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Causality of the Coulomb field of relativistic electron bunches

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Abstract Recent experiments, performed by Prof. Pizzella's team with relativistic electron bunches, indicate that Coulomb field is rigidly attached to the charge's instantaneous position. Despite a widespread opinion, this fact does not violate causality in moving reference frames. To see that, one should apply the Wigner – Dirac theory of relativistic dynamics and take into account that the Lorentz boost generator depends on interaction.

1 Experiment at Frascati

In 2012 the team of Prof. Pizzella at the Frascati National Laboratory performed a remarkable experiment [1] that observed rigid connection between electric charge and its Coulomb field. In order to see how surprising this result is from the point of view of the traditional Maxwell–Liénard–Wiechert electrodynamics, let us turn to Fig. 1.

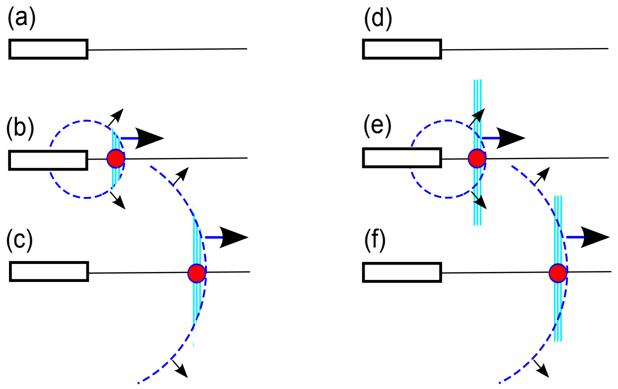

Fig. 1 Time evolution of the electric field of an electron bunch (small circle) leaving the accelerator pipe (rectangle): (a)→(b)→(c) Maxwell–Liénard–Wiechert theory; (d)→(e)→(f) Frascati experiment. Broken line circle fragments show the expanding spherical electromagnetic wave. Parallel lines perpendicular to the beam's axis represent the Coulomb field disk viewed from its side

In three panels 1 (a)→(b)→(c) we show the sequence of events expected in the traditional theory. The snapshot 1 (a) is taken just before the electron bunch left the accelerator's pipe. In this case the bunch's field is shielded by the pipe's metal, so there is no electric field in the surrounding space.

In two frames 1 (b) - (c) the bunch has left the pipe. The emerged force field has two components. First, there is an expanding spherical electromagnetic wave (radiation field) centered at the pipe's exit. Second, there is a non-radiating bound Coulomb field, which is squeezed (or Lorentz-contracted [2]) in the direction of the bunch's motion, thus forming a narrow disk. There is also an associated disk of a bound magnetic field, but we will not discuss it in this Letter.

According to special relativity, electric field lines of the Coulomb disk cannot "leak" outside the "light cone" bounded by the expanding sphere of the electromagnetic wave. Therefore, the transverse dimension d of the disk grows with time according to formula

$$d \approx 2ct/\gamma \approx 2L/\gamma \qquad (1)$$

where L is the distance traveled by the bunch from the pipe's exit and $\gamma \equiv (1-v^2/c^2)^{-1/2}$ is the usual relativistic factor. In the Frascati experiment, the energies of electrons in the beam were 500 MeV, which corresponded to the Lorentz contraction factor of $\gamma \approx 1000$. At the longest experimental travel distance of 5.5 m the expected size of the Coulomb disk was not greater than 11 mm — too small to have any effect on the field sensors placed around the beam.

In the parallel sequence of frames 1 (d)→(e)→(f) we present true results of the Frascati experiment. In contrast to theoretical expectations, the size of the Coulomb field's disk did not grow in a linear fashion (1). Instead, measurements

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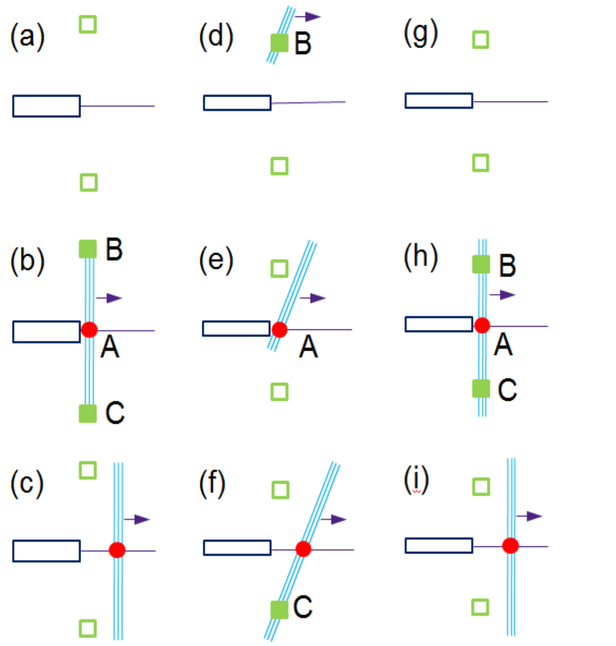

