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Special Relativity solved examples using an Electrical Analog Circuit

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Mourici Shachter

mourici@gmail.com
mourici@walla.co.il
ISRAEL, HOLON
054-5480550

Introduction

In this paper, I develop a simple analog electrical circuit. And I use this circuit to solve some classical problems in special relativity.

	Definitions
m_0 , m_T	Rest mass and moving mass
c	speed of light
v	mass velocity relative to the lab
$\beta = \frac{v}{c}$	slip
L_X	Inductance
α	Inductor quality factor

Step 1

The electrical analog circuit for Relativistic Mass And hidden parameters in Special Relativity

According to Special Relativity, elementary particles have a rest mass m_0. When the particle moves with a "slip" β it's mass m_T is growing according to

1] $$m_T = \frac{m_0}{\sqrt{1-\beta^2}}$$

Because of the square root in the equation, the "slip" β is limited to $-1 < \beta < 1$ which means that the particle velocity is always less then the speed of light

Now I multiply both side of eq.1 by α and get rid of the square root

2] $$\alpha^2 m_T^2 = \frac{\alpha^2 m_0^2}{1-\beta^2} = \frac{\alpha^2 m_0^2}{(1-\beta)(1+\beta)} = \frac{\alpha^2 m_0^2}{2}\cdot\left[\frac{1}{1-\beta}+\frac{1}{1+\beta}\right]$$

Eq.2 in contrary to eq.1, is a "continuous function" with only two "bad" points $\beta \neq \pm 1$
According to eq. 2 we can pass the speed of light,

Step 2

Deriving an Electrical Equivalent Circuit that helps understand Special Relativity

Let go back to eq. 2 and write it as follows

3] $$\alpha^2 m_T^2 = \alpha^2 m_-^2 + \alpha^2 m_+^2 = \frac{\alpha^2 m_0^2}{2}\cdot\left[\frac{1}{1-\beta}+\frac{1}{1+\beta}\right]$$

Equation 3 is divided into two parts that now on **must be treated separately**

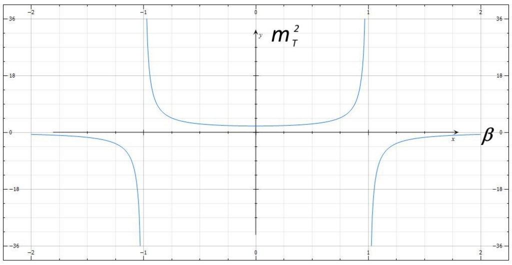
4] $$\alpha^2 m_-^2 = \frac{\alpha^2 m_0^2}{2}\cdot\left[\frac{1}{1-\beta}\right]$$
$$\alpha^2 m_+^2 = \frac{\alpha^2 m_0^2}{2}\cdot\left[\frac{1}{1+\beta}\right]$$

The total relativistic mass is computed using Pythagoras

5] $$\alpha^2 m_T^2 = \alpha^2 m_-^2 + \alpha^2 m_+^2$$

- mourici@gmail.com , mourici@walla.co.il Israel tel 972- 054-5480550

The following graph is for m_T^2 against β



Step 3

Making the equation a bit more physical

In eq.1 or eq. 2 we dont see any familiar physical phenomena. And the idea that speed of light is the same in all directions is not intuitive So, I am going to turn those equations into conventional not weird and intuitive physical equation that every student meets during his studies.

