Two conjectures on the numbers obtained concatenating the squares of two primes q and r where r = q+18n

Abstract. In this paper I state the following two conjectures: (I) For any prime q greater than 5 there exist an infinity of primes p obtained subtracting the square of q or the square of r from the number obtained concatenating the square of q with the square of r, where r prime, r = q + 18*n, and adding 1 (for example, p = 121841 – 121 + 1 = 121721, prime, also p = 121841 – 841 + 1 = 121001, prime, where q^2 = 11^2 = 121, r^2 = 29^2 = 841 and 29 = 11 + 18*1); (II) For any positive integer n there exist an infinity of triplets of primes [p, q, r] such that r = q + 18*n and p is obtained subtracting the square of q or the square of r from the number obtained concatenating the square of q with the square of r and adding 1.

Conjecture 1:

For any prime q greater than 5 there exist an infinity of primes p obtained subtracting the square of q or the square of r from the number obtained concatenating the square of q with the square of r, where r prime, r = q + 18*n, and adding 1.

Example:

: p = 121841 – 121 + 1 = 121721, prime, also p = 121841 – 841 + 1 = 121001, prime, where q^2 = 11^2 = 121, r^2 = 29^2 = 841 and 29 = 11 + 18*1.

Examples of primes p for q = 7:

: p = 490001 = 491849 – 1849 + 1, where [r, n] = [43, 2];
: p = 496193 = 496241 – 49 + 1, where [r, n] = [79, 4];
: p = 499361 = 499409 – 49 + 1, where [r, n] = [97, 5];
: p = 4900001 = 4922801 – 22801 + 1, where [r, n] = [151, 8].

Examples of primes p for q = 11:

: p = 121721 = 121841 – 121 + 1, where [r, n] = [29, 1];
: p = 121001 = 121841 – 841 + 1, where [r, n] = [29, 1];

: p = 12136361 = 12136481 - 121 + 1, where [r, n] = [191, 10].

Examples of primes p for q = 13, 17, 19, 23, 29, 31, 37:

: p = 16919153 = 16919321 - 169 + 1, where [q, r, n] = [13, 139, 7];
: p = 2890001 = 2892809 - 289 + 1, where [q, r, n] = [17, 53, 2];
: p = 3614969 = 3615329 - 361 + 1, where [q, r, n] = [19, 73, 3];
: p = 5292953 = 5293481 - 529 + 1, where [q, r, n] = [23, 59, 2];
: p = 8410001 = 8412209 - 2209 + 1, where [q, r, n] = [29, 47, 1];
: p = 96100001 = 96110609 - 10609 + 1, where [q, r, n] = [31, 103, 4];
: p = 13693961 = 13695329 - 1369 + 1, where [q, r, n] = [37, 73, 2].

Conjecture 2:

For any positive integer n there exist an infinity of triplets of primes [p, q, r] such that r = q + 18*n and p is obtained subtracting the square of q or the square of r from the number obtained concatenating the square of q with the square of r and adding 1.

Examples of primes p for n = 0:

: p = 121001 = 121121 - 121 + 1, where r = q = 11;
: p = 289001 = 289289 - 288 + 1, where r = q = 17;
: p = 361001 = 361361 - 360 + 1, where r = q = 19.

Examples of primes p for n = 1:

: p = 121721 = 121841 - 121 + 1, where [q, r] = [11, 29];
: p = 121001 = 121841 - 841 + 1, where [q, r] = [11, 29];
: p = 8410001 = 8412209 - 2208 + 1, where [q, r] = [29, 47];
: p = 18490001 = 18493721 - 3721 + 1, where [q, r] = [43, 61].

Examples of primes p for n = 2:

: p = 490001 = 491849 - 1849 + 1, where [q, r] = [7, 43];
: p = 8410001 = 8412209 - 2209 + 1, where [q, r] = [29, 47];

: p = 13693961 = 13695329 - 1369 + 1, where [q, r] = [37, 73];
: p = 18490001 = 18496241 - 6241 + 1, where [q, r] = [43, 79].

Examples of primes p for n = 3:

: p = 2894753 = 2895041 - 289 + 1, where [q, r] = [17, 71];
: p = 2890001 = 2895041 - 5041 + 1, where [q, r] = [17, 71].

Examples of primes p for n = 4, 5, 6, 7, 8, 9, 10:

: p = 496193 = 496241 - 49 + 1, where [q, r, n] = [7, 79, 4];
: p = 499361 = 499409 - 49 + 1, where [q, r, n] = [7, 97, 5];
: p = 96100001 = 96119321 - 19321 + 1, where [q, r, n] = [31, 139, 6];
: p = 16919153 = 16919321 - 169 + 1, where [q, r, n] = [13, 139, 7];
: p = 4900001 = 4922801 - 22801 + 1, where [q, r, n] = [7, 151, 8];
: p = 96136289 = 96137249 - 961 + 1, where [q, r, n] = [31, 193, 9];
: p = 12136361 = 12136481 - 121 + 1, where [q, r, n] = [11, 191, 10].

Observation:

Because a pair [q, r] of emirps (reversible, but different, primes, see A006567 in OEIS) respects always the relation r = q + 18*n, there exist also primes p obtained as above; examples:

: p = 2894753 = 2895041 - 289 + 1, where [q, r] = [17, 71];
: p = 2890001 = 2895041 - 5041 + 1, where [q, r] = [17, 71];
: p = 13693961 = 13695329 - 5041 + 1, where [q, r] = [37, 73];
: p = 24649539353 = 24649564001 - 24649 + 1, where [q, r] = [157, 751];
: p = 39601942481 = 39601982081 - 39600 + 1, where [q, r] = [199, 991];
: p = 39601000001 = 39601982081 - 982081 + 1, where [q, r] = [199, 991];
: p = 22201000001 = 22201885481 - 885480 + 1, where [q, r] = [149, 941].