Non-Metric Microscopic Formulation of Relativity*

Alan M. Kadin
Princeton Junction, NJ 08550 USA
amkadin@alumni.princeton.edu
March 2016

Abstract: *The elusive unity of microscopic and macroscopic physics may reflect a profound misunderstanding of the microscopic world. We start with a real, deterministic de Broglie wave, and show how this leads to the equations of both special and general relativity, providing a unified picture for physics at all scales. In this picture, transformations of ω and k are more fundamental than spacetime transformations of t and r. Local metrics compatible with general relativity may be derived, but are not necessary for calculations. Furthermore, coupling constants and h are invariant, while c, G, e, and particle rest masses are not. Such variations cannot be observed using local measurements, due to the principle of relativity.*

Introduction

It is conventional to view time and space as fundamental physical parameters. In classical physics, time and space are abstract but uniform and universal. In special relativity, time and space become non-universal, with coupled spacetime transformations and the invariance of the speed of light *c*. In general relativity, time and space become non-uniform, connected via a spacetime metric, and the invariance of *c* is maintained. But the deflection of light in a gravitational field clearly suggests a variable speed of light, at least from the viewpoint of a flat reference space and time. This was evident to Einstein prior to general relativity, but this is hidden by the use of curved spacetime.

Matter waves were first derived by de Broglie using special relativity. We show below that this relation may be logically inverted, and special relativity derived from quantum waves. The key point is that quantum waves have a characteristic frequency and wavelength, in contrast to light waves. Since all clocks and rulers are made of quantum waves, these provide the macroscopic functional definition of time and space. Further, the characteristic parameters are modulated by gravitational fields, giving rise to the physics of general relativity. Thus, time and space are not abstract, but follow directly from the behavior of quantum waves. Remarkably, this approach shows that many of the fundamental physical constants may vary in space and time. This has important implications for our fundamental understanding of gravitation and cosmology.

Quantum Waves and Special Relativity

The relativistic wave equation for de Broglie waves is the Klein-Gordon differential equation,

$$\partial^2\Psi/\partial t^2 = c^2\nabla^2\Psi - (mc^2/\hbar)^2\,\Psi, \qquad (1)$$

for which the dispersion relation is:

* *Essay Written for the Gravity Research Foundation 2016 Awards for Essays on Gravitation*

$$\omega^2 = k^2c^2 + \omega_0^2 \qquad (2)$$

where $\omega_0 = mc^2/\hbar$. For $\omega_0 \neq 0$, this may also be written

$$(\omega T)^2 = (kL)^2 + 1, \qquad (3)$$

where L and T are the characteristic length and time for the matter wave, with $T = 1/\omega_0$, $L = \hbar/mc$ is the (reduced) Compton wavelength, and $L/T = c$. Conventionally, c and ω_0 are assumed to be universal constants, but we will allow them to vary (in space and/or time) due to gravity. Note that ω_0 may differ for different types of "particles" (electron, quark, etc.), but c is the same for all particles in a local region (even if it varies in space or time).

The velocity of a wave packet from Eq. (2) is the group velocity

$$v = dr/dt = \partial\omega/\partial k = kc^2/\omega = c[1-(\omega_0/\omega)^2]^{1/2} . \qquad (4)$$

The solution can also be written $\omega = \gamma\omega_0$ and $k = \gamma\omega v/c^2$, where as usual $\gamma = (1-v^2/c^2)^{-1/2}$. This is indeed a de Broglie wave; in the non-relativistic limit, $k = \omega_0 v/c^2 = mv/\hbar$. The original Klein-Gordon equation was for scalar waves; we suggest that vector waves are more appropriate, but the dispersion relation remains the same as in Eq. (2).

De Broglie himself believed in the pilot-wave picture, with both a point particle and a wave, but we propose an alternative picture whereby on the microscopic level, only quantized wave packets (representing fundamental particles such as electrons and quarks) are present [Kadin 2011]. So an electron at rest consists of a wave packet with a real coherently rotating vector field (at a frequency mc^2/h), with distributed angular momentum corresponding to a total quantized spin $\hbar/2$. Without point particles, this picture requires a soliton-like nonlinear self-interaction to maintain "particle" integrity. But once spin is quantized, the linear equations (2) and (3) should continue to be valid. Classical physics follows directly from the trajectories of elementary wave packets, without uncertainty, entanglement, or decoherence. Composite particles are not de Broglie waves, but are just bags of elementary wave packets.