To do so, Let define

6]

$$j=\sqrt{-1}\ ,\quad f_0\quad \text{to be any frequency}\ ,\quad \omega_0=2\pi f_0\ ,\quad T_0=\frac{1}{f_0}\quad ,\quad k_0=\frac{2\pi}{\lambda_0}$$

$$\frac{1}{L_0}=\frac{\alpha^2 m_0^2}{2}\quad ,\quad \frac{1}{L_-}=\alpha^2 m_-^2\ ,\quad \frac{1}{L_+}=\alpha^2 m_+^2\quad ,\quad \frac{1}{L_T}=\alpha^2 m_T^2$$

let substitute those definitions in eq. 4, and divide each equation by non zero term $j\omega_0$

I define some new terms such as: Z_0 , Z_- , Z_+, Z_T and put them in eq. 3 and eq. 4

7]

$$\alpha^2 m_-^2=\frac{\alpha^2 m_0^2}{2}\cdot\left[\frac{1}{1-\beta}\right]$$

$$\frac{1}{Z_-}=\frac{\alpha^2 m_-^2}{j\omega_0}=\frac{1}{j\omega_0 L_-}=\frac{1}{j\omega_0\left(1-\beta\right)}\cdot\frac{\alpha^2 m_0^2}{2}=\frac{1}{j\omega_0\left(1-\beta\right)L_0}$$

$$\alpha^2 m_+^2=\frac{\alpha^2 m_0^2}{2}\cdot\left[\frac{1}{1+\beta}\right]$$

$$\frac{1}{Z_+}=\frac{\alpha^2 m_-^2}{j\omega_0}=\frac{1}{j\omega_0 L_+}=\frac{1}{j\omega_0\left(1+\beta\right)}\cdot\frac{\alpha^2 m_0^2}{2}=\frac{1}{j\omega_0\left(1+\beta\right)L_0}$$

What we get is something more familiar to electrical engineers

- mourici@gmail.com , mourici@walla.co.il Israel tel 972- 054-5480550

8]

$$\alpha^2 m_T^2 = \alpha^2 m_-^2 + \alpha^2 m_+^2 = \frac{\alpha^2 m_0^2}{2} \cdot \left[\frac{1}{1-\beta} + \frac{1}{1+\beta} \right]$$

$$\frac{1}{Z_T} = \frac{1}{Z_-} + \frac{1}{Z_+} = \frac{\alpha^2 m_T^2}{j\omega_0} = \frac{\alpha^2 m_-^2}{j\omega_0} + \frac{\alpha^2 m_-^2}{j\omega_0}$$

$$\frac{1}{Z_T} = \frac{1}{Z_-} + \frac{1}{Z_+} = \frac{1}{j\omega_0 L_T} = \frac{1}{j\omega_0 L_-} + \frac{1}{j\omega_0 L_+} = \frac{1}{j\omega_0 (1-\beta) L_0} + \frac{1}{j\omega_0 (1+\beta) L_0}$$

and from the last line in eq 8

9] $$\frac{1}{L_T} = \frac{1}{L_-} + \frac{1}{L_+} = \frac{1}{(1-\beta) L_0} + \frac{1}{(1+\beta) L_0}$$

According to electrical engineering the last equation describes the equivalent value L_T of two inductors L_- and L_+ in parallel. But it is wrong to compute parallel value of inductors that react with different frequencies.

10] $$\frac{1}{j\omega_0 L_T} = \frac{1}{j\omega_0 L_-} + \frac{1}{j\omega_0 L_+} = \frac{1}{j\left[\omega_0 (1-\beta)\right] L_0} + \frac{1}{j\left[\omega_0 (1+\beta)\right] L_0}$$

equation 9 and 10 are correct numerically but wrong physically because the frequencies are not the same in L_- and L_+

This lead to the conclusion that professor Albert Einstein missed something that will be explained now

Every electrical engineer will say that $L_T, L_-, L_+, \mathrm{L}_0$ are electrical inductors.
And The impedance of an inductor is given in general by

