular momentum is really so weak. Therefore, we



**Fig. 9f**

cannot expect to have a spiral arms flowing out at the middle of the bar. The Far 3kpc Arm and
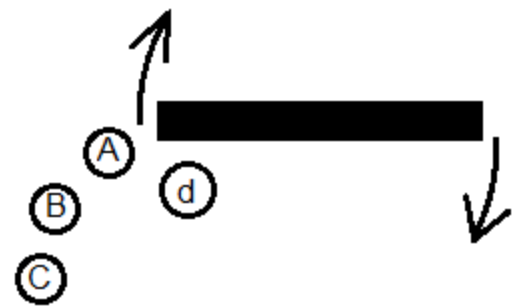
the Near 3kkpc Arm in *Fig. 9a* are produced by materials not having enough angular momentum but entering the off-axis effect region prematurely.  Object *d* shown in **Fig. 9f** is an example for such premature entry. Clearly shown in the picture, both the 3kpc Arms do not stem from the middle of the Bar.   On the other hand, **Fig. 9a** does show that the on-axis effect having captured higher concentration of materials at each end of the Bar.



Galaxy rotation curve for the Milky Way. Vertical axis is speed of rotation about the Galactic Center. Horizontal axis is distance from the Galactic Center in kpcs. The Sun is marked with a yellow ball. The observed curve of speed of rotation is blue. The predicted curve based upon stellar mass and gas in the Milky Way is red. Scatter in observations roughly indicated by gray bars. The difference is due to dark matter.[34][145][146]

**Fig. 9b**

Source of photo and illustration: **Wikipedia**

**Case 5c**

The blue curve in Fig. 9b shows the observed speed distribution of materials in the Milky Way.   At distance up to about 3 kpcs from the galactic center, the curve shows that the material movements obey what the gravity generated by a point mass would command.  However, beginning from the s kpcs point, such a point mass domination is abruptly interrupted.  This is because, beginning from this point, the on-axis and off-axis effect begin to dominate, and the so called flat rotation begins to be prominent.   One of the reasons for the flat rotation to occupy such an extensively large area (from 3 kpcs to 17 kpcs in radius) out of the entire Milky Way is like this:  Materials rotating about a center would receive stronger gravitational influence from those in the inner circles closer to the center than those from the outer circle.
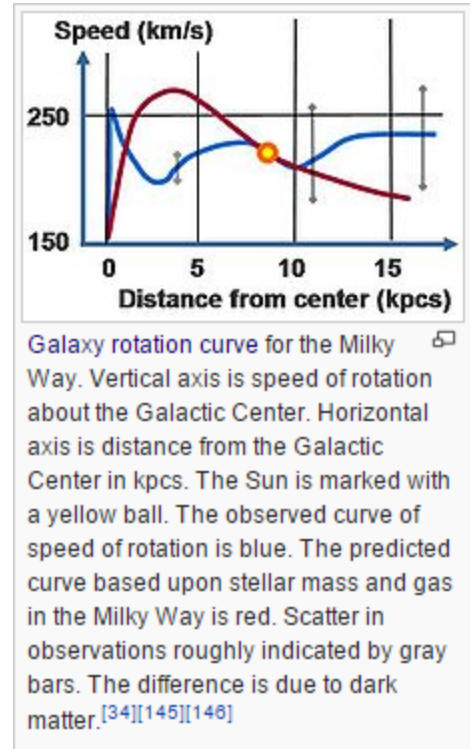
**Case 6   The Theoretical Impossibility for the Magellanic Clouds to Move on a Close Orbit about the Milky Way.**

Had the Magellanic Clouds ever been some satellites of the Milky Way, their current location and movement would only indicate that they have now been far away from the point called **Periapsis,** which is the point on the Clouds' supposed elliptical orbit but closest to the mass center of the Milky Way.
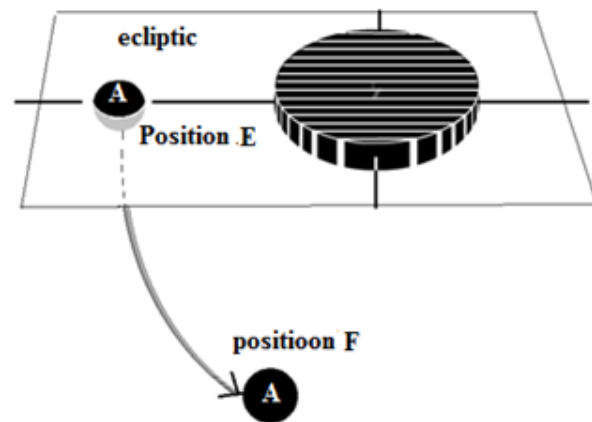


