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The Electromagnetic Wave in Alternating Current Wire

Annotation

A solution of Maxwell equations for alternating current wire is presented. The structure of currents and energy flows is reviewed.

Contents

1. Introduction

In [1] presents a noncontradictory solution of Maxwell equations for vacuum. Here we offer a similar solution of Maxwell equations for an alternating current wire.

The Maxwell equations in general in GHS system have the following form [2]:

$$\mathrm{rot}(E)+\frac{\mu}{c}\frac{\partial H}{\partial t}=0, \tag{1}$$

$$\mathrm{rot}(H)-\frac{\varepsilon}{c}\frac{\partial E}{\partial t}-\frac{4\pi}{c}J=0, \tag{2}$$

$$\mathrm{div}(E)=0, \tag{3}$$

$$\mathrm{div}(H)=0, \tag{4}$$

$$J=\frac{1}{\rho}E, \tag{5}$$

where

J, H, E - conduction current, magnetic and electric intensity accordingly ,

$\varepsilon, \ \mu, \ \rho$ - dielectric permittivity, permeability, specific resistance of the wire's material

Further these equations are used for analyzing the structure of Alternating Current in a wire. For sinusoidal current in a wire with specific inductance L and specific resistance ρ intensity and current are related in the following way:

$$J = \frac{1}{\rho + i\omega L} E = \frac{\rho - i\omega L}{\rho^2 + (\omega L)^2} E .$$

Hence for $\rho << \omega L$ we find:

$$I \approx \frac{-i}{\omega L} E .$$

Therefore for analyzing the structure of sinusoidal current in the wire for a sufficiently high frequency the condition (5) can be neglected. При этом is necessary to solve the equation system (1-4), where the known value is the current J_z flowing among the wire, i.e. the projection of vector J on axis oz.

2. Solution of Maxwell's equations

Let us consider the solution of Maxwell equations system (1.1-1.4) for the wire. In cylindrical coordinates system $r, \ \varphi, \ z$ these equations look as follows [3]:

$$\frac{E_r}{r} + \frac{\partial E_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial E_\varphi}{\partial \varphi} + \frac{\partial E_z}{\partial z} = 0, \tag{1}$$

$$\frac{1}{r} \cdot \frac{\partial E_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z} = v \frac{dH_r}{dt}, \tag{2}$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = v \frac{dH_\varphi}{dt}, \tag{3}$$

$$\frac{E_\varphi}{r} + \frac{\partial E_\varphi}{\partial r} - \frac{1}{r} \cdot \frac{\partial E_r}{\partial \varphi} = v \frac{dH_z}{dt}, \tag{4}$$

$$\frac{H_r}{r} + \frac{\partial H_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial H_\varphi}{\partial \varphi} + \frac{\partial H_z}{\partial z} = 0, \tag{5}$$

$$\frac{1}{r} \cdot \frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\varphi}{\partial z} = q \frac{dE_r}{dt} \tag{6}$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = q \frac{dE_\varphi}{dt}, \tag{7}$$

$$\frac{H_\varphi}{r}+\frac{\partial H_\varphi}{\partial r}-\frac{1}{r}\cdot\frac{\partial H_r}{\partial \varphi}=q\frac{dE_z}{dt}+\frac{4\pi}{c}J_z, \tag{8}$$

where

$$v=-\frac{\mu}{c}, \tag{9}$$

$$q=\frac{\varepsilon}{c}, \tag{10}$$

Further we shall consider only monochromatic solution. For the sake of brevity further we shall use the following notations:

$$co=\cos(\alpha\varphi+\chi z+\omega t), \tag{11}$$

$$si=\sin(\alpha\varphi+\chi z+\omega t), \tag{12}$$

where $\alpha,\ \chi,\ \omega$ – are certain constants. Let us present the unknown functions in the following form:

$$H_r.=h_r(r)co, \tag{13}$$

$$H_\varphi.=h_\varphi(r)si, \tag{14}$$

$$H_z.=h_z(r)si, \tag{15}$$

$$E_r.=e_r(r)si, \tag{16}$$

$$E_\varphi.=e_\varphi(r)co, \tag{17}$$

$$E_z.=e_z(r)co, \tag{18}$$

$$J_r.=j_r(r)co, \tag{19}$$

$$J_\varphi.=j_\varphi(r)si, \tag{20}$$

$$J_z.=j_z(r)si, \tag{21}$$

where $h(r),\ e(r),\ j(r)$ - certain function of the coordinate r.

By direct substitution we can verify that the functions (13-21) transform the equations system (1-8) with four arguments $r,\ \varphi,\ z,\ t$ into equations system with one argument r and unknown functions $h(r),\ e(r),\ j(r)$.

Further it will be assumed that there exists only the current (21), directed along the axis z. This current is created by an external source. It is shown that the presence of this current is the cause for the existence of electromagnetic wave in the wire.