Fig. 2 Time evolution of the bunch's instantaneous Coulomb field: (a)→(b)→(c) in the rest frame; (d)→(e)→(f) in the moving frame, as predicted by Lorentz transformations (2) – (5); (g)→(h)→(i) in the moving frame, as predicted by the Wigner–Dirac theory. Squares mark positions of electric field sensors. Filled squares indicate "clicking" sensors

suggested that the field's disk emerged from the accelerator fully formed in the entire space (even beyond the light cone) and did not change with time, apart from the uniform movement together with the bunch.

This fact can be interpreted as an indication of an instantaneous force between charges in the bunch and in the field sensors. The ideas about electromagnetic interactions being composed of both instantaneous (bound) and retarded (radiation) parts are not new. They were repeatedly expressed theoretically [3–5], and electromagnetic superluminal effects were seen in experiments as well [6–8]. However, these ideas and experiments are usually met with scepticism, because they violate the special relativistic ban on faster-than-light propagation of signals.

2 Ban of superluminal signals

To understand why special relativity does not tolerate such superluminal effects, consider the time evolution of the instantaneous Coulomb field[1] shown in three consecutive snapshots 2 (a)→(b)→(c). Frame 2 (a) is taken just before the bunch has left the accelerator's pipe, frame 2 (b) shows the exact instant of the exit, and 2 (c) is taken a moment after this event. This time, for clarity, we chose not to show the spherical electromagnetic waves, because according to the experiment [1], their signals are weaker by an order of magnitude. We also placed two electric field sensors next to the pipe's exit.

From the point of view of the observer at rest O, the field's disk forms instantaneously (panel 2 (b)), so the three events A (the bunch's exit from the pipe), B and C (clicks of the sensors) occur simultaneously, despite the fact that B and C are caused by the event A and separated from it by a considerable distance.

In three frames 2 (d)→(e)→(f) we show how this situation looks from the point of view of an inertial observer O' moving with high velocity in the vertical direction from the bottom of the page to its top.[2] To draw these panels we employed Lorentz transformations of special relativity

$$t' = t\cosh\theta - (y/c)\sinh\theta \qquad (2)$$

$$x' = x \qquad (3)$$

$$y' = y\cosh\theta - ct\sinh\theta \qquad (4)$$

$$z' = z \qquad (5)$$

where parameter θ was related to the observer's velocity by formula $v = c\tanh\theta$.

The absurdity of these drawings is clear already from the panel 2 (d), where a portion of the Coulomb field's disk emerged even before the bunch has left the accelerator's pipe. This means that the "effect" B occurred earlier than the "cause" A, in a clear violation of the principle of causality. Two other panels show the further sequence of events: the bunch leaves the accelerator 2 (e), the lower sensor clicks 2 (f).

To avoid such blatant violations of causality, special relativity forbids faster-than-light formation of the field's disk. However, in a clear disrespect of this ban, the Frascati experiment did show exactly such a behavior in the laboratory. This extraordinary observation demands an explanation.

3 Relativistic interactions between particles

Our claim is that charges in the bunch and in field sensors may, indeed, interact via an instantaneous action-at-a-distance potential. The usual attitude is that such potentials are impossible because (i) they cannot be relativistically invariant and (ii) they violate causality. We will disprove the latter statement in section 4. Here we would like to mention that the former statement is not correct as well.

First explicit construction of a relativistically invariant multiparticle model with instantaneous forces was undertaken by Bakamjian and Thomas [9]. They used the the-

[1] same as in Fig. 1 (d)→(e)→(f)

[2] To be consistent with [1], we call it y-direction. For added realism, the whole picture has been Lorentz-contracted in this direction.

ory of unitary representations of the Poincaré group developed earlier by Wigner [10] and Dirac [11]. According to this theory [12], interactions must modify generators of the Poincaré group representation, so that the commutators of generators remain intact. The full Hamiltonian (= the generator of time translations) is obtained by adding the "potential energy" operator V to the free Hamiltonian: $\mathrm{H} = \mathrm{H}_0 + \mathrm{V}$. In the Dirac's instant form of relativistic dynamics, the relativistic invariance of the theory is achieved by also adding an interacting "potential boost" $\mathbf{Z}$ to the noninteracting boost generator: $\mathbf{K} = \mathbf{K}_0 + \mathbf{Z}$. Generators of space translations and rotations remain interaction-free: $\mathbf{P} = \mathbf{P}_0$, $\mathbf{J} = \mathbf{J}_0$.

