Planck Quantization of Newton and Einstein Gravitation

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Abstract

In this paper we rewrite the gravitational constant based on its relationship with the Planck length and, based on this, we rewrite the Planck mass in a slightly different form (that gives exactly the same value). In this way we are able to quantize a series of end results in Newton and Einstein's gravitation theories. The formulas will still give exactly the same values as before, but everything related to gravity will then come in quanta. Numerically this only has implications at the quantum scale; for macro objects the discrete steps are so tiny that they are close to impossible to notice. Hopefully this can give additional insight into how well or not so well (ad hoc) quantized Newton and Einstein's gravitation are potentially linked with the quantum world.

Key words: Quantized gravitation, gravitational constant, escape velocity, gravitational time dilation, Schwarzschild radius, Planck length, bending of light, Planck mass, Planck length.

1 Foundation

We suggest that the gravitational constant should be written as a function of Planck's reduced constant

$$G_p = \frac{\aleph^2 c^3}{\hbar} \tag{1}$$

where $\hbar$ is the reduced Planck's constant and c is the well tested round-trip speed of light. We could call this Planck's form of the gravitational constant. The parameter $\aleph$ is an unknown constant that is calibrated so that G_p matches our best estimate (measurement) for the gravitational constant.

As shown by Haug (2016), the Planck form of the gravitational constant enables us to rewrite the Planck length as

$$l_p = \sqrt{\frac{\hbar G_p}{c^3}} = \sqrt{\frac{\hbar \frac{\aleph^2 c^3}{\hbar}}{c^3}} = \aleph \tag{2}$$

and the Planck mass as

$$m_p = \sqrt{\frac{\hbar c}{G_p}} = \sqrt{\frac{\hbar c}{\frac{\aleph^2 c^3}{\hbar}}} = \frac{\hbar}{\aleph}\frac{1}{c} \tag{3}$$

Using the gravitational constant in the Planck form, as well as the rewritten Planck units, we are easily able to modify a series of end results from Newton and Einstein's gravitational theories to contain quantization as well.

2 Newton Universal Gravitational Force

The Newton gravitational force is given by

$$F_G = G_p \frac{m_1 m_2}{r^2} \tag{4}$$

*e-mail espenhaug@mac.com. Thanks to Victoria Terces for helping me edit this manuscript. In version 5 a mathematical typo in the gravitational acceleration and Newtons version of Kellers third law was fixed. If you find this paper of interest you will possibly also find my recent paper in the Relativity and Cosmology section "The Collapse of the Schwarzschild Radius: The End of Black Holes" of interest.

Using the gravitational constant of the form $G_p = \frac{\aleph^2 c^3}{\hbar}$ and the Planck mass of $m_p = \frac{\hbar}{\aleph}\frac{1}{c}$ we can rewrite the Newton gravitational force for two Planck masses as

$$\begin{aligned} F_{G_P} &= G_p \frac{m_p m_p}{r^2} \\ F_{G_P} &= \frac{\aleph^2 c^3}{\hbar} \frac{\frac{\hbar}{\aleph}\frac{1}{c}\frac{\hbar}{\aleph}\frac{1}{c}}{r^2} = \frac{\hbar c}{r^2} \end{aligned} \tag{5}$$

In the special case where $r = \aleph$ we get

$$F_{G_p} = \frac{\hbar}{\aleph}\frac{c}{\aleph} \tag{6}$$

It seems from this that gravity could be interpreted as hits per second. For large masses the form will be

$$\begin{aligned} F_{G_p} &= G_p \frac{N_1 m_p N_2 m_p}{r^2} \\ F_{G_p} &= G_p \frac{N_1 N_2 m_p^2}{r^2} \\ F_{G_p} &= N_1 N_2 \frac{\hbar}{r^2} c \end{aligned} \tag{7}$$

where N_1 is the number of Planck masses in object one and N_2 is the number of Planck masses in object two. In the case when the two masses are of equal size we have

