Comments on "Thermal Implications of the New Relaxed IEEE RF Safety Standard for Head Exposures to Cellular Telephones at 835 and 1900 MHz"

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WE read with interest the article [1] about determination of thermal response due to electromagnetic exposure based on numerical methods. The bio-heat transfer equation for homogeneous material model can be easily calculated by using second order finite difference approximation to discretize the spatial derivatives and explicit finite-difference time-domain (FDTD) scheme for time domain discretization. Mr. Gandhi and colleagues solved the bio-heat equation for inhomogeneous models utilizing implicit finite-difference method. Whereas we appreciate their research, we would like to address a few issues that may help further clarify or confirm the research.

First, in the ref. [1-3], instead of showing a discretization of the differential equation by their methods "an implicit finite-difference method that achieves a higher order accuracy of the Crank-Nicholson formulation", however, Mr. Gandhi and colleagues gave the following equation (1) by solely subscripting all variables of the partial differential equation with voxel indices i,j,k.

$$\begin{aligned} m_{i,j,k}C_{i,j,k}\frac{\partial T_{i,j,k}}{\partial t} = \big[\nabla(k_{i,j,k}\nabla T_{i,j,k}) + h_{m_{i,j,k}} + \\ h_{EM_{i,j,k}} + b_{f_{i,j,k}}C_b(T_b - T_{i,j,k})\big]\, V_{i,j,k} \\ - h_{RAD_{i,j,k}} - h_{CONV_{i,j,k}} - h_{E_{i,j,k}} \end{aligned} \tag{1}$$

The expression $\nabla T_{i,j,k}$ in (1) is incorrect since $T_{i,j,k}$ is not a function of space but a numerical approximation of the function "T" at the voxel (i, j, k). It will be elaborated below.

In the three-dimensional Cartesian coordinate system, the gradient of scalar temperature function T is given by $\nabla T = (\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z})$. For a voxel (i, j, k):

$$(\frac{\partial T}{\partial x})_{i,j,k} = \frac{T_{i+1,j,k} - T_{i,j,k}}{\Delta x} \tag{2}$$

$$(\frac{\partial T}{\partial y})_{i,j,k} = \frac{T_{i,j+1,k} - T_{i,j,k}}{\Delta y} \tag{3}$$

$$(\frac{\partial T}{\partial z})_{i,j,k} = \frac{T_{i,j,k+1} - T_{i,j,k}}{\Delta z} \tag{4}$$

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The approximation for partial derivatives of temperature function e.g. (2) is obtained from Taylor's theorem:

$$\begin{aligned} T_{i+1,j,k} = & T_{i,j,k} + \frac{\Delta x}{1!}(\frac{\partial T}{\partial x})_{i,j,k} + \frac{(\Delta x)^2}{2!}(\frac{\partial^2 T}{\partial x^2})_{i,j,k} \\ & + \frac{(\Delta x)^3}{3!}(\frac{\partial^3 T}{\partial x^3})_{i,j,k} + \cdots \end{aligned} \tag{5}$$

$T_{i,j,k}$ in (5) is a numerical approximation of temperature function T at the point $(i\Delta x, j\Delta y, k\Delta z)$. It is a fixed value.

The flawed equation (1) will result in a set of algebraic equations below which are all wrong, for $\nabla T_{1,1,1}$ in (6) equals zero (Gradient of a constant function would be a 0 vector), so do $\nabla T_{2,1,1}$ in (7), $\cdots$, $\nabla T_{i,j,k}$ in (8), $\cdots$

$$\begin{aligned} m_{1,1,1}C_{1,1,1}\frac{\partial T_{1,1,1}}{\partial t} = \big[\nabla(k_{1,1,1}\nabla T_{1,1,1}) + h_{m_{1,1,1}} + \\ h_{EM_{1,1,1}} + b_{f_{1,1,1}}C_b(T_b - T_{1,1,1})\big]\, V_{1,1,1} \\ - h_{RAD_{1,1,1}} - h_{CONV_{1,1,1}} - h_{E_{1,1,1}} \end{aligned} \tag{6}$$

$$\begin{aligned} m_{2,1,1}C_{2,1,1}\frac{\partial T_{2,1,1}}{\partial t} = \big[\nabla(k_{2,1,1}\nabla T_{2,1,1}) + h_{m_{2,1,1}} + \\ h_{EM_{2,1,1}} + b_{f_{2,1,1}}C_b(T_b - T_{2,1,1})\big]\, V_{2,1,1} \\ - h_{RAD_{2,1,1}} - h_{CONV_{2,1,1}} - h_{E_{2,1,1}} \end{aligned} \tag{7}$$

