

Every large prime must lie on a Diriclet's arithmetic sequence and a simple method to identify such arithmetic progressions

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Abstract: Dirichlet's theorem establishes that every arithmetic progression of the form $a+nb$ where $\gcd(a,b)=1$ contains infinitely many primes for positive integers a and b and $n=1,2,3,4,\dots$. We show a simple proof for the existence of such an arithmetic progression for every large prime. This also reveals a method to identify arithmetic progressions on which a particular prime will appear.

Results:

Let p be a very large prime. Then there must exist primes p_1, p_2, \dots, p_k that are less than p . Therefore for every such prime we may express p as the following combination of positive integers

$$p = p_1 q_1 + r_1 \text{ such that } 0 < r_1 < p_1$$

$$p = p_2 q_2 + r_2 \text{ such that } 0 < r_2 < p_2$$

.....

$$p = p_k q_k + r_k \text{ such that } 0 < r_k < p_k$$

Each of these expressions for p represent an arithmetic progression of the form $a+nb$ where $a=r_i$, $b=p_i$ and $n=q_i$ and where $i=1,2,\dots,k$

Alternatively we can also consider the arithmetic progressions where $a=r_i$, $n=p_i$ and $b=q_i$ where $i=1,2,\dots,k$

For every p_i, q_i, r_i , $\gcd(p_i, q_i, r_i) = 1$

Thus every large prime must lie on a Diriclet's arithmetic sequence.