In Appendix 1 it is shown that for system (1.1-1.4) at the conditions (13-21) there **exists** a solution of the following form:

$$e_\varphi(r)=Ar^{\alpha-1}, \tag{22}$$

$$e_r(r)=e_\varphi(r), \tag{23}$$

$$e_z(r) = \hat{\chi}\frac{(M-1)}{\sqrt{M}}\frac{\omega\sqrt{\varepsilon\mu}}{\alpha c} r e_\varphi(r), \qquad (24)$$

$$h_r(r) = \hat{\chi}\sqrt{\frac{\varepsilon}{M\mu}} e_\varphi(r), \qquad (25)$$

$$h_\varphi(r) = -h_r(r), \qquad (26)$$

$$h_z(r) = 0, \qquad (27)$$

$$j_z(r) = \frac{\varepsilon\omega}{4\pi} e_z(r) = \frac{\chi\varepsilon\omega}{2\pi\alpha} A r^\alpha, \qquad (28)$$

where A, c, α, ω – constants.

Let us compare this solution to the solution obtained in [1] for vacuum – see Table 1. Evidently (despite the identity of equations) these solutions differ greatly. These differences are caused by the presence of external electromotive force with $e_z(r) \neq 0$. It causes a longitudinal displacement current which changes drastically the structure of electromagnetic wave.

Table 1.

	Vacuum	**Wire**
χ	$\hat{\chi}\frac{\omega}{c}\sqrt{\varepsilon\mu}$	$\hat{\chi}\frac{\omega}{c}\sqrt{M\varepsilon\mu}$, $\hat{\chi} = \pm 1$
j_z	**0**	$\frac{\varepsilon\omega}{4\pi} e_z(r)$
e_r, e_φ	$Ar^{\alpha-1}$	$Ar^{\alpha-1}$
e_z	**0**	$\hat{\chi}\frac{(M-1)}{\sqrt{M}}\frac{\omega\sqrt{\varepsilon\mu}}{\alpha c} r e_\varphi(r)$
h_r	$-e_\varphi(r)$	$\hat{\chi}\sqrt{\frac{\varepsilon}{M\mu}} e_\varphi(r)$
h_φ	$-h_r(r)$	$-h_r(r)$
h_z	**0**	**0**

3. Intensities and currents in the wire

Further we shall consider only the functions $j_z(r)$, $e_r(r)$, $e_\varphi(r)$, $e_z(r)$, $h_r(r)$, $h_\varphi(r)$, $h_z(r)$. Fig. 1 shows, for example, the graphs of these functions for $A=1,\ \alpha=3,\ \mu=1,\ \varepsilon=1,\ \omega=300$. The value $j_z(r)$ is shown in units of (A/mm^2) - in contrast to all the other values shown in system SI. <u>The increase of function</u> $j_z(r)$ <u>at the radius increase explains the skin-effect.</u>

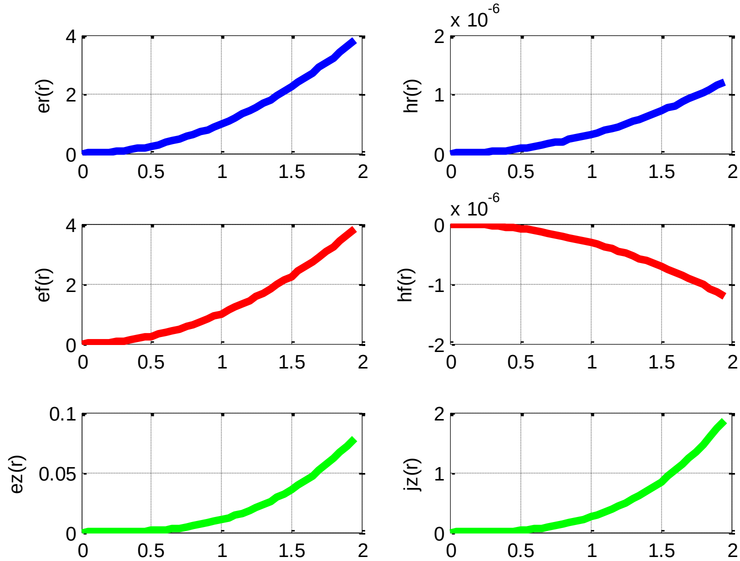

Fig.1. (SSMB)

The energy density of electromagnetic wave is determines as the sum of modules of vectors E, H from (2.13, 2.14, 2.16, 2.17, 2.23, 2.24) and is equal to

$$W = E^2 + H^2 = (e_r(r)si)^2 + (e_\varphi(r)si)^2 + (h_r(r)co)^2 + (h_\varphi(r)co)^2$$

or

$$W = (e_r(r))^2 + (e_\varphi(r))^2 \tag{1}$$

- see also Fig. 1. Thus, the <u>density of electromagnetic wave energy is constant in all points of a circle of this radius</u>.

To demonstrate that <u>the components of the wave are shifted in phase</u>, in Fig. 2 shows the functions

$$co = \cos(\alpha\varphi + \chi z + \omega t), \quad si = \sin(\alpha\varphi + \chi z + \omega t)$$

or equivalent to them at $z = ct$ function

$$co = \cos\left(\alpha\varphi + \frac{2\omega}{c}z\right), \qquad si = \sin\left(\alpha\varphi + \frac{2\omega}{c}z\right). \tag{2}$$

At $\varphi = 0,\ 2\omega/c = 0.1$ these functions take the form $co = \cos(z)$, $si = \sin(z)$ and shown in Fig. 2.