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$$\begin{aligned} m_{i,j,k}C_{i,j,k}\frac{\partial T_{i,j,k}}{\partial t} = \big[\nabla(k_{i,j,k}\nabla T_{i,j,k}) + h_{m_{i,j,k}} + \\ h_{EM_{i,j,k}} + b_{f_{i,j,k}}C_b(T_b - T_{i,j,k})\big]\, V_{i,j,k} \\ - h_{RAD_{i,j,k}} - h_{CONV_{i,j,k}} - h_{E_{i,j,k}} \end{aligned} \tag{8}$$

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The expression $\nabla T_{i,j,k}$ in (1) is incorrect. The gradient of the temperature function T of the voxel (i, j, k) can be expressed as $\nabla T|_{i,j,k}$ or $(\nabla T)_{i,j,k}$,

$$\nabla T|_{i,j,k} = (\frac{T_{i+1,j,k} - T_{i,j,k}}{\Delta x}, \frac{T_{i,j+1,k} - T_{i,j,k}}{\Delta y}, \frac{T_{i,j,k+1} - T_{i,j,k}}{\Delta z}) \quad (9)$$

Similarly, the correct gradient expression in (10) [4,5] should be $\nabla T|_i$ or $(\nabla T)_i$.

$$\rho_i c_i \frac{\partial T_i}{\partial t} = \nabla(k_i \nabla T_i) + h_{m_i} + h_{em_i} - h_{e_i} - h_{rad_i} - h_{conv_i} + Q_{b_i}(T_b - T_i) \quad (10)$$

Our second doubt is what is the purpose of the pointless subscripting when a fully discretized difference equation for thermal model is not even shown? If it were not necessary to show a discretization by their methods, it would be more unnecessary to solely subscript all variables of the original partial differential equation with the voxel indices i,j,k. Neither the spatial derivatives nor the time derivative was discretized, the equation (1) is still a partial differential equation. A partial differential equation with subscripted variables can never represent a finite difference numerical form. Furthermore, there were no spatial discretization process and no time domain discretization in their method, it is quite unclear how they achieved the simulation results presented in their papers [1-5]. By expanding it in its finite-difference approximation, the flawed equation (1) can be written as

$$T^{n+1}_{i,j,k} = T^n_{i,j,k} + \frac{\delta_t V_{i,j,k}}{m_{i,j,k} C_{i,j,k}} \left[h_{m_{i,j,k}} + h_{EM_{i,j,k}} + b_{f_{i,j,k}} C_b (T_b - T^n_{i,j,k}) \right] - \frac{\delta_t h_{RAD_{i,j,k}}}{m_{i,j,k} C_{i,j,k}} - \frac{\delta_t h_{CONV_{i,j,k}}}{m_{i,j,k} C_{i,j,k}} - \frac{\delta_t h_{E_{i,j,k}}}{m_{i,j,k} C_{i,j,k}} \quad (11)$$

where n is the time step index and δ_t is the time step size.

In (11), the new value of a temperature at any voxel depends only on its previous value. The bio-heat conduction equation has become a non-conductive heat equation.

In general, commenting on a wrong gradient expression in the technical literature is not a pleasant issue. However, Mr. Gandhi and colleagues wrote the false equation (1) or similar wrong equation (10) for no less than 5 times [1-5], with due respect, perhaps an improvement should be carried out.

Readers may expect to see how the gradient operators in (1) and (10) can be discretized, as that is the only difficulty to the solution of bio-heat equation for inhomogeneous model utilizing implicit FDTD scheme. Therefore, it would be interesting if the authors were to show discretizations by their methods — "a modification of Douglas and Rachford that achieves higher-order accuracy of the Grank-Nicholson formulation. The unconditionally stable system similar to that of Douglas and Rachford method had the advantage of using larger time steps, while the unknown temperatures are computed explicitly at the advanced time level, similar to that in the Grank-Nicholson formulation having higher-order accuracy" [4], "The heat conduction equation is solved in rectangular coordinates by the standard implicit finite difference technique which is stable for all size time steps" [5] and "an implicit finite-difference method that achieves a higher order accuracy of the Crank-Nicholson formulation" [1,2].

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