$$F_{G_P} = N^2 \frac{\hbar}{r^2} c \tag{8}$$

3 Escape Velocity at the Quantum Scale

The traditional escape velocity is given by

$$v_e = \sqrt{\frac{2GM}{r}} \tag{9}$$

where G is the traditional gravitational constant and M is the mass of the object we are "trying" to escape from, and r is the radius of that object. In other words, we stand at the surface of the object, for example a hydrogen atom or a planet. Based on the gravitational constant written in the Planck form we can find the escape velocity at Planck scale; see also the Appendix for a derivation from "scratch". It must be

$$\begin{aligned} v_{e,p} &= \sqrt{\frac{2G_p N m_p}{r}} \\ v_{e,p} &= \sqrt{\frac{2N\frac{\aleph^2 c^3}{\hbar}\frac{\hbar}{\aleph}\frac{1}{c}}{r}} \\ v_{e,p} &= \sqrt{\frac{2N\aleph c^2}{r}} \\ v_{e,p} &= c\sqrt{\frac{2N\aleph}{r}} \end{aligned} \tag{10}$$

where N is the number of Planck masses in the planet or mass in question.

A particularly interesting case is when we only have one Planck mass $N = 1$ and $r = 2\aleph$ (this is actually the Schwarzschild radius of a Planck mass object). This gives us

$$\begin{aligned} v_{e,p} &= c\sqrt{\frac{2 \times 1 \times \aleph}{2\aleph}} \\ v_{e,p} &= c \end{aligned} \tag{11}$$

as the escape velocity for a particle with Planck mass with radius $2\aleph$ is c. Next we will see if the formula above can also be used to calculate the escape velocity of Earth. The Earth's mass is 5.972×10^{24} kg. We must convert this to the number of Planck masses. The Planck mass is

$$m_p = \frac{\hbar}{\aleph}\frac{1}{c} \approx 2.17651 \times 10^{-8}$$

The Earth's mass in terms of the numbers of Planck masses must be $\frac{5.972\times10^{24}}{2.17651\times10^{-8}} \approx 2.74388 \times 10^{32}$. Further the radius of the Earth is $r \approx 6\ 371\ 000$ meters. We can now just plug this into the Planck scale escape velocity:

$$\begin{aligned} v_{e,p} &= c\sqrt{\frac{2N\aleph}{r}} \\ v_{e,p} &= 299\ 792\ 458 \times \sqrt{\frac{2 \times 2.74388 \times 10^{32} \times 1.61622837 \times 10^{-35}}{6\ 371\ 000}} \approx 11\ 185.7 \text{ meters/second} \end{aligned}$$

which is equal to 40,269 km/h, the well-known escape velocity from the Earths gravitational field. We think our new way of looking at gravity could have consequences for the understanding of gravity. Gravitation must come in discrete steps and the escape velocity must also come in discrete steps for a given radius; this is because the amount of matter likely comes in discrete steps.

4 Orbital Speed

The orbital speed is given by

$$v_o \approx \sqrt{\frac{GM}{r}} \tag{12}$$

We can rewrite this in the form of the Planck gravitational constant and the Planck mass as

$$\begin{aligned} v_{o,p} &\approx \sqrt{\frac{G_p N m_p}{r}} \\ v_{o,p} &\approx \sqrt{\frac{\frac{\aleph^2 c^3}{\hbar} N \frac{\hbar}{\aleph}\frac{1}{c}}{r}} \\ v_{o,p} &\approx c\sqrt{\frac{N\aleph}{r}}. \end{aligned} \tag{13}$$

This can also be written as

$$v_{o,p} \approx \frac{v_e}{\sqrt{2}} = c\sqrt{\frac{N\aleph}{r}} \tag{14}$$

5 Gravitational Acceleration

The gravitational acceleration field in modern physics is given by

$$g = \frac{GM}{r^2} \tag{15}$$

This can be rewritten in quantized form as

$$\begin{aligned} g &= \frac{G_p M}{r^2} \\ g &= \frac{\frac{\aleph^2 c^3}{\hbar} N \frac{\hbar}{\aleph}\frac{1}{c}}{r^2} \\ g &= \frac{N\aleph c^2}{r^2} \end{aligned} \tag{16}$$