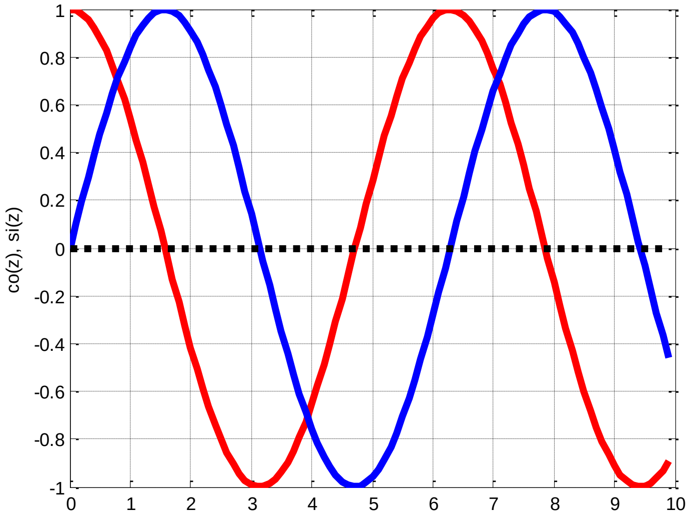

Fig.2. (SSMB)

Let us find the average value of current amplitude density in a wire of radius R:

$$\overline{J_z} = \frac{1}{\pi R^2} \iint_{r,\varphi} [J_z] dr \cdot d\varphi . \tag{5}$$

Taking into account (2.21), we find:

$$\overline{J_z} = \frac{1}{\pi R^2} \iint_{r,\varphi} [j_z(r) si] dr \cdot d\varphi = \frac{1}{\pi R^2} \int_0^R j_z(r) \left(\int_0^{2\pi} (si \cdot d\varphi) \right) dr \cdot$$

Taking into account (2), we find:

$$\overline{J_z} = \frac{1}{\alpha \pi R^2} \int_0^R j_z(r) \left(\cos(2\alpha\pi + \frac{2\omega}{c} z) - \cos(\frac{2\omega}{c} z) \right) dr$$

or

$$\overline{J_z} = \frac{1}{\alpha \pi R^2} (\cos(2\alpha\pi) - 1) \cdot J_{zr} , \tag{6}$$

where

$$J_{zr} = \int_0^R j_z(r) dr . \tag{7}$$

Taking into account (2.28), we find:

$$J_{zr} = \frac{A\chi\varepsilon\omega}{2\pi\alpha} \int_0^R (r^\alpha) dr \tag{9}$$

or

$$J_{zr} = \frac{A\chi\varepsilon\omega}{2\pi\alpha(\alpha+1)} R^{\alpha+1}. \qquad (10)$$

Fig. 3 shows the function $\overline{J_z}(\alpha)$ (6, 10) for $A=1$. On this Figure the dotted and solid lines are related accordingly to $R=2$ and $R=1.75$. From (6, 8) and Fig. 3 it follows that for a certain distribution of the value $j_z(r)$ the average value of the amplitude of current density $\overline{J_z}$ depends significantly of α.

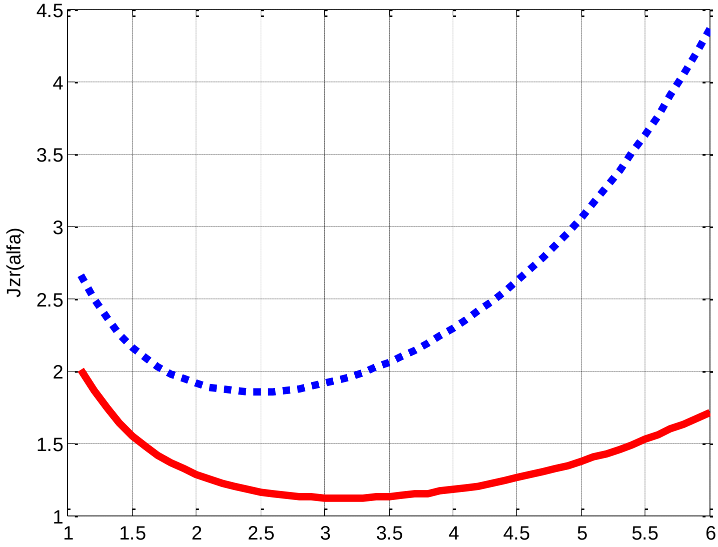

Fig.3. (SSMB)

The current is determined as

$$J = \frac{\varepsilon}{c}\frac{\partial E}{\partial t}, \qquad (9)$$

or, taking into account (2.13-2.21):

$$J_r. = \frac{\varepsilon\omega}{c} e_r(r) co, \qquad (10)$$

$$J_\varphi. = \frac{\varepsilon\omega}{c} e_\varphi(r) si, \qquad (11)$$

$$J_z. = \left(\frac{\varepsilon\omega}{c} e_z(r) + j_z\right) si. \qquad (12)$$

You can talk about the lines of these currents. Thus, for instance, the current $J_z.$ flows along the straight lines parallel to the wire axis. We shall look now on the line of summary current.

It can be assumed that the speed of displacement current propagation does not depend on the current direction. In particular, for a fixed radius the path traversed by the current along a circle, and the path traversed by it along a vertical, would be equal. Consequently, for a fixed radius we can assume that

$$z = \gamma \cdot \varphi \tag{13}$$

where γ is a constant. Based on this assumption we can convert the functions (4b) into

$$co = \cos(\alpha\varphi + 2\chi\gamma\varphi), \qquad si = \sin(\alpha\varphi + 2\chi\gamma\varphi) \tag{14}$$

and build an appropriate trajectory for the current. Fig. 4 shows two spiral lines of summary current described by the functions of the form

$$co = \cos((\alpha + 2)\varphi), \qquad si = \sin((\alpha + 2)\varphi).$$

On Fig. 4 the thick line is built for $\alpha = 1.8$ and a thin line for $\alpha = 2.5$.

