New Einstein gravity field equation and Dark matter problem

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ABSTRACT

In the general relativity theory, we discover New Einstein's gravity field equation. We solve the dark matter problem of the cosmology by the New gravity field equation.

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1.Introduction

We solve the dark matter problem of the cosmology by using New gravity field equation.

New gravity field equation is

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R - T^{\lambda}{}_{\lambda} \frac{C_1 \pi G^4 h^2 k_0}{c^{16}} j^{\mu} j_{\mu} g_{\mu\upsilon}$$

$$= R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R + T^{\lambda}{}_{\lambda} \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} \rho^{+}{}_0{}^2 g_{\mu\upsilon} = -\frac{8\pi G}{c^4} T_{\mu\upsilon}$$

$$T^{\lambda}{}_{\lambda} = g^{\mu\upsilon} T_{\mu\upsilon}, \quad C_1 < 0,$$

4-vector Convection-Currents $j^{\mu} = \rho^{+}{}_0 \frac{dx^{\mu}}{d\tau}$, The law of Conservation $j^{\mu}{}_{;\mu} = 0, j_{\mu;\mu} = 0$

$$j^{\mu} j_{\mu} = \rho^{+}{}_0{}^2 \frac{dx^{\mu} dx_{\mu}}{d\tau^2} = \rho^{+}{}_0{}^2 \frac{ds^2}{d\tau^2} = -c^2 \rho^{+}{}_0{}^2$$

$\rho^{+}{}_0$ is the density of charge, h is plank constant, c is light speed.

$k_0 = \frac{1}{4\pi\varepsilon_0}$, ε_0 is the permittivity constant

(1)

Eq(1) is

$$(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R)_{;\mu} + T^{\lambda}{}_{\lambda;\mu} \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} \rho^{+}{}_0{}^2 g_{\mu\upsilon} = -\frac{8\pi G}{c^4} T_{\mu\upsilon;\mu} = 0$$

$$g^{\mu\upsilon}(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R) + T^{\lambda}{}_{\lambda} \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} \rho^{+}{}_0{}^2 g^{\mu\upsilon} g_{\mu\upsilon}$$

$$= R - 2R + 4T^{\lambda}{}_{\lambda} \rho^{+}{}_0{}^2 \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} = -\frac{8\pi G}{c^4} T^{\lambda}{}_{\lambda},$$

$$R = \frac{8\pi G}{c^4} T^{\lambda}{}_{\lambda} + 4T^{\lambda}{}_{\lambda} \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} \rho^{+}{}_0{}^2 \quad (2)$$

Hence,

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^{\lambda}{}_{\lambda}) + \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} T^{\lambda}{}_{\lambda} \rho^{+}{}_0{}^2 g_{\mu\upsilon} \quad (3)$$

2.Newton limitation and Weak gravity field approximation

In this theory, Newton limitation is

$$g_{\mu\upsilon} \approx \eta_{\mu\upsilon}, \; |T_{ij}| << T_{00}$$

$$R_{ij}-\frac{1}{2}g_{ij}R\approx 0\rightarrow R_{ij}\approx\frac{1}{2}\delta_{ij}R$$
$$R\approx -R_{00}+\sum_{i=1}^{3}R_{ii}=-R_{00}+\frac{3}{2}R$$
$$R\approx 2R_{00} \tag{4}$$

Hence, Newton limitation of Eq(1)

$$R_{0000}\approx 0, R_{i0j0}\approx\frac{1}{2}\frac{\partial^2 g_{00}}{\partial x^i\partial x^j}$$
$$R_{00}-\frac{1}{2}g_{00}R+T^{\lambda}{}_{\lambda}g_{00}\frac{C_1\pi G^4h^2k_0}{c^{14}}\rho^{+}{}_{0}{}^{2}$$
$$\approx R_{00}+\frac{1}{2}R\approx 2R_{00}\approx\nabla^2 g_{00}\approx -\frac{8\pi G}{c^4}T_{00} \tag{5}$$