If this construction is performed in a two-particle system,[3] then the Hamiltonian H is a function of positions and momenta of both particles. Time dependencies of these observables are obtained by standard quantum mechanical formulas

$$\begin{aligned} \mathbf{r}_j(t) &= e^{\frac{i}{\hbar}\mathrm{H}t}\mathbf{r}_j e^{-\frac{i}{\hbar}\mathrm{H}t} \\ \mathbf{p}_j(t) &= e^{\frac{i}{\hbar}\mathrm{H}t}\mathbf{p}_j e^{-\frac{i}{\hbar}\mathrm{H}t} \end{aligned} \tag{6}$$

where $j = 1,2$ is particle label. These expressions can be viewed either as quantum mechanical time-dependent operators in the Heisenberg picture or as particle trajectories in the $\hbar \to 0$ classical limit. In the latter case one should replace quantum commutators with classical Poisson brackets.

The force acting on the particle 2

$$\begin{aligned} \mathbf{f}_2(t) &= \frac{d}{dt}\mathbf{p}_2(t) = -\frac{i}{\hbar}\left[\mathbf{p}_2(t), \mathrm{H}\right] \\ &\equiv \mathbf{f}_2(\mathbf{r}_1(t), \mathbf{p}_1(t); \mathbf{r}_2(t), \mathbf{p}_2(t)) \end{aligned} \tag{7}$$

depends on positions and momenta of both particles at the same time instant t. This demonstrates that interaction propagates instantaneously in the reference frame O at rest.

4 Causality

Now consider the above two-particle system from the point of view of the moving reference frame O'. Particle trajectories in this frame are[4]

$$\begin{aligned} \mathbf{r}_j(\theta, t') &= e^{-\frac{ic}{\hbar}\mathrm{K}_y\theta} e^{\frac{i}{\hbar}\mathrm{H}t'}\mathbf{r}_j e^{-\frac{i}{\hbar}\mathrm{H}t'} e^{\frac{ic}{\hbar}\mathrm{K}_y\theta} \\ \mathbf{p}_j(\theta, t') &= e^{-\frac{ic}{\hbar}\mathrm{K}_y\theta} e^{\frac{i}{\hbar}\mathrm{H}t'}\mathbf{p}_j e^{-\frac{i}{\hbar}\mathrm{H}t'} e^{\frac{ic}{\hbar}\mathrm{K}_y\theta} \end{aligned} \tag{8}$$

The Hamiltonian in the reference frame O' is $\mathrm{H}(\theta) = \exp\left(-\frac{ic}{\hbar}\mathrm{K}_y\theta\right)\mathrm{H}\exp\left(\frac{ic}{\hbar}\mathrm{K}_y\theta\right)$. Therefore, the force acting on the particle 2

$$\begin{aligned} &\mathbf{f}_2(\theta, t') \\ &= \frac{d}{dt'}\mathbf{p}_2(\theta, t') = -\frac{i}{\hbar}\left[\mathbf{p}_2(\theta, t'), \mathrm{H}(\theta)\right] \\ &= -\frac{i}{\hbar}\left[e^{-\frac{ic}{\hbar}\mathrm{K}_y\theta} e^{\frac{i}{\hbar}\mathrm{H}t'}\mathbf{p}_2 e^{-\frac{i}{\hbar}\mathrm{H}t'} e^{\frac{ic}{\hbar}\mathrm{K}_y\theta}, e^{-\frac{ic}{\hbar}\mathrm{K}_y c\theta}\mathrm{H}e^{\frac{ic}{\hbar}\mathrm{K}_y\theta}\right] \\ &= -\frac{i}{\hbar}e^{-\frac{ic}{\hbar}\mathrm{K}_y\theta}\left[e^{\frac{i}{\hbar}\mathrm{H}t'}\mathbf{p}_2 e^{-\frac{i}{\hbar}\mathrm{H}t'}, \mathrm{H}\right]e^{\frac{ic}{\hbar}\mathrm{K}_y\theta} \\ &= -\frac{i}{\hbar}e^{-\frac{ic}{\hbar}\mathrm{K}_y\theta}\left[\mathbf{p}_2(0, t'), \mathrm{H}\right]e^{\frac{ic}{\hbar}\mathrm{K}_y\theta} \\ &= e^{-\frac{ic}{\hbar}\mathrm{K}_y\theta}\mathbf{f}_2(0, t')e^{\frac{ic}{\hbar}\mathrm{K}_y\theta} \\ &= e^{-\frac{ic}{\hbar}\mathrm{K}_y\theta}\mathbf{f}_2\left(\mathbf{r}_1(0, t'), \mathbf{p}_1(0, t'); \mathbf{r}_2(0, t'), \mathbf{p}_2(0, t')\right)e^{\frac{ic}{\hbar}\mathrm{K}_y\theta} \\ &= \mathbf{f}_2(\mathbf{r}_1(\theta, t'), \mathbf{p}_1(\theta, t'); \mathbf{r}_2(\theta, t'), \mathbf{p}_2(\theta, t')) \end{aligned} \tag{9}$$