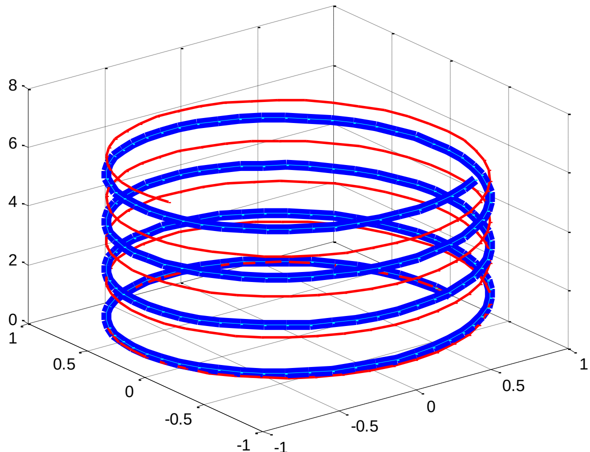

Fig.4. (SSMB)

From (2.19-2.21, 14) follows that the currents will keep their values for given r, φ (independently of z) if only the following value is constant

$$\beta = (\alpha + 2\chi\gamma). \tag{15}$$

Further, based on (14, 15) we shall be using the formula

$$co = \cos(\beta\varphi), \qquad si = \sin(\beta\varphi). \tag{16}$$

4. Energy Flows

The density of electromagnetic flow is Pointing vector

$$S = \eta E \times H, \tag{1}$$

where

$$\eta = c/4\pi. \tag{2}$$

In cylindrical coordinates $r,\ \varphi,\ z$ the density flow of electromagnetic energy has three components $S_r,\ S_\varphi,\ S_z$, directed along вдоль the axis accordingly. They are determined by the formula

$$S = \begin{bmatrix} S_r \\ S_\varphi \\ S_z \end{bmatrix} = \eta(E \times H) = \eta \begin{bmatrix} E_\varphi H_z - E_z H_\varphi \\ E_z H_r - E_r H_z \\ E_r H_\varphi - E_\varphi H_r \end{bmatrix}. \qquad (4)$$

From (2.13-2.18) follows that the flow passing through a given section of the wave in a given moment, is:

$$\overline{S} = \begin{bmatrix} \overline{S_r} \\ \overline{S_\varphi} \\ \overline{S_z} \end{bmatrix} = \eta \iint_{r,\varphi} \begin{bmatrix} s_r \cdot si^2 \\ s_\varphi \cdot si \cdot co \\ s_z \cdot si \cdot co \end{bmatrix} dr \cdot d\varphi. \qquad (5)$$

where

$$\begin{aligned} s_r &= (e_\varphi h_z - e_z h_\varphi) \\ s_\varphi &= (e_z h_r - e_r h_z). \\ s_z &= (e_r h_\varphi - e_\varphi h_r) \end{aligned} \qquad (6)$$

It is values density of the energy flux at a predetermined radius which extends radially, circumferentially along, the axis oz respectively. Fig. 5 shows the graphs of these functions depending on the radius at $A=1,\ \alpha=3,\ \mu=1,\ \varepsilon=1,\ \omega=300$.

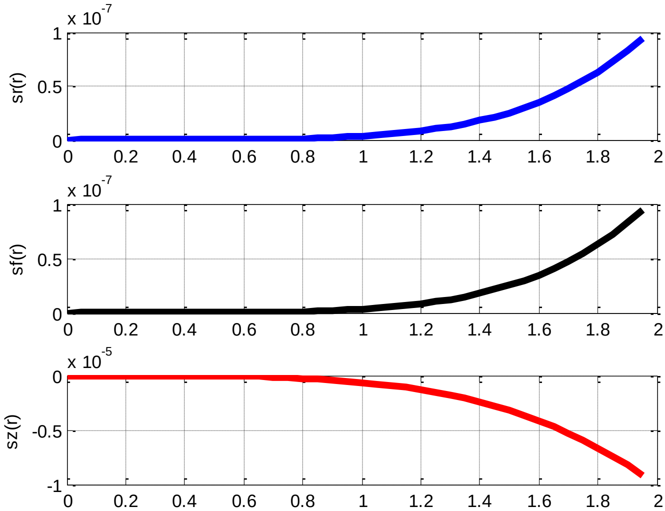

Fig.5. (SSMB)

The flow of energy along the axis oz is

$$\overline{S}=\overline{S_z}=\eta\iint\limits_{r,\varphi}[s_z\cdot si\cdot co]dr\cdot d\varphi. \tag{7}$$

We shall find s_z. From (6, 2.22, 2.23, 2.26), we obtain:

$$s_z=-2e_\varphi h_r=-\hat{\chi}\sqrt{\frac{\varepsilon}{M\mu}}e_\varphi^2(r) \tag{9}$$

or

$$s_z=Qr^{2\alpha-2}, \tag{10}$$

while

$$Q=A^2\hat{\chi}\sqrt{\frac{\varepsilon}{M\mu}}. \tag{11}$$

In Appendix 2 of article [1] shows that from (7) implies that

$$\overline{S}=\frac{c}{16\alpha\pi}(1-\cos(4\alpha\pi))\int\limits_r(s_z(r)dr). \tag{12}$$

