# Pi Formulas , Part 10

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abstract

In this note we give some formulas related to the constant Pi

# NÚMERO $\pi$, CONSTANTES , FÓRMULAS

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**Resumen**
Se muestra una colección de fórmulas que involucran la constante $\pi$

## 1 Introducción

En esta nota se muestra una colección de fórmulas que involucran la constante $\pi$, y otras constates como son: $e, G, \gamma$, etc.En algunas fórmulas aparecen las funciones zeta de Riemann $\zeta(x)$ , Beta de Dirichlet $\beta(x)$, los números de Bernoulli $B_n$:

$$B_n = \left\{ \frac{1}{6}, \frac{1}{30}, \frac{1}{42}, \frac{1}{30}, \frac{5}{66}, \frac{691}{2730}, ... \right\}$$

Los números armonicos $H_{n,m} = \sum_{k=1}^{n} \frac{1}{k^m}$ , la función zeta de Hurwitz $\zeta(s,a)$ , la función Gama $\Gamma(x)$, las funciones parte real $\Re(z) \equiv Re(z)$ , e imaginaria $\Im(z) \equiv Im(z)$ de un número complejo .

## 2 Fórmulas

$$\frac{\pi}{2} = 1 + \sqrt{-1} - \sqrt{-1} \sum_{n=1}^{\infty} \left( n \left( \sqrt[n+1]{\sqrt{-1}} - \sqrt[n]{\sqrt{-1}} \right) + \sqrt[n+1]{\sqrt{-1}} - 1 \right) \tag{2.1}$$

$$\frac{8\sqrt{\pi}}{\left(\Gamma\left(1/4\right)\right)^2}exp\left(-i\left(\frac{\pi}{4}-ln\left(\frac{\left(\Gamma\left(1/4\right)\right)^2}{\pi\sqrt{2\pi}}\right)\right)\right)=\left(1-i\right)\prod_{n=1}^{\infty}\left(\frac{n+1}{n}\right)^{i^n} \tag{2.2}$$

$$i=\sqrt{-1}$$

$$G+\frac{\pi}{4}=\int_0^1\frac{ln\left(e/x\right)}{1+x^2}dx \tag{2.3}$$

$$G-\frac{\pi}{4}=-\int_0^1\frac{ln\left(ex\right)}{1+x^2}dx \tag{2.4}$$

$$G-\frac{\pi}{4}=-e\int_0^e\frac{ln\left(x\right)}{e^2+x^2}dx \tag{2.5}$$

$$G+\frac{\pi}{4}=e\int_e^{\infty}\frac{ln\left(x\right)}{e^2+x^2}dx \tag{2.6}$$

$$\frac{\pi}{24}+\frac{1}{8}sen^{-1}\left(\frac{8}{\sqrt{3}}\prod_{n=1}^{\infty}\frac{\left(1+\left(-1\right)^n\left(1/3\right)^n\right)^8}{\left(1+\left(1/3\right)^{2n-1}\right)^4}\right)=\sum_{n=2}^{\infty}\left(-1\right)^n tan^{-1}\left(\sqrt{3}\left(\frac{1}{3}\right)^n\right) \tag{2.7}$$

$$\frac{\pi}{4}=ln\left(\prod_{n=0}^{\infty}ch\left(\frac{1}{2n+1}\right)\right)+ln\left(\prod_{n=0}^{\infty}\left(1+\left(-1\right)^n th\left(\frac{1}{2n+1}\right)\right)\right) \tag{2.8}$$

$$\frac{1-x^2}{2\left(1+x^2\right)}ln\left(\frac{1-x}{1+x}\right)+\frac{x}{1+x^2}ln\left(x\right)+\frac{\pi}{4}=$$

$$=\sum_{n=0}^{\infty}\frac{1}{\left(4n+1\right)\left(4n+3\right)}\left(\frac{4x^3}{1+x^2}x^{4n}+\frac{2\left(1-x\right)^3}{\left(1+x\right)\left(1+x^2\right)}\left(\frac{1-x}{1+x}\right)^{4n}\right) \qquad (2.9)$$