Weak gravity field approximation is

$$g_{\mu\upsilon}=\eta_{\mu\upsilon}+h_{\mu\upsilon}$$
$$R_{\mu\upsilon}=-\frac{8\pi G}{c^4}(T_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda})+\frac{C_1\pi G^4h^2k_0}{c^{14}}T^{\lambda}{}_{\lambda}\rho^{+}{}_{0}{}^{2}g_{\mu\upsilon}$$
$$R_{\mu\upsilon}=-\frac{8\pi G}{c^4}S_{\mu\upsilon}$$
$$S_{\mu\upsilon}=T_{\mu\upsilon}-\frac{1}{2}\eta_{\mu\upsilon}T^{\lambda}{}_{\lambda}+\frac{1}{8}\frac{C_1G^3h^2k_0}{c^{10}}T^{\lambda}{}_{\lambda}\eta_{\mu\upsilon}\rho^{+}{}_{0}{}^{2}$$
$$h_{\mu\upsilon}(t,\vec{x})=\frac{4G}{c^2}\int d^4x'\frac{S_{\mu\upsilon}(t-|\vec{x}-\vec{x}'|,\vec{x}')}{|\vec{x}-\vec{x}'|}$$
$$h_{00}(\vec{x})=\frac{4G}{rc^2}\int d^3x'[T_{00}-\frac{1}{2}T_{00}+\frac{C_1G^3h^2k_0}{8c^{10}}T_{00}\rho^{+}{}_{0}{}^{2}]$$
$$\approx\frac{4G}{rc^2}\int d^3x'[\frac{1}{2}T_{00}]=\frac{2GM}{rc^2}$$
$$h_{ij}(\vec{x})=\frac{4G}{rc^2}\int d^3x'[T_{ij}+\frac{1}{2}\delta_{ij}T_{00}-\frac{C_1G^3h^2k_0}{8c^{10}}T_{00}\delta_{ij}\rho^{+}{}_{0}{}^{2}]$$
$$\approx\frac{4G}{rc^2}\int d^3x'[\frac{1}{2}\delta_{ij}T_{00}]=\frac{2GM}{rc^2}\delta_{ij}$$
$$c^2d\tau^2=-g_{\mu\upsilon}dx^{\mu}dx^{\upsilon}\approx(1-\frac{2GM}{rc^2})c^2dt^2-(1+\frac{2GM}{rc^2})\delta_{ij}dx^idx^j \tag{6}$$

In Eq(3), if $T_{\mu\upsilon}=0$,

$$R_{\mu\upsilon} = 0 \qquad (7)$$

The solution of Eq(7) is Schwarzschild solution.

$$c^2 d\tau^2 = -g_{\mu\upsilon} dx^{\mu} dx^{\upsilon} = (1 - \frac{2GM}{rc^2}) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \qquad (8)$$

3.Solving Dark matter problem

The cosmologic theory is Robertson-Walker soloution.

$$c^2 d\tau^2 = c^2 dt^2 - \Omega^2(t) [\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \qquad (9)$$

$$T^{\mu\upsilon} = pg^{\mu\upsilon} + (p / c^2 + \rho) U^{\mu} U^{\upsilon}$$

$$U^{\mu} = (c,0,0,0), T_{00} = \rho(t) c^2, \; T_{ij} = p(t) g_{ij}$$

$$T_{00} = \rho(t) c^2 \;\;, \;\; T_{0i} = 0, \; T_{ij} = p(t) g_{ij}$$

$$T^{\lambda}{}_{\lambda} = -\rho(t) c^2 + 3p(t) \text{-}$$

$$R_{00} = 3 \frac{\ddot{\Omega}}{\Omega} \;\;, \;\; R_{0i} = 0,$$

$$R_{ij} = -(\Omega \ddot{\Omega} + 2\dot{\Omega}^2 + 2k) \frac{g_{ij}}{\Omega^2} \qquad (10)$$