Classical waves are distortions of a medium, and the fixed medium is the preferred reference frame. Matter waves, like light waves, travel in a vacuum, with no preferred reference frame. By the principle of relativity, Eqs. (2) and (3) should remain the same in any inertial reference frame. Just as the Lorentz transformation is the only transformation that preserves c in all reference frames, it is also the only transformation for which Eqs. (2) and (3) are properly invariant:

$$\omega' = \gamma(\omega + \mathbf{k}.\mathbf{v}) \qquad (5)$$

$$\mathbf{k}' = \gamma(\mathbf{k} + \omega\mathbf{v}/c^2) \qquad (6)$$

where here v is the relative speed of the primed and unprimed reference frames. So $\omega^2\text{-}k^2c^2 = \omega_0^2$ is a Lorentz invariant. This also corresponds to time dilation and length contraction, but one can view the real microscopic waves as primary quantities, from which macroscopic time and space are derived. The invariance of c is due not to the special role of light waves, but rather to the fact that c is the characteristic reference velocity for all quantum waves.

One can convert from the intrinsic quantities ω and $\mathbf{k}$ to extrinsic quantities E and $\mathbf{p}$ using the Einstein-deBroglie relations $E = \hbar\omega$ and $\mathbf{p} = \hbar\mathbf{k}$. These relations should follow from the equations for the primary quantum fields, once spin quantization is established. For example, for a circularly polarized classical electromagnetic wave representing a photon, where the electric field is a vector rotating at ω, the energy density $\mathcal{E}$ is related to the angular momentum density $\mathcal{L}$ by $\mathcal{E} = \mathcal{L}\,\omega$. If the quantum wave self-organizes into localized domains with total spin $S = \hbar$, then $E = \hbar\omega$ and $\mathbf{p} = \hbar\mathbf{k}$ follow automatically.

Quantum Waves and General Relativity

Gravity can be easily introduced to Eq. (2) by noting that a classical potential energy of a test mass m in the field of a large mass M should shift the total energy by $m\Phi = -mMG/r$. So

$$mc^2 = m_0 c_0^2 + m\Phi, \qquad (7)$$

which can be rewritten (to first order in the dimensionless potential $\phi = \Phi/c^2$) as

$$T = T_0(1-\phi) \text{ and } \omega_0 = \omega_{00}(1+\phi) \qquad (8)$$

Substituting this into Eq. (2) or Eq. (3) creates the proper gravitational time dilation, since ϕ is always negative. This also changes the speed of light, since $c = L/T$.

This substitution (which is not complete, as shown below) permits one to solve the modified Eq. (1) using the classical Hamiltonian approach [Kadin 2014]. For a wave packet trajectory in a linear medium, the frequency is a constant of the motion:

$$d\omega/dt = 0 = (\partial\omega/\partial\mathbf{r}).(d\mathbf{r}/dt) + (\partial\omega/\partial\mathbf{k}).(d\mathbf{k}/dt) = \mathbf{v}.(\partial\omega/\partial\mathbf{r} + d\mathbf{k}/dt) \qquad (9)$$

so that $d\mathbf{k}/dt = -\,\partial\omega/\partial\mathbf{r}$. This leads directly to the classical Hamiltonian relation $d\mathbf{p}/dt = -\partial H/\partial\mathbf{r}$, where $E = H(\mathbf{p},\mathbf{r})$.

Such a Hamiltonian approach can only be applied to a space with a uniform time reference. Within this picture, a photon may be emitted from an atom with angular frequency ω_1 and wavevector $k_1 = c_1\omega_1$. If it moves to a region 2 with lightspeed c_2, then the photon still has ω_1, but $k = c_2\omega_1 = k_2$. So the gravitational red shift is due to the change in the wavelength of a photon, rather than a change in its frequency, at least from the point of view of any flat reference spacetime.