$$0\leq x\leq 1$$

$$\frac{3}{4}ln\left(\frac{1}{3}\right)+ln\left(\frac{1}{2}\right)+\frac{5\pi}{8}=\sum_{n=0}^{\infty}\frac{\left(1/2\right)^{4n}+\left(1/3\right)^{4n+1}}{\left(4n+1\right)\left(4n+3\right)} \qquad (2.10)$$

$$\pi\, cos^{-1}\left(x\right)=\sum_{n=0}^{\infty}\frac{2^{n+1}\left(n!\right)^2}{\left(2n+2\right)!}\left(\left(1+\sqrt{1-x^2}\right)^{n+1}-\left(1-\sqrt{1-x^2}\right)^{n+1}\right) \qquad (2.11)$$

$$0<x<1$$

$$\frac{\pi}{\sqrt{k^2-m^2}}cos^{-1}\left(\frac{m}{k}\right)=\sum_{n=0}^{\infty}\frac{2^{n+1}\left(n!\right)^2u_n\left(m,k\right)}{\left(2n+2\right)!\,k^{n+1}} \qquad (2.12)$$

$$u_{n+2}\left(m,k\right)=2ku_{n+1}\left(m,k\right)-m^2u_n\left(m,k\right)$$

$$u_0\left(m,k\right)=2 \quad , \quad u_1\left(m,k\right)=4k \quad ,m,k\in\mathbb{N},m<k$$

$$\frac{2^{2k-1}B_k}{\left(2k\right)!}\pi^{2k}=\sum_{n=1}^{\infty}\left(-\frac{1}{2k}\right)^{n-1}\zeta\left(2k\left(n+1\right)\right)+\sum_{n=1}^{\infty}\frac{2k}{2kn^{2k}+1} \qquad (2.13)$$

$$k\in\mathbb{N}$$

$$\frac{\pi^2}{6}=\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n-1}\zeta\left(2n+2\right)+\sum_{n=1}^{\infty}\frac{2}{2n^2+1} \tag{2.14}$$

$$\frac{\pi^4}{90}=\sum_{n=1}^{\infty}\left(-\frac{1}{4}\right)^{n-1}\zeta\left(4n+4\right)+\sum_{n=1}^{\infty}\frac{4}{4n^4+1} \tag{2.15}$$

$$\frac{e^{-\left(2k+1\right)\gamma}}{\pi}=\frac{\left(\frac{1}{2}\right)_{k+1}^{2}\left(k+\frac{1}{2}\right)_{n+1}^{2}e^{-\left(2k+1\right)H_n}}{\left(n!\right)^2}\prod_{m=n+1}^{\infty}\left(1+\frac{2k+1}{2m}\right)^2 e^{-\left(2k+1\right)/m} \tag{2.16}$$

$$n\in\mathbb{N},k\in\mathbb{N}_0,H_n=\sum_{k=1}^{n}\frac{1}{k}$$

$$\frac{\pi}{4}=\sum_{n=0}^{\infty}\frac{\left(-1\right)^n\left(\frac{1}{3}\right)_n\left(\frac{2}{3}\right)_n 3^{3n}}{\left(\frac{3}{2}\right)_n n!\,2^{2n}}P\left(n\right) \tag{2.17}$$

$$P\left(n\right)=\sum_{k=0}^{2n}\frac{\binom{2n}{k}\left(2\left(\frac{1}{5}\right)^{2n+2k+1}+\left(\frac{1}{7}\right)^{2n+2k+1}+2\left(\frac{1}{8}\right)^{2n+2k+1}\right)}{2n+2k+1}$$

$$\frac{\pi}{4}=-tan^{-1}\left(\frac{1}{x_1}\right)-tan^{-1}\left(\frac{1}{x_2}\right)-tan^{-1}\left(\frac{1}{x_3}\right) \tag{2.18}$$

