Eight formulas that generate semiprimes pq such that n=q-p+1 is prime respectively n=p+q-1 is prime

Abstract. In this paper I list a number of eight formulas that generate two certain types of semiprimes, i.e. semiprimes p*q with the property that n = q – p + 1 is prime respectively with the property that P + q – 1 is prime.

Formulas that generate semiprimes m = p*q such that n = q – p + 1 is prime

(1) m = 30*x*y + 1, where y – x + 1 is prime.

Examples:

: m = 30*7*173 + 1 (173 – 7 + 1 = 167, prime); it can be seen that m = 36331 = 47*773 and n = 773 – 47 + 1 = 727, prime);
: m = 30*7*263 + 1 (263 – 7 + 1 = 257, prime); it can be seen that m = 55231 = 11*5021 and n = 773 – 47 + 1 = 727, prime);
: m = 30*7*269 + 1 (269 – 7 + 1 = 263, prime); it can be seen that m = 56491 = 17*3323 and n = 3323 – 17 + 1 = 3307, prime);
: m = 30*7*509 + 1 (509 – 7 + 1 = 503, prime); it can be seen that m = 106891 = 139*769 and n = 769 – 139 + 1 = 631, prime).

(2) m is the number obtained concatenating to the right a square of prime with the digit 1.

Examples:

: for 11^2 = 121, m = 1211 = 7*173 and n = 173 – 7 + 1 = 167, prime;
: for 13^2 = 169, m = 1691 = 19*89 and n = 89 – 19 + 1 = 11, prime;
: for 17^2 = 289, m = 2891 = 49*59 and n = 59 – 49 + 1 = 11, prime;
: for 31^2 = 961, m = 9611 = 7*1373 and n = 1373 – 7 + 1 = 1367, prime;
: for 53^2 = 2809, m = 28091 = 7*4013 and n = 4013 – 7 + 1 = 4007, prime;
: for 59^2 = 3481, m = 34811 = 7*4973 and n = 4973 – 7 + 1 = 4967, prime;
: for 67^2 = 3721, m = 37211 = 127*293 and n = 293 – 127 + 1 = 167, prime;
: for 79^2 = 6241, m = 62411 = 139*449 and n = 449 - 139 + 1 = 311, prime.

(3) m is the product of the numbers obtained concatenating to the right a multiple of 3 with 1 respectively with 11.

Examples:
: m = 31*311 and n = 311 - 31 + 1 = 281, prime;
: m = 151*1511 and n = 1511 - 151 + 1 = 1361, prime;
: m = 211*2111 and n = 2111 - 211 + 1 = 1901, prime;
: m = 271*2711 and n = 2711 - 271 + 1 = 2441, prime;
: m = 691*6911 and n = 6911 - 691 + 1 = 6221, prime;
: m = 1291*12911 and n = 12911 - 1291 + 1 = 11621, prime;
: m = 1831*18311 and n = 18311 - 1831 + 1 = 16481, prime;
: m = 2251*22511 and n = 22511 - 2251 + 1 = 20261, prime.

(4) m = (n + 1)*p^2 - n, where p is prime and n positive integer.

Examples:
: for n = 5 the formula becomes m = 6*p^2 - 5:
  : for p = 11, m = 721 = 7*103 and n = 103 - 7 + 1 = 97, prime;
  : for p = 41, m = 10081 = 17*593 and n = 593 - 17 + 1 = 577, prime;
: for n = 6 the formula becomes m = 7*p^2 - 6:
  : for p = 13, m = 1177 = 11*107 and n = 107 - 11 + 1 = 97, prime;
  : for p = 37, m = 9577 = 61*157 and n = 157 - 61 + 1 = 97, prime;
  : for p = 61, m = 11761 = 19*619 and n = 619 - 19 + 1 = 601, prime;
: for n = 7 the formula becomes m = 8*p^2 - 7:
  : for p = 17, m = 2305 = 5*461 and n = 461 - 5 + 1 = 457, prime.

(5) m is the number obtained concatenating two Poulet numbers.

Examples:
: for P1 = 561 and P2 = 1729, m = 5611729 = 73*76873 and n = 76873 - 73 + 1 = 76801, prime;
: for P1 = 561 and P2 = 4033, m = 5614033 = 643*8731 and n = 8731 - 643 + 1 = 8089;
: for P1 = 4033 and P2 = 561, m = 4033561 = 7*576223 and n = 576223 - 7 + 1 = 576217, prime;
: for P1 = 645 and P2 = 1729, m = 6451729 = 571*11299 and n = 11299 - 571 + 1 = 10729, prime;
: for P1 = 645 and P2 = 4033, m = 6454033 = 17*379649 and n = 379649 - 17 + 1 = 379633, prime.

(6) m is the number obtained concatenating two squares of primes.

Examples:

: for 19^2 = 361 and 11^2 = 121, m = 361121 = 331*1091 and n = 1091 - 331 + 1 = 761, prime;
: for 11^2 = 361 and 47^2 = 2209, m = 1212209 = 97*12497 and n = 12497 - 97 + 1 = 12401, prime;
: for 41^2 = 1681 and 31^2 = 961, m = 1681961 = 367*4583 and n = 4583 - 367 + 1 = 4217, prime.

7) m is the number 1abc obtained concatenating 1 with a, prime, then with b prime such that b = a + 6 and then with c prime such that c = b + 6.

Examples:

: for [a, b, c] = [31, 37, 43], m = 1313743 = 17*77279 and m = 77279 – 17 + 1 = 77263, prime;
: for [a, b, c] = [47, 53, 59], m = 1475359 = 127*11617 and m = 11617 – 127 + 1 = 11491, prime;
: for [a, b, c] = [97, 103, 109], m = 197103109 = 7*28157587 and m = 28157587 – 7 + 1 = 28157581, prime.