As was shown earlier, this solution yields a deflection of light and rotation of the perihelion of Mercury, similar to that observed in accordance with standard tests of general relativity. However, a quantitative comparison with general relativity, using the Parameterized Post-Newtonian (PPN) approximation, shows that these trajectories are not quite correct. Just as for special relativity, for consistency one needs to include gravitational length contraction as well as time dilation. This corresponds to

$$L = L_0(1+\phi), \qquad (10)$$

which for negative ϕ is indeed length contraction. Unlike the time correction (which corresponds to the classical gravitational potential), the space correction has no classical analog. This is because the potential energy term due to L has a factor of v^2/c^2, which disappears in the classical limit. Note that to the same lowest order in ϕ, we now have $c = c_0(1+2\phi)$, so c decreases sharply in a (negative) gravitational potential. This also corresponds to a gravitational index of refraction

$$n = c_0/c = (1-2\phi)>1. \qquad (11)$$

This will indeed cause an optical beam to deflect toward a gravitational potential well, as predicted by general relativity and observed.

With both T and L modified, the resulting trajectories can be calculated from the dispersion relation adapted from Eq. (3):

$$(\omega T_0)^2(1-2\phi) = (kL_0)^2(1+2\phi) + 1, \qquad (12)$$

using the Hamiltonian approach in a flat spacetime, without direct consideration of metrics. The results match those of general relativity to first order, within the PPN approximation. Eq. (12) shows the same dependences as in the isotropic form of the Schwarzschild metric, to lowest order in ϕ:

$$c^2 d\tau^2 = c^2\, dt^2\, (1-2GM/rc^2) - dr^2\, (1 + 2GM/rc^2) \qquad (13)$$

where τ is the usual relativistic proper time.

In order to determine the gravitational dependence of m, we need to know that of $\hbar$. Since angular momentum is one of the few physical quantities that is also Lorentz invariant (e.g., invariance of photon spin), and $\hbar$ is at the heart of quantum mechanics, it should also be gravitationally invariant. So we have

$$mc^2 = m_0 c_0^2\, (1 + \phi) \qquad (14)$$

and therefore

$$m = m_0\, (1- 3\phi) \qquad (15)$$

One of the guiding principles in general relativity is the principle of equivalence, that the gravitational trajectories are independent of the mass. According to the Hamiltonian approach, the trajectory follows a path of fixed ω (fixed total energy) with varying ϕ. Different values of ω_0 correspond to different values of mass, but the path of fixed ω will be the same. Further, the velocity along the path will also be the same, as Eq. (4) indicates, independent of ω_0 or mass.

Variable and Invariant Constants

Consider a region with a uniform gravitational potential, which shifts times and distances, but has no gravitational forces. The principle of relativity requires that all dimensionless constants be gravitationally invariant, including specifically the electromagnetic and gravitational coupling constants. Otherwise, one could determine the gravitational potential by a local measurement. Although others have previously derived trajectories within a gravitationally flat spacetime with varying m and c similar to that above (see, e.g., Krogh 2006), the important implications for the coupling constants have previously been overlooked.

Consider first the electromagnetic fine structure constant. That represents the normalized shift in energy (or frequency) for two electrons separated by a reduced Compton wavelength L. There are two distinct expressions of α in two different systems of units: SI and Gaussian CGS:

$$\alpha = (e^2/\hbar c) = 1/137 \text{ in Gaussian CGS} \qquad (16)$$

$$\alpha = (e^2/\hbar c)(1/4\pi\varepsilon_0) = (e^2/\hbar c)\,(\mu_0 c^2/4\pi) = (\mu_0/4\pi)\,(e^2 c/\hbar) = 1/137 \text{ in SI.} \qquad (17)$$

With respect to the latter expression, $\mu_0/4\pi = 10^{-7}$, which must be invariant by definition. Thus, e^2/c should be invariant in Gaussian units, while e^2c should be invariant in SI units. This requires that $e=e_0(1+\phi)$ (Gaussian), while $e = e_0(1-\phi)$ (SI). While it may seem odd that the electric charge can have two different gravitational variations depending on which system of units is used, the forms of Maxwell's equations are also different in the two systems, and in any case, real physical measurements involve mechanical quantities, which scale in the same way for the two systems.