$$x_1=\sqrt[3]{11+10\sqrt[3]{11+10\sqrt[3]{11+\ldots}}}$$

$$x_2 = -\frac{x_1}{2} + \frac{1}{2}\sqrt{10 - \frac{33}{x_1}}$$

$$x_3 = -\frac{x_1}{2} - \frac{1}{2}\sqrt{10 - \frac{33}{x_1}}$$

$x_1, x_2, x_3$ , son las raíces de la ecuación $x^3 - 10x - 11 = 0$

$$\frac{\pi^2}{6} + \zeta(4m) = 2\zeta(2m+1) + \sum_{n=2}^{\infty}\left(\frac{n^{2m} - n}{n^{2m+1}}\right)^2 \quad , m \in \mathbb{N} \tag{2.19}$$

$$G + \sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{n}\beta(n+1) = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} ln\left(\frac{2n+2}{2n+1}\right) \tag{2.20}$$

$$\frac{\pi}{4} = \int_0^a \frac{e^x(1+x)}{1+x^2e^{2x}}dx \tag{2.21}$$

$$a = e^{-e^{-e^{-\cdots}}} = 0.567143...$$

$$\frac{\pi}{4} = \int_1^e \frac{1}{x\left(1+(ln(x))^2\right)}dx \tag{2.22}$$

$$\frac{\pi}{\sqrt{6}} = 1 + \sum_{n=1}^{\infty} \frac{n!}{(n+1)\left((n+1)\sqrt{h_n} + \sqrt{h_{n+1}}\right)} = \\ = 1 + \frac{1!}{2\left(2\sqrt{1}+\sqrt{5}\right)} + \frac{2!}{3\left(3\sqrt{5}+\sqrt{49}\right)} + \dots \tag{2.23}$$

$$h_n = \sum_{k=1}^{n} \left(\frac{n!}{k}\right)^2$$

$$\frac{\pi\sqrt{3}}{16} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{5}\right)_n \left(\frac{2}{5}\right)_n \left(\frac{3}{5}\right)_n \left(\frac{4}{5}\right)_n}{\left(\frac{1}{2}\right)_n \left(\frac{3}{4}\right)_n \left(\frac{5}{4}\right)_n n!} \left(\frac{3125}{2304}\right)^n a_n \tag{2.24}$$

$$a_n = \sum_{k=0}^{4n} \frac{(-1)^k \binom{4n}{k} (n+k+1)}{3^{2k} (4n+4k+1)(4n+4k+3)}$$

$$\frac{\pi^2}{2} + \frac{\Gamma'(1/2)}{\sqrt{\pi}} = \frac{\pi^2}{2} - \gamma - 2\ln(2) = \sum_{n=1}^{\infty} \left(1 + \zeta\left(\frac{n+1}{n}, \frac{1}{2}\right) - \zeta\left(\frac{n+2}{n+1}, \frac{1}{2}\right)\right) \tag{2.25}$$

$$\zeta(s,a) = \sum_{n=0}^{\infty} (a+n)^{-s} \ , \Re(s) > 1, a > 0$$

$$\frac{\pi}{4} = \int_{b}^{\infty} \frac{e^x - 1}{e^{2x} - 2e^x (a+x) + (a+x)^2 + 1} dx \tag{2.26}$$

$$a > 0 \ , b = \ln(1+a+b)$$

$$b = \ln\left(1 + a + \ln\left(1 + a + \ln\left(1 + a + \ldots\right)\right)\right)$$

$$\frac{\pi}{2} + \ln\left(\frac{1+x}{x(1-x)}\right) = 4\sum_{n=1}^{\infty}\frac{1}{4n-3}\left(x^{4n-3} + \left(\frac{1-x}{1+x}\right)^{4n-3}\right) \tag{2.27}$$

$$0 < x < 1$$

$$\frac{\pi}{2} + \ln(6) = 4\sum_{n=1}^{\infty}\frac{1}{4n-3}\left(2^{-(4n-3)} + 3^{-(4n-3)}\right) \tag{2.28}$$