Let R be <u>the radius of the circular front of the wave.</u> Then from (12) we obtain, as in [1],

$$S_{\text{int}}=\int\limits_{r=0}^{R}(s_z(r)dr)=\frac{Q}{2\alpha-1}R^{2\alpha-1}, \tag{13}$$

$$S_{alfa}=\frac{1}{\alpha}(1-\cos(4\alpha\pi)), \tag{14}$$

$$\overline{S}=\frac{c}{16\pi}S_{alfa}S_{\text{int}}. \tag{15}$$

Combining formulas (11-15), we get:

$$\overline{S_z}=\frac{c}{16\pi}\frac{1}{\alpha}(1-\cos(4\alpha\pi))A^2\sqrt{\frac{\varepsilon}{M\mu}}\frac{\hat{\chi}}{2\alpha-1}R^{2\alpha-1}$$

or

$$\overline{S_z}=\frac{\hat{\chi}A^2c(1-\cos(4\alpha\pi))}{8\pi\alpha(2\alpha-1)}\sqrt{\frac{\varepsilon}{M\mu}}R^{2\alpha-1}. \tag{16}$$

This energy flow does not depend on the coordinates, and so it keeps its value along all the length of wire.

Fig. 7 shows the function $\overline{S}(\alpha)$ (15) for $Q=1$. On Fig. 7 the dotted and the solid lines refer respectively to $R=2$ and $R=1.8$.

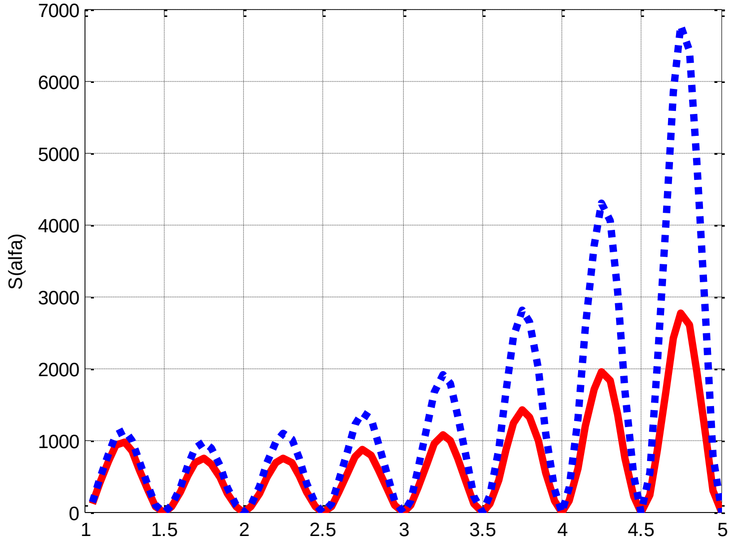

Fig.7. (SSMB)

Since the energy flow and the energy are related by the expression $S = W \cdot c$, then from (15) we can find the energy of a wavelength unit:

$$\overline{W} = \frac{A}{16\pi} S_{alfa} S_{\text{int}}. \tag{17}$$

It follows from (7, 3.16), the energy flux density on the circumference of the radius defined function of the form

$$\overline{S}_{rz} = s_z \sin(2\beta\varphi). \tag{18}$$

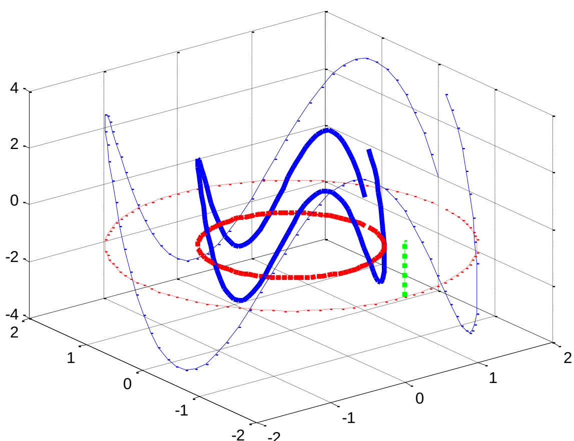

Fig.8. (SSMB)

Fig. 8 shows this function (18) for $A=1$, $\chi=1$, $\alpha=1.4$, $\beta=1.6$ and for two values of radius: $r=1$ and $r=2$. На рис. 8 показана функция (18) at $s_z=r^{2\alpha-2}$ - see (10). Shows two curves for two values at $\alpha=1.4$ and at two values of radius $r=1$ (thick line) and $r=2$ (thin line).