A similar analysis requires that the gravitational coupling constant $Gm^2/\hbar c$ (~ 10^{-45}) be invariant. This requires that $G = G_0(1+8\phi)$. The factor of 8 suggests that the conventional form of the gravitational interaction may not be the most fundamental. If we consider that gravity really involves modulation of the frequency of quantum waves by all other quantum waves, the interaction can be rewritten as

$$\Delta\omega_0/\omega_0 = \phi = -(G\hbar/c^4)\,\Sigma(\omega_i/r_i), \qquad (18)$$

where $\Delta\omega_0/\omega_0$ is the relative shift of a given quantum wave, and the sum is over all other quantum waves with frequencies ω_i and distance r_i. Note that the prefactor $G\hbar/c^4$ is now gravitationally invariant. If this is rewritten in term of fundamental units L and T, the prefactor becomes the coupling constant $Gm^2/\hbar c$, which is also invariant.

Discussion and Conclusions

This analysis has focused on the case of a weak gravitational potential $\phi \ll 1$. A picture in which time and space are based on real physical waves suggests that there should be no singularities for strong potentials. This would tend to argue against the existence of black holes and event horizons. While the astronomical evidence indicates the presence of gravitationally compact objects with large values of ϕ, their interpretation as black holes is neither quantitative nor definitive. Measurements that go beyond the linear, small ϕ regime are not yet available, and orthodox theory provides little guidance. The analysis presented here would be consistent with the formulas of orthodox general relativity, but would also be consistent with those of exponential gravity (see, e.g., Ben-Amots 2011), where a factor of $(1\pm\phi)$

would be the first term in exp($\pm\phi$). Such an exponential factor would have dramatic implications in modifying gravitational collapse or the early stages of a big-bang cosmology. Further progress will require measurements and observations that can definitively distinguish between such alternatives.

In conclusion, this analysis suggests that the primary difficulty in achieving unity between microphysics and macrophysics has been the belief that quantum waves are abstract, indeterminate, and entangled. In contrast, a simple, consistent picture of both the microworld and the macroworld follows from the consideration of real, deterministic localized quantum waves on the microscopic level. These quantum waves have characteristic frequency and wavevector that define local time and space, and provide the basis for both special and general relativity. This is a non-metric formulation, although it appears to be equivalent to the conventional metric formulation. This suggests that gravity is fundamentally a modulation of quantum rotation by other quantum rotations. Gravitational trajectories may be obtained using a simple classical Hamiltonian formalism that is conceptually and mathematically simpler than the tensor formalism of orthodox general relativity. Within this picture, some fundamental constants may vary in space and time, while others are truly invariant.

References:

N. Ben-Amots (2011), "Some Features and Implications of Exponential Gravitation", *J. Phys. Conf. Ser.* 330, 012017. http://iopscience.iop.org/article/10.1088/1742-6596/330/1/012017/pdf

A.M. Kadin (2014), "Understanding Gravity on the Microscopic Level", Essay Submitted to Gravity Research Foundation Essay Contest, http://vixra.org/pdf/1405.0276v1.pdf .

A.M. Kadin (2011), "Waves, Particles, and Quantized Transitions: A New Realistic Model of the Microworld", ArXiv Physics Preprint, http://arxiv.org/abs/1107.5794.

K. Krogh (2006), "Gravitation without Curved Space-Time", http://arxiv.org/pdf/astro-ph/9910325.pdf.

Wikipedia articles (2016): Equivalence principle, Fine-structure constant, Gravitational coupling constant, Gravitational time dilation, Group velocity, Hamiltonian mechanics, Klein-Gordon equation, Matter wave, Parameterized post-Newtonian formalism, Photon polarization, Pilot wave, Principle of relativity, Schwarzschild metric, Soliton, Tests of general relativity, Variable speed of light, Wave packet

About the Author:

Dr. Alan M. Kadin is an applied physicist (Ph.D. in Physics, Harvard, 1979), specializing in superconducting devices, who has worked both in industry and academia, including at the University of Rochester and at Hypres, Inc. He is the author of a textbook, *Introduction to Superconducting Circuits*, (Wiley, 1999), and more than 100 publications. He has also maintained an interest in the foundations of physics, going back to his senior thesis at Princeton on hidden variables in quantum mechanics. Dr. Kadin is now an independent consultant living in Princeton Junction, New Jersey, USA.