11] $$Z = j\omega L$$

And the current passes through the inductor is given by Ohm's law

12] $$I = \frac{V}{Z} = \frac{V}{j\omega L}$$

we found in eq. 8 that

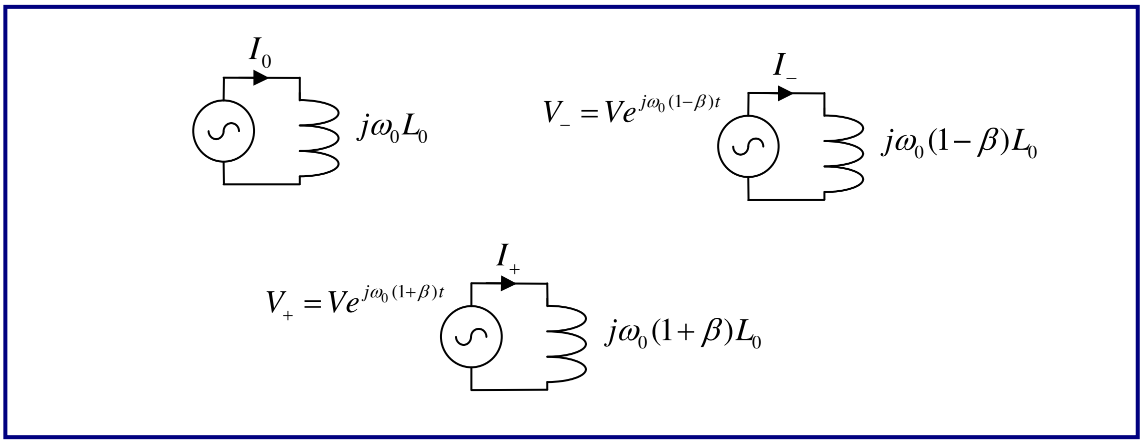
13] $$\frac{1}{Z_T} = \frac{1}{Z_-} + \frac{1}{Z_+}$$

but this equation is wrong because of the different frequencies in each inductor.
The reason for that is that **the frequency of the current and voltage must be the same and the same frequency must also appear in the impedance of the inductor**
This is because for an inductor

14] $$v = L\frac{di}{dt}$$

- mourici@gmail.com , mourici@walla.co.il Israel tel 972- 054-5480550

So we must compute the currents in each inductor separately according to the electrical circuits in the picture below and eq. 15



[15]

$$I_- = \frac{V_-}{Z_-} = \frac{V_-}{j\omega_0 L_-} = \frac{V_-}{j\omega_0(1-\beta)L_0} = \frac{Ve^{j(\omega_0(1-\beta)t)}}{j\omega_0(1-\beta)L_0}$$

$$I_+ = \frac{V_+}{Z_+} = \frac{V_+}{j\omega_0 L_+} = \frac{V_+}{j\omega_0(1+\beta)L_0} = \frac{Ve^{j(\omega_0(1+\beta)t)}}{j\omega_0(1+\beta)L_0}$$

$$I_0 = \frac{V_0}{Z_0} = \frac{V_0}{j\omega_0 L_0} = \frac{Ve^{j(\omega_0 t)}}{j\omega_0 L_0}$$

where

16]

$$V_0 = Ve^{j\omega_0 t} \qquad V_- = Ve^{j\omega_0(1-\beta)t} \qquad V_+ = Ve^{j\omega_0(1+\beta)t}$$

The conclusion is that when the mass is moving and $\beta \neq 0$ we have two frequencies So

16]

$$I_T = I_- + I_+$$

The "current" I_T Is a superposition of two different currents I_- and I_+ with different frequencies and amplitudes?

Before continuing with the theory Let solve some classical examples

Time Dilation

Consider a pendulum clock sitting stationary in the frame S_0 the pendulum ticks at intervals of T_0. This means that the tick events in frame S_0 occur at the same place. In a moving frame S, the interval between ticks is due to time dilation

$$T = \frac{T_0}{\sqrt{1-\beta^2}}$$

- mourici@gmail.com , mourici@walla.co.il Israel tel 972- 054-5480550

And every tick occurs at a different place.

If

$$\beta \le 1 \quad \text{then } T \ge T_0$$

A moving clock runs more slowly. The correct interpretation is that time itself runs more slowly in moving frames.