$$\ln(2) = \sum_{n=1}^{\infty}\tan^{-1}\left(\frac{n+1}{(2n-1)\left(2n^2+1\right)+n}\right) \tag{2.29}$$

$$\frac{\pi}{4} - \ln(2) = \tan^{-1}\left(\frac{1}{3}\right) - \sum_{n=2}^{\infty}\tan^{-1}\left(\frac{n+1}{(2n-1)\left(2n^2+1\right)+n}\right) \tag{2.30}$$

$$\frac{\pi}{4} - \ln(2) = \sum_{n=2}^{\infty}(-1)^n \tan^{-1}\left(\frac{n}{n^2+1+(-1)^n}\right) \tag{2.31}$$

$$\frac{\pi}{4} = \sum_{n=0}^{\infty}\tan^{-1}\left(\frac{Im\left(\left(x+i(1-x)\right)^{2^n}\right)}{1+Re\left(\left(x+i(1-x)\right)^{2^n}\right)}\right) \tag{2.32}$$

$$0 < x < 1$$

$$\frac{\pi}{4}=\sum_{n=0}^{\infty} tan^{-1}\left(\frac{Im\left(\left(p+i\left(q-p\right)\right)^{2^n}\right)}{q^{2^n}+Re\left(\left(p+i\left(q-p\right)\right)^{2^n}\right)}\right) \tag{2.33}$$

$$0<p<q\ ;\ p,q\in\mathbb{N}$$

$$\frac{2^{m+1}\overbrace{\sqrt{2-\sqrt{2+\ldots+\sqrt{2}}}}^{m-radicales}}{\pi\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{m+1-radicales}}=\prod_{n=1}^{\infty}\left(1-\frac{1}{2^{2m+4}n^2}\right) \tag{2.34}$$

$$m\in\mathbb{N}$$

$$\pi^4\prod_{n=1}^{\infty}\left(1-\frac{1}{2^{2m+6}n^2}\right)^4= \\ =2^{4m+8}\left(6+\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{m-radicales}-4\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{m+1-radicales}\right) \tag{2.35}$$

$$m\in\mathbb{N}$$

$$\pi^4\left(2^{m+2}-1\right)^4\prod_{n=1}^{\infty}\left(1-\frac{\left(2^{m+2}-1\right)^2}{2^{2m+6}n^2}\right)^4= \\ =2^{4m+8}\left(6+\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{m-radicales}+4\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{m+1-radicales}\right) \tag{2.36}$$

$$m\in\mathbb{N}$$

$$\pi = \frac{1}{2}\left(\frac{(m-1)!}{\left(\frac{1}{2}\right)_m}\right)^2 \prod_{n=0}^{\infty}\left(\prod_{k=0}^{n}(k+2m)^{(-1)^k\binom{n}{k}}\right)^{2^{-n}} \tag{2.37}$$

$$m \in \mathbb{N}$$

$$\frac{1}{\pi} = \frac{1}{2}\left(\frac{\left(\frac{1}{2}\right)_{m-1}}{(m-1)!}\right)^2 \prod_{n=0}^{\infty}\left(\prod_{k=0}^{n}(k+2m-1)^{(-1)^k\binom{n}{k}}\right)^{2^{-n}} \tag{2.38}$$

$$m \in \mathbb{N}$$

$$G = \cfrac{1}{1+\cfrac{1^4}{8+\cfrac{3^4}{16+\cfrac{5^4}{24+\cfrac{7^4}{32+\ldots}}}}} \tag{2.39}$$

$$\frac{\pi^2}{16}+\frac{G}{2} = \cfrac{1}{1-\cfrac{1^4}{1^2+5^2-\cfrac{5^4}{5^2+9^2-\cfrac{9^4}{9^2+13^2-\ldots}}}} \tag{2.40}$$

$$\frac{\pi^2}{16}-\frac{G}{2}=\cfrac{1}{1-\cfrac{3^4}{3^2+7^2-\cfrac{7^4}{7^2+11^2-\cfrac{11^4}{11^2+15^2-\dots}}}} \tag{2.41}$$

