Each system is Hamiltonian, and it is quantizable. Quantum systems are classical systems

Abstract

I prove that the classical trajectories are a projection of an Hamiltonian trajectory of higher dimension.

Hamiltonian System

Each trajectory in a $\mathcal{N}$-dimensional space can be written:

$$\begin{cases} y^1 = f^1(t) \\ \vdots \\ y^{\mathcal{N}} = f^{\mathcal{N}}(t) \end{cases} \tag{1}$$

each coordinates motion is the solution of a linear differential equation (there is ever a high order linear differential equation that have the solution f^s, because the differential equation have solution a sum of Taylor, Fourier and Laplace series, and a non-linear differential equation is a best approximation); so:

$$0 = \mathcal{F}^c\left(f^c, \dot{f}^c, \ddot{f}^c, \cdots\right) = a^c_{10\cdots} + a^c_{010\cdots} f^c + a^c_{0010\cdots}\dot{f}^c + \cdots + a^c_{0101\cdots} f^c \ddot{f}^c + \cdots \tag{2}$$

$$0 = \mathcal{F}^c\left(f^c, \dot{f}^c, \ddot{f}^c, \cdots\right) = \sum_{i_0,\cdots,i_n} a^c_{i_0,\cdots,i_n} \frac{d^{i_0} f^c}{dt^{i_0}} \cdots \frac{d^{i_n} f^c}{dt^{i_n}} \tag{3}$$

the derive of the differential equation is linear in the higher derivative:

$$0 = \frac{d\mathcal{F}^c\left(f^c, \dot{f}^c, \ddot{f}^c, \cdots\right)}{dt} = \frac{d}{dt} \sum_{i_0,\cdots,i_n} a^c_{i_0,\cdots,i_n} \prod_{s=1}^{n} \left(\frac{d^s f^c}{dt^s}\right)^{i_s} \tag{4}$$

$$0 = \sum_{k,i_0,\cdots,i_n} a^c_{i_0,\cdots,i_n} \prod_{s=1}^{n} i_k \left(\frac{d^s f^c}{dt^s}\right)^{i_s - \delta_{sk}} \frac{d^{k+1} f^c}{dt^{k+1}} \tag{5}$$

$$\frac{d^{\mathcal{N}} f^c}{dt^{\mathcal{N}}} = \mathcal{G}^c\left(f^c, \frac{df^c}{dt}, \ldots, \frac{d^{\mathcal{N}-1} f^c}{dt^{\mathcal{N}-1}}\right) \tag{6}$$

$$\frac{d^{\mathcal{N}} y^c}{dt^{\mathcal{N}}} = \mathcal{G}^c\left(y^c, \frac{dy^c}{dt}, \ldots, \frac{d^{\mathcal{N}-1} y^c}{dt^{\mathcal{N}-1}}\right) \tag{7}$$

so that each polynomial differential equation can be write linearly in the maximum derivative; so that:

$$\begin{cases} y^c = y_0^c \\ \frac{dy_0^c}{dt} = y_1^c \\ \vdots \\ \frac{dy_{s-1}^c}{dt} = y_s^c \\ \vdots \\ \frac{dy_{\mathcal{N}-2}^c}{dt} = y_{\mathcal{N}-1}^c \\ \frac{dy_{\mathcal{N}-1}^c}{dt} = \mathcal{G}^c(y_0^p, \cdots, y_{\mathcal{N}-2}^c) = y_{\mathcal{N}}^c \end{cases} \tag{8}$$

this system is the half of an Hamiltonian system $\mathcal{H}$, that have $\mathcal{N}$ new momenta:

$$\mathcal{H} = \sum_c \sum_{i=0}^{\mathcal{N}-1} p_i^c \left\{ y_{i+1}^c + \delta_{i,\mathcal{N}-1} \left[-y_{i+1}^c + \mathcal{G}^c \right] \right\} = \sum_c \left(\sum_{i=0}^{\mathcal{N}-2} p_i^c y_{i+1}^c + p_{\mathcal{N}-1}^c \mathcal{G}^c \right)$$

$$\begin{cases} \frac{dy_{j \neq \mathcal{N}-1}^c}{dt} = \frac{\partial \mathcal{H}}{\partial p_j^c} = y_{j+1}^c \\ \frac{dy_{\mathcal{N}-1}^c}{dt} = \frac{\partial \mathcal{H}}{\partial p_{\mathcal{N}-1}^c} = \mathcal{G}^c \\ \frac{dp_{j \neq \mathcal{N}-1}^c}{dt} = -\frac{\partial \mathcal{H}}{\partial y_j^c} = -p_{\mathcal{N}}^c \frac{\partial \mathcal{G}^c}{\partial y_j^c} - p_{j-1}^c \\ \frac{dp_{\mathcal{N}-1}^c}{dt} = -\frac{\partial \mathcal{H}}{\partial y_{\mathcal{N}-1}^c} = -p_{\mathcal{N}-2}^c \end{cases} \tag{9}$$

the volume of the phase space is an invariant and the sum of the areas is invariant, because of there is a momenta compensation.

The quantum system is obtained using the correspondence principle:

$$\begin{array}{c} \mathcal{H} = \sum_{ci} p_i^c y_i^c \\ i\hbar \frac{\partial \psi}{\partial t} = -i\hbar \sum_{ic} y_i^c \frac{\partial \psi}{\partial y_i^c} \\ \boxed{0 = \frac{\partial \psi}{\partial t} + \sum_{ic} y_i^c \frac{\partial \psi}{\partial y_i^c}} \end{array} \tag{10}$$

The Hamilton-Jacobi equation, that give the classical solution of the Hamiltonian, is:

$$\begin{array}{c} \mathcal{H} = \sum_{ic} p_i^c y_i^c \\ \frac{\partial \psi}{\partial t} + H(p_i^c = \frac{\partial \psi}{\partial y_i^c}, y_i^c) = 0 \\ \boxed{0 = \frac{\partial \psi}{\partial t} + \sum_{ic} y_i^c \frac{\partial \psi}{\partial y_i^c}} \end{array} \tag{11}$$

in this case the function ψ permit to calculate the momenta values like a gradient of the ψ function. Also in this case the classical solution, and the quantum solution, coincide; and the equation for the amplitude, or the probability, are equal because of the linearity of the equation.