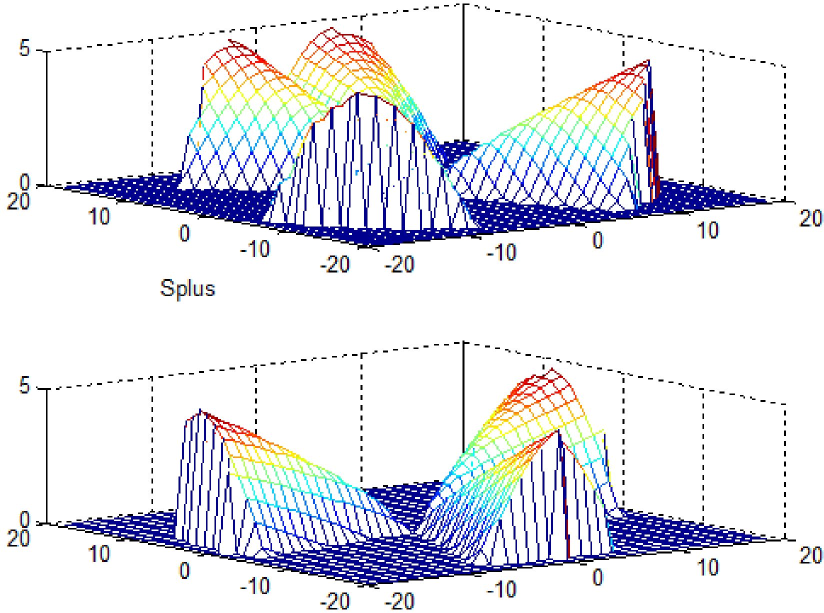

Fig.9. (SSMB)

Fig. 9 shows the function S (18) on the whole plane of wire section for $A=1$, $\chi=1$, $\alpha=1.4$, $\beta=1.6$, $R=19$. Fig. 9 shows the function S (18) on the entire section plane of wire at $s_z=r^{2\alpha-2}$ and at $\alpha=1.4$.The upper window shows the part of function S graph for which $S>0$ - called $Splus$, and the lower window shows the part S graph for which $S<0$ - called $Sminus$, and this part for clarity is shown with the opposite sign. This figure shows that

$$S = Splus + Sminus > 0,$$

i.e. the summary vector of flow density is directed toward the increase of z - toward the load. However there are two components of this vector: the $Splus$ component, directed toward the load, and $Sminus$ component, directed toward the source of current. These components of the flow transfer the active and reactive energies accordingly.

It follows that

- flux density is unevenly distributed over the flow cross section – there is a picture of the distribution of flow density by the cross section of the wave
- this picture is rotated while moving on the axis OZ;
- the flow of energy (15), passing through the cross-sectional area, not depend on $t,\ z$; the main thing is that the value does not change with time, and this complies with the Law of energy conservation.
- the energy flow has two opposite directed components, which transfer the active and reactive energies; thus, there is no need in the presentation of an imaginary Pointing vector.

5. Current and energy flow in the wire

One can say that the flow of mass particles (mass current) *"bears"* a flow of kinetic energy that is released in a collision with an obstacle. Just so the electric current *"bears"* a flow of electromagnetic energy released in the load. This assertion is discussed and substantiated in [4-9]. The difference between these two cases is in the fact that value of mass current fully determines the value of kinetic energy. But in the second case value of electrical current DOES NOT determine the value of electromagnetic energy released in the load. Therefore the transferred quantity of electromagnetic energy – the energy flow, - is being determined by the current structure. Let us show this fact.

As follows from (3.10), the average value of amplitude density of current $\overline{J_z}$ in a wire of radius R depends on two parameters: α and A. For a given density one can find the dependence between these parameters, as it follows from (3.10):

$$A = \frac{2\pi\alpha(\alpha+1)}{\chi\varepsilon\omega} R^{-\alpha-1} J_{zr}\,. \tag{1}$$

As follow from (4.16), the energy flow density along the wire also depends on two parameters: α and A. Fig. 10 shows the dependencies (1) and (4.16) for given $\overline{J_z} = 2,\ R = 2$. Here the straight line depicts the constant current density (in scale 1000), solid line – the flow density, dotted line – parameter A in scale (in scale 1000). Here A calculated according to (1), the energy flux density - to (4.16) for a given A One can see that for the same current density the flow density can take absolutely different values.

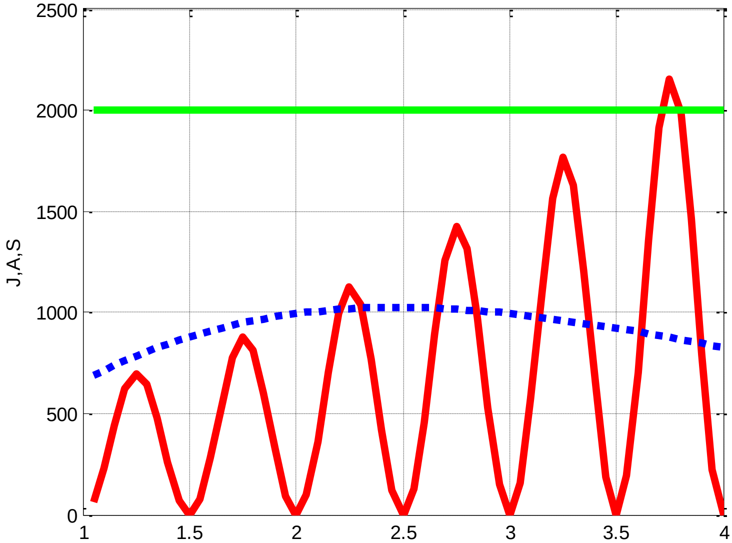

Fig.10. (SSMB)

6. Discussion

It was shown that an electromagnetic wave is propagating in an alternating current wire, and the mathematic description of this wave is given by the solution of Maxwell equations.

This solution largely coincides with the solution found before for an electromagnetic wave propagating in vacuum [1].