$$\pi=4-1\frac{1}{3}+\sum_{n=1}^{\infty}\left(\frac{1}{n\frac{1}{4}}-\frac{1}{n\frac{3}{4}}\right)= \\ =4-1\frac{1}{3}+\frac{1}{1\frac{1}{4}}-\frac{1}{1\frac{3}{4}}+\frac{1}{2\frac{1}{4}}-\frac{1}{2\frac{3}{4}}+\frac{1}{3\frac{1}{4}}-\frac{1}{3\frac{3}{4}}+\dots \tag{2.42}$$

$$\frac{\pi}{4}=\sum_{n=0}^{m}\frac{(-1)^n}{2n+1}-\cfrac{(-1)^m}{1+\cfrac{(3+2m)^2}{2+\cfrac{(5+2m)^2}{2+\cfrac{(7+2m)^2}{2+\dots}}}} \tag{2.43}$$

$$m\in\mathbb{N}$$

$$\frac{\pi}{3}=ln\left(\frac{(1+a)\left(1+a+\sqrt{3}(a-1)\right)}{(1-a)\left(1-a+\sqrt{3}(a+1)\right)}\right)-\sum_{n=3}^{\infty}2^{n-1}\int_{a}^{\frac{1+a\sqrt{3}}{\sqrt{3}-a}}\frac{1}{1+x^{2^{n-1}}}dx \tag{2.44}$$

$$1<a<\sqrt{3}$$

$$\pi = 2\ln\left(\left(\frac{a+1}{a-1}\right)^2 \frac{(2a-1)(3a-1)}{(2+a)(3+a)}\right) - 4\sum_{n=1}^{\infty} 2^n \left( \int_a^{\frac{1+2a}{2-a}} \frac{dx}{1+x^{2^{n+1}}} + \int_a^{\frac{1+3a}{3-a}} \frac{dx}{1+x^{2^{n+1}}} \right) \quad (2.45)$$

$$1 < a < 2$$

$$\pi = 4\ln\frac{10}{3} - 4\sum_{n=1}^{\infty} 2^n \left( \int_{3/2}^{8} \frac{dx}{1+x^{2^{n+1}}} + \int_{3/2}^{11/3} \frac{dx}{1+x^{2^{n+1}}} \right) \quad (2.46)$$

$$\tan^{-1}\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right) = \frac{\pi}{6} + \sum_{n=1}^{\infty} (-1)^n \tan^{-1}\left( \frac{\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)^{3^n}}{1-\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)^{2\cdot 3^n}} \right) \quad (2.47)$$

$$\begin{aligned}
&\tan^{-1}\left(\left(\sqrt{2}-1\right)^{3^{-m}}\right) + \tan^{-1}\left(\left(\frac{\sqrt{13}-3}{2}\right)^{3^{-m}}\right) = \\
&= \sum_{n=0}^{m-1} (-1)^n \left( \tan^{-1}\left( \frac{\left(\sqrt{2}-1\right)^{3^{n-m}}}{1-\left(\sqrt{2}-1\right)^{2\cdot 3^{n-m}}} \right) + \tan^{-1}\left( \frac{\left(\frac{\sqrt{13}-3}{2}\right)^{3^{n-m}}}{1-\left(\frac{\sqrt{13}-3}{2}\right)^{2\cdot 3^{n-m}}} \right) \right) + \\
&+ (-1)^m \frac{\pi}{4} + \\
&+ \sum_{n=+m+1}^{\infty} (-1)^n \left( \tan^{-1}\left( \frac{\left(\sqrt{2}-1\right)^{3^{n-m}}}{1-\left(\sqrt{2}-1\right)^{2\cdot 3^{n-m}}} \right) + \tan^{-1}\left( \frac{\left(\frac{\sqrt{13}-3}{2}\right)^{3^{n-m}}}{1-\left(\frac{\sqrt{13}-3}{2}\right)^{2\cdot 3^{n-m}}} \right) \right)
\end{aligned} \quad (2.48)$$