is a function of positions and momenta of both particles at the same time instant t'. Thus, from the point of view of the moving observer O', the interaction propagates instantaneously, exactly as for the observer at rest O. Moreover, in agreement with the principle of relativity, the force function $\mathbf{f}_2$ in (9) has the same form as in the rest frame (7).

In regard to the Frascati experiment, this means that clicks of the sensors (events B and C) in the frame O' must occur exactly at the time of the bunch's exit from the pipe (event A). This prediction of the Wigner – Dirac theory is shown in the sequence of snapshots 2 (g)→(h)→(i). In other words, in this theory the three events A, B, C remain simultaneous in all frames,[5] the "effects" B and C never occur before the "cause" A, and instantaneous potentials do not violate the principle of causality.

5 Discussion

The above discussion implies that in the presence of interaction ($V \neq 0, \mathbf{Z} \neq 0$), particle trajectories in the moving frame (8) cannot be obtained from rest-frame trajectories (6) by applying Lorentz formulas (2) - (5). One can say that interacting Wigner – Dirac theory does not have "invariant worldlines" [13, 14]. This idea is highly controversial, and the majority of theoreticians believe that the "invariant worldline" condition must be respected in all dynamical theories. Based on this belief, a multitude of approaches were formulated [15–22], which deviated from the clear path of Hamilton, Poincaré, Wigner, and Dirac. So far, the predictive power of these approaches remains rather limited.

Another idea is that particle equations of motion (including boost transformations) should be extracted from our most successful theory – quantum electrodynamics (QED). At the first sight it seems that Maxwell's equations, as well

[3] For example, we can assume that charge 1 represents an electron in the bunch and charge 2 is a part of the electric field sensor.

[4] Here t' is time measured by the clock of the observer O'.

[5] Note that this universal simultaneity is valid only for events linked by interaction. For unrelated pairs of events the usual "relativity of simultaneity" still applies.

as Liénard–Wiechert retarded potentials, must follow from the QED Lagrangian in the classical limit. However, this conclusion is not so obvious, because time evolution in QED is an ill-defined concept as this theory is formulated in terms of fictitious bare particles, and the Hamiltonian contains divergent renormalization counterterms. These interpretational difficulties are conducive to opinions that observables (positions and momenta) of interacting particles have no meaning in QED [23] or even that *"there are no particles, there are only fields"* [24]. For more works on the same theme see [25–28].

A somewhat different and more pragmatic attitude is taken in research programs that try to replace the field-based language of quantum field theory with ideas of physical particles interacting through effective relativistic potentials. One way to obtain such potentials is to fit them to the known renormalized QED S-matrix. This can be done not only in the lowest second perturbation order, resulting in the classic Coulomb–Darwin–Breit potential [23, 29], but also taking into account radiative corrections [30–33]. Another way forward is to apply a unitary dressing transformation [34–36] to the QED Hamiltonian. This method has the added advantage that it explicitly preserves the relativistic invariance of the theory [37].

It appears that the effective electromagnetic interactions between physical charges separate into radiation and bound types. As expected, the radiation force field is transmitted by real photons and propagates with the speed of light [38, 39]. However, this is not true for the bound (Coulomb and magnetic) potentials. Even in high perturbation orders they are expressed by functions that depend on instantaneous positions and momenta of the charges. These potentials are well suited to describe the field dynamics of the relativistic electron beam observed in Frascati and they are in full agreement with causality.

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