6 Gravitational Parameter

The standard gravitational parameter is given by

$$\mu = GM \tag{17}$$

This can be rewritten in quantized form as

$$\begin{aligned}\mu_p &= G_p M\\ \mu_p &= G_p N m_p\\ \mu_p &= \frac{\aleph^2 c^3}{\hbar} N \frac{\hbar}{\aleph}\frac{1}{c}\\ \mu_p &= N\aleph c^2\end{aligned} \tag{18}$$

7 Kepler's Third Law of Motion

The Newton "mechanics version" of Kepler's third law of motion for a circular orbit is given by

$$\frac{P^2}{a^3} = \frac{4\pi^2}{G(M_s + m)} \tag{19}$$

Where M_s is the mass of the Sun, m the mass of the planet, P is the period, and a is the semi-major axis. This can be re-written as

$$\begin{aligned}\frac{P^2}{a^3} &= \frac{4\pi^2}{G_p(N_1 m_p + N_2 m_p)}\\ \frac{P^2}{a^3} &= \frac{4\pi^2}{\frac{\aleph^2 c^3}{\hbar}\left(N_1 \frac{\hbar}{\aleph}\frac{1}{c} + N_2 \frac{\hbar}{\aleph}\frac{1}{c}\right)}\\ \frac{P^2}{a^3} &= \frac{4\pi^2}{\aleph c^2 (N_1 + N_2)}\end{aligned} \tag{20}$$

where N_1 is the number of Planck masses in the mass of the Sun M_s and N_2 is the number of Planck mass of the planet m. In the case the planets mass is much smaller than the Suns mass, we can use the following approximation

$$\frac{P^2}{a^3} \approx \frac{4\pi^2}{\aleph c^2 N} \tag{21}$$

where N is now the number of Planck masses in the Sun.

8 Gravitational Time Dilation at Planck Scale

Einstein's gravitational time dilation is given by

$$t_0 = t_f\sqrt{1 - \frac{2GM}{rc^2}} = t_f\sqrt{1 - \frac{v_e^2}{c^2}} \tag{22}$$

where v_e is the traditional escape velocity. We can rewrite this in the form of quantized escape velocity (derived above).

$$\begin{aligned}t_o &= t_f\sqrt{1 - \frac{v_{e,p}^2}{c^2}}\\ t_o &= t_f\sqrt{1 - \frac{\left(c\sqrt{2N\frac{\aleph}{r}}\right)^2}{c^2}}\\ t_o &= t_f\sqrt{1 - \frac{2N\aleph}{r}}\end{aligned} \tag{23}$$

Let's see if we can calculate the time dilation at, for example, the surface of the Earth from Planck scale gravitational time dilation. The Earth's mass is 5.972×10^{24} kg. And again, the Earth's mass in terms of the Planck mass must be $\frac{5.972\times10^{24}}{2.17651\times10^{-8}} \approx 2.74388 \times 10^{32}$. Further, the radius of the Earth is $r \approx 6\ 371\ 000$ meters. We can now just plug this into the quantized gravitational time dilation

$$
\begin{aligned}
t_o &= t_f\sqrt{1-\frac{2N\aleph}{r}} \\
t_o &= t_f\sqrt{1-\frac{2\times 2.74388\times10^{32}\times 1.61622837\times10^{-35}}{6\ 371\ 000}} \approx t_f \times 0.999999999303915
\end{aligned}
$$

That is for every second that goes by in outer space (a clock far away from the massive object), 0.99999999930391500 seconds goes by on the surface of the Earth. That is for every year in in outer space (very far from the Earth), there are about 22 milliseconds left to reach an Earth year. This is naturally the same as we would get with Einstein's formula. Still, the new way of writing the formula gives additional insight.