Muons Life Time

Time dilation is tested most accurately is in particle accelerators where elementary particles routinely reach speeds close to c.

The effect of time dilation is particularly vivid on unstable particles which live much longer in the lab frame than in their own rest frame. For example, The muons are unstable particles. They decay into an electron, together with a couple of neutrinos, with a half-life of $\tau_0 \approx 2\cdot 10^{-6}s$ Muons are created when cosmic rays hit the atmosphere, and subsequently rain down on Earth.

Yet to make it down to sea level, it takes about $T \approx 7\cdot 10^{-6}s$, somewhat longer than their lifetime. Given this, why are there any muons detected on Earth at all? Surely they should have decayed. The reason that they do not decay is because the muons are traveling at a speed close to the speed of light,. From the muon's perspective, the journey takes only $T_0 \approx 7\cdot 10^{-7}s$, and $T_0 \le \tau_0$ is less than their lifetime.

The equation of time dilation are represented in a more intuitive form

$$\alpha T = \frac{\alpha T_0}{\sqrt{1-\beta^2}}$$

squaring and dividing by $j\omega_0$

$$\frac{1}{j\omega_0 L} = \frac{\alpha^2 T^2}{j\omega_0} = \frac{\alpha^2 T_0^2}{j\omega_0\left(1-\beta^2\right)} = \frac{\alpha^2 T_0^2}{j\omega_0 2}\left[\frac{1}{1-\beta}+\frac{1}{1+\beta}\right] = \frac{1}{j\omega_0 L_0}\left[\frac{1}{1-\beta}+\frac{1}{1+\beta}\right]$$

where

$$L_0 = \frac{2}{\alpha^2 T_0^2} \qquad , \qquad L = \frac{1}{\alpha^2 T^2}$$

and therefore the two inductors in parallel are

$$\frac{1}{j\omega_0 L} = \frac{1}{j\omega_0\left(1-\beta\right)L_0} + \frac{1}{j\omega_0\left(1+\beta\right)L_0}$$

and the inductors with their voltage source

$$\frac{V^{j\omega_0 t}}{j\omega_0 L} = \frac{V^{j\omega_0(1-\beta)t}}{j\omega_0\left(1-\beta\right)L_0} + \frac{V^{j\omega_0(1+\beta)t}}{j\omega_0\left(1+\beta\right)L_0}$$

- mourici@gmail.com , mourici@walla.co.il Israel tel 972- 054-5480550

Numerical Example

The average lifetime of $\mu-$mesons with a speed of 0.95c is measured to be $\tau \approx 6\cdot 10^{-6} s$. Compute the average lifetime of $\mu-$mesons in a system in which they are at rest

$$\tau = \frac{\tau_0}{\sqrt{1-\beta^2}}$$

$$6\cdot 10^{-6} s = \frac{\tau_0}{\sqrt{1-0.95^2}}$$

$$\tau_0 = 6\cdot 10^{-6} s\sqrt{1-0.95^2} = 1.87\cdot 10^{-6}$$

But if we use the electric analog circuit

$$\frac{\alpha^2 T^2}{j\omega_0} = \frac{\alpha^2 T_0^{\ 2}}{j\omega_0\left(1-\beta^2\right)} = \frac{\alpha^2 T_0^{\ 2}}{j\omega_0 2}\left[\frac{1}{1-\beta}+\frac{1}{1+\beta}\right] = \frac{1}{j\omega_0 L_0}\left[\frac{1}{1-\beta}+\frac{1}{1+\beta}\right]$$

$$L_0 = \frac{2}{\alpha^2 T_0^{\ 2}} \qquad , \qquad L = \frac{1}{\alpha^2 T^2}$$

$$\frac{V^{j\omega_0 t}}{j\omega_0 L} = \frac{V^{j\omega_0(1-\beta)t}}{j\omega_0\left(1-\beta\right)L_0} + \frac{V^{j\omega_0(1+\beta)t}}{j\omega_0\left(1+\beta\right)L_0}$$

$$\frac{L_0 V^{j\omega_0 t}}{j\omega_0} = \frac{L V^{j\omega_0(1-\beta)t}}{j\omega_0\left(1-\beta\right)} + \frac{L V^{j\omega_0(1+\beta)t}}{j\omega_0\left(1+\beta\right)}$$

$$\frac{2}{j\omega_0\alpha^2\tau_0^{\ 2}} = \frac{1}{j\omega_0\alpha^2\tau^2}\left[\frac{1}{1-\beta}+\frac{1}{1+\beta}\right]$$