**Fig. 10**

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The Milky Way disk can be considered as being composed of many bars like what is shown in **Fig. 10**. When a massive body, called *A*, moves near the bars, it must receive certain on-axis effect of gravity from each bar. If *A* ever moves along an elliptical orbit about one bar, and the axis of the bar lies in the orbital plane, we have several situations as shown in **Fig. 11a**, **11b**, and **11c**.
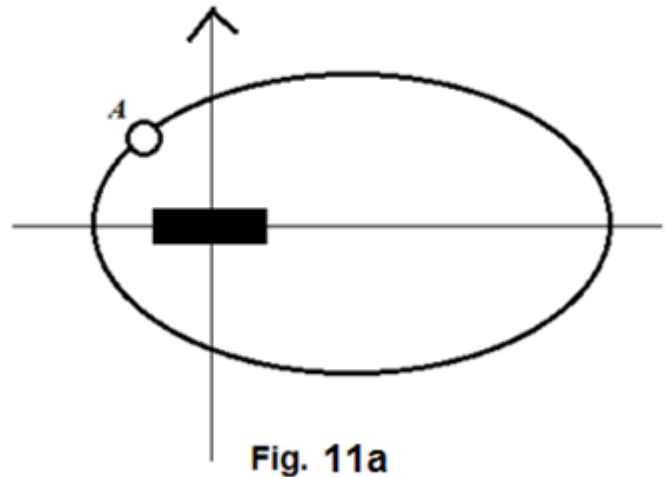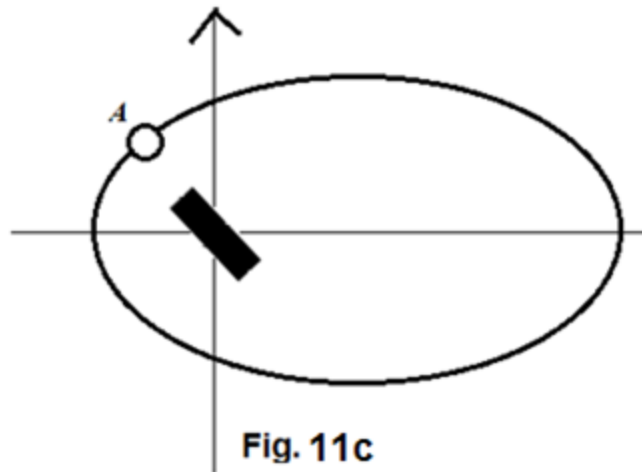
Comparison between **Fig. 11a**, **11b**, and **11c**, should lead us to visualize that **Fig 11a** is the most probable situation to happen. In **Fig 11a**, body *A* will receive the strongest gravitational force around the bar because of the on-axis effect when it migrates crossing the bar's axis, or at the point of periapsis. Subsequently, *A* has the highest speed here in the entire orbit.



Fig. 11a
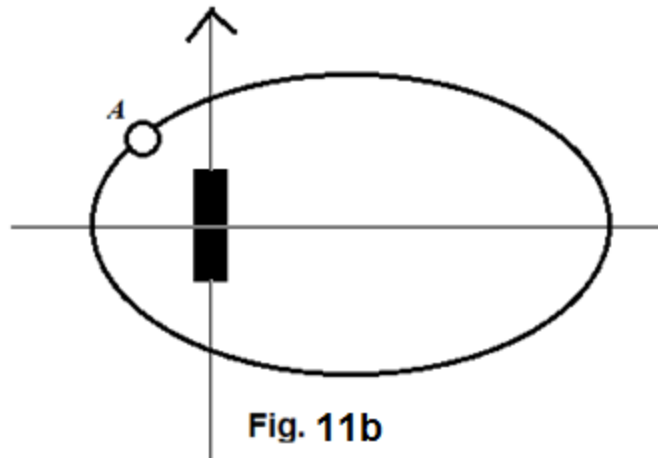
The problem is that, when *A* leaves the periapsis, it would enter a region where the off-axis is getting more and more prominent, thus the gravitational pull from the bar reduces more and more. However, the angular momentum with which *A* survives the gravitational pull at the periapsis remains the same. In other words, body *A* has more and more excessive momentum in responding to the gravitational pull of the bar after it leaves the periapsis. Any excessive momentum thus resulted must derail *A* from the supposed close orbit; any moving object considered to be a satellite of something else must have a close orbit.



Fig. 11b

The Milky Way as an entirety can be regarded as a collection of bars laid side by side but within the ecliptic. The on-axis effect of gravitational influence from each bar on the Magellanic Clouds is fundamentally the same, although the farther away a bar is from the galaxy center, the less prominent the on-axis effect would be. As the Magellanic Clouds move to a location like what position F indicates in **Fig. 10**, the off-axis effect between it and each bar would have been quite pronounced, or the gravitational pull from the Milky Way would have been quite weak. Then the only destination for the Clouds is to fly away from the Milky Way.

Therefore, we can claim with confidence that the Magellanic Clouds are visitors to the Milky Way only once in the Milky Way's life time, and in the Clouds' life time as well. Given that the current speed of the Large Magellenic Cloud is 378km/sec and the speed of the Small Cloud is 302 km/sec, if the universe has an age of 13.5 billion years, their birth place should have been no more than 17 million light years away from the current position, and about 100 times of the current distance between them and the Milky Way, provided that nothing has ever altered their movement during their entire journey, and that their journey is a straight line.



Fig. 11c

**Reference**:

http://phys.org/news/2007-01-magellanic-clouds.html

http://phys.org/news/2007-01-magellanic-clouds.html#jCp

Milky Way, http://Wikipedia.org