Circular orbits gravitational time dilation

The time dilation for a clock at circular orbit[1] is given by

$$t_0 = t_f\sqrt{1-\frac{3}{2}\frac{2GM}{rc^2}} = \sqrt{1-\frac{3}{2}\frac{v_e^2}{c^2}} \tag{24}$$

where v_e is the traditional escape velocity. We can rewrite this in the form of quantized escape velocity (derived above).

$$
\begin{aligned}
t_o &= t_f\sqrt{1-\frac{3}{2}\frac{v_{e,p}^2}{c^2}} \\
t_o &= t_f\sqrt{1-\frac{3}{2}\frac{\left(c\sqrt{2N\frac{\aleph}{r}}\right)^2}{c^2}} \\
t_o &= t_f\sqrt{1-\frac{3N\aleph}{r}} \qquad (25)
\end{aligned}
$$

9 The Schwarzschild Radius

The Schwarzschild radius of a mass M is given by

$$r_s = \frac{2GM}{c^2} \tag{26}$$

Rewritten into the quantum realm as described in this article it must be

$$
\begin{aligned}
r_s &= \frac{2G_pM}{c^2} \\
r_s &= \frac{2G_pNm_p}{c^2} \\
r_s &= \frac{2\frac{\aleph^2c^3}{\hbar}N\frac{\hbar}{\aleph}\frac{1}{c}}{c^2} \\
r_s &= 2N\aleph \qquad (27)
\end{aligned}
$$

For a clock at the Schwarzschild radius we get a time dilation of

$$t_o = t_f\sqrt{1-\frac{2N\aleph}{2N\aleph}} = 0. \tag{28}$$

At the Schwarzschild radius, time stands still. For a radius shorter than that the gravitational time dilation equation above breaks down.[2]

[1] At orbital radius larger then $\frac{3}{2}r_s$

[2] Except if we assume the $\aleph$ represents the radius of an indivisible particle. Thus if we move away from the point particle concept, this would simply mean that we could not go below the Planck scale Schwarzschild Radius.

Mass in Schwarzschild meter

The Schwarzschild mass in terms of meters is given by

$$\text{meter} = \frac{GM}{c^2} \tag{29}$$

This can be re-written as

$$\begin{aligned}
\text{meter} &= \frac{G_p N m_p}{c^2} \\
\text{meter} &= \frac{\frac{\aleph^2 c^3}{\hbar} N \frac{\hbar}{\aleph}\frac{1}{c}}{c^2} \\
\text{meter} &= N\aleph
\end{aligned} \tag{30}$$

10 Quantized Gravitational Bending of Light

The angle of deflection in Einstein's General relativity theory is given by

$$\delta_{GR} = \frac{4GM}{c^2 r}$$

This can be rewritten as

$$\begin{aligned}
\delta_{GRH} &= \frac{4G_p M}{c^2 r} \\
\delta_{GRH} &= \frac{4G_p N m_p}{c^2 r} \\
\delta_{GRH} &= \frac{4\frac{\aleph^2 c^3}{\hbar} N \frac{\hbar}{\aleph}\frac{1}{c}}{c^2 r} \\
\delta_{GRH} &= 4N\frac{\aleph}{r}
\end{aligned} \tag{31}$$

where N is the number of Planck masses making up the mass we are interested in. From the formula above, this means that the deflection of angles comes in quanta. Lets also "control" that our Planck scale deflection rooted in Planck and GR is consistent for large bodies like the Sun, for example. The solar mass is $M_s \approx 1.988 \times 10^{30}$ kg. The Sun's mass in terms of the number of Planck masses must be $\frac{1.988 \times 10^{30}}{2.17651 \times 10^{-8}} \approx 9.134 \times 10^{37}$. Further, the radius of the Sun is $r_s \approx 696\ 342\ 000$ meters. We can just plug this into the Planck scale deflection:

$$\delta_{GRH} = \frac{4N\aleph}{r} = \frac{4 \times 9.134 \times 10^{37} \times 1.61622837 \times 10^{-35}}{696\ 342\ 000} \approx 8.48 \times 10^{-06} \tag{32}$$

If we multiply this by $\frac{648\ 000}{\pi}$ we get a bending of light of about 1.75 arcseconds or $\frac{1.75}{3\ 600}$ of a degree. This is the same as has been confirmed by experiments and helped make Einstein famous, as Newton gravitation supposedly only predicted half of the bending of light. Newton bending of light is given by

$$\delta_{Newton} = \frac{2GM}{c^2 r} \tag{33}$$

See for example Soares (2009) and Momeni (2012) for derivations of bending of light under Newton gravitation.