It appears that the current propagates in the wire along a spiral trajectory, and the density of the spiral depends on the flow density of electromagnetic energy transferred along the wire to the load, i.e. on the transferred power. And the main flow of energy is propagated along and inside the wire.

Appendix 1

Let us consider the solution of equations (2.1-2.10) in the form of (2.13-2.23). Further the derivatives of r will be designated by strokes. We write the equations (2.1-2.10) in view of (2.11, 2.12) in the form

$$\frac{e_r(r)}{r} + e_r'(r) - \frac{e_\varphi(r)}{r}\alpha - \chi \cdot e_z(r) = 0, \tag{1}$$

$$-\frac{1}{r} \cdot e_z(r)\alpha + e_\varphi(r)\chi - \frac{\mu\omega}{c} h_r = 0, \tag{2}$$

$$e_r(r)\chi - e_z'(r) + \frac{\mu\omega}{c} h_\varphi = 0, \tag{3}$$

$$\frac{e_\varphi(r)}{r}+e'_\varphi(r)-\frac{e_r(r)}{r}\cdot\alpha+\frac{\mu\omega}{c}h_z=0, \tag{4}$$

$$\frac{h_r(r)}{r}+h'_r(r)+\frac{h_\varphi(r)}{r}\alpha+\chi\cdot h_z(r)=0, \tag{5}$$

$$\frac{1}{r}\cdot h_z(r)\alpha-h_\varphi(r)\chi-\frac{\varepsilon\omega}{c}e_r=0, \tag{6}$$

$$-h_r(r)\chi-h'_z(r)+\frac{\varepsilon\omega}{c}e_\varphi=0, \tag{7}$$

$$\frac{h_\varphi(r)}{r}+h'_\varphi(r)+\frac{h_r(r)}{r}\cdot\alpha+\frac{\varepsilon\omega}{c}e_z(r)=\frac{4\pi}{c}j_z(r), \tag{8}$$

We multiply (8) on $(-\chi)$ and take into account (9). Then we get:

We multiply (5) on $\left(-\frac{\mu\omega}{c\chi}\right)$. Then we get:

$$-\frac{\mu\omega}{c\chi}\frac{h_r(r)}{r}-\frac{\mu\omega}{c\chi}h'_r(r)-\frac{\mu\omega}{c\chi}\frac{h_\varphi(r)}{r}\alpha-\frac{\mu\omega}{c}h_z(r)=0. \tag{9}$$

Comparing (4) and (9), we see that they are the same, if

$$\left\{\begin{array}{l} h_z\neq 0 \\ -\frac{\mu\cdot\omega}{c\chi}h_\varphi(r)=e_r(r), \\ \frac{\mu\cdot\omega}{c\chi}h_r(r)=e_\varphi(r), \end{array}\right\} \tag{9a}$$

or, if

$$\left\{\begin{array}{l} h_z=0, \\ -M\frac{\mu\cdot\omega}{c\chi}h_\varphi(r)=e_r(r), \\ M\frac{\mu\cdot\omega}{c\chi}h_r(r)=e_\varphi(r), \end{array}\right\} \tag{9в}$$

where M - constant. Next, we use formulas

$$-M\frac{\mu\cdot\omega}{c\chi}h_\varphi(r)=e_r(r), \tag{10}$$

$$M\frac{\mu\cdot\omega}{c\chi}h_r(r)=e_\varphi(r), \tag{11}$$

where $M=1$ in the case of (9a). Rewrite (2, 3, 6, 7) in the form:

$$e_z(r)=\frac{\chi r}{\alpha}e_\varphi(r)-\frac{r}{\alpha}\frac{\mu\omega}{c}h_r(r), \tag{12}$$

$$e_z'(r)=e_r(r)\chi+\frac{\mu\omega}{c}h_\varphi(r), \tag{13}$$

$$h_z(r)=\frac{\chi r}{\alpha}h_\varphi(r)+\frac{r}{\alpha}\frac{\varepsilon\cdot\omega}{c}e_r(r), \tag{14}$$

$$h_z'(r)=-h_r(r)\chi+\frac{\varepsilon\cdot\omega}{c}e_\varphi(r), \tag{15}$$

Substituting (10, 11) in these equations (12, 13), we get:

$$e_z(r)=\left(\chi-\frac{\chi}{M}\right)\frac{r}{\alpha}e_\varphi(r)=\frac{(M-1)}{M}\frac{\chi r}{\alpha}e_\varphi(r), \tag{16}$$

$$e_z'(r)=\left(\chi-\frac{\chi}{M}\right)e_r(r)\chi=\frac{(M-1)}{M}\chi e_r(r). \tag{17}$$

Substituting (10, 11) in these equations (14, 15), we get:

$$h_z(r)=\left(\chi-M\frac{\varepsilon\cdot\omega}{c}\frac{\mu\cdot\omega}{c\chi}\right)\frac{r}{\alpha}h_\varphi(r)=\frac{r}{\alpha c^2\chi}\left(c^2\chi^2-M\varepsilon\mu\omega^2\right)h_\varphi(r), \tag{18}$$

$$h_z'(r)=\left(-\chi+M\frac{\varepsilon\cdot\omega}{c}\frac{\mu\cdot\omega}{c\chi}\right)h_r(r)=\frac{-1}{c^2\chi}\left(c^2\chi^2-M\varepsilon\mu\omega^2\right)h_r(r). \tag{19}$$