$$\frac{\pi}{6}=\sum_{n=0}^{\infty}3^{n}\int_{0}^{1/\sqrt{3}}\frac{x^{2\left(3^{n}-1\right)}\left(1-2x^{2\cdot 3^{n}}\right)}{1-x^{2\cdot 3^{n}}+x^{4\cdot 3^{n}}}dx \tag{2.49}$$

$$\tan^{-1}\left(\frac{2\,sh\left(\frac{\pi}{3}\right)}{\sqrt{3}}\right)=\frac{\pi}{4}-\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{1}{18n^{2}}\right)+\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{2}{9\left(2n-1\right)^{2}}\right) \tag{2.50}$$

$$\tan^{-1}\left(\sqrt{2}sh\left(\frac{\pi}{4}\right)\right)=\frac{\pi}{4}-\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{1}{32n^{2}}\right)+\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{1}{8\left(2n-1\right)^{2}}\right) \tag{2.51}$$

$$\tan^{-1}\left(2sh\left(\frac{\pi}{6}\right)\right)=\frac{\pi}{4}-\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{1}{72n^{2}}\right)+\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{1}{18\left(2n-1\right)^{2}}\right) \tag{2.52}$$

$$\tan^{-1}\left(\frac{2sh\left(\frac{\pi}{2^{k+1}}\right)}{\underbrace{\sqrt{2-\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}}_{k-radicales}}\right)=\frac{\pi}{4}-\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{1}{2^{2k+3}n^{2}}\right)+ \\ +\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{1}{2^{2k+1}\left(2n-1\right)^{2}}\right) \tag{2.53}$$

$$k\in\mathbb{N}$$

$$\frac{\pi}{6}+\sum_{n=2}^{\infty}(-1)^{n-1}\tan^{-1}\left(\frac{\sqrt{3}}{3^n}\right)= \\ =\sqrt{3}\sum_{n=1}^{\infty}\frac{3^{2n-2}}{(4n-3)\left(3^{4n-3}+1\right)}-\frac{3^{2n-1}}{(4n-1)\left(3^{4n-1}+1\right)} \tag{2.54}$$

$$\frac{\pi}{6\sqrt{3}}+\frac{1}{\sqrt{3}}\sum_{n=2}^{m}\tan^{-1}\left(\frac{1}{n\sqrt{3}}\right)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}H_{m,2n-1}}{(2n-1)3^n} \tag{2.55}$$

$$m\in\mathbb{N}$$

$$\frac{\pi}{4}+\sum_{n=2}^{m}\tan^{-1}\left(\frac{5n}{6n^2-1}\right)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}H_{m,2n-1}}{2n-1}\left(\left(\frac{1}{2}\right)^{2n-1}+\left(\frac{1}{3}\right)^{2n-1}\right) \tag{2.56}$$

$$m\in\mathbb{N}$$

$$\frac{\pi}{4}+\sum_{n=2}^{\infty}\tan^{-1}\left(\frac{\Im\left((x-1+ix)^n\right)}{1+\Re\left((x-1+ix)^n\right)}\right)= \\ =\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\frac{\Im\left((x-1+ix)^n\right)}{\left(1-\Re\left((x-1+ix)^n\right)\right)^2+\left(\Im\left((x-1+ix)^n\right)\right)^2}= \\ =\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\frac{\Im\left((x-1+ix)^n\right)}{1+\left(1-2x+2x^2\right)^2-2\Re\left((x-1+ix)^n\right)} \tag{2.57}$$

$$0<x<1$$

$$\frac{\pi\sqrt{2}}{6}=\sqrt{2}\,ln\left(2+\sqrt{3}\right)-\sum_{n=0}^{\infty}\frac{(2n)!}{2^{3n}(n!)^2}\sum_{k=0}^{n}\sum_{m=0}^{k}\frac{(-1)^k\binom{n}{k}\binom{k}{m}}{(2m+1)(2k-2m+1)} \tag{2.58}$$

# 3 Referencias


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