$\alpha = 1 \qquad \beta = 0.95$

$$\frac{2}{\tau_0^{\ 2}} = \frac{1}{\left(6\cdot 10^{-6}\right)^2}\left[\frac{1}{1-0.95}+\frac{1}{1+0.95}\right]$$

$$\tau_0 \approx \sqrt{\frac{1}{\left(6\cdot 10^{-6}\right)^2}\frac{2}{\left[\frac{1}{1-0.95}+\frac{1}{2}\right]}} = 1.874\cdot 10^{-6} s$$

Of course we get the same result

- mourici@gmail.com , mourici@walla.co.il Israel tel 972- 054-5480550

Conclusion

For particles that move close to the speed of light Time dilation can be computed with

$$L_0 = \frac{2}{\alpha^2 T_0^2} \quad , \quad L = \frac{1}{\alpha^2 T^2}$$

$$\frac{V^{j\omega_0 t}}{j\omega_0 L} = \frac{V^{j\omega_0(1-\beta)t}}{j\omega_0(1-\beta)L_0}$$

or

$$\frac{1}{L} = \frac{1}{(1-\beta)L_0}$$

Those who learned electricity understand better the meaning of my equations

Length Contraction

Length or more accurately "proper length" is defined as the distance between two coordinates measured at the same time. It is measured at the same time in the moving frame $S^{\#}$ and the rest frame S

According to Lorentz transformation

$t_B = t_A$ measured at the same time

$$\Gamma_0 = x_B^{\#} - x_A^{\#} = \frac{(x_B - x_A) - v(t_B - t_A)}{\sqrt{1-\beta^2}} = \frac{\Gamma - v(0)}{\sqrt{1-\beta^2}} = \frac{\Gamma}{\sqrt{1-\beta^2}}$$

$$\Gamma = x_B - x_A$$

It is clear that

If

$$\beta \le 1 \quad \text{then } \Gamma \le \Gamma_0$$

Professor Albert Einstein believed that Length contraction can not be measured (Ladder Paradox)

The solution of a problem with length contraction looks the same as a problem with time dilation

- mourici@gmail.com , mourici@walla.co.il Israel tel 972- 054-5480550

Relativistic dynamic

A completely inelastic collision

Let's apply the conservation of energy and momentum to a specific case.
A ball of bubble gum with rest mass 16 kg, and another ball of bubble gum with rest mass 9 kg, speeds toward each other as shown:
Before collision

$$m_{0a} = 16kg \quad , \quad m_{0b} = 9kg \quad , \quad \beta_a = 3/5 \quad , \quad \beta_b = 4/5$$

After collision: The two balls stick together.
The total (horizontal) momentum and the total energy before and after collision are as follow (don't miss any line because the solution is not conventional)

$$E = \frac{m_0 c^2}{\sqrt{1-\beta^2}}$$

$$E^2 = \frac{m_0{}^2 c^4}{1-\beta^2} = \frac{m_0{}^2 c^4}{2}\left[\frac{1}{1-\beta} + \frac{1}{1+\beta}\right]$$

$$m_{0a} = 16 \quad \beta = 3/5 \qquad\qquad m_{0b} = 9 \quad \beta_b = 4/5$$

$$E_a{}^2 = \frac{16^2 c^4}{2}\left[\frac{1}{1-3/5} + \frac{1}{1+3/5}\right] = (320+80)c^4 = 400c^4$$

