

## Four conjectures on the numbers $2(p \cdot q \cdot r) \pm 1$ where $p$ and $q=p+6$ and $r=q+6$ are odd numbers

**Abstract.** In this paper I make the following four conjectures: (I) there exist an infinity of primes of the form  $2 \cdot (p \cdot q \cdot r) + 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6 \cdot k - 1$ ; (II) there exist an infinity of semiprimes  $m \cdot n$  of the form  $2 \cdot (p \cdot q \cdot r) + 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6 \cdot k - 1$ , semiprimes having the property that  $n - m + 1$  is prime; (III) there exist an infinity of primes of the form  $2 \cdot (p \cdot q \cdot r) - 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6 \cdot k + 1$ ; (IV) there exist an infinity of semiprimes  $m \cdot n$  of the form  $2 \cdot (p \cdot q \cdot r) + 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6 \cdot k + 1$ , semiprimes having the property that  $n - m + 1$  is prime;

### Conjecture 1:

There exist an infinity of primes  $m$  of the form  $2 \cdot (p \cdot q \cdot r) + 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6 \cdot k - 1$ .

The sequence of primes  $m$ :

:  $m = 1871$ , for  $(p, q, r) = (5, 11, 17)$ ;  
:  $m = 22679$ , for  $(p, q, r) = (17, 23, 29)$ ;  
:  $m = 46691$ , for  $(p, q, r) = (23, 29, 35)$ ;  
:  $m = 83231$ , for  $(p, q, r) = (29, 35, 41)$ ;  
:  $m = 1403531$ , for  $(p, q, r) = (83, 89, 95)$ ;  
:  $m = 2442383$ , for  $(p, q, r) = (101, 107, 113)$ ;  
:  $m = 2877659$ , for  $(p, q, r) = (107, 113, 119)$ ;  
:  $m = 3361751$ , for  $(p, q, r) = (113, 119, 125)$ ;  
:  $m = 4486751$ , for  $(p, q, r) = (125, 131, 137)$ ;  
:  $m = 5132843$ , for  $(p, q, r) = (131, 137, 143)$ ;  
:  $m = 6605171$ , for  $(p, q, r) = (143, 149, 155)$ ;  
:  $m = 7436591$ , for  $(p, q, r) = (149, 155, 161)$ ;  
:  $m = 10342979$ , for  $(p, q, r) = (167, 173, 179)$ ;  
:  $m = 11457791$ , for  $(p, q, r) = (173, 179, 185)$ ;  
:  $m = 15276563$ , for  $(p, q, r) = (191, 197, 203)$ ;  
(...)

### Conjecture 2:

There exist an infinity of semiprimes  $m \cdot n$  of the form  $2 \cdot (p \cdot q \cdot r) + 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6 \cdot k - 1$ , semiprimes having the property that  $n - m + 1$  is prime.

The sequence of semiprimes  $m \cdot n$ :

:  $m \cdot n = 8603 = 7 \cdot 1229$  where  $1229 - 7 + 1 = 1223$ ,  
 prime, for  $(p, q, r) = (11, 17, 23)$ ;  
 :  $m \cdot n = 406511 = 7 \cdot 58073$  where  $58073 - 7 + 1 = 58067$ ,  
 prime, for  $(p, q, r) = (53, 59, 65)$ ;  
 :  $m \cdot n = 18243611 = 139 \cdot 131249$  where  $131249 - 139 + 1 =$   
 $131111$ , prime, for  $(p, q, r) = (203, 209, 215)$ ;  
 (...)

### Conjecture 3:

There exist an infinity of primes  $m$  of the form  $2 \cdot (p \cdot q \cdot r) - 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6 \cdot k + 1$ .

The sequence of primes  $m$ :

:  $m = 3457$ , for  $(p, q, r) = (7, 13, 19)$ ;  
 :  $m = 57349$ , for  $(p, q, r) = (25, 31, 37)$ ;  
 :  $m = 98641$ , for  $(p, q, r) = (31, 37, 43)$ ;  
 :  $m = 449569$ , for  $(p, q, r) = (65, 71, 73)$ ;  
 :  $m = 1222129$ , for  $(p, q, r) = (79, 85, 91)$ ;  
 :  $m = 2178037$ , for  $(p, q, r) = (97, 103, 109)$ ;  
 :  $m = 4087621$ , for  $(p, q, r) = (121, 127, 133)$ ;  
 :  $m = 2178037$ , for  $(p, q, r) = (139, 145, 151)$ ;  
 :  $m = 8649757$ , for  $(p, q, r) = (157, 163, 169)$ ;  
 (...)

### Conjecture 4:

There exist an infinity of semiprimes  $m \cdot n$  of the form  $2 \cdot (p \cdot q \cdot r) - 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6 \cdot k + 1$ , semiprimes having the property that  $n - m + 1$  is prime.

The sequence of semiprimes  $m \cdot n$ :

:  $m \cdot n = 12349 = 53 \cdot 233$  where  $233 - 53 + 1 = 181$ ,  
 prime, for  $(p, q, r) = (13, 19, 25)$ ;  
 :  $m \cdot n = 596701 = 7 \cdot 85243$  where  $85243 - 7 + 1 = 85237$ ,  
 prime, for  $(p, q, r) = (61, 67, 73)$ ;  
 :  $m \cdot n = 772777 = 23 \cdot 33599$  where  $33599 - 23 + 1 =$   
 $33577$ , prime, for  $(p, q, r) = (67, 73, 79)$ ;  
 :  $m \cdot n = 18121 = 193 \cdot 18313$  where  $18313 - 193 + 1 =$   
 $33577$ , prime, for  $(p, q, r) = (115, 121, 127)$ ;  
 :  $m \cdot n = 7728481 = 149 \cdot 51869$  where  $51869 - 149 + 1 =$   
 $51721$ , prime, for  $(p, q, r) = (151, 157, 163)$ ;  
 :  $m \cdot n = 9641449 = 107 \cdot 90107$  where  $90107 - 107 + 1 =$   
 $90001$ , prime, for  $(p, q, r) = (163, 169, 175)$ ;  
 :  $m \cdot n = 10706149 = 1481 \cdot 7229$  where  $7229 - 1481 + 1 =$   
 $5749$ , prime, for  $(p, q, r) = (169, 175, 181)$ ; (...)