Three conjectures on the numbers 6pq+1 where p and q primes and q=2p-1

Abstract. In this paper I make the following three conjectures on the numbers of the form n = 6*p*q + 1, where p and q are primes and q = 2*p – 1: (I) There exist an infinity of n primes; (II) There exist an infinity of n semiprimes; (III) There exist an infinity of n composites with three or more prime factors, 7 being one of them. Note that for all the first 46 pairs of primes [p, q] with the property mentioned (see the sequence A005382 in OEIS for these primes) the number n obtained belongs to one of the three sequences considered by the three conjectures above.

Conjecture I:

There exist an infinity of primes **n of the form** n = 6*p*q + 1, where p and q are primes and q = 2*p – 1.

The sequence of these primes is: 547 (= 6*7*13 + 1), 4219 (= 6*19*37 + 1), 74419 (= 6*79*157 + 1), 112327 (= 6*97*193 + 1), 627919 (= 6*229*457 + 1), 879667 (= 6*271*541 + 1), 2310019 (= 6*439*877 + 1), 5725627 (= 6*691*1381 + 1), 6337987 (= 6*727*1453 + 1), 16447867 (= 6*1171*2341 + 1), 23478019 (= 6*1399*2797 + 1), 32937847 (= 6*1657*3313 + 1)...

Conjecture II:

There exist an infinity of semiprimes **n of the form** n = 6*p*q + 1, where p and q are primes and q = 2*p – 1.

The sequence of these semiprimes is: 11347 (= 7*1621 = 6*31*61 + 1), 16207 (= 19*853 = 6*37*73 + 1), 1129147 (= 79*14293 = 6*307*613 + 1), 1312747 (= 43*30529 = 6*331*661 + 1), 2985019 (= 163*18313 = 6*499*997 + 1), 4330807 (= 13*333139 = 6*601*1201 + 1), 4417747 (= 19*232513 = 6*607*1213 + 1), 5239087 (= 7*748441 = 6*661*1321), 7887787 (= 151*52237 = 6*811*1621 + 1), 9224287 (= 211*43717 = 6*877*1753 + 1), 10530007 (= 1279*8233 = 6*937*1873 + 1), 13706719 (= 13*1054363 = 6*1069*2137 + 1), 18354607 (= 1153*15919 = 6*1237*2473 + 1), 19622419 (= 61*19622419 = 6*1279*2557 + 1), 20178727 (= 37*545371 = 6*1297*2593 + 1), 24495919 (= 7*3499417 = 6*1429*2857 + 1), 28118347 (= 19*1479913 = 6*1531*3061 + 1), 31056919 (= 1993*15583 = 6*1609*3217 + 1)...

Conjecture III:

There exist an infinity of n composites with three or more prime factors, 7 being one of them, **of the form** n = 6*p*q + 1, where p and q are primes and q = 2*p - 1.

The sequence of these numbers is: 294847 (= 7*73*577 = 6*157*313 + 1), 474019 (= 7*13*5209 = 6*199*397 + 1), 532987 (= 7*13*5857 = 6*211*421 + 1), 1360807 (= 7*31*6271 = 6*337*673 + 1), 1614067 (= 7*13*17737 = 6*367*733 + 1), 1721419 (= 7^2*19*43^2 = 6*379*757 + 1), 3587227 (= 7*31*61*271 = 6*547*1093 + 1), 3991687 (= 7^2*81463 = 6*577*1153 + 1), 4594219 (= 7*19*34543 = 6*619*1237 + 1), 8241919 (= 7*73*127^2 = 6*829*1657 + 1) 11215267 (= 7^2*228883 = 6*967*1933 + 1), 11922127 (= 7*79*21559 = 6*997*1993 + 1), 12210919 (= 7*61*28597 = 6*1009*2017 + 1), 31755787 (= 7*433*10477 = 6*1627*3253 + 1)...

Note:

For all the first 46 pairs of primes [p, q] with the property mentioned (see the sequence A005382 in OEIS for these primes) the number n obtained belongs to one of the three sequences considered by the three conjectures above.