$$E_b{}^2 = \frac{9^2 c^4}{2}\left[\frac{1}{1-4/5} + \frac{1}{1+4/5}\right] = (202.5+22.5)c^4 = 225c^4$$

$$E_T{}^2 = E_a{}^2 + E_b{}^2 = 400c^4 + 225c^4 = 625c^4 = 25^2 c^4 = \frac{m_{0T}{}^2 c^4}{2}\left[\frac{1}{1-\beta} + \frac{1}{1+\beta}\right]$$

$$\beta_{afterc} = 0 \qquad\qquad m_{0T} = 25$$

The momentum is computed twice, so keep paying attention

$$p_a = \frac{m_{0a}\beta_a c}{\sqrt{1-\beta_a{}^2}} = \frac{16(3/5)c}{\sqrt{1-(3/5)^2}} = 12c \qquad , \qquad p_b = \frac{m_{0b}\beta_b c}{\sqrt{1-\beta_b{}^2}} = \frac{9(-4/5)c}{\sqrt{1-(-4/5)^2}} = -12c$$

$$m_{0T} = m_{0a} + m_{0b} = 16 + 9 = 25kg$$

$$p_T = p_a + p_b = \frac{m_{0T}\beta_T c}{\sqrt{1-\beta_T{}^2}}$$

$$p_T = \frac{(m_{0a}+m_{0b})\beta_T c}{\sqrt{1-\beta_T{}^2}} = \frac{25\beta_T c}{\sqrt{1-\beta_T{}^2}} = 12c - 12c = 0 \quad \rightarrow \beta_T = 0$$

The total momentum after collision is zero. Therefore after collision masses doesn't move

- mourici@gmail.com , mourici@walla.co.il Israel tel 972- 054-5480550

Now I solve the same example using electrical analog circuit

$$\alpha^2 p_a{}^2 = \frac{\alpha^2 m_{0a}{}^2 \beta_a{}^2 c^2}{1-\beta_a{}^2} = \frac{\alpha^2 m_{0a}{}^2 c^2 \beta_a{}^2}{2}\left[\frac{1}{1-\beta_a}+\frac{1}{1+\beta_a}\right] = \frac{1}{L_{a0}}\left[\frac{1}{1-\beta_a}+\frac{1}{1+\beta_a}\right]$$

$$L_{a0} = \frac{2}{\alpha^2 m_{0a}{}^2 c^2 \beta_a{}^2} \quad , \quad L_a = \frac{1}{\alpha^2 p_a{}^2}$$

$$\frac{1}{L_a} = \frac{1}{(1-\beta_a)L_{a0}} + \frac{1}{(1+\beta_a)L_{a0}}$$

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assume $\alpha = 1$

$$p_a{}^2 = \frac{16^2 c^2 \left(3/5\right)^2}{2}\left[\frac{1}{1-\left(3/5\right)}+\frac{1}{1+\left(3/5\right)}\right] = 144c^2 = 12^2 c^2$$

$$\alpha^2 p_b{}^2 = \frac{\alpha^2 m_{0b}{}^2 \beta_b{}^2 c^2}{1-\beta_b{}^2} = \frac{\alpha^2 m_{0b}{}^2 c^2 \beta_b{}^2}{2}\left[\frac{1}{1-\beta_b}+\frac{1}{1+\beta_b}\right] = \frac{1}{L_{b0}}\left[\frac{1}{1-\beta_b}+\frac{1}{1+\beta_b}\right]$$

$$p_b{}^2 = \frac{9^2 c^2 \left(16/25\right)}{2}\left[\frac{1}{1-\left(-4/5\right)}+\frac{1}{1+\left(-4/5\right)}\right] = 144c^2 = 12^2 c^2$$

****

The sum and diference of the momentum are hiden in the next expressions

$$\alpha^2 p_a{}^2 - \alpha^2 p_b{}^2 = (\alpha p_a + \alpha p_b)(\alpha p_a - \alpha p_b) = (12c+12c)(12c-12c) = 0$$