11 Gravitational Redshift

The Einstein gravitational redshift is given by

$$\lim_{r \to +\infty} z(r) = \frac{1}{\sqrt{1 - \frac{\frac{2GM}{c^2}}{R_e}}} - 1 \tag{34}$$

where R_e is the distance between the center of the mass of the gravitating body and the point at which the photon is emitted. This we can rewrite as

$$\lim_{r\to+\infty} z(r) = \frac{1}{\sqrt{1-\frac{\frac{2GM}{c^2}}{R_e}}} - 1$$
$$\lim_{r\to+\infty} z(r) = \frac{1}{\sqrt{1-\frac{2\frac{\aleph^2 c^3}{\hbar}N\frac{\hbar}{\aleph}\frac{1}{c}}{\frac{c^2}{R_e}}}} - 1$$
$$\lim_{r\to+\infty} z(r) = \frac{1}{\sqrt{1-\frac{2N\aleph}{R_e}}} - 1 \quad (35)$$

Further in the Newtonian limit when R_e is sufficiently large compared to the Schwarzschild radius we can approximate the above expression with

$$\lim_{r\to+\infty} z(r) \approx \frac{GM}{c^2 R_e}$$
$$\lim_{r\to+\infty} z(r) \approx \frac{\frac{\aleph^2 c^3}{\hbar}N\frac{\hbar}{\aleph}\frac{1}{c}}{c^2 R_e}$$
$$\lim_{r\to+\infty} z(r) \approx \frac{N\aleph}{R_e} \quad (36)$$

12 Einstein's Field Equation

And finally we get to Einstein's field equation. It is given by

$$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi G}{c^4}T_{\mu v} \quad (37)$$

I am far from an expert on Einstein's field equation, but based on the Planck gravitational constant given in this paper we can rewrite it as

$$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi G_p}{c^4}T_{\mu v}$$
$$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi\frac{\aleph^2 c^3}{\hbar}}{c^4}T_{\mu v}$$
$$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi\aleph^2}{\hbar c}T_{\mu v} \quad (38)$$

Bear in mind $\hbar = \frac{h}{2\pi}$ and based on this we can alternatively write Einstein's field equation as

$$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{16\pi^2\aleph^2}{hc}T_{\mu v} \quad (39)$$

The potential interpretation and usefulness of this rewritten version of Einstein's field equation we leave up to other experts to consider. An interesting question is naturally whether or not it is consistent with some of the derivations given above in this form.

13 Table Summary

The table below summarizes our rewriting of some gravitational formulas. The output is still the same, but based on this view of gravity, masses, gravitational time dilation, and even escape velocity all come in discrete steps.

14 Conclusion

By making the gravitational constant a function form of the reduced Planck constant one can easily rewrite many of the end results from Newton and Einstein's gravitation in quantized form. Even if this is seen as an ad hoc method, it could still give new insight into what degree quantized Newton's gravitation and General relativity are consistent with the quantum realm.

Table 1: The table shows some of the standard gravitational relationships given by Newton and Einstein and their expression in quantized form.