Formulas that generate semiprimes m = p*q such that n = P + q + 1 is prime

(1) m = 30*x*y - 1, where x + y - 1 is prime.

Examples:

: m = 30*7*257 - 1 (257 + 7 - 1 = 263, prime); it can be seen that m = 53969 = 29*1861 and n = 29 + 1861 - 1 = 1889, prime);
: m = 30*7*433 - 1 (433 + 7 - 1 = 439, prime); it can be seen that m = 90929 = 79*1151 and n = 79 + 1151 - 1 = 1229, prime);
: m = 30*7*563 - 1 (563 + 7 - 1 = 569, prime); it can be seen that m = 118229 = 191*619 and n = 191 + 619 - 1 = 809, prime);
: m = 30*7*571 - 1 (571 + 7 - 1 = 577, prime); it can be seen that m = 119909 = 19*6311 and n = 19 + 6311 - 1 = 6329, prime).

(2) m is the number obtained concatenating to the right a square of prime with the digit 1.

Examples:

: for 11^2 = 121, m = 1211 = 7*173 and n = 173 + 7 - 1 = 179, prime;

: for 13^2 = 169, m = 1691 = 19*89 and n = 89 + 19 - 1 = 109, prime;
: for 17^2 = 289, m = 2891 = 49*59 and n = 59 + 49 - 1 = 109, prime;
: for 19^2 = 361, m = 3611 = 23*157 and n = 157 + 23 - 1 = 179, prime;
: for 29^2 = 841, m = 8411 = 13*647 and n = 647 + 13 - 1 = 659, prime;
: for 53^2 = 2809, m = 28091 = 7*4013 and n = 4013 + 7 - 1 = 4019, prime;
: for 67^2 = 3721, m = 37211 = 127*293 and n = 293 + 127 - 1 = 419, prime;
: for 79^2 = 6241, m = 62411 = 139*449 and n = 449 + 139 - 1 = 587, prime.

(3) m = (n + 1)*p^2 – n, where p is prime and n positive integer.

Examples:
: for n = 6 the formula becomes m = 6*p^2 – 5:
  : for p = 11, m = 721 = 7*103 and n = 103 + 7 - 1 = 109, prime;
  : for p = 31, m = 5761 = 7*823 and n = 823 + 7 - 1 = 829, prime;
: for n = 8 the formula becomes m = 8*p^2 – 7:
  : for p = 19, m = 2881 = 43*67 and n = 67 + 43 - 1 = 109, prime.

(4) m is the number obtained concatenating two Poulet numbers.

Examples:
: for P1 = 1905 and P2 = 341, m = 1905341 = 251*7591 and n = 7591 + 251 - 1 = 7841, prime;
: for P1 = 561 and P2 = 1387, m = 5611387 = 337*16651 and n = 16651 + 337 - 1 = 16987, prime;
: for P1 = 2701 and P2 = 561, m = 2701561 = 43*62827 and n = 62827 + 43 - 1 = 62869, prime;
: for P1 = 2465 and P2 = 341, m = 2465341 = 1237*1993 and n = 1993 + 1237 - 1 = 3229, prime.

(5) m is the number obtained concatenating two squares of primes.

Examples:
: for 13^2 = 169 and 7^2 = 49, m = 16949 = 17*997 and n = 997 + 17 - 1 = 1013, prime;
: for 7^2 = 49 and 31^2 = 961, m = 49961 = 47*1063 and n = 1063 + 47 - 1 = 1109, prime;
: for 11^2 = 121 and 41^2 = 1681, m = 1211861 = 709*1709 and n = 1709 + 709 - 1 = 2417, prime;

: for 13^2 = 169 and 31^2 = 961, m = 169961 = 11*15451 and n = 15451 + 11 - 1 = 15461, prime;
: for 23^2 = 289 and 53^2 = 2809, m = 2892809 = 1217*2377 and n = 1217 + 2377 - 1 = 3593, prime;
: for 43^2 = 1849 and 19^2 = 361, m = 1849361 = 23*80407 and n = 23 + 80407 - 1 = 80429, prime;
: for 29^2 = 841 and 31^2 = 961, m = 841961 = 23*36607 and n = 23 + 36607 - 1 = 36629, prime;
: for 43^2 = 1849 and 29^2 = 841, m = 1849841 = 7*264263 and n = 7 + 264263 - 1 = 264269, prime;
: for 43^2 = 1849 and 31^2 = 961, m = 1849961 = 41*45121 and n = 41 + 45121 - 1 = 45161, prime;
: for 37^2 = 1369 and 43^2 = 1849, m = 13691849 = 89*153841 and n = 89 + 153841 - 1 = 153929, prime.

(6) m is a number formed from one digit or a group of digits concatenated with itself an odd number of times then to the left and to the right with digit 1.

Examples:
: m = 133333331 = 11287*11813 and 11287 + 11813 – 1 = 23099, prime;
: m = 14441 = 7*2063 and 7 + 2063 – 1 = 2069, prime;
: m = 15555555551 = 1709*9102139 and 1709 + 9102139 – 1 = 9103847, prime;
: m = 13434341 = 373*36017 and 373 + 36017 – 1 = 36389, prime;
: m = 14343431 = 59*243109 and 59 + 243109 – 1 = 243167, prime;
: m = 13535351 = 61*221891 and 61 + 221891 – 1 = 221951, prime;
: m = 17676761 = 3529*5009 and 5009 + 3529 – 1 = 8537, prime;
: m = 18989891 = 131*144961 and 131 + 144961 – 1 = 145091, prime;
: m = 13343343341 = 20047*665603 and 20047 + 665603 – 1 = 685649, prime.