We know also that energy and momentum depend according to

$$E^2 = p^2c^2 + E_0{}^2 \qquad \text{where} \qquad E_0 = m_0 c^2$$

$$E_a{}^2 = p_a{}^2 c^2 + E_{a0}{}^2$$

$$400c^4 = p_a{}^2 c^2 + \left(16c^2\right)^2$$

$$p_a{}^2 c^2 = 400c^4 - 256c^4 = 144c^4 \qquad \rightarrow p_a = 12c$$

$$E_b{}^2 = p_b{}^2 c^2 + E_{b0}{}^2$$

$$225c^4 = p_b{}^2 c^2 + \left(9c^2\right)^2$$

$$p_b{}^2 c^2 = 225c^4 - 81c^4 = 144c^4 \qquad \rightarrow p_b = 12c$$

- mourici@gmail.com , mourici@walla.co.il Israel tel 972- 054-5480550

And if we compute the sum and difference of energy and momentum we get again previous results

$E^2 = p^2c^2 + E_0{}^2 \quad , \quad E_0 = m_0c^2$

$E_a{}^2 = p_a{}^2c^2 + E_{a0}{}^2 \quad , \quad E_b{}^2 = p_b{}^2c^2 + E_{b0}{}^2$

and the energy sum is;

$E_{T+}{}^2 = p_T{}^2c^2 + E_{T0}{}^2$

so

$E_{T+}{}^2 = p_{T+}{}^2c^2 + E_{T+0}{}^2 = E_a{}^2 + E_b{}^2 = \left(p_a{}^2 + p_b{}^2\right)c^2 + E_{a0}{}^2 + E_{b0}{}^2$

$E_{T+}{}^2 = p_{T+}{}^2c^2 + E_{T+0}{}^2 = 400c^4 + 225c^4 = \left(p_a{}^2 + p_b{}^2\right)c^2 + 256c^4 + 81c^4$

$625c^4 = \left(p_a{}^2 + p_b{}^2\right)c^2 + 337c^4$

$288c^4 = \left(p_a{}^2 + p_b{}^2\right)c^2$

$p_a{}^2 + p_b{}^2 = 288c^2$

$p_a = 12c \quad , \quad p_b = 12c$ as before

****

Now, let find the energy and momentum difference

$E_{T-0}{}^2 = E_{a0}{}^2 - E_{b0}{}^2 \quad , \quad p_{T-}{}^2 = p_a{}^2 - p_b{}^2$

$E_{T-}{}^2 = p_{T-}{}^2c^2 + E_{T-0}{}^2$

$E_{T-}{}^2 = \left(E_a{}^2 - E_b{}^2\right) = \left(p_a{}^2 - p_b{}^2\right)c^2 + \left(E_{a0}{}^2 - E_{b0}{}^2\right) = \left(p_a - p_b\right)\left(p_a + p_b\right)c^2 + \left(E_{a0}{}^2 - E_{b0}{}^2\right)$

$E_{T-}{}^2 = 400c^4 - 225c^4 = 175c^4 = \left(p_a{}^2 - p_b{}^2\right)c^2 + 256c^4 - 81c^4 = \left(p_a - p_b\right)\left(p_a + p_b\right)c^2 + 175c^4$

$\left(p_a - p_b\right)\left(p_a + p_b\right)c^2 = 0 \quad \rightarrow \quad p_a = p_b$

Conclusions

It is well understood that the electrical analog circuit help solving problems in special relativity.
The analog electrical circuit also explain special relativity not the way professor Albert Einstein did

To see detail of this explanation you have to read a short paper

http://vixra.org/abs/1602.0282

Special Relativity Electromagnetic and Gravitation Combined Into One Theory

- mourici@gmail.com , mourici@walla.co.il Israel tel 972- 054-5480550