Units:	**Newton and Einstein form:**	**Quantized-form:**
Gravitational constant	$G \approx 6.67408 \times 10^{-11}$	$G_p = \frac{\aleph^2 c^3}{\hbar}$
Newton's gravitational force	$F_G = G\frac{M_1 M_2}{r^2}$	$F_G = N_1 N_2 \frac{\hbar c}{r^2}$
Newton's gravitational force	$F_G = G\frac{m_p m_p}{\aleph^2}$	$F_G = G_p \frac{m_p m_p}{r^2} = \frac{\aleph^2 c^3}{\hbar} \frac{\frac{\hbar}{\aleph}\frac{1}{c}\frac{\hbar}{\aleph}\frac{1}{c}}{\aleph^2} = \frac{\hbar}{\aleph}\frac{c}{\aleph}$
Kepler's third law	$\frac{P^2}{a^3} = \frac{4\pi^2}{G(M_s+m)}$	$\frac{P^2}{a^3} = \frac{4\pi^2}{\aleph c^2 (N_1+N_2)}$
Newton's Escape velocity from any mass	$v_e = \sqrt{\frac{2GM}{r}}$	$v_{e,p} = c\sqrt{\frac{N_2}{N_1}\frac{2\aleph}{r}}$
Orbital velocity for any mass	$v_o \approx \sqrt{\frac{GM}{r}}$	$v_{o,p} \approx c\sqrt{\frac{N_2}{N_1}\frac{\aleph}{r}}$
Gravitational acceleration field	$g = \frac{GM}{r^2}$	$g_p = \frac{N\aleph c^2}{r^2}$
Gravitational parameter	$\mu = GM$	$\mu_p = N\aleph c^2.$
Gravitational time dilation	$t_0 = t_f\sqrt{1 - \frac{2GM}{c^2}} = \sqrt{1 - \frac{v_e^2}{c^2}}$	$t_o = t_f\sqrt{1 - \frac{2N\aleph}{r}}$
Orbital time dilation	$t_0 = t_f\sqrt{1 - 3\frac{GM}{c^2}} = \sqrt{1 - 3\frac{v_e^2}{c^2}}$	$t_o = t_f\sqrt{1 - \frac{3N\aleph}{r}}$
Schwarzschild radius	$r_s = \frac{2GM}{c^2}$	$r_s = 2N\aleph$
Bending of light	$\delta_{GR} = \frac{4GM}{c^2 r}$	$\delta_{GRH} = \frac{4N\aleph}{r}$
Black holes	Possible	Depends on quantum interpretation

Appendix: Escape velocity

Derivation of the escape velocity from Planck scale

$$\begin{aligned}
E &\approx \frac{1}{2}mv^2 - \frac{GmM}{r} \\
E &\approx \frac{1}{2}N_1 m_p v^2 - \frac{GN_1 m_p N_2 m_p}{r} \\
E &\approx \frac{1}{2}N_1 \frac{\hbar}{\aleph}\frac{1}{c}v^2 - \frac{N_1 \frac{\aleph^2 c^3}{\hbar}\frac{\hbar}{\aleph}\frac{1}{c} N_2 \frac{\hbar}{\aleph}\frac{1}{c}}{r} \\
E &\approx \frac{1}{2}N_1 \frac{\hbar}{\aleph}\frac{1}{c}v^2 - N_1 N_2 \frac{\hbar}{r}c
\end{aligned} \tag{40}$$

where N_1 is the number of Planck masses in the smaller mass m (for example a rocket) and N_2 is the number of Planck masses in the other mass. This we have to set to 0 and solve with respect to v to find the escape velocity:

$$\begin{aligned}
\frac{1}{2}N_1 \frac{\hbar}{\aleph}\frac{1}{c}v^2 - N_1 N_2 \frac{\hbar}{r}c &= 0 \\
v^2 &= 2\frac{N_1 N_2 \frac{\hbar}{r}c}{N_1 \frac{\hbar}{\aleph}\frac{1}{c}} \\
v^2 &= 2N_2 \frac{\aleph c^2}{r} \\
v &= c\sqrt{N_2 \frac{2\aleph}{r}}
\end{aligned} \tag{41}$$

This is a quantized escape velocity. Since N_1 cancels out we can simply call N_2 for N and write the escape velocity as

$$v = c\sqrt{N\frac{2\aleph}{r}} \tag{42}$$

where N is the number of Planck masses in the mass we are trying to escape from.

References

Einstein, A. (1916): "Näherungsweise Integration der Feldgleichungen der Gravitation," *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin.*

Haug, E. G. (2016): "The Gravitational Constant and the Planck Units. A Deeper Understanding of the Quantum Realm," *www.ViXra.org March 13 2016*.

Momeni, D. (2012): "Bending of Light a Classical Analysis," *Working paper*, pp. 1–4.

Newton, I. (1686): *Philosophiae Naturalis Principia Mathematics.* London.

Planck, M. (1901): "Ueber das Gesetz der Energieverteilung im Normalspectrum," *Annalen der Physik*, 4.

Schwarzschild, K. (1916a): "Über das Gravitationsfeld einer Kugel aus Inkompressibler Flussigkeit nach der Einsteinschen Theorie," *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik*, p. 424.

——— (1916b): "Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie," *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik*, p. 189.

Soares, D. S. L. (2009): "Newtonian Gravitational Deflection of Light Revisited," *Working Paper version 4*.