Differentiating (16) and comparing with (17), we find:

$$\frac{(M-1)}{M}\frac{\chi}{\alpha}\left(re_\varphi(r)\right)'=\frac{(M-1)}{M}\chi e_r(r)$$

or

$$\left(re_\varphi(r)\right)'=\alpha e_r(r)$$

or

$$\left(e_\varphi(r)+r\cdot e_\varphi'(r)\right)=\alpha e_r(r). \tag{20}$$

From (1, 16), we find:

$$\frac{e_r(r)}{r}+e_r'(r)-\frac{e_\varphi(r)}{r}\alpha-\frac{(M-1)}{M}\chi^2\frac{r}{\alpha}e_\varphi(r)=0 \tag{23}$$

From physical considerations we must assume that

$$h_z(r)=0. \tag{24}$$

Then from (18) we find

$$\left(c^2\chi^2-\varepsilon\mu\omega^2\right)=0$$

or

$$\chi=\hat{\chi}\frac{\omega}{c}\sqrt{M\varepsilon\mu},\quad \hat{\chi}=\pm1. \tag{25}$$

From (16, 25), we find:

$$e_z(r) = (M-1)\frac{\chi r}{\alpha} e_\varphi(r) = \frac{(M-1)}{M}\hat{\chi}\frac{\omega}{c}\sqrt{M\varepsilon\mu}\,\frac{r}{\alpha} e_\varphi(r)$$

or

$$e_z(r) = \hat{\chi}\frac{(M-1)}{\sqrt{M}}\frac{\omega\sqrt{\varepsilon\mu}}{\alpha c} r e_\varphi(r) \tag{25a}$$

For $\omega << c$ from (25) we find that

$$|\chi| << 1. \tag{26}$$

Then in the equation (23) we can neglect the value χ^2 and obtain an equation of the form

$$\alpha \cdot e_\varphi(r) = e_r(r) + r \cdot e_r'(r). \tag{27}$$

From (27, 20) due to the symmetry we find:

$$e_r(r) = e_\varphi(r), \tag{28}$$

$$\alpha \cdot e_\varphi(r) = e_\varphi(r) + r \cdot e_\varphi'(r). \tag{29}$$

The solution of this equation is as follows:

$$e_\varphi(r) = Ar^{\alpha-1}, \tag{30}$$

which can be checked by substitution of (30) into (29). From (11, 25), we find

$$h_r(r) = \hat{\chi}\sqrt{\frac{\varepsilon}{M\mu}} e_\varphi(r), \tag{31}$$

and from (10, 28), we find

$$h_\varphi(r) = -h_r(r). \tag{32}$$

Finally, from (8, 32), we find

$$j_z(r) = \frac{c}{4\pi}\left(-\frac{h_r(r)}{r} - h_r'(r) + \frac{h_r(r)}{r}\cdot\alpha + \frac{\varepsilon\omega}{c} e_z(r)\right) \tag{33}$$

Taking into account (30.31), we note that the sum of the first three terms is equal to zero, and then

$$j_z(r) = \frac{\varepsilon\omega}{4\pi} e_z(r). \tag{34}$$

So, we finally obtain:

$$e_\varphi(r) = Ar^{\alpha-1}, \tag{30}$$

$$e_r(r) = e_\varphi(r), \tag{28}$$

$$e_z(r) = \hat{\chi}\frac{(M-1)}{\sqrt{M}}\frac{\omega\sqrt{\varepsilon\mu}}{\alpha c} r e_\varphi(r) \tag{25a}$$

$$h_r(r) = \hat{\chi}\sqrt{\frac{\varepsilon}{M\mu}} e_\varphi(r), \tag{31}$$

$$h_\varphi(r) = -h_r(r). \tag{32}$$

$$h_z(r) = 0. \tag{24}$$

$$j_z(r) = \frac{\varepsilon\omega}{4\pi} e_z(r). \tag{34}$$

The accuracy of the solution

To analyze the accuracy of the solution may be for given values of all constants to find the residual equation (1-7). Fig. 0 shows the logarithm of the mean square residual of the parameter α - $\ln N = f(\alpha)$, when $A=1,\ \omega=300,\ \mu=1,\ \varepsilon=1$.

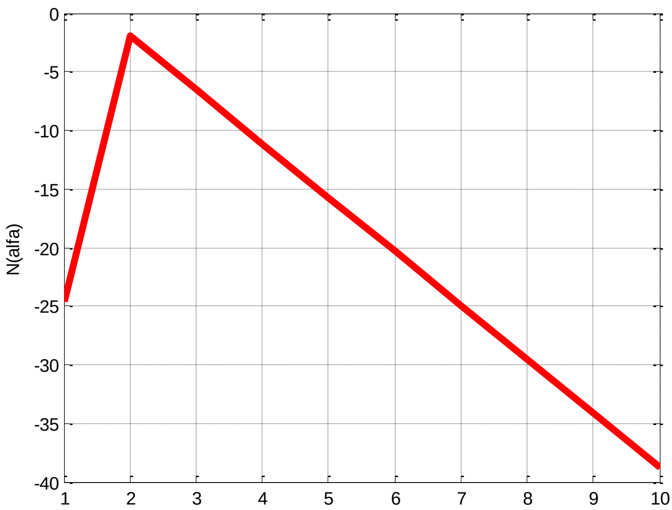

Fig.0. (SSMB)

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