Hence,

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4} (T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^{\lambda}{}_{\lambda}) + \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} T^{\lambda}{}_{\lambda} {\rho^{+}{}_0}^2 g_{\mu\upsilon}$$

$$3 \frac{\ddot{\Omega}}{\Omega} = -\frac{4\pi G}{c^4} (\rho c^2 + 3p) - \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} (-\rho c^2 + 3p) {\rho^{+}{}_0}^2 \qquad (11)$$

$$-(\Omega \ddot{\Omega} + 2\dot{\Omega}^2 + 2k) \frac{1}{\Omega^2} = -\frac{4\pi G}{c^4} (\rho c^2 - p) + \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} (-\rho c^2 + 3p) {\rho^{+}{}_0}^2 \qquad (12)$$

Eq(11)+ $3 \times$ Eq(12) is

$$-6 \frac{(\dot{\Omega}^2 + k)}{\Omega^2} = -16 \frac{\pi G}{c^4} \rho c^2 + 2 \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} (-\rho c^2 + 3p) {\rho^{+}{}_0}^2$$

$$\rightarrow \quad \frac{(\dot{\Omega}^2 + k)}{\Omega^2} = \frac{8}{3} \frac{\pi G}{c^4} \rho c^2 - \frac{1}{3} \frac{C_1 \pi G^4 h^2 k_0}{c^{14}} (-\rho c^2 + 3p) {\rho^{+}{}_0}^2 \qquad (13)$$

Hence,

$$(\frac{8\pi G}{3c^2} + \frac{C_1 \pi G^4 h^2 k_0}{3c^{12}} {\rho^+}_0{}^2)\rho(t) \approx \frac{8\pi G}{3c^2}\rho(t)$$

$$= \frac{(\dot{\Omega}^2 + k)}{\Omega^2} + p(t)\frac{C_1 \pi G^4 h^2 k_0}{c^{14}} {\rho^+}_0{}^2$$

$$\to \quad \rho(t) \approx \frac{3c^2}{8\pi G}[(\frac{\dot{\Omega}}{\Omega})^2 + \frac{k}{\Omega^2} + p(t)\frac{C_1 \pi G^4 h^2 k_0}{c^{14}} {\rho^+}_0{}^2] \quad (14)$$

In this time,

The present time of Universe $t_0 \approx \frac{\Omega(t_0)}{\dot{\Omega}(t_0)} = H_0^{-1}$ (15)

Therefore, Eq(14) is

$$\rho(t_0) \approx \frac{3c^2}{8\pi G}[(\frac{\dot{\Omega}(t_0)}{\Omega(t_0)})^2 + \frac{k}{\Omega(t_0)^2} + p(t_0)\frac{C_1 \pi G^4 h^2 k_0}{c^{14}} {\rho^+}_0{}^2]$$

$$\approx \frac{3c^2}{8\pi G}[{H_0}^2 + \frac{k}{\Omega(t_0)^2} + p(t_0)\frac{C_1 \pi G^4 h^2 k_0}{c^{14}} {\rho^+}_0{}^2] \ ,$$

$$C_1 < 0 \quad (16)$$

4. Conclusion

Therefore,

$$\rho_c = \frac{3c^2}{8\pi G}{H_0}^2 \approx 5 \times 10^{-30} gm/cm^3, \ \ \rho(t_0) \approx 2 \times 10^{-31} gm/cm^3$$

According to Eq(16), the present universe's pressure $p(t_0)$ has to be huge.

Reference

[1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[2]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[3]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[4]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[5]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)

[6]E.Hubble,Proc. Nat.Acad. Sci. U. S. 15, 169(1929)

[7]A.Sandage,"Distances to Galaxies:the Hubble Constant, the Friedmann Time and the Edge of World"' in Proceedings of the Symposium on the Galaxy and the Distant Scale, Essex, England(1972)

[8]A.Sandage, Astrophys, J. 178, 1(1972)

[9]E.Kasner, Am. J. Math. 43, 217(1921)

[10]G.Birkoff,Relativity and Modern Physics(Harvard University Press,1923),p.253