An algorithm for general solution of Monkey and Coconut problem

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Abstract: In this paper I discuss an algorithm which will solve a very famous puzzle involving a monkey, few men and some coconuts. The puzzle involves a group of n men who have an unknown number of coconuts among them. At night, while the others are asleep, one of the men divides the coconuts in n parts and hides his share. While dividing, he discovers that there is one extra coconut, which he gives away to a monkey. Exactly the same thing happens with the rest of the men, one by one. They all hide their share, are left with one extra coconut that cannot be divided, which they give to the monkey. The next morning they again divide the coconuts together equally among themselves, with no extra coconut remaining this time. The puzzle is to find out the initial number of coconuts.

Index Terms— Algorithm, Puzzle, Number Theory

I. Introduction

THE difficulty of this puzzle is computational in nature. This puzzle can be reduced to finding the solution of a simple equation (in the manner I will presently discuss in VERIFICATION section), which is,

$$\frac{am+b}{c} = x \qquad \ldots(1)$$

Here $a, b, c, m \text{ and } x$ are integers. Also, the values of $a, b \text{ and } c$ are known, and depend upon the number of men in our puzzle. Here x denotes the initial number of coconuts in the puzzle and m denotes the final number of coconuts that each of the men *receive* as equal share in the following morning. The problem is to find an integral value of m for which x is also an integer for the given values of $a, b \text{ and } c$. The usual method of finding an integral x is repeatedly plugging in increasing integral values for m and checking if x is an integer for that value or not. This method is called heat and trial method and large calculative difficulties arise for large values of $a, b \text{ and } c$ if we use this method.

In this paper, I will discuss a simpler method for solving this puzzle.

Please note that infinitely many solutions of the equation (1) are possible. Our task is to find the simplest possible solution.

II. THE ALGORITHM

Here is the algorithm:

Let $a = (g_1.c) - h_1$, $\qquad \ldots(2)$

Here $g_1 \in N$ and $0 < h_1 < c$

Similarly, let, $a = (g_2.h_1) - h_2$

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Here $g_2 \in N \text{ and } 0 < h_2 \leq h_1$

…
…
…

Let, $a = (g_i . h_{i-1}) - h_i$

Here $g_i \in N \text{ and } h_i = 0$

Now, let,

$$b = p_0 . c + p_1 . h_1 + p_2 . h_2 + \cdots + p_{i-1} . h_{i-1} \quad \text{... (3)}$$

Here $A = \{p_0, p_{1,} \dots p_{i-1}\} \subseteq Z$. Note that there exist infinitely many sets A satisfying the above equation since there exist infinitely many solution to our question, but it is a simple procedure to find the simpler ones.

Now,

$$x = p_0 + p_1 . g_1 + p_1 . g_1 . g_2 + \cdots + p_{i-1} . g_1 . g_2 \dots g_{i-1} \quad \dots (4)$$

Hence, we have arrived at an expression for the number of coconuts in our puzzle.

Now, I will like to prove that there exists at least one set A satisfying equation (3). We can do this in following manner:

Theorem 1: Given two distinct integers $x \text{ and } y$, where both $x \text{ and } y$ are not simultaneously even, then all other integers $n \in Z$ can be expressed as $n = rx + sy$, where $r, s \in Z$.

Theorem 2: Given two distinct *even* integers $x \text{ and } y$; $n = rx + sy$ for some $r, s \in Z$ iff n is an even integer.

(The proof of these theorems is left for the readers.)

Now, b from equation (3) can be expressed as,

$$b = p_0 . c + p_1 . h_1 \quad \text{... (6)}$$

Referring to equation (1) and equation (2), we notice that if $a \text{ and } c$ are both even simultaneously then b is also even, implying that h_1 is also even. Hence, we can say that

1. If $c \text{ and } h_1$ in $equation\,(6)$ are *not* simultaneously even, then according to $theorem\,(1)$, all other integers $b \in Z$ can be expressed as $b = p_0 . c + p_1 . h_1$, where $p_0, p_1 \in Z$.
2. If $c \text{ and } h_1$ are both simultaneously even, then according to $theorem\,(2)$, all even integers b can be expressed as $b = p_0 . c + p_1 . h_1$, where $p_0, p_1 \in Z$. We also note that if $c \text{ and } h_1$ are both simultaneously even, then this implies that b is also even.

This suggests that we can always find a set $A = \{p_0, p_{1,} \dots p_{i-1}\}$ such that A satisfies equation (3). Hence, it is proven.

III. VERIFICATION

Let us assume, for simplicity, a situation in which there are just three men in our original question. Let us assume that initially there exist x coconuts. Also, let the number of coconuts hidden by the first, the second and the third man be $j, k \text{ and } l$ respectively. Let the number of coconuts they finally receive after sharing it equally the next morning is m. Then:

$$x = 3j + 1$$

$$2j = 3k + 1$$

$$2k = 3l + 1$$

$$2l = 3m$$

After expressing x in terms of m, we get:

$$x = \frac{81m + 38}{8} \quad \text{... (7)}$$

Equation (7) is of the same form as equation (1), as I have mentioned earlier.

Now, applying the algorithm to this situation:

Here, $a = 81, b = 38 \text{ and } c = 8$.

Now,

1. $81 = 8 \times 11 - 7$
2. $81 = 7 \times 12 - 3$
3. $81 = 3 \times 27 - 0$

Hence, $g_1 = 11, g_2 = 12, g_3 = 27, h_1 = 7, h_2 = 3$ *and h3=0*

Now, b can be expressed as:

$$b = 38 = 5 \times h_1 + 1 \times h_2$$

Hence, $p_1 = 5 \text{ and } p_2 = 1$.

Now, solving for x using equation (4),

$$x = p_0 + p_1 . g_1 + p_1 . g_1 . g_2 + \cdots p_{i-1} . g_1 . g_2 \dots g_{i-1}$$
$$= 5 \times 11 + 1 \times 11 \times 12$$
$$= 187$$

Putting the value of $x = 187$ in equation…(7), we find that it indeed gives an integral value for $m = 18$. Hence, our algorithm is verified!

Note that in this situation b can also be expressed as:

$$b = 4 \times c + 2 \times h_2$$

Which will give $x = 268$. This value is slightly larger than the previous one, yet correct. In fact, one has to creatively think of ways to express b in different manners and one will get infinitely many values of x.

One can notice that the method I discuss to solve this puzzle has application beyond this puzzle.

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REFERENCES

[1] B.A Williams, "The Saturday evening post," October 9, 1926.