Conjecture on the numbers 6(m+n)+1 where m and n are the two prime factors of a 2-Poulet number

Abstract. In this paper I make the following conjecture: there exist an infinity of 2-Poulet numbers P = m*n having the property that the number q = 6*(m + n) + 1 is prime.

Conjecture 1:

There exist an infinity of 2-Poulet numbers P = m*n having the property that the number q = 6*(m + n) + 1 is prime.

The sequence of 2-Poulet numbers (A214305 in OEIS):

: 341, 1387, 2047, 2701, 3277, 4033, 4369, 4681, 5461, 7957, 8321, 10261, 13747, 14491, 15709, 18721, 19951, 23377, 31417, 31609, 31621, 35333, 42799, 49141, 49981, 60701, 60787, 65077, 65281, 80581, 83333, 85489, 88357, 90751, 104653, 123251, 129889, 130561, 150851, 162193, 164737, 181901, 188057, 194221, 196093, 215749, 219781, 220729 (...)

The sequence of primes q:

: q = 673 for P = 2047 = 23*89;
: q = 661 for P = 2701 = 37*73;
: q = 853 for P = 3277 = 29*113;
: q = 877 for P = 4033 = 37*109;
: q = 1093 for P = 4681 = 31*151;
: q = 1021 for P = 5461 = 43*127;
: q = 1093 for P = 7957 = 73*109;
: q = 1753 for P = 13747 = 59*233;
: q = 2281 for P = 14491 = 43*337;
: q = 1741 for P = 18721 = 97*193;
: q = 2113 for P = 19951 = 71*281;
: q = 2029 for P = 23377 = 97*241;
: q = 3037 for P = 31609 = 73*433;
: q = 2917 for P = 35333 = 89*397;
: q = 4621 for P = 65281 = 97*673;
: q = 8293 for P = 80581 = 61*1321;
: q = 4513 for P = 90751 = 151*601;
: q = 2889 for P = 123251 = 59*2089;
: q = 5197 for P = 129889 = 193*673;
: q = 5113 for P = 150851 = 251*601;
: q = 13477for P = 220729 = 103*2143;
(...)

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:    q = 2112493  for P = 27547091281 = 117361*234721;
:    q = 2581933  for P = 27547169521 = 78241*352081;
:    q = 2012401  for P = 27549352151 = 143743*191657;
:    q = 20127253 for P = 27550161997 = 8233*3346309;
:    q = 14751637 for P = 27603401233 = 11279*2447327;
:    q = 7326733  for P = 27604984321 = 23041*1198081;
:    q = 4681921  for P = 27615119599 = 37159*743161;
:    q = 3324613  for P = 27632967001 = 55411*498691;
:    q = 2421733  for P = 27642607621 = 87421*316201;
:    q = 2240317  for P = 27653056649 = 101833*271553;
:    q = 4017697  for P = 27685810639 = 44273*625343;
:    q = 2038693  for P = 27708447397 = 135913*203869;
:    q = 2680141  for P = 27712970209 = 74449*372241;
     (...)
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Note the chain of four consecutive 2-Poulet numbers 27547091281, 27547169521, 27549352151, 27550161997 for which q is prime.